

Federal Reserve Bank of Cleveland Working Paper Series

Nominal Rigidities and the Term Structures of Equity and Bond Returns

Pierlauro Lopez, David Lopez-Salido, and Francisco Vazquez-Grande

Working Paper No. 23-11

May 2023

Suggested citation: Lopez, Pierlauro, David Lopez-Salido, and Francisco Vazquez-Grande. 2023. "Nominal Rigidities and the Term Structures of Equity and Bond Returns." Working Paper No. 23-11. Federal Reserve Bank of Cleveland. <u>https://doi.org/10.26509/frbc-wp-202311</u>.

Federal Reserve Bank of Cleveland Working Paper Series ISSN: 2573-7953

Working papers of the Federal Reserve Bank of Cleveland are preliminary materials circulated to stimulate discussion and critical comment on research in progress. They may not have been subject to the formal editorial review accorded official Federal Reserve Bank of Cleveland publications.

See more working papers at: <u>www.clevelandfed.org/research.</u> Subscribe to email alerts to be notified when a new working paper is posted at: <u>https://www.clevelandfed.org/subscriptions</u>.

Nominal Rigidities and the Term Structures of Equity and Bond Returns

Pierlauro Lopez^{a,1}, David Lopez-Salido^b, and Francisco Vazquez-Grande^b

^aFederal Reserve Bank of Cleveland ^bFederal Reserve Board

Abstract

We present a production economy with nominal price rigidities that explains several asset pricing facts, including a downward-sloping term structure of the equity premium, upwardsloping term structures of nominal and real interest rates, and the cyclical variation of the term structures. In the model, after a productivity shock a countercyclical labor share exacerbates the procyclicality of dividends, and hence their riskiness, and generates countercyclical inflation. The dividend share gradually increases after a negative productivity shock as the price level increases sluggishly, so the payoffs of short-duration dividend claims (bonds) are more (less) procyclical than the payoffs of long-duration claims (bonds). A slow-moving external habit then produces large and countercyclical prices for these risks as well as high risk premia at very long horizons. In bad times, the slope of equity (bond) yields for the observable maturities becomes more negative (more positive), but risk premia also increase at longer horizons, and market equity premia end up increasing by more than short-run equity premia. The simultaneous presence of market and home consumption habits allows for uniting habits and a production economy without compromising the model's ability to fit macroeconomic variables. The central bank's anti-inflationary stance plays a key role in shaping equity and bond prices.

JEL classification: E43; E44; G12.

Keywords: Structural term structure modeling, Equity and bond yields, Habit formation, Nominal rigidities, Macro-finance separation.

Email addresses: pierlaurolopez@gmail.com (Pierlauro Lopez), david.j.lopez-salido@frb.gov (David Lopez-Salido), francisco.vazquez-grande@frb.gov (and Francisco Vazquez-Grande)

¹This paper was previously circulated under the title "Macro-Finance Separation by Force of Habit." We would like to thank Jarda Borovička, John Cochrane, Niels Gormsen, François Gourio, Joe Haubrich, Urban Jermann, Ivan Jaccard, Ralph Koijen, Roberto Marfè, Anna Orlik, Roberto Steri, and Eric Swanson for insightful discussions, and seminar participants at several institutions and conferences for very useful comments. The views presented here are those of the authors and not necessarily those of any institution of the Federal Reserve System.

1. Introduction

Recent evidence shows that discount rates of financial claims differ across maturities as well as across asset classes. In particular, the maturity structure and time variation of equity and bond risk premia contain rich information to understand investors' marginal utility of wealth and their expectations about future macroeconomic variables. The expected returns of nominal and real bonds increase with the maturity on average, while claims to short-term dividends have higher excess returns than the aggregate stock market, implying a term structure of the equity premium that is downward-sloping on average over the observable maturities (Binsbergen et al., 2012). Such a slope characterizes the term structures of both one-period equity returns and yields, which differ however in their cyclicality. During recessions, the term structure of equity yields becomes more negatively sloped (Binsbergen et al., 2013), while the term structure of one-period equity returns becomes less so, as the equity premium can increase by more than short-duration equity premia (Gormsen, 2021). Finally, at the bottom of stock market recessions, the term structure of bond yields becomes steeper, as it predicts higher future growth and rates (an observation that goes back to at least Estrella and Hardouvelis, 1991). A general-equilibrium explanation of the forces driving such features of equity and bond prices is still missing.

Our first contribution is to offer an explanation of the macroeconomic forces that drive these empirical features of equity and bond markets. We present a New Keynesian model with habit formation that relies on *two key ingredients*. The first ingredient, a textbook *sticky-price production economy*, provides a familiar model of output and inflation that matches standard macroeconomic data and rationalizes the payoffs of nominal bonds (inverse inflation) and dividends as a levered version of consumption. The second ingredient, *slowmoving external habit formation* à la Campbell and Cochrane (1999), generates realistically large and time-varying discount rates.

Our second contribution is to illustrate how to unite nonlinear consumption habits and a production economy without compromising the model's ability to fit macroeconomic variables—a challenge documented by Lettau and Uhlig (2000) and Rudebusch and Swanson (2008). We avoid that difficulty by including a *second* habit in effective leisure, or home consumption. Intuitively, after a bad productivity shock, both market and home consumption drop close to their habit levels, with offsetting effects on the labor choice, thereby neutralizing the undesirable effect of habits on production. In fact, we show how one can approximately preserve a macroeconomist's preferred model of quantities, in this case of the New Keynesian production economy, while using nonlinear habits to produce realistic asset prices. This *macro-finance separation* result—whereby the states that drive variation in discount rates beyond the usual CRRA preferences do not drive consumption, hours, and inflation—implies that habits do not affect the well-known properties of quantities in the macro model. It also follows that we can effectively inspect the mechanism by considering in isolation the role of the two ingredients.

Our first ingredient, the production economy with sticky prices, provides a macroeconomic model that matches the observed volatility and autocorrelation of cash flows (consumption growth, dividend growth, and inflation). Because of the approximate macro-finance separation,

the model's quantity and inflation implications and responses to a productivity shock are standard. The central mechanism relies on nominal rigidities that produce countercyclical labor shares after a productivity shock, which imply procyclical corporate profits and countercyclical inflation. Therefore, dividend claims and nominal bonds pay off badly in a downturn when marginal utility is high, and are therefore risky investments. However, since the labor share is stationary, and hence it mean reverts, the payoffs of long-duration dividend strips (nominal bonds) are less (more) procyclical: corporate profits and the price level increase after a bad transitory shock as more and more firms are able to adjust their prices to mark them up over marginal costs.

Our second ingredient, the slow-moving external habit formation, then magnifies these cash flow risks into large and countercyclical risk premia. Moreover, for sufficiently long durations, the model's discount rate does more than simply amplify risk premia; all claims are risky in the very long run. In fact, in a downturn, habits make prices drop more the longer the claim's duration, because people will slowly get used to the lower consumption level, so people will want to anticipate consumption and will require compensation for shifting resources in the future, even if the shock to consumption is permanent. Because of this *habit effect*, we produce a term structure of the equity premium that is U-shaped—with a negative slope in the short to medium run, driven by the cyclicality of dividends, and a positive slope for longer maturities, driven by the habit effect. By the same habit effect, our model produces a positively sloped term structure of real rates, thereby avoiding a real bond premium puzzle (Backus et al., 1989), while the cyclicality of inflation implies a positive inflation risk premium at all horizons, and hence a positively sloped nominal term structure.

The nonlinear habits also generate the cyclicality of the term structures documented in the data. In bad times, as consumption falls close to habits and dividends drop, risk premia increase and future dividends are expected to recover; hence, the slope of equity yields for the observable maturities becomes more negative, but risk premia also increase for longer horizons and, consequently, market equity premia turn out to increase by more than short-run equity premia in the model. At the same time, inflation is expected to increase sluggishly, and hence the slope of bond yields becomes more positive.

This paper offers a structural story that captures several of the empirical properties of equity and bond prices that so far only the descriptive, no-arbitrage models of Lettau and Wachter (2007, 2011) and Gormsen (2021) have tried to capture. The model fits the listed term structural facts despite being parameterized to match macroeconomic quantities. Furthermore, our framework preserves the main achievements of Campbell and Cochrane (1999), including a solution to the average equity premium and the risk-free rate puzzles, long-horizon predictability of excess stock returns, and the countercyclical variation of stock market returns and volatility. All these phenomena arise naturally as we unite slow-moving countercyclical discount rates and New Keynesian cash flows.

The choice of external habits to explain the term structure evidence may seem surprising at first, as some authors document the challenges of the habit framework in producing a downward-sloping term structure (e.g., Binsbergen et al., 2012), even though they are naturally consistent with the countercyclicality of one-period equity term premia (Gormsen, 2021) due to the habit effect at very long horizons. Those results, however, are derived in endowment economies with random-walk dividend streams. Once we inject a mean-reverting component into dividends, as endogenously generated by the production economy, we depart from those benchmark models. In particular, by using the model to make the properties of cash flows match their volatility and autocorrelation in the data, we are able to naturally generate a downward and procyclical slope at the observable end of the term structure of equity yields while preserving the property of habits that generates countercyclical one-period equity term premia.

Furthermore, the results in Gormsen (2021) rule out Epstein-Zin preferences as an obvious ingredient. In fact, the habit effect at long horizons is preserved in the production economy. In contrast, with Epstein-Zin preferences, the ingredients necessary for flipping the sign of the slope of the equity term structure will tend to operate also at long horizons. Indeed, Gormsen shows how recent examples in Hasler and Marfè (2016) and Ai et al. (2018), who are able to generate a downward-sloping equity term structure by changing the cash flow process, display as a consequence the wrong cyclicality of the term structure of equity premia. The extension of those setups to a nominal production economy, therefore, seems to be a challenging avenue.

Even though we operate under approximate macro-finance separation, the nonlinearity of habits still calls for an accurate nonlinear solution method. In particular, we solve the model by a global solution spanned by a basis of high-order polynomials and confirm that macro-finance separation holds almost exactly. Furthermore, while we focus on evidence that goes back to the 1980s or early 1990s, and while the facts we are after have also been documented in periods where the federal funds rate was not constrained by the zero lower bound on the nominal interest rate, we also solve the model subject to a zero-lower-bound constraint. Our results remain similar.

Our emphasis is on the effect of productivity shocks in our simple framework, which, as we show, goes a long way in explaining several asset pricing facts. A full-fledged model would include more shocks, including demand shocks, to capture more comprehensively the data. For example, as argued by Campbell et al. (2020), the presence of a mix of demand and supply shocks can capture changing correlation patterns between consumption and inflation and between stock and bond returns. Therefore, we extend our model to include demand shocks and parameterize their size to match the observed correlation between consumption growth and inflation, which is too low in a model with only productivity shocks. In line with the evidence in Campbell et al., we find that the model can easily produce decade-long spells with negative correlations between stock and bond returns. In this context, while the presence of demand shocks partly offsets the term structural properties generated by supply shocks, the properties of interest remain consistent with the data. That is, the model augmented with demand shocks displays a similar cash flow mean reversion and similar slopes of the term structures of the equity premium and interest rates, although flatter than in the baseline model. The cyclicality of the term structure of equity is likewise preserved when we add demand shocks.

Finally, we use our setup to quantify the role played by nominal rigidities and monetary policy in shaping asset returns. Indeed, since we captured several stylized facts of equity and bond markets in a New Keynesian model, it follows that the degree of nominal price stickiness in the economy and the monetary policy stance of the central bank will affect the properties of the term structures. The model predicts that a higher degree of nominal rigidities exacerbates the downward slope of the equity term structure and flattens the bond term structures, as the countercyclicality of the labor share is stronger and the response of inflation is more sluggish. Vice versa, a lower degree of price stickiness increases the average slopes of all term structures. Similarly, we find that as the systematic response of the central bank to inflation increases, the slopes of the equity and bond term structures increase. Therefore, in the model, monetary policy is not just an important driver of economic activity and inflation, but also of equity and bond prices.

Relationship to the literature

Our paper subscribes to a recent literature that focuses on risk pricing across maturities (see, for example, Lettau and Wachter, 2007, 2011; Binsbergen et al., 2012, 2013; Borovicka and Hansen, 2014; Belo et al., 2015; Hasler and Marfè, 2016; Marfè, 2017; Ai et al., 2018; Lopez, 2021; Weber, 2018; Bansal et al., 2021; Gormsen, 2021). While the search for a structural explanation of the positive slope and cyclicality of the term structure of interest rates has a rather long history, the search for a structural explanation for the negative slope of the term structure of equity has only recently received attention (see Binsbergen and Koijen, 2017, for a survey). While a relatively small sample and potential liquidity issues introduce some controversy as to the sign of the average slope, as argued by Bansal et al. (2021), the cyclical properties of the term structure of equity are so far less controversial. These more recent facts add to the more classic facts documenting a large and countercyclical equity premium and a low and stable risk-free rate. Our simple story captures these empirical regularities, which so far only the descriptive models in Lettau and Wachter (2007, 2011) and Gormsen (2021) have tried to capture, although outside a general-equilibrium context that spells out preferences and production choices.

We likewise subscribe to a growing literature focusing on the asset pricing implications of nominal rigidities, including Rudebusch and Swanson (2008, 2012); Bekaert et al. (2010); Li and Palomino (2014); Weber (2014); Kung (2015); Gorodnichenko and Weber (2016); Gourio and Ngo (2016); Campbell et al. (2020); Pflueger and Rinaldi (2022). None of the cited papers focuses on the joint properties of the term structures of equity and interest rates. Moreover, Campbell et al. (2020) and Pflueger and Rinaldi (2022) also study the effects of Campbell-Cochrane-type habits in the presence of sticky prices and emphasize the interaction of countercyclical risk premia generated by habits with inflation and monetary policy. However, they do so with a reduced-form specification of the production side of the economy and of the shocks that hit the economy. In this context, as we show how to integrate Campbell-Cochrane habits into production economies, we not only respond to the critique by Lettau and Uhlig (2000) but also show how the additional structure that comes from the production side of the economy, which endogenizes both inflation and dividends, jointly delivers realistic properties of the three term structures.

2. The Model

Our setup is a textbook DSGE model with nominal rigidities that we augment with nonlinear habits in market and home consumption. As we unite habit formation and a production economy, however, their interaction in general equilibrium could have counterfactual implications. In fact, as we will show, in a production economy habits affect the intertemporal rate of substitution, which drives consumption-saving and investment decisions, and the intratemporal rate of substitution, which controls the link between consumption and labor supply. The consequence is that equilibrium quantities depend on the state variables that drive risk premia and that will be volatile for realistic asset prices. This situation can be associated with counterfactually large fluctuations in some real variables or with small risk premia as households absorb aggregate shocks by varying labor to achieve an extremely stable consumption path (Lettau and Uhlig, 2000).²

To avoid this well-known quantity puzzle, we introduce nonlinear habits in two consumption goods, one purchased in the market and the other produced at home, and in the spirit of Campbell and Cochrane (1999), we engineer restrictions on the habit dynamics to control the intertemporal and static spillovers of habits onto macroeconomic quantities. We therefore extend to the habit formation setting a macro-finance separation result analogous to the one that Tallarini (2000) described in a setting with Epstein-Zin preferences.

2.1. Households

As in Greenwood and Hercowitz (1991), identical households derive utility from two sources: nondurable goods purchased in the market and goods produced at home. Our households get used to an accustomed standard of living as represented by some particular level of consumption of the market-purchased and the home-produced goods. Accordingly they have preferences

$$U_0(j) = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{[C_t(j) - X_t^c]^{1-\gamma} - 1}{1-\gamma} + \chi \frac{[H_t(j) - X_t^h]^{1-\gamma} - 1}{1-\gamma} \right)$$
(1)

where $C_t(j)$ is real consumption purchased in the market by household $j \in (0, 1)$ and $H_t(j)$ denotes the consumption produced at home. As described later, X_t^c and X_t^h represent external habit levels that are nonlinear functions of contemporaneous and past aggregate consumption. The parameter β is the subjective discount rate and χ controls the steady-state value of hours, while the curvature of the utility function in market and home consumption is the same to ensure balanced growth.

As in Campbell and Cochrane (1999), we ensure well-behaved marginal utilities in this context of additive habits by assuming that the law of motion of habits is specified indirectly

²See also Jermann (1998); Uhlig (2007); Rudebusch and Swanson (2008, 2012); Swanson (2012); Jaccard (2014) for illustrations of the difficulties in reconciling business cycle facts with habit formation models in production economies.

through the processes for log surplus market consumption ratio s_t and log surplus home consumption ratio z_t , defined as

$$s_t \equiv \ln\left[\frac{C_t - X_t^c}{C_t}\right], \quad z_t \equiv \ln\left[\frac{H_t - X_t^h}{H_t}\right]$$

Their laws of motion are driven by aggregate market and home consumption, $C_t \equiv \int C_t(j)dj$ and $H_t \equiv \int H_t(j)dj$; since each individual agent has zero mass, she takes the habit levels as external to her consumption decisions. The dynamics for the logarithms of aggregate surplus ratios are

$$s_{t+1} = s + \rho_s \hat{s}_t + \Lambda_{c,t} (c_{t+1} - E_t c_{t+1})$$

$$z_{t+1} = z + \rho_s \hat{z}_t + \Lambda_{h,t} (h_{t+1} - E_t h_{t+1})$$
(2)

where lower-case letters denote logarithms and a hatted variable \hat{x} represents the deviation of variable $x \equiv \ln(X)$ from its steady state. We specify below the parametric shape of the sensitivity functions (Λ_c and Λ_h) as well as the calibration of the steady-state levels of the surplus variables that ensure well-defined habits.

The home consumption good is produced by households with technology

$$H_t(j) = A_t[1 - N_t(j)]$$

where $N_t(j)$ is the amount of time spent working outside the home, the total time endowment is normalized to 1, and A_t is the productivity in home-produced goods. Note that there are at least two reasons to focus on home consumption rather than standard leisure. First, once it is accepted that people get used to an accustomed market consumption level, it is only natural to assume that people also develop a habit in home consumption. Second, the inclusion of preferences in home consumption in form (1) implies separability between consumption and labor choices while remaining consistent with balanced growth; therefore, we can keep the elasticity of intertemporal substitution as a free parameter while preserving the form in Campbell and Cochrane (1999) for the stochastic discount factor.

Consumers trade in complete financial markets and choose market and home consumption to maximize the intertemporal objective (1) subject to the present-value budget constraint

$$E_0 \sum_{t=0}^{\infty} M_{0,t} C_t(j) = \frac{B_{-1}(j)}{P_0} + E_0 \sum_{t=0}^{\infty} M_{0,t} \left(\frac{W_t}{P_t} N_t(j) + D_t\right)$$

with t-period real contingent claims prices $M_{0,t}$, where W_t is the nominal wage rate, P_t is the price index, D_t is the dividend consumers receive from owning the aggregate firm, and B_t denotes their nominal holdings of one-period nominal government bonds with unit price $\exp(-i_t)$. The budget constraint implicitly includes other arbitrary claims in zero net supply, which we can therefore price by relying on no-arbitrage relations, including one-period noncontingent claims with unit price $\exp(-r_{f,t}) = E_t M_{t+1}$.

Optimality implies that consumers equalize intertemporal rates of substitution and

contingent claims price ratios, and hence the t-period stochastic real discount factor

$$M_{0,t} = \beta^t \left(\frac{C_t S_t}{C_0 S_0}\right)^{-\gamma},\tag{3}$$

that their bond holdings satisfy the Euler equation

$$i_t = -\ln E_t(\beta e^{-\gamma \Delta c_{t+1} - \gamma \Delta s_{t+1} - \pi_{t+1}}),$$
(4)

where $\pi_t \equiv \ln(P_t/P_{t-1})$ defines the inflation rate, and that the rate of substitution between labor and consumption equals the wage rate

$$w_t - p_t = \ln(\chi) + a_t + \gamma(c_t - h_t + s_t - z_t)$$
(5)

Equation (3) shows how, through the process s_t , habits generate additional discountrate variation relative to a CRRA specification of preferences. But habits also affect the consumption-saving tradeoff in (4) as well as the consumption-labor tradeoff in (5), with a spillover controlled by the process $s_t - z_t$. Intuitively, the reason we introduce a second habit in home consumption is to offset the effect of the market consumption habit on the labor choice, thus avoiding the outlandish implications for labor and consumption documented by Lettau and Uhlig (2000) in a production economy with only market consumption habits.

2.2. Firms

The production side of the economy is characterized by a unit mass of monopolistically competitive firms indexed by $i \in [0, 1]$. They discount future profits by the households' stochastic discount factor $M_{0,t}$ and choose nominal prices $\{P_t^*(i)\}$ and labor demand $\{N_t(i)\}$ to maximize the intertemporal objective

$$E_0 \sum_{t=0}^{\infty} M_{0,t} D_t(i), \qquad D_t(i) = \frac{P_t(i)}{P_t} Y_t(i) - (1-\tau) \frac{W_t}{P_t} N_t(i) - T_t$$

subject to Calvo nominal price rigidities

$$P_t(i) = \begin{cases} P_t^*(i), & \text{with probability } 1 - \eta \\ e^{\pi^*} P_{t-1}(i), & \text{with probability } \eta \end{cases}$$

where the *i*th good sells for the nominal price $P_t(i)$ and $P_t \equiv [\int_0^1 P_t(i)^{1-\varepsilon} di]^{1/(1-\varepsilon)}$ is the price index. We allow for indexation to a positive steady-state inflation rate π^* to produce a realistic level of the nominal yield curve. Firms operate with production technology

$$Y_t(i) = [e^{\mu t} \widetilde{A}_t N_t(i)]^{1-\alpha} K_t(i)^{\alpha}$$

where Y_t is real output, N_t is the labor input, $e^{\mu t} \tilde{A}_t$ denotes the exogenous labor-augmenting productivity level, and $K_t = e^{\mu t}$ is the deterministic capital stock, which grows at rate μ on

a balanced-growth path.³ In the model we define market equity as the value of the aggregate firm, which pays out to households per-period equilibrium profits $D_t(i)$ as dividends.

Owing to the price stickiness, each firm *i* can reset prices at any given time only with probability $1 - \eta$ and faces the demand curve for the good it produces $C_t(i) = [P_t(i)/P_t]^{-\varepsilon}C_t$, which arises as the cost-minimizing plan of individual consumers who bundle the continuum of goods via a Dixit-Stiglitz aggregator with constant elasticity of substitution between goods, ε . To single out the role of nominal rigidities, we assume that the government levies lump-sum taxes T_t on each firm to finance an employment subsidy, τ , which reduces the unit nominal cost of labor and is in place to offset any steady-state distortions caused by monopolistic competition.

In this context, a firm's optimal labor demand schedule implies real marginal costs

$$mc_t(i) = w_t - p_t + \ln(1 - \tau) - \ln(1 - \alpha) - y_t(i) + n_t(i)$$
(6)

while a nonlinear New Keynesian Phillips curve describes the optimal price-setting behavior of a firm that reset prices at time t as the condition linking inflation and marginal costs,

$$\left(\frac{1-\eta e^{(\varepsilon-1)(\pi_t-\pi^*)}}{1-\eta}\right)^{\frac{1}{1-\varepsilon}} = \frac{E_t \sum_{j=0}^{\infty} (\beta\eta)^j e^{y_{t+j}-\gamma c_{t+j}-\gamma \hat{s}_{t+j}+\varepsilon \sum_{h=1}^j (\pi_{t+h}-\pi^*)+mc_{t+j}^*-\ln((\varepsilon-1)/\varepsilon)}}{E_t \sum_{j=0}^{\infty} (\beta\eta)^j e^{y_{t+j}-\gamma c_{t+j}-\gamma \hat{s}_{t+j}+(\varepsilon-1) \sum_{h=1}^j (\pi_{t+h}-\pi^*)}}$$
(7)

where the real marginal cost at date t+j for a firm that reset prices at date t is, in equilibrium,

$$mc_{t+j}^* = mc_{t+j} - \frac{\alpha\varepsilon}{(1-\alpha)(1-\varepsilon)} \ln\left(\frac{1-\eta e^{(\varepsilon-1)(\pi_t-\pi^*)}}{1-\eta}\right) + \frac{\alpha\varepsilon}{1-\alpha} \sum_{h=1}^j (\pi_{t+h} - \pi^*) - \Delta_{t+j}$$
(8)

where we used optimality condition (6) and the market clearing condition described below. Note how each firm faces the same problem, so we dropped the i index from equation (7). The aggregate marginal cost in equation (8) integrates equation (6) over i.

2.3. Government

Monetary policy is described by a simple Taylor rule for the nominal interest rate that reacts to inflation and the output gap relative to its stochastic trend, \bar{y}_t , described below,

$$i_t = i^* + \phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - \bar{y}_t) \tag{9}$$

for an interest rate level i^* consistent with the positive steady-state inflation rate π^* .

Fiscal policy runs a balanced budget in that $P_t T_t = \tau W_t N_t$.

³Our benchmark economy abstracts from the dynamic effects of capital accumulation, as they do not change the core features of the model. In the online appendix we allow for nontrivial capital accumulation and describe one last spillover, controlled by the curvature of capital adjustment costs $\xi_3 \in \mathbb{R}_+$, which affects the consumption-investment tradeoff. This last spillover becomes zero as adjustment costs go to infinity, in which case the model reduces to the one described here.

2.4. Technology

As in Campbell and Ludvigson (2001), the same productivity process governs both the household's and the firm's production functions to ensure a balanced-growth path. In particular, the relationship between productivity in market- and home-produced goods is $A_t = e^{\mu t} \tilde{A}_t^{1-\alpha}$. The logarithm of the growth rate of productivity evolves by the process

$$\Delta a_{t+1} = \mu + u_t + \sigma \varepsilon_{t+1}$$

$$u_{t+1} = \rho_u u_t - \phi \sigma \varepsilon_{t+1}$$
(10)

where ε_t is i.i.d. normally distributed with mean 0 and variance 1. This structure nests both random-walk ($\phi = 0$) and AR(1) specifications ($\phi = 1 - \rho_u$) with drift μ . Our estimation strategy will pin down the exact dynamics to correctly capture the volatility and autocorrelation of consumption, dividends, and the CPI and hence of the cash flow processes we are interested in pricing.

Note that the Beveridge-Nelson trend \bar{a}_t in productivity evolves as $\Delta \bar{a}_{t+1} = \mu + [1 - \phi/(1 - \rho_u)]\sigma\varepsilon_{t+1}$, and hence the stochastic trend of output can be written as

$$\bar{y}_t = y + \bar{a}_t$$

2.5. Competitive equilibrium

Market clearing for each good *i* implies market clearing at the aggregate level, $y_t = c_t$. Market clearing in the labor market $N_t = \int_0^1 N_t(i) di$ implies

$$y_t = a_t + (1 - \alpha)(n_t - \Delta_t)$$

where cross-sectional price dispersion $\Delta_t \equiv \ln \int_0^1 [P_t(i)/P_t]^{-\varepsilon/(1-\alpha)} di$ evolves according to

$$e^{\Delta_t} = \eta e^{\frac{\varepsilon}{1-\alpha}(\pi_t - \pi^*) + \Delta_{t-1}} + (1-\eta) \left(\frac{1-\eta e^{(\varepsilon-1)(\pi_t - \pi^*)}}{1-\eta}\right)^{\frac{\varepsilon}{(1-\alpha)(\varepsilon-1)}}$$
(11)

The aggregate market production function then implies the aggregate home production

$$e^{h_t} = e^{a_t} \left(1 - e^{\frac{1}{1-\alpha}(y_t - a_t) + \Delta_t} \right)$$
(12)

and the aggregate dividend

$$D_t = \int_0^1 D_t(i)di = \int_0^1 \left[\frac{P_t(i)}{P_t}Y_t(i) - (1-\tau)\frac{W_t}{P_t}N_t(i) - T_t\right]di = Y_t - W_t N_t$$

or, using the equilibrium condition for labor demand,

$$d_t = c_t + \ln[1 - (1 - \alpha)e^{mc_t - mc^*}]$$
(13)

We are thus ready to define the competitive equilibrium allocation of this economy:

Definition (Competitive equilibrium). For a specified policy process $\{i_t\}_{t=0}^{\infty}$, endogenous state vector $\{s_t, z_t, \Delta_t\}_{t=0}^{\infty}$ and exogenous state vector $\{a_t, u_t\}_{t=0}^{\infty}$, the competitive equilibrium is an allocation $\{c_t, h_t, n_t\}_{t=0}^{\infty}$ and a price system $\{w_t - p_t, \pi_t, r_{f,t}\}_{t=0}^{\infty}$ such that for each date t, (a) the choice of prices and labor demand solves the individual firm's problem, (b) the choice of market and home consumption solves the individual consumer's problem, (c) the goods and labor markets clear, (d) market and home consumption habits evolve according to (2), (e) price dispersion evolves according to (11), (f) the nominal rate follows rule (9), and (g) the fiscal authority runs a balanced budget.⁴

2.6. Macro-finance separation with habit formation

We now specify the functional form of the sensitivity functions, Λ_c and Λ_h , in the surplus consumption dynamics (2). In particular, to preserve the desirable implications for observed macroeconomic quantities of the textbook New Keynesian model with CRRA preferences, we show that there exists a parameterization of the two habit levels that approximately implies macro-finance separation, which we define as follows:

Definition (Macro-finance separation). Our competitive equilibrium is macro-financially separate if the equilibrium allocation $\{c_t, h_t, n_t\}$ and the price system $\{w_t - p_t, \pi_t, r_{f,t}\}$ are the same as in the model with CRRA utility such that $X_t^c = X_t^h = 0$ at all dates.

In focusing on this case we are taking to its logical extreme the critique of DSGE models with habit formation in the spirit of Campbell and Cochrane (1999) made by Lettau and Uhlig (2000), and revived by Uhlig (2007) and Rudebusch and Swanson (2008). Our strategy implies that we can approximately preserve a macroeconomist's preferred model of quantities, with the associated well-tested empirical properties for quantities, including inflation.

While we are not denying the possibility that a more volatile discount factor better fits quantity dynamics,⁵ we argue that the first step of the modeling exercise of incorporating volatile discount factors into a macro model should be to keep the spillovers on quantities under control. We can then allow for an arbitrary spillover and a role for habits in the determination of quantity dynamics.

2.6.1. Sources of financial spillovers

In a production economy, habits affect intertemporal and static decisions by their effect on the marginal utilities of market and home consumption. Intertemporally, the consumptionsaving tradeoff is described by equation (4), which implies a potential dependence of the real risk-free rate on surplus consumption. Intratemporally, equation (5) shows how the surplus consumption processes have an effect on the consumption-labor tradeoff that makes the real wage, and therefore inflation and dividends, depend on the ratio of the surplus processes. Surplus consumption drives risk prices and is very volatile; therefore, the spillover of this

⁴We choose the textbook equilibrium selection strategy under a Taylor rule, and hence focus on the unique locally bounded solution (see Cochrane, 2011, for a critical discussion).

⁵For example, such a spillover generates hump-shaped dynamics in Boldrin et al. (2001), and is essential to generate realistic unemployment fluctuations in Kehoe et al. (2022).

state variable on real variables by these two channels can induce counterfactual business cycle properties.

To handle the *inter*temporal spillover (4), we follow Campbell and Cochrane (1999) and choose the market consumption sensitivity function to satisfy the following conditions: (i) the market consumption habit does not produce a risk-free rate puzzle by having intertemporal substitution and precautionary saving effects offset each other; (ii) the habit is approximately a linear habit that adjusts slowly to unanticipated movements in market consumption; and (iii) the habit is locally predetermined and moves nonnegatively with consumption near the steady state. While the first condition describes how habits must be engineered for intertemporal neutrality, the remaining conditions represent a minimal microfoundation to guarantee a sensible notion of habit.

By analogous logic, we choose the home consumption sensitivity function to handle the *intra*temporal spillover (5) as follows: (i) the home consumption habit does not produce a quantity puzzle by having the habit-related effects of shocks on the marginal utility of market and home consumption offset each other; (ii) the habit is approximately a linear habit that adjusts slowly to unanticipated movements in home consumption; and (iii) the habit is locally predetermined and moves nonnegatively with home consumption near the steady state. As in the case of market consumption habits, the last three conditions can be interpreted as local microfoundations.

To achieve these objectives, we parameterize the surplus consumption dynamics as follows. First, the surplus market consumption sensitivity function Λ_c and steady-state level S are:

$$\Lambda_{c,t} = \begin{cases} \sqrt{\frac{var(\varepsilon^c)}{var_t(\varepsilon_{t+1}^c)}} \frac{1}{S}\sqrt{1-2\hat{s}_t} - 1, & \hat{s}_t \le \frac{1}{2}(1-S^2)\\ 0 & \hat{s}_t > \frac{1}{2}(1-S^2) \end{cases}, \qquad S = \sqrt{\frac{\gamma \ var(\varepsilon^c)}{1-\rho_s - \xi_1/\gamma}} \tag{14}$$

for a free parameter $\xi_1 < \gamma(1 - \rho_s)$, where $\varepsilon_t^x \equiv (E_t - E_{t-1})x_t$ denotes innovations in variable x. Appendix A proves the desired properties (i)-(iii) of the implied market consumption habits.

The motivation for the choice of S follows Campbell and Cochrane (1999), and it is made to control the direct spillover of state s_t on the risk-free rate. The choice of sensitivity function and the value for S keep this spillover as close as possible to $\xi_1 \hat{s}_t$ in a mean-squared sense, and exactly equal to it when consumption innovations are conditionally Gaussian.

Second, we specify the surplus home consumption sensitivity function Λ_h and steady-state level Z as

$$\Lambda_{h,t} = \frac{S}{1-S} \frac{1-Z}{Z} \frac{cov_t(\varepsilon_{t+1}^c, \varepsilon_{t+1}^h)/var_t(\varepsilon_{t+1}^h)}{cov(\varepsilon^c, \varepsilon^h)/var(\varepsilon^h)} \Lambda_{c,t}, \quad Z = \left(1 + \frac{1-S}{S}(1+\xi_2)\frac{cov(\varepsilon^c, \varepsilon^h)}{var(\varepsilon^h)}\right)^{-1}$$
(15)

for a free parameter ξ_2 . Appendix A proves the desired properties (ii)-(iii) of the implied home consumption habits.

The choice of Z is instead motivated to control the direct spillover of surplus consumption on the marginal rate of substitution between consumption and labor. Namely, since our choice of Z is such that

$$Z = \arg\min var_t [\Lambda_{h,t} \varepsilon_{t+1}^h - (1+\xi_2)\Lambda_{c,t} \varepsilon_{t+1}^c]$$
(16)

it follows that the term $\hat{s}_t - \hat{z}_t$ in (5) is as close as possible to $-\xi_2 \hat{s}_t$ in a mean-squared error sense.

We have therefore gained a handle on both spillovers through the choice of values of S and Z or, equivalently, through the choice of the free parameters ξ_1 and ξ_2 .

2.6.2. Macro-finance separation: An illustration

To illustrate our theoretical reconciliation of habit formation with business cycle facts, consider the flexible-price competitive equilibrium. Intratemporally, in the online appendix we verify that our choice of the process for home and market consumption habits, together with the production functions and market clearing, implies that in equilibrium

$$z_t - z = (1 + \xi_2)(s_t - s)$$

and therefore the parameter ξ_2 can be used to control the effect that the surplus consumption processes have on inflation, consumption, and dividends. The case $\xi_2 = 0$ implies a consumption-labor tradeoff (5) equivalent to the one that would hold in a model without habits. The flexible-price competitive equilibrium for quantities would become $c_t = const. + a_t$ and $n_t = n$.

Intuitively, in the presence of a negative market consumption shock, the home consumption habit makes the substitution effect toward home consumption dominate the income effect, making households choose not to absorb the movement in consumption by significantly increasing their labor effort. With home consumption habits, the Frisch labor supply elasticity scales by Z_t and hence it scales down by a factor $Z \in (0, 1)$ relative to the case without habits, and it drops in a recession; households become particularly sensitive to fluctuations in *both* market and home consumption during a downturn.

Intertemporally, the lognormality of consumption implies that the consumption-saving tradeoff is described by

$$r_{f,t} = -\ln(\beta) + \gamma E_t \Delta c_{t+1} + \gamma E_t \Delta s_{t+1} - \frac{1}{2} \gamma^2 (1 + \Lambda_{c,t})^2 var_t(\varepsilon_{t+1}^c)$$

= $-\ln(\beta) - \frac{\gamma (1 - \rho_s - \xi_1 / \gamma)}{2} + \gamma E_t \Delta c_{t+1} - \xi_1 \hat{s}_t$

where we used specification (14), and so parameter ξ_1 controls the spillover to consumptionsaving decisions by balancing intertemporal substitution and precautionary saving motives, as in Campbell and Cochrane (1999) and Wachter (2006).

The case $\xi_1 = \xi_2 = 0$ implies therefore consumption-savings and consumption-labor tradeoffs (4) and (5) that reduce to the ones that would hold in a model without habits, and hence macro-finance separation. Therefore, the separation result holds exactly in the flexible-price economy. In the case of our sticky-price model, consumption shocks are not lognormal and homoskedastic, and the spillovers cannot be completely eliminated. In practice, we calibrate $\xi \equiv [\xi_1; \xi_2]$ so that the model is macro-financially separate to a first-order approximation around the risky steady state, as defined for example in Lopez et al. (2022). Using a global solution, we later confirm the limited quantitative impact of higher-order terms on the equilibrium allocation.

3. Quantitative Results: Quantities

We parameterize the model by a standard calibration of all parameters except the productivity process, which we estimate by a generalized method of moments (GMM) that matches the observed volatility at different horizons, and hence the serial correlation, of the cash flows of equities and bonds. This strategy ensures that the model displays a realistic degree of mean reversion.

We solve the model numerically using a collocation method over a tensor grid to project the global solution of our model onto the subspace spanned by a basis of Chebyshev polynomials of up to degree 15. We consider large boundaries for the grid, especially in the endogenous states, motivated by Wachter (2005), who shows how the best practice in solving models with Campbell-Cochrane preferences is to consider a large and fine grid that places many grid points close to zero in the space of surplus market consumption, which mostly drives the stochastic discount factor.

To gain insight into the equilibrium relationships of our model we also use a risk-adjusted linear approximation that provides analytical expressions.

3.1. Parameterization

Table 1 lists all deep parameters in the model and their calibrated values. To calibrate our model we focus on data since the 1980s to focus on a historical period that can be appropriately described by a New Keynesian model with determinate dynamics (Clarida et al., 2000) and over which the stylized facts about the term structure of the equity premium have been documented.

We calibrate all parameters of the production side of the economy according to a standard New Keynesian model; we pick values from Galí (2008). Parameter α matches a labor share in value added of 2/3; the elasticity of substitution in the CES aggregator is $\varepsilon = 6$; the average duration of prices is $(1 - \eta)^{-1} = 9$ months; the interest rate rule coefficients attached to inflation and the output gap are 1.5 and 0.5/12. We assume steady-state hours N = 1/2, so the Frisch elasticity of labor supply equals the elasticity of intertemporal substitution, as in Galí (2008). The steady-state inflation rate is calibrated to the sample's average inflation rate of 2.2 percent p.a. over the 1980-2019 period.

We set all parameters related to preferences, and hence to the pricing kernel, following the same calibration strategy used by Campbell and Cochrane (1999). Namely, we set a value for the elasticity of intertemporal substitution of .5 for a maximum Sharpe ratio of .42 on an annual basis; habit persistence matches an annual autocorrelation coefficient of the CRSP stock market price-dividend ratio of .905; the subjective discount factor matches an

	Parameter		Value	Method
New	$1 - \alpha$	Labor share in value added	2/3	Galí (2008)
Keynesian	ε	Elasticity of substitution in CES aggregator	6	Galí (2008)
block	$1/(1 - \eta)$	Average price duration (in months)	9	Galí (2008)
	ϕ_{π}	Policy response coefficient to inflation	1.5	Galí (2008)
	ϕ_y	Policy response coefficient to output	0.5/12	Galí (2008)
	π^*	Average inflation rate $(\%)$	0.55	Cal.
Preference	β	Subjective discount factor	.994	Cal.
block	$1/\gamma$	Elasticity of intertemporal substitution (EIS)	1/2	Cal.
	$ ho_s$	Habit persistence	.992	Cal.
	ξ_1	Intertemporal financial spillover	.0001	MFS
	ξ_2	Static financial spillover	013	MFS
Exogenous	μ	Mean productivity growth rate (%)	.13	Cal.
block	$ ho_u$	Persistence of conditional mean growth rates	.813	GMM
	σ	Conditional volatility of productivity	1.07	GMM
	ϕ	Relative volatility of predictable productivity	.134	GMM

Cal.: steady-state calibration.

MFS: value chosen to ensure approximate macro-finance separation.

GMM: estimated by a generalized method of moments to match realized 1- to 5-year horizon volatility of per capita real consumption growth, real dividend growth, and inflation. The calibration for ξ_1 and ξ_2 implies S = 0.064 and Z = 0.257.

Table 1: Deep parameters and their calibration (monthly frequency). Data for consumption growth and inflation use BEA data over the period 1985-2019 for personal consumption expenditure on nondurables and services and the CPI; dividend growth uses dividend payouts (aggregated without reinvestment) of the end-of-month CRSP value-weighted stock index; the real interest rate is the CRSP 1-month Treasury bill rate minus expected CPI inflation. Weighting matrices for the two-stage GMM estimates are the identity matrix (first step) and the spectral density at frequency zero (second step) constructed as the Newey-West HAC estimator with a Bartlett kernel and an automatic bandwidth selection criterion.

$\sqrt{\frac{1}{n}var(g_{t+n}-g_t)}$		1	Hor 2	izon (ye 3	ears) 5	7
Consumption $g = c$,	model:	.0122	.0115	.0112	.0109	.0108
	data:	.0116	.0142	.0155	.0168	.0166
Dividends $g = d$,	model: data:	$.170 \\ .145$.125 .133	.102 .129	$.079 \\ .108$.066 .091
Inverse CPI $g = -p$,	model:	.0098	.0110	.0114	.0118	.0119
	data:	.0113	.0114	.0114	.0116	.0124

Table 2: Standard deviations of simulated and historical cash flow data at different horizons.

annualized average real risk-free rate of 0.69 percent. The spillover parameter $\xi = [\xi_1, \xi_2]$ is set to ensure approximate macro-finance separation as discussed in the previous section.

To pin down the exogenous process that drives the model's dynamics, we use a generalized method of moments strategy to estimate the parameter values that capture the volatility at different horizons of the cash flows we are interested in pricing—consumption and dividend growth and inflation. Note that our process for productivity allows for flexible dynamics encompassing in particular both random-walk and AR(1) processes. We therefore estimate the degree of mean reversion in productivity, which will be a key factor behind the slopes of the term structures at the short end, to capture the serial correlation in the data. Namely, we rely on monthly data over the period 1980-2019 on personal consumption expenditure on nondurable goods and services from the BEA and on dividend payouts on the CRSP value-weighted stock index covering all firms continuously listed on the NYSE, AMEX, and NASDAQ; we rely on annual BEA data on the CPI as our preferred measure of the price level.

Table 2 and Figure 1a show how our simple production economy under approximate macro-finance separation is a realistic model of cash flows along many dimensions, with a consumption process close to a random walk, much more volatile and strongly mean-reverting dividends, and a positively autocorrelated inflation rate. Figure 1a also shows that the macro-finance separation is nearly exact, since a version of the model with the parameters in Table 1 but with CRRA preferences, plotted by the dotted lines, has virtually indistinguishable implications for cash flows. Note that the estimated productivity process is clearly not a random walk but neither is it trend-stationary since $\phi < 1 - \rho_u$.

3.2. Inspecting the mechanism

Even though we will solve the model by a global projection method, it is useful to gain analytic insight with a first-order approximation around the risky steady state (see Lopez et al., 2022), which accounts for time-varying components contained in conditional second moments that are crucial in a world with Campbell and Cochrane (1999) habits, and that are otherwise captured only by conventional approximations around the deterministic steady state of third order.

To a first-order perturbation around the risky steady state, there are particular values for the free parameters ξ_1 and ξ_2 such that the approximate solutions for consumption and



(a) Fit to the GMM objective; markers are moments in the data, curves are moments in the model. Dotted lines are the implications of the model under CRRA preferences.

(b) Generalized impulse response to -1 standard deviation productivity shock; percentage changes in cash flows over a given horizon.

Figure 1: Dynamic properties of cash flows.

inflation are

$$c_t - a_t = (1 - \alpha)n + \psi_{cu}u_t, \qquad \pi_t = \pi^* + \psi_{\pi u}u_t$$
(17)

In words, the equilibrium dependence of consumption and inflation on surplus consumption is zero—what we previously called macro-finance separation. Note also that to a first-order approximation price dispersion is a trivial process, $\Delta_t = 0$, and hence we omit it as a state variable.

As described in Table 1, the specific calibration of the spillover parameters ξ_1 and ξ_2 that achieves an approximate macro-finance separation is close to the point $\xi_1 = \xi_2 = 0$,⁶ and implies (17) with the approximate equilibrium coefficients, derived in the online appendix,

$$\psi_{cu} = \frac{1}{1 - \rho_u} \frac{[\gamma(1 - \rho_u) + \phi_y](1 - \beta e^{(1 - \gamma)\mu}\rho_u) + \varphi(\phi_\pi - \rho_u)}{[\gamma(1 - \rho_u) + \phi_y](1 - \beta e^{(1 - \gamma)\mu}\rho_u) + \kappa(\phi_\pi - \rho_u)}$$

$$\psi_{\pi u} = \frac{1}{1 - \rho_u} \frac{(\kappa - \varphi)[\gamma(1 - \rho_u) + \phi_y]}{[\gamma(1 - \rho_u) + \phi_y](1 - \beta e^{(1 - \gamma)\mu}\rho_u) + \kappa(\phi_\pi - \rho_u)}$$
(18)

Coefficients κ and φ are functions of other deep parameters and reduce to $\kappa = (1 - \beta e^{(1-\gamma)\mu}\eta)(1-\eta)[\gamma(2-\alpha)+\alpha]/\eta(1-\alpha+\alpha\varepsilon)$ and $\varphi = 0$ when $\sigma = 0$, as in conventional

⁶Namely, the spillover parameters are

$$\xi_1 = \frac{\gamma}{S} (1 + \psi_{cu}\theta)\psi_{\pi u}\theta\sigma^2, \qquad \qquad \xi_2 = \frac{(e^{-L_3} - e^{-L_2})\beta e^{(1-\gamma)\mu}\eta(1-\rho_s)}{(e^{-L_3} - \beta e^{(1-\gamma)\mu}\eta\rho_s)(e^{-L_2} - \beta e^{(1-\gamma)\mu}\eta)}$$

where L_2 and L_3 are small terms proportional to σ^2 .

linearizations around the deterministic steady state of the basic New Keynesian model with CRRA utility (e.g., Galí, 2008).

These coefficients highlight in particular the role played by nominal rigidities and by the anti-inflationary stance of the central bank. Higher nominal rigidities (or a higher parameter η) imply a lower κ , and hence a greater elasticity of consumption, ψ_{cu} , and a smaller elasticity of inflation, $\psi_{\pi u}$, to changes in u_t . In contrast, a stronger anti-inflationary stance (or a higher parameter ϕ_{π}) implies lower elasticities of both consumption and inflation to changes in u_t . We will exploit these results in Section 5 when discussing the role of monetary policy in shaping asset markets.

3.3. Model intuition

The New Keynesian framework models endogenously a difference between the cash flow processes paid out by real and nominal bonds and by consumption and dividend strips. In fact, the labor share fluctuates when prices are sticky, and those movements are responsible for movements in inflation as well as for breaking down the equality between dividend growth and consumption growth. Intuitively, corporate profits are low when marginal costs are high, while inflation is high when firms expect high marginal costs, in which case resetting firms choose a price above the index to realign their marginal costs to the desired level.

Figure 1b summarizes the main differences among the cash flows of consumption and dividend claims and of nominal bonds by plotting the anticipated reaction of the main cash flow processes to a negative productivity shock. All cash flows drop during a downturn, and so all three assets are risky in this sense. The negative productivity shock depresses output but signals higher future growth because the process is partially mean reverting. Consumption drops on impact and then recovers, although to a limited extent, reflecting an estimated productivity process with a sizable permanent component. Dividends drop by a multiple of consumption and quickly rebound and, owing to their cointegration with consumption, experience the same long-run decline as consumption. The price level increases sluggishly, and hence the nominal bonds' payoff decreases sluggishly.

The intuition behind this reaction of cash flows can be understood from the approximate equilibrium equation for the real risk-free rate, up to an irrelevant constant,

$$r_{f,t} = \gamma [1 - (1 - \rho_u)\psi_{cu}]u_t - \xi_1 \hat{s}_t$$

The drop in productivity combined with expectations of better future growth (i.e., a higher u_t) prompts households to anticipate consumption and command a higher interest rate to save. Yet, the real rate increases less than it would without monetary frictions, as $\psi_{cu} = 0$ in the flexible price equilibrium, and hence the incentives to save remain too low; so demand and output rise above potential and exert upward pressure on marginal costs. This cost effect depresses corporate profits by equation (13) while causing inflationary pressure by equation (7) as resetting firms raise prices to realign profits to the desired level. Profits are then expected to jump back up as the excessive production is corrected; hence, dividends are expected to grow more than consumption, while positive inflation persists for a while as more and more firms get a chance to reset their prices upward. Finally, note that, when



(a) Term structures of one-period returns. (b) Term structures of yields.

Figure 2: Unconditional term structures of equity and interest rates. Different lines are associated with term structures of different cash flow claims: real bonds (dashed), nominal bonds (dash-dotted), and market equity (solid line).

 $\xi_1 > 0$, the drop in productivity causes a discount-rate effect that increases the real rate, as consumers' incentives to save for precautionary reasons increase. However, because of macro-finance separation, such an effect will be trivial on quantities.

Higher nominal rigidities exacerbate these effects, as inflation takes longer to manifest when rigidities are stronger, and hence profits must absorb a larger share of the shocks that hit the economy, with a stronger contractionary effect on dividends. Note how the New Keynesian model endogenously generates operating leverage that makes dividend growth more volatile and more risky than consumption growth, as in the data. In this sense we have an endogenous mechanism by which dividends are a levered version of consumption, as routinely assumed in endowment-economy asset pricing models.

4. Quantitative Results: Asset Prices

This section studies the quantitative predictions for stock and bond markets. We emphasize how our model's implications for the term structures of asset prices represent information that was not used in the parameterization step. In this sense, the model naturally reproduces the empirical regularities of interest in equity and bond markets.

4.1. Results

4.1.1. Average asset pricing moments

Figure 2a reports the average term structures of equilibrium one-period risk premia of equities and of real and nominal interest rates. The average annualized premium on the market portfolio is 6.2 percent in the model, close to the 7.2 percent in the data and considerably less than the premium commanded by short-term equities, which can reach up to 13 percent on average, consistent with observed strip returns (e.g., Binsbergen et al., 2012). For example, in the model the premium at a 6-month maturity is 12.0 percent, close to the 13.3 percent reported in the data by Lopez (2021) over the 1990-2019 period. The model therefore predicts a downward-sloping average term structure of dividend strip returns. It also reproduces upward-sloping average term structures of bonds with a sizable gap between the two, reflecting an inflation risk premium that is positive at all maturities, between 0 and 1 percent per year. Note that we match the level of the bond term structures because we parameterized the model to match average risk-free and inflation rates of .69 percent and 2.2 percent, respectively.

At the longer end, note that the term structure of equity displays an important nonmonotonicity. The term structure of equity is U-shaped, with premia that start to increase beyond 5 or so years, and become especially large in the long run. Our model therefore reflects the familiar property of habit models, already documented by Binsbergen et al. (2012), of implying large risk premia for long-duration claims. This U-shape is driven by the dependence of yields on the two states, namely, expected productivity growth u_t , which mainly accounts for the shape of the short end of the curve, and the surplus consumption ratio \hat{s}_t , which mainly accounts for the shape of the long end of the curve. As we discuss below, since aggregate price-dividend ratios are mostly driven by the state \hat{s}_t , such a U-shape is key to reconciling the evidence of large short-run premia and yields with a larger correlation of long-run returns than of short-run returns with aggregate price-dividend ratios.

Figure 2b reports the average term structures of equity and bond yields, which reflect properties similar to those of the average term structures of one-period returns. Namely, the equity term structure is downward sloping with realistically large values at the short end (Binsbergen et al., 2013), and it becomes upward sloping after 12 years, beyond the maximum horizon for which we have dividend strip data. The term structures of real and nominal interest rates are upward sloping on average, with a positive difference between the nominal and real term structures reflecting sizable inflation risk premia and the positive steady-state inflation rate π^* .

4.1.2. Aggregate price-dividend ratios

We look next at the implications of our model for the aggregate stock market. We reproduce the main appealing properties of Campbell and Cochrane (1999), including countercyclical financial market volatility and risk premia, as well as the long-horizon predictability of excess stock returns. Similar to Campbell and Cochrane, aggregate pricedividend ratios have a mean similar to that in the data and around half the observed variance, and, in line with the evidence, move mostly (94 percent) on news about future returns and only to a limited extent (7 percent) on news about future dividend growth. Table 3 reports these properties.

Overall, the model of cash flows plays an important role in driving the equity term structure at the short end of the term structure, but a minor one in driving the behavior of the aggregate stock market in the sense that it preserves the aggregate stock market properties in Campbell and Cochrane. In particular, we preserve their successes in modeling

	Model		Data		
Variable	mean	s.d.		mean	s.d.
r_f (p.a.)	.007	.024		.007*	.024*
$r_{e,m}$ (p.a.)	.062	.169		.072	.153
pd (annualized)	3.52	.185		3.75	.382
	Lag (in years)				
pd	1	2	3	5	7
Autocorrelation, model:	.87	.82	.74	.61	.51
Autocorrelation, data:	.90*	.84	.79	.59	.43
		Returns		Dividends	
Variance decomposition:	on: 94%		%	7%	
	Lag (in years)				
Long-run return regressions	1	2	3	5	7
$10 \times \text{coefficient}$	-1.1	-2.2	-3.1	-4.6	-5.9
R^2	4.7%	8.7%	12%	17%	22%

Table 3: Mean, standard deviations and autocorrelation of simulated and historical data over 1980-2019: Asset prices. $r_{e,m}$ and pd denote the excess return and the price-dividend ratio of the aggregate stock market portfolio; r_f is the real risk-free rate. The long-run return regressions run the predictive regression $\sum_{j=1}^{h} r_{m,t+j} - r_{f,t+j} = b_0 + b_1 p d_t + e_{t+j}$. Variance decomposition indicates the percentage of var(pd) accounted for by covariance with dividend growth $\sum_{j=1}^{h} \delta^j \Delta d_{t+j}$ and returns $\sum_{j=1}^{h} \delta^j r_{m,t+j}$, where $\delta = PD/(1 + PD)$, $PD = E(e^{pd})$ and large h. An asterisk denotes a moment that was matched during our estimation using the approximate solution (any discrepancy between a simulated moment and target is due to the distance between the global and the approximate solution).

aggregate price-dividend ratios.

4.1.3. Cyclicality of asset prices

The literature suggests an important correlation in the data between aggregate pricedividend ratios and equity yields and one-period returns. Namely, consistent with the evidence in (Gormsen, 2021), long-term one-period returns are more correlated than short-term returns with movements in the aggregate price-dividend ratio, with the term structure of one-period returns even becoming upward sloping in bad times. In contrast, the term structure of equity yields is more downward sloping in downturns and becomes upward sloping in normal times, consistent with the evidence in Binsbergen et al. (2013).

Figure 3 plots the term structures of expected one-period returns and yields conditional on different values of market price-dividend ratios. The top panel replicates Gormsen's Figure 1 by regressing simulated one-period returns of a short-maturity claim, $\frac{1}{N} \sum_{n=1}^{N} r_{t,t+12}^{(n)}$ for different yearly values of N, and of the market portfolio on a constant and log price-dividend ratios, and by plotting averages and the first and fifth quintiles of the fitted values of the two regressions. In the model, the market portfolio is more elastic than short-maturity claims, including at the 7-year horizon reported by Gormsen, and the term structure of one-period

equity returns becomes more downward sloping in good times.

The bottom panel of Figure 3 shows the cyclicality of yields generated by the model. Bad times, as indicated by low values of price-dividend ratios, scale up the level and slope of the term structures. The term structure of equity yields becomes more downward sloping in *bad* times. Moreover, our simple model is able to predict a sign shift in the slope of the term structures of yields in good times, as high price-dividend ratios forecast lower inflation and growth rates, and hence better payoffs of and stronger intertemporal substitution motives to invest into long-duration bonds. For equity yields, these properties are consistent with the procyclicality of equity yield spreads documented by Binsbergen et al. (2013).

These properties are also consistent with the observed behavior of the term structure of nominal bond yields. As in the data, the model predicts a term structure of bond yields that becomes more upward sloping in times of low aggregate price-dividend ratios, which are associated with increasing expected future interest and inflation rates, and high term and inflation risk premia. The term structure of bond yields inverts in good times by the opposite logic. While this stylized fact goes back at least to Estrella and Hardouvelis (1991), we confirm it by estimating in the data a correlation of -.38 versus a correlation of -.35 in the model between price-dividend ratios and the 10-year to 3-month slope of the nominal yield curve.

	$\beta_1^{mkt,2}$	$\gamma_1^{5,1}$	$\phi_1^{5,1}$
Model	0.07	1.12	-0.32
95%-confidence interval	[0.02; 0.17]	[0.95; 1.28]	[-0.59; -0.14]
Gormsen (2021)	0.29	1.10	-0.15
95%-confidence interval	[0.15; 0.45]	[0.58; 1.76]	[-0.18; -0.11]

Table 4: Time variation of the equity term structure

Finally, we further examine the model's ability to match the cyclicality of the equity term structure by replicating the simulation study conducted with the reduced-form model of Gormsen (2021) in his Table X. Namely, we run 10,000 simulations of 100 years of artificial data and calculate median estimates for the regression coefficients of the difference between annual market returns and annual 2-year strip returns on the aggregate dividend-price ratio $(\beta_1^{\text{mkt},2})$, of the current 5- minus 1-year yield spread on the aggregate dividend-price ratio $(\phi_1^{5,1})$, and of the 4-year yield in one year minus the current 1-year yield on the current 5-minus 1-year spread $(\gamma_1^{5,1})$. Table 4 reports the results. Consistent with the results previously described, the model captures the countercyclicality of equity term premia (reflected by a positive $\beta_1^{\text{mkt},2}$), the procyclicality of yield spreads (reflected by a negative $\phi_1^{5,1}$), and a coefficient $\gamma_1^{5,1}$ larger than one, which implies that yield spreads are negatively related to future term premia. The model generates less countercyclical variation in term premia than the model in Gormsen (2021) but the predictions of the two models are in line with each other once we consider dispersion in the estimates across simulations.



(a) One-period returns: Short-maturity equity claims are the average return to dividend strips of 1 to 3.5 years maturity.

(b) One-period returns: Short-maturity equity claims are the average return to dividend strips of 1 to 7 years maturity.



Figure 3: Time variation of the term structures of one-period returns and yields. Bad (good) times are periods in which the aggregate price-dividend ratio is below (above) the bottom (top) quartile of the time series.

4.2. Inspecting the mechanism

Our model approximately implies a two-factor structure for yields and a one-factor structure for one-period risk premia, as revealed by a first-order approximation around the risky steady state, which captures time variation in risk premia.

First recall the behavior of cash flows during a downturn, as illustrated in Figure 1b. Consumption and surplus consumption fall on impact, and hence exposure to the shock is risky. However, the negative shock makes expected productivity growth u_t increase; since productivity is partly mean reverting, better times are ahead. In this context, an asset with a sufficiently large cash flow exposure to u_t serves as insurance. Indeed, dividends are expected to recover fast, while the payoff of nominal bonds is expected to fall more and more as time passes. From a purely cash flow perspective, a claim to dividends a few years out is less risky than a claim to tomorrow's dividends, while a claim to a unit of numeraire a few years out is more risky than a claim to tomorrow's numeraire. This intuition is key to understanding the downward- and upward-sloping term structures of equity and interest rates. Still, it is incomplete, because it disregards the discount-rate effect of a shock, to which we turn next.

To formalize these results, we set up the approximate solution for cash flows as

$$\Delta g_{t+1} = \mu_g + C_{g,u}u_t + C_{g,s}\hat{s}_t + D_g\varepsilon_{t+1}$$
$$u_{t+1} = \rho_u u_t - \phi\sigma\varepsilon_{t+1}$$
$$\hat{s}_{t+1} = \rho_s\hat{s}_t + \Lambda_{c,t}D_c\varepsilon_{t+1}$$

where $g \in \{d, c, 0, -p\}$ denotes the four different cash flow processes (dividends, consumption, the numeraire, and the inverse of the price level), and C_u , C_s , and D contain the reduced-form coefficients of the model's solution. We also rewrite the one-period stochastic discount factor as

$$m_{t+1} = \ln(\beta) - \gamma E_t \Delta c_{t+1} + \gamma (1 - \rho_s) \hat{s}_t - x_t \varepsilon_{t+1}$$

where we define $x_t \equiv \gamma (1 + \Lambda_{c,t}) D_c$ as the price of risk.

To gain intuition we can solve approximately for the no-arbitrage price of a claim to cash flow g that will be realized in n periods, $P_{g,t}^{(n)} = E_t(M_{t,t+n}G_{t+n}) = E_t(M_{t,t+n}P_{g,t+1}^{(n-1)})$ with $P_{g,t}^{(0)} = G_t$, and the associated one-period log return $r_{g,t+1}^{(n)} = p_{g,t+1}^{(n-1)} - p_{g,t}^{(n)}$ and yield $y_{g,t}^{(n)} = -\ln(P_{g,t}^{(n)}/G_t)/n$.

4.3. Model intuition: Yields

We again rely on the risk-adjusted approximation to gain intuition for our numerical results. Namely, the log price-dividend ratio at time t on a claim to cashflow G_{t+n} , denoted $pd_{g,t}^{(n)} = \ln(P_{g,t}^{(n)}/G_t)$, takes the approximate form

$$pd_{g,t}^{(n)} = A_g^{(n)} + B_{g,u}^{(n)}u_t + B_{g,s}^{(n)}\hat{s}_t$$

with $pd_{g,t}^{(0)} = 0$, where the coefficients have the form

$$\begin{aligned} A_g^{(n)} &= A_g^{(n-1)} + \ln(\beta) + (1-\gamma)\mu + \frac{1}{2} \Big(D_g - B_{g,u}^{(n-1)}\phi\sigma - B_{g,s}^{(n-1)}D_c - \frac{\gamma - B_{g,s}^{(n-1)}}{S}D_c \Big)^2 \\ B_{g,u}^{(n)} &= (C_{g,u} - \gamma C_{c,u}) \frac{1-\rho_u^n}{1-\rho_u} \\ B_{g,s}^{(n)} &= \gamma + \rho_s (B_{g,s}^{(n-1)} - \gamma) + C_{g,s} - \frac{1-\rho_s}{\gamma^2} \left(B_{g,s}^{(n-1)} - \gamma \right)^2 - \frac{D_c (D_g - B_{g,u}^{(n-1)}\phi\sigma - B_{g,s}^{(n-1)}D_c)}{S} \left(B_{g,s}^{(n-1)} - \gamma \right) \end{aligned}$$

with $A_g^{(0)} = B_{u,s}^{(0)} = B_{g,s}^{(0)} = 0$. (The online appendix derives these formulas.) Yields relate to these log price-dividend ratios as $y_{g,t}^{(n)} = -pd_{g,t}^{(n)}/n$.

Figure 4 plots these coefficients. Figure 4a shows constant terms that decrease with the maturity, with a speed that reflects the average payoff growth and the riskiness of the security. More important for our purpose, the elasticities of strips to the productivity state u, plotted in Figure 4b, are negative and decreasing for consumption claims and bonds, but positive and increasing for dividend claims. These properties combine the elasticity of cash flows with respect to u, $C_{g,u}$, described above and in Figure 1a, and the elasticity of discount rates with respect to u, $-\gamma C_{c,u}$. A higher value of u implies higher future cash flows and inflation and higher discount rates, as consumers prefer to anticipate consumption. For bonds, these effects imply lower valuations. The valuations of consumption claims also decrease, as the discount-rate effect outweighs the cash flow effect because the elasticity of intertemporal substitution is greater than unity. For dividend claims, however, the positive cash flow effect is so strong as to outweigh the discount-rate effect, and valuations are higher.

The elasticities of strips to the surplus consumption ratio s, plotted in Figure 4c, are positive and monotonically increasing for bonds and equities towards $B_{g,s}^{(\infty)} = \gamma$ —a feature we previously called the habit effect. Indeed, whenever surplus consumption is low, households forecast lower subsequent marginal utility, since their habits will adjust to the lower consumption, and the more so the farther out the horizon; hence, they require compensation to shift resources forward to a time in which they will be less hungry. It follows that the price of long-duration claims is low in a recession, and the more so the longer the claim's duration.

Note that we can write the aggregate equity log price-dividend ratio as

$$pd_t = \ln \sum_{n=1}^{\infty} e^{pd_{g,t}^{(n)}} = \ln \sum_{n=1}^{\infty} e^{A_g^{(n)} + B_{g,u}^{(n)}u_t + B_{g,s}^{(n)}\hat{s}_t}$$

which is driven by both states. In particular, a high productivity state u and a high surplus consumption ratio s both imply high aggregate price-dividend ratios, even though u and s are negatively correlated, since u is high after a bad productivity shock. Quantitatively, however, price-dividend ratios move mostly with surplus consumption, which tends to be low after negative productivity shocks, when u is high. As a consequence, in the model, low price-dividend ratios predict higher future price-dividend ratios, as in Campbell and Cochrane (1999), but also higher future inflation and dividend growth.

Accordingly, as in Figure 3, when price-dividend and surplus consumption ratios are low, nominal yields slope up, predicting higher future rates and inflation, and equity yields slope more negatively on the observable end, predicting higher future dividends.

4.4. Model intuition: One-period returns

The approximate one-period excess return $r_{g,t+1}^{e,(n)} = r_{g,t+1}^{(n)} - r_{f,t}$ associated with the approximate expression for price-dividend ratios of strips can be written as

$$r_{g,t+1}^{e,(n)} = E_t r_{g,t+1}^{e,(n)} + V_{g,n-1,t} \varepsilon_{t+1}$$

where the stochastic vector

$$V_{g,n-1,t} \equiv \underbrace{D_g}_{\substack{\text{short-run}\\ \text{cash flow risk}}} + \underbrace{B_{g,u}^{(n-1)}(-\phi\sigma)}_{\substack{\text{long-run cash flow}\\ \text{and discount rate risk}}} + \underbrace{B_{g,s}^{(n-1)}\Lambda_{c,t}D_c}_{\substack{\text{habit-related}\\ \text{discount rate risk}}}$$
(19)

represents the quantity of risk in the nth cash flow strip.

The closed-form approximate solution (19) provides insight into the determinants of the term structures of risk premia on different cash flow claims. The first term in equation (19) is entirely due to the one-period-ahead volatility in cash flows. The second term captures the effect that news about expected productivity growth has on tomorrow's prices through its effect on future cash flows and discount rates. The third term reflects the effect of movements in risk aversion on tomorrow's prices through their effect on cash flows and, chiefly, on long-run discount rates by the habit effect.

4.4.1. A 3-factor decomposition: Level, short-run, and long-run slope

Risk premia are the product of the systematic exposure of each strip to the structural shock and the price of a unit exposure to the structural shock, x_t . We can therefore use the approximation (19) to decompose risk premia as

$$\ln E_t R_{g,t+1}^{e,(n)} = cov(-m_{t+1}, \Delta g_{t+1} + pd_{g,t+1}^{(n-1)}) = x_t V_{g,n-1,t}$$

into three determinants: a level factor $x_t D_{g,t}$, a factor that controls the short end of the curve, $x_t B_{g,u}^{(n-1)} \phi \sigma$, and a factor that controls the long end of the curve, $x_t B_{g,s}^{(n-1)} \Lambda_{c,t} D_c$.

Level. The short end of the term structures depends primarily on the loadings on short-term cash flow risk through D_g , which controls the level of the term structures, whose initial value is

$$cov_t(-m_{t+1}, \Delta g_{t+1}) = x_t D_q$$

This level factor in the term structures of risk premia is depicted in Figure 4d, evaluated at the risky steady state under our baseline calibration.

The level of the term structure of dividend strips can be very high because of the high leverage in corporate profits, which fluctuate more than consumption, as nominal rigidities



Figure 4: 3-factor decomposition of log price-dividend ratios (top row), one-period risk premia (middle row), and Borovicka-Hansen dynamic value decomposition (bottom panel) of one-period risk premia of different zero-coupon cash flow claims. The bottom row plots annualized shock-exposure and shock-price elasticities after a marginal increase in exposure along the direction x_t . Thin solid lines in the plots of shock-price elasticities represent the interquartile range for the elasticities of real bonds. The decompositions are such that one-period risk premia in Figure 2 = [(d)+(e)+(f)] = [(g)+(h)-(i)].

force firms to act on quantities rather than on prices to absorb the economic shocks. The first dividend strip tends to have a dramatically low payoff precisely in those states in which households are hit by negative consumption shocks. Relatedly, the first nominal interest rate is strictly positive owing to the negative correlation between inflation and consumption news.

Short-run slope. An expected productivity growth above average tomorrow signals good future cash flows, which increase prices, but also lower future marginal utility, which decreases prices as households want to anticipate consumption. This discount-rate effect dominates the cash flow effect for all claims considered except for market equities, whose future prices therefore increase after a positive shock to u_t , and the more so the longer the strip duration. Since positive shocks to u_t arrive together with bad consumption news, it follows that this effect generates a negative slope in the term structure of equity and upward slopes in the remaining term structures.

In particular, we are able to generate a downward-sloping short end of the term structure of market equity for any calibration such that $B_{u,g}^{(n)}$ is sufficiently positive. In fact, for dividend claims the exposure to u_t commands a price

$$cov_t(-m_{t+1}, B_{g,u}^{(n)}u_{t+1}) = -x_t(C_{g,u} - \gamma C_{c,u})\frac{1 - \rho_u^n}{1 - \rho_u}\phi\sigma = -x_t\left(1 - \gamma + \frac{2\gamma}{\alpha}(1 - \rho_u)\psi_{cu}\right)\frac{1 - \rho_u^n}{1 - \rho_u}\phi\sigma$$

which is a negative number for sufficiently rigid prices, as ψ_{cu} starts at 0 when prices are flexible and increases with the degree of price stickiness. Therefore, the risk premium due to exposure to u_t commanded by dividend strips is a negative and convex function of maturity, and the analogous factor in consumption strips and zero-coupon bonds (real and nominal) is a positive and concave function of maturity, as shown in Figure 4e under our baseline calibration.

Long-run slope. The loading of tomorrow's yields on surplus consumption captures the properties of the premium commanded by long-duration claims; all term structures display an upward slope at the long end, a property that is driven by the perfectly negative correlation between shocks to consumption and to the price of risk. Tomorrow's price of long-duration claims is low, and hence one-period returns are low, precisely in those states of the world in which surplus consumption is low, and the more so the longer the claim's duration.

In particular, the loadings of yields on surplus consumption converge to the positive number $B_{s,d}^{(\infty)} = \gamma$ for any dividend process, with the speed of convergence controlled by the persistence of habits. Since shouldering surplus-consumption shocks is equivalent to shouldering consumption shocks, the habit-related loading of infinite-duration zero-coupon cash flow claims commands a strictly positive price

$$cov_t(-m_{t+1}, B_{s,g}^{(\infty)}\hat{s}_{t+1}) = \gamma^2 \Lambda_{c,t}(1 + \Lambda_{c,t}) D_c^2$$

Figure 4f plots these loadings under our baseline calibration.

4.4.2. Dynamic value decomposition: Borovicka-Hansen elasticities

The 3-factor decomposition of the one-month-ahead volatility in strip returns is deeply linked with the shock-exposure and shock-price elasticities proposed by Borovicka and Hansen (2014) as measures to quantify the exposure of cash flows over alternative horizons to shocks and the corresponding compensation commanded by investors. In particular, we can write one-period risk premia as

$$\ln E_t R_{g,t+1}^{e,(n)} = \underbrace{\varepsilon_{g,t}^{(n)}}_{\text{shock-exposure}} - \underbrace{\varepsilon_{p,t}^{(n)}}_{\text{elasticity}} + \underbrace{var_t(m_{t+1})}_{\text{precautionary}}_{\text{motive}}$$

where $\varepsilon_{g,t}^{(n)} = \frac{d}{dr} \ln E_t \left[\frac{D_{t+n}}{D_t} e^{rx_t \varepsilon_{t+1} - \frac{r^2}{2} x_t^2} \right]_{r=0}$ and $\varepsilon_{p,t}^{(n)} = \varepsilon_{g,t}^{(n)} - \frac{d}{dr} \ln E_t \left[M_{t,t+n} \frac{D_{t+n}}{D_t} e^{rx_t \varepsilon_{t+1} - \frac{r^2}{2} x_t^2} \right]_{r=0}$ denote the elasticities of expected future cash flows and of expected future returns to a marginal increase in exposure at t+1 along direction x_t , i.e., after a discount rate shock increasing marginal utility. Therefore, one-period risk premia are equivalent to a strictly positive level factor (households require some compensation to save when facing uncertainty about future marginal utility), plus the elasticity of future cash flows to a discount-rate shock (cash flow effect of the shock) less the elasticity of future investors' required compensation for exposure to the shock (discount-rate effect of the shock).

A marginal increase in exposure with the same direction as a discount-rate shock recovers the movement in expected cash flows and returns associated with that shock. Figures 4g to 4i plot these elasticities. The precautionary motive component in Figure 4g accounts for a large component of risk premia that is constant across maturities and asset classes. Figure 4h, which is essentially the flipped version of the impulse response functions in Figure 1b, shows how a positive discount-rate innovation is associated with positive and partially mean-reverting dividend and consumption news as well as with disinflationary news. Figure 4i shows how a positive discount-rate innovation is associated with lower marginal utility in the near future driven by higher future growth as well as with higher marginal utility in the very long run owing to a habit level slowly growing toward the higher consumption level.

Tomorrow's cash flow and discount-rate effects combine to account for one-period risk premia. The cash flow effects revealed by the shock-exposure elasticities show how they are the dominating components behind the shape of the term structure over the 1- to 5-year horizon. Shock-price elasticities show instead how the discount-rate effects dominate the long-term pricing of equity and bond claims.

4.4.3. Understanding the cyclicality

For equity claims, the structure for one-period risk premia is U-shaped on average, downward-sloping at first and, after 5 years or so, upward-sloping. The downward slope at the short end of the curve is driven by the mean-reverting productivity state, which drives aggregate equity yields only to a limited extent, while, through the surplus consumption ratio, habits are responsible for the upward slope at the long end of the curve and are the main drivers of aggregate price-dividend ratios. Under the approximate solution for one-period expected returns we have that, for any $n, m \ge 0$ with n > m, the equity term premium can be written as

$$TP_{d,t}^{n,m} \equiv \ln E_t R_{d,t+1}^{(n+1)} - \ln E_t R_{d,t+1}^{(m+1)} = x_t \left(V_{d,n,t} - V_{d,m,t} \right)$$

Therefore, we have the term premium and its elasticity, both evaluated at the steady state,

$$TP_{d,t}^{n,m} = x \left(V_{d,n} - V_{d,m} \right), \qquad \frac{\partial TP_{d,t}^{n,m}}{\partial x_t} = V_{d,n} - V_{d,m} + \frac{1}{S} (B_{d,s}^n - B_{d,s}^m)$$

Importantly, there is a disconnect between the two objects. In particular, the term $V_{d,n} - V_{d,m}$ in the elasticity reflects how the term premium changes with the price of risk and has the same sign as the average term premium, while the term $(B_{d,s}^n - B_{d,s}^m)/S$ reflects how the quantity of risk in future prices changes after a shock. Since, as in Figure 4f, when n > m we have $B_{d,s}^n - B_{d,s}^m > 0$ and S is a small positive number, we can have $V_{d,n} - V_{d,m} < 0$, and hence a downward-sloping term structure; yet $\partial T P_{d,t}^{n,m}/\partial x_t > 0$, and hence the countercyclical term premia documented by Gormsen (2021). This derivative will be indeed captured by the correlation between the term premium and the aggregate price-dividend ratio because, as we saw, aggregate price-dividend ratios are mostly driven by surplus consumption, as the elasticity of equity strips on surplus consumption is large at long maturities.

4.5. Additional diagnostics

The dynamics of our model's stochastic discount factor satisfy the properties postulated in Alvarez and Jermann (2005) and Hansen and Scheinkman (2009). Our parameterizations are associated with unit-root dynamics in the marginal utility of wealth with some amount of mean reversion, and hence with a model of the stochastic discount factor that displays three key realistic features in the language of Alvarez and Jermann: a time-varying permanent component, a time-varying transient component, and time variation in the relative importance of the permanent and transient components.

In the context of an approximation around the risky steady state, the martingale component of the stochastic discount factor can be shown to be

$$m_{t+1}^{P} = \begin{cases} -\frac{1}{2}x_{t}^{2} - x_{t}\varepsilon_{t+1}, & \text{if } \phi = \xi_{1} = 0\\ -\frac{1}{2}\gamma^{2}\left(1 - \frac{\phi}{1 - \rho_{u}}\right)\sigma^{2} - \gamma\left(1 - \frac{\phi}{1 - \rho_{u}}\right)\sigma\varepsilon_{t+1} & \text{otherwise} \end{cases}$$

which is discontinuous at $\phi = \xi_1 = 0$, has trivial properties only under trend-stationary productivity, and implies the approximate entropy ratio

$$\frac{var_t(m_{t+1}^P)}{var_t(m_{t+1})} = \begin{cases} 1 & \text{if } \phi = \xi_1 = 0\\ \frac{\gamma(1 - \frac{\phi}{1 - \rho_u})\sigma^2}{(1 - \rho_s)(1 - 2\hat{s}_t)}, & \text{otherwise} \end{cases}$$
(20)

(The online appendix derives both formulas.)

The martingale component of the stochastic discount factor reveals a permanent com-

ponent in the marginal utility of consumption, such that shocks to surplus consumption (if $\phi = \xi_1 = 0$) or shocks to the predictable component of consumption (if $\phi \neq 0$ or $\xi_1 \neq 0$) have a permanent effect on the marginal utility of wealth, even though both risk aversion and the predictable component of consumption are stationary.

Consider two extreme cases: $\phi = 1 - \rho_u$ (trend-stationary productivity) and $\phi = 0$ (random-walk productivity). The case of trend-stationary productivity implies $m_{t+1}^P = 0$ because there are no permanent shocks to the marginal utility of wealth, and it would be at odds with the evidence in Alvarez and Jermann (2005), who find that the average variance ratio (20) should be large, indicating a large permanent component in the stochastic discount factor. The case of random-walk productivity, combined with a zero spillover parameter $\xi_1 = 0$, implies instead a variance ratio (20) constant at unity, and hence a trivial transient component of the stochastic discount factor. Since a variance ratio of one would predict a perfectly flat real bond term structure, this case is also at odds with the evidence.

In this context, our estimated productivity process is neither a random walk nor trendstationary. It follows that the marginal utility of wealth includes both permanent and transitory components, in line with Alvarez and Jermann (2005). Indeed, quantitatively, using a 20-year bond as a proxy for the infinite-duration bond as done by Alvarez and Jermann, our model generates a variance ratio quite close to unity (76 percent).

5. Policy Experiments and Robustness

The previous sections rationalized some rich features of equity and bond markets using a simple New Keynesian model in which nominal rigidities play a central role and in which central bank policy has a nontrivial effect on the equilibrium allocations and asset prices. This section conducts some policy experiments where we investigate the role of central bank policy—including the presence of a lower bound constraint on monetary policy—in shaping the properties of equity and bond prices. We additionally investigate the robustness of our results to the inclusion of a more realistic shock structure, in particular demand shocks.

5.1. Varying the degree of nominal rigidity

Figure 5 shows the effect of price stickiness and highlights its role in generating an initially downward-sloping term structure of market equity and in flattening the bond yield curve. While we already discussed the intuition for the role of nominal rigidities in previous sections (see equation 18), the figure quantifies their role. Equilibrium risk premia on zero-coupon equities shift upward as the degree of nominal price rigidity increases, whereas the opposite occurs for zero-coupon nominal bonds. In the limiting case as nominal rigidities disappear (price duration $\approx 1 \text{ month}$), there is no endogenous difference between the term structures of consumption and market equity.

The effect on the term structures is mainly driven by cash flows, as stickier prices make dividends more volatile, which exacerbates the negative slope in the term structure of equity, and the conditional mean of consumption growth and inflation more stable, which flattens the term structure of nominal interest rates and reduces the inflation risk premium. Note how a similar flattening of the term structure occurs also for zero-coupon real bonds.



Figure 5: Term structures of one-period dividend strip, nominal bond, and real bond returns for different degrees of price stickiness. Different lines represent different calibrations for the average price duration: one month (dashed), four months (dash-dotted), and nine months (solid line). As we change the degree of price stickiness, we adjust the habit parameters ξ_1 and ξ_2 to maintain approximate macro-finance separation.



Figure 6: Term structures of one-period dividend strip, nominal bond, and real bond returns for different policy rule parameterizations. Different lines represent different calibrations for the anti-inflationary stance: the baseline value ($\phi_{\pi} = 1.5$, solid), a stronger stance ($\phi_{\pi} = 3$, dash-dotted), and an aggressive stance ($\phi_{\pi} = 6$, dashed line). As we change the policy response to inflation, we adjust the habit parameters ξ_1 and ξ_2 to maintain approximate macro-finance separation.

It is also worth noting the highly nonlinear effect of increasing the degree of price rigidities, which stems from the convexity of the equilibrium coefficients on the key parameters, including, for example, the approximate elasticity ψ_{cu} of detrended consumption on the predictable component of productivity growth, u_t , that we characterized above in (18).

5.2. Varying the monetary policy stance

Figure 6 shows the endogenous effect of monetary policy on the term structures of equity and interest rates. The effect of a weaker anti-inflationary stance, here modeled as a lower Taylor rule coefficient ϕ_{π} , is similar to the effect of larger nominal rigidities, except for its effect on the inflation risk premium. Equilibrium risk premia and volatilities on zero-coupon equities shift downward as the monetary policy rule responds more aggressively to inflation, whereas the opposite occurs for zero-coupon nominal bonds.

Once again, the effect on the term structures is mainly driven by the effect of monetary policy on how cash flows respond to shocks. A more aggressive anti-inflationary stance stabilizes movements in marginal costs and the labor share. Consequently, dividends become less volatile, thereby reducing the negative slope in the term structure of equity. Similarly, the conditional mean of consumption growth becomes less stable as it moves closer to its flexible price value, which increases the term structure of nominal and real interest rates owing to a stronger discount-rate effect. At the same time, inflation also becomes more stable, and hence the inflation risk premium becomes smaller.

5.3. Nonlinearities caused by the zero lower bound

We also explore the robustness of our results to accounting in the model for a zero-lowerbound constraint on the nominal interest rate set by the central bank. In fact, the model spends around 10 percent of the time at the zero lower bound. Accordingly, we consider a version of the model in which we restrict monetary policy to the interest rate rule constrained by the zero lower bound, $i_t = \max(i_t^*, 0)$, where the latent interest rate i_t^* follows rule (9). Note that the zero lower bound introduces additional equilibria associated with a deflationary regime, but we focus here on the global solution in the so-called anchored-expectations regime, as defined, for example, in Aruoba et al. (2018), who find limited evidence in favor of a deflationary regime for the US during the ZLB episode after the great financial crisis.

We maintain the same parameterization and solve the model using the same solution strategy described above. With this additional nonlinearity, the model now spends 13.5 percent of the time at the zero lower bound, slightly more than in our baseline version. The inflation rate is 3.8 percent lower on average and has 9.8 percent more variance, consistent with lower and more elastic inflation when the interest rate is constrained, and the real risk-free rate is 3.6 percent higher on average and has 14.6 percent less variance, consistent with a central bank that is unable to lower the real risk-free rate sufficiently when constrained. The consumption and dividend growth rates are unchanged on average, and their variances are 4.6 percent lower and 0.4 percent higher, respectively.

These changes in cash flows are associated with small changes to the term structures. The online appendix reports these changes at all frequencies for one-period risk premia and yields. The term structures of the equity premium shift down between 3 and 30 basis points



(a) Term structures of one-period returns. (b) Term structures of hold-to-maturity returns.

Figure 7: Unconditional term structures of equity and interest rates, model with supply and demand shocks. Different lines are associated with term structures of different cash flow claims: real bonds (dashed), nominal bonds (dash-dotted), and market equity (solid line). The lines in gray are the curves in the baseline version of the model.

depending on the maturity, a small number that affects none of our previous results. The equity premium is correspondingly reduced by about 8 basis points. Similarly, the term structures of real bond risk premia and yields shift down by 0 to 10 basis points, and the term structures of nominal bond risk premia and yields shift down by 0 to 20 basis points. Our results when we account for the additional nonlinearity caused by the zero lower bound are therefore very similar.

5.4. Incorporating demand shocks

So far we have isolated a simple mechanism that focuses on the effects of productivity shocks. While this mechanism goes a long way in explaining several asset pricing facts, a full-fledged model would include more shocks, including demand shocks to capture more comprehensively the data. For example, as argued by Campbell et al. (2020), the presence of a mix of demand and supply shocks can capture changing correlation patterns between consumption growth and inflation and between stock and bond returns.⁷

Our results are robust to including demand shocks to capture these changing correlations.

⁷Campbell et al. (2020) work within a reduced-form model and attribute observed changes in the stockbond return correlation to a change in the correlation of shocks in the model. Our model produces this result in general equilibrium; thus, through the lens of the model, the cause of the change in those correlations is produced by the different realized mix of productivity and preference shocks over different periods of time.

We extend our model to include demand shocks, Φ_t . Preferences are now described by

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \Phi_t \left(\frac{[C_t - X_t^c]^{1-\gamma}}{1-\gamma} + \chi \frac{[H_t - X_t^h]^{1-\gamma}}{1-\gamma} \right)$$

where Φ_t is an exogenous preference shock, whose log growth rate $\phi_t = \Delta \ln(\Phi_t)$ follows the autoregressive process

$$\phi_{t+1} = \rho_{\phi}\phi_t + \gamma(1 + \Lambda_{c,t})\varepsilon^{\phi}_{t+1} \tag{21}$$

where $\varepsilon_{\phi t}$ is i.i.d. normally distributed with mean 0 and variance σ_{ϕ}^2 , and is independent of the productivity shock. Under the specification for the sensitivity function

$$\Lambda_{c,t} = \begin{cases} \sqrt{\frac{var(\varepsilon^c - \varepsilon^{\phi})}{var_t(\varepsilon_{t+1}^c - \varepsilon_{t+1}^{\phi})}} \frac{1}{\bar{S}} \sqrt{1 - 2\hat{s}_t} - 1, & \hat{s}_t \le \frac{1}{2}(1 - \bar{S}^2) \\ 0 & \hat{s}_t > \frac{1}{2}(1 - \bar{S}^2) \end{cases}, & \bar{S} = \sqrt{\frac{\gamma \ var(\varepsilon^c - \varepsilon^{\phi})}{1 - \rho_s - \xi_1/\gamma}} \end{cases}$$

the spillover of s_t on the real rate is once again controlled by the free parameter ξ_1 , by analogous logic as in the baseline model.

We set the parameters ρ_{ϕ} and σ_{ϕ} as follows. We set the persistence $\rho_{\phi} = \rho_u$, to preserve the estimated serial correlation of cash flows of the baseline model. Furthermore, since the model with only productivity shocks understates the correlation between consumption growth and inflation (-49 percent in the model vs. -38 percent in the data over the 1980-2019 period), we set the standard deviation σ_{ϕ} to match the observed correlation between consumption growth and inflation.

In line with the evidence in Campbell et al. (2020), the model can now generate decadelong spells with negative correlations between stock and bond returns. Namely, we generate a long simulation of the model and compute in 10,000 monthly samples with a length of 10 years the correlation between 5-year nominal bond excess returns and the aggregate stock market excess return. In 9.9 percent of the samples, the model generates a negative correlation. Accordingly, the correlation between the output gap relative to the stochastic trend, $y_t - \bar{y}_t$, and inflation turns positive in 12.5 percent of the samples.

We then turn to the term structural implications of the model. Importantly, Figure 7 shows that the properties of both one-period returns and hold-to-maturity returns remain consistent with the data, with markedly downward-sloping term structures of equity and an upward sloping term structure of real interest rates, even though the presence of demand shocks reduces sizably the riskiness of nominal bonds at the 5-15 year horizon. Therefore, while the presence of demand shocks makes the downward slope in the term structure of equity and the upward slope in the term structure of interest rates less marked, the main properties remain similar to our baseline in a model that includes a realistic amount of demand shocks.

6. Conclusion

We incorporate the variation in discount rates arising from Campbell-Cochrane external habits in a standard macro model with nominal rigidities. We propose a method to break the apparent tradeoff between matching either the dynamics of macroeconomic variables or asset pricing dynamics in nonlinear habit models. The notion of macro-finance separation and small departures from it are shown to be useful for incorporating large discount-rate variation in a DSGE framework while preserving the model's ability to fit quantities.

We derive testable implications for the term structures of equity and interest rates that conform with recent capital market evidence, including the average and cyclicality of the slope of the term structures of equity and of nominal and real interest rates, as well as with more established evidence about the average and cyclicality of the equity premium and the risk-free rate. We showed that our simple theory can go a long way in capturing this rich set of asset pricing facts while preserving a familiar model of macroeconomic quantities.

By focusing on the effects of productivity and demand shocks, we have purposely kept our model and shock structure simple so as to isolate the mechanism, and we have therefore abstracted from a host of potentially relevant extensions of the macroeconomic model, which we leave for future work. For example, a richer shock structure would capture more comprehensively the post-war data, as asset price movements would help the identification of macroeconomic shocks. Overall, our results show that integrating asset pricing in New Keynesian production economies is both a tractable and a promising avenue of future research to understand the effect of macroeconomic shocks and monetary policy on quantities and asset prices.

References

- H. Ai, M. M. Croce, A. M. Diercks, and K. Li. News shocks and the production-based term structure of equity returns. *Review of Financial Studies*, 31(7):2423–2467, 2018. doi: 10.1093/rfs/hhy015.
- F. Alvarez and U. J. Jermann. Using asset prices to measure the persistence of the marginal utility of wealth. *Econometrica*, 73(6):1977–2016, 2005. doi: 10.1111/j.1468-0262.2005.00643.x.
- B. S. Aruoba, P. Cuba-Borda, and F. Schorfheide. Macroeconomic dynamics near the ZLB: A tale of two countries. The Review of Economic Studies, 85(1):87–118, 2018. doi: 10.1093/restud/rdx027.
- D. Backus, A. W. Gregory, and S. E. Zin. Risk premiums in the term structure: Evidence from artificial economies. *Journal of Monetary Economics*, 24(3):371–399, 1989. doi: 10.1016/0304-3932(89)90027-5.
- Ravi Bansal, Shane Miller, Dongho Song, and Amir Yaron. The term structure of equity risk premia. Journal of Financial Economics, 142(3):1209–1228, December 2021. doi: 10.1016/j.jfineco.2021.05.043.
- G. Bekaert, S. Cho, and A. Moreno. New Keynesian macroeconomics and the term structure. Journal of Money, Credit and Banking, 42(1):33–62, 2010. doi: 10.1111/j.1538-4616.2009.00277.x.
- F. Belo, P. Collin-Dufresne, and R. S. Goldstein. Dividend dynamics and the term structure of dividend strips. *Journal of Finance*, 70(3):1115–1160, 2015. doi: 10.1111/jofi.12242.
- J. H. v. Binsbergen and R. S. J. Koijen. The term structure of returns: Facts and theory. Journal of Financial Economics, 124(1):1–21, April 2017. doi: 10.1016/j.jfineco.2017.01.009.
- J. H. v. Binsbergen, M. Brandt, and R. S. J. Koijen. On the timing and pricing of dividends. American Economic Review, 102(4):1596–1618, 2012. doi: 10.1257/aer.102.4.1596.
- J. H. v. Binsbergen, W. H. Hueskes, R. S. J. Koijen, and E. B. Vrugt. Equity yields. Journal of Financial Economics, 110(3):503–519, December 2013. doi: 10.1016/j.jfineco.2013.08.017.

- M. Boldrin, L. J. Christiano, and J. D. M. Fisher. Habit persistence, asset returns, and the business cycle. American Economic Review, 91(1):149–166, 2001. doi: 10.1257/aer.91.1.149.
- J. Borovicka and L. P. Hansen. Examining macroeconomic models through the lens of asset pricing. Journal of Econometrics, 183(1):67–90, November 2014. doi: 10.1016/j.jeconom.2014.06.010.
- J. Y. Campbell and J. H. Cochrane. By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy*, 107(2):205–251, 1999. doi: 10.1086/250059.
- J. Y. Campbell and S. Ludvigson. Elasticities of substitution in Real Business Cycle models with home production. Journal of Money, Credit, and Banking, 33(4):847–875, 2001. doi: 10.2307/2673926.
- J. Y. Campbell, C. Pflueger, and L. M. Viceira. Macroeconomic drivers of bond and equity risks. Journal of Political Economy, 128(8):3148–3185, 2020. doi: 10.1086/707766.
- R. Clarida, J. Galí, and M. Gertler. Monetary policy rules and macroeconomic stability: Evidence and some theory. Quarterly Journal of Economics, 115(1):147–180, 2000. doi: 10.1162/003355300554692.
- J. H. Cochrane. Determinacy and identification with Taylor rules. Journal of Political Economy, 119(3): 565–615, 2011. doi: https://doi.org/10.1086/660817.
- A. Estrella and G. A. Hardouvelis. The term structure as a predictor of real economic activity. Journal of Finance, 46(2):555–576, 1991. doi: 10.1111/j.1540-6261.1991.tb02674.x.
- J. Galí. Monetary Policy, Inflation, and the Business Cycle. An Introduction to the New Keynesian Framework. Princeton University Press, Princeton, 2008.
- Niels Joachim Gormsen. Time variation of the equity term structure. The Journal of Finance, 76(4): 1959–1999, 2021. doi: 10.1111/jofi.13020.
- Y. Gorodnichenko and M. Weber. Are sticky prices costly? Evidence from the stock market. American Economic Review, 106(1):165–199, 2016. doi: 10.1257/aer.20131513.
- F. Gourio and P. Ngo. Risk premia at the ZLB: A macroeconomic interpretation. Manuscript, 2016.
- J. Greenwood and Z. Hercowitz. The allocation of capital and time over the business cycle. Journal of Political Economy, 99(6):1188–1214, 1991. doi: 10.1086/261797.
- L. P. Hansen and J. A. Scheinkman. Long term risk: An operator approach. *Econometrica*, 77(1):177–234, 2009. doi: 10.3982/ECTA6761.
- M. Hasler and R. Marfè. Disaster recovery and the term structure of dividend strips. Journal of Financial Economics, 122(1):116–134, October 2016. doi: 10.1016/j.jfineco.2015.11.002.
- I. Jaccard. Asset returns and labor supply in a production economy. Journal of Money, Credit and Banking, 46(5):889–919, 2014. doi: 10.1111/jmcb.12133.
- U. J. Jermann. Asset pricing in production economies. Journal of Monetary Economics, 41:257–75, 1998. doi: https://doi.org/10.1016/S0304-3932(97)00078-0.
- P. J. Kehoe, P. Lopez, V. Midrigan, and E. Pastorino. Asset prices and unemployment fluctuations. *Review of Economic Studies*, forthcoming, 2022. doi: https://doi.org/10.1093/restud/rdac048.
- H. Kung. Macroeconomic linkages between monetary policy and the term structure of interest rates. *Journal of Financial Economics*, 115(1):42–57, January 2015. doi: 10.1016/j.jfineco.2014.09.006.
- M. Lettau and H. Uhlig. Can habit formation be reconciled with business cycle facts? Review of Economic Dynamics, 3(1):79–99, January 2000. doi: 10.1006/redy.1998.0035.
- M. Lettau and J. A. Wachter. Why is long-horizon equity less risky? A duration-based explanation of the value premium. *Journal of Finance*, 62(1):55–92, 2007. doi: 10.1111/j.1540-6261.2007.01201.x.
- M. Lettau and J. A. Wachter. The term structures of equity and interest rates. Journal of Financial Economics, 101(1):90–113, July 2011. doi: 10.1016/j.jfineco.2011.02.014.
- E. X. N. Li and F. Palomino. Nominal rigidities, asset returns, and monetary policy. Journal of Monetary Economics, 66:210–225, September 2014. doi: 10.1016/j.jmoneco.2014.05.004.
- P. Lopez. Welfare implications of asset pricing facts: Should central banks fill gaps or remove volatility? Federal Reserve Bank of Cleveland, WP 21-16, 2021.
- P. Lopez, D. Lopez-Salido, and F. Vazquez-Grande. Accounting for risk in a linearized solution: How to approximate the risky steady state and around it. Federal Reserve Bank of Cleveland, WP 22-14, 2022.
- A. W. Lynch and O. Randall. Why surplus consumption in the habit model may be less persistent than you think. Working paper 16950, National Bureau of Economic Research, April 2011.

- R. Marfè. Income insurance and the equilibrium term structure of equity. *Journal of Finance*, 72(5): 2073–2130, 2017. doi: 10.1111/jofi.12508.
- C. Pflueger and G. Rinaldi. Why does the Fed move markets so much? Journal of Financial Economics, 146 (1):71–89, October 2022. doi: 10.1016/j.jfineco.2022.06.002.
- G. D. Rudebusch and E. T. Swanson. Examining the bond premium puzzle with a DSGE model. Journal of Monetary Economics, 55(Supplement):S111–S126, October 2008. doi: 10.1016/j.jmoneco.2008.07.007.
- G. D. Rudebusch and E. T. Swanson. The bond premium in a DSGE model with long-run real and nominal risk. American Economic Journal: Macroeconomics, 4(1):105–143, 2012. doi: 10.1257/mac.4.1.105.
- E. T. Swanson. Risk aversion and the labor margin in dynamic equilibrium models. American Economic Review, 102(4):1663–1691, 2012. doi: 10.1257/aer.102.4.1663.
- T. D. Jr. Tallarini. Risk-sensitive real business cycles. Journal of Monetary Economics, 45(3):507–532, June 2000. doi: 10.1016/S0304-3932(00)00012-X.
- H. Uhlig. Explaining asset prices with external habits and wage rigidities in a DSGE model. American Economic Review P&P, 97(2):239-243, 2007. doi: 10.1257/aer.97.2.239.
- J. A. Wachter. Solving models with external habit. Finance Research Letters, 2(4):210–226, December 2005. doi: 10.1016/j.frl.2005.08.001.
- J. A. Wachter. A consumption-based model of the term structure of interest rates. Journal of Financial Economics, 79(2):365–399, February 2006. doi: 10.1016/j.jfineco.2005.02.004.
- M. Weber. Nominal rigidities and asset pricing. 2014 Meeting Papers 53, Society for Economic Dynamics, 2014. URL https://ideas.repec.org/p/red/sed014/53.html.
- M. Weber. Cash flow duration and the term structure of equity returns. Journal of Financial Economics, 128(3):486–503, June 2018. doi: 10.1016/j.jfineco.2018.03.003.

Appendix

A. Motivation for Our Habit Specifications

The two habits satisfy the same local condition for a sensible habit as in Campbell and Cochrane (1999).

A.1. Campbell-Cochrane habit specification in a production economy

The law of motion of surplus consumption assumed by Campbell and Cochrane (1999) in their endowment economy with random-walk consumption can be cast in three *equivalent* specifications:

$$\hat{s}_{t+1} = \rho_s \hat{s}_t + \Lambda(\hat{s}_t) (E_{t+1} - E_t) c_{t+1}$$
(A.1a)

$$= \rho_s \hat{s}_t + \Lambda(\hat{s}_t)(\Delta c_{t+1} - \mu) \tag{A.1b}$$

$$= \rho_s \hat{s}_t + \Lambda E_t (\Delta c_{t+1} - \mu) + \Lambda (\hat{s}_t) (E_{t+1} - E_t) c_{t+1}$$
(A.1c)

where $\mu = E(\Delta c)$. The equality breaks down, however, once we allow for a predictable component in consumption growth, consistent with a generic production economy. Specifications (A.1a) and (A.1b) are commonly found in the literature; still, specification (A.1c) has also been used (e.g., Lynch and Randall, 2011).

We will argue that there is a strong reason to prefer specification (A.1a). To understand which specification we should retain in a production economy it is useful to remember the motivation of the specification in Campbell and Cochrane (1999). First, Campbell and Cochrane pick a specification for habits that implies a risk-free rate that adds to the expression under CRRA utility a linear term in surplus consumption, or, assuming normal consumption innovations,

$$r_{f,t} = r_f + \gamma E_t (\Delta c_{t+1} - \mu) - \xi_1 (s_t - s)$$
(A.2)

with $r_f = -\ln(\beta) + \gamma \mu - .5\gamma(1 - \rho_s - \xi_1/\gamma)$ and ξ_1 a free parameter that controls the volatility of the risk-free rate. Importantly, Campbell and Cochrane's specification breaks the equivalence between the inverse elasticity of intertemporal substitution γ and the risk aversion coefficient that drives the maximum Sharpe ratio, $\gamma(1 + \Lambda_t)$, a required property to generate discount-rate variation without a risk-free rate puzzle.

Second, for a meaningful notion of habit, Campbell and Cochrane's habit is a slow-moving average of past consumption that is predetermined, at least in a steady state in which all shocks are zero, and that has a nonnegative derivative with respect to consumption.

A.2. No risk-free rate puzzle

With normal consumption innovations, the equilibrium risk-free rates under specifications (A.1a), (A.1b) and (A.1c) are⁸

$$r_{f,t} = r_f + \gamma E_t (\Delta c_{t+1} - \mu) \tag{A.2a}$$

$$r_{f,t} = r_f + \gamma (1 + \Lambda_t) E_t (\Delta c_{t+1} - \mu)$$
(A.2b)

$$r_{f,t} = r_f + \gamma (1 + \Lambda) E_t (\Delta c_{t+1} - \mu) \tag{A.2c}$$

As shown by equations (A.2b) and (A.2c), specifications (A.1b) and (A.1c) imply a distorted dynamic IS equation relative to a CRRA specification that would imply a risk-free rate puzzle. Note in fact how a large price of risk $\gamma(1 + \Lambda) = \gamma/S$ is necessary to generate a large equity premium; the parameterization S < 1 is the element that amplifies the coefficient of risk aversion (see the online appendix) while remaining neutral on the risk-free rate and, hence, that allows for breaking the tradeoff between solving the equity premium and the risk-free rate puzzles in the habit framework.

We therefore discard specifications (A.1b) and (A.1c) on the grounds that in production economies they are generically inconsistent with the central idea of the Campbell-Cochrane habits. We thus retain specification (A.1a) and the associated dynamic IS equation (A.2a).

A.3. Local structure and predeterminedness

The market consumption habit specified indirectly by surplus consumption process (A.1a) using $x_t^c = c_t + \ln[1 - \exp(s_t)]$ is a nonlinear function of current and past consumption. However, it is approximately a predetermined, slow-moving average of past consumption, as required for a sensible notion of habit. In fact, a first-order approximation of the expression

⁸For simplicity, we turn off the spillover parameter ξ_1 since it adds nothing to the argument.

around $x_{ct} - c_t = \ln(1 - S)$ yields

$$x_{t+1}^{c} \approx \ln(1-S) + c_{t+1} + \rho_s(x_{ct} - c_t) - \varepsilon_{ct+1}$$
$$= \ln(1-S) + c_{t+1} - \sum_{j=0}^{\infty} \rho_s^j \varepsilon_{ct-j+1}$$

The habit is predetermined because $x_{ct+1} = E_t x_{ct+1}$, and past consumption shocks receive their full weight only asymptotically. Unanticipated movements in consumption move consumption away from habits; surplus consumption is thus essentially detrended consumption.

Symmetrically, the home consumption habit can be written locally as

$$x_{t+1}^{h} = \ln(1-Z) + h_{t+1} - \sum_{j=0}^{\infty} \rho_s^{j} \varepsilon_{ht-j+1}$$

and hence we can prove that the home consumption habit is a predetermined, slow-moving average of past home consumption.

A.4. Home consumption habits

Our home consumption habits can produce a macro-finance separation, and hence break the quantity puzzle, because the same state drives both surplus market and home consumption; so the respective effects on consumption-labor decisions can offset one another.

The local microfoundations of our home consumption habit parallel those of the market consumption habit. Furthermore, we already described in (16) how the choice of steady-state surplus home consumption ratio Z offsets the effect of habits on labor supply.

B. GMM Estimation Strategy

Given the time series of cash flow data $\{X_t\}_{t=1}^T$, with $X_t \equiv [c_t; d_t; \pi_t]$, we construct the GMM estimator of the vector of deep parameters $\theta \equiv [\rho_u; \sigma; \phi]$ as

$$\theta^* \equiv \arg\min E_T[f(X_t;\theta)]' W^{-1} E_T[f(X_t;\theta)]$$

$$f(X_t;\theta) = \begin{bmatrix} \frac{1}{12n} (c_t - c_{t-12n} - E_T[c_t - c_{t-12n}])^2 - var_{c,12n} \\ \frac{1}{12n} (d_t - d_{t-12n} - E_T[d_t - d_{t-12n}])^2 - var_{d,12n} \\ \frac{1}{12n} (p_t - p_{t-12n} - E_T[p_t - p_{t-12n}])^2 - var_{p,12n} \end{bmatrix}$$

for n = 1, ..., 5, with $E_T[\cdot] \equiv \frac{1}{T} \sum_{t=1}^{T} \cdot_t$ and where

$$var_{d,n}(\theta) \equiv \frac{1}{n}var\left[\ln\left(\frac{D_{t+n}}{D_t}\right)\right] = \frac{1}{n}\frac{\|C_d(\theta)\|^2}{1-\rho_u^2} \left\|\frac{1-\rho_u^n}{1-\rho_u}B(\theta)\right\|^2 + \frac{1}{n}\sum_{j=0}^{n-1}\left\|D_d(\theta) + C_d(\theta)\frac{1-\rho_u^j}{1-\rho_u}B(\theta)\right\|^2$$

are the relevant analytical moments computed with a risk-adjusted first-order approximation.

The weighting matrices for the two-stage GMM estimates are the identity matrix for first-step estimates and the spectral density at frequency zero for second-step estimates, which we construct as the Newey-West HAC estimator with a Bartlett kernel and an automatic bandwidth selection criterion.