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Online Appendix

Improving Inflation Forecasts Using Robust Measures*

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Contents

A.1. Data description

A.2. BVAR Model Details

A.3. Additional Skewness Plots

A4. Density Forecasting Results

A5. Forecasting Results Based on Bowley Skew

A6. A Bias-Adjusted Alternative (Two-Step Algorithm)

A7. Skewness and Other Plots by Goods and Services PCE Breakdown

A8. Forecasting Performance: July 2021 through December 2022

A.1. Data description

All of the empirical analysis in this appendix and in the main paper uses data at a monthly frequency spanning January 1978 through June 2021. We use data on the personal consumption expenditures price index (PCE), PCE excluding food and energy components (core PCE), and data on both price indexes and nominal expenditure shares of 181 components of PCE.

All of the PCE data are available from the Bureau of Economic Analysis (and retrieved from Haver Analytics). The monthly series of the unemployment rate (16 years plus) is available from the Bureau of Labor Statistics (BLS). Trimmed-mean PCE inflation is obtained from the website of the Federal Reserve Bank of Dallas. Median PCE inflation is obtained from the website of the Federal Reserve Bank of Cleveland. The measure of long-run inflation (denoted PTR) is obtained from the FRB/US model of the Federal Reserve Board. The measure of the natural rate of unemployment is obtained from the Congressional Budget Office (CBO). We compute skewness measures based on both month-to-month inflation rates and on 12-month inflation rates.

Table A1

Data Series	**Transformation**	**Source**
Unemployment rate	Level	BLS
Long-run estimate of unemployment rate	Level	CBO
PTR – long-run inflation expectations series	Level	Federal Reserve Board website
Median PCE price index	12-month trailing rate; month-to month rate	Federal Reserve Bank of Cleveland
Trimmed-mean PCE price index	12-month trailing rate; month-to-month rate	Federal Reserve Bank of Dallas
Core PCE price index	12-month trailing rate	BEA
Headline PCE price index	12-month trailing rate	BEA
Price indexes of the disaggregates of PCE price index; see the listing in Table A2	12-month trailing rate; month-to-month rate	BEA
Nominal expenditure shares of the disaggregates listed in Table A2	Level	BEA

List of disaggregate components

Table A2

1	Personal Consumption Expenditures: New Domestic Autos (SAAR, Mil.$)
2	Personal Consumption Expenditures: New Foreign Autos (SAAR, Mil.$)
3	Personal Consumption Expenditures: New Light Trucks (SAAR, Mil.$)
4	Personal Consumption Expenditures: Used Autos (SAAR, Mil.$)
5	Personal Consumption Expenditures: Used Light Trucks (SAAR, Mil.$)
6	Personal Consumption Expenditures: Tires (SAAR, Mil.$)
7	Personal Consumption Expenditures: Accessories and Parts (SAAR, Mil.$)
8	Personal Consumption Expenditures: Furniture (SAAR, Mil.$)
9	PCE: Clocks, Lamps, Lighting Fixtures & Other HH Decorative Items (SAAR, Mil.$)
10	PCE: Carpets & Other Floor Coverings (SAAR, Mil.$)
11	Personal Consumption Expenditures: Window Coverings (SAAR, Mil.$)
12	Personal Consumption Expenditures: Major Household Appliances (SAAR, Mil.$)
13	Personal Consumption Expenditures: Small Elec Household Appliances (SAAR, Mil.$)
14	Personal Consumption Expenditures: Dishes and Flatware (SAAR, Mil.$)
15	PCE: Nonelectric Cookware & Tableware (SAAR, Mil.$)
16	Personal Consumption Expenditures: Tools, Hardware, and Supplies (SAAR, Mil.$)
17	Personal Consumption Expenditures: Outdoor Equipment and Supplies (SAAR, Mil.$)
18	Personal Consumption Expenditures: Televisions (SAAR, Mil.$)
19	Personal Consumption Expenditures: Other Video Equipment (SAAR, Mil.$)
20	Personal Consumption Expenditures: Audio Equipment (SAAR, Mil.$)
21	PCE: Audio Discs, Tapes, Vinyl and Permanent Digital Downloads (SAAR, Mil.$)
22	PCE: Video Discs, Tapes and Permanent Digital Downloads (SAAR, Mil.$)
23	Personal Consumption Expenditures: Photographic Equipment (SAAR, Mil.$)
24	PCE: Personal Computers/Tablets and Peripheral Equip (SAAR, Mil.$)
25	PCE: Computer Software & Accessories (SAAR, Mil.$)
26	PCE: Calculators, Typewriters & Other Info Processing Equip (SAAR, Mil.$)
27	PCE: Sporting Equip, Supplies, Guns & Ammunition (SAAR, Mil.$)
28	Personal Consumption Expenditures: Motorcycles (SAAR, Mil.$)
29	Personal Consumption Expenditures: Bicycles and Accessories (SAAR, Mil.$)
30	Personal Consumption Expenditures: Pleasure Boats (SAAR, Mil.$)
31	Personal Consumption Expenditures: Pleasure Aircraft (SAAR, Mil.$)
32	Personal Consumption Expenditures: Other Recreational Vehicles (SAAR, Mil.$)
33	Personal Consumption Expenditures: Recreational Books (SAAR, Mil.$)
34	Personal Consumption Expenditures: Musical Instruments (SAAR, Mil.$)
35	Personal Consumption Expenditures: Jewelry (SAAR, Mil.$)
36	Personal Consumption Expenditures: Watches (SAAR, Mil.$)
37	Personal Consumption Expenditures: Therapeutic Medical Equipment (SAAR, Mil.$)
38	PCE: Corrective Eyeglasses & Contact Lenses (SAAR, Mil.$)
39	Personal Consumption Expenditures: Educational Books (SAAR, Mil.$)
40	PCE: Luggage & Similar Personal Items (SAAR, Mil.$)

41	PCE: Telephone and Related Communication Equipment (SAAR, Mil.$)
42	Personal Consumption Expenditures: Cereals (SAAR, Mil.$)
43	Personal Consumption Expenditures: Bakery Products (SAAR, Mil.$)
44	Personal Consumption Expenditures: Beef and Veal (SAAR, Mil.$)
45	Personal Consumption Expenditures: Pork (SAAR, Mil.$)
46	Personal Consumption Expenditures: Other Meats (SAAR, Mil.$)
47	Personal Consumption Expenditures: Poultry (SAAR, Mil.$)
48	Personal Consumption Expenditures: Fish and Seafood (SAAR, Mil.$)
49	Personal Consumption Expenditures: Fresh Milk (SAAR, Mil.$)
50	Personal Consumption Expenditures: Processed Dairy Products (SAAR, Mil.$)
51	Personal Consumption Expenditures: Eggs (SAAR, Mil.$)
52	Personal Consumption Expenditures: Fats and Oils (SAAR, Mil.$)
53	Personal Consumption Expenditures: Fresh Fruit (SAAR, Mil.$)
54	Personal Consumption Expenditures: Fresh Vegetables (SAAR, Mil.$)
55	Personal Consumption Expenditures: Processed Fruits and Vegetables (SAAR, Mil.$)
56	Personal Consumption Expenditures: Sugar and Sweets (SAAR, Mil.$)
57	PCE: Food Products, Not Elsewhere Classified (SAAR, Mil.$)
58	PCE: Coffee, Tea & Other Bev Mtls (SAAR, Mil.$)
59	PCE: Mineral Waters, Soft Drinks & Vegetable Juices (SAAR, Mil.$)
60	Personal Consumption Expenditures: Spirits (SAAR, Mil.$)
61	Personal Consumption Expenditures: Wine (SAAR, Mil.$)
62	Personal Consumption Expenditures: Beer (SAAR, Mil.$)
63	PCE: Food Produced & Consumed on Farms (SAAR, Mil.$)
64	Personal Consumption Expenditures: Women's and Girls' Clothing (SAAR, Mil.$)
65	Personal Consumption Expenditures: Men's and Boys' Clothing (SAAR, Mil.$)
66	PCE: Children's & Infants' Clothing (SAAR, Mil.$)
67	Personal Consumption Expenditures: Clothing Materials (SAAR, Mil.$)
68	PCE: Standard Clothing Issued to Military Personnel (SAAR, Mil.$)
69	Personal Consumption Expenditures: Shoes and Other Footwear (SAAR, Mil.$)
70	Personal Consumption Expenditures: Gasoline and Other Motor Fuel (SAAR, Mil.$)
71	Personal Consumption Expenditures: Lubricants and Fluids (SAAR, Mil.$)
72	Personal Consumption Expenditures: Fuel Oil (SAAR, Mil.$)
73	Personal Consumption Expenditures: Other Fuels (SAAR, Mil.$)
74	Personal Consumption Expenditures: Prescription Drugs (SAAR, Mil.$)
75	Personal Consumption Expenditures: Nonprescription Drugs (SAAR, Mil.$)
76	Personal Consumption Expenditures: Other Medical Products (SAAR, Mil.$)
77	Personal Consumption Expenditures: Games, Toys, and Hobbies (SAAR, Mil.$)
78	Personal Consumption Expenditures: Pets and Related Products (SAAR, Mil.$)
79	PCE: Flowers, Seeds & Potted Plants (SAAR, Mil.$)
80	Personal Consumption Expenditures: Film and Photographic Supplies (SAAR, Mil.$)
81	Personal Consumption Expenditures: Household Cleaning Products (SAAR, Mil.$)
82	Personal Consumption Expenditures: Household Paper Products (SAAR, Mil.$)
83	Personal Consumption Expenditures: Household Linens (SAAR, Mil.$)

84	Personal Consumption Expenditures: Sewing Items (SAAR, Mil.$)
85	PCE: Miscellaneous Household Products (SAAR, Mil.$)
86	PCE: Hair, Dental, Shaving & Misc Personal Care Prods ex Elec Prods (SAAR, Mil.$)
87	PCE: Cosmetic/Perfumes/Bath/Nail Preparations & Implements (SAAR, Mil.$)
88	PCE: Elec Appliances for Personal Care (SAAR, Mil.$)
89	Personal Consumption Expenditures: Tobacco (SAAR, Mil.$)
90	Personal Consumption Expenditures: Newspapers and Periodicals (SAAR, Mil.$)
91	Personal Consumption Expenditures: Stationery & Misc Printed Materials (SAAR, Mil.$)
92	Personal Consumption Expenditures: Tenant-Occupied Mobile Homes (SAAR, Mil.$)
93	PCE: Tenant-Occupied, Stationary Homes & Landlord Durables (SAAR, Mil.$)
94	Personal Consumption Expenditures: Owner-Occupied Mobile Homes (SAAR, Mil.$)
95	Personal Consumption Expenditures: Owner-Occupied Stationary Homes (SAAR, Mil.$)
96	Personal Consumption Expenditures: Rental Value of Farm Dwellings (SAAR, Mil.$)
97	Personal Consumption Expenditures: Group Housing (SAAR, Mil.$)
98	PCE: Water Supply & Sewage Maintenance (SAAR, Mil.$)
99	Personal Consumption Expenditures: Garbage and Trash Collection (SAAR, Mil.$)
100	Personal Consumption Expenditures: Electricity (SAAR, Mil.$)
101	Personal Consumption Expenditures: Natural Gas (SAAR, Mil.$)
102	Personal Consumption Expenditures: Physician Services (SAAR, Mil.$)
103	Personal Consumption Expenditures: Dental Services (SAAR, Mil.$)
104	Personal Consumption Expenditures: Paramedical Services (SAAR, Mil.$)
105	PCE: Nonprofit Hospitals' Services to Households (SAAR, Mil.$)
106	Personal Consumption Expenditures: Proprietary Hospitals (SAAR, Mil.$)
107	Personal Consumption Expenditures: Government Hospitals (SAAR, Mil.$)
108	Personal Consumption Expenditures: Nursing Homes (SAAR, Mil.$)
109	PCE: Motor Vehicle Maintenance & Repair (SAAR, Mil.$)
110	Personal Consumption Expenditures: Motor Vehicle Leasing (SAAR, Mil.$)
111	Personal Consumption Expenditures: Motor Vehicle Rental (SAAR, Mil.$)
112	Personal Consumption Expenditures: Parking Fees and Tolls (SAAR, Mil.$)
113	Personal Consumption Expenditures: Railway Transportation (SAAR, Mil.$)
114	Personal Consumption Expenditures: Intercity Buses (SAAR, Mil.$)
115	Personal Consumption Expenditures: Taxicabs (SAAR, Mil.$)
116	Personal Consumption Expenditures: Intracity Mass Transit (SAAR, Mil.$)
117	PCE: Other Road Transportation Service (SAAR, Mil.$)
118	Personal Consumption Expenditures: Air Transportation (SAAR, Mil.$)
119	Personal Consumption Expenditures: Water Transportation (SAAR, Mil.$)
120	PCE: Membership Clubs & Participant Sports Centers (SAAR, Mil.$)
121	PCE: Amusement Parks, Campgrounds & Related Recreational Services (SAAR, Mil.$)
122	Personal Consumption Expenditures: Motion Picture Theaters (SAAR, Mil.$)
123	Personal Consumption Expenditures: Live Entertainment, excl Sports (SAAR, Mil.$)
124	Personal Consumption Expenditures: Spectator Sports (SAAR, Mil.$)
125	Personal Consumption Expenditures: Museums and Libraries (SAAR, Mil.$)
126	PCE: Audio-Video, Photographic & Info Processing Services (SAAR, Mil.$)

127	Personal Consumption Expenditures: Casino Gambling (SAAR, Mil.$)
128	Personal Consumption Expenditures: Lotteries (SAAR, Mil.$)
129	Personal Consumption Expenditures: Pari-Mutuel Net Receipts (SAAR, Mil.$)
130	PCE: Veterinary & Other Services for Pets (SAAR, Mil.$)
131	Personal Consumption Expenditures: Package Tours (SAAR, Mil.$)
132	PCE: Maintenance & Repair of Recreational Vehicles & Sports Equip (SAAR, Mil.$)
133	PCE: Elementary & Secondary School Lunches (SAAR, Mil.$)
134	Personal Consumption Expenditures: Higher Education School Lunches (SAAR, Mil.$)
135	Personal Consumption Expenditures: Other Purchased Meals (SAAR, Mil.$)
136	Personal Consumption Expenditures: Alcohol In Purchased Meals (SAAR, Mil.$)
137	Personal Consumption Expenditures: Food Supplied to Civilians (SAAR, Mil.$)
138	Personal Consumption Expenditures: Food Supplied to Military (SAAR, Mil.$)
139	Personal Consumption Expenditures: Hotels and Motels (SAAR, Mil.$)
140	Personal Consumption Expenditures: Housing at Schools (SAAR, Mil.$)
141	Personal Consumption Expenditures: Commercial Banks (SAAR, Mil.$)
142	PCE: Other Depository Institutions & Regulated Investment Companies (SAAR, Mil.$)
143	Personal Consumption Expenditures: Pension Funds (SAAR, Mil.$)
144	PCE: Financial Service Charges, Fees & Commissions (SAAR, Mil.$)
145	Personal Consumption Expenditures: Life Insurance (SAAR, Mil.$)
146	Personal Consumption Expenditures: Net Household Insurance (SAAR, Mil.$)
147	Personal Consumption Expenditures: Net Health Insurance (SAAR, Mil.$)
148	PCE: Net Motor Vehicle & Other Transportation Insurance (SAAR, Mil.$)
149	Personal Consumption Expenditures: Communication (SAAR, Mil.$)
150	PCE: Proprietary & Public Higher Education (SAAR, Mil.$)
151	PCE: Nonprofit Pvt Higher Education Services to Households (SAAR, Mil.$)
152	PCE: Elementary & Secondary Schools (SAAR, Mil.$)
153	Personal Consumption Expenditures: Day Care and Nursery Schools (SAAR, Mil.$)
154	PCE: Commercial & Vocational Schools (SAAR, Mil.$)
155	Personal Consumption Expenditures: Legal Services (SAAR, Mil.$)
156	PCE: Tax Preparation & Other Related Services (SAAR, Mil.$)
157	Personal Consumption Expenditures: Employment Agency Services (SAAR, Mil.$)
158	PCE: Other Personal Business Services (SAAR, Mil.$)
159	Personal Consumption Expenditures: Labor Organization Dues (SAAR, Mil.$)
160	Personal Consumption Expenditures: Professional Association Dues (SAAR, Mil.$)
161	Personal Consumption Expenditures: Funeral and Burial Services (SAAR, Mil.$)
162	PCE: Hairdressing Salons & Personal Grooming Establishments (SAAR, Mil.$)
163	Personal Consumption Expenditures: Misc Personal Care Services (SAAR, Mil.$)
164	PCE: Laundry & Dry Cleaning Services (SAAR, Mil.$)
165	PCE: Clothing Repair, Rental & Alterations (SAAR, Mil.$)
166	Personal Consumption Expenditures: Repair and Hire of Footwear (SAAR, Mil.$)
167	Personal Consumption Expenditures: Child Care (SAAR, Mil.$)
168	Personal Consumption Expenditures: Social Assistance (SAAR, Mil.$)
169	PCE: Social Advocacy & Civic & Social Organizations (SAAR, Mil.$)

170	Receipts From Sales: Religious Organizations' Services to HH (SAAR, Mil.$)
171	Sales Receipts: Foundations & Grant Making & Giving Svcs to HH (SAAR, Mil.$)
172	Personal Consumption Expenditures: Domestic Services (SAAR, Mil.$)
173	PCE: Moving, Storage & Freight Services (SAAR, Mil.$)
174	PCE: Repair of Furniture, Furnishings & Floor Coverings (SAAR, Mil.$)
175	Personal Consumption Expenditures: Repair of Household Appliances (SAAR, Mil.$)
176	Personal Consumption Expenditures: Other Household Services (SAAR, Mil.$)
177	PCE: Foreign Travel by US Residents (SAAR, Mil.$)
178	PCE: Less: Expenditures in the US by Nonresidents (SAAR, Mil.$)
179	PCE: Expenditures Abroad by US Residents Price Index (SA, 2012=100)
180	PCE: Less: Personal Remittances in Kind to Nonresidents Price Idx (SA, 2012=100)
181	Final Consumption Expenditures of Nonprofit Instns Serving HH (SAAR, Mil.$)

A.2. BVAR Model Details [as in Knotek and Zaman, 2019]

A general representation of a VAR(p) model can be written as:

$$Y_t = A_c + A_1 Y_{t-1} + ... + A_p Y_{t-p} + u_t \quad \text{(A1)}$$

where t=1,…,T, $Y_t = [y_{1,t}, y_{2,t}, ..., y_{n,t}]$ is an n×1 data vector of n random variables, $A_c = [c_1, c_2, ..., c_n]$ is an n×1 vector of constants, $A_1, ..., A_p$ are n×n matrices of VAR coefficients, and u_t is an n×1 vector of normally distributed error terms with zero mean and covariance matrix $\Sigma = Eu_t u_t'$. In this n dimensional VAR, each equation has m=np+1 regressors, and with n equations, there are n×m parameters to be estimated. In our exercises, n will range from 2 to 3, and we set the number of lags, p, to 3 to be consistent with the benchmark AR(3) model. The system in equation (A1) can be written in a stacked compact form as:

$$Y = XA + U . \quad \text{(A2)}$$

We use Normal-inverse Wishart (N-IW) conjugate priors to characterize our beliefs about the coefficient estimates in $A_1, ..., A_p$ and Σ.[1] The prior beliefs for the mean and variances of the

[1] The N-IW prior is computationally convenient both for estimation and for performing Bayesian inference

coefficient matrices are:

$$E[A_k^{(i,j)}] = \begin{cases} \delta_i & \text{if } i = j,\ k = 1 \\ 0 & \text{otherwise} \end{cases}$$
$$Var[A_k^{(i,j)}] = \lambda^2 \frac{1}{k^2} \frac{\sigma_i^2}{\sigma_j^2},\ k = 1, \ldots, p \qquad (1)$$

We model inflation in gap form using deviations from its long-run trend, based on work by Faust and Wright (2013) and Zaman (2013), among others, that documents improvements in forecast accuracy from following this approach.[2] Since we are working with stationary data (in gaps), we set δ_i=0.0. The scale factor $1/k^2$ helps impose the prior belief that recent lags play a more influential role compared with more distant lags by proportionally shrinking the variances on the more distant lags (centered on a prior mean of zero). The prior parameter σ_i is set equal to the standard deviation of the residuals obtained from regressing the variable y_i on its own p lags and a constant over the sample period up to any point in time t. The hyperparameter λ governs the tightness of our priors. As $\lambda \to 0$, the prior dominates and the posterior equals the prior; that is, the data have no say. On the other hand, as $\lambda \to \infty$, the prior has no influence and posterior estimates converge to OLS estimates. The prior belief for the residual variance-covariance matrix Σ is set such that the expectation of Σ is equal to $diag(\sigma_1^2, \ldots, \sigma_n^2)$. As in Bańbura, Giannone, and Reichlin (2010), these priors for the coefficient estimates in $A_1, \ldots, A_p$ and Σ are implemented by augmenting equation (A2) with dummy observations.

The above-mentioned BVAR studies document further gains in forecast accuracy by imposing a "sum of coefficients" (SOC) prior on the equations of the VAR. Although this prior

compared with other prior choices. Koop (2013) documents the forecast accuracy of BVARs estimated with N-IW priors compared with other families of prior distributions.

[2] As in Knotek, Zaman, and Clark (2015), the long-run trend for inflation comes from splicing the long-term inflation expectations series from the Federal Reserve Board of Governors' FRB/US econometric model, denoted PTR, with the long-run inflation expectations series from the Survey of Professional Forecasters.

is more relevant when working with data in levels (or log-levels), since it imposes the belief that coefficients on own lags sum to one (or zero when working with stationary data), for purposes of generality, we nevertheless include this prior, but make it very loose. In essence, under the SOC prior, a reasonable forecast of the future level of a variable is the average of that variable's lagged values. The hyperparameter μ governs the tightness of the SOC prior. To implement the SOC prior, letting $\overline{y}_{0i}$ denote the average of the initial lagged p values for variable y_i, we further augment the system in equation (A2) with dummy observations:

$$
\begin{aligned}
Y^{SOC}(i,j) &= \begin{cases} \overline{y}_{0i} / \mu & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \\
X^{SOC}(i,r) &= \begin{cases} \overline{y}_{0i} / \mu & \text{if } i = j, r < m \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
\tag{A3}
$$

where i=1,…,n, j=1,…,n, and r=1,…,m.

A.3. Additional Skewness Plots

Figure A1: Cross-sectional asymmetry in PCE inflation (month-to-month %)

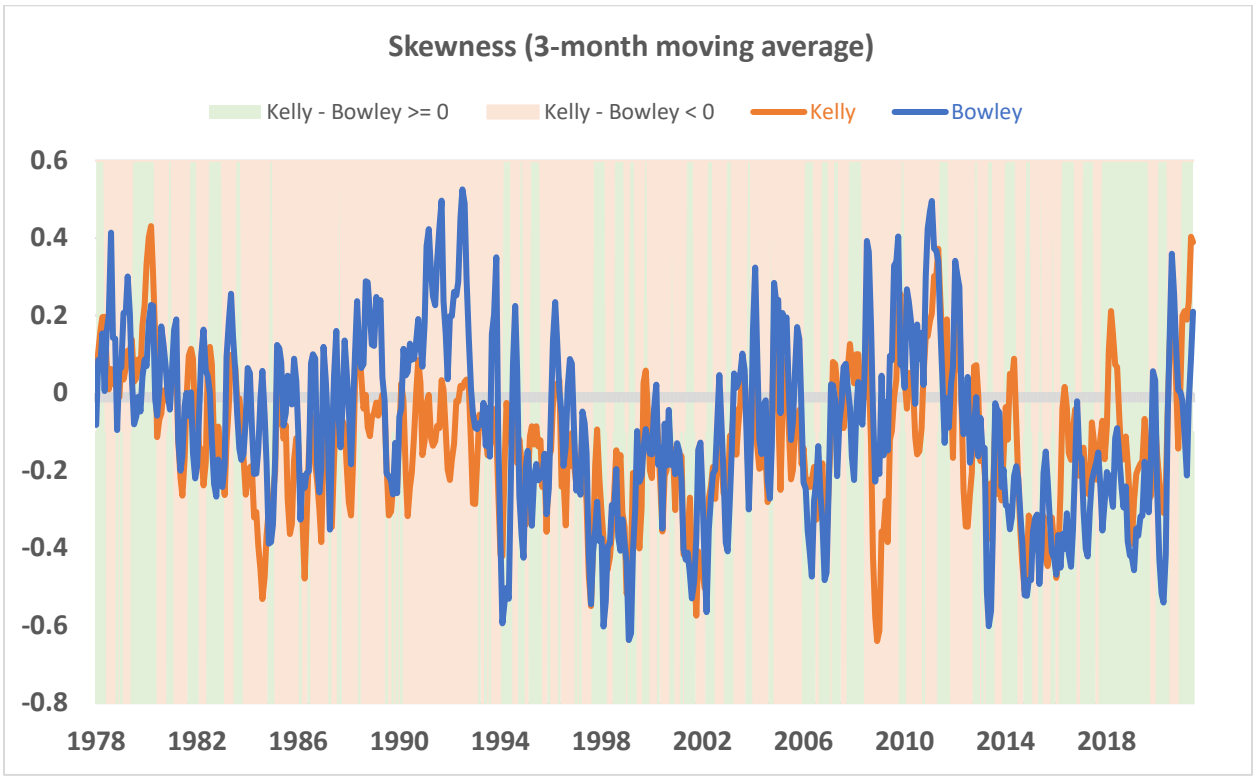


Figure A2: Cross-sectional asymmetry (*Bowley*) in PCE inflation (month-to-month %)

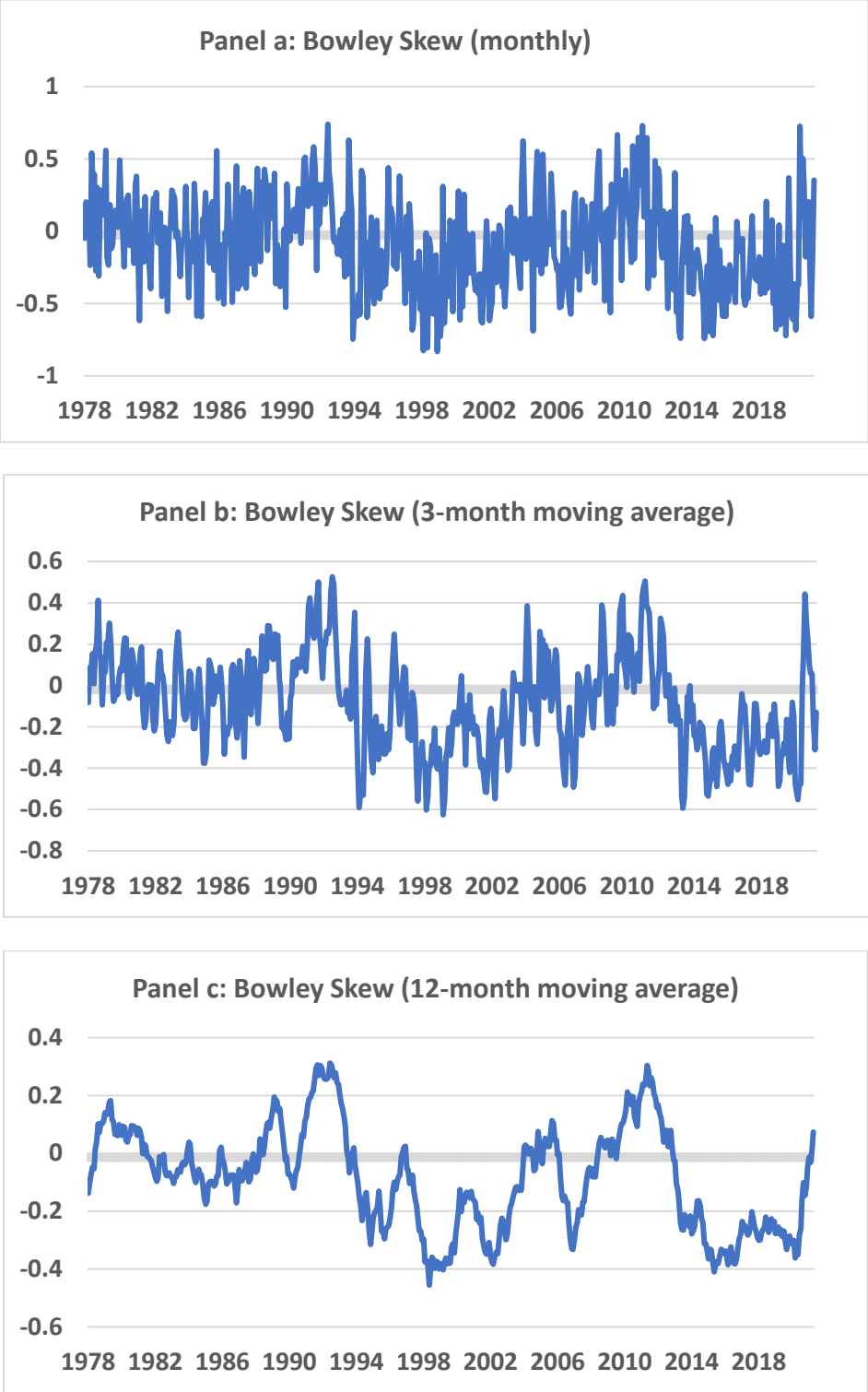
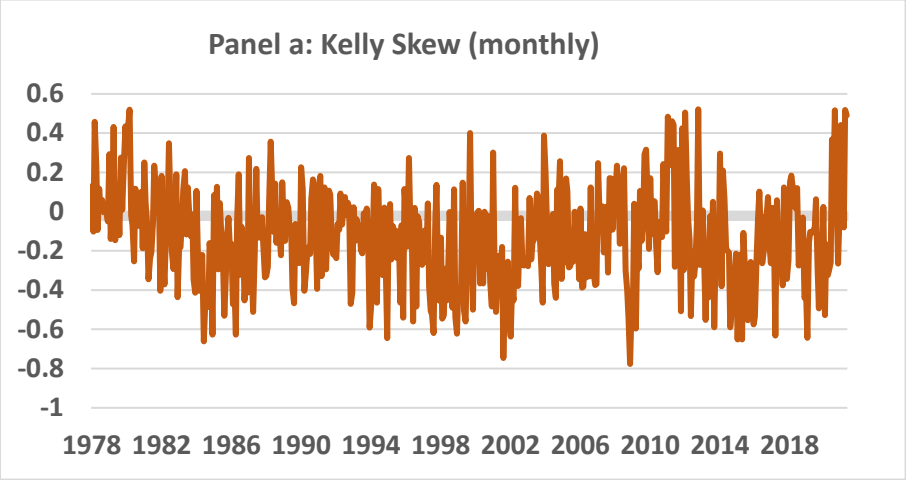
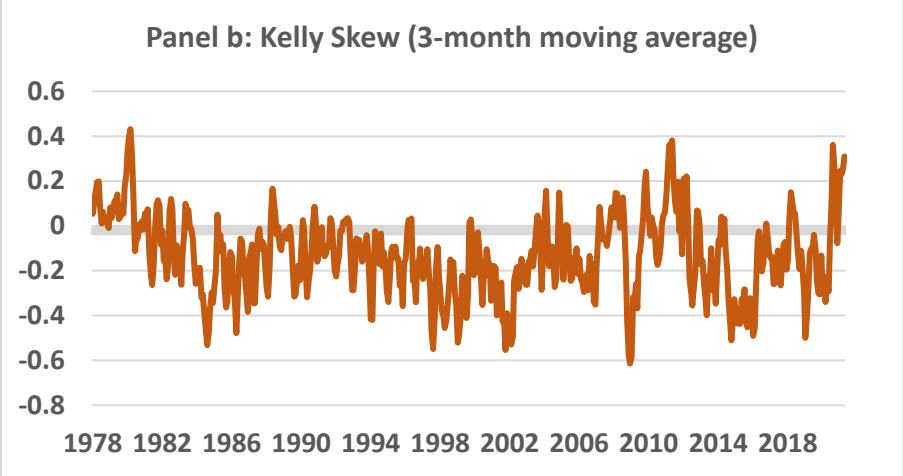
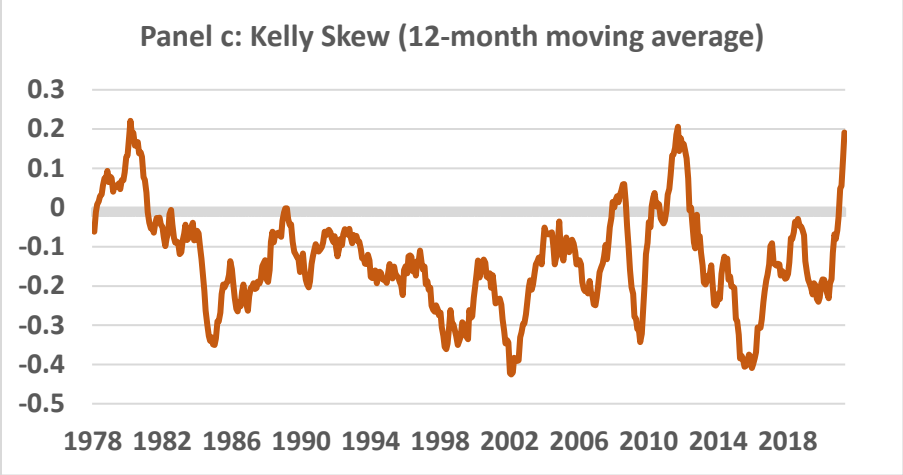


Figure A3: Cross-sectional asymmetry (Kelly Skew) in PCE inflation (month-to-month %)







A4. Density Forecasting Results

Table A3 reports the density forecast evaluation results. The first row in each panel reports the log-score of the density forecasts from the AR(3) inflation gap model. The higher the log-score, the more accurate the density forecast. All other rows report relative log-scores (i.e., log-score of the model being compared minus the log-score of the AR(3) inflation gap model). Negative entries indicate that the univariate inflation in the gap model is more accurate on average than the model being compared. Because most entries in the table are positive (except for one month ahead) and those for horizons 18 months ahead and beyond are statistically significant, it indicates that the addition of the trimmed-mean measures contributes to the increased accuracy of the headline PCE inflation density forecasts. In contrast, adding the skewness measures helps slightly at select forecast horizons. These results are consistent with the point forecast evaluation results.

Table A3: PCE inflation out-of-sample density forecasting comparison
[Skew constructed based on month-over-month inflation rates]

Full sample (January 1994 – June 2021)

	h=1M	h=6M	h=1Y	h=18M	h=2Y	h=30M	h=3Y
AR(3)-PCE LPS	-0.096	-1.302	-1.600	-1.516	-1.526	-1.543	-1.545
Relative LPS							
BVAR: PCE + Skew (K)	-0.013	0.027*	0.009	0.010*	0.019*	0.025*	0.027*
BVAR: PCE + Median	-0.019*	0.004	0.073*	0.072*	0.069*	0.065*	0.074*
BVAR: PCE + Median + Skew (K)	-0.001	0.057	0.070	0.068*	0.069*	0.065*	0.075*
BVAR: PCE + Trim	-0.017	0.004	0.055	0.051	0.054	0.061*	0.071*
BVAR: PCE + Trim + Skew (K)	0.000	0.052	0.060	0.047	0.057	0.063*	0.074*
BVAR: PCE + Core	-0.017	0.003	0.008	0.011*	0.017*	0.025*	0.027*
BVAR: PCE + Core + Skew (K)	-0.011	0.026	0.007	0.009	0.018*	0.025*	0.028*

Pre-financial crisis sample (January 1994 – December 2007)

	h=1M	h=6M	h=1Y	h=18M	h=2Y	h=30M	h=3Y
AR(3)-PCE LPS	-0.010	-0.875	-1.235	-1.335	-1.426	-1.471	-1.494
Relative LPS							
BVAR: PCE + Skew (K)	-0.015	0.001	0.003	0.011*	0.018*	0.023*	0.028*
BVAR: PCE + Median	-0.017	-0.011	0.059	0.086*	0.099*	0.097*	0.099*
BVAR: PCE + Median + Skew (K)	-0.010	-0.005	0.057	0.081*	0.095*	0.094*	0.095*
BVAR: PCE + Trim	-0.015	-0.015	0.038	0.062	0.084*	0.095*	0.104*
BVAR: PCE + Trim + Skew (K)	-0.009	-0.005	0.035	0.057	0.082*	0.092*	0.102*
BVAR: PCE + Core	-0.010	-0.008	-0.010	-0.003	0.004	0.011*	0.017*
BVAR: PCE + Core + Skew (K)	-0.009	-0.012	-0.014	-0.007	0.002	0.010	0.016*

Financial crisis and onward sample (January 2008 – June 2021)

	h=1M	h=6M	h=1Y	h=18M	h=2Y	h=30M	h=3Y
AR(3)-PCE LPS	-0.183	-1.748	-1.948	-1.565	-1.445	-1.467	-1.429
Relative LPS							
BVAR: PCE + Skew (K)	-0.008	0.075*	0.039	0.017*	0.028*	0.027*	0.028*
BVAR: PCE + Median	-0.024	0.041	0.073	0.050	0.105*	0.116*	0.097*
BVAR: PCE + Median + Skew (K)	0.009	0.130	0.076	0.050	0.111*	0.119*	0.104*
BVAR: PCE + Trim	-0.026	0.021	0.071	0.049	0.118*	0.133*	0.100*
BVAR: PCE + Trim + Skew (K)	-0.003	0.110	0.070	0.048	0.124*	0.137*	0.105*
BVAR: PCE + Core	-0.023*	0.015	0.035*	0.034*	0.033*	0.035*	0.035*
BVAR: PCE + Core + Skew (K)	-0.013	0.064*	0.035*	0.031*	0.035*	0.034*	0.035*

Notes: The numbers reported in the first row of each panel are the logarithmic predictive score (LPS) from the univariate AR PCE inflation in gaps (3-lag specification), while the rows below it are relative logarithmic predictive scores (relative to LPS from the AR(3) PCE inflation in gaps). Thus, a relative LPS that is negative indicates that the univariate inflation in gaps model is more accurate on average than the model being compared. Similarly, the positive value of relative LPS indicates the model being compared is more accurate on average. The forecast performance is based on an expanding window of estimation spanning the period January 1994 through June 2021 (full sample), and January 1994 through December 2007 (pre-financial crisis sample). * indicates statistical significance up to the 10% level and is based on the LR test of Amisano and Giacomini (2007).

A5. Forecasting Results Based on Bowley Skew

Table A4: PCE inflation out-of-sample point forecasting comparison
[Skew measures constructed based on month-over-month inflation rates]

Full sample (January 1994 – June 2021)

	h=1M	h=6M	h=1Y	h=18M	h=2Y	h=30M	h=3Y
AR(3)-PCE RMSE	0.265	0.858	1.126	1.064	1.075	1.077	1.044
Relative MSE							
BVAR: PCE + Skew (B)	1.043*	0.988	0.988	0.975	0.975	0.964	0.968
BVAR: PCE + Median	1.046*	0.991	0.893	0.882*	0.879*	0.898*	0.887*
BVAR: PCE + Median + Skew (B)	1.039*	0.948	0.881	0.878*	0.883*	0.900*	0.884*
BVAR: PCE + Trim	1.045*	0.997	0.891	0.918	0.913	0.916	0.913*
BVAR: PCE + Trim + Skew (B)	1.040*	0.964	0.884	0.915	0.913	0.911	0.907*
BVAR: PCE + Core	1.045*	1.010	1.008	0.997	0.980	0.967*	0.973
BVAR: PCE + Core + Skew (B)	1.050	1.009	1.015	0.996	0.986	0.963*	0.964
BVAR: PCE + UR	1.109*	1.181	1.320*	1.485*	1.628*	1.634*	1.612*

Pre-financial crisis sample (January 1994 – December 2007)

	h=1M	h=6M	h=1Y	h=18M	h=2Y	h=30M	h=3Y
AR(3)-PCE RMSE	0.245	0.553	0.806	0.870	0.941	0.955	0.930
Relative MSE							
BVAR: PCE + Skew (B)	1.006	0.988	1.008	1.001	0.994	0.995	1.008
BVAR: PCE + Median	1.024*	1.053	0.883	0.815	0.787*	0.795*	0.796*
BVAR: PCE + Median + Skew (B)	0.998	0.987	0.887	0.817	0.787*	0.794*	0.796*
BVAR: PCE + Trim	1.019	1.076	0.951	0.910	0.860	0.838*	0.814*
BVAR: PCE + Trim + Skew (B)	0.994	1.008	0.948	0.914	0.862	0.839*	0.815*
BVAR: PCE + Core	1.005	1.030	1.031	1.018	1.004	0.997	1.006
BVAR: PCE + Core + Skew (B)	1.006	1.022	1.050*	1.036	1.019	1.008	1.015
BVAR: PCE + UR	1.016	1.220	1.375	1.602*	1.648*	1.769*	1.979*

Financial crisis and onward sample (January 2008 – June 2021)

	h=1M	h=6M	h=1Y	h=18M	h=2Y	h=30M	h=3Y
AR(3)-PCE RMSE	0.285	1.087	1.359	1.097	0.975	0.972	0.833
Relative MSE							
BVAR: PCE + Skew (B)	1.071*	0.988	0.978	0.941*	0.939*	0.936	0.962
BVAR: PCE + Median	1.063*	0.976	0.906	0.932	0.793*	0.742*	0.790*
BVAR: PCE + Median + Skew (B)	1.070*	0.941	0.893	0.921	0.794*	0.745*	0.792*
BVAR: PCE + Trim	1.065*	0.980	0.883	0.933	0.774*	0.709*	0.808
BVAR: PCE + Trim + Skew (B)	1.075*	0.958	0.875	0.919	0.770*	0.708*	0.808*
BVAR: PCE + Core	1.076*	1.004	0.997	0.974*	0.946*	0.942*	0.954*
BVAR: PCE + Core + Skew (B)	1.084*	1.005	0.998	0.949*	0.941*	0.938*	0.954*
BVAR: PCE + UR	1.180*	1.179	1.347*	1.603	1.913	1.807	1.894

Notes: The numbers reported in the first row of each panel are the root mean squared error (RMSE) from the univariate AR PCE inflation in gaps (3-lag specification), while the rows below it are ratios that report relative MSEs (relative to the MSE from the AR(3) PCE inflation in gaps). Thus, a ratio of more than 1 indicates that the univariate inflation in gaps model is more accurate on average than the model being compared. The forecast performance is based on an expanding window of estimation spanning the period January 1994 through June 2021 (full sample), and January 1994 through December 2007 (pre-financial crisis sample). * indicates statistical significance up to the 10% level and is based on the Diebold-Mariano West test.

Table A5: PCE inflation out-of-sample density forecasting comparison

Full sample (January 1994 – June 2021):
[Skew measures constructed based on month-over-month inflation rates]
[Results using Bowley Skew]

	h=1M	h=6M	h=1Y	h=18M	h=2Y	h=30M	h=3Y
AR(3)-PCE LPS	-0.096	-1.302	-1.600	-1.516	-1.526	-1.543	-1.545
Relative LPS							
BVAR: PCE + Skew (B)	-0.021	0.007	0.010	0.012*	0.014*	0.023*	0.024*
BVAR: PCE + Median	-0.019*	0.004	0.073*	0.072*	0.069*	0.065*	0.074*
BVAR: PCE + Median + Skew (B)	-0.017	0.029	0.077*	0.072*	0.067*	0.062*	0.071*
BVAR: PCE + Trim	-0.017	0.004	0.055	0.051	0.054	0.061*	0.071*
BVAR: PCE + Trim + Skew (B)	-0.015	0.019	0.064	0.051	0.054	0.061*	0.070*
BVAR: PCE + Core	-0.017	0.003	0.008	0.011*	0.017*	0.025*	0.027*
BVAR: PCE + Core + Skew (B)	-0.020	0.005	0.008	0.011	0.014*	0.022*	0.025*

A6. A Bias-Adjusted Alternative (Two-Step Algorithm)

An alternative approach to examining the efficacy of skewness in improving the accuracy of inflation forecasts is to create a bias-adjusted trimmed-mean measure – where the bias adjustment is informed using the skewness – and then evaluate this measure's predictive ability versus its non-bias-adjusted counterpart. In principle, this bias-adjusted measure will embed both direct and implicit information about the skewness. We perform this analysis as a robustness check.

This approach is implemented using a two-step algorithm. In the first step, an estimate for the bias, defined as the moving average of the gap between the trimmed-mean inflation measure and the headline inflation measure, is computed.

$$Gap_t^{TMeasurePCE} = \pi_t^{TMeasurePCE} - \pi_t^{HeadlinePCE} \qquad (3)$$

To compute the moving average of the gap, a 36-month window is adopted, which is commonly used in the literature for trend estimation (see Rich, Verbrugge, and Zaman, 2022, and Verbrugge, 2021, among others):

$$Bias_t^{TMeasurePCE} = \frac{1}{36}\sum_{s=t}^{t-36} Gap_s^{TMeasurePCE} \qquad (4)$$

In the second step, the bias computed in the previous step is then regressed on the skewness to compute the bias, which is then applied to the trimmed-mean measure to construct the (bias) adjusted trimmed-mean inflation measure.

$$Bias_t^{TMeasurePCE} = \theta + \lambda(skew_t) + \varepsilon_t \qquad (5)$$

where $skew$ refers to a 12-month moving average of skewness.

$$\pi_t^{TMeasurePCE,Bias-adjusted} = \pi_t^{TMeasurePCE} - \theta - \lambda(skew_t) \qquad (6)$$

The bias-adjusted trimmed-mean measure is added alongside the headline inflation measure to construct a bi-variate model, whose accuracy is then compared to that of the bi-variate model of headline inflation and the (unadjusted) trimmed-mean inflation measure. The comparison of the forecast accuracy of headline inflation between these two bi-variate models indicates the marginal value of skewness.

The two-step approach yields very similar inferences. Table A6 reports the results.

Table A6: PCE inflation out-of-sample point forecasting comparison
[Skew constructed based on month-over-month inflation rates]
[**Two-step algorithm**]

Full sample (January 1994 – June 2021)

	h=1M	h=6M	h=1Y	h=18M	h=2Y	h=30M	h=3Y
AR(3)-PCE RMSE	0.265	0.858	1.126	1.064	1.075	1.077	1.044
Relative MSE							
BVAR: PCE + Skew (K)	1.028	0.957*	0.988	0.976	0.959*	0.959*	0.967*
BVAR: PCE + Median	1.046*	0.991	0.893	0.882*	0.879*	0.898*	0.887*
BVAR: PCE + Median Adjusted	1.045*	0.972	0.881	0.918*	0.924*	0.928	0.924*
BVAR: PCE + Trim	1.045*	0.997	0.891	0.918	0.913	0.916	0.913*
BVAR: PCE + Trim Adjusted	1.044*	0.988	0.900	0.973	0.969	0.953	0.963*

Pre-financial crisis sample (January 1994 – December 2007)

	h=1M	h=6M	h=1Y	h=18M	h=2Y	h=30M	h=3Y
AR(3)-PCE RMSE	0.245	0.553	0.806	0.870	0.941	0.955	0.930
Relative MSE							
BVAR: PCE + Skew (K)	1.009	1.006	0.998	0.989	0.980	0.981	0.995
BVAR: PCE + Median	1.024*	1.053	0.883	0.815	0.787*	0.795*	0.796*
BVAR: PCE + Median Adjusted	1.018	0.979	0.847	0.812*	0.800*	0.807*	0.820*
BVAR: PCE + Trim	1.019	1.076	0.951	0.910	0.860	0.838*	0.814*
BVAR: PCE + Trim Adjusted	1.007	0.970	0.895	0.895	0.878	0.868*	0.866*

Financial crisis and onward sample (January 2008 – June 2021)

	h=1M	h=6M	h=1Y	h=18M	h=2Y	h=30M	h=3Y
AR(3)-PCE RMSE	0.285	1.087	1.359	1.097	0.975	0.972	0.833
Relative MSE							
BVAR: PCE + Skew (K)	1.043*	0.943*	0.982	0.953	0.908*	0.924*	0.947*
BVAR: PCE + Median	1.063*	0.976	0.906	0.932	0.793*	0.742*	0.790*
BVAR: PCE + Median Adjusted	1.052*	0.943	0.893	0.928	0.726*	0.695*	0.757*
BVAR: PCE + Trim	1.065*	0.980	0.883	0.933	0.774*	0.709*	0.808
BVAR: PCE + Trim Adjusted	1.054*	0.947	0.875	0.930	0.718*	0.691*	0.805*

Notes: The numbers reported in the first row of each panel are the root mean squared error (RMSE) from the univariate AR PCE inflation in gaps (3-lag specification), while the rows below it are ratios that report relative MSEs (relative to the MSE from the AR(3) PCE inflation in gaps). Thus, a ratio of more than 1 indicates that the univariate inflation in gaps model is more accurate on average than the model being compared. The forecast performance is based on an expanding window of estimation spanning the period January 1994 through June 2021 (full sample), and January 1994 through December 2007 (pre-financial crisis sample). * indicates statistical significance up to the 10% level and is based on the Diebold-Mariano West test

A7. Skewness and Other Plots by Goods and Services PCE Breakdown

Figure A4 plots the (Kelly) skewness measures computed separately by services and goods categories. Also plotted is the aggregate skewness, which is computed using all the PCE components.[3] The skewness measures plotted correspond to the 12-month moving average of the monthly skewness estimates. A few observations immediately stand out. First, it is common to observe positive skewness in services inflation, whereas it is rare in goods inflation. Second, overall negative skewness is driven both by the negative skewness in the goods inflation over most of the sample and by the fact that goods inflation has been negative. Negative goods inflation influences overall skewness because, given typically positive overall inflation, components within the goods category have generally fallen in the left tail of the price change distribution and thus contribute to the negative skew in aggregate PCE inflation. Third, in recent months, both goods and services inflation have been experiencing positive skewness, contributing to the positive skewness in aggregate PCE inflation. Interestingly, this positive skewness in goods inflation coincides with sharp increases in goods inflation, driven by the dramatic shift in consumption away from services and toward goods in conjunction with supply-chain pressures. This represents a notable shift from the negative goods inflation observed over the past three decades.

[3] Because aggregate, goods, and services skewness measures are constructed separately, the aggregate skewness is not the sum of the skewness in goods PCE and the skewness in services PCE.

Figure A4: (Kelly) Skew by goods and services

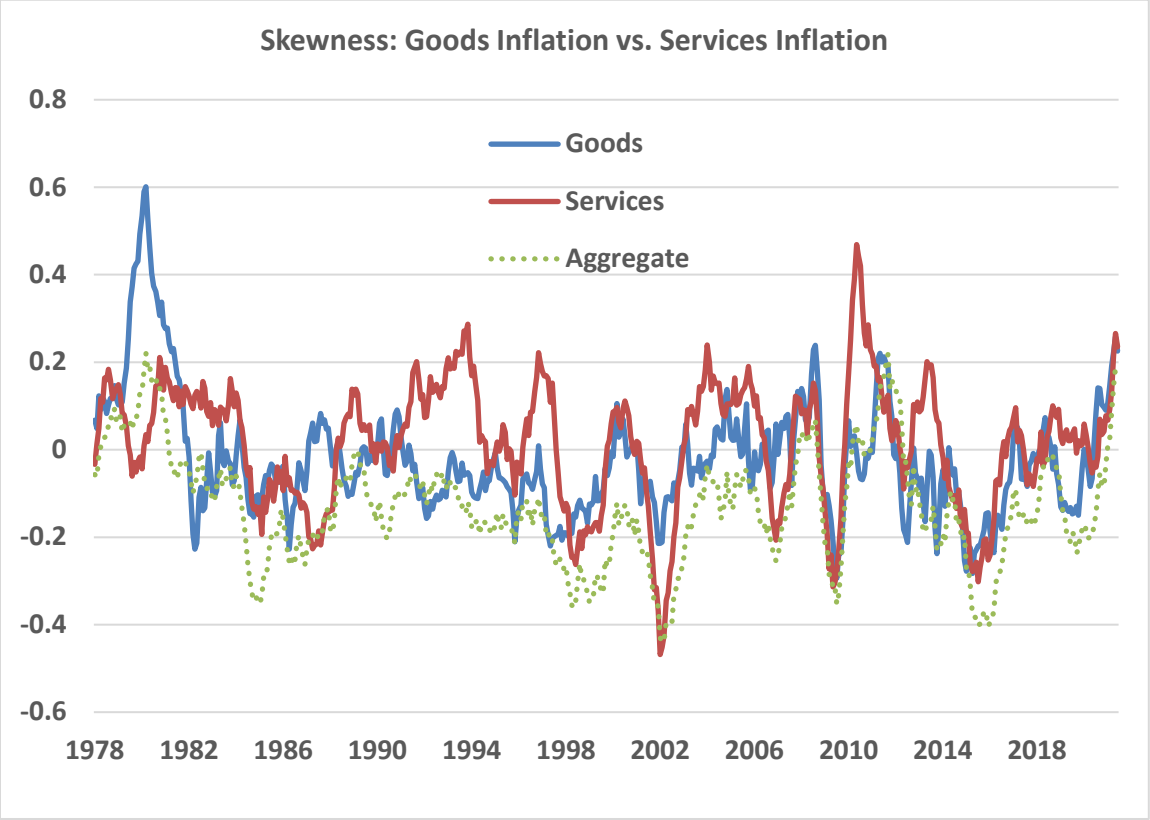
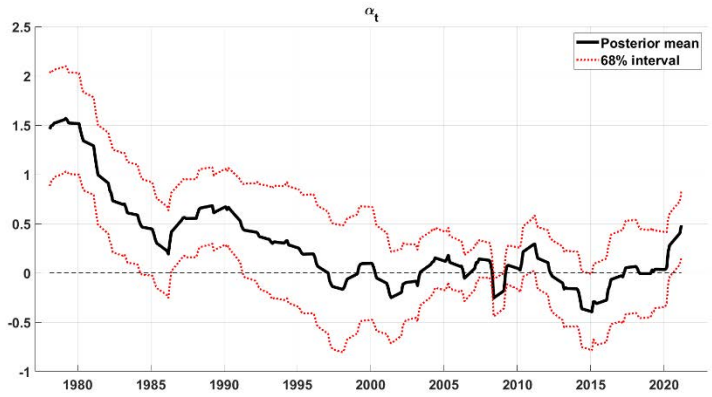
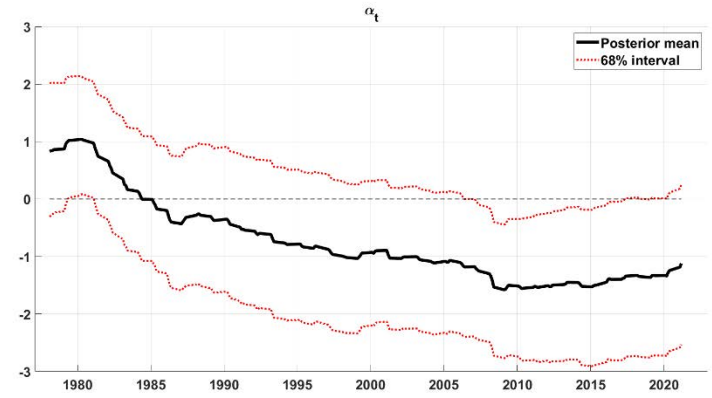


Figure A5: Estimated relationship between inflation and inflation volatility/uncertainty

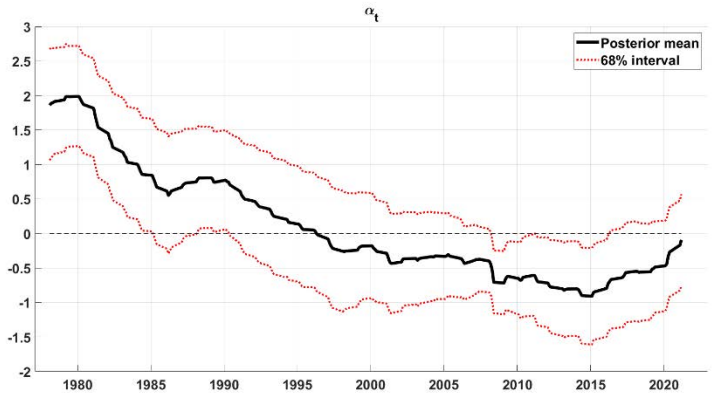
(a) Goods PCE inflation: Time-varying parameter alpha



(b) Services PCE inflation: Time-varying parameter alpha



(c) Headline PCE inflation: Time-varying parameter alpha



A8. Forecasting Performance: July 2021 through December 2022

Since the conclusion of our paper, more data have become available (July 2021 through December 2022) that permit a forecasting examination of our models and the usefulness of robust measures over this extended period. The period from July 2021 onward is unique, as this period is associated with strongly rising inflation until mid-2022, and slow moderation since then. A natural question is: how well did the robust measures do in predicting the developments in inflation? Before we provide a preview of our predictive examination, we provide a little context about inflation developments. Sometime in the middle of 2021, a few components of the PCE basket experienced outsized increases, such as the prices of used and new cars (driven by a mix of supply and demand factors), that pushed aggregate inflation higher. As the months rolled forward, higher inflation spread to several other components, resulting in a broad-based surge in inflation. The surge continued through the middle of 2022, but then as supply chain disruptions improved and monetary policy tightened to bring demand in better alignment with supply, inflation began to moderate. However, inflation remains elevated as of December 2022.

Our forecast examination reveals that in mid-2021, as the surge in inflation picked up, our models thought the surge would be transitory, as did most forecasters at the time. It took some time for the models to realize that the surge in inflation was persistent. Figure A6 below shows the evolution of the forecast for December 2022, made over the period July 2021 (using data through June 2021) through December 2022 (based on data through November 2022) using an AR gap with skewness model specification. As can be seen from the figure, inflation data through June 2021 indicated inflationary pressures were limited to just a few components; hence, the model(s) projected inflation would moderate to 2.4 percent by December 2022. The actual data came in at 4.9 percent. As we rolled forward, the accuracy of the model shown here (and of other models not shown) improved. In early 2022, the model's projection (based on estimating with data through March 2022) tracked actual inflation remarkably well. Figure A7 shows the evolution of the forecast paths over this same period from the AR gap with skewness model. An interesting and insightful finding is that model specifications with skewness are found to be more accurate than models without it over this period. The bi-variate model specifications entertaining trimmed-mean estimators, median PCE, or trimmed-mean PCE performed comparably to the

univariate AR gap model. This reflects the fact that over most of this period, i.e., when inflation became broad-based sometime in September 2021, headline, core, median, and trimmed-mean PCE behaved similarly, and so provided very similar inference about the future path of headline PCE inflation. However, model specifications with skewness indicated an inflation path that was higher than model specifications without skewness and proved to be more accurate. Table A7 reports the relative forecast accuracy comparing model specifications with and without the skewness measure. As can be seen, bringing information from skewness is beneficial to improving accuracy, with a couple of exceptions. This is a very interesting result because prior to this recent period – as indicated in the main paper – the marginal contribution of the skewness measure(s) above and beyond that of trimmed-mean indicators to forecast accuracy was minimal. But in the recent period of elevated inflation, its marginal role in improving accuracy is nontrivial.

These results, combined with the results documented in the main paper, reinforce our recommendation in the paper: "…one is better off incorporating information from trimmed-mean estimators (and Kelly skewness) in constructing forecasts of PCE inflation using popular time-series models."

Figure A6: Forecast evolution for forecast month December 2022

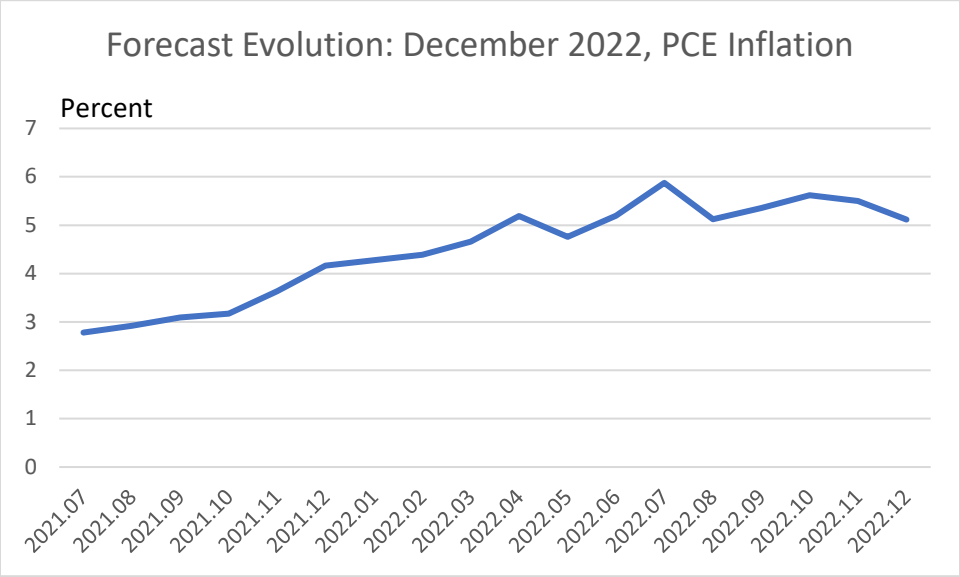


Figure A7: Forecast evolution across pseudo-real time vintages

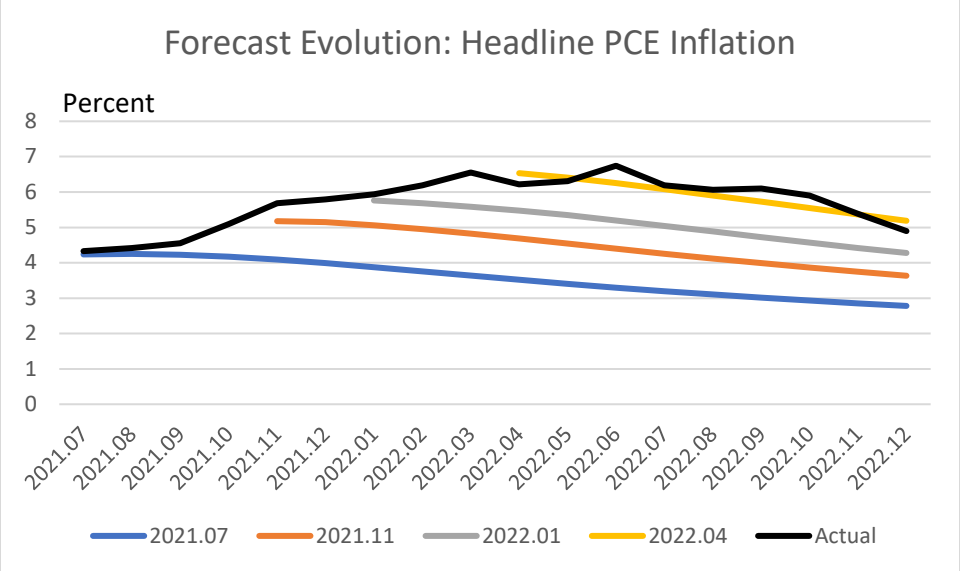


Table A7: Relative forecast accuracy, July 2021 to December 2022

Relative MSE	h=1M	h=3M	h=6M	h=9M	h=12M
$\frac{MSE\ of\ AR(3)\ PCE + Skew}{MSE\ of\ AR\ (3)\ PCE}$	0.875	0.734	0.805	0.903	0.964
$\frac{MSE\ of\ BVAR: PCE + Median\ +\ Skew}{MSE\ of\ BVAR: PCE + Median}$	0.832	0.765	0.876	1.000	1.077
$\frac{MSE\ of\ BVAR: PCE + Trimm\ +\ Skew}{MSE\ of\ BVAR: PCE + Trimm}$	0.854	0.781	0.878	0.994	1.064
$\frac{MSE\ of\ BVAR: PCE + Core\ +\ Skew}{MSE\ of\ BVAR: PCE + Core}$	0.853	0.678	0.716	0.794	0.853

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