Appendices to: The Intermittent Phillips Curve: Finding a Stable (But Persistence-Dependent) Phillips Curve Model Specification

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Working Paper No. 19-09R2 (appendices)

February 2023

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APPENDIX A: OTHER INFLATION INDICATORS, AND CBO GAP

In Table A1 we present the results using the CBO $u_t^*$ estimate rather than that of Tasci (2018) (in column 2), and then using several other inflation indicators. We provide the trimmed PCE results from Table 1 in column 1 for comparison. The final two rows in the table refer to GR and DM forecast comparisons against the baseline CBO model for the same dependent variable.

<table>
<thead>
<tr>
<th>Table A.1. Other Inflation Indicators.</th>
<th>Trimmed PCE</th>
<th>Trimmed PCE with CBO gap</th>
<th>Median PCE</th>
<th>Core PCE</th>
<th>Median CPI</th>
<th>Trimmed CPI</th>
<th>Core CPI</th>
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<tr>
<td>$\lambda_1^+$</td>
<td>0.03</td>
<td>0.01</td>
<td>0.04</td>
<td>0.03</td>
<td>0.05</td>
<td>0.03</td>
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<td>t-stat</td>
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<td>0.33</td>
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<td>0.57</td>
<td>1.22</td>
<td>0.51</td>
<td>0.06</td>
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<td>$\lambda_1^-$</td>
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<td>-0.35</td>
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<td>$\lambda_2^+$</td>
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<td>0.19</td>
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<td>0.07</td>
<td>1.65</td>
<td>5.76</td>
<td>1.16</td>
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<td>H0: $\lambda_1^+ = \lambda_1^-$</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.04</td>
</tr>
<tr>
<td>H0: $\lambda_3^+ = \lambda_3^-$</td>
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<td>0.04</td>
<td>0.27</td>
<td>0.32</td>
<td>0.01</td>
<td>0.01</td>
<td>0.16</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>Adjusted R-squared</td>
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<td>0.72</td>
<td>0.78</td>
<td>0.42</td>
<td>0.76</td>
<td>0.56</td>
<td>0.52</td>
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<td>&gt;0.10</td>
<td>&lt;0.05</td>
<td>&gt;0.10</td>
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<td>&lt;0.05</td>
<td>&lt;0.10</td>
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<tr>
<td>DM Test p-value</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
<td>0.11</td>
</tr>
</tbody>
</table>

1 With median CPI, the Equation (3) forecast outperforms the baseline CBO forecast until mid-2018, when the baseline model briefly dominates it at the 10 percent level. Interestingly, over this same period, the Equation (3) forecast with trimmed mean – which uniformly dominates the linear CBO forecast – actually achieves its best relative performance versus the baseline CBO forecast over the same period.
In short, our results do not hinge on using the trimmed-mean PCE as the inflation indicator and, broadly speaking, are robust to using different inflation indicators. The exception is core PCE inflation, a topic we turn to next.
APPENDIX B: DEFICIENCIES IN CORE PCE

We acknowledge that the core PCE tests do not reject symmetry in the persistent and moderately persistent components. Like Ball and Mazumder (2019), we suggest that this “puzzling” result stems from deficiencies of core PCE inflation as a measure of trend inflation. Theory predicts two major deficiencies of less-food-and-energy (“core”) inflation indexes, and both were exhibited in the post-1985 period. First, because the core PCE price index simply excludes items from the basket, core PCE inflation can be subject to bias over prolonged periods. And as Carroll and Verbrugge (2019) indicate, this bias has also been highly unstable over time. For example, between 1995 and 2007, core PCE inflation was downwardly biased by 0.25 percentage points, while it was upwardly biased by 0.3 percentage points between 1980 and 1985. This fact alone raises some doubts about its ability to truly match trend inflation. Second, despite their moniker, core inflation indexes are subject to large idiosyncratic transitory shocks that distort the estimate of trend inflation. (Indeed, the standard deviation of core inflation measures is so large that they are almost always examined in time-averaged form.) Large shocks are not confined to food and energy components. This sensitivity to transitory noise is significant in the present study: transitory shocks can occur at any time, but in the context of analyses that distinguish between phases of the business cycle, these shocks will be especially detrimental if they are correlated with the phase within the sample. One aspect of core PCE inflation is noteworthy: as discussed below, core PCE inflation is sensitive to the movements of prices that are not market-determined, and such movements may well be systematically related to the business cycle. In terms of its ability to reliably reflect trend inflation, as discussed above, when core PCE inflation departs from trimmed-mean PCE inflation, it is core PCE inflation that adjusts to eliminate the gap.

There were only three NBER recessions post-1985. This implies that the moderately persistent component experienced only three nonzero episodes after 1985: starting in 1991, starting in 2001, and starting in mid-2007. During two of these recoveries, core PCE inflation experienced dynamics that were at odds with limited-influence trend inflation indicators such as
the trimmed-mean PCE or the median CPI, and even with the other prominent “less food and energy” series, the core CPI. During the aftermath of the 2001 recession, year-over-year core PCE inflation displayed a prominent rebound from early 2002 to early 2003, including one month with 2.4 percent inflation, a reading not seen since the early 1990s.\(^2\) Other limited-influence trend inflation indicators displayed an essentially monotonic decline from 2001Q3 to 2003Q4. During the Great Recession, while other (year-over-year) trend inflation measures displayed an essentially monotonic decline from 2008Q4 through 2010, core PCE inflation again exhibited a strong rebound in the middle of this episode: starting from below 1 percent in September 2009, it rapidly rose to 1.7 percent during the first few months of 2010, then fell gradually back down to end below 1 percent in 2010Q4.

Conversely, during both of these episodes, inflation in the *market-based* core PCE\(^3\) displayed dynamics that were *similar* to other limited-influence trend inflation indicators; see Figure 4. This indicates that core PCE’s unusual dynamics during both of these episodes stemmed from the behavior of prices that were not market-determined.\(^4\) In short, core PCE inflation was evidently subject to countervailing idiosyncratic influences during the aftermath of both the 2001 recession and the Great Recession that all but masked trend inflation movements

\(^{2}\) This may have been due to insurance payments related to 9/11 that caused m/m core PCE inflation to run negative in the fall, which showed up in y/y core PCE inflation a year later.

\(^{3}\) The PCE market-based price index is based primarily on observed market transactions for which there are corresponding price measures. It includes owners’ equivalent rent, but excludes most imputed expenditures, such as “financial services furnished without payment,” most insurance purchases, gambling, margins on used light motor vehicles, and expenditures by US residents working and traveling abroad.

\(^{4}\) See also Peach et al. (2013), who display a decomposition into goods and services. The anomalous movements during the Great Recession were almost entirely driven by imputed financial services price movements.
during critical periods. The anomalous behavior of core PCE inflation during these crucial episodes surely calls into serious question its usefulness as a trend inflation estimator.  

![Figure A.1: Four Inflation Measures](image)

We plot four trend inflation indicators in Figure A.1, from 1988 onward (when XFE market-based PCE became available). Only one of these, “core” PCE, displays a notable inflation rebound in 2010. This came from the unusual behavior of nonmarket goods – in particular, imputed financial services. The susceptibility of core PCE inflation to such movements reduces its usefulness as a trend inflation measure.

Stock and Watson (2020) also eschew core inflation, and stress the importance of using appropriate inflation measures. Similarly, both the Reserve Bank of New Zealand and the Bank of Canada now shun the use of “core” (exclusion-based) inflation measures as measures of trend inflation. See Carroll and Verbrugge (2019) and Verbrugge and Zaman (2022) for additional evidence regarding the superior forecasting ability of the trimmed mean PCE and median PCE over core PCE in forecasting headline PCE movements. Further, when trimmed mean PCE diverges from core PCE, it is the latter that moves to eliminate the gap. (This fact was first noted by Boston Fed President Rosengren 2019.) Note that limited-influence trend inflation estimators,

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5 For more details, see Verbrugge (2022), which extends the assessment of Luciani and Trezzi (2019) and discusses the findings of Crone et al. (2013). And for more on the findings of the latter paper, see Verbrugge and Zaman (2021).
in practice, tend to place more weight on services, which have a more stable relationship to slack (see, e.g., Tallman and Zaman 2017, Zaman 2019, and Stock and Watson 2020). For a deeper look at deficiencies in core PCE, see Verbrugge (2022).
APPENDIX C: COMPARISON TO STOCK-WATSON MODELS:
RECESSION GAP MODEL AND UCSV MODEL

For the post-1985 period, Figure A.2 plots the 12-month trimmed PCE inflation rate along with the (monthly) SW recession gap term and the positive part of the moderately persistent unemployment rate fluctuations.\(^6\)

Figure A.2: Stock-Watson Recession Gap and Rescaled Moderately Persistent Gap vs. Inflation

In this figure, the latter series has been scaled by multiplying it by 6 so as to render its peak magnitude comparable to that of the recession gap during the middle two recession episodes. Regarding ocular econometrics, the moderately persistent “bust” gap has an edge in timing, in that the peak inflation deceleration is relatively close to the peak of the bust gap (but well prior to the peak of the recession gap) and ends roughly when the bust gap vanishes (while the recession gap stays significantly positive for much longer). However, this is merely

\(^6\) It is worth mentioning that the unemployment gap resulting from the year-over-year filter in Stock and Watson (2020) bears a notable resemblance to the moderately persistent component in our partitioning. This is not surprising, given the frequency gain of the year-over-year filter.
suggestive. We now provide out-of-sample forecast evidence that our specification is superior: at least over the post-1985 period, the bust gap better captures the impact of recessions on inflation dynamics.

In Figure A.3, panel (A), we display the Giacomini-Rossi forecast comparison results from our Equation (3) model versus the Stock and Watson recession gap model. The Equation (3) model outperforms the recession gap model over the entire period – the %FOOS statistic is generally well above 0 – with forecast gains that are statistically significant at the 5 percent level in mid-2010, and from mid-2011 to mid-2012, and gains that are statistically significant at the 10 percent level for most of 2017. The Diebold-Mariano test, with a p-value of 0.01, indicates that the gain from our model is quite compelling when considering the sample as a whole.

![Graph](image.png)

**Figure A.3: GR Fluctuation Tests, Equation (3) versus: (A), Stock-Watson Gap Model; and (B), UCSV Model**

In Figure A.3, panel (B), we display the Giacomini-Rossi forecast comparison results from our Equation (3) model versus the Stock-Watson (2007) UCSV model, specified in terms of 12-month changes in trimmed-mean PCE. The UCSV model is often taken to be a benchmark forecasting model. Our Equation (3) forecasts are better on average over the entire

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7 Our UCSV forecast ends in 2018:4. Accordingly we use a somewhat shorter window of 4 years, so the comparison starts in 2009:1.
comparison period (though clearly not superior, from mid-2014 through 2016) and these gains are statistically significant at the 5 percent level from mid-2010 to mid-2012. For this OOS forecast comparison as well, the Diebold-Mariano rejection p-value is 0.01, indicating that the gain from our model is quite compelling when considering the sample as a whole.
APPENDIX D: RELATIONSHIP TO SOME OTHER FINDINGS IN THE LITERATURE

Our results reinterpret a finding in Coibion and Gorodnichenko (2015). These authors constructed the “NAIRU” implied by their estimated model that would be necessary to explain the “missing disinflation” during the Great Recession. While differing in details, the gap implied by the Coibion-Gorodnichenko NAIRU has broad similarities to our bust gap: it starts opening up shortly after the unemployment rate started rising rapidly in 2008 but was virtually back to zero by mid-2009. (We emphasize that inflation data are not used in constructing our gap measures.) These authors concluded that these dynamics were implausible for a NAIRU. However, our findings indicate that the implausibility of their estimate stemmed not from the possibility that a NAIRU might have dynamics that were at such great odds with conventional estimates, but rather with the notion that a NAIRU is just another way to describe the natural rate of unemployment. As we have noted above, there is no reason that these concepts should coincide.8 Implicitly, Coibion and Gorodnichenko (2015) provide evidence supportive of our findings.

A mismeasured gap will likely lead to the conclusion that forecasting performance is episodic (for example, Stock and Watson 2009) or that there is time variation in the inflation process. We discuss the literature focusing on time variation via unobserved components below. Relevant to this literature, however, Stock and Watson (2020) note some disadvantages to specifying an explicit (semi-structural) model for trends in the data, since the model could be misspecified. Instead, these authors explicitly eliminate trends (low-frequency variation), via their time-series filters, so as to focus solely on the business-cycle-frequency relationship. As noted above, their findings are consistent with our results: by restricting attention solely to the relationship at business-cycle frequencies, these authors find a strong, and more stable, Phillips curve relationship (see also Lee and Nelson 2007). The effectiveness of their approach is diminished by its a priori restriction to a consideration of the Phillips curve relationship at only

8 See further discussion in Tasci and Verbrugge (2014).
this particular range of frequencies, however, and their use of a two-sided bandpass filter (for most of their results) is subject to the criticisms discussed briefly above, and in more detail in Ashley and Verbrugge (2009) and Ashley et al. (2020).

Our findings also reconcile evidence in, for example, Laxton et al. (1995), Debelle and Laxton (1997), Filardo (1998), Laxton et al. (1999), Barnes and Olivei (2003), Huh and Jang (2007), Baghli et al. (2007), Stock and Watson (2009), Fuhrer and Olivei (2010), Peach, Rich, and Cororaton (2011), Peach et al. (2013), Babb and Detmeister (2017), and Murphy (2017) that the PC is either convex or “convex-concave” (see also Xu et al. 2015 and Forbes, Gagnon and Collins 2022). These studies, among others already noted above, find a steepening of the Phillips curve as slack becomes negative. Similarly, our findings are also consistent with regime-switching studies, such as Huh et al. (2009) or Donayre and Panovska (2016),9 that find three regimes in the wage Phillips curve. Our viewpoint, though, is that previous studies somewhat mischaracterize the reduced-form Phillips relationships, first because none (aside from SW) can well approximate the positive part of our moderately persistent component, and second because they typically estimate a fixed lower threshold for slack rather than allowing for a time-varying natural rate of unemployment. In sum, the form of nonlinearity we uncover is well-supported in the data and is consistent with economic theory (see Appendix H), yet is not cleanly captured by the standard sorts of nonlinearity that most models admit.

In addition to the recent studies (such as Angeletos et al. 2020 and Beaudry et al. 2020) demonstrating the usefulness of exploring relationships across the frequency domain,10 numerous recent studies find evidence for persistence-dependence in macroeconomic relationships. For example, Blundell et al. (2013) find that at the micro level, consumption responds to persistent movements in after-tax income; Arellano et al. (2018) find that persistent earned income shocks are harder to insure against; and Ciner (2015) finds that stock returns have

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9 See also Nalewaik (2016) for a rich regime-switching approach.

10 Beaudry et al. (2020) state: “Therefore, in order to evaluate business cycle properties, one needs to find a way to extract properties of the data that are unlikely to be contaminated by the lower-frequency forces that are not of direct interest.” Below, we discuss how standard approaches will typically lead one astray.
a persistence-dependent relationship to inflation movements. Cochrane (2018) and Ashley and Verbrugge (2015) find that the velocity of money has a persistence-dependent relationship to interest rates; Ashley et al. (2020) find that historical FOMC policy responses to inflation and unemployment are persistence-dependent, with persistent movements in the unemployment rate, and transitory movements in inflation, being ignored by policymakers. Caraiani and Gupta (2018) find that the Bank of England responds only to persistent movements in the real exchange rate; Ashley and Li (2014) find that state-level consumption has a persistence-dependent relationship to movements in both housing wealth and stock wealth; Yanfeng (2013) finds that Japanese industrial activity and inflation have a persistence-dependent relationship to oil prices. Tan and Ashley (1999) find that the relationship between consumption and income is frequency-dependent, as suggested by the permanent income hypothesis. Finally, Benati (2009) finds that inflation is mainly related to low-frequency movements in money; Reynard (2007) finds that the money-inflation relationship depends upon persistence-dependence in velocity; and Cochrane (1989) finds that interest rates respond negatively to transitory movements in money growth.

There is a large and growing literature using semi-structural unobserved-component (UC) models to study the Phillips curve, trend inflation, and the natural rate of unemployment (see, e.g., Kim and Nelson (1999), Lee and Nelson (2007), Kang et al. (2009), Kim et al. (2014), and Bradley et al. (2015)).

A somewhat more recent literature examines time-varying parameter (TVP) models, often in conjunction with UC and often including stochastic volatility; see Stella and Stock (2013), Chan, Koop and Potter (2016), Karlsson and Österholm (2018), and Vlekke, Koopman and Mellens (2021), who all locate evidence for time variation in the slope of the Phillips curve (see also Clark and McCracken 2006, Stock and Watson 2007, Ball and Mazumder 2011, and Luengo-Prado et al. 2018 for evidence on time variation, versus Stock and Watson 2020, who find stability at business-cycle frequencies). Zaman (2022) implicitly detrends inflation, but allows for a “persistent transitory component” as well, and relates the inflation gap to an output gap; this in some sense focuses attention on business-cycle frequencies.
Not all studies find time variation: Coibion and Gorodnichenko (2015) and Fu (2020) estimate many variants of a standard expectations-augmented Phillips curve, using several different measures of inflation expectations, and find that evidence for changes in the slope of the Phillips curve is mixed. Finally, Berger, Everaert and Vierke (2016) cannot reject stability in the Phillips curve slope; their point estimates do vary over time, but the standard error estimates are large. Other authors, such as Chan et al. (2016), Nakajima and West (2013), and Belmonte, Koop and Korobilis (2014), emphasize (and address) the potential for over-parameterization in time-varying parameter models; see also Stock and Watson (2020).

As noted in the main body, our contribution to this debate is that our respecification of the Phillips curve provides one explanation as to why we might observe variation in coefficient estimates over time, and we provide an “observed” decomposition of the unemployment gap that accords well with intuition and that we view as a simple and intuitively appealing way to resolve many Phillips curve puzzles.
APPENDIX E: RELATIONSHIP BETWEEN MODERATELY PERSISTENT AND TRANSIENT COMPONENTS, AND ESTIMATED IMPACT OF THREE NONZERO RELATIONSHIPS

Figure A.4: Moderately Persistent and Transient Components of Unemployment

Figure A.4 plots \( \left( gap_{\text{mod. persist}, t}^+, \text{Gap}_t^+ \right) \), here labelled Mod_Persist Gap+, and \( \left( gap_{\text{transient}, t}^+, \text{Gap}_t^+ \right) \). These series are positively correlated, especially at the beginning of a recession.

Figure A.5: Estimated Force on Inflation over the next 12 months from Components:

\[
\text{FORCE1} = \lambda^- \left( gap_{\text{persist}, t}^- \right), \quad \text{FORCE2} = \lambda^+_2 \left( gap_{\text{mod. persist}, t}^+ \right), \quad \text{and} \quad \text{FORCE3} = \lambda^+_3 \left( gap_{\text{transient}, t}^+ \right)
\]

Figure A.5 plots the estimated impact (over time) on subsequent 12-month inflation in the trimmed mean PCE from \( \left( gap_{\text{persist}, t}^- \right), \left( gap_{\text{mod. persist}, t}^+ \right), \) and \( \left( gap_{\text{transient}, t}^+ \right) \), based upon their values over the sample, and the estimated coefficients \( \lambda^- \), \( \lambda^+_2 \), and \( \lambda^+_3 \). The two chief Phillips curve relationships are associated with \( \left( gap_{\text{persist}, t}^- \right) \) and \( \left( gap_{\text{mod. persist}, t}^+ \right) \), although \( \left( gap_{\text{transient}, t}^+ \right) \) plays a supporting role during recessions.
Figure A.6: More Conditional Recursive Forecasts: Forecasts Based upon 2006:12 Estimated Coefficients, and Forecasts Based upon 2019:12 Estimated Coefficients, Using Only 2006:12 Inflation Data

Notice, comparing the solid red and red dotted lines, that the conditional forecasts of our persistence-dependent model are quite close to each other. In keeping with the results of our coefficient stability tests, the forecasts are similar regardless of whether we estimate coefficients in 2006 or 2019.

Conversely, conditional forecasts from a standard linear model are notably different if one estimates the Phillips curve in 2019:12 rather than 2006:12. This reflects the purported weakening of the Phillips curve; \( \hat{\lambda} \) equals -0.21 when estimated in 2006:12, but equals -0.08 when estimated in 2019:12. And even with coefficients estimated in 2019:12, the fit to the evolution of inflation is still rather poor. The fit is particularly poor at the beginning of the Great Recession – this is the downward speed puzzle highlighted by Clark (2014).
Notice, comparing the solid red and dotted red lines, that the conditional forecasts of our persistence-dependent model are very similar, regardless of whether we estimate coefficients in 2006 or 2019. Conversely, conditional forecasts from a standard linear model are notably different if one estimates the Phillips curve in 2019:12 rather than 2006:12. This reflects the purported weakening of the Phillips curve. With coefficients estimated in 2019:12, the fit to the evolution of inflation is still poor, with a far worse fit at the beginning of the Great Recession.

Figure A.7 below plots these deviations. The average Equation (3) forecast deviation is -0.06 (with the biggest deviation at -0.38), and the average Equation (1) forecast deviation is -0.40 (with the biggest deviation at -0.90).

Figure A.7: Deviations Between Forecasts Based upon 2006:12 Estimated Coefficients, and Forecasts Based upon 2019:12 Estimated Coefficients
APPENDIX G: COMPARISON TO WAGE PC OF MORRIS, RICH, AND TRACY

In the table below, we compare the wage Phillips curve parameter estimates of Morris, Rich and Tracy (2019) (MRT) to the parameter estimates derived from a similarly specified price Phillips curve. The dependent variable in MRT is a four-quarter growth in average wages term, constructed in MRT on the basis of CPS data, detrended using long-run SPF inflation expectations as in this paper. The specification in MRT is

\[
GAW_{t+4} - \pi_t^* = \alpha_0 + \alpha_1 \left( prod_{t}^{\text{trnd}} \right) + \alpha_1^* \left( un_t^Q - un_t^{*,CBO} \right) + \alpha_2 \left( un_t^Q - un_t^{*,CBO} \right) + \alpha_2^* \left( \Delta \left( un_t^Q - un_t^{*,CBO} \right) \right) + \alpha_3 \left( \Delta \left( un_t^Q - un_t^{*,CBO} \right) \right) + \alpha_4 \left( \Delta \left( un_t^Q - un_t^{*,CBO} \right) \right) + \varepsilon_t,
\]

where \( GAW_{t+4} \) refers to growth in average wages (constructed from CPS data, as detailed in MRT), \( un_t^Q \) is quarterly unemployment, \( un_t^{*,CBO} \) is the CBO estimate of the natural rate of unemployment, and \( prod_{t}^{\text{trnd}} \) refers to trend productivity growth at time \( t \). For comparison to (6), we re-specify Equation (3) alternatively as

\[
\left( \pi_{t+12} - \pi_t^* \right) = \alpha + \beta_1 \left( \pi_{t+12} - \pi_{t+12}^* \right) + \beta_2 \left( \pi_{t+12} - \pi_{t-24}^* \right) + \lambda_t^* \left( un_t^3 - un_t^* \right) + \lambda_t^* \left( un_t^3 - un_t^* \right) + \varepsilon_t,
\]

or in a slightly simplified form,

\[
\left( \pi_{t+12} - \pi_t^* \right) = \alpha + \beta_1 \left( \pi_{t+12} - \pi_{t+12}^* \right) + \beta_2 \left( \pi_{t+12} - \pi_{t-24}^* \right) + \lambda_t^* \left( un_t^3 - un_t^* \right) + \lambda_t^* \left( un_t^3 - un_t^* \right) + \varepsilon_t,
\]

where \( un_t^3 \) is a three-month moving average of the real-time unemployment rate.

It is interesting to note that estimates of this wage Phillips curve yield qualitatively similar results, though they differ in details. For instance, there is a modest, but nonzero, downward force on wage growth when labor force slack is high. But the strongest pressures occur on wage growth from upward movements in the unemployment rate (though this force is not as strong as the downward force on prices during these periods), and from overheating (and this force is much stronger than the upward force on prices during these periods).
We again see clear evidence of nonlinearity in the Phillips curve. However, while appealing in their greater simplicity, Equations (7) and (8) do not fit the data quite as well as (3), nor do they yield quite as accurate a conditional forecast of inflation over the post-2006 period.

One might have guessed this, given the estimated differential between $\lambda_2^+$ and $\lambda_3^+$ in conjunction with Figure A.4 above. It would be hard for a single term in $\Delta un$ to capture what appears to be two different relationships.
APPENDIX H: THEORY

H.1: Overheating and Inflation

From its inception in Phillips (1958), it was generally believed (see also Lipsey (1960)) that the general shape of the Phillips curve is convex, so that a negative unemployment gap (an overheating economy) has a bigger price impact than the same percentage positive unemployment gap (slack). Many theories naturally give rise to a convex wage Phillips curve. Layard, Nickell, and Jackman (1991) demonstrate that the shirking model of Shapiro and Stiglitz (1984) implies a nonlinear wage Phillips curve. The “bottlenecks” model of Evans (1985) and the bargaining model of Blanchflower and Oswald (1990) also imply a nonlinear wage Phillips curve. We would expect such convexity to spill over into convexity in the price Phillips curve.

Harding et al. (2022) demonstrate that the mere inclusion of real rigidities (using the Kimball aggregator) induces a convex shape in the Phillips curve, but this will be erroneously missed unless one eschews linearization and instead uses a nonlinear solution method; see also Gasteiger and Grimaud (2020).

A convex shape to the price Phillips curve is suggested by models in which prices are downwardly rigid, such as Ball, Mankiw, and Romer (1988).\footnote{Downward nominal wage rigidity is a classic explanation for a convex wage Phillips curve (see Phillips (1958) and Daly and Hobijn (2014)), and see Dupraz, Nakamura and Steinsson (2019) for a recent model delivering asymmetric unemployment fluctuations.} In this model, which features menu costs of price adjustment in the presence of generally positive inflation, prices are more sticky downward because the relative price declines can “automatically” occur via inflation. Thus, even if a firm desires a relative price decline, it will optimally choose inaction and wait for inflation to deliver that decline in the near future.

In the standard New Keynesian model, the output gap maps directly into inflationary pressure. In the standard DMP model, the value of unemployment determines the worker’s outside option. Moscarini and Postel-Vinay (2017) draw attention to the fact that individual wage growth co-varies more strongly with the aggregate job-to-job transition rate than with the...
aggregate unemployment rate. Moscarini and Postel-Vinay (2019) provide a New Keynesian job-
ladder model that is consistent with this fact and that explains how an overheating labor market
can translate into price pressures. In this model, workers’ bargaining power derives from the
ability to receive outside offers, not from the unemployment outside option. After a downturn,
many employed workers are mismatched and easily poachable, and numerous unemployed
workers are profitably hired. But late in the cycle, the stiff competition for employed productive
workers leads to many outside wage offers being matched by current employers, and these wage
increases effectively become cost-push shocks.

Another class of models that naturally deliver a Phillips curve relationship of this sort –
that is, strong upward price pressure when the economy is overheating – is capacity-constraints
models.\textsuperscript{12} Bils and Klenow (1998) find procyclical relative price and TFP movements in highly
procyclical consumption good sectors and argue that this suggests the existence of varying
capacity utilization with occasionally binding capacity constraints. Capacity constraints naturally
induce business-cycle asymmetries (Hansen and Prescott, 2005). In the New Keynesian model of
Alvarez-Lois (2004), the Phillips curve becomes

\[
\pi_t = \beta E \pi_{t+1} + \mu (\hat{\theta}_t + m\hat{e}_t)
\]

\textsuperscript{12} The models expounded in Clark and Laxton (1997) or Clark, Laxton and Rose (2001) also feature capacity
constraints. Alan Greenspan seems to have believed in a convex Phillips curve arising from capacity constraints. For
example, in his testimony to the Subcommittee on Economic Growth and Credit Formation (Greenspan 1994b,
p.12), he stated: “If the economy were nearing capacity, we would expect to see certain patterns in the statistical and
anecdotal information ... To attract additional workers, employers would presumably step up their use of want-ads
and might begin to use nonstandard techniques...All of these steps in themselves could add to costs and suggest
developing inflationary imbalances.” In his testimony before the Joint Economic Committee in January 1994, he
noted: “History suggests, however, that higher price inflation tends to surface rather late in the business cycle...”
(Greenspan 1994a, p.6). In his testimony before the Committee on Finance in January 1995, he stated: “Knowing in
advance our true growth potential obviously would be useful in setting policy because history tells us that economies
that strain labor force and capital stock limits tend to engender inflationary instabilities.”
where \( \theta_t \) is the share of firms in the economy that are operating at full capacity. (See also Alvarez-Lois (2005, 2006) for related models, and Mikosch (2012) and Kuhn and George (2019) for alternative New Keynesian models with capacity constraints.) There is supportive evidence. The paper by Lein and Köberl (2009) is a micro study of Swiss manufacturing firms. These authors find evidence of a strong relationship between price increases and being capacity constrained (either due to labor or due to technical capacity).13

**H.2: Busts and Inflation**

It has been thought puzzling that large labor market slack does not weigh on inflation, leading to the famous inflation puzzle of the Great Recession. Not only is this suggested by a conventional Phillips curve, it is ostensibly an implication of standard New Keynesian theory (see, for example, King and Watson 2012). That paper demonstrates, though, that the low-frequency movements in inflation should line up with low-frequency movements in real unit labor costs. Most of the empirical work in the New Keynesian paradigm has used a variant of labor’s share as the proxy for real marginal costs, but Bils (1987), Petrella and Santoro (2012), and Madeira (2014) demonstrate that this can be a misleading proxy. Petrella and Santoro (2012) use the income share of intermediate goods (and stress the importance of disaggregated data; see also Bouakez, Cardia, and Ruge-Murcia 2014); Madeira (2014) constructs a proxy using overtime costs. Both alternatives improve the fit of New Keynesian Phillips curves.

Standard industrial organization theory predicts that, at the onset of a recession, we might see an initial drop in inflation, but not continued downward pressure – even though slack (as conventionally measured) remains high. In particular, the received wisdom in the industrial organization literature is that demand shortfalls tend to provoke price wars. But this behavior is

---

13 Using these same data, Köberl and Lein (2011) find that an aggregated capacity constraint measure is useful in a Phillips curve. Similarly, at the micro level, Mikosch (2012) finds that the slope of the micro Phillips curve is increasing as capacity constraints become tighter, although this effect disappears for firms facing intense competition.
forward-looking, and price declines are front-loaded. After a time, the price war effect ceases, and prices then start to drift slowly upward again. More generally, as is well known, countercyclical markups will mitigate aggregate price drops during recessions. Fernández et al. (2015) demonstrate that, in Spain, average markups rose in half of the sectors after 2008. Faccini and Melosi (2020) provide a theory that generates low inflationary pressures over most of the recovery. Gilchrist et al. (2017) develop a New Keynesian model, extended in Gilchrist et al. (2018), that builds upon these insights, and provides supportive empirical evidence. These authors draw attention to the standard IO theory, but further note a nuance to this basic relationship. In customer markets, pricing decisions are investment decisions, and factors that influence investment will influence pricing. Thus, in the theory of Gottfries (1991) and Chevalier and Scharfstein (1996), under financial frictions, constrained firms in customer markets facing a fall in demand may find it optimal to maintain, or even increase, their prices to boost cash flow and avoid costly external financing. Financially unconstrained firms have the opportunity to reduce prices and invest in market share. In the model of Gilchrist et al. (2017), financial frictions imply that markups remain elevated after the initial adverse demand (or financial) shock. Evidence in both Gilchrist et al. (2017) and Gilchrist et al. (2018) is supportive; for instance, financially constrained firms in the US, on average, raised prices at the onset of the Great Recession, while other firms dropped prices aggressively and increased their market share. Prices remained flat for about a year, then began to rise again. The resulting changes in

\[ \text{equation} \]

\[ \text{equation} \]

\[ \text{equation} \]

14 In the price experimentation model of Bachmann and Moscarini (2012), a recession might trigger some firms to optimally increase prices, as the costly acquisition of information might allow them to “gamble for resurrection.”

15 This is not the same mechanism as in Christiano, Eichenbaum, and Trabandt (2015), in which a jump in credit spreads increases the cost of working capital, increasing marginal costs. Klemperer (1995) also draws attention to the notion of market share as an investment good, with the concomitant influence of the interest rate on prices. For a model featuring countercyclical markups driven by exit, in the absence of financial frictions, see Cheremukhin and Tutino (2016).

16 Asplund, Ericksson, and Strand (2005), Lundin et al. (2009), and Montero and Urtasun (2014) find similar evidence. Gilchrist et al. (2018) find a similar dichotomy between firms in financially weak versus financially strong
market share were persistent. See also Hong (2019), who finds that markups are countercyclical
(with cyclicality varying systematically across firms) and who develops a customer-capital
variant of a Hopenhayn (1992) model consistent with his findings. Finally, Alves (2019)
demonstrates that a reduction in job-to-job flows during the recovery can worsen labor
productivity, providing an upward force on inflation that is absent in standard models.

countries in Europe. They further find that the deviations of price trajectories from the predictions of a standard
Phillips curve can be related to financial constraints.
I.1 Description of One-Sided Filtering

In brief, we compute one-sided filtering by running a window through the data. Over each window, we save the decomposition at the final data point in the window. Then we increment the window by one month. However, each window includes not just data, but also a second component that is a forecast. In other words, each window includes data augmented with a forecast.

To explain this in more detail, consider Figure A.8. We wish to compute the decomposition of the unemployment rate at time $s+\kappa$. As is well-known, obtaining the decomposition at $s+\kappa$ by using a two-sided filter from time $s$ to time $s+\kappa+m$ would yield estimates with very poor properties. In particular, the resultant time series would (for most filters) incorporate a pronounced phase shift, in addition to being highly inaccurate; this inaccuracy is due to the well-known “edge effect” problem plaguing all filters.

Both the phase-shift and edge effect problems are addressed by augmenting the data within a window with forecasts. In particular, as in Dagum (1978), Stock and Watson (1999), Kaiser and Maravall (1999), Mise, Kim and Newbold (2005), and Clark and Kozicki (2005) – and as is done routinely in seasonal-adjustment procedures – we augment the window sample...
data with forecasted data. In the situation depicted in Figure A.8, we have \( \kappa \) sample data points (from time period \( s \) to time period \( s+\kappa \)), and \( m \) months of projections, yielding a \((\kappa+m)\)-month window (from time period \( s \) to time period \( s+\kappa+m \)). Decompositions by frequency must be performed on trendless data, and accordingly, frequency-domain filters detrend sample data prior to the decomposition, and add the trend back afterwards. Our procedure is no different in this respect, except that detrending must occur within each sample window. Consequently, for each window, we linearly detrend the data over the window. Then we use a two-sided filter to partition that window into persistence components, and then save the partition at date \( s+\kappa \); notice that this is a one-sided partition, since no data after date \( s+\kappa \) are used. The trend estimate at \( s+\kappa \) is added to the lowest-frequency band at \( s+\kappa \); thus, this first component includes all variation at frequencies so low (i.e., reversion periods so large) that they exceed the length of the window. To obtain the partition at date \( s+\kappa+1 \), we repeat this procedure, obtaining a forecast from data \( s+\kappa+1 \) to data \( s+\kappa+1+m \), then use a two-sided filter over dates \( s+1 \) to \( s+\kappa+m+1 \) and saving the partition at date \( s+\kappa+1 \). This procedure also gracefully allows us to use real-time data. In our experience (and see Ashley and Verbrugge 2022b), as long as one uses at least a year of projections within the window, this procedure results in acceptable persistence-component decompositions and phase shifts (and turning point distortions) that are quite limited for all persistence components. (Of course, in accordance with the information available to economic agents, turning points will typically be detected with a lag, unless one uses forecasts that detect such turning points rapidly.)

We note in passing that our results are very similar if we directly partition the unemployment gap, rather than partitioning \( u \) and then subtracting off \( u^* \) from the most persistent component.

**I.2 Testing for Persistence-Dependence**

How does testing proceed? In the present case, we wish to test whether the Phillips curve is persistence-dependent. Thus, we partition the unemployment rate \( un \) into three components (say): \( un^1 \), \( un^2 \), and \( un^3 \). Then we replace \( un \) in the Phillips curve specification with its three
components. Then, following Tan and Ashley (1999), one may readily test for persistence-dependence using a standard Chow test. Since the components sum to the original series and are based upon one-sided filtering, the causality structure and the properties of the error term are preserved. For more details, see the appendix to Ashley, Tsang and Verbrugge (2020).

**I.3 Sensitivity to Forecasts and Filter**

In our experience, the resultant persistence-decomposition is not very sensitive to the number of forecast periods chosen, as long as at least 12 months of projections are used, nor to the details of how these forecasts are produced (as long as they are reasonably accurate); see Ashley and Verbrugge (2022b) for more details. In this study, forecasts are derived from the Survey of Professional Forecasters, but we obtain similar results if we base our forecasts on simple univariate time-series models. (Having said this, better forecasts are obviously preferred.)

We obtain good results using either the Christiano-Fitzgerald filter, the Iacobucci-Noullez filter, or the Ashley-Verbrugge filter. What is crucial is to *partition* the explanatory variables into an interpretably small set of frequency/persistence components that add up to the original data, using moving windows passing through the data so that the filtering is done in a backward-looking or one-sided manner. There are some pragmatic issues of importance, which we discuss in Section I.4.

But bearing in mind such details, what is of practical macroeconometric importance is to allow for frequency/persistence dependence in the relationship, not – so long as one is mindful of the basic desiderata delineated above – the technical details of precisely how the explanatory variable is partitioned into its frequency/persistence components. We find that alternative techniques usually yield quite similar empirical results in practice; see Ashley and Verbrugge (2022b) for more details. RATS, Stata, and matlab code to accomplish this type of one-sided decomposition (using simple univariate or multivariate forecasts) is available from the authors.
I.4 Rationale for Partitioning, One-Sided Filtering, and Filtering Only Explanatory Variables

Why are partitioning, one-sided filtering, and restriction of the filtering solely to the $u_t - u_t^*$ data all essential? Partitioning is necessary in order that these three components of the unemployment rate gap add up to the original data, so that it is easy to test the null hypothesis that the coefficients with which these three components enter a regression model for the inflation rate are all equal. One-sided filtering is necessary for two reasons. First, in the present case, one-sided filtering – and only one-sided filtering – sensibly comports with the use of real-time unemployment rate data. Second, more fundamentally, two-sided filtering – such as ordinary HP-filtering or ordinary spectral analysis – inherently mixes up future and past values of the unemployment rate gap in obtaining the persistence components, distorting the causal meaning of inference in the resulting inflation model and limiting its use for practical forecasting and/or policy analysis (although we hasten to add that the present work is primarily intended as a contribution to our understanding of inflation dynamics, not as proposing a new forecasting model). These distortions from the use of two-sided filtering are particularly severe when the dependent variable is also filtered and when the key relationship likely (as here) involves feedback from the dependent variable (inflation) to the (filtered) components of $u_t - u_t^*$ being used as explanatory variables. Fundamentally, this is because filtering the dependent variable in a regression model implies that the model error term is similarly filtered. For more details, see Ashley and Verbrugge (2009, 2022a); and for a “practical” comparison of methods, including the Hamilton (2018) filter, see Ashley and Verbrugge (2022b).

How about two-sided spectral estimates or filtering with wavelets? These are two-sided methods, so the same criticisms apply. Hence, both two-sided cross-spectral estimates and filtering with wavelets are ruled out for analyses of the present sort. As noted above, our decomposition still yields consistent parameter estimation where (as is typically the case with economic relationships) one cannot rule out feedback (or bi-directional) causality. Further, our moving-window approach applies gracefully to real-time economic data.
Regarding spectral methods, even absent feedback, transfer function gain and phase plots are substantially more challenging to interpret than our approach. Outside of simple cases, gain and phase are notoriously opaque. Even without the presence of feedback, Granger describes interpretation of such plots as “difficult or impossible” (Granger, 1969). As gain is nonnegative, an inverse relationship at one frequency is “interpreted” as a phase shift of 0.5; this is mathematically correct, but damages intuition. In fact, as Engle (1976) points out, since the phase is only known up to adding or subtracting an integer, even the lead-lag relationship is not known for sure. These interpretive difficulties underscore a chief advantage of our approach: straightforward interpretability.

1.5 Comparison to Trend-Cycle Decomposition Methods

In contrast to trend-cycle decomposition methods (e.g., Beveridge-Nelson), our frequency-filtering approach is not limited to decomposing an explanatory variable like $u_t - u_t^*$ into just two components: an arbitrarily-persistent – i.e., I(1) or I(1)-like – trend, and a stationary I(0) fluctuation. Our decomposition instead produces several components; these components span the complete range of persistence levels by construction. Since these components add up to $u_t - u_t^*$, replacing them with $u_t - u_t^*$ in the relevant regression model allows the data themselves to quantify the degree of persistence-dependence in the relationship – with a good deal less in the way of a priori restrictions imposed by the analytical framework.

On the other hand, this flexibility requires the analyst to choose the number of persistence bands and the concomitant persistence cut-offs. For the present study we chose three persistence bands with two cut-offs – at $\tau_{\text{persist}} = 48$ months and $\tau_{\text{transient}} = 12$ months. In point of fact, our Phillips curve regression results are not greatly sensitive to these particular choices, as indicated by the results presented in Appendix J below.
J.1 Full Set of Exhibits: 5-year window

Herein, we provide exhibits pertaining to results from an alternative decomposition of the unemployment gap, namely one based upon a five-year window (48 months of data, + 12 months of forecast), where we maintain the assumption that $\tau_{\text{persist}} = 48$, that is, the lowest frequency band corresponds to fluctuations greater than 48 months. Similar to the decomposition in the main body, we use the CF filter to partition the unemployment rate, and the forecasts are from Blue Chip. We further provide a summary table of results pertaining to a large number of modeling alternatives. Results are qualitatively very similar to those of our baseline, reported in the main body.

Figure A.9: One-Sided Partition of the Unemployment Rate Gap (5 yr window)
In-sample results are also very similar; see Table A.3.

Table A.3. Phillips Curve Regression Estimation Results

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Specif. (1)</th>
<th>Specif. (2)</th>
<th>Specif. (3)</th>
<th>Specif. (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_t - u_t^* )</td>
<td>( \lambda )</td>
<td>-0.08 (-1.54)</td>
<td>( \lambda_1^+ )</td>
<td>0.04 (0.84)</td>
</tr>
<tr>
<td></td>
<td>(t-stat)</td>
<td></td>
<td>(-1.27)</td>
<td>(-3.36)</td>
</tr>
<tr>
<td>Persistent component</td>
<td></td>
<td></td>
<td></td>
<td>(-3.31)</td>
</tr>
<tr>
<td>of ( u_t - u_t^* )</td>
<td>( \lambda_1^- )</td>
<td>-0.07 (-1.27)</td>
<td>-0.29 (-3.36)</td>
<td>-0.27 (-3.31)</td>
</tr>
<tr>
<td></td>
<td>(t-stat)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moderately persistent component</td>
<td>( \lambda_2^+ )</td>
<td>-0.93 (-3.61)</td>
<td>-1.33 (-8.37)</td>
<td>-1.26 (-9.79)</td>
</tr>
<tr>
<td>of ( u_t - u_t^* )</td>
<td>(t-stat)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \lambda_2^- )</td>
<td>0.25 (0.52)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transient component</td>
<td>( \lambda_3^+ )</td>
<td>-0.34 (-1.44)</td>
<td>-0.79 (-2.19)</td>
<td>-0.70 (-1.97)</td>
</tr>
<tr>
<td>of ( u_t - u_t^* )</td>
<td>(t-stat)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \lambda_3^- )</td>
<td>0.25 (1.28)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lagged inflation</td>
<td>( \beta_1 )</td>
<td>0.48 (9.65)</td>
<td>0.30 (3.13)</td>
<td>0.26 (3.31)</td>
</tr>
<tr>
<td></td>
<td>(t-stat)</td>
<td></td>
<td>(3.19)</td>
<td>(5.18)</td>
</tr>
<tr>
<td></td>
<td>( \beta_2 )</td>
<td>0.09 (1.11)</td>
<td>0.30 (3.19)</td>
<td>0.31 (5.18)</td>
</tr>
<tr>
<td></td>
<td>(t-stat)</td>
<td></td>
<td>(3.19)</td>
<td>(8.20)</td>
</tr>
<tr>
<td>constant</td>
<td>-0.08 (-0.88)</td>
<td>0.31 (1.07)</td>
<td>-0.05 (-0.49)</td>
<td>-0.06 (-0.69)</td>
</tr>
<tr>
<td></td>
<td>(t-stat)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.55</td>
<td>0.66</td>
<td>0.73</td>
<td>0.73</td>
</tr>
<tr>
<td>BIC</td>
<td>0.96</td>
<td>0.72</td>
<td>0.52</td>
<td>0.49</td>
</tr>
<tr>
<td>Hypothesis Test</td>
<td></td>
<td></td>
<td></td>
<td>&lt;0.005</td>
</tr>
<tr>
<td>Rejection P-Values</td>
<td></td>
<td></td>
<td>&lt;0.005</td>
<td>&lt;0.005</td>
</tr>
<tr>
<td>( H_0: \lambda_1^+ = \lambda_1^- )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( H_0: \lambda_2^+ = \lambda_2^- )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( H_0: \lambda_3^+ = \lambda_3^- )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( H_0: \lambda_1^+ = \lambda_2^+ = \lambda_2^- = \lambda_3^+ = \lambda_3^- )</td>
<td>&lt;0.005</td>
<td>&lt;0.005</td>
<td>&lt;0.005</td>
<td></td>
</tr>
<tr>
<td>( H_0: (Chow test): {coefficients unchanged before and after 2006:12} )</td>
<td>0.03</td>
<td>0.49</td>
<td>0.10</td>
<td>0.69</td>
</tr>
</tbody>
</table>

NOTE: Figures in parentheses are estimated t-statistics, based on (13-month) HAC standard error estimates. Given this choice, diagnostic checks regarding heteroscedasticity are not quoted. Fitting errors for Equation (2), (3) and (4) display no evidence of notable outliers.

The main difference between these results and the Table 1 results in the main body is the Chow test for Equation (3). This results from instability in the three statistically insignificant coefficients, \( \lambda_1^+ \), \( \lambda_2^- \), and \( \lambda_2^- \). Dropping these terms, the Chow test p-value is 0.40.
Out-of-sample results are also very similar. For instance, the conditional recursive forecast figure is quite similar.

Figure A.10: Conditional Recursive Forecasts from Equations (1) and (3), and from a UCSV model (5-yr window)

…as are the analogues to Figures A.6 and A.7, depicted in Figure A.11.
Figure A.11: Panel (A): More Conditional Recursive Forecasts: Forecasts Based upon 2006:12 Estimated Coefficients, and Forecasts Based upon 2019:12 Estimated Coefficients, Using Only 2006:12 Inflation Data. Panel (B): Deviations Between Forecasts Based upon 2006:12 Estimated Coefficients, and Forecasts Based upon 2019:12 Estimated Coefficients. (Both from 5-yr window).
Results from out-of-sample forecast tests are also very similar. In each case, there is a 5 percent result over some span of time.

Figure A.11. GR Fluctuation Tests, Equation (3) versus: (A) Baseline CBO Model; (B) Atkeson-Ohanian Model; (C) UCSV; (D) SW Recession Gap

Over the full sample, the p-values of the DM test statistics are: 0.03 vs. CBO Model; 0.01 vs. AO Model; 0.03 vs. UCSV; and 0.02 vs. SW Recession Gap.
J.2 Robustness Results with Respect to Persistence Component Details, Conditional

In Table A.5, we report results pertaining to a wide range of modeling alternatives while holding $\tau_{\text{persist}} = 48$. We include here, and in Section J.3 below, results pertaining to the use of two additional filters: the Ashley/Verbrugge (2008) filter, and the Iacobucci/Noullez filter (Iacobucci and Noullez, 2005). Table A.4 lists the various modeling choices that correspond to each column. We repeat the baseline results from the main body in columns a) – c). These results are substantially equivalent to our baseline results, which is reassuring since results should be largely insensitive to variation along these dimensions.

Table A.4. Model Specification: Window size, Filter, Forecast Details

<table>
<thead>
<tr>
<th>a) Baseline: CBO gap model</th>
</tr>
</thead>
<tbody>
<tr>
<td>b) Baseline: Equation (3) model: 48 month window, CF filter, 12 month forecast, Blue Chip forecasts.</td>
</tr>
<tr>
<td>c) Baseline: Equation (4) model: 48 month window, CF filter, 12 month forecast, Blue Chip forecasts</td>
</tr>
<tr>
<td>d) Equation (3) model: 48 month window, AV filter, 12 month forecast, Blue Chip forecasts</td>
</tr>
<tr>
<td>e) Equation (3) model: 48 month window, IN filter, 12 month forecast, Blue Chip forecasts</td>
</tr>
<tr>
<td>f) Equation (3) model: 60 month window, IN filter, 12 month forecast, Blue Chip forecasts</td>
</tr>
<tr>
<td>g) Equation (3) model: 60 month window, CF filter, 12 month forecast, Blue Chip forecasts</td>
</tr>
<tr>
<td>h) Equation (3) model: 60 month window, CF filter, 12 month forecast, univariate AR(2) in levels</td>
</tr>
</tbody>
</table>
Table A.5. Phillips Curve Regression Estimation and Forecasting Results

<table>
<thead>
<tr>
<th>Model</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
<th>(g)</th>
<th>(h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda ) or ( \lambda^+ )</td>
<td>-0.08</td>
<td>0.03</td>
<td>0.04</td>
<td>0.02</td>
<td>0.02</td>
<td>0.04</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>(t-stat)</td>
<td>(-1.54)</td>
<td>(0.87)</td>
<td>(0.92)</td>
<td>(0.66)</td>
<td>(0.68)</td>
<td>(1.21)</td>
<td>(1.07)</td>
<td></td>
</tr>
<tr>
<td>( \lambda^- )</td>
<td>-0.29</td>
<td>-0.27</td>
<td>-0.33</td>
<td>-0.30</td>
<td>-0.31</td>
<td>-0.33</td>
<td>-0.26</td>
<td></td>
</tr>
<tr>
<td>(t-stat)</td>
<td>(-3.57)</td>
<td>(-3.50)</td>
<td>(-3.71)</td>
<td>(-4.04)</td>
<td>(-4.38)</td>
<td>(-4.49)</td>
<td>(-3.69)</td>
<td></td>
</tr>
<tr>
<td>( \lambda^+ )</td>
<td>-1.81</td>
<td>-1.72</td>
<td>-1.27</td>
<td>-2.39</td>
<td>-2.03</td>
<td>-1.55</td>
<td>-1.81</td>
<td></td>
</tr>
<tr>
<td>(t-stat)</td>
<td>(-9.16)</td>
<td>(-9.68)</td>
<td>(-6.20)</td>
<td>(-6.31)</td>
<td>(-7.11)</td>
<td>(-7.53)</td>
<td>(-8.19)</td>
<td></td>
</tr>
<tr>
<td>( \lambda^- )</td>
<td>0.23</td>
<td>0.43</td>
<td>1.30</td>
<td>0.95</td>
<td>0.57</td>
<td>-0.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(t-stat)</td>
<td>(0.31)</td>
<td>(0.93)</td>
<td>(2.48)</td>
<td>(1.75)</td>
<td>(1.14)</td>
<td>(-0.24)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lambda^+ )</td>
<td>-0.53</td>
<td>-0.51</td>
<td>-0.47</td>
<td>-0.35</td>
<td>-0.54</td>
<td>-0.99</td>
<td>-1.59</td>
<td></td>
</tr>
<tr>
<td>(t-stat)</td>
<td>(-2.00)</td>
<td>(-2.11)</td>
<td>(-0.99)</td>
<td>(-0.93)</td>
<td>(-1.36)</td>
<td>(-2.23)</td>
<td>(-3.03)</td>
<td></td>
</tr>
<tr>
<td>( \lambda^- )</td>
<td>0.07</td>
<td>-0.08</td>
<td>-0.15</td>
<td>-0.12</td>
<td>-0.01</td>
<td>0.29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(t-stat)</td>
<td>(0.68)</td>
<td>(-0.32)</td>
<td>(-0.81)</td>
<td>(-0.94)</td>
<td>(-0.08)</td>
<td>(0.58)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **H_0:** \( \lambda^+_1 = \lambda^-_1 < 0.01 \)
- **H_0:** \( \lambda^+_2 = \lambda^-_2 < 0.01 \)
- **H_0:** \( \lambda^+_3 = \lambda^-_3 < 0.01 \)

<table>
<thead>
<tr>
<th>( { \lambda_{2006} = \lambda_{2019} \ for \ \lambda^- \ &amp; \lambda^+_2 \ &amp; \lambda^+_3 } )</th>
<th>( H_0: )</th>
<th>( H_0: )</th>
<th>( H_0: )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( { \lambda_{2006} = \lambda_{2019} \ for \ \lambda^- \ &amp; \lambda^+_2 \ &amp; \lambda^+_3 } )</td>
<td>0.03</td>
<td>0.36</td>
<td>0.02</td>
</tr>
</tbody>
</table>

| RMSE vs. Inflation, '07-'19 | 0.29 | 0.18 | 0.17 | 0.19 | 0.19 | 0.19 | 0.20 | 0.16 |
| DM p-value vs. CBO gap model | 0.02 | <0.01 | 0.04 | 0.01 | 0.01 | 0.02 | 0.01 |
| DM p-value vs. AO model | 0.77 | <0.01 | <0.01 | 0.10 | 0.02 | 0.01 | 0.03 | <0.01 |
| DM p-value vs. UCSV model | 0.83 | 0.01 | <0.01 | 0.28 | 0.09 | 0.05 | 0.08 | 0.02 |
| DM p-value vs. SW RG model | 0.89 | 0.01 | <0.01 | 0.03 | <0.01 | <0.01 | 0.02 | 0.01 |
J.3 Robustness Results with Respect to Persistence Component Details, For Differing Settings of $\tau_{\text{persist}}$.

In Table A.8, we report results pertaining to a wide range of modeling alternatives, this time including variation in $\tau_{\text{persist}}$. These results are substantially equivalent to our results for $\tau_{\text{persist}} = 48$. However, unlike the robustness checks investigated in Section J.2 above, we had no a priori reason to believe that this would be so – since results should in principle be sensitive to variation in this value. Table A.7 lists the various modeling choices that correspond to each column. We repeat the baseline (main body) results again in columns a) – c).
Table A.7. Model Specification: Window size, $\tau$, Filter, Forecast Details

<table>
<thead>
<tr>
<th>Model Specification</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Baseline: CBO gap model</td>
<td></td>
</tr>
<tr>
<td>b) Baseline: Equation (3) model: 48 month window, $\tau_{\text{persist}} = 48$ mo; CF filter, 12 month forecast, Blue Chip forecasts.</td>
<td></td>
</tr>
<tr>
<td>c) Baseline: Equation (4) model: 48 month window, $\tau_{\text{persist}} = 48$ mo; CF filter, 12 month forecast, Blue Chip forecasts</td>
<td></td>
</tr>
<tr>
<td>d) Equation (3) model: 72 month window, $\tau_{\text{persist}} = 60$ mo.; CF filter, 18 month forecast, univariate AR(2) in levels</td>
<td></td>
</tr>
<tr>
<td>e) Equation (3) model: 60 month window, $\tau_{\text{persist}} = 60$ mo.; IN filter, 12 month forecast, Blue Chip forecasts</td>
<td></td>
</tr>
<tr>
<td>f) Equation (3) model: 60 month window, $\tau_{\text{persist}} = 60$ mo.; AV filter, 12 month forecast, Blue Chip forecasts</td>
<td></td>
</tr>
<tr>
<td>g) Equation (3) model: 72 month window, $\tau_{\text{persist}} = 60$ mo.; IN filter, 12 month forecast, univariate AR(2) in levels</td>
<td></td>
</tr>
<tr>
<td>h) Equation (3) model: 60 month window, $\tau_{\text{persist}} = 60$ mo.; CF filter, 12 month forecast, univariate AR(2) in levels</td>
<td></td>
</tr>
<tr>
<td>i) Equation (3) model: 60 month window, $\tau_{\text{persist}} = 60$ mo.; CF filter, 12 month forecast, Blue Chip forecasts</td>
<td></td>
</tr>
<tr>
<td>j) Equation (3) model: 72 month window, $\tau_{\text{persist}} = 72$ mo.; CF filter, 18 month forecast, univariate AR(2) in levels</td>
<td></td>
</tr>
</tbody>
</table>
Table A.8. Phillips Curve Regression Estimation and Forecasting Results

<table>
<thead>
<tr>
<th>Model</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
<th>(g)</th>
<th>(h)</th>
<th>(i)</th>
<th>(j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda \text{ or } \lambda^+ ) (t-stat)</td>
<td>-0.08</td>
<td>0.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lambda^- ) (t-stat)</td>
<td>-0.29</td>
<td>-0.27</td>
<td>-0.27</td>
<td>-0.31</td>
<td>-0.27</td>
<td>-0.29</td>
<td>-0.27</td>
<td>-0.28</td>
<td>-0.28</td>
<td>-0.24</td>
</tr>
<tr>
<td>( \lambda^+ ) (t-stat)</td>
<td>-1.81</td>
<td>-1.72</td>
<td>-1.19</td>
<td>-2.03</td>
<td>-1.08</td>
<td>-1.49</td>
<td>-1.27</td>
<td>-1.30</td>
<td>-1.30</td>
<td>-1.22</td>
</tr>
<tr>
<td>( \lambda^- ) (t-stat)</td>
<td>0.23</td>
<td>0.31</td>
<td>-0.02</td>
<td>0.95</td>
<td>0.47</td>
<td>-0.14</td>
<td>-0.15</td>
<td>0.23</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>( \lambda^+ ) (t-stat)</td>
<td>-0.53</td>
<td>-0.51</td>
<td>-0.88</td>
<td>-0.54</td>
<td>-0.68</td>
<td>-0.90</td>
<td>-1.00</td>
<td>-0.66</td>
<td>-0.13</td>
<td></td>
</tr>
<tr>
<td>( \lambda^- ) (t-stat)</td>
<td>0.07</td>
<td>0.68</td>
<td>0.20</td>
<td>-0.12</td>
<td>0.21</td>
<td>0.32</td>
<td>0.79</td>
<td>0.32</td>
<td>0.38</td>
<td></td>
</tr>
</tbody>
</table>

| H0: | \( \lambda^+_1 = \lambda^-_1 \) | <0.01 | <0.01 | 0.01 | <0.01 | <0.01 | <0.01 | <0.01 | 0.01 | |
| H0: | \( \lambda^+_2 = \lambda^-_2 \) | <0.01 | <0.01 | <0.01 | <0.01 | <0.01 | <0.01 | <0.01 | 0.01 | |
| H0: | \( \lambda^+_3 = \lambda^-_3 \) | 0.02 | | | | | | | | |

| RMSE vs. Inflation, 07-19 | 0.29 | 0.18 | 0.17 | 0.16 | 0.19 | 0.18 | 0.16 | 0.15 | 0.18 | 0.15 |
| DM p-value vs. CBO gap model | 0.02 | <0.01 | 0.03 | 0.01 | 0.03 | 0.02 | 0.05 | 0.03 | 0.06 | |
| DM p-value vs. AO model | 0.77 | <0.01 | <0.01 | 0.01 | 0.01 | 0.04 | <0.01 | 0.03 | 0.01 | 0.05 |
| DM p-value vs. UCSV model | 0.83 | 0.01 | <0.01 | 0.02 | 0.05 | 0.13 | 0.02 | 0.06 | 0.03 | 0.09 |
| DM p-value vs. SW RG model | 0.89 | 0.01 | <0.01 | 0.03 | <0.01 | 0.03 | 0.02 | 0.05 | 0.02 | 0.07 |


**DATA REFERENCES**


9. US Congressional Budget Office. “Natural Rate of Unemployment (Long-Term) [NROU],” Retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/NROU.