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# Loan Modifications and the Commercial Real Estate Market\*

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## Abstract

Banks modify more CRE loans than CMBS, contributing to better loan performance when property incomes decline. However, banks have higher delinquency rates for less-stressed loans, consistent with modification policies encouraging strategic default. Motivated by these facts, we develop a trade-off theory model in which lenders vary in their modification technologies. Modification frictions discourage strategic renegotiation, enabling CMBS to offer higher LTV loans and attract borrowers seeking higher leverage. The model produces cross-lender differences in LTVs and spreads consistent with the data. Reducing modification frictions at CMBS decreases welfare by restricting debt capacity for the borrowers that value it most.

**Keywords:** commercial real estate, modifications, LTV

**JEL Classification:** G21, G22, G23, R33

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## 1. INTRODUCTION

Commercial real estate (CRE) lenders differ notably in their tendency to modify loans. Before the COVID-19 pandemic, bank loans were modified at a rate of around 2 percent per quarter, well above the 0.1 percent rate for loans in commercial mortgage-backed securities (CMBS) pools. This difference was even more dramatic during the COVID-19 pandemic, when the share of bank loans receiving a modification rose and remained high, reaching a peak of over 17 percent. Meanwhile, CMBS modification rates rose only briefly and never breached 5 percent. These differences in modification rates are likely driven by institutional factors that create barriers for the modification of CMBS loans.<sup>1</sup>

The benefits of reducing barriers to modification are ambiguous. Modifications of truly distressed loans can prevent foreclosures or mitigate other costs associated with default. However, the prospect of a modification may cause borrowers to default with the express purpose of securing a modification (strategic defaults). The ability to commit ex-ante to not modifying loans can therefore reduce overall costs to the lender, resulting in more favorable loan terms for borrowers. Ultimately, the desirability of the ability to modify loans depends on the extent to which borrowers value the upfront benefits from commitment relative to the dynamic benefits from modification options.

In this paper, we investigate these trade-offs associated with differences in modification frictions. Our first contribution is to use loan-level data to document how CRE loan modifications differ across lenders. We demonstrate that banks modify CRE loans more often and more preemptively (that is, for less-stressed properties) compared to CMBS. These liberal modifications appear to bolster loan performance, as bank CRE loans are less likely to become delinquent when stressed (measured by either the COVID-19 shock or weak property cash flows). However, banks have higher rates of both delinquency and modification for loans against less-stressed properties, suggesting that their willingness to modify loans encourages strategic default.

Our second contribution is to develop a model of CRE loan underwriting and renegotiation that is consistent with the empirical findings. We use the calibrated model to address several questions about the broader implications

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<sup>1</sup>We provide the details pertaining to factors restricting the modification of CMBS loans in Appendix A.

of these differences. How do modification frictions affect the origination terms offered by different lenders? How do these differences in terms affect the sorting of borrowers into lenders? What are the welfare implications of reducing modification frictions?

In the model, lenders are able to modify required loan payments, helping to reduce the risk of inefficient liquidations. However, the prospect of a favorable modification causes some borrowers to renegotiate loans unnecessarily, increasing modification rates for loans against some modestly stressed properties. In turn, lenders with lower modification frictions have lower delinquency rates for stressed properties but higher strategic defaults for some less-stressed properties, consistent with the empirical findings.

In equilibrium, high-modification lenders offer contracts with stricter loan-to-value (LTV) limits to mitigate their higher renegotiation risk. The key trade-off from a borrower's perspective is then between the higher debt capacity at low-modification lenders and the increased downside protection at high-modification lenders. This trade-off induces borrowers with higher demand for leverage to sort into low-modification lenders.

We calibrate the model to match moments related to underwriting terms and modification rates observed in the data. The calibrated model produces (untargeted) cross-lender differences in average LTVs and spreads that are consistent with the data. CMBS loans have higher spreads and LTVs, on average, reflecting the willingness of CMBS to make high-LTV loans. Though banks require higher spreads for any given contract—compensation for their higher expected modification costs—they make fewer loans to borrowers that will pay a premium to increase leverage, as such borrowers are better served by CMBS. This sorting effect causes CMBS loans to have higher LTVs and spreads than bank loans.

The endogenous sorting of borrowers into lenders in the model is also critical for evaluating welfare. Motivated by a temporary easing of modification restrictions during the COVID-19 pandemic, we examine the effect of reducing modification frictions at CMBS.<sup>2</sup> Under our calibration, though most borrowers benefit from lower frictions, this is not the case for most CMBS borrowers. Easing modification restrictions reduces the availability of high-LTV CMBS loans and thus lowers average welfare by restricting leverage for those who most benefit from it.

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<sup>2</sup>The IRS took steps to temporarily allow more modifications under REMIC laws in the COVID-19 crisis. See the discussion in Appendix A for further details.

Our work contributes to the literature examining differences in the loan portfolios of various types of CRE lenders. [Glancy et al. \(2019\)](#) show that such differences in bank, CMBS, and life insurer portfolios can be explained by supply-side factors affecting their competitiveness for different market segments. Our model effectively endogenizes such segmentation by LTV, showing that modification frictions can explain the cross-lender LTV differences in the data. Also related, [Ghent and Valkanov \(2016\)](#) show that CMBS disproportionately hold loans against larger properties than banks. Lenders differ along other margins that we do not consider in this paper. As examples, [Black et al. \(2017, 2020\)](#) provide evidence that banks make comparatively riskier loans, and [Downs and Xu \(2015\)](#) show that time to resolution of distressed loans is comparatively longer for CMBS.<sup>3</sup>

Our work also contributes to a large theoretical literature on loan renegotiation and resolution in corporate finance and CRE. Our modeling of modifications follows [Hackbarth et al. \(2007\)](#) in that borrowers make a take-it-or-leave-it offer to reduce flow debt service costs.<sup>4</sup> More specific to the CRE market, [Riddiough and Wyatt \(1994\)](#) examine a game where lenders and borrowers decide between a loan workout and foreclosure, demonstrating that workouts are feasible and desirable in the presence of foreclosure costs. Our model differs in that we allow modifications to break down and result in a foreclosure, enabling us to analyze the effects of modification frictions.

Last, we contribute to the empirical literature on renegotiation vs. default in both residential real estate (RRE) and CRE loan markets. The work on CRE has historically relied on life insurer data ([Snyderman, 1990](#); [Brown et al., 2006](#)). More recent work analyzes renegotiations for bank and CMBS loans. [Flynn Jr. et al. \(2021\)](#) examine the impact of a 2009 change in real estate mortgage investment conduit (REMIC) laws on CMBS modification rates. [Glancy et al. \(2021\)](#) show that recourse mitigates renegotiation risk for bank loans and expands the range of contracts available to borrowers. Motivated by this latter finding, we allow lenders in the model to differ in their use of recourse and show that accounting for the effects of recourse on bank underwriting results in a better match to the data.

Our empirical analysis illustrates some key differences between CRE and RRE

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<sup>3</sup>Though we focus on differences in the demand for leverage, our model could be extended to account for other such heterogeneity. Variation in time to resolution could be modeled as different foreclosure costs by lender type, and variation in borrower risk could be modeled as different net operating income volatilities.

<sup>4</sup>[Hackbarth et al. \(2007\)](#) is also a useful reference for its thorough examination of related theories.

loan modifications. During the housing bust in the 2000s, RRE modification rates were generally small, with only modest differences between securitized and portfolio loans (Adelino et al., 2013; Agarwal et al., 2011). We find much higher modification rates for bank CRE loans (17 percent per quarter *overall* for bank CRE loans during the COVID-19 pandemic, compared to under 10 percent for *distressed* RRE loans during the housing bust) and much larger differences across lender types. We argue that supply-side institutional factors drive CMBS to modify very few loans relative to banks. Our theory does not speak to why banks would modify so many more loans in CRE than in RRE, but a possible candidate is differences in information frictions.

The outline for the rest of the paper is as follows. In Section 2, we present empirical evidence on differences in CRE loan modification rates across lender types. In Section 3, we write down the model. In Section 4, we present the model calibration, quantitative results, and welfare counterfactuals. In Section 5, we conclude.

## 2. CRE LOAN MODIFICATIONS IN THE DATA

In this section, we use loan-level data from banks and CMBS to better understand differences in their modification and delinquency rates. We show that bank CRE loan modifications are both more preemptive, supporting the less-troubled loans that CMBS rarely modify, and more responsive to stress, expanding notably when strains emerge. Banks have lower delinquency rates on more-distressed loans, suggesting that modifications bolster loan performance, but higher delinquency rates on less-distressed loans, consistent with borrowers strategically defaulting to secure a modification.

### 2.1. Data Sources

We rely on two data sources: monthly data on CMBS loans from Trepp and quarterly data on CRE loans held by large US banks from Federal Reserve Y-14Q filings.<sup>5</sup> Each data source provides information on loan terms, property characteristics, and loan performance over time.

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<sup>5</sup>The Y-14 reporting panel consists of banks with consolidated assets of \$50 billion (\$100 billion starting in 2019). Banks report loans with a committed balance of \$1 million or more. The data are at the facility level, but, as most facilities have only one loan, we treat the data as being at the loan level.

We include in the analysis first-lien commercial loans secured by stabilized, non-owner-occupied, nonresidential properties in the United States.<sup>6</sup> We exclude construction and land development loans and owner-occupied CRE loans—loan types predominantly provided by banks—to maintain a similar sample of loans for banks and CMBS.<sup>7</sup> We exclude loans secured by multifamily properties, as government-sponsored enterprises account for a large share of such lending and terms differ notably from those for other property types. We also exclude some minor property types (for example, healthcare) for which there is no consistent categorization across banks and CMBS. These filters limit our sample to loans backed by industrial, lodging, office, and retail properties. Finally, we exclude loans that are cross-collateralized or are missing information on the location of the collateral. Table 1 provides information on origination characteristics for this sample of loans by property and lender type.

The identification of loan modifications differs for the two types of lenders. For CMBS, modification dates and some details on the type of modification are either directly reported by the servicers or derived by our data vendor (Trepp). This information includes whether the modification involved a maturity date extension, a principal reduction, a rate reduction, the capitalization of interest or principal payments, forbearance, or a combination of various modification types. For banks, we impute modifications by identifying changes in loan terms over time, similar to the methodology of [Adelino et al. \(2013\)](#). Specifically, a loan is considered modified if it switched from being amortizing to interest only, if the committed balance rises (indicating interest payments are added to the loan balance as part of a forbearance plan), if the committed balance falls in tandem with a positive cumulative charge-off (indicating a write-off), if the maturity date is extended (outside of a pre-negotiated renewal), or if the loan enters troubled debt restructuring.<sup>8</sup>

For all lender types, we subdivide modifications into two broad types: those that result in a reduction in payments and those that do not. The latter category is

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<sup>6</sup>Additionally, for CMBS loans, we limit our sample to conduit, single asset-single borrower, or large loan deals.

<sup>7</sup>We drop any construction and land development loan or any loan for which the reported property value is an estimate for the property once it is completed or stabilized (as opposed to the value being reported “as is”).

<sup>8</sup>Additionally, we consider changes in origination dates, which occur when there is a substantial change in a loan’s terms.

largely made up of loan extensions.<sup>9</sup> This category can also include other changes, such as adding or removing recourse or cross-collateralization from a loan, though in our data these modifications are rare. Modifications that result in payment changes include interest rate reductions, changes in the amortization schedule (including a switch to interest only), forbearance, and more substantial loan restructurings, such as an A/B split for a CMBS loan.<sup>10</sup> While we provide descriptive information for overall modification rates, we focus most of our attention on payment modifications. Nonpayment modifications—most notably, extensions—might occur for reasons besides preventing default. For example, banks might be willing to extend a loan at the end of its term because it has good risk characteristics.

The two performance measures of interest are whether a loan is modified or 90 days delinquent in a quarter. Delinquency and modification rates are not always directly comparable across lenders: a single bank modification can appear multiple times (for example, if a forbearance period spans quarter-end), and delinquency rates are affected by the duration with which delinquent loans are reported. For this reason, our primary analysis predicts whether loans that had not been previously modified or 90 days delinquent become so in a given quarter. This measure of first modification or delinquency is not sensitive to reporting differences and thus better reflects the rate at which such events occur.

We analyze how loan performance across lenders differs by the degree of stress the loan is experiencing. In the time-series analysis, this amounts to studying changes in modifications and delinquency during the pandemic (covering 2020:Q1 to 2021:Q2). In our cross-sectional analysis we look at loan performance across two different dynamic measures of loan risk: LTV and the debt-service coverage ratio (DSCR). LTV, defined as the ratio of the loan balance to the most recent appraised property value, reflects the ability of the borrower to pay back the loan by selling or refinancing the property. DSCR, defined as the ratio of the collateral's net operating income (NOI) to annual debt obligations, measures how well the property can support the debt service costs associated with the loan. We calculate DSCR ourselves using estimated annual debt service obligations and the reported current NOI. To account for the fact that NOI is necessarily a backward-looking measure, we calculate DSCR using the year-ahead NOI.

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<sup>9</sup>Loan extensions allow a borrower to avoid needing to refinance to make a balloon payment at maturity.

<sup>10</sup>Figure D.2 in the appendix provides details on the share of outstanding loan balances that have received modifications by lender type.



## 2.2. *Modification and Delinquency Rates Over Time*

From Figure 1, we see that CRE loans held in banks' portfolios are modified much more frequently than loans in CMBS pools. In the quarters leading up to the pandemic, banks modified loans at a rate of about 1.5 percent per quarter, while modifications of CMBS loans were almost nonexistent. By contrast, rates at which loans become 90 days delinquent were modestly higher for CMBS.

During the pandemic, these differences widened in absolute terms. Transitions into delinquency spiked for CMBS, reaching a peak of around 5 percent per quarter in 2020:Q3, while remaining under 1 percent for bank loans.<sup>11</sup> Bank loans instead saw a spike in modification rates. Bank loans received modifications at a rate of nearly 10 percent per quarter in 2020:Q1, rising to a rate of 17 percent by the end of 2020. Meanwhile, the CMBS modification rate remained under 5 percent for all quarters.

In Table 2, we disaggregate the information in Figure 1 by property type and modification type. Banks are much more likely to modify loans across property types, with modification rates in 2018 and 2019 that range from 1.3 to 3.2 percent across property types, compared to under 0.1 percent for CMBS. Banks also experienced a larger increase in their modification rates during the pandemic, driven predominantly by payment modifications (mainly forbearances). The modification rate for bank lodging loans rose to 16 percent per quarter during the first year and a half of the pandemic. For other property types, modification rates still rose to near 10 percent per quarter. Meanwhile, for CMBS, modification rates only rose to around 4 percent for lodging loans while remaining under 1 percent for other property types. In the last column of Table 2 we show the share of loans that received either a payment modification or became 90 days delinquent, thus measuring the share of loans not making promised payments either due to delinquency or modification.<sup>12</sup> Modifications for bank loans are high enough that these overall distress rates are much higher than those for loans in CMBS pools, both before and during the pandemic, despite the higher delinquency rates for CMBS.

To get a more accurate estimate of the difference in probability of receiving

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<sup>11</sup>We define a loan as delinquent when it is 90+ days past due. Therefore, the loans entering delinquency in the third quarter generally started to miss payments in the second quarter.

<sup>12</sup>Since loans can both be modified and become 90 days delinquent within a quarter, this rate may not be the exact sum of the rate of payment modifications and delinquencies.

a modification for bank portfolio loans versus those in CMBS, we pool data across lenders and estimate linear probability models predicting modification and delinquency with lender type, while controlling for an array of risk characteristics. Our regressions take the following form:

$$\begin{aligned} \text{Mod}_{i,t} \times 100 = & \beta_1 \text{CMBS}_i + \beta_2 \text{CMBS}_i \times \text{COVID}_t \\ & + \alpha_1 \mathbf{X}_{i,t} + \alpha_2 \mathbf{X}_{i,t} \times \text{COVID}_t + \gamma_t + \nu_i + \delta_i + \zeta_i + \epsilon_{i,t}, \end{aligned} \quad (1)$$

where  $\text{CMBS}_i$  and  $\text{COVID}_t$  are indicators of whether loan  $i$  is funded by CMBS and whether quarter  $t$  is 2020:Q1 or later, respectively.  $\mathbf{X}_{i,t}$  contains the following loan-level controls: log origination amount, term in years, an indicator for whether the loan is interest only, current LTV and DSCR, and LTV and DSCR at origination. We also include time fixed effects ( $\gamma_t$ ), origination year fixed effects ( $\nu_i$ ), state fixed effects ( $\delta_i$ ), and property-type fixed effects ( $\zeta_i$ ). The dependent variable is multiplied by 100, so that the coefficients provide predicted effects in percentage points.

Our left-hand-side variables are indicators for whether loan  $i$  was modified or became 90 days delinquent in quarter  $t$ . To account for differences in the reporting of modifications or delinquency, in each regression we remove observations after the first instance of the outcome of interest. That is, delinquency regressions predict whether previously performing loans first become seriously delinquent in time  $t$ , and modification regressions similarly predict the occurrence of a modification for previously unmodified loans.<sup>13</sup> As a result, our sample size varies slightly in each column.

We present results from these regressions in columns (1)-(3) of Table 3. The results confirm the general patterns shown in Figure 1. Column (1) shows that after controlling for loan-level characteristics, banks and CMBS have similar delinquency rates pre-COVID, with CMBS loans having delinquency rates that are 0.06 percentage points lower. However, CMBS see a much larger spike during the pandemic, with the delinquency rate rising 0.29 percentage points more than for banks. Column (2) shows that CMBS have modification rates that are 1.5 percentage points below banks in normal times, with the difference rising by an additional 4.4 percentage points during the pandemic. These results are similar

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<sup>13</sup>Allowing for multiple modifications results in larger differences between bank and CMBS modification rates.

for payment modifications, shown in column (3).

### 2.3. *Modification and Delinquency Rates by Property Performance*

The time-series evidence suggests that banks modify loans more than CMBS, increase modifications more in times of stress, and provide more preemptive modifications—modifying loans even for less troubled property types. Here, we examine the extent to which such patterns hold in the cross-section by looking at the propensity of lenders to modify loans when the property securing them experiences stress.

Figure 2 displays delinquency, modification, and overall distress rates (defined as either a delinquency or a modification) by current DSCR or LTV. Each panel shows a binned scatterplot, where each bin reflects the average within a quantile of observations based on DSCR or LTV. Reported values are residualized on quarter, property type, origination year, and state  $\times$  CBSA fixed effects.

The two left-hand panels of Figure 2 show delinquency rates by DSCR and LTV for banks and CMBS. Each panel tells a similar story: when property performance metrics look favorable (DSCRs are above 1.5 or LTVs are below 60), bank loans are more likely to become delinquent. However, when conditions deteriorate, CMBS are more likely to become delinquent. For both lenders, high LTVs or low DSCRs increase the likelihood of delinquency, but the effects are much stronger for CMBS.

The middle two panels show modification rates for the two lender types. These illustrate three characteristics of banks' modification behavior that parallel the time-series results. First, banks provide modifications to loans across the entire spectrum of DSCR and LTV. Second, banks are more preemptive in their modifications. For example, the modification rate for bank loans starts to increase sharply as LTVs rise above 75 percent, whereas modifications are fairly flat for CMBS until LTVs reach around 100 percent. Third, modification rates for bank loans that are in clear distress (those with low DSCRs or high LTVs) are much higher than rates for CMBS loans in the same range. As such loans are likely to benefit from a modification, the lack of modifications for CMBS is likely indicative of constraints on the part of CMBS servicers in modifying such loans.

Last, the two right panels compare loan distress rates across the two lender types. We define a distressed loan as one that becomes delinquent or receives a payment modification in a given quarter. Either way, this reflects the rate at which borrowers cease to maintain promised loan payments due to a delinquency or a

lender-provided modification.

For loans that are clearly vulnerable (DSCRs below 1 and LTVs above 100), distress rates on CMBS and bank loans are broadly similar. The key difference between banks and CMBS for such loans is the composition of why borrowers are not maintaining payments: distressed CMBS loans are mostly delinquent, while distressed bank loans are mostly modified. This result suggests that bank loans in this range are only avoiding default because of active modifications provided by banks. In contrast, the higher distress rate for bank loans that are not observably troubled is suggestive of strategic behavior on the part of borrowers to obtain a modification when it is not necessarily needed.

These findings can also be demonstrated through regressions similar to those from equation (1) but with the CMBS dummy interacted with the loan's LTV and DSCR rather than the COVID dummy. Columns (4)-(6) of Table 3 present the results of this analysis.<sup>14</sup> The main findings from Figure 2 broadly hold. Low DSCRs increase the likelihood of modification or delinquency, with banks seeing a larger increase in modifications and CMBS seeing a larger increase in delinquency. Higher LTVs raise the likelihood of delinquency but raise the likelihood of modification only for banks.<sup>15</sup>

#### 2.4. Discussion

To summarize, relative to CMBS, we have shown that (1) banks modify more loans overall; (2) banks modify loans preemptively; (3) banks have higher modification and delinquency rates for less-stressed loans, consistent with modifications encouraging strategic default; and (4) bank borrowers cease making promised loan payments at similar rates for more-stressed loans, but these occurrences mostly consist of modifications for banks and delinquencies for CMBS.

Why would CRE lenders differ so substantially in the propensity to modify their loans? While it is possible that there are fundamental differences between bank and CMBS borrowers that affect the returns to modification, the fact that modification rates differ so substantially, even for clearly distressed borrowers,

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<sup>14</sup>To focus on cross-sectional differences, we restrict the sample to the pre-COVID period. LTV and DSCR are demeaned so that the coefficient on  $CMBS_i$  reflects the predicted effect for a loan at the average LTV and DSCR.

<sup>15</sup>There is little difference in the effects of higher LTVs on delinquency across lenders. This may be because property values are only updated when there is a new appraisal, hindering the ability of the LTV measure to accurately measure stresses in property valuations.

indicates that lenders differ in their ability to modify loans.

A review of institutional factors affecting the lenders supports this hypothesis. CMBS are restricted in their ability to modify loans by both their pooling and servicing agreements (which define the rights and responsibilities of the mortgage servicer) and by IRS policies (which define when a modified mortgage would constitute a new loan), thereby threatening the securitization vehicle's REMIC status and subjecting it to federal taxation.<sup>16</sup> In contrast, the other major CRE lenders are typically the sole debt holder, face minimal restrictions on loan modifications, and were encouraged by regulators to modify loans during the pandemic. In March 2020, banks' regulators issued a joint statement actively encouraging banks to take "proactive actions that can manage or mitigate adverse impacts [of COVID-19] on borrowers." Life insurers, which we emphasize less due to data limitations, were similarly encouraged "to work with borrowers who are unable, or may become unable, to meet their contractual payment obligations because of the effects of COVID-19."<sup>17</sup> In short, institutional differences between CMBS and balance-sheet lenders plausibly result in these lenders differing in loan modification technologies.

In the remainder of this paper, we explore how these differences in modification ability affect the broader CRE market. While the ability to modify loans may benefit borrowers in times of stress, the specter of strategic renegotiation may restrict the range of contracts that banks are willing to offer. That is, if bank borrowers cannot commit to not strategically negotiating lower loan payments, banks may require larger down payments to mitigate this modification risk. Indeed, Table 1 shows that CMBS loans have higher LTVs, on average, than bank loans for the same property type, and Table 4 demonstrates that these differences hold, controlling for other observable loan or property type characteristics.<sup>18</sup>

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<sup>16</sup>We provide more details on the regulatory environment affecting modifications for banks and CMBS in Appendix A, including a more thorough discussion of how tax considerations restrict CMBS modification options.

<sup>17</sup>We do not emphasize life insurers in Section 2 because the limited detail on loan terms and low reporting frequency for life insurer data prevent us from accurately identifying modifications. The statement from the National Association of Insurance Commissioners is available at <https://content.naic.org/sites/default/files/inline-files/INT%2020-03%20%20-%20TDR%20for%20COVID-19%3B%20Consolidated%20Appropriations%20Act%20Update.pdf>.

<sup>18</sup>The regressions predict the effect of the loan being in a CMBS pool on LTV, controlling for other observable characteristics. Two controls stand out as affecting the results: spreads and recourse. First, higher CMBS LTVs partially reflect higher spreads. This result is consistent with CMBS borrowers having a higher demand for leverage, a pattern that endogenously comes out of the model studied in Section 3. Second, predicted LTV differences across lenders are larger when we account for recourse, which enables some bank borrowers to have higher LTVs (Glancy et al., 2021).

### 3. MODEL

In the previous section, we presented empirical evidence that there is heterogeneity in the propensity to modify troubled loans across lender types. Motivated by this finding, we now present a trade-off theory model adapted to aspects of the CRE market in which lenders differ in their ability to modify loans.

We start by deriving expressions for the values of equity and debt in this environment. We then solve for the equilibrium modification strategies, the set of contracts (LTVs and spreads) offered by a competitive loan market, and the loan contracts optimally chosen by borrowers. We then derive how borrowers optimally sort into lenders, which differ in modification ability. Finally, we aggregate across heterogeneous borrowers to solve for lenders' equilibrium loan portfolios, accounting for both differences in loan offers across lenders and the endogenous sorting of borrowers into lenders.

#### 3.1. *Environment and Value Functions*

We start by considering the problem of a particular property investor negotiating a loan contract from a particular lender. At time  $t = 0$ , the investor buys a property partially using perpetual, defaultable debt with a flow coupon payment of  $C$  (to be endogenized later). Let the after-tax NOI from this property at time  $t$  (denoted  $X_t$ ) follow a geometric-Brownian motion process:

$$\frac{dX_t}{X_t} = \mu dt + \sigma dZ_t.$$

Lenders and property investors are risk neutral and discount cash flows at

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To account for this mechanism, we allow lenders in the model to vary in recourse use as well as modification ability.

the risk-free rate  $r$ . Therefore, the present value of promised coupon payments is  $\frac{C}{r}$  and the present value of future NOI is  $\frac{X_t}{r-\mu}$ . Investors earn a flow return of  $X_t - (1 - \tau)C$ , where  $\tau$  is the effective tax rate that determines the tax advantage of debt and thus the demand for leverage.<sup>19</sup>

In the event of default at time  $t$ , the lender can foreclose on the property and recover the unleveraged property value, less a proportional foreclosure cost  $\alpha^F \in [0, 1)$ . In addition, motivated by the finding that loans with recourse are less likely to be modified (Glancy et al., 2021), we allow for the availability of recourse to affect loan recoveries. Specifically, lenders can claim a fraction  $\theta \in [0, 1 - \tau)$  of the present value of promised debt payments from a deficiency judgment, paying a proportional cost of  $\alpha^D \in [0, 1]$ .<sup>20</sup>

The recovery in the event of foreclosure,  $R(X)$ , is therefore

$$R(X) = (1 - \alpha^F) \frac{X}{r - \mu} + (1 - \alpha^D) \theta \frac{C}{r}.$$

The deadweight costs of foreclosure leave room for mutually beneficial loan modifications with the purpose of forestalling loan defaults. In these loan renegotiations, borrowers can make a take-it-or-leave-it offer to the lender to lower their debt service at time  $t$ , following Hackbarth et al. (2007). Therefore, if a borrower seeks a modification, they choose a strategic debt service offer  $S(X)$  so as to make the lender indifferent between foreclosing and accepting the modification. When NOI falls below an endogenous threshold  $X_n$ , lenders become willing to accept a debt service payment that is sufficiently low for borrowers to renegotiate their loan. As a result, there are two regions in the model: a low region (denoted  $L$ ) where  $X \leq X_n$  and lenders receive loan payments  $S(X)$ , and a high region (denoted  $H$ ) where  $X > X_n$  and lenders receive loan payments  $C$ .

We make one key departure from Hackbarth et al. (2007) in how renegotiations work: while the loan is in the renegotiation region and paying  $S(X) < C$ , negotia-

<sup>19</sup>The effective tax rate,  $\tau$ , is a standard parameter in trade-off theory models.  $\tau$  determines the size of the tax shield and, hence, the demand for leverage. It can stand in more generally for other factors that affect the demand for leverage, such as liquidity needs or wedges in the required return on equity between borrowers and lenders.

<sup>20</sup> $\theta = 0$  for non-recourse loans, such as most CMBS or life insurer loans. For recourse loans (the majority of bank loans),  $\theta$  reflects how much borrowers actually expect to pay in a deficiency judgment. Even a full recourse loan would have a low  $\theta$  if the borrower has few outside assets.  $\theta$  is bounded above by  $1 - \tau$  to ensure that there exists a value of  $X_t > 0$  such that borrowers choose to renegotiate their loan.

tions break down at an exogenous rate  $\lambda$ , resulting in foreclosure.<sup>21</sup> Therefore, by varying  $\lambda$ , one can study how differences in modification frictions affect outcomes in the market.

In Appendix B, we derive the equilibrium debt service offer when loans are renegotiated and the values of debt and equity in this environment. Borrowers offer a modified payment  $S(X)$  satisfying

$$S(X) = (1 - \alpha^F)X + (1 - \alpha^D)\theta C.$$

Given this renegotiation outcome, we can solve for the following functions defining the value of the loan,  $D(X)$ , and the value of the equity in the property investment,  $E(X)$ , in the  $H$  and  $L$  regions:

$$\begin{aligned} D_H(X; C, X_n) &= \frac{C}{r} - \left(\frac{X}{X_n}\right)^{-\gamma} \left[ (1 - (1 - \alpha^D)\theta) \frac{C}{r} - (1 - \alpha^F) \frac{X_n}{r - \mu} \right] \\ D_L(X; C, X_n) &= (1 - \alpha^F) \frac{X}{r - \mu} + (1 - \alpha^D)\theta \frac{C}{r} \\ E_H(X; C, X_n) &= \frac{X}{r - \mu} - \frac{(1 - \tau)C}{r} - \left(\frac{X}{X_n}\right)^{-\gamma} \left[ \beta_x \frac{X_n}{r - \mu} - \beta_c \frac{C}{r} \right] \\ E_L(X; C, X_n) &= \frac{1 - (1 - \alpha^F)(1 - \tau)}{r + \lambda - \mu} X - \frac{\lambda \theta C}{r(r + \lambda)} - \frac{(1 - \tau)(1 - \alpha^D)\theta C}{r + \lambda}, \end{aligned} \tag{2}$$

where

$$\begin{aligned} \beta_c &\equiv \frac{\lambda(1 - \tau - \theta) + r(1 - \tau)(1 - (1 - \alpha^D)\theta)}{r + \lambda} \\ \beta_x &\equiv \frac{\lambda + (1 - \alpha^F)(1 - \tau)(r - \mu)}{r + \lambda - \mu} \\ \gamma &= \left( \mu - .5\sigma^2 + \sqrt{(.5\sigma^2 - \mu)^2 + 2\sigma^2 r} \right) / \sigma^2 \end{aligned}$$

are positive constants determined by parameter values.  $\beta_c$  and  $\beta_x$  reflect the sensitivities of  $E(X)$  to property values and coupon payments at the renegotiation threshold, respectively, and  $\gamma$  reflects the inverse of the risk of downward movements in NOI.

We can then determine the renegotiation threshold,  $X_n$ , from the smooth-

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<sup>21</sup>Our model also differs from [Hackbarth et al. \(2007\)](#) in that all loans are first lien; that is, we are not studying differences in debt priority structure.



pasting condition that  $\frac{\partial E_H(X_n)}{\partial X} = \frac{\partial E_L(X_n)}{\partial X}$ :

$$\frac{X_n}{r - \mu} = \frac{\gamma}{1 + \underbrace{\gamma \beta_x}_{\equiv \rho(\lambda)}} \frac{\beta_c}{r} C. \quad (3)$$

Equation 3 implies that borrowers choose to renegotiate a loan when the value of the unlevered property falls below a fraction  $\rho(\lambda)$  of the present value of promised debt service payments. In Appendix C.1, we analytically characterize  $\rho$ . We show that the modification boundary is decreasing in  $\lambda$ , meaning that borrowers are more willing to continue making promised debt payments when modifications are less certain.

That  $\frac{\partial \rho}{\partial \lambda}$  is negative is consistent with the patterns for modification and delinquency rates shown in Figure 2.  $\rho(\lambda)$  determines  $\frac{X_n}{C}$ —the threshold DSCR below which borrowers modify loans. Since  $\rho$  decreases with  $\lambda$ , there is an intermediate range of DSCRs such that borrowers from banks (low  $\lambda$  lenders) would modify, and sometimes go delinquent, while borrowers from CMBS would continue making promised payments. Figure 3 demonstrates this fact. The figure plots debt service payments ( $S(X)$  or  $C$ ) as a function of  $X_t$  for two loans that are identical except for  $\lambda$ . The cross-hatched region shows the range of  $X_t$  such that only bank loans undergo renegotiation and possible default (consistent with banks' higher rates of delinquency and modification for less-stressed properties). For  $X_t$  below this range, all borrowers renegotiate, but more negotiations fail for CMBS (consistent with CMBS' higher delinquency rates and lower modification rates for stressed properties).

### 3.2. Lender Pricing of LTV

Having solved for borrowers' optimal renegotiation strategy (the modification threshold and strategic debt service amount), we can determine the contracts offered by a competitive loan market. Substituting (3) into (2), we can solve for the values of debt and equity for a given NOI and coupon payment. Since lenders will not originate a loan that borrowers would immediately renegotiate, available loan

terms are determined by the valuation of loans in the  $H$  region, which is given by

$$D_H(X; C) = \frac{C}{r} \left( 1 - \underbrace{\left( \frac{\frac{X}{r-\mu}}{\rho \frac{C}{r}} \right)^{-\gamma}}_{\equiv s} \chi \right), \quad (4)$$

where  $\rho$  is as in (3),  $s$  is the loan rate spread, and  $\chi$  reflects the lender's loss from modifying the loan:

$$\begin{aligned} \chi &\equiv \frac{\frac{C}{r} - D(X_n)}{\frac{C}{r}} \\ &= 1 - (1 - \alpha^D)\theta - (1 - \alpha^F)\rho. \end{aligned} \quad (5)$$

Given the expression for  $\chi$  in (5), we see that  $s$  has the intuitive interpretation of being the product of the likelihood of modification and the loss given modification.<sup>22</sup>

Since loans initially price at par, the initial loan balance will be  $D_H(X_0; C)$ , making the coupon payment  $C = r^m D_H(X_0; C)$ , where  $r^m$  is the mortgage rate. Evaluating at  $X_0$  and substituting in for  $C$ , equation (4) can be rearranged to express LTV as a function of loan rate spreads:

$$LTV(s) = \frac{s^{\frac{1}{\gamma}}(1-s)}{\chi^{\frac{1}{\gamma}}\rho}, \quad (6)$$

where  $LTV \equiv \frac{D_H(X_0; C)}{X_0/(r-\mu)}$  is the ratio of loan size to the unlevered property value. Also, note that with this substitution,  $s = \frac{r^m - r}{r^m}$ .<sup>23</sup>

The above expression is effectively the credit supply curve: it determines the schedule of loan terms that lenders are willing to offer property investors. It is clear that lenders are willing to offer higher LTVs for a given spread when borrowers are more willing to maintain promised debt payments instead of seeking a modification ( $\rho$  is low) or when their losses from a modification are lower ( $\chi$  is low). In Appendix C.2, we present the comparative statics of this supply curve

<sup>22</sup>More formally, the first term gives the fair price of a security that pays 1 at the time of modification.

<sup>23</sup>This concept of spreads is convenient for presenting the expressions that follow. When we take the model to the data, we use the more conventional spreads measure  $r^m - r = \frac{s}{1-s}r$ .

with respect to  $\lambda$ . We show that lenders are willing to offer higher LTVs for loans with higher modification frictions because of how modification frictions affect the modification boundary.

In short, while the ability to modify loans provides borrowers some insurance against downward movements in NOI, this gain comes at a cost. Lenders anticipate losses from strategic modification requests and provide less favorable loan terms at origination. Lenders are unwilling to offer high-LTV, easily modified loans, as borrowers would immediately be able to negotiate more favorable terms. Borrowers thus need to provide some protection from strategic renegotiation, either through a high down payment or frictional modifications.

To understand how borrowers evaluate these trade-offs and ultimately choose which type of lender to borrow from, we need to solve for which available contracts borrowers choose and evaluate welfare at the optimal contract. We do this in the next subsection.

### 3.3. Equilibrium Pricing, LTV, and Welfare

Firms choose the debt contract that maximizes firm value  $v(X; C) = E_H(X; C) + D_H(X; C)$ . Substituting in equations (2) and (3) and simplifying,  $v(x; C)$  can be written as

$$v(X; C) = \frac{X}{r - \mu} + \frac{\tau C}{r} - \left( \frac{X}{\rho \frac{C}{r}} \right)^{-\gamma} \Lambda \frac{C}{r}, \quad (7)$$

where

$$\Lambda \equiv \tau\chi + \frac{\lambda}{r + \lambda} \theta(\alpha^D + \tau(1 - \alpha^D)) + \frac{\lambda}{r + \lambda - \mu} \rho(\alpha^F + \tau(1 - \alpha^F))$$

is the deadweight cost from entering the modification region (as a share of  $\frac{C}{r}$ ).  $\tau\chi$  is the lost tax shield due to the lower coupon rate at modification, and the rest of the expression reflects the expected loss due to modifications breaking down (deadweight recovery costs and the loss of the remaining tax shield).<sup>24</sup>

<sup>24</sup>It is useful to note that when  $\mu = 0$ ,  $\Lambda$  can be written as

$$\Lambda |_{\mu=0} = \underbrace{\tau \frac{C - S(X_n)}{C}}_{\text{Lost debt shield from mod}} + \underbrace{\frac{\lambda}{r + \lambda}}_{\text{Pricing of breakdown risk}} \left[ \underbrace{\theta\alpha^D + \rho\alpha^F}_{\text{Foreclosure costs}} + \underbrace{\tau \frac{S(X_n)}{C}}_{\text{Lost debt shield from mod breakdown}} \right].$$

Taking the first-order condition of (7) with respect to  $C$ , we can show that borrowers choose the contract with a spread:

$$s^* = \frac{\tau\chi}{(1+\gamma)\Lambda}, \quad (8)$$

where  $\Lambda$  is defined in (7).

Having now found the optimal spread chosen by the borrower, we can close the model and present closed-form expressions for the LTV chosen by the borrower and for borrower welfare. To find the equilibrium LTV, evaluate the supply function from equation (6) at the chosen spread from equation (8) and obtain

$$LTV = \frac{1}{1+\gamma} \left( \frac{\tau}{(1+\gamma)\Lambda} \right)^{\frac{1}{\gamma}} \rho^{-1} \left( \gamma + \frac{\Lambda - \tau\chi}{\Lambda} \right). \quad (9)$$

Recovering  $C^*$  from the expression for  $s$  in (4) using (8) and then substituting  $C^*$  into (7), we obtain the value of the property investment for the optimal loan contract:

$$v(X_0) = \frac{X_0}{r - \mu} \left[ 1 + \underbrace{\tau \frac{\gamma}{1+\gamma} \left( \frac{\tau}{(1+\gamma)\Lambda} \right)^{\frac{1}{\gamma}} \rho^{-1}}_{\equiv v} \right]. \quad (10)$$

### 3.4. Choice of Lenders

The results thus far determine how a particular borrower  $i$ , defined by a set of risk characteristics and leverage preferences, chooses loan terms from a particular lender  $j$ , defined by  $\lambda_j$ . In this subsection, we model the selection of borrowers into different lenders, effectively endogenizing  $\lambda$  as the optimal choice from a menu of contracts offered by different types of lenders.

First, consider a borrower  $i$  with a particular set of characteristics  $\mathbf{b}_i \equiv (\tau_i, \sigma_i, \mu_i, \alpha_i^F)$ . This borrower needs to choose a particular lender  $j \in J$  to borrow from, with each  $j$  defined by a particular  $(\lambda_j, \theta_j)$ .<sup>25</sup> The borrower does this so as to maximize the value of a property investment with a mortgage from  $j$ . From

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That is, when modifications do not break down, the deadweight loss comes from the lost tax shield due to lower debt payments. Deadweight losses rise as  $\lambda$  rises because, in addition to the lower tax shield, there is a risk of modifications breaking down, resulting in foreclosure costs being realized and the remaining debt shield also being removed.

<sup>25</sup>Given banks are the main recourse lender in the CRE market, we make  $\theta$  a lender characteristic.

equation (10), this amounts to maximizing  $v_{i,j} \equiv \tau_i \frac{\gamma_i}{1+\gamma_i} \left( \frac{\tau}{(1+\gamma_i)\Lambda_{i,j}} \right)^{\frac{1}{\gamma_i}} \rho_{i,j}^{-1}$ , where  $i$  and  $j$  subscripts refer to functions evaluated for borrower and borrower-lender characteristics, respectively.

In reality, one would not expect all sorting in the CRE market to be driven by differences in modification frictions or the use of recourse. CRE lenders may also differ in risk tolerance, desired investment horizons, or various other dimensions (Glancy et al., 2019). To reflect these unmodeled factors affecting sorting, we add unobserved heterogeneity in preferences for CRE lenders so that borrowers match to lenders probabilistically based on their value from borrowing from a particular lender (instead of matching perfectly to the lender with the highest  $v_{i,j}$ ).

In particular, we assume that  $i$  chooses  $j$  if  $v_{i,j}z_{i,j} \geq v_{i,k}z_{i,k} \forall k \in J$ , where  $z_{i,k}$  is an i.i.d., Fréchet distributed random variable reflecting unobserved preferences with CDF  $P(Z < z) = \exp(-z^{-\epsilon})$ . With this setup, the probability that borrower  $i$  chooses lender  $j$  is<sup>26</sup>

$$P_j(\mathbf{b}_i) = \frac{v_{i,j}^\epsilon}{\sum_{k \in J} v_{i,k}^\epsilon}. \quad (11)$$

In short,  $v_{i,j}$  determines the average benefit that  $i$  gets from obtaining a mortgage from  $j$ . This amount reflects how well a particular lender's available terms match a borrower's preferences. Borrowers seeking high-LTV loans (those with a high  $\tau$ ) may like the higher debt capacity that can be found from lenders with a higher  $\lambda$ , while other borrowers may prefer the downside protection offered by lenders with a lower  $\lambda$ .

### 3.5. Aggregation

Having now determined how borrowers sort into particular lenders, we can solve for the portfolio characteristics of different lenders. Let  $f(\mathbf{b})$  denote the probability density function of borrower characteristics.<sup>27</sup> Given the sorting implied by equation (11), the distribution of borrower characteristics for the loans made by

<sup>26</sup>Note that as  $\epsilon \rightarrow \infty$ , the probability of choosing the lender with the highest  $v_{i,j} \rightarrow 1$ . That is, in the limit, this setup encapsulates the situation where lenders maximize welfare as measured in equation (10).

<sup>27</sup>In Section 4, we will quantitatively explore heterogeneity in  $\tau$  to analyze the effects of sorting based on leverage demand. However, here we consider the more general case with heterogeneity in other borrower characteristics.

a particular lender  $j$  will be  $f_j(\mathbf{b}) = \frac{P_j(\mathbf{b})f(\mathbf{b})}{\int P_j(\mathbf{b})f(\mathbf{b})d\mathbf{b}}$ .

We obtain the average characteristics for the loans of a given lender by integrating over this distribution. For example, the average unlevered LTV for lender  $j$  would be  $\int \text{LTV}_j(\mathbf{b})f_j(\mathbf{b})d\mathbf{b}$ , where  $\text{LTV}_j(\mathbf{b})$  comes from equation (9) evaluated at a particular set of borrower and lender characteristics.

This expression shows that lenders' portfolios will differ for two reasons. First, lenders offer different terms, reflecting differences in  $\lambda$  and the effect  $\lambda$  has on loan outcomes. That is, lenders differ in the loans that would be made to an identical borrower. Second, lenders differ in which borrowers they serve. Borrowers disproportionately sort into the lenders that better match their preferences, creating differences in, for example, borrowers' willingness to accept higher spreads to achieve higher leverage.

## 4. QUANTITATIVE RESULTS

We now examine the quantitative implications of the model. Lenders differ in their ability to modify loans, resulting in a varied willingness to make high-LTV loans. Borrowers are heterogeneous in their demand for debt, causing higher demand borrowers to sort into lenders offering higher debt capacity. The first section presents results from a two-lender calibration, where lenders differ only in their ability to modify loans. The second section adds a recourse lender to the calibration, improving the model's ability to hit untargeted moments. The final subsection examines the welfare implications of reducing modification frictions in CMBS.

### 4.1. *Two-Lender Calibration of the Model*

We start by calibrating parameters for a two-lender calibration of the model, where borrowers choose between banks and CMBS, which differ only in the rate at which modifications break down. We then investigate how modification frictions affect LTVs and spreads for loans from these lenders. This calibration is less realistic quantitatively than the calibration considered in the next subsection, but it provides a useful first step in understanding the mechanics of the model.

### 4.1.1. Calibration

To provide a broad overview of the calibration, we directly set  $\mu$ ,  $\lambda_j$ , and some parameters of  $f(\mathbf{b})$  based on values from the data or the related literature. We then jointly calibrate the remaining parameters to match relevant moments in the data.

Regarding lender parameters, we set  $\lambda_j$  for each lender to equate  $\frac{\lambda_j}{r}$  to the delinquency-to-modification rates reported in Table 5 (0.64 for banks, 7.76 for CMBS). In this version of the calibration, both lenders are considered to be non-recourse ( $\theta_j=0$ ), so any difference between the lenders reflects the effects of modification frictions.

Regarding the borrower parameters, we will start by discussing parameters related to the distribution of borrower characteristics, as other moments involve integrating over this distribution. We allow  $\tau$  to be heterogeneous so as to study how borrowers sort into lenders based on their demand for debt. We assume that  $\tau_i \sim \beta(a, b, \underline{\tau}, \bar{\tau})$  and calibrate these parameters to match the distribution of LTVs in CMBS pools, omitting the highest and lowest percentiles to reduce the effects of reporting errors and outliers.<sup>28</sup>  $\underline{\tau}$  and  $\bar{\tau}$  are set to match the lowest and highest CMBS LTVs in the data (30 percent and 75 percent, respectively). The shape parameters,  $a$  and  $b$ , come from the joint calibration, with the mean and residual standard deviation of CMBS LTV as the corresponding target moments.<sup>29</sup> We assume that the value from leverage is capitalized into appraisals and transaction prices, so that the true LTV for a property is  $\frac{LTV_j(s^*)}{1+v_{i,j}}$ , where  $LTV(s^*)$  is the optimal unlevered LTV from equation (9), and  $v_{i,j}$  is the markup on the property value due to the benefits of leverage from equation (10).

Turning to the remaining parameters, we set  $\mu = .01$  so that average NOI growth matches the 1 percent average rent growth in An et al. (2016).<sup>30</sup>  $r$  is targeted to match the 5.5 percent national cap rates in CBRE Econometric Advisors data.<sup>31</sup>  $\alpha^F$  is targeted to produce the 30 percent average foreclosure cost in Brown

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<sup>28</sup>We focus on CMBS, since the lack of recourse or relationship lending means the data-generating process for CMBS likely aligns best with the factors incorporated into the model, with loan underwriting and performance driven by the cash flows of the underlying property.

<sup>29</sup>Some variation in CMBS LTVs reflects factors that are not accounted for in the model, for example, differences in LTV limits by property types. Since all of the variation in CMBS LTVs in the model reflects borrower preferences, we target the residual standard deviation after controlling for size, amortization, duration, pari passu status, and year and property type fixed effects.

<sup>30</sup>An et al. (2016) use panel data on property-level rents from 2001:Q2 to 2010:Q2 to estimate their model. See Table 3 for the GLS estimate of long-term average rent growth we use.

<sup>31</sup>The mean national cap rate (NOI as a fraction of property value) in the CBRE data is 5.5

et al. (2006).<sup>32</sup>  $\sigma$  targets the 2.51 percent average spread on CMBS loans. Finally, we calibrate  $\epsilon$ , which reflects the sensitivity of market shares to changes in  $v_{i,j}$ , to match the elasticity of CMBS market shares with respect to loan rates in Glancy et al. (2019).<sup>33</sup>

We present the results from our calibration in Table 6. The top panel reports parameters that are either directly set or exactly determined by other parameters, while the bottom panel reports parameters determined in the joint calibration.

$\tau$  is estimated to range from 0.04 to 0.45, with a distribution that is right-skewed. Given the estimated required return of 7 percent, the modification breakdown rates are calibrated as 0.05 and 0.55 for banks and CMBS, respectively. NOI is estimated as having a volatility of 27 percent, and the calibrated  $\alpha^F$  implies that recoveries average 78 percent of the unlevered property value.<sup>34</sup>

The right-most columns indicate that the model is successful at fitting the targeted moments. The targeted moments in the joint calibration—cap rates, foreclosure costs, CMBS spreads, the mean and dispersion of CMBS LTVs, and the sensitivity of CMBS market shares to rate shocks—are all hit within at least two decimal places.

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percent, averaging over property types and quarters from 2012 to 2019. The cap rate in the model is  $\frac{r-\mu}{1+v_{i,j}}$ . Since the numerator is heterogeneous, the target is the average over borrowers and lenders.

<sup>32</sup>Based on a sample of distressed life-insurer-owned commercial properties, Brown et al. (2006) find that sales prices were about 30 percent lower than transfer values after accounting for capital expenditures. Note that  $1 - \alpha^F$  is the recovery as a share of the unlevered property value, so the foreclosure cost relative to the actual property value is  $1 - \frac{1-\alpha^F}{1+v_{i,j}}$ .

<sup>33</sup>In Table 6, the authors estimate that a 25 basis point increase in CMBS loan rates—equivalent to a 1 percentage point origination fee per a common heuristic—causes about a quarter of CMBS borrowers to switch to banks. We calibrate  $\epsilon$  so that such a decline in the value of borrowing from CMBS reduces the CMBS market share by about a quarter. That is, a 25 basis point shock reduces values by 1 percent of the loan size (or the LTV ratio as a percent of the property value). For example, the shock would be the equivalent of a 0.65 percentage point decline in  $v_{i,CMBS}$  for a 65 percent LTV loan.

<sup>34</sup>Note that this is the volatility of NOI of a single property, so it will naturally be higher than estimates using index data.



#### 4.1.2. *Effects of Modification Frictions on LTVs and Spreads*

With the calibrated model, we can now investigate how modification frictions affect CRE loan market outcomes. Figure 4 plots how market shares of banks and CMBS (depicted by the blue and red areas, respectively) vary by  $\tau$ . The figure additionally plots LTVs (left panel) and spreads (right panel) as functions of  $\tau$  for both lenders (shown by the equivalent color lines). This figure therefore displays both how underwriting terms vary for a particular borrower (different terms given  $\tau$ ) and how borrowers sort into lenders (different market shares by  $\tau$ ).

LTVs for CMBS loans are more responsive to differences in  $\tau$  than for banks loans. The bank LTV function is increasing but flattens out quickly, reflecting the tight limits banks need to impose to prevent strategic renegotiation. The CMBS LTV function is steeper, meaning that CMBS increase LTV more for borrowers seeking leverage. This pattern results in CMBS having higher LTVs than banks for loans to high  $\tau$  borrowers. In contrast, CMBS loans have lower LTVs for low  $\tau$  borrowers, as difficulty modifying loans increases the risks associated with leverage.

Though CMBS do not uniformly have higher LTVs for all borrowers, variation in market shares causes CMBS to have more high-LTV loans in their portfolio. High  $\tau$  borrowers, unable to receive high-LTV loans from banks, disproportionately borrow from CMBS, as shown by CMBS market shares increasing in  $\tau$ . Simply put, the higher debt capacity at CMBS is valued by high-demand borrowers, causing CMBS to make proportionally more loans to such borrowers.

Differences in spreads across lenders are more consistent; banks require a premium in order to offset expected future declines in cash flows from modifications. As a result, banks charge higher spreads for all  $\tau$ s. However, while banks require higher spreads for all borrowers, there are offsetting compositional effects. Spreads increase monotonically in  $\tau$  since high  $\tau$  borrowers choose high-spread, high-LTV loans. Since CMBS make more loans to the types of borrowers that choose high-spread loans, they can still have higher spreads, on average, if the sorting effect is strong enough.

Table 7 shows the average LTV and spreads by lender type for the two-lender calibration. Differences in these averages reflect both variation in loan outcomes at a particular  $\tau$  and the sorting effects from lenders serving different customers. CMBS have LTVs of 64 percent and spreads of 2.43, as in the data. Of greater interest are the bank results, as those moments are not targeted in the calibration.

The calibrated model is also successful at reproducing bank LTVs: bank LTVs are 58 percent in the data and 59 percent in the model. However, the model misses with spreads: spreads on bank CRE loans are 16 basis points below CMBS in the data but are 12 basis points above CMBS in the model. That is, the premium banks charge to modify loans is more than enough to offset the sorting effects, resulting in banks having higher spreads than CMBS, contrary to the data.

Overall, this calibration is useful for understanding the effects of modification frictions. Since the lenders differ only in  $\lambda$ , all of the differences between banks and CMBS documented here reflect the effects that modifications have on loan underwriting and lender selection. This analysis clearly shows that modification frictions enable higher LTV lending and disproportionately attract borrowers seeking higher leverage.

However, quantitatively the model misses in some dimensions: bank spreads are too high (by nearly 30 basis points) and their LTV limits are too low (no bank loans have LTVs above 65 percent). These results suggest that bank loans have other characteristics that mitigate the effects strategic renegotiation has on loan pricing and LTV limits. To better match the data, we next add in a recourse lender. Section 3 shows that recourse acts as a substitute for modification frictions by discouraging strategic renegotiations, increasing debt capacity, and lowering the cost of bank CRE loans. As most bank loans have recourse (Glancy et al., 2021), failing to account for these effects may contribute to the overly high bank spreads in the model.

## 4.2. *Three-Lender Calibration of the Model*

In this section, we add a recourse lender to the calibration and show that the model comes very close to reproducing the average LTVs and spreads in the data.

### 4.2.1. *Calibration*

Relative to the calibration in Section 4.1.1, we make two major changes. First, we expand the set of lenders ( $J$ ) that borrowers can choose from. We consider three lender types, differing in both modification frictions and recourse, that broadly span the various kinds of credit available from the major CRE lenders:  $(\lambda_{\text{Bank}}, \theta)$  represents modifiable, recourse loans such as typical bank loans;  $(\lambda_{\text{CMBS}}, 0)$  represents low-modification, non-recourse loans such as CMBS loans; and  $(\lambda_{\text{Bank}}, 0)$

represents modifiable, non-recourse loans such as those provided by life insurers and some banks.<sup>35</sup> For brevity, we refer to these three lenders as banks, CMBS, and life insurers, respectively, though banks provide both recourse and non-recourse credit.

The second major change is that two more variables now need to be added to the joint calibration:  $\theta$  and  $\alpha^D$ . The recourse parameter,  $\theta$ , generally determines the effects of recourse on supply, and the cost of deficiency judgments,  $\alpha^D$ , generally determines the extent to which borrowers respond to recourse by choosing lower spreads or higher LTVs. We thus estimate  $\theta_{\text{Bank}}$  and  $\alpha^D$  to match the 20 basis point effect of recourse on spreads and 2.8 percentage point effect of recourse on LTVs found in [Glancy et al. \(2021\)](#).<sup>36</sup> We additionally alter the target change in market share from a 25 basis point CMBS shock to reflect the fact that there is another lender that borrowers can switch to. Instead of targeting the roughly one-quarter of borrowers that switch to banks, we now target the 37.5 percent of CMBS borrowers that switch to either banks or life insurers in [Glancy et al. \(2019\)](#).

We present results from the three-lender calibration in Appendix Table [D.1](#). Most of the parameters are in line with those from Table [6](#). The right-most columns indicate that the model is still successful at fitting the targeted moments beyond those that are set directly. Regarding the new parameters, the value for  $\theta$  indicates that banks expect to lose about 7.5 percent of the present value of promised debt payments from a deficiency judgment upon foreclosure, while the value for  $\alpha^D$  indicates that banks expect to lose over 40 percent of this due to the costs of collecting a deficiency judgment.

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<sup>35</sup>We do not emphasize life insurers in Section [2](#) because of data limitations. We treat life insurers as identical to banks in terms of modifications, as their regulators also encouraged them to provide accommodation to stressed borrowers, and they also saw little increase in loan delinquency during the pandemic. Despite the inferior data, accounting for life insurers is relevant as they are one of the three major CRE lenders, with a 15 percent market share, roughly comparable to CMBS ([Glancy et al., 2019](#)).

<sup>36</sup>The authors use loan-level data from bank CRE portfolios to identify these effects, exploiting cross-loan variation in recourse controlling for other loan and property characteristics. Since the study is of bank loans, the model moment is the difference in LTVs and spreads for loans with and without recourse for the borrowers that sort into banks.

#### 4.2.2. Average LTVs and Spreads

Figure 5 plots how market shares, LTVs, and spreads vary by  $\tau$  for the three lenders. The figure tells a story similar to the one portrayed in Figure 4. As before, LTVs at CMBS are more responsive to borrower demand, resulting in higher CMBS LTVs for high  $\tau$  borrowers relative to other lenders. CMBS also continue to achieve higher market shares at higher  $\tau$ s and to provide lower spreads throughout the distribution.

While the differences between high and low  $\lambda$  lenders are similar, there is now variation within the low  $\lambda$  lenders. Recourse lenders provide higher LTVs and lower spreads than non-recourse lenders throughout the distribution. LTV limits for the recourse lender are less tight, resulting in that lender making loans with LTVs above the maximum LTV provided by the non-recourse lender. In turn, this availability of higher-LTV loans allows the recourse lender to achieve a greater market share at intermediate levels of demand (though the highest-demand borrowers still predominantly go to CMBS).

What do these patterns mean for the average portfolio characteristics of the lenders? Table 8 shows the average LTVs and spreads by lender type for the three-lender calibration. The results align well with the averages for the primary lenders in the market. Average LTV differences in the model are as expected given the sorting effects and differences in LTVs displayed in Figure 5: CMBS have the highest LTVs at 64 percent, followed by the recourse lender at 60 percent, and then the non-recourse balance-sheet lender at 56 percent. These match up well with the data as banks have an average LTV of 58 percent (in between that of the recourse and non-recourse lender), and life insurers have an average LTV of 56 percent (equaling that of the non-recourse balance-sheet lender).<sup>37</sup>

Spreads are also reasonably close to those in the data. Average spreads for balance-sheet lenders in the model and data all fall within a 9 basis point range, running from 2.18 percent for life insurers and 2.27 percent for banks, with the LTVs for the balance-sheet lenders in the model falling in between. In the model, the direct effect of recourse on loan rate spreads roughly offsets the sorting effect

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<sup>37</sup>Appendix Figure D.3 plots the distribution of at-origination LTV for loans from banks, CMBS, and life insurers. As discussed in Glancy et al. (2019), CMBS loans tend to receive higher LTVs, with modal LTVs around 70 percent, compared to around 65 percent for banks. These higher LTVs for bank loans are not due to differences in other observable characteristics, as CMBS loans are predicted to have higher LTVs even controlling for location, property type, loan size, amortization, and origination year, as shown in Table 4.

from the recourse lender serving more high  $\tau$  borrowers, resulting in loan rate spreads that are similar.<sup>38</sup>

Overall, the three-lender calibration is successful at capturing patterns in the data. Accounting for the effects of recourse reduces the overly high spreads for modifiable loans in the two-lender calibration. Adding the recourse lender lowers spreads for balance-sheet lenders because most such loans now either have recourse (providing protection from renegotiation) or go to borrowers seeking low LTVs. Finally, recourse increases debt capacity and thus addresses the very tight LTV limits for bank loans implied by the two-lender calibration.

### 4.3. Welfare

With a quantitatively reasonable calibrated model in hand, we can now investigate the welfare implications of changing modification frictions. We focus on the effects of reducing frictions at CMBS, as those frictions to some degree reflect policy choices that can be altered. Indeed, the IRS issued guidance to enable more modifications during the pandemic, likely contributing to the decline in the delinquency-to-modification ratio shown in Table 5 and the spike in CMBS forbearances shown in Appendix Figure D.2.<sup>39</sup> Were such an easing of modification restrictions to be made permanent, how would this affect the welfare of those subsequently seeking a commercial mortgage?

In the model, welfare is reflected in  $v_{i,j}$ —that is, the increase in property value (relative to the unlevered value) achieved with a loan from  $j$ . Higher spreads, lower allowable leverage, or a greater risk of losing the property in a foreclosure reduce this value. Consequently, the welfare implications of changing modification frictions depend on the counteracting effects frictions have in easing underwriting terms but reducing protection against price declines.

Figure 6 plots  $v_{i,\text{Bank}}$  and  $v_{i,\text{CMBS}}$ , normalized to  $v_{i,\text{Life}}$ , for different values of  $\tau$ . The line for banks, in blue, thus shows how borrowers view recourse (since banks and life insurers differ only in  $\theta_j$ ), while the line for CMBS, in red, shows how borrowers value modification frictions (since CMBS and life insurers differ

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<sup>38</sup>Life insurers have risk-sensitive capital requirements that cause them to concentrate in safer loans (Glancy et al., 2019). As the model only accounts for differences in the use of recourse and loan modifications across lenders, this mechanism does not explain the slightly lower spreads at life insurers.

<sup>39</sup>The IRS took steps to allow more modifications under REMIC laws during the pandemic. See IRS guidance available at <https://www.irs.gov/pub/irs-drop/rp-20-26.pdf>.

only in  $\lambda_j$ ). Both lines are increasing in  $\tau$ , reflecting the fact that recourse and modification frictions both facilitate higher LTV lending by discouraging strategic default. Consistent with the market shares shown in Figure 5, life insurers are preferred at the lowest  $\tau$ s, CMBS at the highest  $\tau$ s, and banks in between.

The dashed red line shows the relative value for CMBS after reducing  $\lambda$  by a factor of 5. Reducing modification frictions rotates the CMBS value function toward that of life insurers. While some borrowers benefit from the reduction in modification frictions—that is, the ones with lower demand for leverage—the overall effect on welfare is negative. Since CMBS make few loans to borrowers with low  $\tau$ s—their value functions are well below those of banks and life insurers for such borrowers—the benefits realized by low  $\tau$  borrowers are small on average. As a result, reducing modification frictions is associated with lower welfare on average.

This effect is seen more clearly in Figure 7, which plots how expected welfare is affected by reducing the modification breakdown rate at CMBS by a factor of 5. Recall from Section 3.4 that borrowers maximize  $z_{i,j}v_{i,j}$ , where  $z_{i,j}$  is a Fréchet distributed random variable. The figure plots  $\bar{v}(\tau_i) = \mathbb{E}(\max_j\{v_{i,j}z_{i,j}\})$  when CMBS modification frictions are reduced by a factor of 5, relative to the expected value in the calibrated model.<sup>40</sup>

This average value reflects how much the value of borrowing from CMBS changes for a given  $\tau$  and how likely CMBS are to lend to different borrowers. The figure shows that while there is a welfare gain for low  $\tau$  borrowers, the gain is small (under 1 percent) since most of these borrowers will not choose CMBS loans. Welfare changes more notably for high  $\tau$  borrowers, who are more reliant on CMBS. Welfare declines by over 4 percent for the borrowers with the highest demand for leverage. In aggregate, averaging across borrowers, this change amounts to a little more than a half percentage point decline in aggregate welfare.<sup>41</sup>

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<sup>40</sup>Integrating over the idiosyncratic lender preferences, we get that the expected welfare for a borrower with a given  $\tau$  is:

$$\bar{v}(\tau_i) = \mathbb{E}(\max_j\{v_{i,j}z_{i,j}\}) = \Gamma\left(\frac{\epsilon-1}{\epsilon}\right)\left(\sum_{j \in J} v_{i,j}^\epsilon\right)^{\frac{1}{\epsilon}},$$

which is increasing in each  $v_{i,j}$ , with a greater influence from the lenders with a higher  $v_{i,j}$ .

<sup>41</sup>The results are qualitatively similar in the two-lender calibration of the model, as the same mechanisms are at play: small gains at low  $\tau$ s are offset by declines for the higher  $\tau$  borrowers that are more likely to use CMBS credit. Quantitatively, the welfare costs of easing modifications are higher in the two-lender calibration (nearly a 1 percent decline in welfare), as non-recourse

Altogether, the welfare exercise demonstrates the importance of variety in loan underwriting. While most borrowers benefit from the ability to modify loans, CMBS serve an important niche in the market. Difficulties in modifying loans enable borrowers to achieve higher leverage than is available from lenders for which strategic renegotiation is more of a concern. Reducing CMBS' advantage in this regard is thus costly, both on average and especially for high-leverage borrowers.

## 5. CONCLUSION

We investigate how differences in the ability to modify loans affect CRE loan outcomes. Empirically, we demonstrate that banks are more likely to modify loans than CMBS and are more willing to offer preemptive modifications. To better understand the equilibrium implications of these modification patterns, we build a tractable trade-off theory model adapted to the CRE market where modification frictions differ between lender types. We show that modification frictions discourage strategic renegotiation and facilitate higher LTV lending. In turn, borrowers demanding higher leverage disproportionately match to lenders with higher modification frictions. The model can thus explain why CMBS loans have higher average LTVs than bank loans. The model also allows us to evaluate the effects of changing modification frictions. Reducing modification frictions at CMBS constricts the range of contracts offered by CMBS and lowers welfare for borrowers seeking higher LTV loans.

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balance-sheet lenders are a worse substitute for high-LTV CMBS loans.

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	Loans (#)	Orig. Amt (Mil.\$)	Orig. LTV	Orig. DSCR	Rate Spread (percent)	Term	IO (percent)	Floating Rate (percent)	Recourse (percent)
<b>Banks</b>									
Industrial	4,809	9	59	2.6	2.28	7	17	53	76
Lodging	1,975	22	58	3.5	2.63	7	27	61	61
Office	8,591	19	60	2.7	2.24	7	27	56	68
Retail	10,690	8	58	2.5	2.27	7	17	51	74
<b>CMBS</b>									
Industrial	1,104	14	63	1.9	2.54	9	52	3	1.9
Lodging	3,233	25	62	2.2	2.81	9	26	5	1.2
Office	4,238	36	62	2.0	2.48	9	66	7	2.4
Retail	6,554	18	64	1.8	2.47	10	55	2	0.6

Table 1: LOAN ORIGINATION CHARACTERISTICS FOR BANK AND CMBS LOANS. *Note:* Limited to loans originated between 2012 and 2019. Bank loans are limited to those originated after a lender begins reporting. All values are unweighted means. IO is interest-only. *Source:* Authors' calculations using Trepp CMBS data and Y-14 H.2 Schedule.

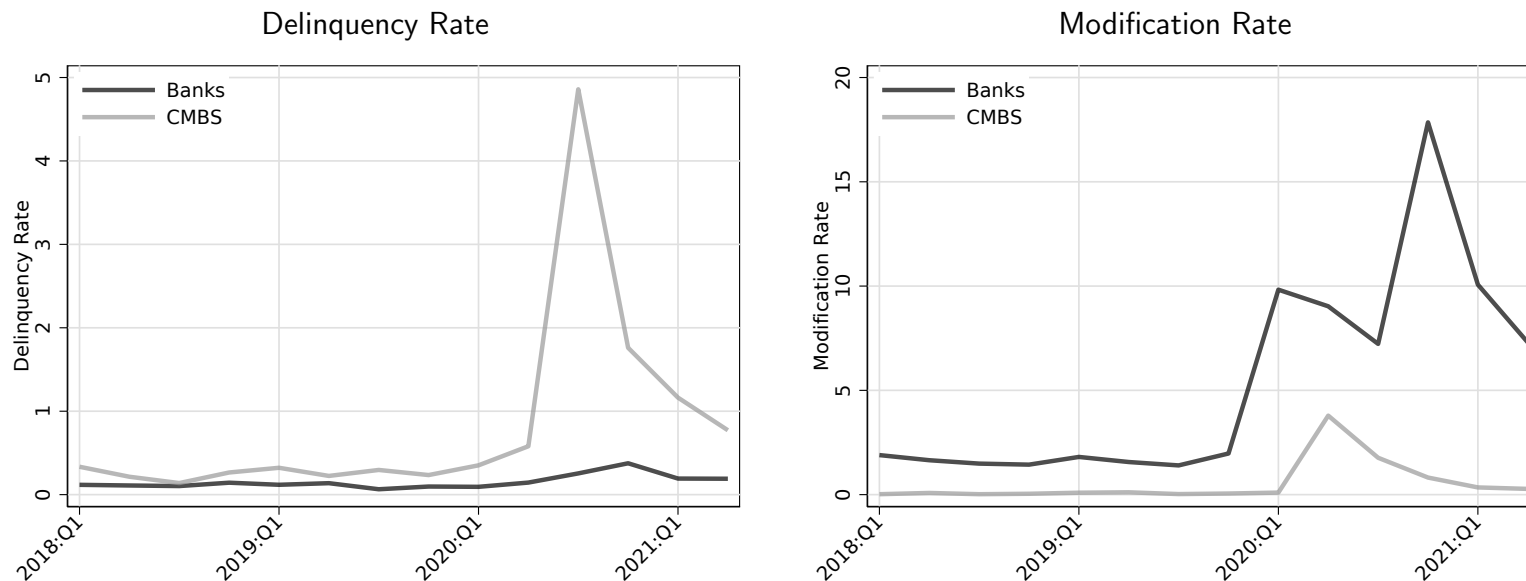


Figure 1: BANK AND CMBS DELINQUENCY AND MODIFICATION RATES. *Note:* Modifications include both payment and nonpayment modifications. Rates are calculated as the share of all outstanding loans in a given quarter that become 90 days delinquent or receive a modification (in percentage terms), where all loans more than 120 days delinquent have been removed from the sample. *Source:* Authors' calculations using Trepp CMBS data and Y-14 H.2 Schedule.

	2018:Q1–2019:Q4					2020:Q1–2021:Q2				
	Mod. Rate			Delinq. Rate	Delinq. or Pay Mod.	Mod. Rate			Delinq. Rate	Delinq. or Pay Mod.
	All	Pay	Other			All	Pay	Other		
<b>Banks</b>										
Industrial	1.36	1.25	1.29	0.09	1.32	9.05	8.92	1.27	0.07	8.98
Lodging	3.18	2.93	1.98	0.24	3.15	16.48	15.82	2.69	0.99	16.50
Office	1.87	1.60	1.69	0.10	1.69	10.51	10.17	1.73	0.13	10.26
Retail	1.42	1.25	1.45	0.11	1.35	9.82	9.23	2.10	0.23	9.43
<b>CMBS</b>										
Industrial	0.06	0.04	0.02	0.24	0.27	0.05	0.02	0.03	0.30	0.31
Lodging	0.05	0.02	0.03	0.30	0.31	4.18	2.71	1.48	4.55	7.16
Office	0.08	0.02	0.06	0.21	0.23	0.23	0.08	0.16	0.39	0.46
Retail	0.05	0.01	0.04	0.26	0.27	0.65	0.42	0.24	1.25	1.64

Table 2: MODIFICATION AND DELINQUENCY RATES. *Note:* Average quarterly modification and 90-day delinquency rates for bank and CMBS portfolios. Modification rates are calculated as the share of loans (in percentage terms) that are less than 120 days delinquent that receive a modification in a given quarter. Delinquency rates are calculated as the share of loans (in percentage terms) that are less than 120 days delinquent and become 90 days delinquent in the given quarter. *Source:* Authors' calculations using Trepp CMBS data and Y-14 H.2 Schedule.

	Delinquency	All Mods	Payment Mods	Delinquency	All Mods	Payment Mods
	(1)	(2)	(3)	(4)	(5)	(6)
CMBS	-0.0579** (0.0243)	-1.467*** (0.0442)	-1.292*** (0.0397)	-0.0499*** (0.0178)	-1.798*** (0.0338)	-1.626*** (0.0299)
CMBS × Covid	0.291*** (0.0588)	-4.374*** (0.108)	-4.163*** (0.0972)			
CMBS × LTV				0.00154 (0.00102)	-0.0111*** (0.00193)	-0.0123*** (0.00171)
CMBS × DSCR				-0.149*** (0.0295)	0.315*** (0.0560)	0.193*** (0.0496)
LTV	0.0126*** (0.00129)	0.00228 (0.00234)	0.000627 (0.00211)	0.0117*** (0.00105)	0.0106*** (0.00200)	0.00954*** (0.00177)
DSCR	-0.231*** (0.0202)	-0.184*** (0.0369)	-0.122*** (0.0332)	-0.164*** (0.0205)	-0.301*** (0.0389)	-0.180*** (0.0344)
N	516,507	515,173	515,392	426,960	426,337	426,480
R2	0.02	0.04	0.03	0.00	0.02	0.01
Mean of Dep. Var. for Banks (%)	.11	2.02	1.69	1.39	1.09	1.0
Quarter FEs	Y	Y	Y	Y	Y	Y
Orig. Year FEs	Y	Y	Y	Y	Y	Y
State FEs	Y	Y	Y	Y	Y	Y
Property Type FEs	Y	Y	Y	Y	Y	Y
Controls and FEs × Covid	Y	Y	Y	-	-	-
Sample	2012:Q1–2021:Q2	2012:Q1–2021:Q2	2012:Q1–2021:Q2	2012:Q1–2019:Q4	2012:Q1–2019:Q4	2012:Q1–2019:Q4

Table 3: LINEAR PROBABILITY REGRESSIONS. *Note:* All regressions are of the form described in equation (1). The sample includes loans that are less than 120 days delinquent with at-origination DSCRs greater than one. Modification regressions predict first modification, so loan-quarter observations after a loan modification are removed from the sample. This causes observation numbers to vary across specifications. The dependent variables of interest are whether a loan goes 90 days delinquent (Columns 1 & 4), receives a modification (Columns 2 & 5), or receives a payment modification (Columns 3 & 6) in a quarter. Columns (1)-(3) include the interaction of the COVID and CMBS indicators. Columns (4)-(6) restrict the sample to the pre-COVID period and instead include the CMBS indicator interacted with the current LTV and DSCR. Dependent variables are multiplied by 100, so coefficients reflect predicted effects in percentage points. *Source:* Authors' calculations using Trepp CMBS data and Y-14 H.2 Schedule.

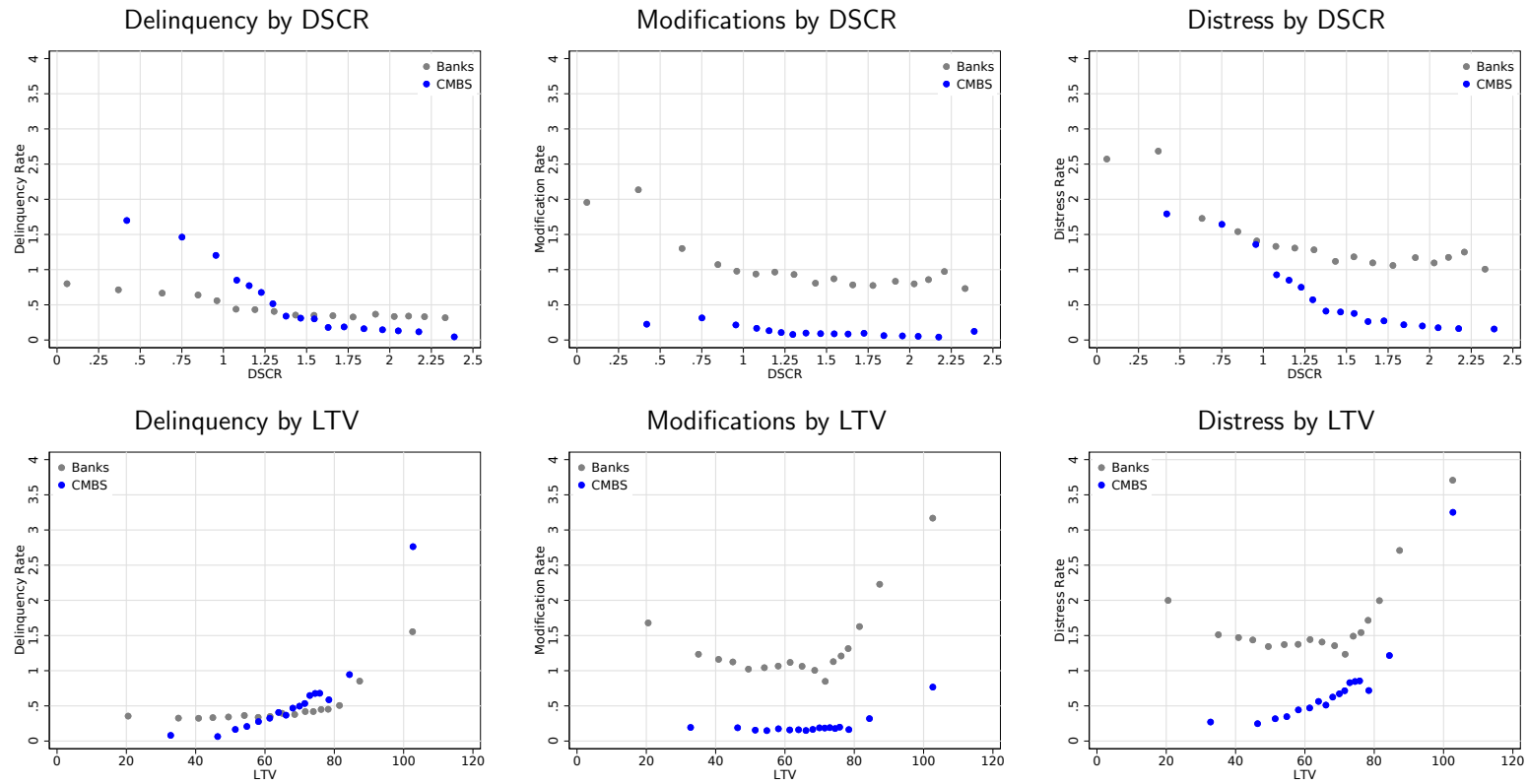


Figure 2: DELINQUENCY AND MODIFICATION RATES BY CURRENT DSCR AND LTV. *Note:* Data include loan-quarter observations in 2012q1–2019q4. Rates are in percentage points. All values are residualized on origination year, quarter, property type, and state by CBSA fixed effects. Plots are binned scatterplots where observations are binned according to the residualized value of the x-axis. When looking across LTV, observations are binned into the following quantiles: {5, 10, 15, 20, 30, 40, 50, 60, 70, 80, 85, 90, 92.5, 95, 97.5, 99}. When looking across DSCR, observations are binned into the following quantiles: {1, 2.5, 5, 7.5, 10, 15, 20, 30, 40, 50, 60, 70, 80, 85, 90, 95}. All loans 120 days or more delinquent are excluded. We remove loans that have a DSCR at origination of less than one. *Source:* Authors' calculations using Trepp CMBS data and Y-14 H.2 Schedule.

	LTV (in percentage points)			
	Full Sample		Non-recourse loans	
	(1)	(2)	(3)	(4)
CMBS	2.397*** (0.178)	1.684*** (0.182)	3.665*** (0.196)	2.923*** (0.199)
Interest Only	-1.853*** (0.185)	-1.959*** (0.188)	-2.131*** (0.206)	-2.071*** (0.209)
ln(Origination Amount)	1.775*** (0.0731)	1.960*** (0.0752)	0.853*** (0.0888)	1.195*** (0.0926)
Interest Rate Spread		2.225*** (0.113)		2.780*** (0.142)
N	45,290	43,103	23,296	22,357
R2	0.16	0.16	0.15	0.17
Orig. Year FEs	Y	Y	Y	Y
Property Type FEs	Y	Y	Y	Y
CBSA × State FEs	Y	Y	Y	Y

Table 4: LTV REGRESSIONS. *Note:* Each column presents a regression predicting at-origination LTV with lender type for the combined sample of bank and CMBS loans. Columns (1) and (2) include all first-lien loans on stabilized properties in the sample, with column (2) adding a control for loan rate spreads. Columns (3) and (4) exclude bank loans with recourse from the sample, with column (4) including the spread control. *Source:* Authors' calculations using Trepp CMBS data and Y-14 H.2 Schedule.

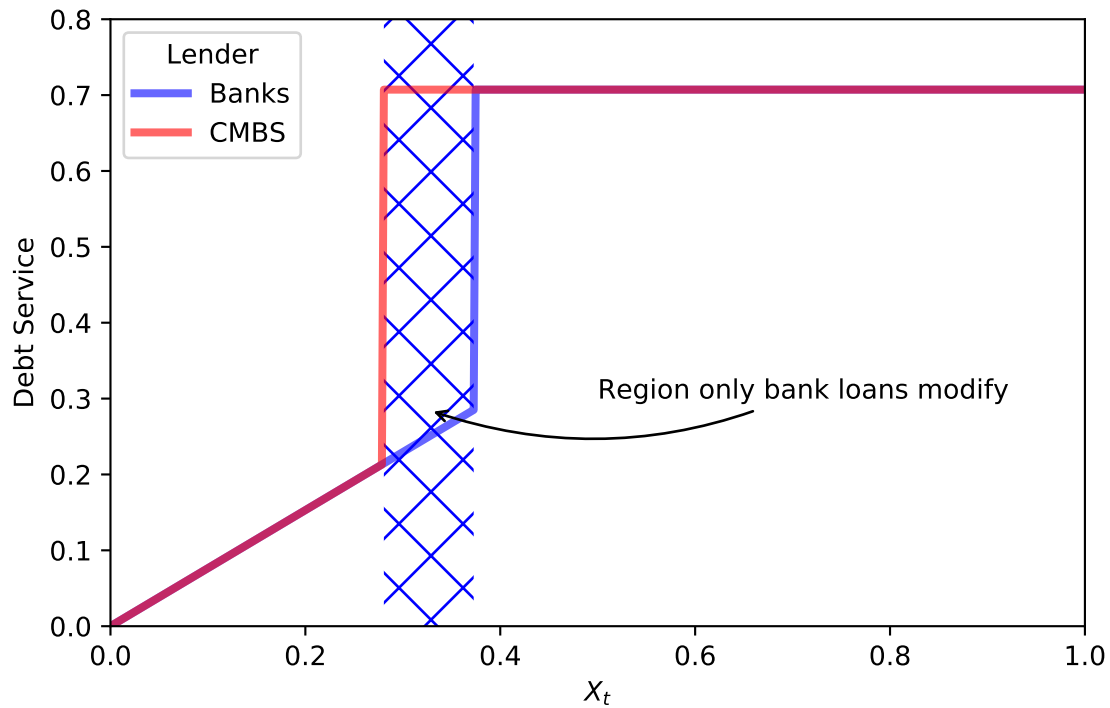


Figure 3: DEBT SERVICE COSTS BY CURRENT NOI. *Note:* This figure plots debt service costs as a function of current NOI ( $X_t$ ) for two lenders with identical promised coupons but different  $\lambda$ s. Payments for lender with a low  $\lambda$  (“banks”) are shown in blue, and payments for the high  $\lambda$  lender (“CMBS”) are shown in red. The cross-hatched region shows the range of incomes where only the low  $\lambda$  loan is modified.



	2012:Q1-2019:Q4	2020:Q1-2021:Q2
Banks	0.64	0.48
CMBS	7.76	1.06

Table 5: DELINQUENCY-TO-MODIFICATION RATIOS. *Note:* Values are the ratio of delinquency rates to modification rates by lender type and time period. We use these values to calibrate  $\frac{\lambda_j}{r}$ , reflecting the breakdown risk in the model. *Source:* Authors' calculations using Trepp CMBS data and Y-14 H.2 Schedule.

Estimated Parameters		Model Fit		
Parameter	Estimate	Moment	Target	Model
<i>Directly Set</i>				
$\mu$	0.010	Rent Growth, An et al. (2016)	1%	1%
$\underline{\tau}$	0.044	Min CMBS LTV	30%	30%
$\bar{\tau}$	0.448	Max CMBS LTV	75%	75%
$\lambda_{\text{Bank}}$	0.046	$\frac{\lambda_{\text{Bank}}}{r}$ = Bank Delinquency-to-Mod Rate	.64	.64
$\lambda_{\text{CMBS}}$	0.552	$\frac{\lambda_{\text{CMBS}}}{r}$ = CMBS Delinquency-to-Mod Rate	7.76	7.76
<i>Jointly Estimated</i>				
$r$	0.071	Average Cap Rate, CBRE	5.50%	5.50%
$\alpha^F$	0.222	30% Foreclosure Cost, Brown et al. (2006)	30%	30%
$\sigma$	0.270	Average Loan Spread	2.43%	2.43%
$\epsilon$	7.009	Effect of 25bp shock on CMBS share	-24.4%	-24.4%
$a$	1.297	Average CMBS LTV	0.64	0.64
$b$	1.727	Dispersion in CMBS LTV	0.08	0.08

Table 6: CALIBRATION RESULTS. *Note:* From left to right, this table presents (1) the variable to be calibrated, (2) the calibrated value, (3) a description of the corresponding target, (4) the targeted moment in the data, and (5) the value of that moment in the calibrated model.

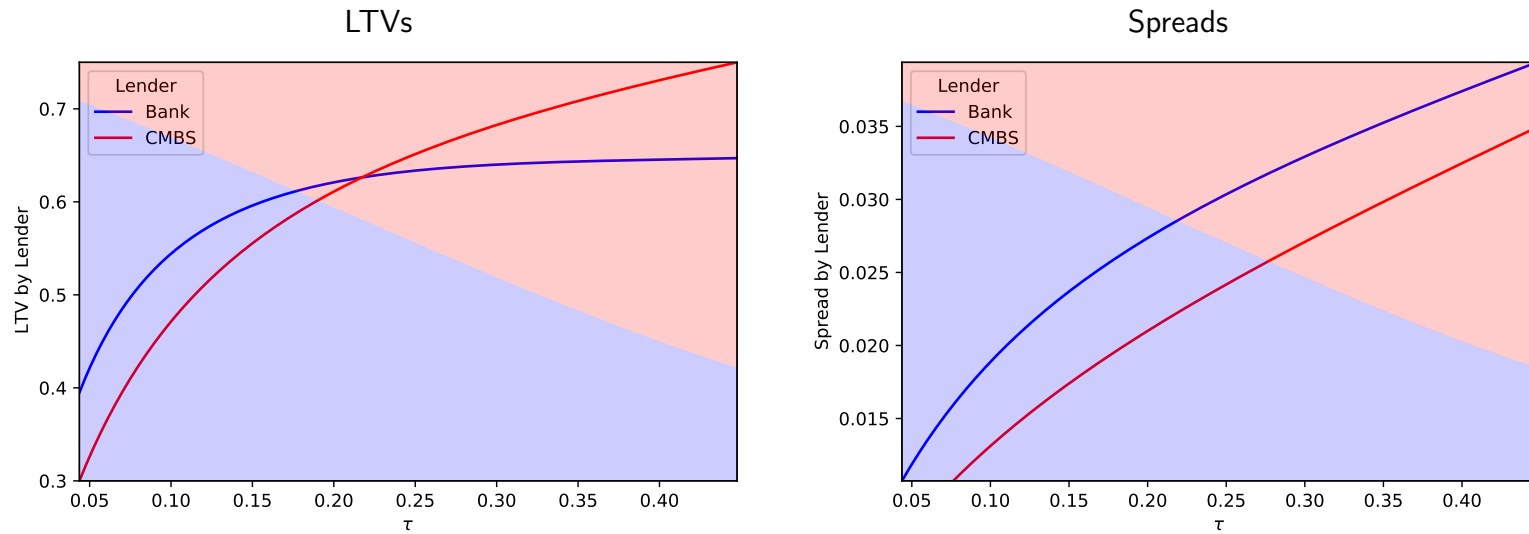


Figure 4: LTVs AND SPREADS BY  $\tau$ , TWO-LENDER CALIBRATION. *Note:* The lines in the figures show either the LTV (left) or loan rate spread (right) chosen by a borrower from a given lender at a given  $\tau$ . The shaded regions show the market share for a given lender. Blue lines and regions show bank underwriting terms and market shares by  $\tau$ , and red lines and areas show these quantities for CMBS.

Lender	Data	Model
<u>LTVs</u>		
Bank	58	59
CMBS	64	64
<u>Spreads</u>		
Bank	2.27	2.55
CMBS	2.43	2.43

Table 7: AVERAGE LTVs AND SPREADS, TWO-LENDER CALIBRATION. *Note:* This table presents average LTV and spreads by lender in the data and the model. *Source:* Authors' calculations using Trepp CMBS data and Y-14 H.2 Schedule.

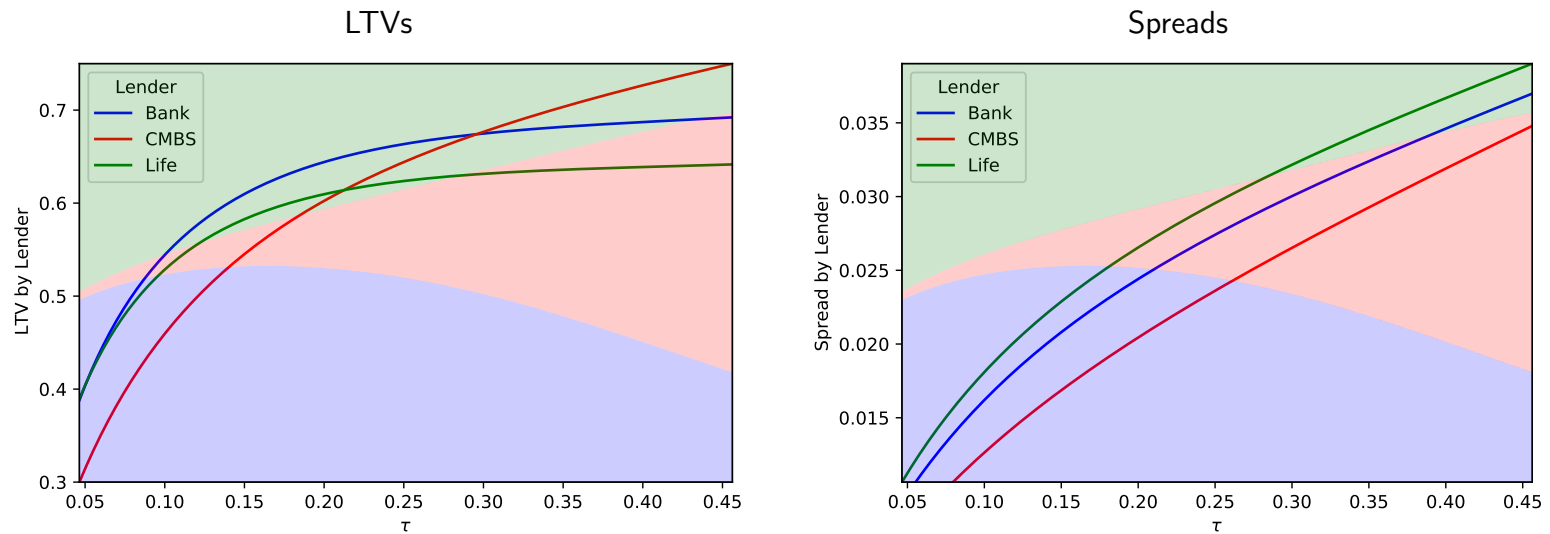


Figure 5: LTV AND SPREAD BY  $\tau$ , THREE-LENDER CALIBRATION. *Note:* The lines in the figures show either the LTV (left) or loan rate spread (right) chosen by a borrower from a given lender at a given  $\tau$ . The shaded regions show the market share by lender. Results for banks, CMBS, and life insurers are shown in blue, red, and green, respectively.

Lender	Data	Model
<u>LTVs</u>		
Banks	58	60
CMBS	64	64
Life	56	56
<u>Spreads</u>		
Bank	2.27	2.21
CMBS	2.43	2.43
Life	2.18	2.25

Table 8: AVERAGE LTVs AND SPREADS, THREE-LENDER CALIBRATION. *Note:* This table presents average LTV and spreads by lender in the data and the model. *Source:* Authors' calculations using Trepp CMBS data, NAIC, and Y-14 H.2 Schedule.

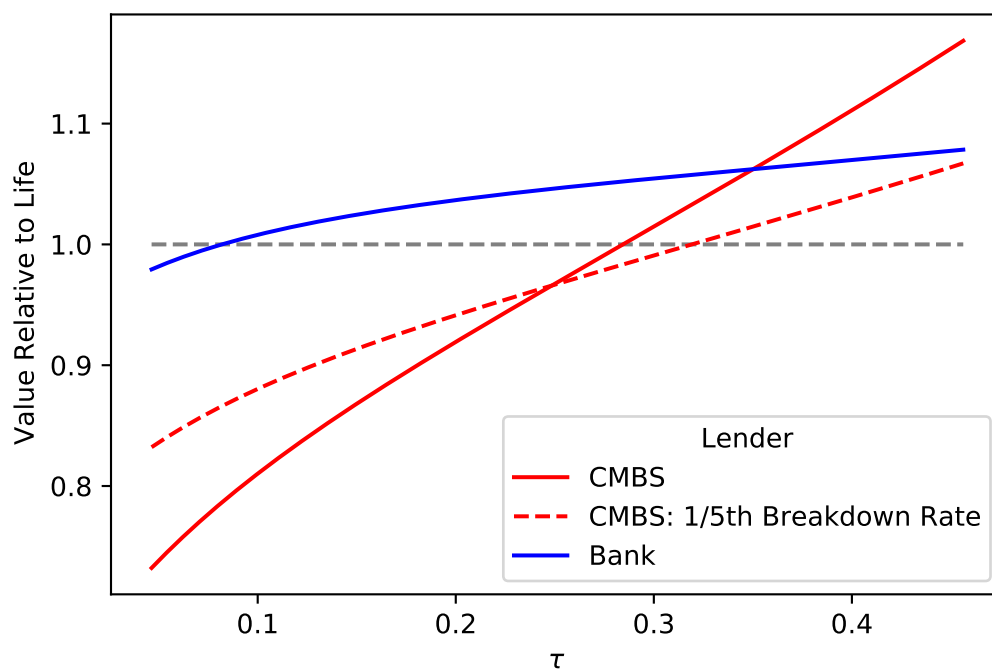


Figure 6: VALUES BY  $\tau$  AND LENDER TYPE. *Note:* The solid lines of this figure show  $v_{i,\text{Bank}}$  and  $v_{i,\text{CMBS}}$ , respectively, normalized to  $v_{i,\text{Life}}$ , for different values of  $\tau$ . The dashed red line shows  $v_{i,\text{CMBS}}$  normalized to  $v_{i,\text{Life}}$ , for different values of  $\tau$  when  $\lambda$  is reduced by a factor of 5.

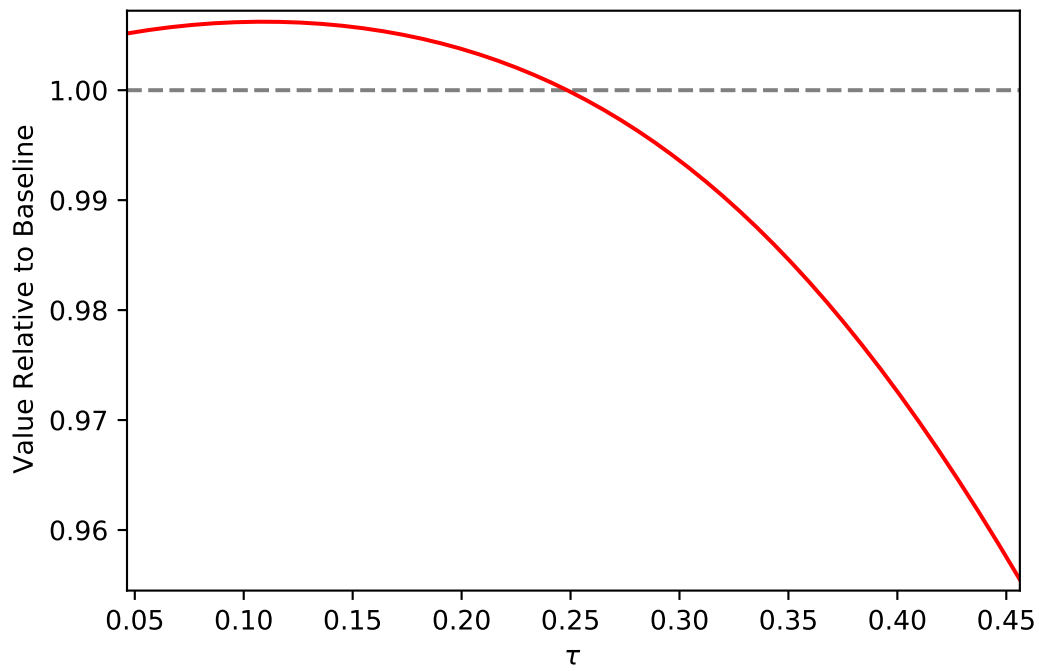


Figure 7: CHANGE IN WELFARE FROM REDUCING  $\lambda_{\text{CMBS}}$  BY FOUR-FIFTHS. *Note:* This figure shows the value of  $\bar{v}(\tau_i)$  over  $\tau$ , normalized by its respective value for the baseline three-lender parameterization when  $\lambda$  is reduced by a factor of 5.



## A. INSTITUTIONAL OVERVIEW: CMBS VS. BANKS

In this appendix, we briefly review the relevant institutional factors affecting the willingness of different lenders to offer CRE loan modifications.

### A.1. CMBS Modification Restrictions

CMBS have always been limited in how easily they can modify loans and the types of modifications they can provide. The two most important reasons for this are that CMBS are REMICs and they have pooling and servicing agreements (PSAs)—legally binding contracts that limit the actions of the parties involved in running the CMBS.<sup>42</sup>

A REMIC is an entity satisfying certain criteria, including having effectively all of its investments in qualified mortgages and real estate property (including property in foreclosure). REMICs are exempt from federal income taxes. This exemption allows them to avoid double taxation when they issue pass-through securities to investors. Qualified mortgages must meet certain criteria, including being transferred to the REMIC on its start-up day.

To maintain REMIC status, the REMIC cannot add new loans or property in years subsequent to its start-up day. This can make loan modifications difficult to pursue as a substantially modified loan may be considered a new loan. This new loan will not have been transferred to the REMIC on its start-up day and, therefore, will jeopardize the entity's REMIC status. There are exceptions to this rule, including modifications that are "occasioned by default or a reasonably foreseeable default," but even if the modification falls into an exception, there is a danger that the modified loan will violate another REMIC requirement.

REMIC rules have changed over time. During the global financial crisis, the IRS updated the rules to allow more flexibility for modifications with the REMIC structure. The rule issued in 2009 relaxed the foreseeable default requirement to allow modifications if the servicer determines that "there is a significant risk of default of the pre-modification loan upon maturity of the loan or at an earlier date." Despite this more lenient rule, there were still concerns about making significant modifications to loans (see [Flynn Jr. et al. 2021](#) for further details on this rule change and its effects). Another major rule change occurred in the pandemic when

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<sup>42</sup>Other aspects of CMBS also make modifications more prohibitive. For example, CMBS receive credit ratings, and certain modifications will require a new rating.

the IRS issued a statement temporarily allowing forbearances within the REMIC structure. This rule change led to a number of forbearances that were historically extremely uncommon in CMBS.<sup>43</sup>

In addition to maintaining REMIC status, each CMBS pool has a PSA that outlines potential additional restrictions that the special servicer must abide by when modifying loans. For example, a 2016 PSA provides specific guidance on the special servicer's ability to defer interest:

The Special Servicer shall use its reasonable efforts to the extent possible to cause each Specially Serviced Loan to fully amortize prior to the Rated Final Distribution Date and shall not agree to a modification, waiver, or amendment of any term of any Specially Serviced Loan if such modification, waiver or amendment would ... provide for the deferral of interest unless interest accrues on the related Mortgage Loan or the related Serviced Whole Loan at the related Mortgage Rate.

PSAs also outline other parties that have the right to consent to modifications. These consent requirements can also complicate, or at least delay, the approval of modifications. This can be particularly problematic when the relevant parties are inundated with requests, as was the case early on in the pandemic.

#### *A.2. Bank Modification Encouragement*

In contrast to CMBS, where modifications can be curtailed by REMIC rules and PSA restrictions, banks have fewer impediments to modification. Since banks are typically the sole holder of the loan, they will rarely have conflicts of interest across different investors to complicate loan negotiations.

Instead, modification decisions are more sensitive to banks' assessments of how a modification would affect the likely recovery from a potentially distressed loan and by the views of supervisors as to the risks associated with such modifications. On this second point, banks' regulatory agencies actively encouraged lenders to work with the customers who were adversely affected by the pandemic.

A joint press release from US bank regulatory organizations in March 2020 read:<sup>44</sup>

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<sup>43</sup>See IRS Rev. Proc 2020-26 at <https://www.irs.gov/pub/irs-drop/rp-20-26.pdf>.

<sup>44</sup>The text from the March 2020 press release is available at <https://www.federalreserve.gov/newsevents/pressreleases/bcreg20200322a.htm>, and a revision to that interagency state-

The agencies view prudent loan modification programs offered to financial institution customers affected by COVID-19 as positive and proactive actions that can manage or mitigate adverse impacts on borrowers, and lead to improved loan performance and reduced credit risk. ... Regardless of whether modifications are considered TDRs or are adversely classified, agency examiners will not criticize prudent efforts to modify terms on existing loans for affected customers.

A follow-up press release in April reaffirmed and further clarified this regulatory stance.

## B. VALUE FUNCTIONS SOLUTIONS

In this section we derive the equilibrium strategic debt service offer from renegotiations,  $S(X)$ , and the functions defining the values of debt and equity as a function of current NOI.

Since lenders and borrowers are risk neutral, the value functions for debt and equity in the  $H$  and  $L$  regions must satisfy the ordinary differential equations (ODEs):

$$\begin{aligned}
 rD_H(X) &= C + \mu X D'_H(X) + \frac{1}{2} \sigma^2 X^2 D''_H(X) \\
 rD_L(X) &= S(X) + \mu X D'_L(X) + \frac{1}{2} \sigma^2 X^2 D''_L(X) \\
 rE_H(X) &= X - (1 - \tau)C + \mu X E'_H(X) + \frac{1}{2} \sigma^2 X^2 E''_H(X) \\
 rE_L(X) &= X - (1 - \tau)S(X) + \mu X E'_L(X) + \frac{1}{2} \sigma^2 X^2 E''_L(X) \\
 &\quad + \lambda \left( -\theta \frac{C}{r} - E_L(X) \right),
 \end{aligned} \tag{12}$$

where  $\lambda \left( -\theta \frac{C}{r} - E_L(X) \right)$  reflects the expected loss to equity holders from renegotiation breaking down.<sup>45</sup>

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ment pertaining to the CARES Act is available at <https://www.federalreserve.gov/newsevents/pressreleases/bcreg20200407a.htm>.

<sup>45</sup>This term does not enter into  $D_L(X)$  because  $S(X)$  is set so that the lender is indifferent between continuation and foreclosure.

First, we determine  $S(X)$  based on the equilibrium condition that lenders are indifferent between modification and foreclosure. We then solve this set of ODEs to find the resultant value functions. Since borrowers make a take-it-or-leave-it offer to their lender, the value of debt must equal the recovery value from foreclosure:  $D_L(X) = (1 - \alpha^F) \frac{X}{r - \mu} + (1 - \alpha^D) \theta \frac{C}{r}$ . We can then substitute  $D_L(X)$ ,  $D'_L(X)$ , and  $D''_L(X)$  into the second line of equation (12) and solve for  $S(X)$  as

$$S(X) = (1 - \alpha^F)X + (1 - \alpha^D)\theta C.$$

Once we substitute this expression for  $S(X)$  into the fourth line of equation (12), we can see that the remaining three ODEs take the form

$$cV(X) = a + bX + V'(X)\mu X + \frac{1}{2}\sigma^2 X^2 V''(X),$$

which has solution

$$V(y) = \frac{a}{c} + \frac{b}{c - \mu}y + A_\gamma y^{-\gamma} + A_\eta y^\eta,$$

where  $\gamma > 0$  and  $\eta > 1$  are functions of  $c$ ,  $\mu$ , and  $\sigma$ , and  $A_\gamma$  and  $A_\eta$  are constants to be pinned down by boundary conditions.<sup>46</sup>

We can solve this set of ODEs as a function of the renegotiation boundary,  $X_n$ , using a set of value-matching and asymptotic conditions. Using the asymptotic

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<sup>46</sup> Note that  $c = r$  in all equations except for the function  $E_L(X)$ , for which  $c = r + \lambda$ . We do not define the exponents  $\gamma$  and  $\eta$  for that equation, because its constants are 0. That is,  $E_L(X)$  is linear.  $\gamma$  and  $\eta$  therefore are defined as the exponents that correspond with the other value functions:

$$\begin{aligned} \gamma &= \left( \mu - .5\sigma^2 + \sqrt{(.5\sigma^2 - \mu)^2 + 2\sigma^2 r} \right) / \sigma^2 > 0 \\ \eta &= - \left( \mu - .5\sigma^2 - \sqrt{(.5\sigma^2 - \mu)^2 + 2\sigma^2 r} \right) / \sigma^2 > 1. \end{aligned}$$

Note that  $\lim_{\sigma \rightarrow 0} \gamma = \infty$  and  $\lim_{\sigma \rightarrow \infty} \gamma = 0$ , so a higher  $\gamma$  means lower volatility.

conditions, we can show that

$$\begin{aligned}
D_H(X) &= \frac{C}{r} + A_\gamma^D X^{-\gamma} \\
D_L(X) &= (1 - \alpha^F) \frac{X}{r - \mu} + (1 - \alpha^D) \theta \frac{C}{r} \\
E_H(X) &= \frac{X}{r - \mu} - \frac{(1 - \tau)C}{r} + A_\gamma^E X^{-\gamma} \\
E_L(X) &= \frac{1 - (1 - \alpha^F)(1 - \tau)}{r + \lambda - \mu} X - \frac{\lambda \theta C}{r(r + \lambda)} - \frac{(1 - \tau)(1 - \alpha^D) \theta C}{r + \lambda}.
\end{aligned} \tag{13}$$

The other nonlinear term in  $D_H(X)$  is eliminated by the condition that  $\lim_{X \rightarrow \infty} D_H(X) = \frac{C}{r}$ .  $D_L(X)$  is determined by the equilibrium condition that banks are indifferent between foreclosure and renegotiation. The other nonlinear term in  $E_H(X)$  is eliminated by the condition that the value of the default option goes to 0 as  $X \rightarrow \infty$ . The non-linear terms in  $E_L(X)$  are eliminated by the conditions that  $\lim_{X \rightarrow 0} E_L(X) = \frac{-\lambda \theta C}{r(r + \lambda)} - \frac{(1 - \tau)(1 - \alpha^D) \theta C}{r + \lambda}$  and  $\lim_{\lambda \rightarrow \infty} E_L(X) = \frac{-\theta C}{r}$ .<sup>47</sup>

The remaining constants ( $A_\gamma^E$  and  $A_\gamma^D$ ) are identified by the value matching conditions that  $D_H(X_n) = D_L(X_n)$  and  $E_H(X_n) = E_L(X_n)$ . For these equations to hold, the value functions in the non-renegotiation region must be

$$\begin{aligned}
D_H(X) &= \frac{C}{r} - \left(\frac{X}{X_n}\right)^{-\gamma} \left[\frac{C}{r} - D_L(X_n)\right] \\
&= \frac{C}{r} - \left(\frac{X}{X_n}\right)^{-\gamma} \left[ (1 - (1 - \alpha^D) \theta) \frac{C}{r} - (1 - \alpha^F) \frac{X_n}{r - \mu} \right] \\
E_H(X) &= \frac{X}{r - \mu} - \frac{(1 - \tau)C}{r} - \left(\frac{X}{X_n}\right)^{-\gamma} \left[ \frac{X_n}{r - \mu} - \frac{(1 - \tau)C}{r} - E_L(X_n) \right] \\
&= \frac{X}{r - \mu} - \frac{(1 - \tau)C}{r} - \left(\frac{X}{X_n}\right)^{-\gamma} \left[ \beta_X \frac{X_n}{r - \mu} - \beta_C \frac{C}{r} \right],
\end{aligned} \tag{14}$$

where  $\beta_c$  and  $\beta_x$  are as in (2). With (13) and (14), we obtain the value functions shown in (2).

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<sup>47</sup>  $\frac{\lambda \theta C}{r(r + \lambda)}$  is the present discounted value of a deficiency judgment payout of  $\frac{\theta C}{r}$  with an exponentially distributed arrival time, and  $\frac{(1 - \tau)(1 - \alpha^D) \theta C}{r + \lambda}$  is the present discounted value of debt payments (excluding tax shields) made before negotiation breaks down. Combined, they give the value of payments by the property investor—the only cash flow when the property is yielding no income. The second condition says that if negotiation breaks down immediately, the value in the renegotiation state is  $\frac{-\theta C}{r}$ , reflecting an immediate deficiency judgment.

## C. COMPARATIVE STATICS AND ANALYTIC RESULTS

In this section, we analyze the comparative statics for key functions in the model.

### C.1. Characterization of the Modification Boundary

#### C.1.1. Comparative statics for recourse ( $\frac{\partial \rho}{\partial \theta}$ )

In the model, we do not explicitly write down  $\rho$ , which is a complicated expression. For these comparative statics, we need this object, so we write it here:

$$\rho(\lambda, \theta) = \frac{\gamma}{1 + \gamma} \frac{r + \lambda - \mu}{r + \lambda} \frac{\lambda(1 - \tau - \theta) + r(1 - \tau)(1 - (1 - \alpha^D)\theta)}{\lambda + (1 - \alpha^F)(1 - \tau)(r - \mu)}. \quad (15)$$

By differentiating equation (15) with respect to  $\theta$ , it is clear that higher recourse discourages borrowers from seeking a modification:

$$\frac{\partial \rho}{\partial \theta} = -\frac{\gamma}{1 + \gamma} \frac{r + \lambda - \mu}{r + \lambda} \frac{\lambda + r(1 - \tau)(1 - \alpha^D)}{\lambda + (1 - \alpha^F)(1 - \tau)(r - \mu)} < 0. \quad (16)$$

The two mechanisms by which recourse affects  $\rho$  are most clearly shown in the numerator of the last expression. The first term ( $\lambda$ ) reflects the fact that firms are less willing to renegotiate because they are concerned that negotiations might break down, causing them to lose a deficiency judgment. This does not depend on  $\alpha^D$  because it reflects the borrower's losses instead of the lender's recoveries. That is, even if lenders cannot recover anything from a deficiency judgment, they can still impose a cost on borrowers, making borrowers more hesitant to force a modification. This effect is higher when negotiations are more likely to break down ( $\lambda$  is high).

The second term ( $r(1 - \tau)(1 - \alpha^D)$ ) reflects the effect of recourse on debt service payments on modified loans. Recourse loans give lenders more bargaining power in a renegotiation due to their higher recovery in foreclosure (note this term is proportional to the recovery rate  $(1 - \alpha^D)$ ). This means that recourse borrowers need to make higher modified loan payments than non-recourse borrowers and thus are less quick to force a modification. This mechanism is more relevant when  $\lambda$  is low, as firms expect to maintain the modified payment terms longer before negotiation potentially breaks down.

C.1.2. *Comparative statics for modification frictions* ( $\frac{\partial \rho}{\partial \lambda}$ )

Since  $\rho = \frac{\gamma}{1+\gamma} \frac{\beta_c}{\beta_x}$ , the sensitivity of the default boundary to modification frictions is

$$\frac{\rho \lambda}{\rho} = \frac{\partial \beta_c}{\partial \lambda} - \frac{\partial \beta_x}{\partial \lambda}.$$

To evaluate this, it is helpful to simplify the expressions for  $\beta_x$  and  $\beta_c$  and differentiate them with respect to  $\lambda$ :

$$\begin{aligned} \beta_c &= 1 - (1 - \alpha^D)\theta - \frac{\lambda}{r + \lambda}(\tau + \alpha^D\theta) \\ \implies \frac{\partial \beta_c}{\partial \lambda} &= \frac{-r}{(r + \lambda)^2}(\tau + \alpha^D\theta) \\ \beta_x &= (1 - \alpha^F)(1 - \tau) + \frac{\lambda}{r + \lambda - \mu} \left(1 - (1 - \alpha^F)(1 - \tau)\right) \\ \implies \frac{\partial \beta_x}{\partial \lambda} &= \frac{r - \mu}{(r + \lambda - \mu)^2} \left(1 - (1 - \alpha^F)(1 - \tau)\right). \end{aligned}$$

Substituting in these expressions, we can solve for  $\frac{\rho \lambda}{\rho}$  as

$$\begin{aligned} \frac{\rho \lambda}{\rho} &= -\frac{r}{r + \lambda} \times \frac{\tau + \alpha^D\theta}{\lambda(1 - \tau - \theta) + r(1 - (1 - \alpha^D)\theta)} \\ &\quad - \frac{r - \mu}{r + \lambda - \mu} \times \frac{1 - (1 - \alpha^F)(1 - \tau)}{\lambda + (r - \mu)(1 - \alpha^F)(1 - \tau)} < 0. \end{aligned}$$

As both terms are negative, this derivative shows that the modification boundary is decreasing in  $\lambda$ .<sup>48</sup> That is, higher modification frictions cause borrowers to be willing to maintain promised debt payments for lower levels of NOI.

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<sup>48</sup>We assume that  $\theta < 1 - \tau$  in order to ensure that default is possible. Otherwise, the combination of the tax shield and recourse would be such that, for a sufficiently high  $\lambda$ , borrowers would not seek a modification even if incomes were 0.

### C.1.3. Characteristics of $\rho$ in the limit

The economic mechanisms affecting modification boundaries are most easily understood in the limiting cases. Taking the limits of equation (15) as  $\lambda$  goes to 0 or  $\infty$ , we can find the modification boundary when modifications never break down, or when they immediately break down:

$$\begin{aligned}\lim_{\lambda \rightarrow 0} \rho &= \frac{\gamma}{1 + \gamma} \frac{1 - (1 - \alpha^D)\theta}{1 - \alpha^F} \\ \lim_{\lambda \rightarrow \infty} \rho &= \frac{\gamma}{1 + \gamma} (1 - \tau - \theta).\end{aligned}\tag{17}$$

At the lower limit for  $\lambda$ , the renegotiation boundary is the same as in [Hackbarth et al. \(2007\)](#) except for the term  $(1 - \alpha^D)\theta$ , reflecting how much recourse affects the negotiation boundary when modifications never break down. Since negotiations never break down at the lower limit, recourse only affects modifications to the extent that it affects the lender's bargaining power. Therefore, the boundary only shifts to the extent that lenders can recover losses from a deficiency judgment. Higher foreclosure costs raise the renegotiation threshold because lenders are willing to accept a lower debt service payment to avoid a foreclosure, motivating borrowers to renegotiate.

At the other limit, as  $\lambda \rightarrow \infty$ , negotiations break down immediately. In this case, the decision to renegotiate is a decision to accept foreclosure. This limit corresponds to the default threshold in [Leland \(1994\)](#)—shifted to reflect recourse—where firms are choosing an optimal default threshold instead of a renegotiation threshold. At this limit, the recourse share matters on its own, instead of the recourse share times the recovery rate. Without modifications, recourse affects the default boundary because it imposes losses on the borrower and discourages them to default. This expression says that borrowers will be willing to maintain debt payments even when the present value of NOI falls below the present value of promised debt payments to preserve the option value of the loan ( $\gamma$  is decreasing in  $\sigma$ ), to preserve their debt shield (the  $\tau$  term), and to avoid a deficiency judgment (the  $\theta$  term). Foreclosure costs no longer matter, as they affect the lender's recovery, not the borrower's loss.

At intermediate values of  $\lambda$ , both sets of mechanisms matter: lenders' potential recoveries affect borrowers' incentives to modify, as this determines payments required on modified loans, while borrowers' losses in foreclosure affect incentives



to modify, as borrowers know that negotiations may break down before exiting the renegotiation region. The extent to which each factor matters depends on how close  $\lambda$  is to either extreme.

### C.2. Comparative Statics for Supply Curves

Here we analyze how recourse and modification frictions affect supply curves—that is, the LTVs that lenders are willing to offer for a given loan rate spread. Comparative statics with respect to  $\theta$  and  $\lambda$  are similar, as both variables affect supply by changing the modification boundary. For this reason, we analyze the effects of these variables together.

Substituting  $\chi$  from (5) into the supply curve defined in (6) and differentiating with respect to  $\theta$  and  $\lambda$ , we can see that recourse or higher modification frictions induce banks to offer higher LTVs for a given spread:

$$\begin{aligned} \frac{\partial LTV(s; \theta, \lambda)}{\partial \lambda} &= LTV(s) \left( \underbrace{\frac{(1 - \alpha^F)\rho - \gamma\chi}{\gamma\chi}}_{(- \text{ for } \lambda > 0)} \underbrace{\frac{\rho_\lambda}{\rho}}_{(-)} \right) > 0 \\ \frac{\partial LTV(s; \theta, \lambda)}{\partial \theta} &= LTV(s) \left( \underbrace{\frac{(1 - \alpha^F)\rho - \gamma\chi}{\gamma\chi}}_{(- \text{ for } \lambda > 0)} \underbrace{\frac{\rho_\theta}{\rho}}_{(-)} + \underbrace{\frac{1 - \alpha^D}{\gamma\chi}}_{(+)} \right) > 0, \end{aligned} \tag{18}$$

where  $\rho_\theta$  and  $\rho_\lambda$  are the partial derivatives of  $\rho$  with respect to  $\theta$  and  $\lambda$ , respectively, which were shown to be negative in Appendix C.1.

As  $1 - \alpha^D$  and  $\gamma\chi$  are positive, it is clear that the sign of the comparative statics depends critically on the sign of  $(1 - \alpha^F)\rho - \gamma\chi$ . This expression measures the sensitivity of loan supply to changes in the modification boundary, accounting for both the direct effects of changing  $\rho$  in equation (6), and the effects operating through  $\chi$ .<sup>49</sup> This sensitivity can be shown to be 0 for  $\lambda = 0$  and positive for  $\lambda > 0$ . To see why, substitute in  $\chi$  from equation (5) and  $\rho(0, \theta)$  from equation (17). This shows that the expression is 0 for  $\lambda = 0$ . Note also that  $(1 - \alpha^F)\rho - \gamma\chi$  is increasing monotonically in  $\rho$  (since  $\rho$  enters negatively in  $\chi$ ). As  $\rho$  is monotonically

<sup>49</sup>These effects work in opposite directions. A lower modification boundary directly increases allowable LTVs; however, as modifications occur at lower property values, loan losses when modifications do occur are higher. We show here that the first effect wins out when  $\lambda > 0$ .

decreasing in  $\lambda$ , the expression is monotonically decreasing in  $\lambda$ . Since  $(1 - \alpha^F)\rho - \gamma\chi = 0$  for  $\lambda = 0$  and is decreasing in  $\lambda$ , it is negative for all  $\lambda > 0$ .

Having derived the direction of the effects of recourse and modification frictions on supply, we now discuss the economics involved. Focusing first on the top line of (18), which shows how modification frictions affect LTV, we can see that  $\lambda$  affects the supply curve entirely by shifting the modification boundary. When  $\lambda$  is higher, the renegotiation threshold ( $\rho$ ) is lower, since the risk of negotiations breaking down discourages renegotiation at the margin. Lenders can therefore offer a higher original LTV and achieve the same risk of modification, and thus allow higher LTVs for a given spread. Overall, this term shows that increased modification frictions ( $\lambda \uparrow$ ) lower the modification boundary ( $\rho \downarrow$ ), which allows borrowers to take out a higher LTV for a given spread ( $LTV(s) \uparrow$ ). That is, credit supply is increasing in  $\lambda$ .

The second line in (18) shows how the availability of recourse affects LTVs. The first term is similar to the previous expression. Increasing recourse shifts the supply curve out by lowering the modification boundary. However, there is one additional term,  $\frac{1-\alpha^D}{\gamma\chi}$ , which reflects the extent to which recourse reduces loss given default. Thus, recourse affects supply in two ways: first, it discourages borrowers from seeking modifications (as with increasing  $\lambda$ ), and, second, it directly affects recoveries when lenders foreclose. Both of these forces contribute to a positive relationship between LTV and recourse.

## D. APPENDIX TABLES AND FIGURES

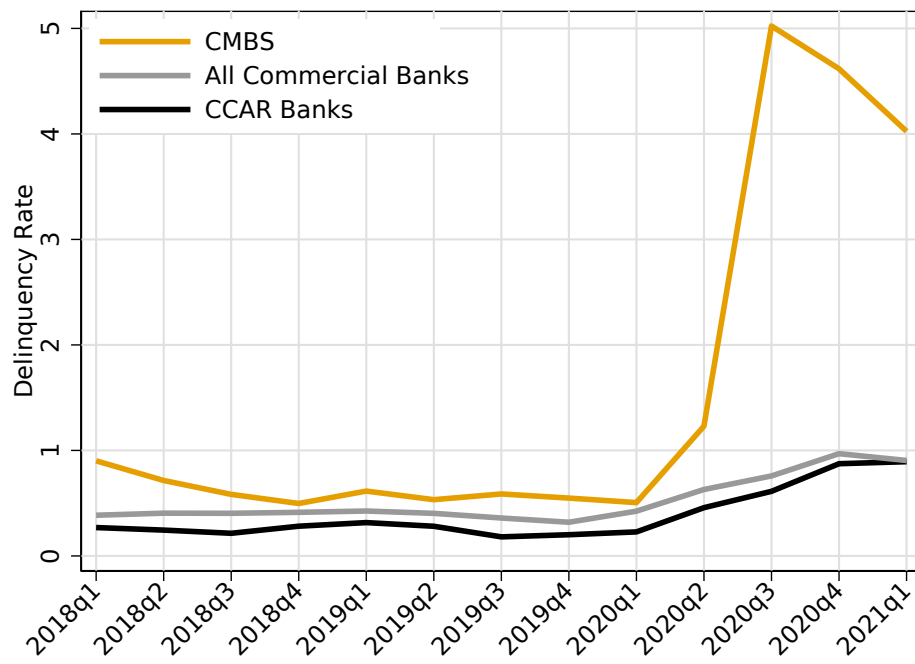


Figure D.1: SHARE OF BALANCES THAT ARE 90+ DAYS DELINQUENT OR IN NON-ACCRUAL. *Note:* Shares are put in percentage terms. *Source:* Authors' calculations using Trepp CMBS data, Call Reports, and Y-14 H.2 Schedule.

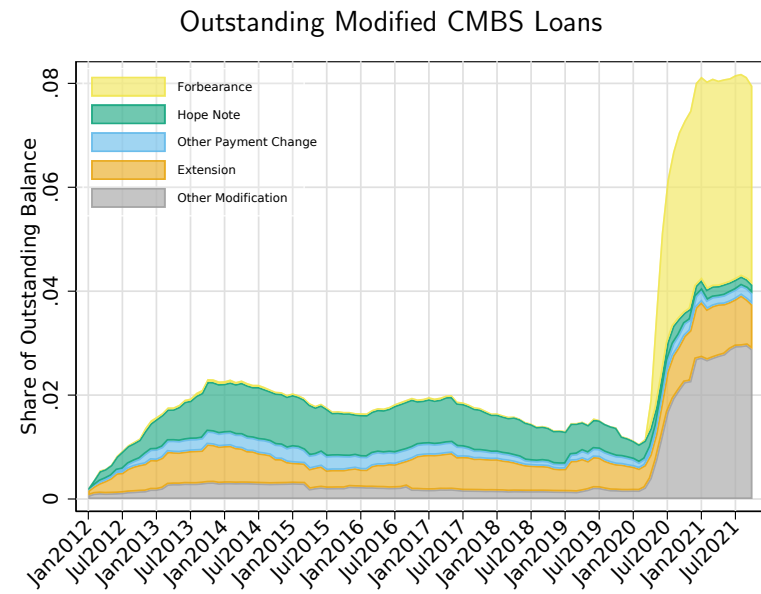
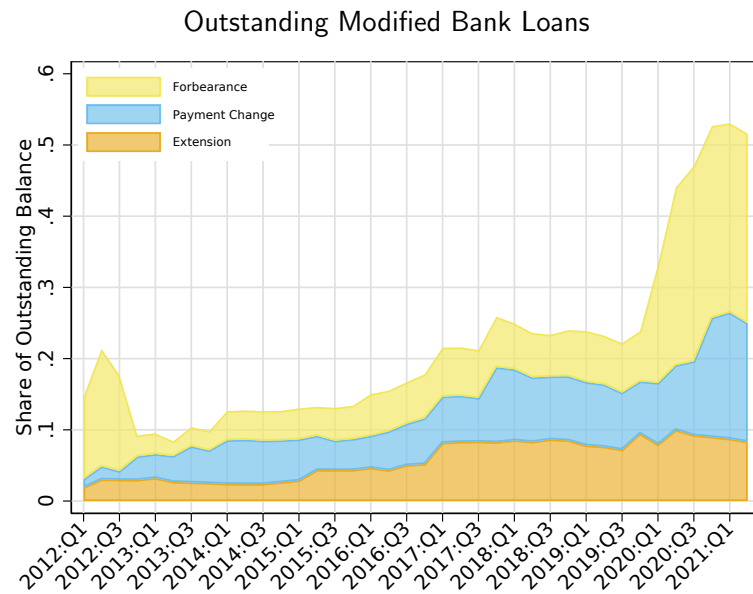


Figure D.2: BANK AND CMBS MODIFICATION TYPES. *Note:* Share of outstanding balances that have received a modification since January 2012. Outstanding balances are limited to loans that are current or less than 120 days delinquent. A “hope note” is a type of CMBS modification where an underwater loan is split into two pari passu pieces, generally also with an equity injection from the borrower, where the A piece is paid off as normal, and the B piece (or hope note) is only repaid if the property value recovers. *Source:* Authors’ calculations using Trepp CMBS data and Y-14 H.2 Schedule.

Estimated Parameters		Model Fit		
Parameter	Estimate	Moment	Target	Model
<i>Directly Set</i>				
$\mu$	0.010	Rent Growth, An et al. (2016)	1%	1%
$\underline{\tau}$	0.046	Min CMBS LTV	30%	30%
$\bar{\tau}$	0.456	Max CMBS LTV	75%	75%
$\lambda_{\text{Bank}}$	0.045	$\frac{\lambda_{\text{Bank}}}{r}$ = Bank Delinquency-to-Mod Rate	.64	.64
$\lambda_{\text{CMBS}}$	0.544	$\frac{\lambda_{\text{CMBS}}}{r}$ = CMBS Delinquency-to-Mod Rate	7.76	7.76
<i>Jointly Estimated</i>				
$r$	0.070	Average Cap Rate, CBRE	5.50%	5.50%
$\alpha^F$	0.235	30% Foreclosure Cost, Brown et al. (2006)	30%	30%
$\sigma$	0.268	Average Loan Spread	2.43%	2.43%
$\epsilon$	10.722	Effect of 25bp shock on CMBS share	-37.5%	-37.5%
$a$	1.042	Average CMBS LTV	0.64	0.64
$b$	1.952	Dispersion in CMBS LTV	0.08	0.08
$\theta$	0.075	Effect of Recourse on LTV	2.80	2.80
$\alpha^D$	0.421	Effect of Recourse on Spreads	-20bp	-20bp

Table D.1: CALIBRATION RESULTS, THREE-LENDER MODEL. *Note:* From left to right, this table presents (1) the variable to be calibrated, (2) the calibrated value, (3) a description of the corresponding target, (4) the targeted moment in the data, and (5) the value of that moment in the calibrated model.

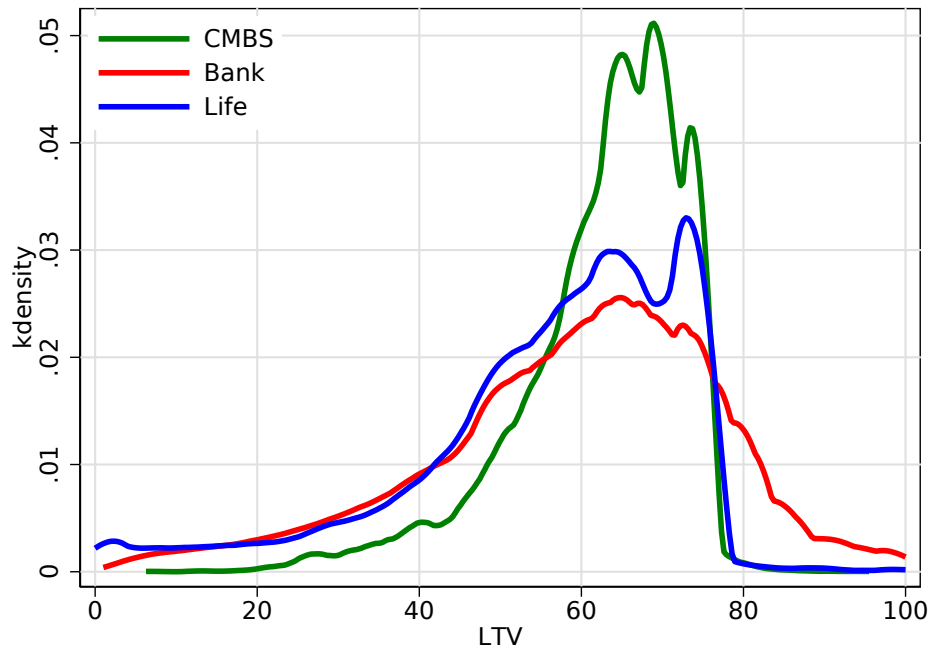


Figure D.3: LTV DISTRIBUTION. *Note:* Distribution of LTVs for loans securitized in CMBS and held on portfolio by commercial banks or life insurance companies. Sample is limited to loans originated between 2012 and 2019 on industrial, lodging, office, or retail properties. Bank loans are limited to first lien loans on stabilized properties. *Source:* Authors' calculations using Trepp CMBS data, NAIC, and Y-14 H.2 Schedule.