



Federal Reserve Bank of Cleveland Working Paper Series

Labor Substitutability among Schooling Groups

Mark Bills, Barış Kaymak, and Kai-Jie Wu

Working Paper No. 22-07

March 2022

Suggested citation: Bills, Mark, Barış Kaymak, and Kai-Jie Wu. 2022. "Labor Substitutability among Schooling Groups." Working Paper No. 22-07. Federal Reserve Bank of Cleveland.
<https://doi.org/10.26509/frbc-wp-202207>.

Federal Reserve Bank of Cleveland Working Paper Series

ISSN: 2573-7953

Working papers of the Federal Reserve Bank of Cleveland are preliminary materials circulated to stimulate discussion and critical comment on research in progress. They may not have been subject to the formal editorial review accorded official Federal Reserve Bank of Cleveland publications.

See more working papers at: www.clevelandfed.org/research. Subscribe to email alerts to be notified when a new working paper is posted at: www.clevelandfed.org/subscribe.

Labor Substitutability among Schooling Groups*

Mark Bills

University of Rochester
FRB Richmond
NBER

Bariş Kaymak

FRB Cleveland
Université de Montréal
CIREQ

Kai-Jie Wu

University of Rochester

March 2022

ABSTRACT

Knowing the degree of substitutability between schooling groups is essential to understanding the role of human capital in income differences and to assessing the economic impact of such policies as schooling subsidies, immigration systems, or redistributive taxes. We derive a lower bound for the substitutability required for worldwide growth in real GDP from 1960 to 2010 to be consistent with a stable wage premium for schooling despite the rapid growth in schooling, assuming no *exogenous* worldwide regress in the technology frontier for workers with only primary schooling. That lower bound for the long-run elasticity of substitution is about 4, which is far higher than values commonly used in the literature. Given our bound, we reexamine the importance of human capital in cross-country income differences and the roles of school quality versus the skill bias of technology in greater efficiency gains from schooling in richer countries.

JEL Codes: E24, J24, O15, O47

Keywords: Elasticity of Skill Substitution, Aggregate Human Capital, Growth and Development

*We thank Hannes Malmberg for his helpful comments. Barış Kaymak acknowledges support from Chaire de la Fondation J.W. McConnell en études Américaines. The views expressed here are those of the authors and do not necessarily reflect the views of the Federal Reserve Banks of Cleveland and Richmond or the Federal Reserve System.

1 Introduction

The impacts of policies such as schooling subsidies, immigration systems, or redistributive taxes hinge on the substitutability in labor demand between schooling groups. [Tinbergen \(1975\)](#) analyzed how expanding the supply of more-educated workers, under imperfect substitution, can drive down the return to schooling, offsetting the impact of skill-biased technological change.¹ Likewise, assessing the wage impact of selective immigration policies or of mass migration events in times of geopolitical distress, because they shift relative employments by skill, requires knowledge of that substitutability.

Substitutability is also key to interpreting data on schooling and wages: e.g. lower degrees of substitutability require more skill-biased advances in technology to explain the rising return to schooling despite the gains in educational attainment in recent U.S. history ([Goldin and Katz, 2010](#)).² Similarly, substitutability plays a prominent role in efforts to quantify human capital's role in economic development as it guides our measures of labor productivity across countries based on their educational attainments and returns to schooling (see [Hsieh and Klenow, 2010](#); [Jones, 2014](#); [Caselli, 2016](#); [Caselli and Ciccone, 2019](#); [Hendricks and Schoellman, forthcoming](#), among others).

The consensus in the literature is that substitutability across workers of differing schooling is quite low. This consensus largely reflects works by [Katz and Murphy \(1992\)](#), [Heckman et al. \(1998\)](#), and [Card and Lemieux \(2001\)](#), all of whom estimate an elasticity of labor demand between high school and college-trained U.S. workers of about 1.5. Each estimates that elasticity first controlling for longer-term trends in relative wages, trends that largely show both rising relative wages and supply for more-educated workers. Therefore, these estimates identify a relatively short-run elasticity. According to the LeChatelier principle ([Samuelson, 1947](#)), a longer-run elasticity would presumably be larger.³ In particular, it will reflect the incentive to innovate technology

¹This reasoning is incorporated more explicitly into general equilibrium models by [Heckman et al. \(1998\)](#) and [Johnson and Keane \(2013\)](#), among others. More generally, if substitutability is low, any policy to redistribute income that reduces the relative hours of more-skilled workers will create an offsetting increase in inequality by raising returns to skill (e.g., [Feldstein, 1973](#)).

²See [Ciccone and Peri \(2005\)](#) for a review of work in this area.

³[Acemoglu and Autor \(2011\)](#) extend the [Katz and Murphy \(1992\)](#) exercise through 2008 U.S. CPS data and estimate elasticities of 1.6 to 2.9 depending on how flexibly they specify trends. [Ciccone and Peri \(2005\)](#) estimate a longer-run elasticity of around 1.5 across U.S. schooling groups based on supply differences instrumented with child-labor and compulsory-schooling laws. But their confidence interval includes considerably larger elasticities, especially if one allows for the strength of their first-stage instruments. [Malmberg \(2018\)](#) infers a low elasticity based on an estimated gravity model of trade driven by factor endowments, which we interpret as a fairly long-run elasticity. [Bowlus et al.'s \(2021\)](#) work is an important counterweight to these earlier estimates. They differ from the prior literature by

toward the expanding groups (e.g., [Caselli and Coleman, 2006](#); [Acemoglu, 2007](#); [Hendricks and Schoellman, forthcoming](#)). For questions regarding development accounting, growth accounting, or the longer-term impact of education policies, it is the longer-run elasticity that is relevant.

Given that countries differ markedly and persistently in schooling attainments, it is natural to look at how returns to schooling differ across countries to gauge long-run substitutability. Many authors have noted that estimated Mincerian returns to schooling are nearly as high in richer countries despite their far greater supply of educated workers. [Bowles \(1970\)](#) and [Psacharopoulos and Hinchliffe \(1972\)](#) interpret this as evidence for high substitutability across schooling groups. But it is now well-recognized that one cannot infer an elasticity from the cross-country relationship between schooling and Mincer returns if richer countries exhibit more skill-biased technology, as in [Caselli and Coleman \(2006\)](#), or better quality of schooling, as in [Jones \(2014\)](#).⁴

We take a different route to gauge substitutability, by focusing on the dramatic worldwide increase in schooling since 1960, an increase that has been accompanied by no decline in Mincerian returns. We derive a lower bound on the substitutability required for growth rates in real GDP to be consistent with that rapid growth in schooling and with the stability of Mincer returns, assuming there was no *exogenous* worldwide contraction of the technology frontier for workers without secondary schooling. We view this assumption as conservative as it covers a significant share of the world's labor force. It also still permits an *endogenous* diversion of technology away from these groups, allowing their wages to potentially decline over time. The required lower bound for the elasticity of substitution is nearly 4. Lower elasticities imply rapid technological regress for a large section of the workforce worldwide from 1960 to 2010, even beyond that from technology shifting endogenously toward workers with more schooling. In particular, for an elasticity of substitution of 1.5 the technology frontier must contract by 98 percent for those without secondary schooling, a group that averaged about 40 percent of the population for our sample of 60 countries.

modeling skill-biased technical change explicitly as a function of investments in information technology. They also adjust skill prices and quantities to account for estimated rates of investment in human capital. As a result, they estimate a much higher elasticity of substitution across U.S. schooling groups of 4 to 5, depending on the precise specification. We interpret their estimated elasticity as applying to the medium term.

⁴Reflecting that identification problem, there is no consensus on why returns are not lower in richer countries. [Jones \(2014\)](#), [Caselli and Ciccone \(2019\)](#), and [Rossi \(2022\)](#) each presume low substitutability. Jones attributes the “flat” Mincer pattern to richer countries having better schooling; Casselli and Ciccone demonstrate that it could equally reflect richer countries’ better schooling or more skill-biased technology; and Rossi, given his measure of schooling quality, attributes the pattern to differential skill bias in technology. By contrast, [Hendricks and Schoellman \(forthcoming\)](#) do not assume low substitutability. In fact, given their measure of schooling quality and some limits on skill bias in technology, they interpret the cross-country Mincer returns as consistent with a high elasticity of substitution.

We exploit our bound on substitutability to reassess the role of schooling in cross-country development accounting. Early papers by [Klenow and Rodrigues-Clare \(1997\)](#) and [Hall and Jones \(1999\)](#) treat schooling groups as perfect, but unequal, substitutes. But, as discussed by [Jones \(2014\)](#), [Caselli and Ciccone \(2019\)](#), and [Hendricks and Schoellman \(forthcoming\)](#), the mapping from Mincerian returns to human capital depends critically on substitutability. If substitutability is low, then the cross-country differences in Mincerian returns largely reflect relative scarcities of workers with more schooling, rather than efficiencies. [Jones \(2014\)](#), [Caselli \(2016\)](#), and [Malmberg \(2018\)](#) each consider imperfect substitutability, focusing on elasticities on the order of 1.5 given the estimates from the literature. This implies much greater differences in worker efficiencies across countries.

Our bound for substitutability implies smaller differences in efficiencies between workers in rich and poor countries. We conduct the exercise in [Jones \(2014\)](#) for $\varepsilon = 1.5$ versus $\varepsilon = 4$. As anticipated by Jones, for $\varepsilon = 1.5$, worker efficiency accounts for most of the income difference between rich and poor countries—in fact, together with differences in physical capital, it explains 123 percent of these differences. By contrast, under our proffered value of $\varepsilon = 4$, it explains much less, accounting for about a fifth of income differences.

Our preferred elasticity of 4 still implies that workers with more schooling are markedly more efficient in richer countries, even relative to those with less schooling. Next, we ask whether this greater efficiency reflects better quality of schooling or a more skill-biased technology frontier, as in [Caselli and Coleman \(2006\)](#) and [Rossi \(2022\)](#). To do so, we examine cross-country estimates of schooling quality based on international test scores or earnings of immigrants to the United States from [Schoellman \(2012\)](#). The distinction is important for deciding whether development policy should focus on technological advancement or on embodied skill development.⁵ For an elasticity of substitution of 4, we find that both higher school quality and more skill-biased technology are factors in the higher efficiency for workers with more schooling in richer countries. But the differential in skill bias across countries is much less than under an elasticity of 1.5.⁶ Elasticity values much higher than 4 instead require the technology frontier to be *less* skill biased in richer countries. If we disallow such a scenario, then we also obtain an upper bound on the elasticity of substitution on the order of 6, so not so far above our lower bound.

⁵[Jones \(2014\)](#) interprets these differences in efficiencies for workers with more schooling as reflecting higher human capital in richer countries. But [Caselli and Ciccone \(2019\)](#) clarify that higher efficiency for workers with more schooling in richer countries could alternatively reflect more skill-favoring technologies in these countries.

⁶Note that this skill bias in the technology frontier is additional to that from technology's endogenous response to the greater supply of skilled workers in richer countries, as in [Caselli and Coleman \(2006\)](#) and outlined in the next section. That response is captured in the long-run elasticity across schooling groups.

As a baseline, we treat production as a CES aggregator of multiple schooling groups. Because the literature sometimes presumes low substitutability across certain groups, e.g., college versus non-college, we examine robustness to allow varying degrees of substitutability among different schooling groups. Our bounding approach does allow for a fairly low degree of substitutability between a subset of schooling categories, provided high substitutability is maintained overall. In particular, development accounting for these more general formulations still implies that schooling differences explain only a fourth, or less, of the income gaps across countries.

The paper proceeds as follows. In Section 2 we lay out our assumptions for production, focusing on long-run substitutability of schooling groups when technology is directed to groups as in [Caselli and Coleman \(2006\)](#). In Section 3 we obtain our lower bound for substitutability by comparing growth rates in human capital constructed under various elasticities of substitution to growth rates dictated by traditional growth accounting. We then examine the implications of our bound for cross-country development accounting in Section 4, and discuss the relative roles of school quality and skill bias in the technology frontier. We consider the robustness of our results to alternative production technologies in Section 5. We conclude in Section 6.

2 Model

Our analysis is based on the relationship between the wage premium for schooling and the production structure that shapes worker productivity. We first present an economy's production as a function of its schooling for a given technology. We then take into account that technology responds to the relative importance of schooling groups, as in [Caselli and Coleman \(2006\)](#), to derive the long-run substitutability among schooling groups, as in [Hendricks and Schoellman \(forthcoming\)](#). Finally, we derive schooling's wage premium and show how its empirical relationship with schooling scarcity may inform us about the long-run substitutability.

2.1 Directed Technology and Long-Run Substitutability

Output for the economy is a Cobb-Douglas function of physical capital, K , and technology-enhanced aggregate labor input, H :

$$Y = K^\alpha H^{1-\alpha}. \tag{1}$$

H aggregates the efficiency units of labor supplied by workers with different schooling levels. Let S be the set of all schooling groups, and L_i be the number of workers with s_i years of schooling for all $i \in S$ with $s_i < s_j$ for $i < j$. Effective labor input is defined by the constant-elasticity-of-substitution (CES) aggregator:

$$H = \left[\sum_{i \in S} \left(A_i q_i L_i \right)^{\frac{\varepsilon_{SR}-1}{\varepsilon_{SR}}} \right]^{\frac{\varepsilon_{SR}}{\varepsilon_{SR}-1}}, \quad (2)$$

where $\varepsilon_{SR} > 1$ is the *short-run* elasticity of substitution between schooling groups. A_i is the level of skill-specific technology and q_i is the human capital associated with schooling level i . q_i reflects both quantity (years) and quality of schooling. The efficiency of schooling group i in production, $A_i q_i$, therefore reflects the extent of that group's skills as well as the technical efficiency attached to those skills.

We follow [Caselli and Coleman's \(2006\)](#) formulation of skill-specific technology investment. Firms choose technologies from a set of possibilities defined by a technology frontier, given the skill endowments in the country. Formally, given the wage rates and the interest rate R , firms rent capital K , adopt technologies A_i , and hire workers L_i from each schooling group to maximize profits ($K^\alpha H^{1-\alpha} - \sum_{i \in S} w_i L_i - RK$) subject to the following technology frontier:

$$\sum_{i \in S} \left(\gamma_i A_i \right)^\omega \leq B,$$

where B determines the level of the technology frontier, which may differ by country. The parameters $\omega > 0$ and γ_i determine the technical trade-off between technologies associated with different schooling groups. If γ_i is increasing with s_i , then investment in technologies assigned to more-educated workers has a higher opportunity cost. ω is the elasticity of substitution between schooling-specific technologies; it shows how fast the marginal cost of technological enhancement rises with the level of the technology. [Caselli and Coleman \(2006\)](#) and [Hendricks and Schoellman \(forthcoming\)](#) assume that ω and γ_i are common to all countries. We maintain a common ω , but allow γ_i to vary by country. This captures, for instance, the idea that richer countries may be better at directing technology to more-schooled workers for reasons exogenous to the model.

To ensure that firms choose $A_i > 0$ for all $i \in S$, we assume that the parameters satisfy:

$$\omega - \varepsilon_{SR} + 1 > 0. \quad (3)$$

As shown in Appendix D.1, this guarantees that firms make the same interior input choices, so we can characterize the equilibrium using the optimality conditions of a representative firm.

The optimal level of technology for schooling group i is dictated by:

$$A_i = \left(\frac{w_i L_i}{\bar{w} L} \right)^{\frac{1}{\bar{\omega}}} \times \left(B^{\frac{1}{\bar{\omega}}} / \gamma_i \right). \quad (4)$$

$B^{\frac{1}{\bar{\omega}}} / \gamma_i$ is the maximal technology achievable for group i , conditional on forgoing any technology for all other groups. We call this the technology frontier for group i and denote it as $b_i = B^{\frac{1}{\bar{\omega}}} / \gamma_i$. Relative to that frontier, the directed component of a group's technology responds positively to its importance in overall earnings. $\frac{1}{\bar{\omega}}$ governs the strength of that response.⁷ Substituting for relative earnings in (4), group i 's technology in reduced form is:

$$A_i = \left(\frac{q_i L_i}{H} \right)^{\frac{\varepsilon_{SR}-1}{\bar{\omega}\varepsilon_{SR}-\varepsilon_{SR}+1}} \times b_i^{\frac{\omega\varepsilon_{SR}}{\bar{\omega}\varepsilon_{SR}-\varepsilon_{SR}+1}}. \quad (5)$$

Technology for schooling group i depends on two factors: its human capital and its technology frontier. Higher group human capital, $q_i L_i$, raises its return to technology investment. Higher b_i reflects better available technology, B , and/or a lower opportunity cost of adoption, γ_i , for group i .

As shown in Hendricks and Schoellman (forthcoming), equation (4) implies that the equilibrium allocation and prices in the labor market are equivalent to those given by the optimality conditions of a representative firm with the following alternative aggregator.⁸

$$H = \left[\sum_{i \in S} \left(q_i b_i L_i \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad (6)$$

where the long-run elasticity of substitution is

$$\varepsilon = \frac{\omega\varepsilon_{SR} - \varepsilon_{SR} + 1}{\omega - \varepsilon_{SR} + 1}.$$

The assumption in (3) guarantees that ε is finite and $\varepsilon > \varepsilon_{SR}$ for all $\varepsilon_{SR} > 0$. Since we assume $\varepsilon_{SR} > 1$, we also have $\varepsilon > 1$. This elasticity of substitution can be considered to be the long-run elasticity of substitution where technology is endogenous.

⁷Given the technology in (2), relative earnings for groups i and j are: $\frac{w_i L_i}{w_j L_j} = \left(\frac{A_i q_i L_i}{A_j q_j L_j} \right)^{1 - \frac{1}{\varepsilon_{SR}}}$.

⁸A derivation is provided in Appendix D.2.

2.2 School Quality, Technology Frontier, and Scarcity in Relative Wages

The wage rate (in logs) for workers in schooling group i implied by the technology in (6) is:

$$\ln w_i = \ln \frac{\partial Y}{\partial H} + \frac{1}{\varepsilon} \ln H + \left(1 - \frac{1}{\varepsilon}\right) \ln (q_i b_i) - \frac{1}{\varepsilon} \ln L_i. \quad (7)$$

Because the first two terms in (6) are common to all workers, the relative wages of any two groups depend only on the last two components—the relative efficiency and relative supply of labor. For instance, the wage premium for group i relative to j is: $\left(1 - \frac{1}{\varepsilon}\right) \ln \left(\frac{q_i b_i}{q_j b_j}\right) - \frac{1}{\varepsilon} \left(\frac{L_i}{L_j}\right)$.

With multiple schooling groups, the wage premium for schooling is typically estimated by projecting log wages on years of schooling, yielding the estimated Mincer return. An important benefit of mapping our framework to the Mincer return is that its estimates are readily available for many countries and time periods from meta studies. Define the (model) Mincer return to schooling, m , as the coefficient of the projection of log wage on years of schooling: $E(\ln w_i | s_i) = \overline{\ln w} + m(s_i - \bar{s})$. The Mincer return is:

$$m = \left(1 - \frac{1}{\varepsilon}\right)(\phi_q + \phi_b) + \frac{1}{\varepsilon} x. \quad (8)$$

ϕ_q and ϕ_b are, respectively, the log-projections of human capital q_i and the technology frontier, b_i , on years of schooling. Intuitively, ϕ_q captures the quality of schooling, while ϕ_b captures the skill bias of the technology frontier. $x = -\phi_L$ is *minus* the regression coefficient from projecting $\ln L_i$ on years of schooling. x gives the average percentage decline in labor supply per year of schooling: $E(\ln L_i | s_i) = \overline{\ln L} - x(s_i - \bar{s})$. It measures the relative scarcity of skilled workers—a large positive x indicates that, on average, the supply of workers rapidly declines with years of schooling.⁹

Equation (8) frames our analysis. Schooling's wage premium reflects the impacts of *long-run efficiency*, $\phi_q + \phi_b$ and *scarcity*, x , with relative weights determined by the inverse elasticity of substitution. The easier the substitutability (lower ε), the higher is the efficiency premium and the lower is the scarcity premium. With directed technologies note that: (a) long-run efficiency reflects skill bias in the technology frontier (ϕ_b), not in the level of technology, and (b) scarcity's weight in the wage premium equals $\frac{1}{\varepsilon} \leq \frac{1}{\varepsilon_{SR}}$, reflecting that the impact on wages of a group's supply is partly offset by improvements to the technology assigned to that group.

⁹If schooling attainment takes only two values from $S = \{1, 2\}$, then: $x = \ln \left(\frac{s_2 - \bar{s}}{\bar{s} - s_1} \right) / (s_2 - s_1)$; so \bar{s} is a sufficient statistic for x . But more generally x varies for a given \bar{s} because it reflects the skewness of the schooling distribution.

3 Substitutability and Growth Accounting

In this section we ask what values of ε are consistent with worldwide trends in schooling attainment and Mincer returns as well as growth rates in labor productivity assuming no worldwide technological regress, decline in b_i , for the bottom schooling group.

3.1 Data and Patterns for Schooling Scarcity and Premia

To exploit the comovements of schooling's scarcity and its wage premium across time and countries, we merge the meta-data set of Mincer returns assembled by [Psacharopoulos and Patrinos \(2018\)](#) with data on schooling distributions from [Barro and Lee \(2013\)](#) for ages 25 to 54 to capture the working-age population. The merger yields an unbalanced panel with 367 observations that consist of 104 countries for the years 1960 to 2010 at 5-year intervals. For growth accounting we require a country to be observed at three or more of the intervals, yielding a smaller sample with 60 countries and 298 observations.¹⁰ We return to the broader sample of 104 countries in Section 4 for conducting the cross-country development accounting. Appendix A provides details on the data sets and a list of countries for each sample.

To measure scarcity of schooling we divide workers into four groups: i) less than secondary, ii) some secondary, iii) completed secondary, and iv) any tertiary (college) education.¹¹ Appendix C examines the consistency of measured scarcity with grouping rules, showing that it is important to have at least three groups to capture scarcity of schooling across countries. Section 5 considers the robustness of our results to alternative technologies that allow for nested-CES aggregators within these schooling groups.

We can measure schooling scarcity, for each country in each year, by regressing the log size of the population in each schooling category on that category's years of schooling. The negative of the estimated coefficient, which shows the average percentage-point decline in labor supply per year of schooling, is our measure of scarcity, x . Note that [Barro and Lee \(2013\)](#) report schooling distributions based on population. Using the population distribution of schooling (rather than employment distribution) could skew scarcity measures if employment-to-population ratios differ systematically with years of schooling. In Appendix B, we show that such a bias is not significant.

¹⁰We drop observations through 1990 for countries that were formerly held in the Soviet Union.

¹¹For our 104 country sample, the average shares by group are respectively 83 percent, 9 percent, 6 percent, and 3 percent in 1965 (weighted by population), and they become 38 percent, 31 percent, 23 percent, 8 percent in 2010.

Figure 1 displays the worldwide trends in average years of schooling, schooling scarcity, and Mincer returns from 1960 to 2010. Averages weight countries by their employment in 2000. As seen in Panel a, the average years of schooling have increased from about 3 in 1960 to 9 in 2010. Corresponding to this trend is the tremendous fall in scarcity shown in Panel b. The worldwide decreases in schooling scarcity, however, have produced no trend in the Mincer return. Average Mincer returns have remained largely stable, decreasing from 1965 to 1975 but rebounding ever since.

These patterns motivate our approach to bound the long-run elasticity of substitution through growth accounting. Recall the relationship between Mincer returns and scarcity in equation (8). Explaining the lack of trend in Mincer returns, despite decreased scarcity, requires faster efficiency growth for groups with more schooling, due to either improved quality of schooling (increase in ϕ_q) or skill-biased shifts in the technology frontier (increase in ϕ_b). If schooling groups are poor substitutes, then this requires spectacularly rapid efficiency gains for those with more schooling. In particular, for $\varepsilon = 1.5$, relative gains in school quality and the technology frontier, $\phi_q + \phi_b$, must be twice the decline in scarcity to keep the Mincer return constant.¹² From 1960 to 2010, average scarcity declined 33 percentage points; so this requires that *each year of schooling* added a 66 percentage-point gain in relative efficiency during this period. This translates to a 700-fold increase between 1960 and 2010 in the efficiency of college graduates relative to workers who completed primary schooling (10 additional years of schooling). But we show next that growth accounting bounds these gains far below such rates given the actual rates of growth in real incomes, assuming that technology for those with less schooling has not *regressed* worldwide. In turn, this implies an elasticity of substitution of 4 across schooling groups, if not higher.¹³

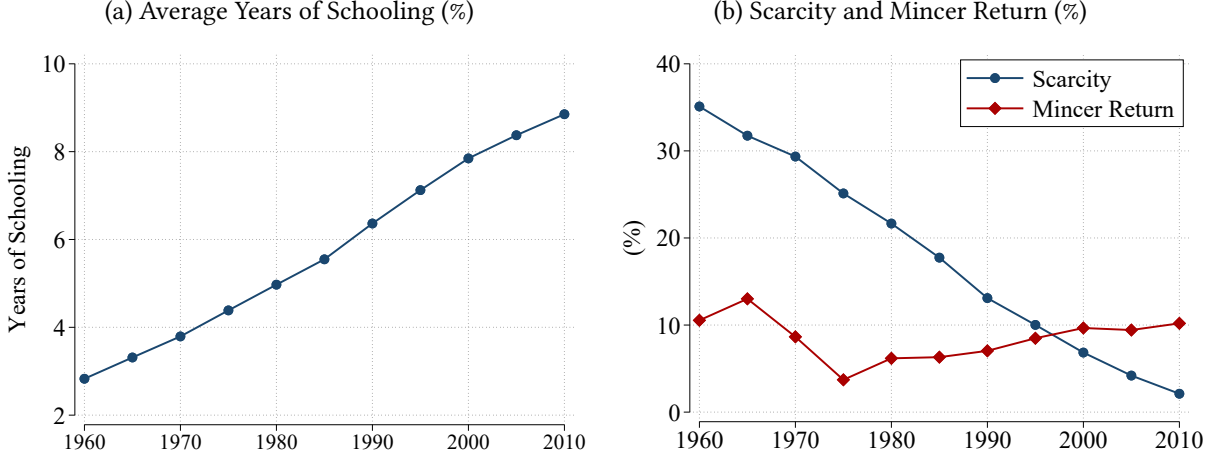
3.2 Reconciling Skill-Biased Efficiency Gains with Growth Accounting

The labor aggregator in equation (6) provides a basis to measure growth in effective labor input, including the impact of labor-augmenting technological change. From (6), effective labor per worker, $h = H/L$, can be expressed in terms of: (a) group 1's schooling and technology frontier, (b) the schooling and technology frontiers of all other groups *relative* to group 1's, and (c) relative

¹²Let Δ denote changes over time. From equation (8), $\Delta m = 0$ implies that $\Delta(\phi_q + \phi_b) = \frac{-1}{\varepsilon-1}\Delta x$, which equals $-2\Delta x$ when $\varepsilon = 1.5$.

¹³For the case of $\varepsilon = 4$, the implied change in $\phi_q + \phi_b$ is 11 percentage points from 1960 to 2010. It then translates to a 3-fold increase in the efficiency of college graduates relative to workers who completed primary schooling during this period.

FIGURE 1: Trends of Schooling, Scarcity, and Mincer Return



Notes: Years of schooling and scarcity are obtained from authors' calculations based on Barro and Lee (2013), where the trends are obtained by taking the average (weighted by employment in 2000) over Barro and Lee's sample. Data on the Mincer return are taken from Psacharopoulos and Patrinos (2018), where the trend is obtained by taking the average (weighted by employment in 2000) over Psacharopoulos and Patrinos' sample.

labor supplies:

$$h(\varepsilon) = q_1 b_1 \left(\frac{L_1}{L} \right) \left[\sum_{i \in S} \left(\frac{q_i b_i}{q_1 b_1} \frac{L_i}{L_1} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}. \quad (9)$$

We write effective labor as $h(\varepsilon)$ to emphasize that its value hinges on the elasticity of substitution ε . From equation (7), $\frac{q_i b_i}{q_1 b_1}$ can be related to the wage rate and to the employment of group i relative to group 1:

$$\frac{q_i b_i}{q_1 b_1} = \left(\frac{w_i}{w_1} \right)^{\frac{\varepsilon}{\varepsilon-1}} \left(\frac{L_i}{L_1} \right)^{\frac{1}{\varepsilon-1}}.$$

Making that substitution in (9) gives:

$$h(\varepsilon) = q_1 b_1 \cdot \left(\frac{\bar{w}}{w_1} \right)^{\frac{\varepsilon}{\varepsilon-1}} \left(\frac{L}{L_1} \right)^{\frac{1}{\varepsilon-1}} = z_1 h_{-z_1}(\varepsilon). \quad (10)$$

Here \bar{w} denotes the average wage in the economy, $\sum_{i \in S} \frac{w_i L_i}{L}$. We approximate \bar{w} relative to group 1's wage based on a country's Mincer return: $\frac{\bar{w}}{w_1} = \sum_{i \in S} \left[e^{m(s_i - s_1)} \frac{L_i}{L} \right]$.

Equation (10) breaks effective labor input per worker into two components: the frontier for

the lowest schooling group, z_1 , and the quantity of effective labor in the economy normalized by z_1 , which we label $h_{-z_1}(\varepsilon)$. $z_1 = q_1 b_1$ is a combination of human capital for the minimal schooling group and its technology frontier. $h_{-z_1}(\varepsilon)$ increases in both the relative wages and employments of other groups compared to group 1. Dramatic growth worldwide in $\frac{L}{L_1}$, together with the stability of Mincer returns, requires an increase in $h_{-z_1}(\varepsilon)$. That increase is especially large if ε is small. In turn, this implies that effective labor input per worker, h , will exhibit growth that is well above and beyond any growth in z_1 for the lowest schooling group through improvement in its human capital or expansion of its technology frontier.

In turn, the growth rate in effective labor input per worker, in log-differences, can be viewed as the sum of growth rates in z_1 and $h_{-z_1}(\varepsilon)$, denoted respectively g_{z_1} and $g_{h-z_1}(\varepsilon)$:

$$g_h(\varepsilon) = g_{z_1} + g_{h-z_1}(\varepsilon). \quad (11)$$

For any assumed elasticity ε , $g_{h-z_1}(\varepsilon)$ can be calculated from how the schooling distribution and Mincer return evolve over time. Our strategy is to construct an implied lower bound for $g_h(\varepsilon)$ under various values for ε by: (a) measuring $g_{h-z_1}(\varepsilon)$ from cross-country data for each ε , and (b) assuming a plausible lower bound for g_{z_1} , the growth in efficiency of the lowest group reflecting changes in its schooling quality and growth in its technology frontier.

We employ $g_{z_1} = 0$ for its lower bound where we treat those with completed primary education or less as group 1. We view this as conservative. For instance, if one assumes no change in labor quality for group 1, this requires no improvement on average from the technological frontier for these workers worldwide since 1960. While declines in school quality are imaginable, we would not anticipate a decline in quality of primary schooling worldwide since 1960.¹⁴ But, regardless, given the limited schooling received by these workers, any such decline in quality should be swamped in importance by worldwide gains in group 1's technology frontier, especially recognizing that this group averaged about 40 percent of the working age population for these countries as of 2000. Therefore, we view $g_{z_1} = 0$ as providing a conservative lower bound for $g_h(\varepsilon)$.

Note that our bounding assumption does allow for technological regress for group 1 in the form of *directed* technological change. In fact, we show below that $g_{z_1} = 0$ implies substantial decline in the *absolute* efficiency of the bottom schooling group, $e_1 = q_1 A_1$, driven by their decline in importance, assuming the elasticity of substitution is lower in the short run than in the long

¹⁴Compositional changes could affect q_1 , as we discuss below in the context of the findings.

run. If one alternatively assumed zero growth in e_1 over time, that would imply a less conservative lower bound. We caution against this assumption. Indeed, using the short-run version of the production function, [Caselli and Coleman \(2002\)](#) and [Acemoglu and Restrepo \(2020\)](#) each find a moderate decline in absolute efficiency for non-college-educated workers in the United States.¹⁵ Our assumption that $g_{z_1} = 0$ *on average* does not restrict how skill bias in technology, the pattern of ϕ_b 's, varies by country. It only requires that, on average, the technology frontier has not regressed worldwide, at least not enough to offset any gains in school quality, for those at the lowest schooling level.

Alternatively g_h can be measured from standard growth accounting, given an economy's growth rates of output and capital. From the aggregate technology in (1), output per worker is $y = k^\alpha h^{1-\alpha}$ where k , like h , denotes input per worker. The growth rate of effective labor is:

$$g_h = \frac{1}{1-\alpha}(g_y - \alpha g_k), \quad (12)$$

where, again, g_h reflects the impact of technical change as well as increases from investments in schooling. Thus, by comparing $g_h(\varepsilon)$ from (11) to its estimate from growth accounting we can judge plausible magnitudes for ε .

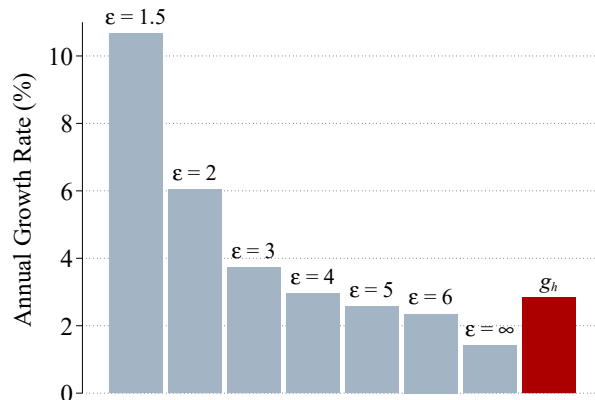
3.3 Results for Lower Bound

In Figure 2, we contrast the worldwide growth rates of effective labor between 1960 and 2010 implied by the two accounting methods, where worldwide means averaging over the 60 countries weighted by employment in 2000.¹⁶ We treat completed primary or less as the minimal schooling group. The data on output and capital per worker behind g_h come from the Penn World Tables 9.1 ([Feenstra et al., 2015](#)). We assume that α , capital's share in output, is 1/3. $g_h(\varepsilon)$ is constructed using data on educational attainments for ages 25 to 54 from [Barro and Lee \(2013\)](#) and Mincer returns from [Psacharopoulos and Patrinos \(2018\)](#). The red (dark) bar to the right shows the growth rate g_h from the growth accounting equation (12). Recall that under our bounding assumption, $g_{h-z_1}(\varepsilon)$ is a lower bound on the implied overall growth rate $g_h(\varepsilon)$. The gray bars show those lower bounds implied by the changes in schooling distributions and Mincer return under differing values of the elasticity of substitution. Each gray bar presumes zero average growth worldwide from 1960

¹⁵[Caselli and Coleman](#) interpret that decline as reflecting directed technological change; [Acemoglu and Restrepo](#) explain it by automatic tasks reducing the demand for less-skilled workers.

¹⁶The annualized growth rates are calculated over 1960 to 2010 for the 60 countries with estimated Mincer returns at three or more points in time. The average beginning and ending years across the countries are 1973 and 2007.

FIGURE 2: Substitutability and the Implied Rate of Effective Labor Input Growth (%)



Notes: Figure depicts average annual growth rate of effective labor across the 60-country sample. The gray (light) bars show the mean $g_h(\epsilon)$ implied by values of the elasticity of substitution (ϵ), given changes in schooling and Mincer returns. The red (dark) bar shows g_h from growth accounting given a capital share of one-third. Countries are weighted by their total employment as of 2000. Source: Authors' calculations based on Psacharopoulos and Patrinos (2018), Barro and Lee (2013), and Penn World Table 9.1.

to 2010 for the lowest schooling group, completed primary or less, from human capital and the technology frontier ($g_{z_1} = 0$).

We see that $g_h(\epsilon)$ is strongly decreasing in the assumed elasticity of substitution. The reasoning is straightforward. The worldwide decrease in the scarcity of higher schooling groups was a force to reduce Mincer returns to schooling, especially if the elasticity of substitution is low. Because Mincer returns were essentially stable despite declining scarcity, higher schooling groups must have become more efficient over time. For low elasticities, that rate of efficiency gain must be extremely rapid. In particular, for $\epsilon = 1.5$, $g_h(\epsilon)$ must average 10.4 percent per year during the 50 years for the 60 countries.

But such rapid growth in effective labor is sharply at odds with actual output growth around the world: A 10.4 percent growth rate in effective labor, given a labor share of two-thirds, produces an annual output growth of 7.0 percent even ignoring capital's growth; that far exceeds actual rates. The (red) dark bar in Figure 2 shows that the rate of effective labor growth that is consistent with the observed output growth is instead 2.8 percent.

Assuming, on average, no growth in human capital or the technology frontier for the lowest schooling group (average $g_{z_1} = 0$), an elasticity of substitution of 4.2 is required to reconcile the growth rate of effective labor from the constructive accounting, $g_h(\epsilon)$, with that from growth

accounting. Furthermore, deviations much from $\varepsilon = 4.2$ imply substantial declines in the technological frontier for group 1. For $\varepsilon = 3$ the frontier must contract at 0.83 percent per year, producing a cumulative contraction of a third *on average* worldwide from 1960 to 2010. For $\varepsilon = 2$ that cumulative contraction is nearly 80 percent, while for $\varepsilon = 1.5$ it is 98 percent. That is, under $\varepsilon = 1.5$, the contraction in the technology frontier for those without any secondary schooling must be so extreme that in 2010 it takes 50 such workers to have the capacity of one in 1960.

We highlight that the bound $g_{z_1} = 0$ does imply technological regress for group 1 via *directed* technological change away from group 1 due to its declining importance. Substituting the optimal choice of technology into the definition of worker efficiency, $e_1 = q_1 A_1$, gives:

$$e_1 = q_1 b_1 \cdot \left(\frac{w_1 L_1}{wL} \right)^{\frac{1}{\omega}} = z_1 \cdot V_1^{\frac{1}{\omega}}.$$

V_1 denotes group 1's earnings as a share of total earnings in the economy. (See Appendix D.3 for a derivation.) Therefore, even assuming $g_{z_1} = 0$, group 1 exhibits a growth rate of $g_{e_1} = \frac{1}{\omega} g_{V_1} = \frac{\varepsilon - \varepsilon_{SR}}{(\varepsilon - 1)(\varepsilon_{SR} - 1)} g_{V_1}$ in its efficiency, where g_{V_1} is the growth rate in its share of payments to labor. For the 60 countries, g_{V_1} averaged -4.5 percent per year from 1960 to 2010. That rapid decline in group 1's relative earnings implies a large decline in the efficiency of group 1 to the extent that ε exceeds ε_{SR} .

Table 1 illustrates the corresponding efficiency drops for values of ε_{SR} ranging from 1.5 up to 4.2, our lower bound for ε , while holding $\varepsilon = 4.2$. For a short-run elasticity of 3, *directed* technological regress yields $g_{e_1} = -0.8$ percent per year, which translates to the lowest schooling group losing 33 percent of its 1960 efficiency by 2010. A short-run elasticity of 2 implies that the first group lost nearly 80 percent of its efficiency. Finally, a short-run elasticity $\varepsilon_{SR} = 1.5$ implies that directing technology away from group 1 reduced its efficiency by 98 percent over the 50 years. For this reason, we would argue that an elasticity as low as 1.5, even in the short to medium run, strains plausibility.

While regress in the technology frontier is unlikely, average skills of the least-educated group could fall over time if they become more negatively selected by ability. Adjusting for plausible selection, however, does not substantially alter the conclusions above. In particular, to rationalize a value for ε of 1.5 requires that selection has reduced the quality of the bottom group by 7.6 percent per year, or by 98 percent in total, between 1960 and 2010. Given that the bottom group shrunk by 65 percent over the entire period, this requires those reductions to fall *on average* on workers who had 76 times the average productivity of the 35 percent that remain

TABLE 1: Decline in Efficiency of Group 1 under $\varepsilon = 4.2$

ε_{SR}	1.5	2	3	4.2
$\frac{e_{1,2010}}{e_{1,1960}}$	0.02	0.21	0.67	1.0

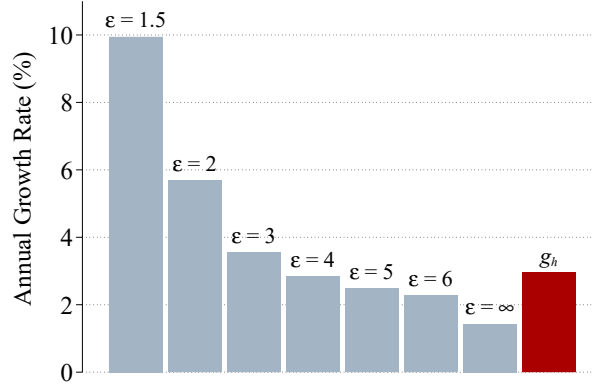
in the bottom group.¹⁷ That is an implausible differential considering the cross-sectional wage dispersion. For comparison, we computed the ratio of average weekly earnings of the highest 65 percent to the lowest 35 percent within the bottom education group for U.S. data from the March supplements to the Current Population Survey (CPS) for the years 1975 to 1985. That ratio is 2.4. This should be an upper bound, for our purposes, because it assumes perfect sorting out of group 1. Even the 90-to-10 ratio for the entire U.S. wage distribution was about 3.5 during the 1980s (Autor et al., 2008). So selection would have to be so extreme that the reductions from the primary schooling group were at the very top of the overall earnings distribution.

Our assumption is that, on average across countries, there was no decline in the combination of human capital and technology frontier for workers with no secondary schooling ($g_{z_1} = 0$). To make clear the conservative nature of this assumption, we note that these workers are a significant share in many countries. Over 1960 to 2010, this group averaged 20 percent or more of the population aged 25-54 in 54 of our 60 countries. To further drive this point home, in Figure 3 we repeat the exercises above, contrasting g_h from growth accounting with the rate $g_h(\varepsilon)$, but now we restrict the sample to the 42 countries for which group 1 constitutes its median schooling level as of 1985, the mid-year of the sample period. Rejecting the bounding assumption of zero combined growth in human capital and technology frontier for those without secondary schooling would thus require regression, on average over 50 years, for the majority of workers in these countries.

Figure 3 presents the worldwide growth rates of effective labor implied by alternative values of ε , where worldwide now means averaging over the 42 countries with median schooling of completed primary or less as of 1985. For lower elasticities, implied annual rates of efficiency gain remain extremely rapid: For $\varepsilon = 1.5$, $g_h(\varepsilon)$ averages 9.7 percent over the 50 years; for $\varepsilon = 2$, it averages 5.5 percent. But these numbers dramatically contradict the rate from growth accounting of 2.9 percent per year (dark bar in figure). To reconcile the growth rate of effective labor from the constructive accounting, $g_h(\varepsilon)$, with that from growth accounting dictates an elasticity of

¹⁷Let q_{out} be the average quality for the 65 percent of group 1 workers who flow out, with q_{stay} that for those who stay. A 98 percent drop in quality requires $q_{stay} = 0.02(0.35 q_{stay} + 0.65 q_{out})$, and hence $q_{out}/q_{stay} \approx 76$.

FIGURE 3: Results for Countries Whose $L_1/L \geq 50\%$ in 1985



Notes: Figure depicts average annual growth rate of human capital across the 42 countries. The gray bars show the mean $g_h(\varepsilon)$ implied by various values of the elasticity of substitution (ε). The red bar shows g_h from growth accounting given a capital share of one-third. Countries are weighted by their total employment as of 2000. Source: Authors' calculations based on Psacharopoulos and Patrinos (2018), Barro and Lee (2013), and Penn World Table 9.1.

substitution of 3.8. This is only modestly smaller than the elasticity of 4.2 required for the 60-country sample. Moreover, significant deviations below $\varepsilon = 3.8$ imply substantial declines in the technological frontier for group 1. For $\varepsilon = 3$ the frontier must contract by nearly a fourth from 1960 to 2010. For $\varepsilon = 2$ and $\varepsilon = 1.5$ the required contractions are 73 percent and 97 percent.

3.4 Cross-Country Results for Lower Bound

Trends *across* countries also reveal little response of Mincer returns to *relative* declines in the scarcity of schooling. Figure 4 plots the average annual change in Mincer return against that in scarcity for our 60-country sample. While scarcity shows a secular decline, consistent with Figure 1, it is far more pronounced in some countries. At the same time, we see no significant correlation between changes in scarcity and changes in Mincer return—the correlation coefficient is -0.12 with a 0.33 p-value. Consider the set of countries that exhibited the steepest declines in scarcity, represented by the top quintile in terms of reduced scarcity. On average, these countries exhibited a 1.2 percentage-point annual decline in scarcity in conjunction with an annual increase in Mincer return of 0.08 percentage points. For $\varepsilon = 1.5$, this requires that each year of schooling be associated with a 132 percentage points gain in relative efficiency on average for these countries from 1960 to 2010.¹⁸ This implies that the efficiency of college graduates, relative to workers with

¹⁸Rearrange equation (8): $\Delta(\phi_q + \phi_b) = \left(\frac{\varepsilon}{\varepsilon-1}\right) \Delta m - \left(\frac{1}{\varepsilon-1}\right) \Delta x$, where Δ denotes change over time.

completed primary, increased by a factor of 500 thousand!

In Figure 5 we plot each country's rate of effective labor growth both from growth accounting, g_h , and from inductive accounting, $g_{h-z_1}(\varepsilon)$, versus its growth rate in schooling scarcity. First consider Panel a, where the triangles illustrate $g_{h-z_1}(\varepsilon)$ calculated for $\varepsilon = 1.5$. The country values for $g_{h-z_1}(1.5)$ lie systematically far above the corresponding rates, illustrated by circles, based on growth accounting. But the size of this discrepancy: (a) differs dramatically across countries, and (b) is larger for countries that exhibited sharper declines in schooling scarcity. On the latter point, a 1 percentage point higher annual growth rate in scarcity is associated with a 12 percentage points lower growth rate in $g_{h-z_1}(1.5)$, despite having no association with effective labor growth as captured by growth accounting. By contrast, in Panel b, with $g_{h-z_1}(\varepsilon)$ calculated for $\varepsilon = 4$, the discrepancy between the two measures of effective labor growth is small on average, much less disperse, and displays no relation to the change in scarcity in a country.

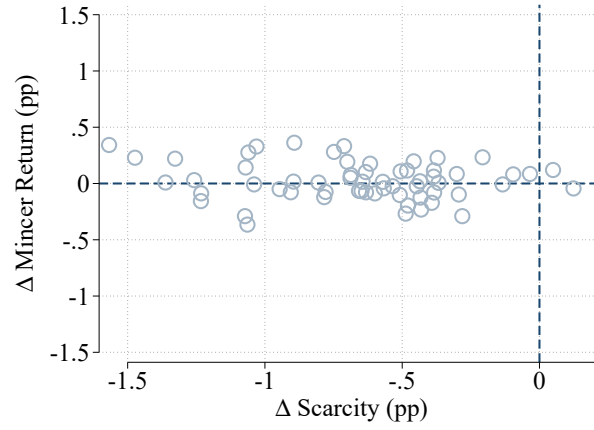
This is further illustrated in Figure 6. The figure shows histograms of the differentials: $g_{z_1} = g_h - g_{h-z_1}(\varepsilon)$, for $\varepsilon = 1.5$ and $\varepsilon = 4$. While our bounding assumption requires these to be non-negative, for $\varepsilon = 1.5$ they are negative for most countries, with discrepancies greater than 10 percent per year for 24 countries. By contrast, under $\varepsilon = 4$ the growth rates, g_{z_1} , required for the two measures to align are fairly tightly grouped near zero. Thus, the cross-country trends in schooling and Mincer returns also support a value for ε of 4, if not higher. In particular, consider choosing ε to minimize the distance between the two measures, g_h and $g_h(\varepsilon)$ across our sample of 60 countries. That exercise weighs matching the cross-country correlation between the two measures as well as their means.¹⁹ This yields $\varepsilon^* = 5.5$, with a 95 percent confidence interval [3.3, 129.9].

4 Implications for Cross-Country Income Accounting

In this section we apply our lower bound for substitutability among schooling groups ($\varepsilon = 4$) to investigate how much of the cross-country income inequality can be potentially attributed to differences in human capital. We revisit Jones's (2014) accounting framework with different values of substitutability ($\varepsilon = 1.5$ and 4) in Section 4.2. As discussed below, as well as in Caselli and Ciccone (2019), that framework may overstate the variation in human capital. We hence propose a revised accounting framework in Section 4.3 that isolates human capital from other

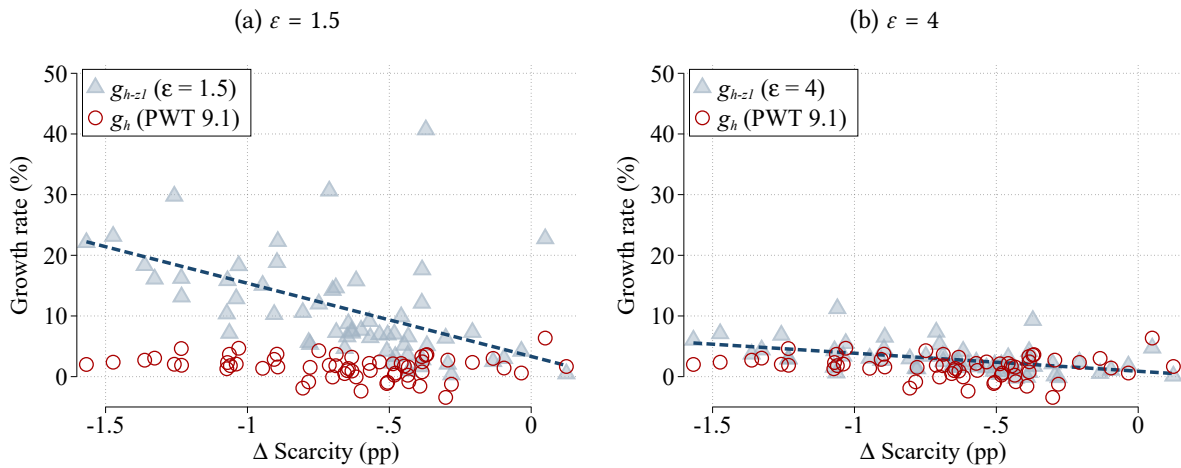
¹⁹Formally, it numerically solves the problem: $\varepsilon^* = \min_{\varepsilon} \sum_c L_c [g_{h,c} - g_{h-z_1,c}(\varepsilon)]^2$, where c denotes a country and L_c is its employment in 2000.

FIGURE 4: Annual Changes in Mincer Return and School Scarcity by Country



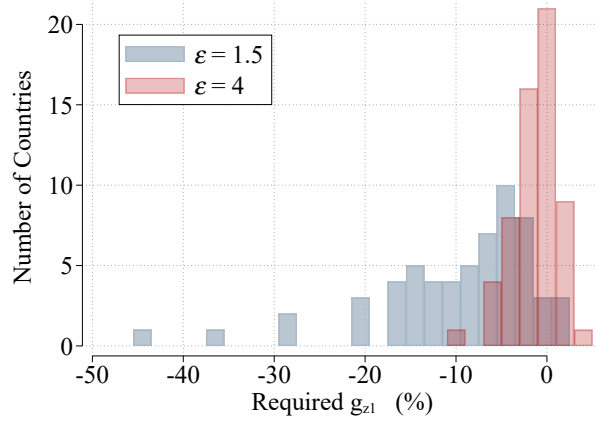
Notes: Figure plots average yearly changes of Mincer return and schooling scarcity for the 60-country sample described in Appendix A. Scarcity is obtained from authors' calculations based on Barro and Lee (2013), and Mincer return is taken from Psacharopoulos and Patrinos (2018). The correlation coefficient is -0.12 (p-value = 0.33).

FIGURE 5: Accounting Results and Decrease in Scarcity



Notes: Figures plot annual growth rates of effective labor, with circles for growth accounting and triangles for inductive accounting, against growth rates of schooling scarcity for the 60-country sample. Source: Authors' calculations based on Psacharopoulos and Patrinos (2018), Barro and Lee (2013), and Penn World Table 9.1.

FIGURE 6: Elasticity of Substitution and Efficiency Regress needed for Group 1



Notes: The figure gives histograms of the frontier growth rates for group 1, g_{z1} , required to reconcile $g_{h-z1}(\epsilon)$ with growth accounting. Histograms are for both $\epsilon = 1.5$ (gray) and for $\epsilon = 4$ (pink). Source: Authors' calculations based on Psacharopoulos and Patrinos (2018), Barro and Lee (2013), and Penn World Table 9.1.

efficiency gains from schooling, such as skill bias in the technology frontier.

4.1 Cross-Country Patterns in Scarcity and Return to Schooling

We first describe how schooling scarcity and Mincer return vary by income per worker in our development accounting sample (104 countries and 367 observations; see Appendix A). Panel a in Figure 7 shows the cross-country distribution of scarcity against log GDP per worker. Not surprisingly, more educated workers (lower scarcity) are more abundant in higher income countries. Recall the relationship between Mincer return and scarcity in equation (8): If wage premiums are driven by scarcity of schooling, then the abundance of schooling should translate to lower wage premiums in these countries. Panel b shows instead that the Mincer return is remarkably flat over income per worker.²⁰ Absent differences in ϕ_q or ϕ_b , this would suggest that the elasticity of substitution must be very high.

To reconcile the patterns in Figure 7 with low substitutability requires that richer countries have sufficiently higher school quality (ϕ_q) or technology frontiers that are sufficiently more skill biased (ϕ_b) to offset the impact of less scarcity. The required efficiency gaps are, however, immense if $\epsilon = 1.5$. On average, a one log point higher income per worker has to be associated with a 17

²⁰Specifically, a one log point higher income is associated with 9 percentage points lower scarcity (standard error = 0.6) and 0.26 percentage points lower Mincer return (standard error = 0.20). We control for year fixed effects in regressions.

FIGURE 7: Mincer Return and Scarcity across Countries



Notes: Data on Mincer return are taken from [Psacharopoulos and Patrinos \(2018\)](#). Scarcity reflects authors' calculations based on [Barro and Lee \(2013\)](#). Log real GDP per worker is from Penn World Tables 9.1. The sample is an unbalanced panel of 104 countries with 367 observations (see Appendix A for detail).

percentage points increase in $\phi_q + \phi_b$. In 2000, for instance, the interquintile range of income in our sample was 1.7 log points, implying a 29 percentage-point difference in $\phi_q + \phi_b$ between the top and bottom 20th percentiles. Note that this productivity gap is per year of schooling! This means that a high school graduate, with 12 years of schooling, in the richer country has 37 times the productivity of a similar worker in the poorer country and a college graduate with 16 years of schooling has 122 times the productivity, even assuming workers without schooling are equally productive.

Such a gap of $\phi_q + \phi_b$ is tantamount to an enormous gap in effective labor input ($h = H/L$) between richer and poorer countries. In contraposition, our lower bound on substitutability therefore places a ceiling on the extent to which schooling can explain cross-country income differences. In the following subsection, we revisit the development accounting proposed by [Jones \(2014\)](#). Under our lower bound of $\varepsilon = 4$, we find that schooling accounts for about 20 percent of income differences. We also show that the commonly used value ($\varepsilon = 1.5$) substantially overstates the role of schooling in development accounting.

4.2 Substitutability and Development Accounting

As derived in Section 3.2, we can write each country's aggregate production function as $y = k^\alpha [z_1 h_{-z_1}(\varepsilon)]^{1-\alpha}$, where $h_{-z_1}(\varepsilon) = (\bar{w}/w_1)^{\frac{\varepsilon}{\varepsilon-1}} (L/L_1)^{\frac{1}{\varepsilon-1}}$ is calculated using data on its schooling and Mincer return for a given value of elasticity of substitution. Taking the log of both sides gives

$$\ln y = \alpha \ln k + (1 - \alpha) \ln h_{-z_1}(\varepsilon) + (1 - \alpha) \ln z_1. \quad (13)$$

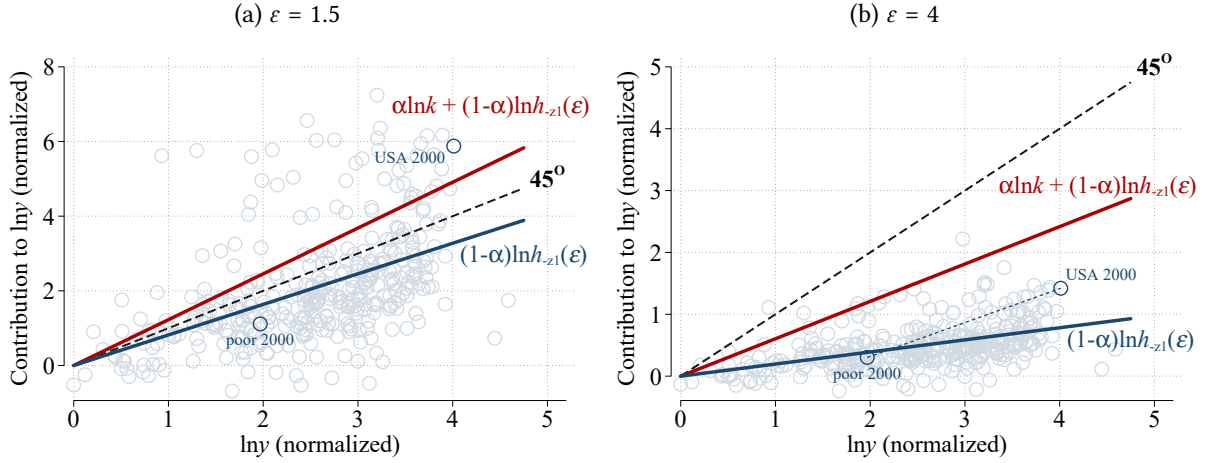
To gauge the importance of human capital, Jones (2014) asks how much of the cross-country differences in log income ($\ln y$) can be accounted for by the term $(1 - \alpha) \ln h_{-z_1}(\varepsilon)$, which captures the contribution of effective labor input to aggregate income per worker. To isolate the role of ε for Jones' results, we ignore potential differences in q_1 across countries.

Figure 8 shows the calculated $(1 - \alpha) \ln h_{-z_1}(\varepsilon)$ on the vertical axis against log income per worker, assuming $\varepsilon = 1.5$ in Panel a and $\varepsilon = 4$ in Panel b. We assume a capital intensity of $\alpha = 1/3$. Variables are normalized to set the predicted value for the poorest country to zero, and we subtract year fixed effects from $(1 - \alpha) \ln h_{-z_1}(\varepsilon)$ to focus on its cross-country variation. Each country-year observation is depicted by a circle, and the solid blue line shows the regression line. Its slope, our coefficient of interest, represents the average share of log income differences explained by the effective labor input. The solid red line similarly depicts the fitted values for $\alpha \ln k + (1 - \alpha) \ln h_{-z_1}(\varepsilon)$. (Country-year observations are not shown.) Its slope captures the total contribution of capital and effective labor. If it exceeds one (red line above the 45-degree line), labor and capital explain more than 100 percent of income gaps, implying that z_1 must decrease with income. We report the slopes of the fitted lines in Table 2 to summarize each factor's contribution to income differences.

Low substitutability greatly widens the $h_{-z_1}(\varepsilon)$ gap between rich and poor countries. For $\varepsilon = 1.5$, effective labor input captures 82 percent of income differences. This is too high, given cross-country differences in incomes and capital stocks. Since capital explains 41 percent of income gaps in our sample, capital and labor together account for 123 percent. An unavoidable implication is that z_1 decreases substantially with development. For instance, workers in the bottom schooling group in a poor country, at the 20th percentile of per capita income, must be 80 percent more efficient than workers of that schooling in a rich country, at the 80th income percentile.²¹

²¹Jones approximates the q_1 gap between rich and poor countries by wage differences among U.S. immigrants. He finds q_1 to be slightly lower in rich countries. Thus, lower efficiency largely translates into a contraction of the technology frontier. For instance, Jones (2014) finds Israel's q_1 to be 17 percent lower than Kenya's. Given Israel has 16.9 times Kenya's income, this explains only $(1 - \alpha) \ln(0.83)/\ln(16.9) = -4$ percent of their income gap, leaving -19 percent to be accounted for by Israel's lower b_1 .

FIGURE 8: Development Accounting Using Jones' (2014) Framework



Notes: Figure plots the effective labor input $(1 - \alpha)h_{-z1}(\varepsilon)$ against log GDP per worker. Variables are normalized to set the predicted value for the poorest country to zero, and year fixed effects are subtracted from the vertical-axis variable. The solid lines depict the OLS fitted values (see description in the text). The poor country is defined as the median country with less than a quarter of US GDP per worker in 2000.

TABLE 2: Share of Income Differences Explained by Factors (%)

ε	1.5	4	∞
$\alpha \ln k$	41	41	41
$(1 - \alpha) \ln h_{-z1}(\varepsilon)$	82	20	7
Total	123	61	48

Notes: Each column reports accounting results given long-run substitutability (ε in column header). Shares are obtained by projecting each factor (specified in row header) on log GDP per worker controlling for year fixed effects.

Our lower bound on substitutability, $\varepsilon = 4$, substantially limits the contribution of effective labor input to 20 percent of income differences. Capital and labor together contribute only 60 percent in this case, implying that z_1 increases with income, accounting for the remaining 40 percent of income differences. In general, using a higher elasticity of substitution results in a lower contribution of labor inputs and a larger contribution of z_1 . For example, early papers treated schooling groups as perfect, but unequal, substitutes (see, e.g., [Klenow and Rodrigues-Clare, 1997](#); [Hall and Jones, 1999](#); [Caselli, 2005](#)). In this case, the contribution of labor input reduces to 7 percent, leaving about half of the income gap to be explained by z_1 . In that sense, development accounting under our lower bound ($\varepsilon = 4$) is much closer to that under perfect substitutes ($\varepsilon = \infty$) than that under $\varepsilon = 1.5$.

Some studies treat the United States as the rich country benchmark. Doing so does not change our conclusion. However, we note that the $h_{-z_1}(\varepsilon)$ for the United States is above average among similarly rich countries. As a result, US comparisons give a larger role for effective labor for any ε . We illustrate this for the year 2000 in [Figure 8](#) by comparing the United States with the median poor country. Poor countries are defined, as in [Hendricks and Schoellman \(forthcoming\)](#), to be those with less than a quarter of the United States' real GDP per worker. We find that effective labor accounts for 55 percent of the income gap under $\varepsilon = 4$, demonstrated by the steeper slope of the dashed blue line in Panel b, and an incredible 233 percent under $\varepsilon = 1.5$.²²

[Jones \(2014\)](#) interprets his framework as accounting for the role of *human capital* in explaining income differences. However, [Caselli and Ciccone \(2019\)](#) point out that this framework counts not only human capital but also all other factors that affect the efficiency gains from schooling, such as skill-biased technology. Through the lens of our model, higher skill premia in richer countries may reflect better schooling (higher ϕ_q) or more skill-biased technology frontiers (higher ϕ_b). [Jones's \(2014\)](#) thought experiment tells us what percentage of the income gap would have been bridged if poor countries could achieve rich countries' educational attainment (L_i), human capital gains from schooling (ϕ_q), as well as the shape of their technology frontier (ϕ_b). Attributing technological gains to human capital overstates the variation in human capital when richer countries' technology frontier is more skill biased. The next subsection provides an alternative accounting that isolates human capital from other skill-biased factors.

²²The contribution of effective labor to the income gap ranges from 48 percent to 65 percent between 1980 and 2000 under $\varepsilon = 4$.

4.3 School Quality and the Skill Bias of Technology Frontier across Countries

To isolate the contribution of human capital from the skill bias of technology frontiers, we employ two measures of school quality. The first is [Schoellman's \(2012\)](#) measure based on immigrants in the United States. This measure reflects the wage premium in the US labor market for education received in the immigrants' countries of origin. Because the wage premia are for the US labor market, their variation reflects the quality of schooling in the country of origin rather than the market-specific factors of technology and schooling scarcity. Specifically, letting c denote the country of origin, we assume that the long-run aggregate labor input in the US is given by:

$$H_{US} = \left[\sum_{i \in S} \left(\sum_{c \in C} b_{i,US} q_{i,c} L_{i,c} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

This formulation assumes that immigrants from different countries are perfect but potentially unequal substitutes. The implied US wage premium is,

$$m_c^{US} = \zeta + \phi_{q,c},$$

where m_c^{US} and $\phi_{q,c}$ are each specific to a worker's country of origin; ζ is a constant reflecting the schooling scarcity and the skill bias of accessible technology in the US (see [Appendix E](#) for derivations). Going forward, we keep the country subscript implicit and refer to US immigrant Mincer return as m^{US} . [Schoellman \(2012\)](#) estimates m^{US} for 131 countries of origin using the 2000 decennial census. Merging his data with PWT 9.1 yields a sample with 116 countries.

Our second measure is based on the test scores from the Programme for International Student Assessment (PISA). PISA strives to evaluate educational systems around the world by testing the scholastic abilities of 15-year-old students along three dimensions: mathematics, science, and reading. We construct a school quality measure based on the micro-level data on test results provided by the OECD for the 2015 wave of tests. The results reported here use test scores for mathematics.²³ We measure school quality by the ratio of the country test score to the modal grade year for those taking the test.²⁴ Expressing the result, which is in units of standardized

²³We pick mathematics because the content is more comparable across countries and because it correlates more strongly with average educational attainment across countries. Measures based on other fields do not vary systematically with income per worker. Following our later discussion, this implies a lower role for human capital and larger role for skill-biased technology frontier.

²⁴Test takers from the same country can be in different grades due to the differing age at which they begin school. We use this variation to construct an alternative measure and summarize the results in [Appendix F](#).

test score, in terms of school quality ϕ_q , which is in wage units, requires a market value for the test score. We calibrate this value to US estimates of the wage return to standardized test score. To that end, we first divide the marginal test score attributable to a year of schooling by the standard deviation of the test score in the United States—this translates schooling quality into units of the standard deviation of US test scores. We then multiply by 15 percent, the wage return to a unit standard deviation of ability in the US as estimated by [Lange \(2007\)](#) using data on the Armed Forces Qualification Test for the 1979 cohort of the National Longitudinal Survey of Youth.²⁵ Merging the PISA-based school quality measures with PWT 9.1 yields a sample with 62 countries.

Given measures of school quality, we can infer the relationship between income and skill bias of technology frontier (ϕ_b). To do so, we first rearrange equation (8) to back out a country's total efficiency gain from schooling ($\phi_q + \phi_b$) implied by its Mincer return (m), scarcity (x), and a particular value of substitutability (ε):

$$\phi_q + \phi_b = \left(\frac{\varepsilon}{\varepsilon - 1} \right) m - \left(\frac{1}{\varepsilon - 1} \right) x. \quad (14)$$

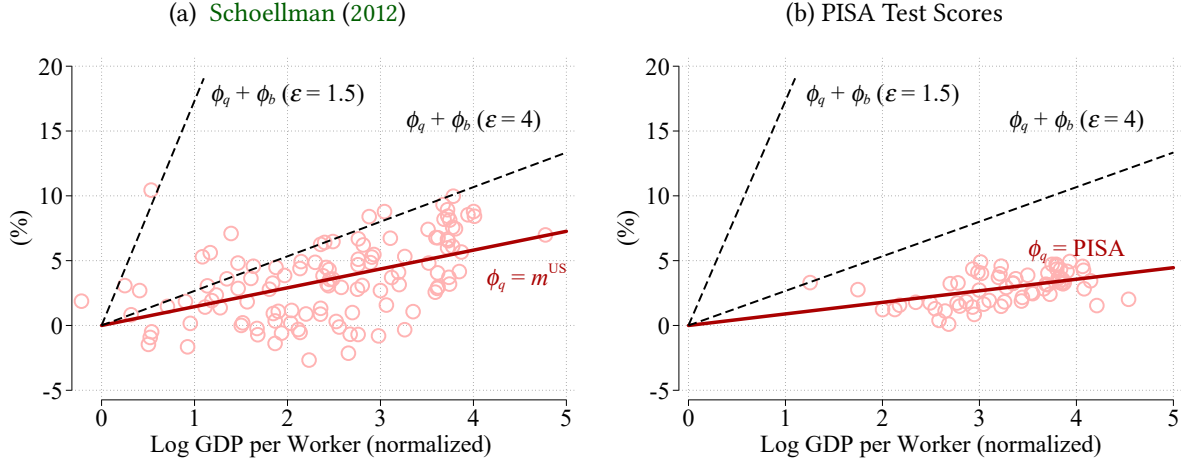
We calculate the right-hand side, denoted as $\tilde{\phi}_z(\varepsilon)$, using our cross-country sample. This allows us to infer the skill bias of technology frontiers by $\tilde{\phi}_z(\varepsilon) - \phi_q$ given each measure of school quality.

Panel a in Figure 9 contrasts [Schoellman's \(2012\)](#) school quality measure on the vertical axis against log GDP per worker, where variables are normalized to set the predicted value for the poorest country equal to zero. The red solid line, which shows the fitted values from a linear regression, displays a positive relationship between a country's income and its school quality. Specifically, each log point increase in income is associated with 1.5 percentage points higher ϕ_q (with a standard error of 0.2). Panel b shows the scatter plot and fitted lines for the PISA-based school quality measure. Under this measure, a one log point higher income is associated with 0.7 percentage points higher ϕ_q (with a standard error of 0.2).

Figure 9 also shows the inferred ϕ_b gap between rich and poor countries. The black dashed lines show the fitted values of $\tilde{\phi}_z(\varepsilon)$ projected on income for $\varepsilon = 1.5$ and $\varepsilon = 4$, respectively (controlling for year fixed effects). The gap between the red solid line and each black dashed line gives an inferred relationship between ϕ_b and income. As discussed in Section 4.1, lower

²⁵[Lange \(2007\)](#) estimates the return to ability in a Mincerian wage regression for workers with different levels of work experience. We pick the return for older workers (Figure 2), which better reflects the impact of ability on worker productivity. The wage return to ability is known to be slightly lower among younger workers due to employer uncertainty regarding worker ability ([Altonji and Pierret, 2001](#)).

FIGURE 9: Measured School Quality vs. Implied $\tilde{\phi}_z(\varepsilon)$



Notes: Figure plots measures for school quality (ϕ_q) against $\ln(\text{GDP})$ per worker across countries. Panel a shows the immigrant-based measure from Schoellman (2012), and Panel b shows the test-score-based measure calculated from PISA. Variables are normalized to set the predicted value for the poorest country to zero. The solid red lines depict the linear fitted values for school quality. The black dashed lines depict the projection of $\tilde{\phi}_z(\varepsilon)$ on log GDP per worker for $\varepsilon = 1.5$ and 4 respectively.

substitutability widens the gap of efficiency gain from schooling. Although we do observe a positive relationship between school quality and income, the gradient would need to be about an order of magnitude steeper to align with the slope implied by $\varepsilon = 1.5$, where a log point increase in income is associated with 17 percentage points higher $\tilde{\phi}_z(1.5)$ (standard error 1.2). Given the strength of the empirical relationship between school quality and income, skill bias of the technology frontier, ϕ_b , must account for much of the immense efficiency gap between rich and poor countries.²⁶

Under an elasticity of 4, by contrast, a log point increase in income is only associated with 2.7 percentage points higher $\tilde{\phi}_z$ (standard error 0.3). In turn, a one log point higher income predicts 1.2 percentage points higher ϕ_b given Schoellman's school quality measure and 2 percentage points higher ϕ_b under the PISA-based measure. So, for $\varepsilon = 4$, school quality and skill-biased technology are roughly comparable in importance in explaining the greater efficiency of more-schooled workers in richer countries, with school quality somewhat more important under the Schoellman measure and technology more important under the test-score measure.

Motivated by this, we ask to what extent cross-country income gaps would be bridged if

²⁶This parallels Rossi (2022). Rossi, using a low short-run substitutability ($\varepsilon_{\text{SR}} = 1.5$ or 2) and an immigrant-based school quality measure, finds large cross-country variations in the skill bias of technology (ϕ_A).

TABLE 3: Share of Income Differences Explained by Human Capital (%)

ε	1.5	4
$(1 - \alpha) \ln h_{-z_1}(\varepsilon)$	82	20
$\phi_q = m^{US}$	18	13
$\phi_q = \text{PISA}$	15	11

Notes: Shares in bottom two rows are obtained by projecting human capital per worker, $(1 - \alpha)\tilde{h}_{-z_1}(\varepsilon)$, on log GDP per worker controlling for year fixed effects. The row header specifies the school quality measure used in calculating human capital, and the column header specifies the long-run substitutability (ε).

poor countries could achieve rich countries' education attainment (L_i) and school quality (ϕ_q) while keeping their current skill bias of technology frontier (ϕ_b). Specifically, we measure human capital, $\tilde{h}_{-z_1}(\varepsilon)$, purged of skill bias in technology as:

$$\tilde{h}_{-z_1}(\varepsilon) = \frac{1}{L} \left[\sum_{i \in S} \left(e^{\hat{\phi}_q(s_i - s_1)} L_i \right)^{\frac{\varepsilon - 1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon - 1}}, \quad (15)$$

where school quality $\hat{\phi}_q$ is predicted by the country's income according to the projection of school quality, measured by m^{US} or PISA, on log income per worker (namely the solid red lines in Figure 9). We project $(1 - \alpha)\tilde{h}_{-z_1}(\varepsilon)$ on log GDP per worker to obtain the contribution of human capital to cross-country income differences.

Table 3 summarizes the contribution of human capital under $\varepsilon = 1.5$ and $\varepsilon = 4$. The first row repeats the results from Table 2 for the broad contribution of schooling to per capita income, reflecting schooling quantity, schooling quality, or skill-biased technology. Rows 2 and 3 report the pure contribution of schooling quantity and quality incorporating the school quality measures from Schoellman (2012) and PISA scores respectively. Low substitutability, $\varepsilon = 1.5$, generates a huge gap between rich and poor countries in the relative efficiency of workers with more schooling. But this gap is not justified by richer countries' advantages in years and measured quality of schooling. For $\varepsilon = 1.5$, the quantity and quality of schooling account for only 15-18 percent of cross-country income differences, depending on the measure of schooling quality. Under our lower bound for the elasticity, $\varepsilon = 4$, the quantity and quality of schooling account for only 11-13 percent.

To sum up, this section contributes to the development accounting literature in two ways. First, we bound the contribution of schooling to cross-country income differences by our find-

ings from Section 3. Second, we apply measures of school quality to assess human capital while keeping the skill bias of technology frontier constant across countries. We find that using our lower bound on substitutability and incorporating school quality measures effectively caps the contribution of human capital to income differences below 15 percent.

We note that how school quality varies by development also implies an *upper* bound on substitutability if we require rich countries' technology frontiers to be more skill biased on average (higher ϕ_b). To see this, recall from Figure 9 that the correlation between skill bias and income (captured by the difference between the slopes of the dashed black lines and the solid red line) is decreasing in ε . For low values of ε , this difference is positive, implying that rich countries have more skill-biased technologies. But when schooling groups are perfect substitutes ($\varepsilon = \infty$), the sum of skill bias and school quality, $\tilde{\phi}_z(\varepsilon) = \phi_q + \phi_b$, simplifies to the Mincer return, which is negatively correlated with income (see Figure 7). Since school quality, ϕ_q is increasing with income in the data, skill bias must then decrease with income. Consequently, there is an upper bound $\bar{\varepsilon} < \infty$ such that, for all $\varepsilon > \bar{\varepsilon}$, the implied skill bias decreases with income on average, i.e., the dashed black line goes below the solid red line. This implied upper bound is 6 when using Schoellman's school quality measure and 10 when using the PISA-based measure.²⁷

5 Robustness to Alternative Production Technologies

In this section, we explore the generality of our results to alternative groupings of the schooling categories in the production function. These groupings allow varying degrees of substitutability among different schooling groups. For instance, the literature has often classified workers into college-trained versus all others. This classification assumes perfect, though possibly unequal, substitution within these groups while allowing for imperfect substitution between them. The upshot of what follows is that our bounding approach is generally consistent with a lower degree of local substitutability, that is, between a subset of schooling categories, provided a sufficiently higher degree of substitutability is adopted among other categories in order that the overall substitutability remains high.

Nevertheless, these different formulations of the long-run production function are nearly identical in terms of their implications for development accounting: Schooling differences explain

²⁷This parallels Hendricks and Schoellman's (forthcoming) identification for long-run substitutability. They assume no cross-country variation in the skill bias of technology frontier, ϕ_b , and use cross-country data on Mincer return, schooling, and school quality measures to identify substitutability and assess human capital variation. We, on the other hand, bound substitutability from below by growth accounting and allow ϕ_b to vary with income.

roughly a fifth of the income gaps across countries. They cast, however, a more nuanced role for human capital, allowing its contribution to vary along the income distribution. We discuss these nuances and caution against the potential pitfalls of using alternative formulations of the production function as a general tool for policy analysis.

5.1 Bounding the Elasticity by Growth Accounting

We begin by noting that our results generalize to the following class of production functions.

$$H = \left[\left(z_1 L_1 \right)^{\frac{\varepsilon-1}{\varepsilon}} + \tilde{H} \left(z_2 L_2, \dots, z_S L_S \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad (16)$$

where \tilde{H} is any constant-returns-to-scale aggregator. This class of functions covers any grouping combinations of the higher schooling categories, with constant or varying elasticities of substitution among them. It includes, among others, a two-group specification where workers with more than primary schooling are perfect substitutes for each other. The only restriction is that the elasticity of substitution between the lowest schooling category and the composite supplied by other workers, \tilde{H} , is constant at ε . The lower bound on that elasticity of substitution is the same as our benchmark CES production function.²⁸ This property is inherent in the accounting approach adopted by Jones (2014). Various elasticities introduced by different formulations of \tilde{H} are reflected in the relative wage term in equation (10) for inference purposes.

Next, we explore the robustness of our results to groupings of the lowest schooling categories in a nested-CES form. Formally, we study the following class of production functions.

$$H = \left[\left(\sum_{j \leq N} (z_j L_j)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1} \cdot \frac{\rho-1}{\rho}} + \tilde{H} \left(z_{N+1} L_{N+1}, \dots, z_S L_S \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}. \quad (17)$$

This specification assumes a constant elasticity of substitution, σ , among the bottom schooling groups and a constant elasticity of substitution of ρ between the composite bottom group and the higher schooling categories. Equation (17) captures a wide variety of commonly used groupings as special cases. If $\sigma = \rho = \varepsilon$ the production function simplifies, yielding our benchmark results. Starting from our four distinct schooling categories, if $N = 3$ and $\sigma = \infty$ then the specification

²⁸Note that equation (16) also generalizes to formulations where higher schooling levels are split further into more than 3 categories.

yields the oft-used college versus non-college grouping that assumes all non-college workers are perfect substitutes. $N = 2$ and $\sigma = \infty$ produces a similar specification for workers with more than versus those with less than a high school education.

We examine the implications of our bounding assumption of no frontier regress for the lowest schooling category, $g_{z_1} \geq 0$, for a range of possible values for σ and ρ . Let G be the compound group of workers with the lowest N schooling levels. Denote the total number of workers in G by $L_G = \sum_{j \leq N} L_j$ and their average wage by $\bar{w}_G \equiv \left(\frac{1}{L_G}\right) \sum_{j \leq N} w_j L_j$. With this notation and the assumption of competitive labor markets, the human capital per worker, $h = H/L$, can be expressed as follows (see Appendix G for the derivation).

$$h(\sigma, \rho) = z_1 \left(\frac{\bar{w}_G}{w_1} \right)^{\frac{\sigma}{\sigma-1}} \left(\frac{L_G}{L_1} \right)^{\frac{1}{\sigma-1}} \left(\frac{\bar{w}}{\bar{w}_G} \right)^{\frac{\rho}{\rho-1}} \left(\frac{L}{L_G} \right)^{\frac{1}{\rho-1}}. \quad (18)$$

Analogous to equation (10) in our benchmark specification, equation (18) expresses average human capital as a function of the technology frontier associated with the lowest schooling level, z_1 , elasticities of substitution, σ and ρ , and the relevant relative wages and labor supplies.

In growth terms, we have:

$$g_h(\sigma, \rho) = g_{z_1} + g_{h-z_1}(\sigma, \rho). \quad (19)$$

We employ our bounding assumption of no regress of the technology frontier, $g_{z_1} \geq 0$ to compute the range of elasticities that are consistent with the growth rates observed in the aggregate data. Since we have four schooling categories, the two relevant alternatives to our benchmark are either $N = 2$, where workers with less than some secondary education, s_1 and s_2 , are included in the bottom nest: $G = \{1, 2\}$, or $N = 3$, where the nest consists of those with a completed high school education or below: $G = \{1, 2, 3\}$.²⁹

The solid curves in Figure 10 show the combinations of substitution elasticities ρ and σ for

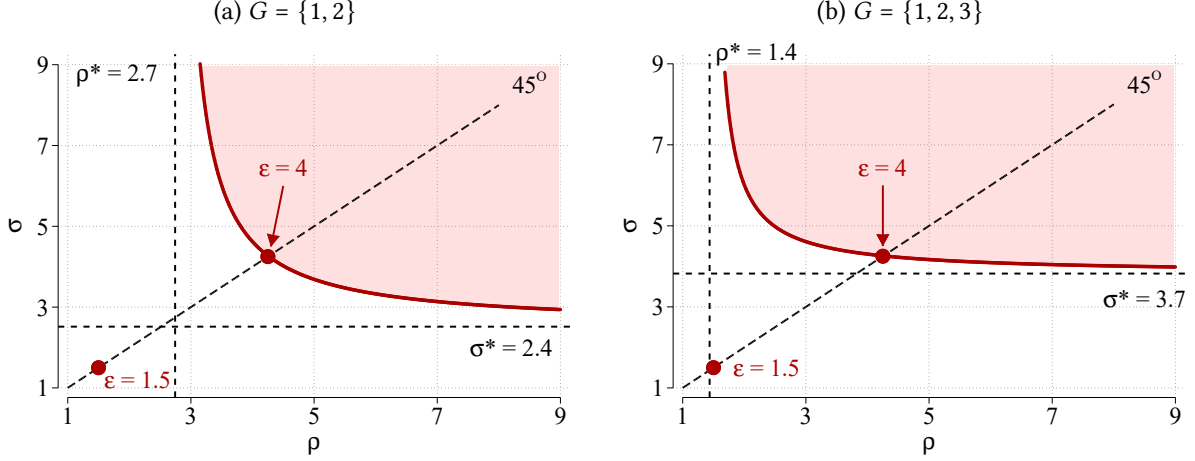
²⁹ The explicit production functions are

$$H = \left[\left[(z_1 L_1)^{\frac{\sigma-1}{\sigma}} + (z_2 L_2)^{\frac{\sigma-1}{\sigma}} + (z_3 L_3)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1} \cdot \frac{\rho-1}{\rho}} + (z_4 L_4)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}},$$

and

$$H = \left[\left[(z_1 L_1)^{\frac{\sigma-1}{\sigma}} + (z_2 L_2)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1} \cdot \frac{\rho-1}{\rho}} + \tilde{H}(z_3 L_3, z_4 L_4)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}.$$

FIGURE 10: Bounding (ρ, σ) by $g_{z_1} \geq 0$.



Notes: Figure shows the combinations of substitution elasticities, ρ and σ , that are consistent with the stable technological capacity associated with the bottom schooling groups: $g_{z_1} = 0$. G denotes the lowest schooling groups nested in the production equation (17). (See footnote 29 for the explicit production functions used.)

equation (19) that are consistent with a stable technological frontier for the lowest schooling category, bounding condition $g_{z_1} = 0$. Panel (a) treats the bottom groups as less than secondary, while Panel (b) treats them as less than college. Combinations that are closer to the origin relative to the curve imply a long-run contraction of the technology frontier, and thus violate our bounding assumption. Our benchmark specification is represented by the intersection of the curve with the 45° line, where $\sigma = \rho = 4$.

Figure 10 depicts the trade-offs in how schooling levels are categorized versus implied substitutability. Generally, an elasticity of substitution lower than 4 among some schooling groups is consistent with our bound only if a higher elasticity is adopted elsewhere along the schooling distribution. In Panel (a), for instance, low substitutability, $\rho < 4$, between high school graduates and above (groups 3 and 4) and those with less than a high school diploma (groups 1 and 2) must be accompanied by easier substitutability between workers with a primary school education and those with some secondary education. Even with perfect substitution within this bottom group ($\sigma = \infty$), however, the lowest value of ρ that is consistent with our bound is 2.7. On the other extreme, the lowest allowable value for σ is 2.4. But it is obtainable only if higher-schooling categories are perfect substitutes with lower-schooling categories ($\rho = \infty$).

Including more schooling categories in the bottom group, shown in Panel (b), allows for $\rho = 1.4$, but only if one assumes that the three bottom groups are perfect substitutes. In our

60-country sample, those three groups represent 90 percent of the work force on average. Thus, this requires that a large fraction of the work force be perfect substitutes. Looking at the other extreme, the lowest permissible value for σ in Panel (b) is still high at 3.7, even though $\rho = \infty$ is adopted.

It is common in the literature to combine schooling categories a priori into two groups that are imperfect substitutes, implicitly assuming perfect substitution within each group. If one of these categories includes a vast majority of workers, as in Panel (b) of Figure 10, then an elasticity of substitution on the order of 1.5 can be consistent with worldwide growth in output and human capital because the worldwide gains in schooling have mostly occurred at lower levels, between primary and secondary education. If these categories are assumed to be perfect substitutes, then these gains do not translate into counterfactually high gains in output.

Specifications with such low degrees of local substitutability may nonetheless lead to stark outcomes when projecting output gains from further achievements in attainment, especially if schooling gains are concentrated in transitions between hard-to-substitute groups. Consider, as an example, the extreme case in Panel (b), where all workers without college are perfect substitutes, but are, as a group, hard to substitute for those with college training: $\sigma = \infty$ and $\rho = 1.4$. The implied output growth under this specification remains too large to reconcile with historical income growth for countries with large transitions between secondary to tertiary education, such as the United States or Japan. Given the growth rates of output and physical capital in these countries, this can only be reconciled if the technology frontier associated with the lowest schooling group, z_1^{US} and z_1^{JPN} , contracted at annual rates of 3.6 percent and 3.4 percent respectively.³⁰

5.2 Revisiting Development Accounting

Despite the seemingly wide variation in degrees of substitutability depicted in Figure 10, the implications of these alternative specifications for development accounting are strikingly similar to those of our benchmark results. In Table 4, we show the average contribution of schooling to cross-country income gaps under two extreme combinations of ρ and σ for each production specification presented in Figure 10. The first row repeats the benchmark results from Table 2. The next row assumes perfect substitution among the bottom schooling categories ($\sigma = \infty$) and sets

³⁰In a similar two-group setup with low substitutability between college and non-college workers, Caselli and Coleman (2002) infer a drop in the technical efficiency of non-college workers for the US since 1980. They interpret this as an endogenous shift in production technologies away from these workers—a drop in their average A_i in our notation.

TABLE 4: Share (%) of $\ln y$ Explained by $(1 - \alpha) \ln h_{-z_1}(\sigma, \rho)$

	$G = \{1, 2\}$	$G = \{1, 2, 3\}$
$\sigma = \rho = 4$	20	20
$\sigma = \infty, \rho = \underline{\rho}$	21	25
$\sigma = \underline{\sigma}, \rho = \infty$	17	18

Notes: Table shows the share of income per capita that can be explained by differences in schooling under alternative production function specifications (see text). Shares represent averages obtained by projections.

substitutability between the bottom group and other groups to the lowest possible value ($\rho = \underline{\rho}$) allowed by our bounding condition. Analogously, the third row sets $\rho = \infty$ and $\sigma = \underline{\sigma}$.³¹ Across different specifications and parameter combinations, schooling differences explain 17-25 percent of income differences across countries. The robustness of the development accounting results reflects the empirical discipline imposed by our bound from growth accounting. Observed output growth puts a limit on the productivity gains that can be attributed to the long-run decline in the scarcity of schooling worldwide. When cross-country human capital differences are computed in a way that is consistent with that insight from growth accounting, the results are comparable across a wide range of production function specifications.

6 Conclusions

We show that growth accounting points to a long-run elasticity of substitution across schooling groups of four, or above. Elasticities that are significantly lower imply rapid technological regress for a large section of the workforce worldwide for 1960 to 2010, even beyond that from technology shifting endogenously away from workers with less schooling due to their declining importance. In particular, under $\varepsilon = 1.5$ the frontier must contract by 98 percent for those without secondary schooling, a group that averaged about 40 percent of the population for our sample of 60 countries.

The elasticity of substitution plays an important role in several quantitative literatures. It is obviously important for understanding the evolution of earnings inequality. It is also central to

³¹With $\sigma = \infty$, the corresponding values for $\underline{\rho}$ are 2.7 when $G = \{1, 2\}$ and 1.4 when $G = \{1, 2, 3\}$. With $\rho = \infty$, the elasticities for $\underline{\sigma}$ are 2.4 when $G = \{1, 2\}$ and 3.7 when $G = \{1, 2, 3\}$.

the study of human capital's role in income differences across countries. [Klenow and Rodrigues-Clare \(1997\)](#) and [Hall and Jones \(1999\)](#), who treat schooling groups as perfect, but unequal, substitutes, find that human capital differences are relatively small compared to the enormous differences in worker productivities across countries. [Jones \(2014\)](#), [Caselli \(2016\)](#), and [Malmberg \(2018\)](#) each allow imperfect substitution. This implies greater differences in worker efficiencies across schooling types in rich countries than in poor ones, much greater for elasticities on the order of $\varepsilon = 1.5$. In turn, this yields much bigger differences in average worker efficiency across countries assuming that less-educated workers are as efficient in rich as in poor countries.

Our bound on the elasticity implies smaller efficiency differences from schooling between rich and poor countries. To illustrate, we conduct the exercise in [Jones \(2014\)](#) for $\varepsilon = 1.5$ versus $\varepsilon = 4$. For $\varepsilon = 1.5$, worker efficiency from schooling accounts for most, 82 percent of the income differences between rich and poor countries, and together with physical capital it accounts for well over 100 percent of those differences. By contrast, under our proffered value of $\varepsilon = 4$, it accounts for 20 percent of income differences, or 60 percent in combination with physical capital.

Our preferred elasticity of 4 does still imply that workers with more schooling are considerably more efficient in richer countries. In fact, we show that an elasticity of 4 is consistent with both better quality of schooling in richer countries, as measured based on international test scores or the earnings of immigrants to the United States from [Schoellman \(2012\)](#), and a more skill-biased technology frontier in richer countries as in [Caselli and Coleman \(2006\)](#) and [Rossi \(2022\)](#).

References

- Acemoglu, Daron (2007). “Equilibrium bias of technology.” *Econometrica*, 75(5), pp. 1371–1409. doi:[10.1111/j.1468-0262.2007.00797.x](https://doi.org/10.1111/j.1468-0262.2007.00797.x).
- Acemoglu, Daron and David Autor (2011). “Skills, tasks and technologies: Implications for employment and earnings.” In *Handbook of Labor Economics*, volume 4, pp. 1043–1171. Elsevier. doi:[10.1016/S0169-7218\(11\)02410-5](https://doi.org/10.1016/S0169-7218(11)02410-5).
- Acemoglu, Daron and Pascual Restrepo (2020). “Unpacking skill bias: Automation and new tasks.” In *AEA Papers and Proceedings*, volume 110, pp. 356–361. doi:[10.1257/pandp.20201063](https://doi.org/10.1257/pandp.20201063).
- Altonji, Joseph G. and Charles R. Pierret (2001). “Employer learning and statistical discrimination.” *Quarterly Journal of Economics*, 116(1), pp. 313–350. doi:[10.1162/003355301556329](https://doi.org/10.1162/003355301556329).
- Autor, David H., Lawrence F. Katz, and Melissa S. Kearney (2008). “Trends in US wage inequality: Revising the revisionists.” *Review of Economics and Statistics*, 90(2), pp. 300–323. doi:[10.1162/rest.90.2.300](https://doi.org/10.1162/rest.90.2.300).
- Barro, Robert J. and Jong Wha Lee (2013). “A new data set of educational attainment in the world, 1950–2010.” *Journal of Development Economics*. doi:[10.1016/j.jdeveco.2012.10.001](https://doi.org/10.1016/j.jdeveco.2012.10.001).
- Bowles, Samuel (1970). “Aggregation of labor inputs in the economics of growth and planning: experiments with a two-level CES function.” *Journal of Political Economy*, 78(1), pp. 68–81. doi:[10.1086/259601](https://doi.org/10.1086/259601).
- Bowlus, Audra, Lance Lochner, Chris Robinson, and Eda Suleymanoglu (2021). “Wages, skills, and skill-biased technical change: The canonical model revisited.” *Journal of Human Resources*, p. 0617. doi:[10.3368/jhr.0617-8889R1](https://doi.org/10.3368/jhr.0617-8889R1).
- Card, David and Thomas Lemieux (2001). “Can falling supply explain the rising return to college for younger men? A cohort-based analysis.” *Quarterly Journal of Economics*, 116(2), pp. 705–746. doi:[10.1162/00335530151144140](https://doi.org/10.1162/00335530151144140).
- Caselli, Francesco (2016). *Technology Differences over Space and Time*. CREI Lectures in Macroeconomics. Princeton University Press. doi:[10.1515/9781400883608](https://doi.org/10.1515/9781400883608).
- Caselli, Francesco (2005). “Accounting for cross-country income differences.” In Philippe Aghion and Steven N. Durlauf, editors, *Handbook of Economic Growth*, volume 1A, chapter 9, pp. 679–741. Amsterdam: Elsevier. doi:[10.1016/S1574-0684\(05\)01009-9](https://doi.org/10.1016/S1574-0684(05)01009-9).

- Caselli, Francesco and Antonio Ciccone (2019). “The human capital stock: A generalized approach: Comment.” *American Economic Review*, 109(3), pp. 1155–1174. doi:[10.1257/aer.20171787](https://doi.org/10.1257/aer.20171787).
- Caselli, Francesco and Wilbur John Coleman, II (2002). “The US technology frontier.” *American Economic Review*, 92(2), pp. 148–152. doi:[10.1257/000282802320189159](https://doi.org/10.1257/000282802320189159).
- Caselli, Francesco and Wilbur John Coleman, II (2006). “The world technology frontier.” *American Economic Review*, 96(3), pp. 499–522. doi:[10.1257/aer.96.3.499](https://doi.org/10.1257/aer.96.3.499).
- Ciccone, Antonio and Giovanni Peri (2005). “Long-run substitutability between more and less educated workers: Evidence from U.S. states, 1950–1990.” *Review of Economics and Statistics*, 87(4), pp. 652–663. doi:[10.1162/003465305775098233](https://doi.org/10.1162/003465305775098233).
- Feenstra, Robert C., Robert Inklaar, and Marcel P. Timmer (2015). “The next generation of the Penn World Table.” *American Economic Review*, 105(10), pp. 3150–3182. doi:[10.1257/aer.20130954](https://doi.org/10.1257/aer.20130954). Available for download at www.ggdc.net/pwt.
- Feldstein, Martin S. (1973). “On the optimal progressivity of the income tax.” *Journal of Public Economics*, 2(4), pp. 357–376. doi:[10.1016/0047-2727\(73\)90026-1](https://doi.org/10.1016/0047-2727(73)90026-1).
- Goldin, Claudia and Lawrence F. Katz (2010). *The Race Between Education and Technology*. Harvard University Press, Cambridge, Mass.
- Hall, Robert E. and Charles I. Jones (1999). “Why do some countries produce so much more output per worker than others?” *Quarterly Journal of Economics*, 114(1), pp. 83–116. doi:[10.1162/003355399555954](https://doi.org/10.1162/003355399555954).
- Heckman, James J., Lance Lochner, and Christopher Taber (1998). “Explaining rising wage inequality: Explorations with a dynamic general equilibrium model of labor earnings with heterogeneous agents.” *Review of Economic Dynamics*, 1(1), pp. 1–58. doi:[10.1006/redo.1997.0008](https://doi.org/10.1006/redo.1997.0008).
- Hendricks, Lutz and Todd Schoellman (forthcoming). “Skilled labor productivity and cross-country income differences.” *American Economic Journal: Macroeconomics*. doi:[10.1257/mac.20200256](https://doi.org/10.1257/mac.20200256).
- Hsieh, Chang-Tai and Peter J. Klenow (2010). “Development accounting.” *American Economic Journal: Macroeconomics*. doi:[10.1257/mac.2.1.207](https://doi.org/10.1257/mac.2.1.207).
- Johnson, Matthew and Michael P. Keane (2013). “A dynamic equilibrium model of the US wage structure, 1968–1996.” *Journal of Labor Economics*, 31(1), pp. 1–49. doi:[10.1086/666698](https://doi.org/10.1086/666698).

- Jones, Benjamin F. (2014). “The Human Capital Stock: A Generalized Approach.” *American Economic Review*. doi:[10.1257/aer.104.11.3752](https://doi.org/10.1257/aer.104.11.3752).
- Katz, Lawrence F. and Kelvin M. Murphy (1992). “Changes in relative wages, 1963-1987: Supply and demand factors.” *Quarterly Journal of Economics*, 107(1), pp. 35–78. doi:[10.2307/2118323](https://doi.org/10.2307/2118323).
- Klenow, Peter J. and Andres Rodrigues-Clare (1997). “The neoclassical revival in growth economics: Has it gone too far?” In Ben S. Bernanke and Julio Rotemberg, editors, *NBER Macroeconomics Annual*, volume 12, pp. 73–114. MIT Press. doi:[10.1086/654324](https://doi.org/10.1086/654324).
- Lange, Fabian (2007). “The speed of employer learning.” *Journal of Labor Economics*, 25(1). doi:[10.1086/508730](https://doi.org/10.1086/508730).
- Malmberg, Hannes (2018). “How does the efficiency of skilled labor vary across rich and poor countries? An analysis using trade and industry data.” Unpublished manuscript, University of Minnesota.
- Psacharopoulos, George and Keith Hinchliffe (1972). “Further evidence on the elasticity of substitution among different types of educated labor.” *Journal of Political Economy*, 80(4), pp. 786–792. doi:[10.1086/259927](https://doi.org/10.1086/259927).
- Psacharopoulos, George and Harry Antony Patrinos (2018). “Returns to investment in education: A decennial review of the global literature.” *Education Economics*. doi:[10.1080/09645292.2018.1484426](https://doi.org/10.1080/09645292.2018.1484426).
- Rossi, Federico (2022). “The relative efficiency of skilled labor across countries: Measurement and interpretation.” *American Economic Review*, 112(1), pp. 235–266. doi:[10.1257/aer.20191852](https://doi.org/10.1257/aer.20191852).
- Samuelson, Paul A. (1947). *Foundations of Economic Analysis*. Harvard University Press, Cambridge, Mass.
- Schoellman, Todd (2012). “Education quality and development accounting.” *Review of Economic Studies*, 79, pp. 388–417. doi:[10.1093/restud/rdr025](https://doi.org/10.1093/restud/rdr025).
- Tinbergen, Jan (1975). *Income Distribution: Analysis and Policies*. North-Holland Publishing Company, New York.

Appendix

A Data Appendix

Sections 3 through 5 use cross-country panel data on schooling attainments and estimated Mincer returns. The data on educational attainment by country are from Barro and Lee (2013) posted online at <http://www.barrolee.com/>. The data include 153 countries with attainments reported at five-year intervals from 1950 to 2010. It contains population frequency distributions over 7 educational categories by broad age groups. We restrict our population sample to those ages 25 to 54. We associate each attainment category with years of schooling using UNESCO Institute for Statistics (<http://data.uis.unesco.org/>) data on the duration of educational categories (in 2010) for each country. Our benchmark case divides workers into four groups: i) completed primary or less, ii) some secondary schooling, iii) completed secondary, and iv) at least some tertiary. To measure scarcity, for each country in each year, we regress the (log) size of the population in each schooling category on the years of schooling for that category. For our 104-country sample for Section 4, the average shares by group are respectively 83 percent, 9 percent, 6 percent, and 3 percent in 1965 (weighted by population), and they become 38 percent, 31 percent, 23 percent, and 8 percent in 2010.

The data on the Mincer return are obtained from Psacharopoulos and Patrinos (2018), who compile 1,120 estimates of Mincer wage equations, from micro data on workers' wages, ages and education, for 139 countries going back before 1960. We use what Psacharopoulos and Patrinos (2018) label the overall Mincerian private return (column G in Annex 2 to their paper.) In cases where multiple Mincer estimates are available for a country at the same five-year intervals, we use the average of those estimates.

The growth and income accounting in 3 and 4 also requires information on real GDP per worker and capital stock per worker for each country in each year. These data are obtained from Penn World Table 9.1 (Feenstra et al., 2015). For growth accounting, we use `rgdpna` and `rnna` for real GDP and capital stock, respectively (i.e., real variables valued at constant 2011 national prices) so that variables are comparable across time with each country. For cross-country income accounting, we use `rgdpo` and `cn` for real GDP and capital stock (i.e., real variables valued at PPPs) so that variables are comparable across countries. We then divide the levels of real income and capital stock by `emp` to obtain their per worker value.

Merging data from Barro and Lee (2013) and Psacharopoulos and Patrinos (2018) with data from the Penn World Table results in an unbalanced panel sample of 367 observations for 104 countries spanning 1960 to 2010 with data on both attainment and the Mincer return to schooling. For the growth accounting results in Section 3, we further require a country to be observed at three or more of the intervals, yielding a smaller sample with 60 countries and 298 observations.^{A1} Figure A1 describes the panel structure of our sample. Panel a depicts the number of countries for each five-year interval. It shows that our observations are mainly concentrated between 1975 and 2010. Panel b shows the frequency distributions of observations per country. Forty-five countries have less than 2 observations and hence are not used for growth accounting. The remaining countries have at least 3 observations during the sample period.

Figure A2 plots the distribution of schooling scarcity (Panel a) and Mincer return (Panel b) in our merged sample. Each light (gray) circle indicates an observation in the 60-country growth accounting sample, and each dark (blue) dot indicates an additional observation of the 105-country income accounting sample. As described in Section 3.1, scarcity trends downward due to increased educational attainment worldwide, while Mincer return stays stable over time. These patterns are consistent in both samples.

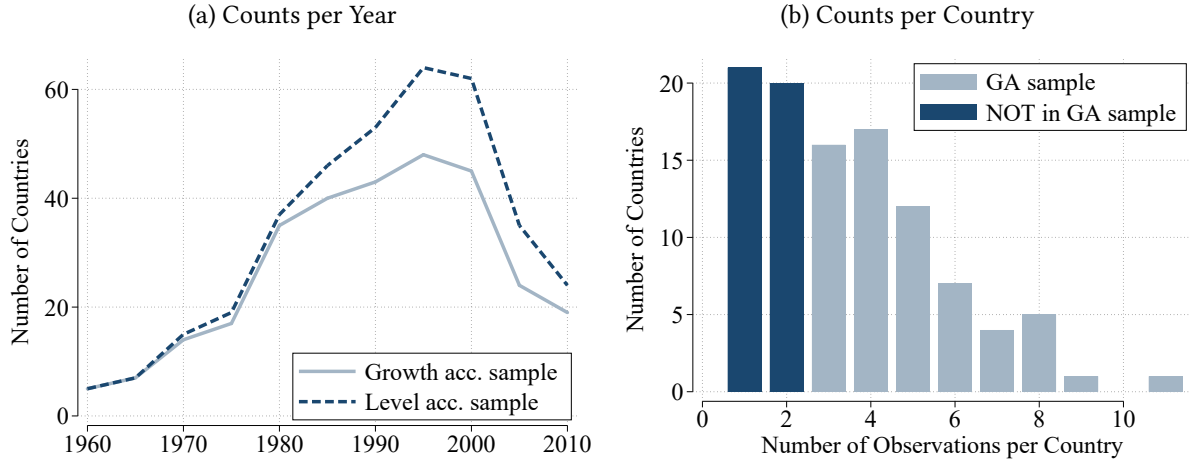
In Section 4.3, we consider two measures of schooling quality across countries. First, we employ Schoellman’s (2012) estimates of a country’s schooling quality in 2000 based on US earnings of immigrants who received all or most of their schooling in their country of birth. Schoellman’s (2012) supplementary data (reported in his Table A1) include school quality estimates for 131 countries in 2000. We merge this sample with the Penn World Table 9.1 to obtain GDP per worker. The merged sample has 116 countries and is used in Section 4.3. There are 51 countries for which we also have estimates of schooling scarcity and Mincer returns in the home countries in 2000.

Our second measure is based on standardized test scores across countries, more precisely on the gradient of the test score with respect to years of schooling by country. The testing is overseen by the Programme for International Student Assessment (PISA). These tests are given to students age 15 in three areas: mathematics, science, and reading. We construct two school quality measures based on the micro-level data from the 2015 wave of the test, as discussed in Section 4.3.^{A2} (We discuss our preferred measure in the text, the alternate only in Section F below.) These data are available from the OECD (<https://www.oecd.org/pisa/data/>). We map these test scores

^{A1}We drop observations through 1990 for countries that were formerly held in the Soviet Union.

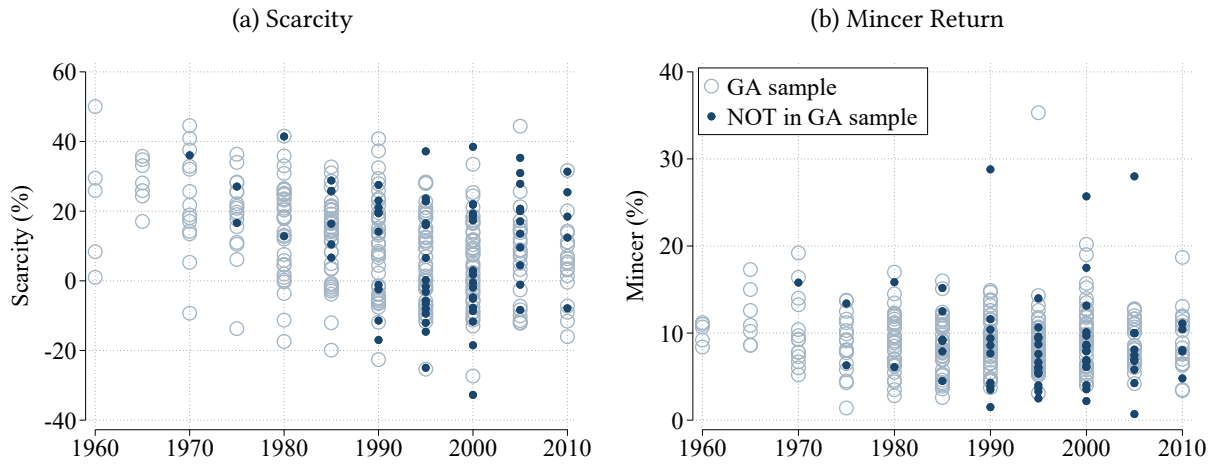
^{A2}See the OECD “PISA 2015 Results in Focus” (<https://www.oecd.org/pisa/pisa-2015-results-in-focus.pdf>).

FIGURE A1: Sample Counts across Years and Countries



Notes: The light (gray) solid line in Panel a shows the total number of observations for each year in the 105-country sample used for income accounting; The dark (blue) dashed line shows the total number of observations for each year for the 60-country sample used for growth accounting (GA sample). Panel b shows the number of observations per country in the sample.

FIGURE A2: Scarcity and Mincer Return in Main Samples



Notes: Figures depict the sample distribution of scarcity and Mincer return across time in our sample. Each light (gray) circle indicates an observation in the 60-country sample used for growth accounting; each dark (blue) dot indicates an observation in the 105-country sample but not used for growth accounting. The plot thickens with time as data become available on more countries.

to their implications for wages based on the relationship between wage rates and a standardized test score in the US as estimated by Lange (2007) for the 1979 cohort of the National Longitudinal Survey of Youth (NLSY, see <https://www.bls.gov/nls/nlsy79.htm>) using Armed Forces Qualification Tests. There are 51 countries from the PISA data for which we can obtain schooling scarcity and Mincer returns from our main sample.

Table A1 lists the countries represented in the empirical results, denoting each exercise for which a country could be utilized.

B Employment-Based Measure of School Scarcity

In our calculations of scarcity, we rely on the population shares of schooling from Barro and Lee (2013), whereas the wage equation in (7) stipulates the relative shares of schooling in the work force. The International Labor Organisation (ILO, <https://ilostat.ilo.org/>) provides employment share by schooling for 121 countries and for the years 1990 to 2018. For most countries, however, data are only available after 2002, which is too recent to line up with the estimated Mincer returns from Psacharopoulos and Patrinos (2018) and hence is not applicable to our main analysis. We therefore use schooling population shares from Barro and Lee (2013) in our analysis.

To gauge the difference between the two measures, we compare employment-based scarcity calculated from the ILO to the population-based scarcity calculated from Barro and Lee (2013) for recent years. Figure A3 shows that the two measures closely align, being concentrated along the dashed 45-degree line with an almost perfect correlation of 0.98, implying that substituting population shares of schooling for employment shares imparts no significant bias.

C Scarcity under Different Grouping Rules

Our benchmark reflects the four schooling groups as listed above. But we consider the sensitivity of measured scarcity to the following alternative groupings:

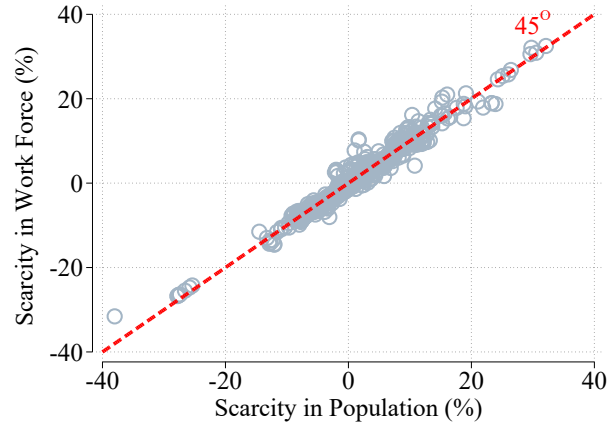
- (a) 2 Groups : less than tertiary; some tertiary or more.
- (b) 2 Groups : less than secondary; some secondary or more.
- (c) 3 Groups: less than secondary; some or complete secondary; some tertiary and above.

TABLE A1: List of Countries in Sample

60 countries for growth and development accounting				
Argentina	Denmark	Iran	Panama	Sweden
Australia	Ecuador	Israel	Peru	Switzerland
Austria	Egypt	Italy	Philippines	Taiwan
Bolivia	Finland	Japan	Poland	Thailand
Brazil	France	Kenya	Portugal	Tunisia
Bulgaria	Germany	Latvia	South Korea	Turkey
Canada	Ghana	Malaysia	Romania	USA
Chile	Greece	Mexico	Slovenia	Uganda
China	Guatemala	Netherlands	South Africa	United Kingdom
Colombia	Hungary	Nicaragua	Spain	Tanzania
Costa Rica	India	Norway	Sri Lanka	Venezuela
Cyprus	Indonesia	Pakistan	Sudan	Viet Nam
44 countries for development accounting only				
Albania	Cote d'Ivoire	Ireland	Mongolia	Singapore
Algeria	Croatia	Jamaica	Morocco	Slovakia
Bangladesh	Czech Republic	Jordan	Namibia	Tajikistan
Belgium	Dominican Republic	Kazakhstan	Nepal	Ukraine
Belize	El Salvador	Kuwait	New Zealand	United Arab Emirates
Botswana	Estonia	Kyrgyzstan	Niger	Uruguay
Cambodia	Gambia	Malawi	Paraguay	Zambia
Cameroon	Honduras	Maldives	Russia	Zimbabwe
Hong Kong	Iraq	Malta	Rwanda	

Notes: Table lists the 104 countries whose Mincer return, schooling distribution, and GDP per worker are observed at least once between 1960-2010. The top panel lists the 60 countries used for growth accounting in Section 3 as well as development accounting in Section 4. The second panel lists the other countries used only for development accounting.

FIGURE A3: Employment and Population Measures of Scarcity



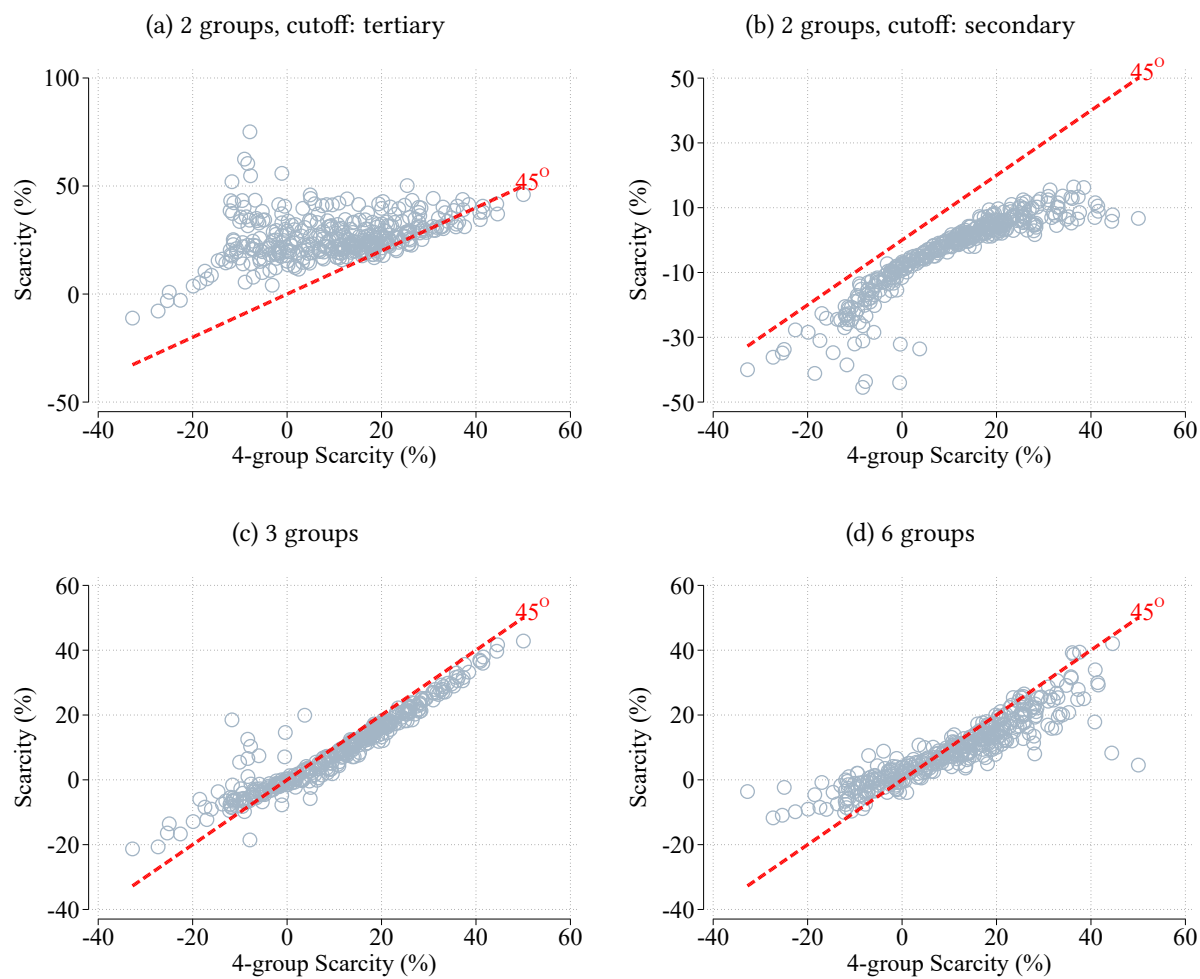
Notes: Schooling scarcity in population (x) reflects authors' calculations based on Barro and Lee (2013). Scarcity in work force is calculated based on data from the International Labor Organisation (ILO). Dashed red line depicts the 45° line.

- (d) 6 Groups: less than complete primary; complete primary; some secondary; complete secondary; some tertiary; complete tertiary.

Figure A4 compares the schooling scarcity calculated under each alternative to our benchmark with 4 groups. The dashed red lines depict the 45° lines. Scarcity measured across three or six levels, Panels c and d, is each similar to that from our four groups. The groupings into two levels, Panels a and b, both diverge from our benchmark, but in different directions reflecting the choice of cutoff. With fewer categories, a larger share of schooling variations are manifested within-group, muting variations in measured scarcity both across countries and over time. This is especially true for only two groups, most notably when the cutoff is further from median schooling as in Panel a. Because of that loss of information on scarcity, we can expect the relationship between scarcity and the Mincer return to appear more blurred, favoring even higher values of substitutability than what we proffer here.

The takeaway from Figure A4 is that distinctions between primary and secondary and between secondary and tertiary schooling are both important components of scarcity. So, at a minimum, three groupings are necessary to capture variations in scarcity relevant to Mincer returns. Also, with three-plus groups, skewness of the schooling distribution, and thus schooling scarcity, varies separately from mean years. This allows us to control for attainment as a proxy for schooling quality and technology bias while relating Mincer returns to scarcity.

FIGURE A4: Scarcity with Differing Number of Groups versus Benchmark 4 Group Scarcity.



Notes: Schooling scarcity in population (x) reflects authors' calculations based on [Barro and Lee \(2013\)](#). Red dash lines depict the 45° lines.

D Deriving the Long-Run Elasticity of Substitution

As in [Caselli and Coleman \(2006\)](#) we consider an economy with a large number of competitive firms, with labor and capital supplied elastically. The representative firm solves the optimization problem:

$$\max_{\{L_i, A_i\}, K} K^\alpha H^{1-\alpha} - \sum_{i \in S} w_i L_i - RK,$$

subject to the technological frontier

$$\sum_{i \in S} (\gamma_i A_i)^\omega \leq B,$$

where effective labor input, H , aggregates labor over skill groups

$$H = \left[\sum_{i \in S} \left(A_i q_i L_i \right)^{\frac{\varepsilon_{SR}-1}{\varepsilon_{SR}}} \right]^{\frac{\varepsilon_{SR}}{\varepsilon_{SR}-1}}. \quad (A1)$$

An equilibrium consists of factor prices $\{w_i\}_{i \in S}$ and R and allocations $\{L_i, A_i\}_{i \in S}$ and K such that input markets clear subject to firms' having optimized at those prices.

We next show the condition for a symmetric equilibrium with an interior solution. It enables us to characterize the equilibrium with the first-order conditions of a representative firm. Then we derive the long-run elasticity of substitution, which parallels [Hendricks and Schoellman's \(forthcoming\)](#) treatment.

D.1 Symmetric Equilibrium with Interior Solution

We want to show that $\omega - \varepsilon_{SR} + 1 > 0$ is a sufficient condition for a symmetric equilibrium with an interior solution. A symmetric equilibrium means all firms choose the same technology bundles, and an interior solution means $A_i > 0$ for all $i \in S$.

First we denote $D_i = A_i^\omega$ and rewrite the firm's optimization problem over technologies, for given values of $K > 0$ and $L_i > 0$, for all $i \in S$, as:

$$\max_{\{D_i\}_S} K^\alpha \left[\sum_{i \in S} D_i^{\frac{\varepsilon_{SR}-1}{\omega \varepsilon_{SR}}} \left(q_i L_i \right)^{\frac{\varepsilon_{SR}-1}{\varepsilon_{SR}}} \right]^{\frac{(1-\alpha)\varepsilon_{SR}}{\varepsilon_{SR}-1}} - \sum_{i \in S} w_i L_i - RK,$$

subject to

$$\sum_{i \in S} \gamma_i^\omega D_i \leq B.$$

The constraint set is convex without additional restrictions on parameters. Now suppose $\omega - \varepsilon_{SR} + 1 > 0$. Then $(\varepsilon_{SR} - 1)/\omega\varepsilon_{SR} < 1$ because $\varepsilon_{SR} > 1$. Under this condition, the objective function is strictly quasi-concave, so the existence and uniqueness of a global maximizer is guaranteed. Additionally, because the marginal profit of investing in D_i goes to infinity when D_i goes to zero, the solution must have $A_i > 0$ for all $i \in S$. The symmetry of equilibrium is directly implied because all firms face the same optimization problem with unique solutions.

D.2 Long-Run Elasticity of Substitution

Rearranging the first-order condition with respect to A_i for each $i \in S$ gives:

$$A_i = \gamma_i^{\frac{-\omega\varepsilon_{SR}}{\omega\varepsilon_{SR}-\varepsilon_{SR}+1}} \left(q_i L_i \right)^{\frac{\varepsilon_{SR}-1}{\omega\varepsilon_{SR}-\varepsilon_{SR}+1}} Q^{\frac{\varepsilon_{SR}}{\omega\varepsilon_{SR}-\varepsilon_{SR}+1}}, \quad (\text{A2})$$

where $Q = (1 - \alpha)K^\alpha H^{1/\varepsilon_{SR}-\alpha}/(\lambda\omega)$ and λ is the Lagrangian multiplier. Note that (A2) can also be written as:

$$\left(\gamma_i A_i \right)^\omega = \left(A_i q_i L_i \right)^{\frac{\varepsilon_{SR}-1}{\varepsilon_{SR}}} Q,$$

for each $i \in S$. Summing up both sides of the equation across skill groups, we have

$$Q = BH^{\frac{1-\varepsilon_{SR}}{\varepsilon_{SR}}}.$$

Substituting for Q in (A2) and letting $b_i = B^{\frac{1}{\omega}}/\gamma_i$, gives the optimal choice of technology:

$$A_i = \left(\frac{q_i L_i}{H} \right)^{\frac{\varepsilon_{SR}-1}{\omega\varepsilon_{SR}-\varepsilon_{SR}+1}} b_i^{\frac{\omega\varepsilon_{SR}}{\omega\varepsilon_{SR}-\varepsilon_{SR}+1}}. \quad (\text{A3})$$

Plugging the optimal technology choice into the labor input aggregator (A1), we get:

$$H = \left[\sum_{i \in S} \left(q_i b_i L_i \right)^{\frac{\omega\varepsilon_{SR}-\omega}{\omega\varepsilon_{SR}-\varepsilon_{SR}+1}} \right]^{\frac{\varepsilon_{SR}-1}{\varepsilon_{SR}}} H^{\frac{1-\varepsilon_{SR}}{\omega\varepsilon_{SR}-\varepsilon_{SR}+1}}.$$

Rearranging the equation to solve for H , we can rewrite the aggregator as:

$$H = \left[\sum_{i \in S} \left(q_i b_i L_i \right)^{\frac{\omega \varepsilon_{SR} - \omega}{\omega \varepsilon_{SR} - \varepsilon_{SR} + 1}} \right]^{\frac{\omega \varepsilon_{SR} - \varepsilon_{SR} + 1}{\omega \varepsilon_{SR} - \omega}}$$

This gives the long-run elasticity of substitution:

$$\varepsilon = \frac{\omega \varepsilon_{SR} - \varepsilon_{SR} + 1}{\omega - \varepsilon_{SR} + 1}.$$

Under the assumption $\omega - \varepsilon_{SR} + 1 > 0$, this long-run elasticity is finite and positive.

Now we can derive the wage-schooling relationship when technology choices are endogenized. Equating group s_i 's wage to its marginal product gives:

$$w_i = \frac{\partial Y}{\partial H} H^{\frac{1}{\varepsilon}} \left(A_i q_i \right)^{\frac{\varepsilon_{SR} - 1}{\varepsilon_{SR}}} L_i^{\frac{-1}{\varepsilon_{SR}}}.$$

Substituting for the optimal technology choice, equation (A3), yields:

$$w_i = \frac{\partial Y}{\partial H} H^{\frac{1}{\varepsilon}} \left(q_i b_i \right)^{\frac{\varepsilon - 1}{\varepsilon}} L_i^{\frac{-1}{\varepsilon}}, \quad (\text{A4})$$

which is equivalent to the first-order condition derived from the long-run aggregator.

D.3 Technology Choice and Wage Shares

In equilibrium, the efficiency of workers in schooling group i can be written as a function of the group's quality q_i , its technology frontier b_i , and its earnings as a share of the total labor income of the economy. Substituting the optimal choice of technology (A3) into the definition of e_i gives

$$e_i = A_i q_i = q_i b_i \left[\frac{q_i b_i L_i}{H} \right]^{\frac{\varepsilon_{SR} - 1}{\omega \varepsilon_{SR} - \varepsilon_{SR} + 1}}.$$

Then we substitute in the long-run labor aggregator H and relabel the parameters.

$$\begin{aligned}
e_i &= q_i b_i \left[\frac{q_i b_i L_i}{\left[\sum_{j \in S} (q_j b_j L_j)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}} \right]^{\frac{1}{\omega} \left(\frac{\varepsilon-1}{\varepsilon} \right)} = q_i b_i \left[\frac{(q_i b_i L_i)^{\frac{\varepsilon-1}{\varepsilon}}}{\sum_{j \in S} (q_j b_j L_j)^{\frac{\varepsilon-1}{\varepsilon}}} \right]^{\frac{1}{\omega}} \\
&= q_i b_i \left(\frac{w_i L_i}{\sum_{j \in S} w_j L_j} \right)^{\frac{1}{\omega}} = q_i b_i \left(\frac{w_i L_i}{\bar{w} L} \right)^{\frac{1}{\omega}}
\end{aligned}$$

The last row is implied by the long-run wage equation (A4).

E Immigrant Mincer Return and Cross-Country Human Capital

There are two sources of efficiency associated with a schooling level: human capital accumulated from the schooling (ϕ_q) and the level of technology accessible with that schooling (ϕ_b). In this paper, we follow [Schoellman \(2012\)](#) by using the Mincer returns that he estimates for immigrants in the United States as a measure of ϕ_q for the immigrants' country of origin. The intuition is that technology reflects a worker's current location, while human capital from schooling was determined by the efficiency of schooling in the country where that investment took place, that being the worker's home country.

To see this, consider the following aggregator extended from (6), where workers in the United States from different home countries $c \in C$ are perfect substitutes provided they have the same educational attainment.

$$H_{US} = \left[\sum_{i \in S} \left(\sum_{c \in C} b_{i,US} q_{i,c} L_{i,c} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Note that immigrant workers work with US technology, so they share a common technology frontier, $b_{i,US}$. On the other hand, the human capital gain from schooling, $q_{i,c}$, depends on the workers' country of origin because immigrant workers accumulated human capital in their home country. The wage for such an immigrant worker is:

$$w_{i,c} = \left(\frac{\partial Y}{\partial H_{US}} H_{US}^{\frac{1}{\varepsilon}} \right) \left(\sum_c b_{i,US} q_{i,c} L_{i,c} \right)^{\frac{-1}{\varepsilon}} b_{i,US} q_{i,c}. \quad (\text{A5})$$

Let m_c^{US} be the Mincer return estimated in the US labor market *across* immigrants from country c . Taking natural logs and projecting on s_i for both sides of equation (A5) gives

$$m_c^{US} = \underbrace{\left[\phi_{b,US} + \frac{1}{\varepsilon} \tilde{x}_{US} \right]}_{\zeta} + \phi_{q,c}, \quad (\text{A6})$$

where \tilde{x}_{US} is the scarcity of more-educated workers in the US in terms of efficiency units, obtained by projecting $-\ln \left(\sum_c b_{i,US} q_{i,c} L_{i,c} \right)$ on s_i . Equation (A6) shows that the cross-country variation in $\phi_{q,c}$ can be captured by the cross-(home)-country variation of immigrants' Mincer return m_c^{US} . In the main text we refer to m_c^{US} as m^{US} , keeping the country subscript implicit.

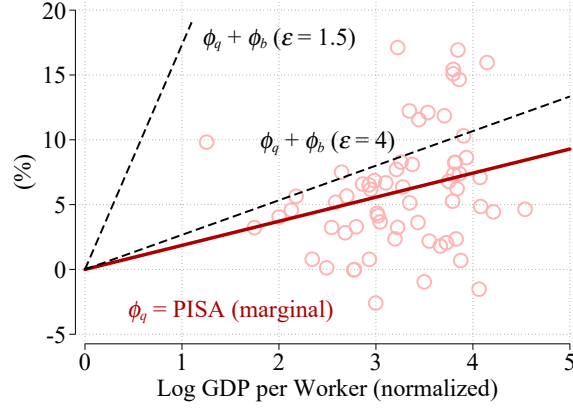
F An Alternative PISA-Based School Quality Measure

Our benchmark quality measure implicitly assumes that the test score prior to schooling is zero. If richer countries have better pre-school training, then the measure is biased up for these countries. To relax this assumption, we construct a second measure of quality.

In the PISA data, students from the same country will be in different grades when taking the test if the age at which they begin school depends on the month of birth or differs across schools, for instance, across regions. We use this variation in schooling to construct an alternative measure. In each country, we regress the test score on the grade year in which the test was taken, controlling for gender. We restrict the sample to native-born students who never repeated a grade. The coefficient on the grade year gives the test-score return to a year of schooling and forms the basis of our second measure. As with our first measure, we divide the resulting per-school-year score by the standard deviation of US test scores and valorize it at 15 percent.

Figure A5 contrasts the alternative PISA-based measure against log GDP per worker. On average, a one log point higher income is associated with a 1.4 percentage-point increase in ϕ_q (with a much larger standard error of 0.7). The gradient is similar to that of Schoellman's immigrant-based measure, and hence their implied ϕ_b variation and human capital are alike (see Panel a of Figure 9). We prefer the benchmark measure because of its lower standard error.

FIGURE A5: PISA-Based School Quality Measure vs. Implied $\tilde{\phi}_z(\varepsilon)$



Notes: Figure plots the alternative PISA-based school quality measure against log GDP per worker. Variables are normalized to set the predicted value for the poorest country to zero. The solid red line depicts the OLS fitted values for school quality. The dashed black lines depict the projections of $\tilde{\phi}_z(\varepsilon)$ on income for $\varepsilon = 1.5$ and 4. Data on income per worker are from PWT 9.1. $\tilde{\phi}_z(\varepsilon)$ reflects authors' calculation based on data from [Barro and Lee \(2013\)](#) and [Psacharopoulos and Patrinos \(2018\)](#).

G Growth Accounting Equations with Nested-CES Production

Let G be the compound bottom group that consists of the first N schooling groups. Consider the nested-CES labor aggregator:

$$H = \left[\left(z_G L_G \right)^{\frac{\varepsilon-1}{\varepsilon}} + \tilde{H} \left(z_{N+1} L_{N+1}, \dots, z_S L_S \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad (\text{A7})$$

where $\tilde{H}(\cdot)$ is constant return to scale, $L_G = \sum_{j \leq N} L_j$, $z_G \equiv Z_G / L_G$ and

$$Z_G \equiv \left[\sum_{j \leq N} (z_j L_j)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \quad (\text{A8})$$

The wage for each schooling group $i > N$ is

$$w_i = \left(\frac{\partial Y}{\partial H} \right) H^{\frac{1}{\varepsilon}} \tilde{H}^{-\frac{1}{\varepsilon}} \tilde{H}'_i z_i. \quad (\text{A9})$$

On the other hand, the wage for each group $j \leq N$ is:

$$w_j = \left(\frac{\partial Y}{\partial H} \right) H^{\frac{1}{\varepsilon}} Z_G^{\frac{\varepsilon-\sigma}{\varepsilon\sigma}} z_j^{\frac{\sigma-1}{\sigma}} L_j^{-\frac{1}{\sigma}}. \quad (\text{A10})$$

Taking the average among the first N groups, we get:

$$\bar{w}_G \equiv \left(\frac{1}{L_G} \right) \sum_{j \leq N} w_j L_j = \left(\frac{\partial Y}{\partial H} \right) H^{\frac{1}{\varepsilon}} z_G^{\frac{\varepsilon-1}{\varepsilon}} L_G^{\frac{-1}{\varepsilon}}. \quad (\text{A11})$$

Now apply (A9) and (A11). We can write the overall average wage as:

$$\begin{aligned} \bar{w} &= \frac{1}{L} \left(\bar{w}_G L_G + \sum_{i>N} w_i L_i \right) = \frac{1}{L} \left(\frac{\partial Y}{\partial H} \right) H^{\frac{1}{\varepsilon}} \left[(z_G L_G)^{\frac{\varepsilon-1}{\varepsilon}} + \tilde{H}^{\frac{-1}{\varepsilon}} \sum_{i>N} \tilde{H}'_i z_i L_i \right] \\ &= \frac{1}{L} \left(\frac{\partial Y}{\partial H} \right) H^{\frac{1}{\varepsilon}} \left[(z_G L_G)^{\frac{\varepsilon-1}{\varepsilon}} + \tilde{H}^{\frac{\varepsilon-1}{\varepsilon}} \right], \end{aligned} \quad (\text{A12})$$

where $\sum_{i>N} \tilde{H}'_i z_i L_i = \tilde{H}$ because $\tilde{H}(\cdot)$ is constant return to scale (Euler's homogeneous function theorem). Combining the previous two equations, we have:

$$\frac{\bar{w} L}{\bar{w}_G L_G} = \left[1 + \left(\frac{\tilde{H}}{z_G L_G} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right].$$

Now turning back to the aggregator (A7), we can get an equation parallel to (10)

$$h \equiv \frac{H}{L} = z_G \left(\frac{L_G}{L} \right) \left[1 + \left(\frac{\tilde{H}}{z_G L_G} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} = z_G \left(\frac{\bar{w}}{\bar{w}_G} \right)^{\frac{\varepsilon}{\varepsilon-1}} \left(\frac{L}{L_G} \right)^{\frac{1}{\varepsilon-1}}. \quad (\text{A13})$$

Likewise, combining equations (A10) and (A11), we have:

$$\frac{\bar{w}_G L_G}{w_1 L_1} = \left[1 + \sum_{2 \leq j \leq N} \left(\frac{z_j L_j}{z_1 L_1} \right)^{\frac{\sigma-1}{\sigma}} \right],$$

which, in turn, gives:

$$z_G = \frac{Z_G}{L_G} = z_1 \left(\frac{L_1}{L_G} \right) \left[1 + \sum_{2 \leq j \leq N} \left(\frac{z_j L_j}{z_1 L_1} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} = z_1 \left(\frac{\bar{w}_G}{w_1} \right)^{\frac{\sigma}{\sigma-1}} \left(\frac{L_G}{L_1} \right)^{\frac{1}{\sigma-1}}. \quad (\text{A14})$$

Combining (A13) and (A14) gives equation (18).