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Chengcheng Jia

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The Informational Effect of Monetary Policy and the Case for Policy Commitment*

Chengcheng Jia†

Federal Reserve Bank of Cleveland

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Abstract

I study how the informational effect of monetary policy changes the optimal conduct of monetary policy. In my model, the private sector extracts information about unobserved shocks from the central bank's interest rate decisions. The central bank optimally changes the informational effect of the interest rate by committing to a state-contingent policy rule, in which case the Phillips curve becomes endogenous. In a dynamic model, the optimal policy rule overshoots the natural-rate shock and gradually responds to the cost-push shock, which makes the interest rate change expected output growth but not expected inflation.

Keywords: monetary policy, information frictions, and policy commitment

JEL classification: E52, E58, E31, D83, D84

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†Federal Reserve Bank of Cleveland. Email: Chengcheng.Jia@clev.frb.org

1 Introduction

When a central bank changes its target for interest rates, is it an exogenous monetary policy shock or an endogenous policy response to changes in the state of the economy? Just as econometricians find exogenous monetary policy shocks hard to identify, so do households and firms in the private sector. When the private sector has imperfect information about the underlying state of the economy, it thinks that changes in monetary policy reveal information about changes in the state of the economy. In this case, monetary policy affects the economy not only through the direct effect of a change in borrowing costs but also through a change in expectations about the state of the economy. Recent studies have documented the effect of monetary policy shocks on expectations in the private sector, which has come to be known as the "informational effect of monetary policy."[1]

This informational effect may lead to a policy dilemma. For example, suppose that the economy is hit by a positive cost-push shock and it is partially observed in the private sector. The central bank now faces a trade-off when making interest rate decisions. If the central bank increases the interest rate, the direct effect of the tightening of monetary policy is to decrease inflation by lowering demand, counteracting the effect of the positive cost-push shock on inflation. At the same time, however, the tightening of monetary policy also reveals information about the realization of the positive cost-push shock, which makes firms increase prices by changing expectations. This informational effect partially offsets the direct effect of the tightening of monetary policy. In this case, should the central bank refrain from changing monetary policy for fear of the informational effect?

This paper studies the optimal monetary policy in the presence of the informational effect of monetary policy. I assume that the private sector has partial information about the realization of underlying shocks, whereas the central bank has perfect information. In this environment, the interest rate becomes a public signal about underlying shocks, and the central bank conveys information through its interest rate decisions. The main contribution of the paper is to show that the central bank can change the informational effect for a given interest rate by committing to a state-contingent policy rule, which makes the Phillips curve endogenous and leads to the informational gains from commitment. In a calibrated dynamic model, I show that the optimal policy rule responds more (less) aggressively to the natural-rate shock (the cost-push shock), which makes the interest rate reveal more (less) precise information about the natural-rate shock (the cost-push shock). Under this policy rule, my model also predicts that only expected output growth, not expected inflation, is

[1] This term was first used in Romer and Romer (2000), and was later adopted in Nakamura and Steinsson (2018). Some other papers on this topic also refer to this effect as the Fed's "signaling effect" (Baeriswyl and Cornand (2010), Melosi (2017), and Tang (2015), among others.)

sensitive to changes in the interest rate, which is consistent with the empirical patterns of the informational effect of monetary policy found in Nakamura and Steinsson (2018).

To model asymmetric information between the central bank and the private sector, I introduce informational frictions to an otherwise canonical New Keynesian model with Calvo price rigidity. There are two types of shocks in the private sector: technology shocks and wage markup shocks. Their aggregate components map to natural-rate shocks in the output gap and cost-push shocks in inflation. Both the household and all firms know the distributions of the shocks, but have partial information on the realization of the shocks. Under rational expectations, the private sector understands the way the interest rate reacts to the two shocks in equilibrium, and extracts information about the realization of the shocks from the interest rate response. Therefore, the interest rate becomes one signal that jointly provides information about two shocks, and how the private sector expects the interest rate to respond to different shocks determines the informational effect for a given interest rate.

I start the analysis from the baseline situation, in which case shocks do not have serial correlation and the current interest rate is the central bank's only policy instrument. These two assumptions make the model static in essence. In this case, although consumption decisions are forward-looking, the output gap is free from expected shocks, because expectations about future equilibrium variables are at their steady-state levels. Therefore, only the direct effect of monetary policy plays a role in determining the output gap. In contrast, the effects of monetary policy on inflation consist of both the direct effect and the informational effect. This is because in a monopolistic competitive market, firms' optimal prices are strategic complements. Each individual firm increases its own price when expecting the aggregate price level to go up. Therefore, when observing an increase in the interest rate, firms regard it as a response to rising inflation. This informational effect increases inflation by increasing expected inflation, which dampens the direct effect of the increase in the interest rate. As a result, the informational effect reduces the slope of the Phillips curve.

A central bank can optimize the interest rate decision either under discretion or under commitment. A discretionary central bank plays a Bayesian Nash game with the private sector. Both the central bank and the private sector expect the other to play its best response in equilibrium, and therefore, the central bank cannot change the private sector's expectations about the interest rate response function. The informational effect for a given interest rate is determined by the expected response function, which the central bank takes as given when choosing the optimal interest rate. Under discretionary optimization, the equilibrium interest rate targets a negative ratio between inflation and the output gap, and the absolute value of the target ratio is greater than that in the case without the informational effect.

The central bank can improve the inflation and output gap trade-off by optimal commitment. The central bank under commitment is a Stackelberg leader. Before the private sector forms expectations, the central bank announces its commitment to a state-contingent policy rule that specifies the response of the interest rate to different shocks. By doing so, the central bank changes the expected interest rate response function, which changes how expectations about different shocks are formed for a given interest rate. As the Phillips curve is affected by expectations, it becomes endogenous to the central bank's optimization problem. I find that the optimal policy rule reduces the degree to which the informational effect dampens the direct effect of monetary policy, which increases the slope of the Phillips curve from the discretionary equilibrium.

I extend the analysis to serially correlated shocks to study the dynamic informational effect. In this case, the private sector forms expectations about current shocks by optimally weighing signals received in the current period and expectations formed in the last period. In this case, the current interest rate has a lagged effect on future equilibrium through its effect on current beliefs. Under discretionary policy, the central bank optimally targets a negative correlation between current and future deviations of the output gap and inflation. Under commitment, the optimal policy rule overshoots the natural-rate shock and gradually responds to the cost-push shock. By doing so, the interest rate reveals more precise information about the natural-rate shock and less precise information about the cost-push shock. In addition, the traditionally studied gains from committing to a delayed response of monetary policy amplify the informational gains from commitment.

Last, I discuss my model predictions in connection to the real world. I show that the optimal central bank communication strategy can be complicated due to the interaction with the informational effect of monetary policy. More precise communication might be detrimental if the informational effect of monetary policy is already very precise. Connecting my model to the empirical literature, my model predicts that only expected output growth, not expected inflation, is sensitive to monetary policy shocks, which closely matches the empirical patterns of the informational effect of monetary policy found in Nakamura and Steinsson (2018).

Relationship to prior work

My paper builds on the growing literature on optimal monetary policy under imperfect information. This field was revived by Woodford (2003), who shows how imperfect information about monetary policy leads to persistent real effects through higher-order beliefs. The majority of papers that study optimal monetary policy under informational frictions assume that the private sector has noisy or delayed information that is exogenous to monetary policy decisions (Ball, Mankiw, and Reis (2005), Adam (2007), Lorenzoni (2009), Angeletos and La'O (2020), and Baeriswyl and Cornand (2010), among others). There are some exceptions, for example, Berkelmans (2011), Tang (2015), and Melosi (2017), that model monetary policy to reveal information about the underlying economy.

The paper most closely related to the present one is Tang (2015), who characterizes the optimal discretionary policy when monetary policy has an informational effect. In Tang (2015), although the Phillips curve is different from the one under perfect information, since the central bank optimizes under discretion, it takes the difference in the Phillips curve as exogenous to its interest rate decisions. My contribution to the literature is to show how policy commitment can change the Phillips curve through commitment.

There are also papers that study the gains from commitment under imperfect information. For example, Svensson and Woodford (2003, 2004) show that the optimal policy under commitment displays considerable inertia relative to the discretionary policy, due to the persistence in the learning process. Paciello and Wiederholt (2013) explore the idea that the central bank is able to change the learning process in the private sector if it is able to commit to completely offsetting inefficient shocks. However, the gains from commitment found in these papers come from the direct effect of monetary policy. To my knowledge, the present work is the first one to show the gains from commitment through the informational effect of monetary policy.

My paper speaks to the literature on the recent changes in the Phillips curve. Previous research has found that accounting for inflation expectations is the key to explaining the changes in the slope of the Phillips curve (Coibion and Gorodnichenko (2015), Blanchard (2016), and Hooper, Mishkin, and Sufi (2020)). I contribute to this literature by showing how expectations in the private sector are shaped by the informational effect of monetary policy.

The information effect of monetary policy is supported by empirical evidence. Romer and Romer (2000) is the first empirical contribution that shows that private inflation forecasts respond to changes in the policy rate after FOMC announcements. More recently, Campbell et al. (2012), Andrade et al. (2019), and Enders, Hünnekes, and Müller (2019) find the informational effect of monetary policy set by the Federal Reserve and the ECB. Jarociński and Karadi (2020) use high-frequency co-movement between interest rates and stock prices to identify the information shock from FOMC statements. Nakamura and Steinsson (2018) show that the expected output growth rates have a significant positive relationship with monetary policy shocks, whereas expected inflation is not sensitive to monetary policy shocks.

2 Private Sector

I add informational frictions in an otherwise standard New Keynesian model with sticky prices. In Section 2.1, I first characterize the optimization problems in the private sector under imperfect information. Then, I specify the information frictions and time protocol in Section 2.2. Last, Section 2.3 solves the equilibrium of aggregate variables.

2.1 Private Sector's Optimization Problem

I model an "islands economy," following lines similar to Phelps (1970), Lucas (1972), Woodford (2003), and Angeletos and La'O (2010). There is a representative household, consisting of a consumer and a continuum of workers. There is a continuum of islands, indexed by j. There is a continuum of monopolistic firms, each located on one island and indexed by the island. Each firm demands labor in the local labor market within the island and produces a differentiated intermediate good. The geographical isolation is a metaphor for information frictions, which I will specify in Section 2.2.

2.1.1 Household

The preferences of the representative household are defined over the aggregate consumption good, C_t, and the labor supplied to each firm, $N_t(j)$, as

$$\mathbb{E}_t^H \Sigma_{t=0}^{\infty} \beta^t \left\{ U(C_t) - \int V(N_t(j))dj \right\}, \tag{1}$$

where $\mathbb{E}_t^H$ denotes the household's subjective expectations conditional on its information set, ω_H. The aggregate good C_t consists of a continuum of intermediate goods:

$$C_t = \left(\int_0^1 C_t(j)^{1-\frac{1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{2}$$

where $C_t(j)$ is the consumption of intermediate good j in period t.

The economy is cashless. The household maximizes expected utility subject to the intertemporal budget constraint:

$$\int_0^1 P_t(j)C_t(j)dj + B_{t+1} \leq \int_0^1 W_t(j)N_t(j)dj + (1+i_t)B_t + \Pi_t, \tag{3}$$

where B_t is the risk-free bond with nominal interest rate i_t, which is determined by the central bank. Π_t is the lump-sum component of household income, which includes dividends from ownership of all firms. $W_t(j)$ and $N_t(j)$ are the labor wage and labor supply for firm j, respectively.

The household's optimization problem can be solved in two stages. First, conditional on the level of aggregate consumption, the household allocates intermediate goods consumption to minimize the cost of expenditure conditional on the level of aggregate goods consumption. The allocation of intermediate goods consumption that minimizes expenditure yields the demand for the intermediate good, which is given by

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} C_t \qquad \forall j, \tag{4}$$

where $P_t = \left[\int_0^1 P_t(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}}$.

In the second stage, conditional on the optimal allocation among intermediate products, the household chooses its aggregate consumption, C_t, its labor supply to all firms, $N_t(j)\ \forall j$, and its savings in the risk-free bond, B_{t+1}. I assume that the utility of aggregate good consumption and the utility of labor supply take the following forms: $U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$, and $V(N_{jt}) = \frac{N_{jt}^{1+\varphi}}{1-\varphi}$, where σ is the inverse of the intertemporal elasticity of substitution and the parameter φ is the inverse of the Frisch elasticity of the labor supply.

The intertemporal consumption decision leads to the following Euler equation:

$$C_t^{-\sigma} = \beta(1+i_t)\,\mathbb{E}_t^H\left(C_{t+1}^{-\sigma}\frac{P_t}{P_{t+1}}\right). \tag{5}$$

Equation (5) shows that the consumption decision is forward-looking. Specifically, current demand depends on expectations on future real consumption and future changes in the aggregate price level.

The intratemporal labor supply decision sets the marginal rate of substitution between leisure and consumption equal to the real wage, which is given by

$$\frac{N_t^{\varphi}(j)}{C_t^{-\sigma}} = \frac{W_t(j)}{P_t}. \tag{6}$$

2.1.2 Firms

Firms are intermediate good producers and subject to both price rigidity and informational frictions. Following the assumptions made in Calvo (1983), when firm j resets its price in period t, it chooses $P_t^*(j)$ to maximize its expectation of the sum of all discounted profits while $P_t^*(j)$ remains effective. The profit-optimization problem is given by

$$max_{P_t^*(j)} \Sigma_{k=0}^{\infty} \theta^k \, \mathbb{E}_t^j \left\{ Q_{t,t+k} \left[P_t^*(j) Y_{t+k}(j) - U_{t+k}^w(j) W_{t+k}(j) N_t(j) \right] \right\}, \tag{7}$$

where θ is the probability that the current price remains effective in the next period. $\mathbb{E}_t^j$ denotes firm j's expectation conditional on its information set, ω_j. $Q_{t,t+k}$ is the stochastic discount factor given by: $Q_{t,t+k} = \beta^k \frac{U'(C_{t+k})}{U'(C_t)} \frac{P_t}{P_{t+k}}$. $U_{t+k}^w(j)$ denotes the wage markup for firm j.

Labor is the only input and each firm produces according to a constant return to scale technology. The production function is assumed to be

$$Y_t(j) = A_t(j) L_t(j), \tag{8}$$

where $A_t(j)$ denotes the technology of firm j.

There are two types of shocks that affect the pricing decisions of each firm: technology shocks and wage markup shocks. I assume that both shocks have an aggregate component and an idiosyncratic component. The idiosyncratic components are drawn independently in every period and are log-normally distributed around zero. Specifically,

$$\begin{aligned} a_t(j) &= a_t + s_t^a(j), \qquad s_t^a(j) \sim N(0,\ \sigma_{sa}^2) \\ u_t^w(j) &= u_t^w + s_t^u(j), \qquad s_t^u(j) \sim N(0,\ \sigma_{su}^2) \end{aligned}$$

where $a_t(j) = log(A_t(j))$ and $u_t^w = log(U_t^w(j))$. I assume that the aggregate components of both shocks follow an AR(1) process:

$$\begin{aligned} a_t &= \phi^a a_{t-1} + v_t^a, \qquad v_t^a \sim N(0,\ \sigma_{va}^2) \\ u_t^w &= \phi^{uw} u_{t-1}^w + v_t^{uw}, \qquad v_t^{uw} \sim N(0,\ \sigma_{vuw}^2) \end{aligned}$$

When choosing the optimal resetting price, firm j expects the demand function of its product as specified in equation (4), and hire labor according to the production function (equation (8)) to meet the demand. The first-order condition for optimal pricing is derived as

$$P_t^*(j) = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_t^j \, \Sigma(\beta\theta)^k u'(C_{t+k}) P_{t+k}^{\epsilon} Y_{t+k} u_{t+k}(j) w_{t+k(j)} A_{t+k}^{-1}(j)}{\mathbb{E}_t^j \, \Sigma(\beta\theta)^k u'(C_{t+k}) P_{t+k}^{\epsilon-1} Y_{t+k}}. \tag{9}$$

Equation (9) implies that the pricing decision of each firm is forward-looking and a strategic complement. Specifically, the optimal resetting price of firm j increases with the expectation of a higher firm-specific marginal cost of production and a higher aggregate price level in both the current and all future periods.

2.2 Information Frictions and Time Protocol

I assume that the private sector has information frictions, whereas the central bank has perfect information. Any period t is divided into three stages. At stage 0, all shocks are realized. At the same time, the private sector forms expectations on the interest rate response function.[2] At stage 1, having observed the realization of shocks and the private sector's expectations on the interest rate response function, the central bank sets the interest rate. In addition, the central bank may also directly communicate with the private sector about the shocks. At stage 2, firms update expectations about underlying shocks using their firm-specific shocks and the public information revealed by the central bank, including both the interest rate decisions and the information from the central bank's direct communication. Each firm does not know about shocks on other islands or decisions made by other firms when it sets its own price. Households observe all prices as well as the interest rate, and they demand goods and supply labor. Firms hire labor and produce to meet the demand from households.

Most papers in the literature assume that households have perfect information.[3] I deviate from this assumption by assuming that households do not observe the firms' technology or wage markup shocks. As I will demonstrate in Section 3, in a static model when shocks have no serial correlation, as long as prices are perfectly observable by households, the consumption decision is the same as the one under perfect information, in which case whether shocks are directly observed by households does not matter. However, in the dynamic case, when households do not observe underlying shocks, their expectations of future equilibrium are different from those under perfect information. Therefore, in the general case, information frictions also affect the output gap.

2.3 Aggregation and Equilibrium in the Private Sector

2.3.1 The Output Gap

Following the New Keynesian tradition, I express output in terms of the output gap, $\hat{y}_t$, which is defined as the difference between the equilibrium output level, y_t, and the natural level of output, y_t^n.[4] The natural level of output is defined as the output level under flexible

[2] The expected interest rate response function depends on whether the central bank optimizes under discretion or under commitment, which I discuss in Section 3.

[3] See Adam (2007), Paciello and Wiederholt (2013), and Melosi (2017), for examples.

[4] Equilibrium variables in the private sector are solved in log deviations from their steady-state values (i.e., $x_t \equiv ln(X_t/X)$), and denoted by lower-case letters. (See Appendix A for details.)

prices and perfect information. In this situation, y_t^n becomes a linear function of a_t, which is given by $y_t^n = \frac{\varphi+\sigma}{1+\varphi} a_t$.

The output gap is derived as follows:

$$\hat{y}_t \equiv y_t - y_t^n = \mathbb{E}_t^H \hat{y}_{t+1} - \frac{1}{\sigma}\left[i_t - \left(\frac{1}{1-\phi} r_t^n - \frac{\phi}{1-\phi} \mathbb{E}_t^H r_t^n \right) - \mathbb{E}_t^H \pi_{t+1} \right], \tag{10}$$

where $\mathbb{E}_t^H \hat{y}_{t+1} = \mathbb{E}_t^H y_{t+1} - \mathbb{E}_t^H y_{t+1}^n = \mathbb{E}_t^H y_{t+1} - \phi \mathbb{E}_t^H y_t^n$. r_t^n denotes the natural rate of interest, which measures the expected growth rate of y_t^n, i.e., $r_t^n = \sigma(\mathbb{E}_t y_{t+1} - y_t^n) = \sigma(\phi - 1)y_t^n$. The natural-rate shock is mapped from the aggregate component of the technology shocks, as $r_t^n = \frac{\varphi+\sigma}{1+\varphi}\sigma(\phi-1)a_t$. By construction, r_t^n follows an AR(1) process, which is given by

$$r_t^n = \phi r_{t-1}^n + v_t, \qquad v_t \sim N(0,\ \sigma_v^2) \tag{11}$$

where $\phi = \phi^a$ and $\sigma_v = \frac{\varphi+\sigma}{1+\varphi}\sigma(\phi-1)\sigma_{va}$.

Notice that equation (10) nests the situation of perfect information. To see this, substitute $\mathbb{E}_t^H r_t^n = r_t^n$, $\mathbb{E}_t^H \hat{y}_{t+1} = \mathbb{E}_t \hat{y}_{t+1}$ and $\mathbb{E}_t^H \pi_{t+1} = \mathbb{E}_t \pi_{t+1}$, which yields

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\sigma}\left[i_t - r_t^n - \mathbb{E}_t \pi_{t+1} \right]. \tag{12}$$

The comparison between equation (10) and equation (12) shows how informational frictions affect the output gap: Suppose there is a positive innovation in r_t^n, which is equivalent to a negative innovation in y_t^n. In the absence of any frictions, the equilibrium price should increase and consumption should decrease. Due to price rigidity, current demand does not decrease sufficiently, which results in a positive output gap. If, in addition to the price rigidity, the household has no information about the change in aggregate technology, it does not adjust demand at all. Consequently, the changes in the output gap due to natural rate shocks are amplified by informational frictions.

This leads to a key implication for optimal monetary policy after the natural-rate shock: Additional information on the natural-rate shock would narrow the output gap. Therefore, if the interest rate has an information effect, it helps stabilize the economy after the natural-rate shock.

2.3.2 Inflation

Under the assumptions of Calvo (1983), the current aggregate price level is the composite of the aggregate price level in the previous period and the average of resetting prices in this period, which is given by

$$p_t = \theta p_{t-1} + (1-\theta) \int p_t^*(j) dj. \tag{13}$$

The aggregation of individual prices may potentially lead to the higher-order beliefs problem. This is because as shown in equation (9), optimal prices of individual firms include expectations about the aggregate price level, which, in turn, includes expectations of optimal prices set by all other firms. Therefore, each firm needs to guess all other firms' expectations about decisions made by all other firms, including the one made by itself. The higher-order beliefs problem applies only when expectations are heterogeneous, which is a result of firms using firm-specific shocks as their private signals to form expectations on the aggregate shocks.

I abstract from this higher-order beliefs problem by assuming that expectations in the private sector are homogeneous. To this end, I assume that firms only use public signals, not private signals, to form expectations on aggregate variables. The public signals include the interest rate decisions and other information communicated by the central bank. This assumption can be thought of as equivalent to the assumption that firms use all available signals, including their firm-specific shocks, but the idiosyncratic components of their firm-specific shocks (in logs) have finite variance. Or, equivalently, aggregate components of the shocks are arbitrarily small relative to the idiosyncratic components.[5] Under this assumption, the private sector holds homogeneous expectations,[6] which I denote as $\mathbb{E}_t^s$.[7]

The aggregation of prices leads to the New Keynesian Phillips curve under information frictions:[8]

$$\pi_t = \beta\theta\, \mathbb{E}_t^s \pi_{t+1} + (1-\theta)\, \mathbb{E}_t^s \pi_t + \kappa\theta \hat{y}_t + u_t, \tag{14}$$

[5]The higher-order beliefs (HOB) problem is solvable if monetary policy does not have an information effect (Woodford (2003), Angeletos and La'O (2009), and Paciello and Wiederholt (2013), among others), or has an information effect but follows a simple rule, such as a Taylor rule (Melosi (2017)). For solution methods to the HOB problem, see Huo and Takayama (2015), Nimark (2017), and Han, Tan, and Wu (2022), among others. In either case, the interest rate is not optimally chosen while the central bank internalizes the fact that the interest rate changes the expectations in the private sector. In this paper, I study the optimal monetary policy under discretion and under commitment when the central bank considers the information effect of its policy decisions, and this is the reason why it is difficult to include the HOB in this paper.

[6]The assumption that households hold the same expectations as firms may seem odd given that households observe all prices and prices may be informative about aggregate shocks. However, following most papers in the literature, I assume that agents only extract information from exogenous signals, not equilibrium variables (prices) that are endogenous decisions by other agents (firms) in the economy. I make only one exception: the interest rate, which is an endogenous decision made by the central bank, is regarded as a signal by the private sector. For examples where agents use information that includes equilibrium endogenous variables, see Amador and Weill (2010) and Han, Tan, and Wu (2022).

[7]$\mathbb{E}_t^s$ denotes the subjective expectations in the private sector, which deviates from the expectations under FIRE (full information rational expectations). Note that the deviation from FIRE is caused by imperfect information, rather than by any other deviations from rational expectations, for example, least-square learning, as in Evans and Honkapohja (2012).

[8]See Appendix A for the detailed derivation.

where $\kappa = \frac{(1-\beta\theta)(1-\theta)(\varphi+\sigma)}{\theta}$, and u_t denotes the cost-push shock, which is linearly mapped from the aggregate component of wage markup shocks as $u_t = (1-\theta)(1-\beta\theta)u_t^w$. By construction, u_t follows an AR(1) process, which is given by

$$u_t = \phi^u u_{t-1} + v_t^u \qquad v_t^u \sim N(0,\ \sigma_{vu}^2) \tag{15}$$

where $\phi_u = \phi^{uw}$ and $\sigma_{vu} = (1-\theta)(1-\beta\theta)\sigma_{vuw}$.

Notice that equation (14) nests the situation of perfect information. To see this, substitute $\mathbb{E}_t^s \pi_{t+1} = \mathbb{E}_t \pi_{t+1}$, $\mathbb{E}_t^s \pi_t = \pi_t$ in equation (14), which leads to

$$\pi_t = \beta\, \mathbb{E}_t \pi_{t+1} + \kappa \hat{y}_t + \frac{1}{\theta} u_t. \tag{16}$$

The comparison between equation (14) and equation (16) shows the effects of information frictions on inflation. After a positive cost-push shock, in the absence of any signals, the effect of the cost-push shock on inflation is reduced from $\frac{1}{\theta}u_t$ to u_t.[9] This is because without information on the aggregate shocks, the strategic complementarity does not play a role in pricing decisions. Each firm changes its price only because of the change in its own cost of production, as it does not have information on changes in the aggregate price level.

This leads to a key implication for optimal monetary policy after the cost-push shock: Information frictions reduce inflation fluctuations due to cost-push shocks. If the interest rate decisions have an information effect, as the central bank tightens monetary policy in response to a positive cost-push shock, firms would positively update their expectations about the cost-push shock and thus inflation rises due to expectations. Therefore, the informational effect dampens the direct effect of the tightening of monetary policy, and makes it harder for the central bank to stabilize the economy.

3 The Baseline Case

In this section, I add two assumptions to make the model static in essence. First, I assume that underlying shocks have no serial correlation. Second, I impose the restriction that the central bank cannot commit across periods, which excludes the traditionally studied gains

[9] Anther way to have homogeneous assumptions in the private sector is to assume that there are only aggregate shocks, for which firms have the same imperfect signals. Under this assumption, however, the shocks can only affect inflation through expectations. Specifically, the last term, u_t, in equation (14) will become $\mathbb{E}_t^j u_t$. In the baseline case where I consider i_t as the only signal (Section 3), this assumption implies that the optimal response of monetary policy to cost-push shocks would be zero. So to make sure that both the actual shock and the expected shock affect equilibrium, I assume that shocks are firm-specific, and yet they are not used as private signals.

from committing to a delayed response. These two assumptions allow me to focus on the within-period informational gains from policy commitment.

Under these assumptions, the expected inflation and the output gap in the next period are at their steady-state levels.[10] Substitute $\phi = \phi^u = 0$ and $\mathbb{E}^s_t \pi_{t+1} = \mathbb{E}^s_t \hat{y}_{t+1} = 0$ into equations (10) and (14) and get:

$$\hat{y}_t = -\frac{1}{\sigma}\left(i_t - r^n_t\right), \tag{17}$$

$$\pi_t = (1-\theta)\,\mathbb{E}^s_t \pi_t + \kappa\theta\hat{y}_t + u_t. \tag{18}$$

Equation (17) shows that the output gap is free from expected shocks, because the household is able to observe the current price level and future equilibrium variables are expected to be at their steady-state levels. In contrast, as shown in equation (18), inflation is affected by subjective expectations, because each individual firm does not observe the aggregate price level when making its own pricing decision.

A few steps of algebra lead to the expression of inflation in terms of the current output gap, the expected shocks, and the realized shocks:[11]

$$\pi_t = \kappa\hat{y}_t + (1-\theta)\frac{\kappa}{\sigma}\left(\mathbb{E}^s_t r^n_t - r^n_t\right) + \frac{1-\theta}{\theta}\,\mathbb{E}^s_t u_t + u_t. \tag{19}$$

Equation (19) shows that the Phillips curve under imperfect information differs from the one under perfect information for two reasons: First is the effect of information frictions, which moves $\mathbb{E}^s_t r^n_t$ and $\mathbb{E}^s_t u_t$ away from r^n_t and u_t. Second is the informational effect of monetary policy, which potentially moves $\mathbb{E}^s_t r^n_t$ and $\mathbb{E}^s_t u_t$ closer to r^n_t and u_t. In this section, I first discuss the effect of information frictions in Section 3.1, and then analyze the information effect of monetary policy in Section 3.2. After that, I characterize the optimal monetary policy with the informational effect both under discretion and under commitment (Section 3.3). Last, I analyze the gains from commitment in Section 3.4.

3.1 The Effects of Information Frictions

For simplicity and to focus on the intuition, this section considers the extreme case in which the private sector does not have any public signals of the aggregate shocks.[12] In this case,

[10] Following the New Keynesian convention, the long-run distortion has been eliminated via a Pigouvian tax as an employment subsidy, so that the steady-state levels of the output gap and inflation are all zero.

[11] This is an intermediate step of the proof of Proposition 1. See Appendix B.1 for derivation details.

[12] Note that firms still have perfect information about their firm-specific shocks. The case where the interest rate becomes a public signal is studied in Section 3.2, and the case where additional public signals become available due to central bank communication is studied in Section 5.1.

firms do not update their beliefs on aggregate shocks, i.e., $\mathbb{E}_t^s r_t^n = 0$ and $\mathbb{E}_t^s u_t = 0$. The following proposition summarizes how information frictions change the Phillips curve.

Proposition 1. *When the private sector has imperfect information about underlying shocks, and monetary policy has no informational effect, the reduced-form Phillips curve becomes*

$$\pi_t = \kappa \hat{y}_t - (1-\theta)\frac{\kappa}{\sigma} r_t^n + u_t, \tag{20}$$

which has

- *the same slope with,*
- *a negative intercept after a positive natural-rate shock, and*
- *a smaller intercept after a positive cost-push shock than*

the Phillips curve under perfect information.

Proof: See Appendix B.1.

The slope of the Phillips curve measures the co-movement in the output gap and inflation following a change in the interest rate. Without the information effect, the interest rate changes inflation only through its direct effect on the output gap. Therefore, the slope stays the same as the one under perfect information.

After a positive natural-rate shock, the intercept represents the equilibrium when the interest rate tracks the natural rate one-to-one, i.e., $i_t = r_t^n$, to completely close the output gap. Unlike in the case of perfect information, the divine coincidence is not achieved under imperfect information. This is because firms only know about the tightening of monetary policy, without knowing that the tightening of monetary policy is a response to the positive natural-rate shock. Therefore, expected demand is lower than actual demand, i.e., $\mathbb{E}_t^s \hat{y}_t = -\frac{1}{\sigma} i_t < \hat{y}_t = 0$. With the expectation of lower demand, firms reduce prices, which results in a negative intercept of the Phillips curve.

After a positive cost-push shock, the intercept represents the equilibrium where the interest rate does not respond, so that the output gap is unchanged. Without any information on the aggregate economy, firms increase prices only because of the increase in their own cost of production, without knowing the increase in the aggregate price level, i.e., $\mathbb{E}_t^s \pi_t = 0 < \pi_t$. Lower expected inflation feeds back to lower actual inflation, which is equivalent to a smaller intercept of the Phillips curve.

The central bank chooses the interest rate to minimize the loss function, which is the weighted sum of squared inflation and the squared output gap for all periods. Due to the static nature of this baseline model, the objective function reduces to minimize current deviations, which is given by

$$min_{i_t} L(t) = \begin{bmatrix} \pi_t & \hat{y}_t \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \pi_t \\ \hat{y}_t \end{bmatrix} + indept.\ terms \tag{21}$$

where ω is the weight on output-gap stabilization versus inflation stabilization, and the central bank is subject to the Phillips curve given by equation (20).

Corollary 1.1. *Without the informational effect of monetary policy, the optimal interest rate targets the same ratio between inflation and the output gap as the one under perfect information.*

Proof: This result is derived from the first-order condition on the interest rate, given by

$$\frac{\pi_t}{\hat{y}_t} = -\left(\frac{\partial \pi_t}{\partial i_t^*}\right)^{-1} \frac{\partial \hat{y}_t}{\partial i_t^*} \omega = -\left(-\frac{\kappa}{\sigma}\right)^{-1} \left(-\frac{1}{\sigma}\right) \omega, \tag{22}$$

which is not affected by information frictions.

Corollary 1.1 implies that the optimal policy without the informational effect can be expressed as the same *flexible* inflation targeting policy as the one under perfect information. The intuition is that the slope of the Phillips curve does not change with information frictions, so that the central bank faces the same sacrifice ratio between inflation and the output gap.

Corollary 1.2. *The interest rate that implements the optimal target between inflation and the output gap is*

$$i_t^* = \left[1 - \frac{(1-\theta)\kappa^2}{\kappa^2 + \omega}\right] r_t^n + \frac{\sigma\kappa}{\kappa^2 + \omega} u_t, \tag{23}$$

which responds less to both the natural-rate shock and the cost-push shock than the optimal interest rate under perfect information.

Proof: See Appendix B.2.

The intuition for Corollary 1.2 is that the intercept of the Phillips curve after either a positive natural-rate shock or a positive cost-push shock is reduced by information frictions (Proposition 1). Since the optimal monetary policy targets the same negative ratio between inflation and the output gap as the one under perfect information (Corollary 1.1), this implies that the equilibrium output gap is higher than the one under perfect information. A higher equilibrium output gap is achieved by a smaller response in the interest rate per the IS curve (equation (17)).

3.2 The Informational Effect of Monetary Policy

Let us now consider the case in which the private sector uses the interest rate as a public signal and has model-consistent expectations about the response function of the interest rate. In this section, I illustrate the informational effect of monetary policy when the private sector expects that the equilibrium interest rate linearly responds to both of the shocks, i.e., $i_t = F_r r_t^n + F_u u_t$ and $F_r \neq 0$, $F_u \neq 0$.[13] In this case, the interest rate is *one signal* that simultaneously provides information about *two shocks*. When the private sector extracts information from the interest rate about one shock, the prior distribution of the other shock becomes the source of noise in this signal.

Agents in the private sector are Bayesian and form their best linear forecasts by optimally weighting their prior beliefs (shocks have zero ex-ante mean) and the current signal (the interest rate). Let K_r and K_u denote the optimal weights on the two state variables when observing the interest rate. Beliefs formed through the Kalman filtering process are given by the following proposition.

Proposition 2. *When the private sector expects the equilibrium interest rate to follow* $i_t = F_r r_t^n + F_u u_t$ *with* $F_r \neq 0$, $F_u \neq 0$, *the expected shocks in the private sector are given by*

$$\begin{bmatrix} E_t^s r_t^n \\ E_t^s u_t \end{bmatrix} = \begin{bmatrix} 1 - K_r \\ 1 - K_u \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} K_r \\ K_u \end{bmatrix} i_t = \begin{bmatrix} K_r F_r & K_r F_u \\ K_u F_r & K_u F_u \end{bmatrix} \begin{bmatrix} r_t \\ u_t \end{bmatrix}, \tag{24}$$

where

$$K_r F_r = \frac{F_r^2 \sigma_r^2}{F_r^2 \sigma_r^2 + F_u^2 \sigma_u^2}, \tag{25}$$

$$K_u F_u = \frac{F_u^2 \sigma_u^2}{F_r^2 \sigma_r^2 + F_u^2 \sigma_u^2}. \tag{26}$$

Corollary 2.1. $\frac{\partial K_u}{\partial F_r} < 0$, *and* $\frac{\partial K_r}{\partial F_u} < 0$.

Corollary 2.1 says that holding the distribution of the shocks fixed, if the central bank increases its response to the natural-rate shock ($\uparrow F_r$), the sensitivity of the expected cost-push shock to the interest rate decreases ($\downarrow K_u$). Intuitively, after observing a change in the interest rate, the private sector infers that such an interest rate change is less likely to be a response to a cost-push shock. Otherwise, provided that F_u becomes smaller relative to F_r, the change in the interest rate has to come from a larger cost-push shock, which is less likely to happen given that the two types of shocks have the same prior distribution.

[13] I verify later in Section 3.3 that this conjecture is satisfied under discretionary optimization.

Corollary 2.1 has a very important implication for optimal monetary policy under the informational effect: The information conveyed by the central bank is tied to its policy actions. Specifically, to provide more (less) information about one type of shock, the central bank needs to make the interest rate less (more) sensitive to the other type of shock.

Corollary 2.2. $\frac{\partial K_r}{\partial \sigma_r} > 0$*,* $\frac{\partial K_r}{\partial \sigma_u} < 0$*,* $\frac{\partial K_u}{\partial \sigma_r} < 0$*, and* $\frac{\partial K_u}{\partial \sigma_u} > 0$*.*

Corollary 2.2 says that holding the interest rate response function fixed, as the standard deviation of the shock increases, the sensitivity of the expected shocks to the interest rate increases for this type of shock and decreases for the other type of shock. Intuitively, when the interest rate is equally sensitive to both shocks ($F_r = F_u$), agents in the private sector put more weight on updating expectations about the shock that has higher ex-ante dispersion, because the ex-ante mean of the shock has a smaller weight in the belief-formation process.

The beliefs-updating process characterized in Proposition 2 shows that the interest rate now has two effects on the equilibrium: the direct effect and the informational effect. The direct effect changes the relative price of consumption between the current and the future period, which changes both the output gap and inflation. The direct effect of monetary policy is summarized as follows:

$$\frac{\partial \hat{y}_t}{\partial i_t}|_{direct} = -\frac{1}{\sigma}, \tag{27}$$

$$\frac{\partial \pi_t}{\partial i_t}|_{direct} = \frac{\partial \pi_t}{\partial \hat{y}_t}\frac{\partial \hat{y}_t}{\partial i_t} = -\frac{\kappa}{\sigma}. \tag{28}$$

Regarding the information effect, since the output gap is free from the expected shocks, monetary policy has no informational effect on the output gap. The informational effect changes inflation as firms use the interest rate to update beliefs about both shocks. The expected natural-rate shock increases inflation, because firms increase prices when expecting a higher demand from the household. The expected cost-push shock also increases inflation, because firms increase prices when expecting a higher aggregate price level. The informational effects of the interest rate on the output gap and on inflation are given as follows:

$$\frac{\partial \hat{y}_t}{\partial i_t}|_{informational} = 0, \tag{29}$$

$$\frac{\partial \pi_t}{\partial i_t}|_{informational} = \frac{\partial \pi_t}{\partial \mathbb{E}_t^s r_t^n}\frac{\partial \mathbb{E}_t^s r_t^n}{\partial i_t} + \frac{\partial \pi_t}{\partial \mathbb{E}_t^s u_t}\frac{\partial \mathbb{E}_t^s u_t}{\partial i_t}. \tag{30}$$

where $\frac{\partial \pi_t}{\partial \mathbb{E}_t^s r_t^n} = (1-\theta)\frac{\kappa}{\sigma}$ and $\frac{\partial \pi_t}{\partial \mathbb{E}_t^s u_t} = \frac{1-\theta}{\theta}$.

The informational effect of monetary policy dampens its direct effect on inflation. Suppose the central bank tightens monetary policy. The direct effect reduces inflation as it lowers demand, but firms think the increase in the interest rate is a result of either a positive demand shock or a positive cost-push shock, and in either case, the aggregate price level is expected to be higher. Therefore, firms increase prices due to higher expected inflation, dampening the direct effect of monetary policy.

The following proposition summarizes the changes in the Phillips curve with the informational effect of monetary policy.

Proposition 3. *When monetary policy has an informational effect, the Phillips curve becomes*

$$\pi_t = \left\{\kappa - \sigma\left[(1-\theta)\frac{\kappa}{\sigma}K_r + \frac{1-\theta}{\theta}K_u\right]\right\}\hat{y}_t + \left\{(1-\theta)\frac{\kappa}{\sigma}(K_r - 1) + \frac{1-\theta}{\theta}K_u\right\}r_t^n + u_t. \tag{31}$$

Compared to the Phillips curve without the informational effect given by equation (20)*, the informational effect of monetary policy*

- *reduces the slope of the Phillips curve,*
- *increases the intercept of the Phillips curve after a positive natural-rate shock, and*
- *does not change the intercept of the Phillips curve after a positive cost-push shock.*

Proof: See Appendix B.3.

What are the intuitions for Proposition 3? First, the flattening of the Phillips curve comes from the fact that the information effect dampens the direct effect of monetary policy on inflation. Second, the intercept after a positive natural-rate shock represents the equilibrium in which the interest rate tracks the natural-rate one-to-one. Firms understand that a positive interest rate is a response to either a positive natural-rate shock or a positive cost-push shock, and increase their expectations on inflation. Therefore, inflation increases with expected inflation, which is equivalent to a higher intercept of the Phillips curve. Third, the intercept after a positive cost-push shock represents the equilibrium in which the interest rate does not change. In the absence of any signals, the private sector does not update expectations about either shock. Consequently, the intercept is the same as the one where monetary policy has no informational effect.

Proposition 3 implies the costs and benefits of the informational effect of monetary policy. The benefit is that it moves $\mathbb{E}_t^s r_t^n$ closer to r_t^n, reducing the deviations due to natural-rate shocks. The cost is that the slope of the Phillips curve is reduced, which requires the central bank to sacrifice more output as the trade-off to stabilize inflation.

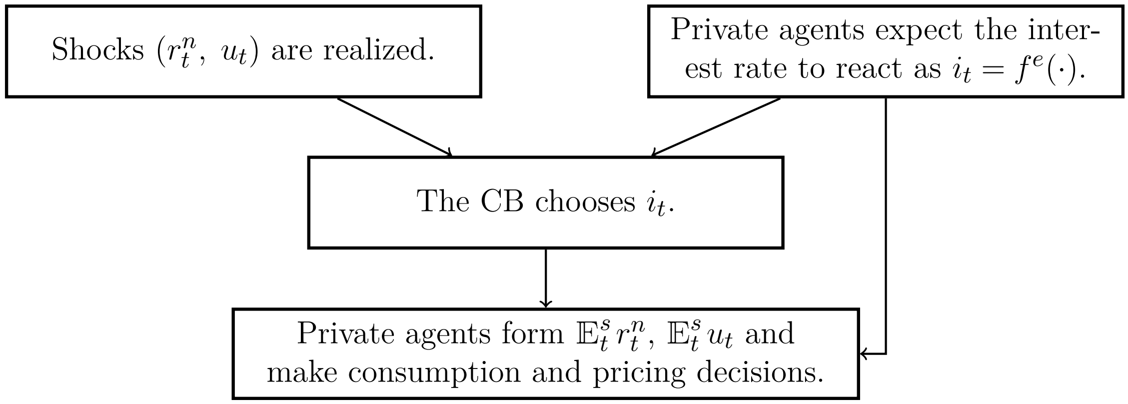

Figure 1: The Sequence of Events under Discretion

3.3 Central Bank's Optimization Problem

This section characterizes the central bank's optimization problem both under discretion and under commitment.

A central bank that optimizes *under discretion* chooses the interest rate ex-post, after the realization of shocks. The discretionary central bank can be viewed as playing a *Nash game* with the private sector. The private sector expects the interest rate to be the central bank's best response in equilibrium, and the central bank cannot change the expected interest rate reaction function. The central bank then chooses the optimal interest rate to maximize its objective function. An equilibrium exists if the actual interest rate response function is the same one expected by private agents. The sequence of events is summarized in Figure 1.

Notice that the private sector receives one piece of information from the central bank: the interest rate decision. The central bank takes two factors as given: the realized shocks and the expected interest rate reaction function. As shown in Proposition 2, the expected interest rate reaction function decides the informational effect for a given change in the interest rate, measured by the Kalman gains, K_r and K_u. Therefore, as the central bank under discretion cannot change the expectations of its policy response function, the central bank takes the informational effect of its interest rate decision as given. In other words, the central bank regards K_r and K_u to be exogenous to its interest rate decisions.

The solution of the equilibrium interest rate under discretion involves a circularity problem, as the optimal interest rate depends on the expected interest rate by the private sector. I follow the method of Svensson and Woodford (2003) to solve the equilibrium interest rate under rational expectations. (See Appendix (C.1) for details.) The following proposition characterizes the optimal discretionary policy under the informational effect.

Proposition 4. *The optimal discretionary monetary policy with informational effect targets* *a negative ratio between inflation and the output gap, given by*

$$R = -\left(-\frac{\kappa}{\sigma} + (1-\theta)\frac{\kappa}{\sigma}K_r + \frac{1-\theta}{\theta}K_u\right)^{-1}\left(-\frac{1}{\sigma}\right)\omega. \tag{32}$$

The absolute value of the targeted ratio is greater than that in the case where monetary policy has no informational effect.[14]

Proof: See Appendix B.4.

The intuition for this proposition is that as the discretionary central bank optimizes at any given state, it chooses the equilibrium as the tangent point between the indifference curve of the central bank's objective function and the Phillips curve. As the slope of the Phillips curve is reduced by the informational effect of monetary policy, the targeted ratio of inflation over the output gap becomes higher in absolute value.

A central bank that optimizes under *commitment* can be viewed as a *Stackelberg leader*. It chooses a state-contingent policy rule ex-ante, before the realization of shocks, and announces the rule to the private sector. The private sector then changes its expected interest rate reaction function to the one that is announced by the central bank. After shocks have been realized, the central bank implements the interest rate implied by the policy rule, which determines the equilibrium output gap and inflation. The sequence of events is summarized in Figure 2.

Notice that the private agents receive two pieces of information from the central bank: the policy rule and the interest rate. The policy rule determines the informational effect for a given change in the interest rate, measured by the Kalman gains, K_r and K_u. In other words, the central bank tells the private sector how to interpret the change in the interest rate by committing to a policy rule.

The key difference between optimal policy under discretion and under commitment is whether the central bank regards the informational effect of the interest rate (measured by K_r and K_u) to be exogenous (in the case under discretion) or endogenous (in the case under commitment). The central bank under commitment changes the informational effect by committing to a policy rule that is different from its best response under discretion. In this way, the central bank chooses a direct mapping from the actual shocks to the expected shocks. The optimal interest rate under commitment can only be solved numerically, which I discuss in the next section.

[14] The targeted ratio without the informational effect is given by equation (22).

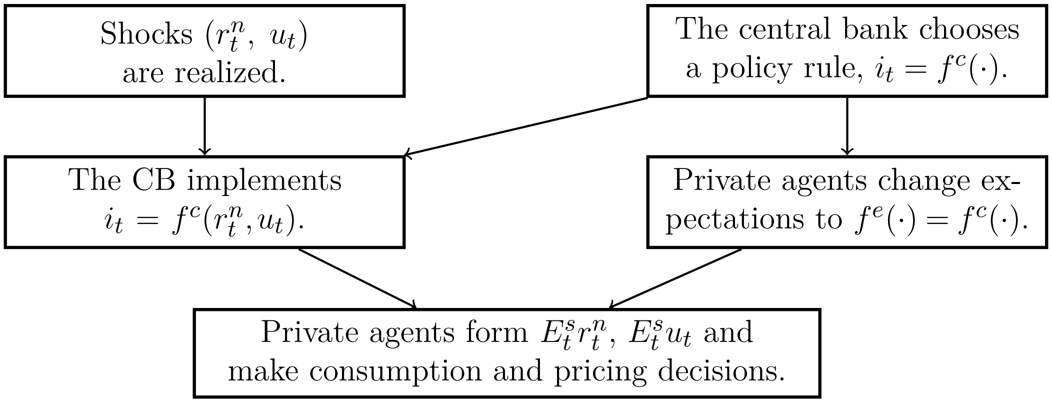

Figure 2: The Sequence of Events under Commitment

3.4 The Informational Gains from Commitment

Before characterizing the optimal monetary policy under commitment, I first use an example of a specific interest rate rule to provide intuition for the informational gains from policy commitment.

Consider the following example: Suppose the central bank commits to the interest rate rule that tracks the natural rate one-to-one and does not respond to the cost-push shock, i.e., $i_t = 1 \cdot r_t^n + 0 \cdot u_t$. In this case, the interest rate provides perfect information about the natural-rate shock and no information about the cost-push shock.[15] Figure 3 compares the Phillips curve under this policy rule and under discretion after a natural-rate shock (left panel) and after a cost-push shock (right panel).

Figure 3 shows that by committing to a policy rule, the central bank can change the Phillips curve, potentially leading to a better available trade-off between inflation and the output gap. Since I consider that the central bank has credible commitment, the central bank does not re-optimize its interest rate decision. Instead, the central bank commits to implementing the equilibrium under the rule, which is denoted with the blue circle.

The left panel of Figure 3 shows that after a natural-rate shock, the Phillips curve crosses the origin, the same as what happens under perfect information. The intuition is that after a natural-rate shock, the interest rate completely offsets its effect on the output gap by tracking it one-to-one, and at the same time, provides perfect information about the shock. Therefore, the divine coincidence can be achieved, as if there are no information frictions. Notice that this is not only due to the policy response to the natural-rate shock, but also due to the commitment of not responding to the cost-push shock. If private agents expect

[15] To see this, substitute $(F_r, F_u) = (1, 0)$ into the belief-updating process given by equation (24), which results in $(K_r, K_u) = (1, 0)$.

(a) After r_t^n (b) After u_t

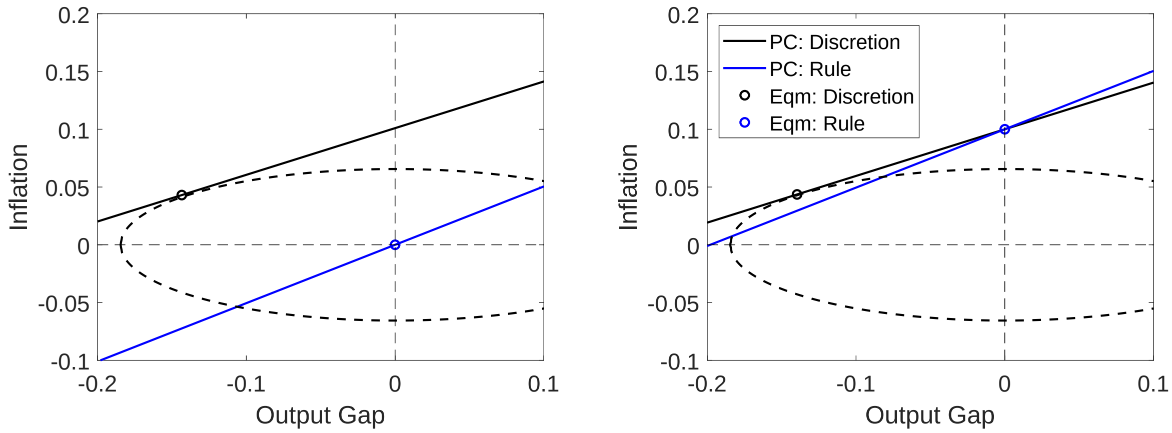

Figure 3: The Phillips Curve under the Rule $i_t = r_t^n$ and under Discretion

Notes: The black line is the Phillips curve for the discretionary central bank. The blue line is the Phillips curve for the central bank following the rule, $i_t = r_t^n$. The black and blue points are the equilibrium under discretion and under the policy rule, respectively. The dotted ellipse denotes the indifference curve of the central bank's loss function.

the interest rate to also react positively to a cost-push shock, the interest rate cannot be a perfect signal of the natural-rate shock, in which case dual stabilization cannot be achieved.

Although this policy rule achieves the first-best after the natural-rate shock, it is not the optimal policy rule. This is because, as indicated by the right panel of Figure 3, the equilibrium under this rule is sub-optimal after cost-push shocks. When the interest rate is completely inelastic to a cost-push shock, it results in an equilibrium with a zero output gap and positive inflation (blue circle), which is much worse than the equilibrium under discretion (black circle).

The optimal policy rule is solved numerically: I search for the best linear interest rate function that minimizes the ex-ante loss function.[16] The intuition for the gains from commitment is that the ability to commit essentially relaxes one constraint for the central bank: The central bank can change the expected interest rate reaction function to any function. In comparison, under discretion, the central bank cannot change the expected interest rate, because private agents always expect the central bank to play its best response after each shock.

The gains from commitment can also be analyzed through the lens of the changes in the Phillips curve, which I characterize in the following proposition.

Proposition 5. *Compared to the equilibrium interest rate under discretion, the optimal pol-*

[16] Detailed characterization of the optimal policy rule is provided in Appendix C.2.

icy rule under commitment increases the slope of the Phillips curve and reduces the intercept of the Phillips curve after the natural rate shock by minimizing the informational effect of monetary policy on inflation, which is given by:

$$\sigma\left[(1-\theta)\frac{\kappa}{\sigma}K_r+\frac{1-\theta}{\theta}K_u\right]. \tag{33}$$

Proof: See Appendix B.5.

The intuition for Proposition 5 is that a better trade-off between inflation and the output gap will become available if the Phillips curve is closer to the origin, the dual stabilization equilibrium point. Proposition 3 shows that after a cost-push shock, the intercept of the Phillips curve is u_t, which is not affected by the informational effect. In this case, the Phillips curve will be closer to the origin if the slope of the Phillips curve is steeper. After a natural-rate shock, minimizing (33) reduces the intercept and steepens the slope at the same time, both of which make the Phillips curve closer to the origin.[17]

The solution to both the optimal discretionary interest rate and the optimal policy rule requires numerical method.[18] With calibrated parameters (see Appendix E for parameter values), the Phillips curves under optimal discretionary policy and under optimal policy rule are given by

$$\pi_t = 0.40\hat{y}_t + 0.10r_t^n + u_t \qquad \text{Discretion} \tag{34}$$

$$\pi_t = 0.46\hat{y}_t + 0.04r_t^n + u_t \qquad \text{Commitment} \tag{35}$$

To better illustrate the differences between the two policies, I compare the equilibrium under commitment and the equilibrium under discretion in Figure 4.

Compared with the Phillips curve under discretion, the Phillips curve under commitment is closer to the origin of the ($\hat{y}_t$, π_t) plane after either a natural-rate shock (left panel) or a cost-push shock (right panel). This shows that an ex-ante commitment to the optimal policy rule can improve the ex-post trade-off faced by the central bank. In the literature, Eggertsson and Woodford (2003) illustrate how committing to future monetary policy can improve the current trade-off.

My contribution to this idea is twofold. First, I show that under the informational effect of monetary policy, not only the intercept but also the slope of the Phillips curve can be changed by commitment. This is because the informational effect affects inflation and the output gap

[17]This situation applies when the Phillips curve has a positive intercept after a natural-rate shock under discretion, which is satisfied under the normal parameter specifications. (See parameter values in Appendix E.)

[18]See Appendix C.1 and Appendix C.2 for details on the solution method.

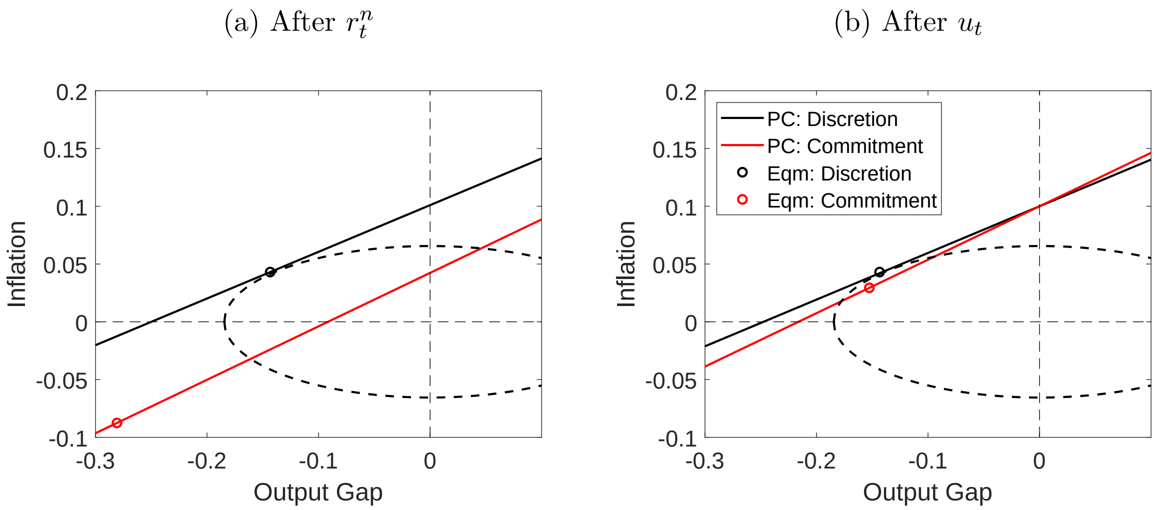

Figure 4: Discretionary Equilibrium vs. Optimal Policy Rule

Notes: The black line is the Phillips curve when the central bank is expected to be discretionary. The red line is the Phillips curve under optimal commitment. The black and red circles are the equilibrium under optimal discretionary policy and under optimal policy rule, respectively. The dotted ellipse denotes the indifference curve of the central bank's loss function.

differently. Second, the change in the Phillips curve through expectations does not rely on the commitment to future monetary policy. To isolate the within-period informational effect, I have imposed the restriction in this section that the policy rule responds only to current shocks. The shift of the Phillips curve is still through changes in expected inflation, but it is expected current inflation, not expected future inflation.

Associated with the gains from commitment is the time-inconsistency problem. After either of the shocks, the equilibrium under the optimal policy rule is not on the tangent point between the Phillips curve and the central bank's objective function. It means that the central bank *commits to be sub-optimal* ex-post. Eggertsson and Woodford (2003) also make a similar point: To fight current deflation, the central bank should commit to a future easing of monetary policy, which will be sub-optimal in the future. Instead of inconsistency across time periods, the inconsistency in my baseline model is across states. The optimal policy rule does not achieve better outcomes than the discretionary equilibrium in every state. Rather, the central bank under optimal commitment sacrifices the outcome after the natural-rate shock (the red circle is on a worse indifference curve than the black circle) to gain a better outcome after a cost-push shock (the red circle is on a better indifference curve than the black circle). The central bank balances outcomes across all states and achieves ex-ante welfare improvement.[19]

[19]In the right panel of Figure 4, the unit of the cost-push shock is chosen to be 0.1 instead of 1. This is to make both figures of the same scale while having the differences between Phillips curves observable.

4 Dynamic Informational Effect

Now, I extend the model to include serially correlated shocks. In this case, the learning process becomes persistent, which leads to the dynamic informational effect of monetary policy. In this section, I first describe the equilibrium in the private sector (Section 4.1), and then characterize the optimal policy under discretion (Section 4.2) and under commitment (Section 4.3).

4.1 The Equilibrium in the Private Sector

With serial correlation, the process of the actual shocks is given by:

$$\begin{bmatrix} r_t^n \\ u_t \end{bmatrix} = \begin{bmatrix} \phi & 0 \\ 0 & \phi^u \end{bmatrix} \begin{bmatrix} r_{t-1}^n \\ u_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_t^r \\ v_t^u \end{bmatrix}. \tag{36}$$

The private sector is Bayesian, and optimally combines its past beliefs and current signals when forming beliefs in the current period. The learning process adds an additional degree of persistence in the private sector, as past expectations affect the current economy through the beliefs-updating process.

The central bank sets the interest rate to stabilize the current economy. Since the current equilibrium is affected by past expectations, I conjecture that the equilibrium interest rate is a linear function of both the actual shocks in period t and beliefs in period $t-1$. Specifically,

$$i_t = F_1 r_t^n + F_2\, \mathbb{E}_{t-1}^s\, r_{t-1}^n + F_3 u_t + F_4\, \mathbb{E}_{t-1}^s\, u_{t-1}. \tag{37}$$

Under rational expectations, the private sector is able to distinguish the fraction of the interest rate that reacts to current shocks from the fraction of the interest rate that reacts to past beliefs. Let $\hat{i}_t$ denote the fraction of i_t that reacts to current shocks, which is given by:

$$\hat{i}_t \equiv i_t - F_3\, \mathbb{E}_{t-1}^s\, r_{t-1}^n - F_4\, \mathbb{E}_{t-1}^s\, u_{t-1} = F_1 r_t^n + F_3 u_t. \tag{38}$$

$\hat{i}_t$ becomes a signal that simultaneously provides information about both shocks.

Denote the unobserved state variables as

$$z_t = \Phi z_{t-1} + v_t, \tag{39}$$

where $z_t = [r_t^n,\ u_t]'$, $\Phi = \begin{bmatrix} \phi & 0 \\ 0 & \phi^u \end{bmatrix}$, and $v_t = [v_t^r,\ v_t^u]'$ with white noise of variance Q. Denote the observable signal as

$$s_t = Dz_t \tag{40}$$

where $s_t = \hat{i}_t$, and $D = [F_1, F_3]'$.

The private sector's expectations formed through the Kalman filtering process is given by

$$E_t^s z_t = \Phi \, \mathbb{E}_{t-1}^s \, z_{t-1}^n + K \left(s_t - D\Phi \, \mathbb{E}_{t-1}^s \, z_{t-1} \right), \tag{41}$$

where the optimal weight, K, is determined by the Ricatti iteration as follows,

$$K = PD'(DPD')^{-1}, \tag{42}$$

$$P = \Phi \left(P - PD'(DPD')^{-1} DP \right) \Phi + Q. \tag{43}$$

The equilibrium in the private sector is described by the system of equations summarizing private-sector optimization decisions in aggregate variables (equations (10) and (14)), the evolution process of the actual shocks (equation (36)), the interest rate reaction function (equation (37)), and the beliefs-updating process characterized in equation (41).

I solve the equilibrium using the method of undetermined coefficients. To economize on the use of notations, I use z_t from now on to denote the vector of predetermined state variables at t, which includes current shocks and past beliefs, i.e., $z_t = \left[r_t^n, \; u_t, \; \mathbb{E}_{t-1}^s \, r_{t-1}^n, \; \mathbb{E}_{t-1}^s \, u_{t-1} \right]$. The equilibrium output gap and inflation are given by

$$\begin{bmatrix} \hat{y}_t \\ \pi_t \end{bmatrix} = \Gamma z_t \tag{44}$$

where the expression of Γ and derivation details are provided in Appendix D.

4.2 Optimal Monetary Policy under Discretion

The central bank has perfect information. The information set of the central bank at t includes the entire history of natural-rate and cost-push shocks upon t and the beliefs formed in the private sector upon $t-1$, i.e.,

$$\mathcal{I}_t = \left\{ r_T^n, \; \mathbb{E}_{T-1}^s \, r_{T-1}^n, \; u_T, \; \mathbb{E}_{T-1}^s \, u_{T-1} \quad \forall T = 1 ... t \right\}.$$

The central bank forms objective expectations under perfect information, which I denote by $\mathbb{E}_t$ to differentiate from the subjective expectations in the private sector, $\mathbb{E}_t^s$. The central bank's objective expectations about the future equilibrium variables are given by

$$\begin{bmatrix} \mathbb{E}_t \pi_{t+j} \\ \mathbb{E}_t \hat{y}_{t+j} \end{bmatrix} = \Gamma \, \mathbb{E}_t \, z_{t+j}, \tag{45}$$

where $\mathbb{E}_t \, z_{t+1} = [\phi r_t^n, \ \mathbb{E}_t \, \mathbb{E}_t^s \, r_t^n, \ \phi^u u_t, \ \mathbb{E}_t \, \mathbb{E}_t^s \, u_t]$. $\mathbb{E}_t \, z_{t+j}$ evolves according to

$$\mathbb{E}_t \, z_{t+j} = \Lambda \, \mathbb{E}_t \, z_{t+j-1} \tag{46}$$

where Λ captures the evolution process of both the actual shocks and the expected shocks through the Kalman filtering process. See Appendix D for the expression of Λ.

The central bank's objective expectations about the future equilibrium variables can be written as

$$\begin{bmatrix} \mathbb{E}_t \pi_{t+j} \\ \mathbb{E}_t \hat{y}_{t+j} \end{bmatrix} = \Gamma \Lambda^{j-1} \, \mathbb{E}_t \, z_{t+1}. \tag{47}$$

Equation (47) suggests that with serially correlated shocks, the current interest rate has a lagged effect. This is because the current interest rate affects current expectations, i.e., i_t affects $\mathbb{E}_t^s \, r_t^n$ and $\mathbb{E}_t^s \, u_t$, and the current expectations affect future equilibrium through the dynamic learning process.

With the dynamic informational effect of monetary policy, the central bank's objective function can no longer be reduced to minimizing fluctuations in the current period. Instead, the discretionary central bank chooses the optimal interest rate in the current period to minimize the following loss function:

$$L(t) = \pi_t^2 + \omega \hat{y}_t^2 + \beta \, \mathbb{E}_t(L(t+1)), \tag{48}$$

I characterize the optimal monetary policy under discretion in the following proposition.

Proposition 6. *With a dynamic informational effect, the optimal discretionary monetary policy is dynamically "leaning against the wind," as it targets a negative correlation between current and future deviations in the output gap and inflation.*

Proposition 6 results from the first-order condition of the central bank's optimization problem, which is given by

$$\left\{ \frac{\partial \pi_t}{\partial i_t^*} \pi_t + \omega \frac{\partial \hat{y}_t}{\partial i_t^*} \hat{y}_t \right\} = -\frac{1}{2} \Sigma_{j=1}^{\infty} \beta^j \left\{ \frac{\partial \, \mathbb{E}_t \, \pi_{t+j}}{\partial i_t^*} \, \mathbb{E}_t \, \pi_{t+j} + \omega \frac{\partial \, \mathbb{E}_t \, \hat{y}_{t+j}}{\partial i_t^*} \, \mathbb{E}_t \, \hat{y}_{t+j} \right\}, \tag{49}$$

If the right-hand side of equation (49) equals zero, then Proposition 6 is equivalent to the within-period "leaning against the wind" policy in the sense of targeting a negative relationship between nominal prices and real output in each period.[20] However, due to the lagged effect of i_t on future equilibrium through its informational effect, the right-hand side of equation (49) does not equal zero. Since the lagged effect of monetary policy comes entirely from the informational effect, Proposition 6 can also be put in the following way:

Corollary 6.1. *The consideration of the dynamic informational effect makes the equilibrium interest rate target beliefs in the private sector in addition to targeting current inflation and the output gap.*

The consideration of the dynamic informational effect consists of two parts. First is the effect on current inflation and the output gap. The consumption and pricing decisions are forward-looking, which allows the dynamic informational effect to change current inflation and the output gap through its effect on expected future equilibrium variables. Second, the central bank also takes into account that $\mathbb{E}_t^s r_t^n$ and $\mathbb{E}_t^s u_t$ become state variables for future equilibrium, which allows the central bank to stabilize future inflation and the output gap through its effect on expectations in the current period.

4.3 Optimal Monetary Policy under Commitment

The central bank under commitment chooses the monetary policy rule prior to the realization of shocks to minimize the ex-ante loss function of the central bank. Given the linear-quadratic nature of the central bank's optimization problem and the beliefs-formation process, I study the best linear interest rate rule that responds to current shocks and past beliefs. Specifically, I assume that the monetary policy rule takes the functional form of $i_t = F_1 r_t^n + F_2 \mathbb{E}_{t-1}^s r_{t-1}^n + F_3 u_t + F_4 \mathbb{E}_{t-1}^s u_{t-1}$. This rule implies that the current interest rate reacts to past shocks only through its reaction to past beliefs. This assumption allows for path dependence without complicating the information revealed by the current state. The private sector can still use the (modified) interest rate as a signal about current shocks given by equation (38).

The optimal commitment policy rule is solved numerically, which is found to be

$$i_t^* = 1.06 r_t^n - 0.05 \mathbb{E}_{t-1}^s r_{t-1}^n + 0.14 u_t + 0.42 \mathbb{E}_{t-1}^s u_{t-1} \tag{50}$$

The optimal policy rule with the informational effect responds to the natural-rate shock more than one-to-one, and responds to the cost-push shock by a much smaller amount. The intuition is that information frictions enlarge the positive output gap after a positive natural-rate shock, so the central bank wants to provide more precise information about the

[20] See Angeletos and La'O (2020) for discussions about the within-period "leaning against" policy that is caused by informational frictions.

natural-rate shock to reduce the output gap. In contrast, providing more precise information about the cost-push shock would increase inflation responses. Therefore, the optimal policy rule reveals more precise information about the natural-rate shock and reveals less precise information about the cost-push shock.

In addition, the optimal policy rule also positively responds to the expected cost-push shock in the past period. By doing so, the traditionally studied gains from committing to a delayed response reinforce the informational gains. After a cost-push shock, the interest rate reveals less information about the realized cost-push shock. In addition, by committing to higher interest rates in future periods, the direct effect of the future tightening of monetary policy decreases expected future inflation. As pricing decisions are forward-looking, expectations of lower future inflation decrease current inflation.

Economists have been debating whether policy rates set by many central banks, including the Federal Reserve, are inertial to changes in the state of the economy, and whether such an inertial response is optimal.[21] My model offers a perspective from the informational effect of the policy rate: The optimal policy rate should overshoot the natural-rate shock to provide more precise information about it, and should be inertial to the cost-push shock to make the interest rate reveal less information about the cost-push shock. The equilibrium output gap and inflation, together with the optimal policy rate, are shown in Figure 5.

5 Discussion

In this section, I discuss the implications of the model. First, in Section 5.1, I study the effects of central bank direct communication, which interacts with the informational effect of interest rates. Then, in Section 5.2, I discuss how my model predictions explain the empirical patterns of the informational effect of monetary policy found in Nakamura and Steinsson (2018).

5.1 Central Bank Direct Communication

Until now, the only information that the private sector receives is the interest rate. In a more realistic setting, the private sector also receives other external signals about the underlying economy. An important source of information is central bank communication. In fact, central

[21] For example, Rudebusch (2002, 2005) finds evidence that financial markets fail to predict future changes in the interest rate target and conclude that interest rate inertia should be explained by the persistence in underlying shocks. However, Coibion and Gorodnichenko (2012) argue that the reason that financial markets fail to predict interest rate inertia is due to the lack of information. In fact, Greenbook forecasts have strong predictive power of future interest rate changes.

(a) After r_t^n (b) After u_t

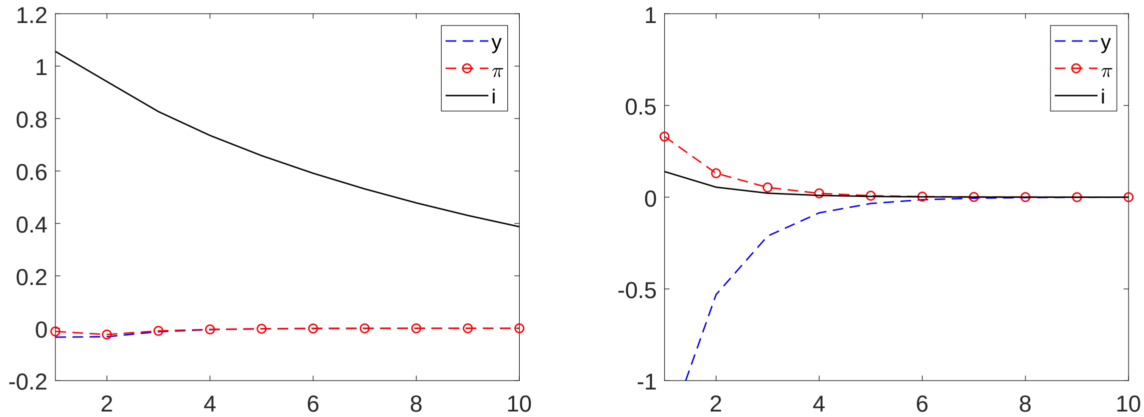

Figure 5: The Impulse Responses under Optimal Policy Rule

Notes: The left figure shows the impulse response of inflation and the output gap after a one unit increase in the natural-rate shock, and the right figure shows the dynamics after a one unit increase in the cost-push shock.

banks around the world have increasingly used direct communication as a monetary policy instrument in addition to setting the current interest rate. This section studies how the optimal communication strategy may depend on the informational effect of monetary policy.

To model the central bank's direct communication, I add public signals of both shocks, and let the central bank control the precision of these signals. Denote the signals sent through the central bank's communications as m_t^r and m_t^u, which distribute log normally around the actual shocks, r_t^n and u_t, with variances of σ_{mr}^2 and σ_{mu}^2, respectively. All the signals received by private agents are summarized as follows:

$$\begin{bmatrix} \hat{i}_t \\ m_t^r \\ m_t^u \end{bmatrix} = \begin{bmatrix} F_1 & F_3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_t^n \\ u_t \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \epsilon_t^r \\ \epsilon_t^u \end{bmatrix} \tag{51}$$

The beliefs formed through the Kalman filtering process are given by

$$\begin{bmatrix} \mathbb{E}_t^s r_t^n \\ \mathbb{E}_t^s u_t \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \end{bmatrix} \begin{bmatrix} i_t \\ m_t^r \\ m_t^u \end{bmatrix} \tag{52}$$

Theoretically, a central bank can provide perfect information about the underlying shocks through direct communication, in which case the informational effect of monetary policy no longer exists. However, is it optimal for a central bank to provide perfect information about both types of shocks?

To answer this question, let us first consider the case in which monetary policy does not have an informational effect. In this case, the optimal strategy for the central bank's direct communication is to provide a perfect signal about the natural-rate shock and provide no signal about the cost-push shock. The intuition is that after a positive natural-rate shock, if the central bank provides a perfect signal about the natural-rate shock, it also reveals the fact that under optimal monetary policy, the central bank is able to achieve dual stabilization, making expected inflation zero. By contrast, after a positive cost-push shock, if the central bank provides any information about the cost-push shock, it also reveals the fact that full stabilization cannot be achieved under optimal monetary policy, making expected inflation positive, and positive expected inflation further increases actual inflation. To minimize deviations caused by underlying shocks, the central bank wants to perfectly reveal natural-rate shocks and completely withhold information about cost-push shocks.[22]

The interaction between the informational effect of monetary policy and central bank direct communication complicates the problem. This interaction makes the central bank unable to separately control the precision of the information about one type of shock. More specifically, the interaction effect is summarized in the following proposition.

Proposition 7. *Providing more precise information about one type of shock through direct communication increases the precision of the informational effect of monetary policy about the other shock.*

Proof: Proposition 7 can be proved by first solving the optimal interest rate (either under discretion or under commitment), and then varying σ_{mu} and σ_{mr} to see how K_{11} and K_{21} change with the signals' precisions.

The intuition for Proposition 7 can be explained by the following example. Suppose that the central bank provides perfect information about the natural-rate shock. Then suppose that the natural-rate shock is realized to be zero and the cost-push shock is realized to be positive. The private sector knows that $r_t^n = 0$ from the central bank's direct communication about the natural-rate shock. In addition, the private sector also observes that the interest rate responds positively, and it can then know for sure that the increase in the interest rate must be a response to the cost-push shock. In this way, the private sector also gets perfect information about the cost-push shock.

In Figure 6, I plot the ex-ante loss function of the central bank at varying precisions of the central bank's direct communication. It shows that more precise communication about either

[22]For a more general discussion on the value of information without the informational effect of monetary policy, see Morris and Shin (2002), Angeletos and Pavan (2007), Angeletos, Iovino, and La'O (2016), for examples. See Blinder et al. (2008) for a survey of the literature on central bank communication.

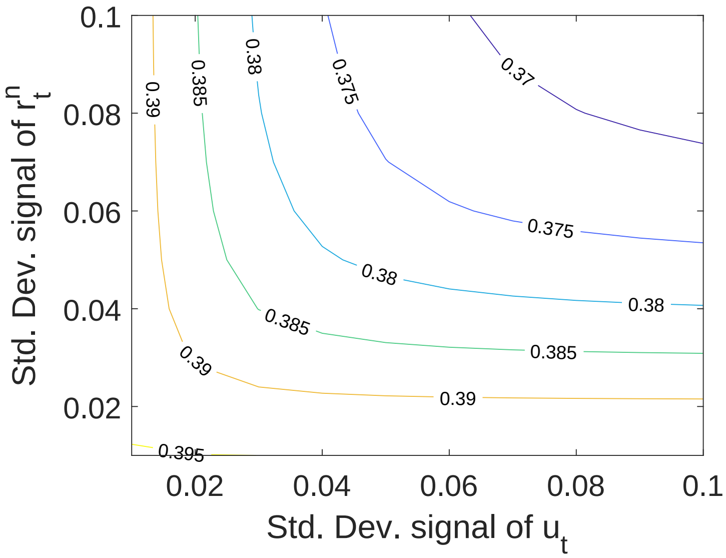

Figure 6: The Value of Central Bank Direct Communication

Notes: This figure plots the value of the ex-ante loss at varying precisions of external signals.

shock (modeled by a decrease in the standard deviation of the external signals) increases the ex-ante loss. This implies that the optimal communication strategy for the central bank is to provide no additional signal about either the natural-rate shock or the cost-push shock. This is because the interaction with the informational effect of the interest rate is very strong. In this case, when the central bank provides more precise information about the natural-rate shock, it also makes the interest rate a more precise signal about the cost-push shock, about which the central bank does not want to provide information.

This model prediction seems counter-intuitive, given the increasing importance of central bank communication in real-world practice. Two factors result in this contradiction. First, I model the content of central bank communication to only include the realization of underlying shocks. Since my model assumes that the private sector has rational expectations, communication about the monetary policy response function or the central bank's inflation target plays no role in my model. In contrast, in the real world, a very important element of central bank communication is its targets and the policy response function. If, however, the monetary policy rule in my model is also not perfectly known to the private sector, then central bank communication regarding its policy rule would have important effects.

In addition, I have assumed so far that there are no exogenous variations in monetary policy. All changes in monetary policy are endogenous responses to underlying shocks. This assumption makes the interest rate a very precise signal about underlying shocks, and the central bank therefore has little need to rely on additional communication. In the real world, monetary policy changes typically have two components: One is the endogenous response to fluctuations in underlying shocks, and the other is a random variable that is usually referred to as an exogenous shock to monetary policy. [23] Allowing for such randomness in monetary policy would make the interest rate less informative about underlying shocks. For this reason, I provide the solution method to the general version of the model, which includes exogenous monetary policy shocks, in Appendix D.

5.2 The Empirical Patterns of the Informational Effect of Monetary Policy

Nakamura and Steinsson (2018) document the empirical patterns of the informational effect of monetary policy. Specifically, the authors show that only the expectations of future output growth are sensitive to monetary policy shocks, whereas expectations of future inflation do not respond to monetary policy shocks. In my model, under optimal commitment, the policy rate reveals more precise information about the natural-rate shock and less precise information about the cost-push shock. This section shows that when mapping the expected shocks to the expected future output growth rate and inflation rate, my model predictions match the empirical patterns found in Nakamura and Steinsson (2018), and are robust at varying degrees of information frictions.

In my model, the informational effect of a positive innovation in monetary policy changes expectations of underlying shocks, $\mathbb{E}_t^s r_t^n$ and $\mathbb{E}_t^s u_t$, and consequently changes the expected output and inflation in the next period. Table 1 presents the model predicted changes in expectations of future output growth and inflation after a positive one percentage point increase in the interest rate.

Table 1 (a) shows that expected inflation is insensitive to changes in monetary policy. In comparison, Table 1 (b) shows that the sensitivity of expected output growth to monetary policy shocks is around one. In addition, this result is robust at varying degrees of information frictions (parameter values of σ_{er} and σ_{eu}). Even when external signals are very precise (in the case of $\sigma_{er} = \sigma_{eu} = 0.03$), the interest rate still significantly changes the expected output growth. These model predictions closely match the empirical results in Nakamura and Steinsson (2018).

Nakamura and Steinsson (2018) provide a theoretical explanation for the above empiri-

[23] The previous literature has offered several interpretations of the policy shock. Christiano, Eichenbaum, and Evans (1999) summarize three types of interpretations, including (1) shocks to the preferences of the members of the FOMC, (2) fluctuations in private agents' expectations to which the monetary authority reacts, and (3) measurement error in the preliminary data available at the time the central bank makes policy decisions.

Table 1: Changes in Expectations After a Positive Innovation in Monetary Policy

(a) Expected Inflation

	$\sigma_{\epsilon r} = 0.03$	$\sigma_{\epsilon r} = 0.1$	$\sigma_{\epsilon r} = 1$
$\sigma_{\epsilon u} = 0.03$	0.0026	0.0126	0.0136
$\sigma_{\epsilon u} = 0.1$	-0.0235	0.0100	0.0120
$\sigma_{\epsilon u} = 1$	-0.1108	0.0449	0.0441

(b) The Expected Output Growth

	$\sigma_{\epsilon r} = 0.03$	$\sigma_{\epsilon r} = 0.1$	$\sigma_{\epsilon r} = 1$
$\sigma_{\epsilon u} = 0.03$	0.9974	0.9874	0.9864
$\sigma_{\epsilon u} = 0.1$	1.0235	0.9900	0.9880
$\sigma_{\epsilon u} = 1$	1.1108	0.9551	0.9559

Notes: The table reports the percentage change in the expected one-period-ahead inflation rate (left panel) and the output growth rate (right panel) after an unexpected one percentage point increase in the interest rate.

cal patterns by modeling that there is one type of shock, the natural-rate shock, that drives variations in monetary policy. My model shows that even after adding a cost-push shock, which potentially allows monetary policy shocks to drive inflation expectations, the model prediction is the same under the optimal policy rule. This is because under optimal commitment, the policy rate reveals more precise information about the natural-rate shock and less precise information about the cost-push shock, which reduces the fluctuations in inflation expectations.

6 Conclusion

This paper studies optimal monetary policy in the presence of the informational effect of monetary policy. In this case, when the central bank adjusts the interest rate to offset the effect caused by underlying shocks, it also reveals information on the realization of the shocks. Therefore, the informational effect dampens the direct stabilizing effect of monetary policy.

The key contribution of the paper is to show that the informational effect of monetary policy results in gains from commitment. The intuition can be shown in the baseline case where the current interest rate is the only signal in the private sector and simultaneously provides information about two types of shocks: the natural-rate shock and the cost-push shock. In this case, the expected interest rate response function determines the informational effect for a given interest rate and can be changed if the central bank commits to a state-contingent policy rule. Under optimal commitment, the central bank decreases the degree to which the informational effect dampens the direct effect of monetary policy, which increases the slope of the Phillips curve from the discretionary equilibrium.

In the dynamic case, my calibrated model shows that the optimal policy rule responds more aggressively to the natural-rate shock and less aggressively to the cost-push shock. By doing so, the interest rate reveals more precise information about the natural-rate shock and withholds information about the cost-push shock. My model also has important implications for the optimal strategy of central bank communication, which depends on the interaction with the informational effect of monetary policy. Bringing my model predictions to the real world, the optimal policy rule under commitment explains the empirical patterns of the informational effect of monetary policy. Specifically, it explains the reason why only expected output growth, not expected inflation, is sensitive to monetary policy shocks.

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Appendices

A Private-Sector Equilibrium and Aggregation

This section provides details of log-linear approximation and aggregation of equilibrium variables in the private sector.

We first work on the household side. The log-linear approximation to the Euler equation is:

$$y_t = \mathbb{E}_t^H y_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t^H \pi_{t+1}\right) \tag{A.1}$$

Next, the log-linear approximation to the individual labor supply (equation (6)) yields

$$\varphi n_t(j) + \sigma y_t = w_t(j), \tag{A.2}$$

where w_t denotes the log approximated real wage, $log(W_t/P_t)$. Recall that the resource constraint implies that $c_t^j = y_t^j \; \forall j$, which further implies $c_t = y_t$. We can then write the labor supply as follows:

$$\varphi n_t(j) + \sigma y_t = w_t(j) \tag{A.3}$$

Integrate the individual labor supply, which is given by

$$\int w_t(j) = \varphi \int n_t(j)dj + \sigma y_t. \tag{A.4}$$

Substitute y_t in the above equation with the demand for intermediate goods, i.e., $y_t(j) - y_t = -\epsilon\,(p_t(j) - p_t)$:

$$\int n_t(j)dj = y_t + \int(-\epsilon)(p_t(j) - p_t) - \int a_t(j) = y_t - a_t \tag{A.5}$$

Substituting $\int w_t(j)$ in (A.4) with (A.5) results in

$$\int w_t(j) - a_t(j) = (\phi + \sigma)y_t - (1+\varphi)a_t \tag{A.6}$$

Define the natural level of output as the equilibrium output level without price rigidity or informational friction. In this case, y_t^n becomes a linear function of the aggregate technology. This allows us to write equation (A.6) in terms of the output gap, which is given by

$$\int w_t(j) - a_t(j) = (\phi + \sigma)(y_t - y_t^n) \tag{A.7}$$

On the firm side, the log-linear approximation to the firm's optimal resetting price is:

$$p_t^*(j) = (1-\beta\theta)\,\mathbb{E}_t^j\left\{\Sigma(\beta\theta)^k\left[p_{t+k} + u_{t+k}(j) + w_{t+k}(j) - a_{t+k}(j)\right]\right\} \tag{A.8}$$

The Calvo assumption implies that the aggregate price index is an average of the price charged by the fraction $1-\theta$ of firms that reset their prices at t, and the fraction θ of firms whose prices remain the same as in the last period. Thus, the aggregate price under log-linear approximation is:

$$p_t = \theta p_{t-1} + (1-\theta)\int p_t^*(j)dj \tag{A.9}$$

Subtract p_{t-1} from both sides and get:

$$\pi_t = (1-\theta)\left(\int p_t^*(j) - p_{t-1}\right) \tag{A.10}$$

Under the assumption that beliefs about aggregate variables are homogeneous, we can simply integrate individual prices without dealing with the higher-order beliefs problem. The integral of individual prices is given by

$$\int p_t^*(j)dj = (1-\beta\theta)\left(\mathbb{E}_t^s p_t + u_t + w_t - a_t\right) + (1-\beta\theta)\Sigma_{k=1}^{\infty}(\beta\theta)^k\,\mathbb{E}_t^s\left(p_{t+k} + w_{t+k} - a_{t+k}\right) \tag{A.11}$$

To write (A.11) in difference equations, we first express

$$\beta\theta\int \mathbb{E}_t^s p_{t+1}^*(j)dj = (1-\beta\theta)\Sigma_{k=1}^{\infty}\,\mathbb{E}_t^s\left(p_{t+k} + u_{t+k} + w_{t+k} - a_{t+k}\right) = \beta\theta\,\mathbb{E}_t^s p_{t+k}^* \tag{A.12}$$

and then subtract (A.12) from (A.11) to get:

$$\int p_t^*(j)dj - \beta\theta\,\mathbb{E}_t^s p_{t+1} = (1-\beta\theta)\,\mathbb{E}_t^s p_t + (1-\beta\theta)u_t + (1-\beta\theta)(\varphi+\sigma)\hat{y} \tag{A.13}$$

Subtract p_{t-1} from both sides and get

$$\int p_t^*(j)dj - p_{t-1} = \beta\theta\left(\mathbb{E}_t^s p_{t+1}^* - \mathbb{E}_t^s p_t\right) + \mathbb{E}_t^s p_t - p_{t-1} + (1-\beta\theta)u_t + (1-\beta\theta)(\varphi+\sigma)\hat{y}_t \tag{A.14}$$

Apply (A.10) and get

$$\pi_t = \beta\theta\,\mathbb{E}_t^s \pi_{t+1} + (1-\theta)\,\mathbb{E}_t^s \pi_t + (1-\theta)(1-\beta\theta)u_t + (1-\beta\theta)(1-\theta)(\varphi+\sigma)\hat{y}_t \tag{A.15}$$

Now, inflation can be written as

$$\pi_t = \beta\theta\,\mathbb{E}_t^s \pi_{t+1} + (1-\theta)\,\mathbb{E}_t^s \pi_t + \kappa\theta\hat{y}_t + u_t \tag{A.16}$$

where $\kappa = \frac{(1-\beta\theta)(1-\theta)(\varphi+\sigma)}{\theta}$, and $u_t = (1-\theta)(1-\beta\theta)u_t$

B Proofs

B.1 Proof of Proposition 1

In this static case, the structural-form Phillips curve is

$$\pi_t = (1-\theta)\,\mathbb{E}_t^s\,\pi_t + \kappa\theta\hat{y}_t + u_t \tag{B.1}$$

To derive the reduced-form Phillips curve under imperfect information, first apply $\mathbb{E}_t^s$ to the structural-form Phillips curve and get

$$\mathbb{E}_t^s\,\pi_t = (1-\theta)\,\mathbb{E}_t^s\,\pi_t + \kappa\theta\left[-\frac{1}{\sigma}\left(i_t - \mathbb{E}_t^s\,r_t^n\right)\right] + \mathbb{E}_t^s\,u_t \tag{B.2}$$

where the second term makes use of the IS curve, $\hat{y}_t = -\frac{1}{\sigma}\left(i_t - r_t^n\right)$, and the assumption that the interest rate is observable, i.e., $\mathbb{E}_t^s\,i_t = i_t$. Substitute $\mathbb{E}_t^s\,\pi_t$ in (B.1) with (B.2) and get

$$\pi_t = (1-\theta)\left(-\frac{\kappa}{\sigma}i_t + \frac{\kappa}{\sigma}\,\mathbb{E}_t^s\,r_t^n + \frac{1}{\theta}\,\mathbb{E}_t^s\,u_t\right) + \kappa\theta\hat{y}_t + u_t \tag{B.3}$$

Next, substitute i_t with $\hat{y}_t$ and get:

$$\mathbb{E}_t^s\,\pi_t = \kappa\hat{y}_t + (1-\theta)\frac{\kappa}{\sigma}\left(\mathbb{E}_t^s\,r_t^n - r_t^n\right) + \frac{1-\theta}{\theta}\,\mathbb{E}_t^s\,u_t + u_t \tag{B.4}$$

Applying the assumption that there are no signals of underlying shocks, i.e., $\mathbb{E}_t^s\,r_t^n = 0$ and $\mathbb{E}_t^s\,u_t = 0$, leads to the reduced-form Phillips curve under information frictions,

$$\pi_t = \kappa\hat{y}_t - (1-\theta)\frac{\kappa}{\sigma}r_t^n + u_t \tag{B.5}$$

To get the reduced-form Phillips curve under perfect information in the static case, simply apply the assumption that $\mathbb{E}_t\,\pi_{t+1} = 0$, which means

$$\pi_t = \kappa\hat{y}_t + \frac{1}{\theta}u_t \tag{B.6}$$

Proposition 1 is then a direct comparison between (B.5) and (B.6).

B.2 Proof of Corollary 1.2

To derive the optimal interest rate, start from the first-order condition on the interest rate (equation (22)). Substitute π_t with (B.5) and get

$$\kappa\hat{y}_t - (1-\theta)\frac{\kappa}{\sigma}r_t^n + u_t = -\frac{\omega}{\kappa}\hat{y}_t. \tag{B.7}$$

Next, substitute $\hat{y}_t$ by i_t and get

$$\left(\kappa + \frac{\omega}{\kappa}\right)\left(-\frac{1}{\sigma}i_t + \frac{1}{\sigma}r_t^n\right) = (1-\theta)\frac{\kappa}{\sigma}r_t^n - u_t \tag{B.8}$$

Rearrange to get the optimal interest rate, given by

$$i_t^* = \left[1 - \frac{(1-\theta)\kappa^2}{\kappa^2+\omega}\right]r_t^n + \frac{\sigma\kappa}{\kappa^2+\omega}u_t, \tag{B.9}$$

Compared to the case under perfect information, in which case $i_t^* = r_t^n + \frac{\sigma\kappa}{\kappa^2+\omega}\frac{1}{\theta}u_t$, the optimal interest rate reacts less to both the natural-rate shock and the cost-push shock under imperfect information.

B.3 Proof of Proposition 3

To derive the Phillips curve with the information effect of monetary policy, substitute $\mathbb{E}_t^s r_t^n$ and $\mathbb{E}_t^s u_t$ with i_t according to the beliefs-updating process in the Phillips curve, which yields

$$\pi_t = \kappa\hat{y}_t + (1-\theta)\frac{\kappa}{\sigma}K_r i_t - (1-\theta)\frac{\kappa}{\sigma}r_t^n + \frac{1-\theta}{\theta}K_u i_t + u_t \tag{B.10}$$

Next, substitute i_t with $\hat{y}_t$ using the IS relationship, which results in

$$\pi_t = \kappa\hat{y}_t + (1-\theta)\frac{\kappa}{\sigma}K_r\left(-\sigma\hat{y}_t + r_t^n\right) - (1-\theta)\frac{\kappa}{\sigma}r_t^n + \frac{1-\theta}{\theta}K_u\left(-\sigma\hat{y}_t + r_t^n\right) + u_t \tag{B.11}$$

Re-arrange and get

$$\pi_t = \left\{\kappa - \sigma\left[(1-\theta)\frac{\kappa}{\sigma}K_r + \frac{1-\theta}{\theta}K_u\right]\right\}\hat{y}_t + \left\{(1-\theta)\frac{\kappa}{\sigma}(K_r-1) + \frac{1-\theta}{\theta}K_u\right\}r_t^n + u_t. \tag{B.12}$$

In equilibrium, the interest rate positively responds to both shocks to offset their effects, which implies $K_r > 0$ and $K_u > 0$. Therefore, the slope of the Phillips curve in equation (B.12) is smaller than the slope without the informational effect of monetary policy, i.e., $\kappa - \sigma\left[(1-\theta)\frac{\kappa}{\sigma}K_r + \frac{1-\theta}{\theta}K_u\right] < \kappa$. For the same reason, the intercept after a positive natural-rate shock is higher than the one without the informational effect of monetary policy, i.e., $(1-\theta)\frac{\kappa}{\sigma}(K_r-1)+\frac{1-\theta}{\theta}K_u > (1-\theta)\frac{\kappa}{\sigma}$.

B.4 Proof of Proposition 4

This proposition is an intermediate step in the solution of discretionary equilibrium. Details of solving the discretionary equilibrium are provided in Appendix C.1.

The optimal monetary policy under discretion is found by taking the first-order condition on i_t, which is given by

$$\pi_t = -\left(\frac{\partial \pi_t}{\partial i_t^*}\right)^{-1} \frac{\partial \hat{y}_t}{\partial i_t^*} \omega \hat{y}_t \equiv= R\hat{y}_t. \tag{B.13}$$

where

$$R = -\left(-\frac{\kappa}{\sigma}+(1-\theta)\frac{\kappa}{\sigma}K_r+\frac{1-\theta}{\theta}K_u\right)^{-1}\left(-\frac{1}{\sigma}\right)\omega. \tag{B.14}$$

Because the equilibrium interest rate positively responds to both shocks to offset their effects, both K_r and K_u are positive. Therefore, $|R| > |R_{no\ info}|$, where R_{full} is the targeted ratio between inflation and the output gap without the informational effect (see Corollary 1.1).

B.5 Proof of Proposition 5

As shown in Proposition 3, the Phillips curve with the informational effect of monetary policy is

$$\pi_t = \left\{\kappa - \sigma\left[(1-\theta)\frac{\kappa}{\sigma}K_r+\frac{1-\theta}{\theta}K_u\right]\right\}\hat{y}_t + \left\{(1-\theta)\frac{\kappa}{\sigma}(K_r-1)+\frac{1-\theta}{\theta}K_u\right\}r_t^n + u_t. \tag{B.15}$$

To move the Phillips curve closer to the origin, which is the dual stabilization equilibrium, the central bank wants to minimize the dampening informational effect on the slope and the on the intercept of the Phillips curve. Minimizing the dampening informational effect on the slope is equivalent to minimizing

$$\sigma\left[(1-\theta)\frac{\kappa}{\sigma}K_r+\frac{1-\theta}{\theta}K_u\right] \tag{B.16}$$

Minimizing the dampening information effect on the intercept after the natural-rate shock is equivalent to minimizing

$$(1-\theta)\frac{\kappa}{\sigma}(K_r-1)+\frac{1-\theta}{\theta}K_u \tag{B.17}$$

Minimizing (B.16) is equivalent to minimizing equation (B.17) if (1) $\sigma > 0$, which is guaranteed, and (2) the intercept after the natural-rate shock is positive (i.e., (B.17) is positive) in the discretionary equilibrium. Since the equilibrium under discretionary optimization is solved as a fixed point, this cannot be proved for all cases. We show that this holds under our calibration, which is illustrated in Figure 4.

C Optimal Monetary Policy in the Baseline Case

This section explains the solution method for the static case when the central bank optimizes under discretion (Section C.1) and under commitment (Section C.2).

C.1 Under Discretion

The objective function of a central bank is to minimize the sum of the squared output gap and squared inflation for all periods. Due to the static nature of this benchmark model, the objective function of the discretionary central bank reduces to minimizing current deviations, which is given by:

$$min_{i_t} L(t) = \begin{bmatrix} \pi_t & \hat{y}_t \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \pi_t \\ \hat{y}_t \end{bmatrix} + indept.\ terms \tag{C.1}$$

subject to

$$\hat{y}_t = -\frac{1}{\sigma}\left(i_t - r_t^n\right) \tag{C.2}$$

$$\pi_t = \kappa \hat{y}_t + (1-\theta)\frac{\kappa}{\sigma}\left(\mathbb{E}_t^s r_t^n - r_t^n\right) + \frac{1-\theta}{\theta}\mathbb{E}_t^s u_t + u_t \tag{C.3}$$

$$\mathbb{E}_t^s r_t^n = K_r\left(F_r^e, F_u^e\right) i_t \tag{C.4}$$

$$\mathbb{E}_t^s u_t = K_u\left(F_r^e, F_u^e\right) i_t \tag{C.5}$$

The solution of the optimal interest rate is a fixed point equilibrium of the following iteration.

1. I conjecture that the interest rate reacts linearly to both shocks.

2. With the conjectured interest rate response function, I solve for the expected natural-rate shock and the expected cost-push shock formed by Kalman filtering.

3. With the expected shocks written as a function of the interest rate, I now express $\hat{y}_t$ and π_t as expected shocks (functions of i_t), actual shocks, and the interest rate.

4. Solve for the optimal interest rate that minimizes the central bank's loss function under the constraint and express the interest rate as a function of actual shocks, i.e.,

$i_t = F_r r_t^n + F_u u_t.$

5. Check if $F_r = F_r^0$ and $F_u = F_u^0$. If not, go back to step 1 and update the values of F_r^0 and F_u^0 in the conjectured function.

6. Iterate the process until convergence.

Details are given as follows:

In step 1, $i_t = F_r^0 r_t^n + F_u^0 u_t$.

In step 2, the expected shocks are solved by the Kalman filtering process, which are given by

$$\mathbb{E}_t^s r_t^n = K_r i_t \tag{C.6}$$
$$\mathbb{E}_t^s u_t = K_u i_t \tag{C.7}$$

where $K_r F_r^0 = \frac{F_r^{02}\sigma_r^2}{F_r^{02}\sigma_r^2 + F_u^{02}\sigma_u^2}$, and $K_u F_u^0 = \frac{F_u^{02}\sigma_u^2}{F_r^{02}\sigma_r^2 + F_u^{02}\sigma_u^2}$.

In step 3, substitute the expected shocks in the output gap and inflation with their expressions in step 2, which yields

$$\hat{y}_t = -\frac{1}{\sigma}\left(i_t - r_t^n\right) \tag{C.8}$$
$$\pi_t = \kappa \hat{y}_t + (1-\theta)\frac{\kappa}{\sigma}\left(\mathbb{E}_t^s r_t^n(i_t) - r_t^n\right) + \frac{1-\theta}{\theta}\mathbb{E}_t^s u_t(i_t) + u_t \tag{C.9}$$

In step 4, first write out the first-order condition of the optimal interest rate, which is given by

$$\pi_t \frac{\partial \pi_t}{\partial i_t} + \omega \hat{y}_t \frac{\partial \hat{y}_t}{\partial i_t} = 0. \tag{C.10}$$

where the partial derivatives are given by

$$\frac{\partial \hat{y}_t}{\partial i_t} = -\frac{1}{\sigma}$$
$$\frac{\partial \pi_t}{\partial i_t} = -\frac{\kappa}{\sigma} + (1-\theta)\frac{\kappa}{\sigma}K_r + \frac{1-\theta}{\theta}K_u$$

Substituting the partial derivatives to the first-order condition leads to the expression of the targeted ratio between inflation and the output gap:

$$R = -\left(-\frac{\kappa}{\sigma} + (1-\theta)\frac{\kappa}{\sigma}K_r + \frac{1-\theta}{\theta}K_u\right)^{-1}\left(-\frac{1}{\sigma}\right)\omega. \tag{C.11}$$

To further solve for the equilibrium interest rate, substitute π_t and $\hat{y}_t$ with equation (C.8) and equation (C.9):

$$\left\{(1-\theta)\frac{\kappa}{\sigma}\left(\mathbb{E}_t^s r_t^n - r_t^n\right) + \frac{1-\theta}{\theta}\mathbb{E}_t^s u_t + u_t\right\}\frac{\partial \pi_t}{\partial i_t} + \left(\omega\frac{\partial \hat{y}_t}{\partial i_t} + \kappa\frac{\partial \pi_t}{\partial i_t}\right)\left\{-\frac{1}{\sigma}\left(i_t - r_t^n\right)\right\} = 0 \quad \text{(C.12)}$$

Substitute $E_t^s r_t^n$ and $E_t^s u_t$ as i_t with equation (C.6) and equation (C.7), which yields

$$\lambda_1 r_t^n + \lambda_2 u_t + \lambda_3 i_t = 0 \qquad \text{(C.13)}$$

where

$$\begin{aligned}
\lambda_1 &= \left\{\left(\kappa\frac{\partial \pi_t}{\partial i_t} + \omega\frac{\partial \hat{y}_t}{\partial i_t}\right)\frac{1}{\sigma} - \frac{\partial \pi_t}{\partial i_t}(1-\theta)\frac{\kappa}{\sigma}\right\} \\
\lambda_2 &= \frac{\partial \pi_t}{\partial i_t} \\
\lambda_3 &= \frac{\partial \pi_t}{\partial i_t}(1-\theta)\frac{\kappa}{\sigma}K_{11} + \frac{\partial \pi_t}{\partial i_t}\frac{1-\theta}{\theta}K_{21} - \left(\kappa\frac{\partial \pi_t}{\partial i_t} + \omega\frac{\partial \hat{y}_t}{\partial i_t}\right)\frac{1}{\sigma}
\end{aligned}$$

Rearrange the above equation to get:

$$i_t = F_1 r_t^n + F_3 u_t \qquad \text{(C.14)}$$

where $F_1 = -\frac{\lambda_1}{\lambda_3}$, and $F_3 = -\frac{\lambda_2}{\lambda_3}$.

In step 5, update the initial conjectured policy function and iterate the above process until $F_r = F_r^0$ and $F_u = F_u^0$.

C.2 Under Commitment

The central bank under commitment chooses the interest rate response function, $i_t = f\left(r_t^n,\ u_t\right)$ ex-ante to minimize the ex-ante loss function of the central bank. Given the linear-quadratic nature of the optimization problem, I consider a linear interest rate rule, and in this case, the central bank's optimization problem is given by

$$min \int\int \pi_t^2(r_t,\ u_t) + \omega \hat{y}_t^2(r_t,\ u_t) dr_t^n du_t, \qquad \text{(C.15)}$$

subject to

$$i_t = F_r^c r_t^n + F_u^c u_t, \tag{C.16}$$

$$\hat{y}_t = -\frac{1}{\sigma}\left(i_t - r_t^n\right) \tag{C.17}$$

$$\pi_t = \kappa \hat{y}_t + (1-\theta)\frac{\kappa}{\sigma}\left(\mathbb{E}_t^s r_t^n - r_t^n\right) + \frac{1-\theta}{\theta}\mathbb{E}_t^s u_t + u_t \tag{C.18}$$

$$\mathbb{E}_t^s r_t^n = K_r(F_r^c,\ F_u^c) F_r^c r_t^n + K_r(F_r^c,\ F_u^c) F_u^c u_t, \tag{C.19}$$

$$\mathbb{E}_t^s u_t = K_u(F_r^c,\ F_u^c) F_u^c r_t^n + K_u(F_r^c,\ F_u^c) F_u^c u_t. \tag{C.20}$$

The optimization problem reduces to searching for (F_r, F_u) that minimizes the ex-ante loss function.

D Optimal Monetary Policy with Dynamic Informational Effect

In this section, I solve for the general version of the model where I have serially correlated shocks, external signals and exogenous monetary policy shocks. Section 4 can be regarded as a special case where the precision of external signals are zero. Section 5.1 can be regarded as a special case where the serial correlation in actual shocks is zero.

The equilibrium in the private sector is described by the system of equations summarizing private-sector optimization decisions in aggregate variables (equations (10) and (14)), the evolution process of the actual shocks (equation (36)), the interest rate reaction function (equation (37)), and the beliefs-updating process characterized in equation (41). As both the household and firms are forward-looking, current equilibrium variables depend on expected future equilibrium variables. I use the method of undetermined coefficients to solve expectations of future equilibrium variables.

The solution method of the equilibrium interest rate under discretion is similar to that for the baseline case, in which I first conjecture the reaction function for the interest rate, and then find the fixed point between the initial guess and the interest rate found in the central bank's optimization problem. The summary of the iteration process is given as follows:

1. I conjecture that the interest rate reacts linearly to predetermined state variables, which include current actual shocks and beliefs in the last period, i.e., $i_t = F_1^0 r_t^n + F_2^0 \mathbb{E}_{t-1}^s r_{t-1}^n + F_3^0 u_t + F_4^0 \mathbb{E}_{t-1}^s u_{t-1}$.

2. With this interest rate, I solve for the expected natural-rate shock and the expected cost-push shock.

3. (Undetermined Coefficient) I conjecture that the current output gap and current inflation are linear functions of current state variables, which include actual shocks and past beliefs. This conjecture allows me to express the expected future output gap and inflation as functions of $[\mathbb{E}_t^s r_t^n,\ \mathbb{E}_t^s u_t]$.

4. I solve for $\hat{y}_t$ and π_t as a function of i_t and other predetermined state variables.

5. Solve for the optimal interest rate that minimizes the loss function and express the interest rate in terms of predetermined state variables.

6. Check if the optimal interest rate is different from the initial conjecture. If not, iterate the process until convergence.

For the optimal commitment policy, I assume the optimal policy rule takes the same functional form as the optimal discretionary policy, i.e., $i_t = F_1^c r_t^n + F_2^c \mathbb{E}_{t-1}^s r_{t-1}^n + F_3^c u_t + F_4^c \mathbb{E}_{t-1}^s u_{t-1}$. Then, using this policy rule, I solve for the expected shocks. With the expected shocks, I solve the equilibrium using the undetermined coefficient method. Last, I search for the policy rule that maximizes the ex-ante objective function of the central bank.

In the rest of this section, I only show the solution method for the discretionary policy. Details are given as follows:

In step 1, I conjecture that $i_t = F_1^0 r_t^n + F_2^0 \mathbb{E}_{t-1}^s r_{t-1}^n + F_3^0 u_t + F_4^0 \mathbb{E}_{t-1}^s u_{t-1}$.

In step 2, the evolution process of actual shocks is given by

$$z_t = \Phi z_{t-1} + v_t \tag{D.1}$$

where $z_t = [r_t^n,\ u_t]'$, $\Phi = \begin{bmatrix} \phi & 0 \\ 0 & \phi^u \end{bmatrix}$ and $v_t = [v_t, v_t^u]$ with the white noise of variance Q.

The set of signals contains the fraction of the interest rate that reacts to current shocks, which is given by

$$\hat{i}_t \equiv i_t - F_2^0 \mathbb{E}_{t-1}^s r_{t-1}^n - F_4^0 \mathbb{E}_{t-1}^s u_{t-1} \tag{D.2}$$

Other signals capture information from central bank direct communication or any other sources of information that are independent of the interest rate. The set of signals is given by

$$\begin{bmatrix} \hat{i}_t \\ m_t^r \\ m_t^u \end{bmatrix} = \begin{bmatrix} F_1^0 & F_3^0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_t^n \\ u_t \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_t \\ \epsilon_t^r \\ \epsilon_t^u \end{bmatrix}, \tag{D.3}$$

which I denote as $s_t = Dz_t + R_t$.

Agents in the private sector are Bayesian, and update their beliefs by the Kalman filtering process, in which they optimally weigh between all current signals and past beliefs. The beliefs follow:

$$\begin{bmatrix} \mathbb{E}_t^s r_t^n \\ \mathbb{E}_t^s u_t \end{bmatrix} = \begin{bmatrix} \phi & 0 \\ 0 & \phi^u \end{bmatrix} \begin{bmatrix} \mathbb{E}_{t-1}^s r_{t-1}^n \\ \mathbb{E}_{t-1}^s u_{t-1} \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \end{bmatrix} \left(\begin{bmatrix} \hat{i}_t \\ m_t^r \\ m_t^u \end{bmatrix} - \begin{bmatrix} F_1^0 & F_3^0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \phi & 0 \\ 0 & \phi^u \end{bmatrix} \begin{bmatrix} \mathbb{E}_{t-1}^s r_{t-1}^n \\ \mathbb{E}_{t-1}^s u_{t-1} \end{bmatrix} \right) \tag{D.4}$$

Write out the expression for $\hat{i}_t$ and collect terms:

$$\begin{aligned} \mathbb{E}_t^s r_t^n = &\left(K_{11}F_1^0 + K_{12}\right) r_t^n + \phi \left(1 - K_{11}F_1^0 - K_{12}\right) \mathbb{E}_{t-1}^s r_{t-1}^n \qquad \text{(D.5)} \\ &+ \left(K_{11}F_3^0 + K_{13}\right) u_t + \phi^u \left(-K_{11}F_3^0 - K_{13}\right) \mathbb{E}_{t-1}^s u_{t-1} + K_{12}\epsilon_t^r + K_{13}\epsilon_t^u + K_{11}e_t \\ \mathbb{E}_t^s u_t = &\left(K_{21}F_1^0 + K_{22}\right) r_t^n + \phi \left(-K_{21}F_1^0 - K_{22}\right) \mathbb{E}_{t-1}^s r_{t-1}^n \qquad \text{(D.6)} \\ &+ \left(K_{21}F_3^0 + K_{23}\right) u_t + \phi^u \left(1 - K_{21}F_3^0 - K_{23}\right) \mathbb{E}_{t-1}^s u_{t-1} + K_{22}\epsilon_t^r + K_{23}\epsilon_t^u + K_{21}e_t \end{aligned}$$

Denote the above equations as

$$\mathbb{E}_t^s r_t^n = \Psi(1) r_t^n + \Psi(2)\, \mathbb{E}_{t-1}^s r_{t-1}^n + \Psi(3) u_t + \Psi(4)\, \mathbb{E}_{t-1}^s u_{t-1} + \Psi(5)\epsilon_t^r + \Psi(6)\epsilon_t^u + \Psi(7) e_t, \tag{D.7}$$

$$\mathbb{E}_t^s u_t = \Psi(8) r_t^n + \Psi(9)\, \mathbb{E}_{t-1}^s r_{t-1}^n + \Psi(10) u_t + \Psi(11)\, \mathbb{E}_{t-1}^s u_{t-1} + \Psi(12)\epsilon_t^r + \Psi(13)\epsilon_t^u + \Psi(14) e_t \tag{D.8}$$

In step 3, I conjecture that equilibrium variables are linear functions of current state variables, which include current actual shocks (r_t^n, u_t), past beliefs $\left(\mathbb{E}_{t-1}^s r_{t-1}^n,\ \mathbb{E}_{t-1}^s u_{t-1}\right)$, and noise in current signals (ϵ_t^r, ϵ_t^u, e_t).

$$\begin{bmatrix} \hat{y}_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 \\ \gamma_8 & \gamma_9 & \gamma_{10} & \gamma_{11} \end{bmatrix} \begin{bmatrix} r_t^n \\ \mathbb{E}_{t-1}^s r_{t-1}^n \\ u_t \\ \mathbb{E}_{t-1}^s u_{t-1} \end{bmatrix} + \begin{bmatrix} \gamma_5 & \gamma_6 & \gamma_7 \\ \gamma_{12} & \gamma_{13} & \gamma_{14} \end{bmatrix} \begin{bmatrix} \epsilon_t^r \\ \epsilon_t^u \\ e_t \end{bmatrix} \tag{D.9}$$

Under this conjecture, the expectations of future equilibrium variables, $\mathbb{E}_t \hat{y}_{t+1}$ and $\mathbb{E}_t^s \pi_{t+1}$ are given by

$$\begin{bmatrix} \mathbb{E}_t^s \hat{y}_{t+1} \\ \mathbb{E}_t^s \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \gamma_1\phi + \gamma_2 & \gamma_3\phi^u + \gamma_4 \\ \gamma_8\phi + \gamma_9 & \gamma_{10}\phi^u + \gamma_{11} \end{bmatrix} \begin{bmatrix} \mathbb{E}_t^s r_t^n \\ \mathbb{E}_t^s u_t \end{bmatrix} \tag{D.10}$$

Substitute $\mathbb{E}_t^s \hat{y}_{t+1}$ and $\mathbb{E}_t^s \pi_{t+1}$ in $\hat{y}_t$ with equation (D.9) and get

$$\begin{aligned}\hat{y}_t = &\left[(\gamma_1\phi+\gamma_2)+\frac{1}{\sigma}(\gamma_8\phi+\gamma_9)-\frac{1}{\sigma}\frac{\phi}{1-\phi}\right]\mathbb{E}_t^s r_t^n \\ &+\left[(\gamma_3\phi^u+\gamma_4)+\frac{1}{\sigma}(\gamma_{10}\phi^u+\gamma_{11})\right]\mathbb{E}_t^s u_t-\frac{1}{\sigma}i_t+\frac{1}{\sigma}\frac{1}{1-\phi}r_t^n\end{aligned} \tag{D.11}$$

To express π_t in terms of predetermined state variables, first write out the expressions for $\mathbb{E}_t^s \hat{y}_{t+1}$ and $\mathbb{E}_t^s \pi_{t+1}$, which are given by

$$\mathbb{E}_t^s \hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\sigma}\left[i_t - \mathbb{E}_t^s r_t^n - \mathbb{E}_t^s \pi_{t+1}\right] \tag{D.12}$$

$$\mathbb{E}_t^s \pi_t = \beta\,\mathbb{E}_t^s \pi_{t+1} + \kappa\left\{\mathbb{E}_t^s \hat{y}_{t+1} - \frac{1}{\sigma}\left[i_t - \mathbb{E}_t^s r_t^n - \mathbb{E}_t^s \pi_{t+1}\right]\right\} + \frac{1}{\theta}\,\mathbb{E}_t^s u_t \tag{D.13}$$

Then, substitute $\mathbb{E}_t^s \hat{y}_{t+1}$ and $\mathbb{E}_t^s \pi_{t+1}$ $\mathbb{E}_t^s \pi_t$ in π_t with equation (D.9) and get

$$\begin{aligned}\pi_t = &\beta\theta\,\mathbb{E}_t^s \pi_{t+1} + (1-\theta)\left\{\beta\,\mathbb{E}_t^s \pi_{t+1} + \kappa\,\mathbb{E}_t^s \hat{y}_t + \frac{1}{\theta}\,\mathbb{E}_t^s u_t\right\} + \kappa\theta\hat{y}_t + u_t \\ &+\left\{(1-\theta)\kappa(\gamma_3\phi^u+\gamma_4)+\frac{1-\theta}{\theta}+\left(\beta+(1-\theta)\frac{\kappa}{\sigma}\right)(\gamma_{10}\phi^u+\gamma_{11})\right\}\mathbb{E}_t^s u_t-(1-\theta)\frac{\kappa}{\sigma}i_t+\kappa\theta\hat{y}_t+u_t\end{aligned} \tag{D.14}$$

For both $\hat{y}_t$ (equation (D.11)) and π_t (equation (D.14)), substitute the expected future equilibrium variables with their conjectured law of motion (equation (D.10)) and then substitute the expected shocks according to the beliefs formation process (equation (D.7)). Then, matching the coefficients of $\hat{y}_t$ and π_t with their conjectures given by equation (D.7) yields the solution for $\hat{y}_t$ and π_t.

In step 5, the central bank's loss function includes the squared output gap and inflation in the current and all future periods, which is given by

$$\mathbb{E}_t L(t) = [\pi_t^2 + \omega\hat{y}_t^2] + \beta\,\mathbb{E}_t(L(t+1)) \tag{D.15}$$

where

$$\begin{aligned}\mathbb{E}_t(L(t+1)) &= \Sigma_{j=1}^{\infty}\beta^j\,\mathbb{E}_t\left\{\begin{bmatrix}\pi_{t+1} & \hat{y}_{t+j}\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & \omega\end{bmatrix}\begin{bmatrix}\pi_{t+j}\\ \hat{y}_{t+j}\end{bmatrix}\right\} \\ &= \Sigma_{j=1}^{\infty}\beta^j\left\{\begin{bmatrix}\mathbb{E}_t \pi_{t+1} & \mathbb{E}_t \hat{y}_{t+j}\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & \omega\end{bmatrix}\begin{bmatrix}\mathbb{E}_t \pi_{t+j}\\ \mathbb{E}_t \hat{y}_{t+j}\end{bmatrix} + \textit{indept. terms}\right\}\end{aligned} \tag{D.16}$$

Let $z_t = \left[r_t^n,\ \mathbb{E}_{t-1}^s r_{t-1}^n, u_t,\ \mathbb{E}_{t-1}^s u_{t-1}\right]'$ denote the persistent state variables. The central bank's expectations for the future output gap and inflation become linear functions of $E_t z_{t+j}$:

$$\begin{bmatrix} \mathbb{E}_t\,\pi_{t+j} \\ \mathbb{E}_t\,\hat{y}_{t+j} \end{bmatrix} = \begin{bmatrix} \gamma_8 & \gamma_9 & \gamma_{10} & \gamma_{11} \\ \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 \end{bmatrix} \mathbb{E}_t\,z_{t+j} \equiv \Gamma\,\mathbb{E}_t\,z_{t+j} \tag{D.17}$$

$E_t z_{t+j}$ follows:

$$\begin{bmatrix} \mathbb{E}_t\,r^n_{t+j} \\ \mathbb{E}_t\,\mathbb{E}^s_{t+j-1}\,r^n_{t+j-1} \\ \mathbb{E}_t\,u_{t+j} \\ \mathbb{E}_t\,\mathbb{E}^s_{t+j-1}\,u_{t+j-1} \end{bmatrix} = \tag{D.18}$$

$$\begin{bmatrix} \phi & 0 & 0 & 0 \\ K_{11}F^0_1 + K_{12} & \phi(1 - K_{11}F^0_1 - K_{12}) & K_{11}F^0_3 + K_{13} & -\phi^u(K_{11}F^0_3 + K_{13}) \\ 0 & 0 & \phi^u & 0 \\ K_{21}F^0_1 + K_{22} & -\phi(K_{21}F^0_1 + K_{22}) & K_{21}F^0_3 + K_{23} & \phi^u(1 - K_{21}F^0_3 - K_{23}) \end{bmatrix} \begin{bmatrix} \mathbb{E}_t\,r^n_{t+j-1} \\ \mathbb{E}_t\,\mathbb{E}^s_{t+j-2}\,r^n_{t+j-2} \\ \mathbb{E}_t\,u_{t+j-1} \\ \mathbb{E}_t\,\mathbb{E}^s_{t+j-2}\,u_{t+j-2} \end{bmatrix}$$

where the second row and the fourth row are results from the Kalman filtering process.

Denote equation (D.18) by $\mathbb{E}_t\,z_{t+j} = \Lambda\,\mathbb{E}_t\,z_{t+j-1}$. The central bank's expectations of future equilibrium variables can now be written as:

$$\begin{bmatrix} \mathbb{E}_t\,\pi_{t+j} \\ \mathbb{E}_t\,\hat{y}_{t+j} \end{bmatrix} = \Gamma\Lambda^{j-1}\,\mathbb{E}_t\,z_{t+1} \tag{D.19}$$

Substituting the expected future equilibrium variables with this expression, the central bank's expected loss at $t+1$ becomes

$$\mathbb{E}_t(L(t+1)) = \Sigma\beta^j\,\mathbb{E}_t\,z'_{t+1}(\Lambda^{j-1})'\Gamma'\Omega\Gamma\Lambda^{j-1}\,\mathbb{E}_t\,z_{t+1} \equiv \Sigma\beta^j\,\mathbb{E}_t\,z'_{t+1}\Theta_{j-1}\,\mathbb{E}_t\,z_{t+1} \tag{D.20}$$

The first-order condition on i^*_t can be written as

$$\left\{ \frac{\partial\,\mathbb{E}_t\,\pi_t}{\partial i^*_t}\,\mathbb{E}_t\,\pi_t + \omega\frac{\partial\,\mathbb{E}_t\,\hat{y}_t}{\partial i^*_t}\,\mathbb{E}_t\,\hat{y}_t \right\} + \frac{1}{2}\Sigma^{\infty}_{j=1}\beta^j\Delta(j-1) = 0 \tag{D.21}$$

where

$$
\begin{aligned}
\Delta_{j-1} = &(\Theta^{21}_{j-1}+\Theta^{12}_{j-1})\phi r^n_t \frac{\partial \mathbb{E}^s_t r^n_t}{\partial i_t} + (\Theta^{32}_{j-1}+\Theta^{23}_{j-1})\phi^u u_t \frac{\partial \mathbb{E}^s_t r^n_t}{\partial i_t} + (\Theta^{42}_{j-1}+\Theta^{24}_{j-1})\,\mathbb{E}^s_t u_t \frac{\partial \mathbb{E}^s_t r^n_t}{\partial i_t} \\
&+ \Theta^{22}_{j-1}\cdot 2\,\mathbb{E}^s_t r^n_t \frac{\partial \mathbb{E}^s_t r^n_t}{\partial i_t} + (\theta^{41}_{j-1}+\Theta^{14}_{j-1})\phi r^n_t \frac{\partial \mathbb{E}^s_t u_t}{\partial i_t} + (\Theta^{43}_{j-1}+\Theta^{34}_{j-1})\phi^u u_t \frac{\partial \mathbb{E}^s_t u_t}{\partial i_t} \\
&+ (\Theta^{42}_{j-1}+\Theta^{24}_{j-1})\,\mathbb{E}^s_t r^n_t \frac{\partial \mathbb{E}^s_t u_t}{\partial i_t} + \Theta^{44}_{j-1}\cdot 2\,\mathbb{E}^s_t u_t \frac{\partial \mathbb{E}^s_t u_t}{\partial i_t} \\
&\equiv \Delta_{j-1}(1) r^n_t + \Delta_{j-1}(2) u_t + \Delta_{j-1}(3)\,\mathbb{E}^s_t u_t + \Delta_{j-1}(4)\,\mathbb{E}^s_t r^n_t \\
&+ \Delta_{j-1}(5) r^n_t + \Delta_{j-1}(6) u_t + \Delta_{j-1}(7)\,\mathbb{E}^s_t r^n_t + \Delta_{j-1}(8)\,\mathbb{E}^s_t u_t
\end{aligned}
$$

To solve for the optimal interest rate, first write out $\hat{y}_t$, π_t, $\mathbb{E}^s_t r^n_t$ and $\mathbb{E}^s_t u_t$ as expected shocks:

$$
\begin{aligned}
\mathbb{E}^s_t r^n_t = &\Big(\phi(1-K_{11}F^0_1-K_{12}) - K_{11}F^0_2\Big)\,\mathbb{E}^s_{t-1} r^n_{t-1} \\
&- \Big(K_{11}F^0_4 + \phi^u(K_{11}F^0_3+K_{13})\Big)\,\mathbb{E}^s_{t-1} u_{t-1} + K_{12} r^n_t + K_{13} u_t + K_{11} i_t \qquad \text{(D.22)} \\
\mathbb{E}^s_t u_t = &\Big(\phi^u(1-K_{21}F^0_3-K_{23}) - K_{21}F^0_4\Big)\,\mathbb{E}^s_{t-1} u_{t-1} \\
&- \Big(\phi(K_{21}F^0_1+K_{22}) + K_{21}F^0_2\Big)\,\mathbb{E}^s_{t-1} r^n_{t-1} + K_{22} r^n_t + K_{23} u_t + K_{21} i_t \qquad \text{(D.23)}
\end{aligned}
$$

Output gap:

$$\hat{y}_t = \Xi(1)\,\mathbb{E}^s_t r^n_t + \Xi(2)\,\mathbb{E}^s_t u_t - \frac{1}{\sigma} i_t + \frac{1}{\sigma}\frac{1}{1-\phi} r^n_t \qquad \text{(D.24)}$$

Inflation:

$$\pi_t = \kappa\theta\hat{y}_t + \Xi(3)\,\mathbb{E}^s_t r^n_t + \Xi(4)\,\mathbb{E}^s_t u_t - (1-\theta)\frac{\kappa}{\sigma} i_t + u_t \qquad \text{(D.25)}$$

Substitute $\hat{y}_t$ and π_t with the above equations in the first-order condition, which yields

$$\lambda_1\,\mathbb{E}^s_t r^n_t + \lambda_2\,\mathbb{E}^s_t u_t + \lambda_3 r^n_t + \lambda_4 u_t + \lambda_5 i_t = 0 \qquad \text{(D.26)}$$

where

$$\lambda_1 = \left(\kappa\theta\frac{\partial \pi_t}{\partial i_t} + \omega\frac{\partial \hat{y}_t}{\partial i_t}\right)\Xi(1) + \frac{\partial \pi_t}{\partial i_t}\Xi(3) + \frac{1}{2}\Sigma\beta^j\left(\Delta_{j-1}(4) + \Delta(7)\right) \tag{D.27}$$

$$\lambda_2 = \left(\kappa\theta\frac{\partial \pi_t}{\partial i_t} + \omega\frac{\partial \hat{y}_t}{\partial i_t}\right)\Xi(2) + \frac{\partial \pi_t}{\partial i_t}\Xi(4) + \frac{1}{2}\Sigma\beta^j\left(\Delta_{j-1}(3) + \Delta(8)\right) \tag{D.28}$$

$$\lambda_3 = \left(\kappa\theta\frac{\partial \pi_t}{\partial i_t} + \omega\frac{\partial \hat{y}_t}{\partial i_t}\right)\frac{1}{\sigma}\frac{1}{1-\phi} + \frac{1}{2}\Sigma\beta^j\left(\Delta_{j-1}(1) + \Delta(5)\right) \tag{D.29}$$

$$\lambda_4 = \frac{\partial \pi_t}{\partial i_t} + \frac{1}{2}\Sigma\beta^j\left(\Delta(2) + \Delta(6)\right) \tag{D.30}$$

$$\lambda_5 = \left(\kappa\theta\frac{\partial \pi_t}{\partial i_t} + \omega\frac{\partial \hat{y}_t}{\partial i_t}\right)\left(-\frac{1}{\sigma}\right) + \frac{\partial \pi_t}{\partial i_t}\left(-(1-\theta)\frac{\kappa}{\theta}\frac{1}{\sigma}\right) \tag{D.31}$$

and partial derivatives are derived as:

$$\frac{\partial\, \mathbb{E}_t^s r_t^n}{\partial i_t} = K_{11} \tag{D.32}$$

$$\frac{\partial\, \mathbb{E}_t^s u_t}{\partial i_t} = K_{21} \tag{D.33}$$

$$\frac{\partial \hat{y}_t}{\partial i_t} = \Xi(1)\frac{\partial\, \mathbb{E}_t^s r_t^n}{\partial i_t} + \Xi(2)\frac{\partial\, \mathbb{E}_t^s u_t}{\partial i_t} - \frac{1}{\sigma} \tag{D.34}$$

$$\frac{\partial \pi_t}{\partial i_t} = \kappa\theta\frac{\partial \hat{y}_t}{\partial i_t} + \Xi(3)\frac{\partial\, \mathbb{E}_t^s r_t^n}{\partial i_t} + \Xi(4)\frac{\partial\, \mathbb{E}_t^s u_t}{\partial i_t} - (1-\theta)\frac{\kappa}{\sigma} \tag{D.35}$$

Further substitute $\mathbb{E}_t^s r_t^n$ with equation (D.22) and $\mathbb{E}_t^s u_t$ with equation (D.23), which turns the first-order condition into

$$\begin{aligned}
0 = \lambda_1\{&\left(\phi(1 - K_{11}F_1^0 - K_{12}) - K_{11}F_2^0\right)\mathbb{E}_{t-1}^s r_{t-1}^n - \left(K_{11}F_4^0 + \phi^u(K_{11}F_3^0 + K_{13})\right)\mathbb{E}_{t-1}^s u_{t-1} \\
&+ K_{12}r_t^n + K_{13}u_t + K_{11}i_t\} \\
&+ \lambda_2\{\left(\phi^u(1 - K_{21}F_3^0 - K_{23}) - K_{21}F_4^0\right)\mathbb{E}_{t-1}^s u_{t-1} - \left(\phi(K_{21}F_1^0 + K_{22}) + K_{21}F_2^0\right)\mathbb{E}_{t-1}^s r_{t-1}^n \\
&+ K_{22}r_t^n + K_{23}u_t + K_{21}i_t\} \\
&+ \lambda_3 r_t^n + \lambda_4 u_t + \lambda_5 i_t
\end{aligned}$$

The optimal interest rate is solved to be

$$i_t = F_1 r_t^n + F_2\,\mathbb{E}_{t-1}^s r_{t-1}^n + F_3 u_t + F_4\,\mathbb{E}_{t-1}^s u_{t-1} \tag{D.36}$$

where

$$F_1 = -\frac{\lambda_1 K_{12} + \lambda_2 K_{22} + \lambda_3}{\lambda_1 K_{11} + \lambda_2 K_{21} + \lambda_5} \tag{D.37}$$

$$F_2 = -\frac{\lambda_1 \left(\phi(1 - K_{11}F_1^0 - K_{12}) - K_{12}F_2^0\right) - \lambda_2 \left(\phi(K_{21}F_1^0 + K_{22}) + K_{21}F_2^0\right)}{\lambda_1 K_{11} + \lambda_2 K_{21} + \lambda_5} \tag{D.38}$$

$$F_3 = -\frac{\lambda_1 K_{13} + \lambda_2 K_{23} + \lambda_4}{\lambda_1 K_{11} + \lambda_2 K_{21} + \lambda_5} \tag{D.39}$$

$$F_4 = -\frac{-\lambda_1 \left(K_{11}F_4^0 + \phi^u(K_{11}F_3^0 + K_{13})\right) + \lambda_2 \left(\phi^u(1 - K_{21}F_3^0 - K_{23}) - K_{21}F_4^0\right)}{\lambda_1 K_{11} + \lambda_2 K_{21} + \lambda_5} \tag{D.40}$$

In step 6, I check if $[F_1, F_2, F_3, F_4] = \left[F_1^0, F_2^0, F_3^0, F_4^0\right]$. If not, iterate the process until convergence.

E Calibration

I adopt parameter values that are aligned with the traditional New Keynesian literature. Specifically, I set $\varphi = 1$ and $\sigma = 1$, assuming a unitary Frisch elasticity of labor supply and log utility of consumption. I use $\beta = 0.99$, which implies a steady-state real return on financial assets of 4 percent. For price rigidity, I set θ, the price stickiness parameter, to be 0.5, which is implied by the average price duration from macro and micro empirical evidence.[24] For the parameter that governs the elasticity of substitution between intermediate goods, I set $\epsilon = 4$, which implies a steady-state price markup of one-third of revenue. For the evolution of underlying shocks, I set the auto-correlation of natural-rate shocks to be 0.9, with a standard deviation of 3 percent, as measured by Laubach and Williams (2003). There is less consensus on the persistence and volatility of cost-push shocks, as they stem from various sources, and I set the auto-correlation for cost-push shocks to be 0.4.

F The Model-Predicted Slope of the Phillips Curve

The expected output growth is the sum of the change in the expected output gap and the change in the expected natural level of output, which is given by

$$\Delta \mathbb{E}_t^s y_{t+1} = (\mathbb{E}_t^s \hat{y}_{t+1} - \mathbb{E}_t^s \hat{y}_t) + \left(\mathbb{E}_t^s y_{t+1}^n - \mathbb{E}_t^s y_t^n\right) \tag{F.1}$$

[24]Sources: Bils and Klenow (2004), Galí and Gertler (1999), and Nakamura and Steinsson (2010)

Table 2: Parameters Values

parameter	description	value
φ	elasticity of labor supply	1
σ	elasticity of intertemporal substitution	1
β	discount factor	0.99
θ	price rigidity	0.5
ϵ	the price elasticity of demand	4
ρ_r	auto-correlation of natural-rate shocks	0 (static)/ 0.9 (dynamic)
ρ_u	auto-correlation of cost-push shocks	0 (static)/ 0.4 (dynamic)
σ_r	std. dev. of natural-rate shocks	0.03
σ_u	std. dev. of cost-push shocks	0.03

Substituting $\mathbb{E}_t^s \hat{y}_t = \mathbb{E}_t^s \hat{y}_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t^s r_t^n - \mathbb{E}_t^s \pi_{t+1}\right)$ and $\mathbb{E}_t^s y_{t+1}^n = \phi \, \mathbb{E}_t^s y_t^n$ results in

$$\begin{aligned} \Delta \mathbb{E}_t^s y_{t+1} &= \frac{1}{\sigma}\left(i_t - \mathbb{E}_t^s r_t^n - \mathbb{E}_t^s \pi_{t+1}\right) + (\phi - 1)\, \mathbb{E}_t^s y_t^n \\ &= \frac{1}{\sigma}\left(i_t - \mathbb{E}_t^s \pi_{t+1}\right) \end{aligned} \tag{F.2}$$

where the second line makes use of the definition of r_t^n, i.e., $r_t^n = \sigma(\phi - 1) y_t^n$.

Next, I calculate the changes in expected one-period-ahead inflation after an innovation in the current interest rate, i.e.,

$$\frac{\partial \, \mathbb{E}_t^s \pi_{t+1}}{\partial i_t} = (\gamma_8 \phi + \gamma_9) \frac{\partial \, \mathbb{E}_t^s r_t^n}{\partial i_t} + (\gamma_{10} \phi^u + \gamma_{11}) \frac{\partial \, \mathbb{E}_t^s u_t}{\partial i_t} \tag{F.3}$$

where γ is determined in equilibrium (see equation (D.10).) The derivatives of the expected shocks to the interest rate are

$$\frac{\partial \, \mathbb{E}_t^s r_t^n}{\partial i_t} = K_{11}, \tag{F.4}$$

$$\frac{\partial \, \mathbb{E}_t^s u_t}{\partial i_t} = K_{21}, \tag{F.5}$$

where K_{11} and K_{21} are the Kalman gains associated with the interest rate. (See Appendix D for details.)