Wealth Effects, Price Markups, and the Neo-Fisherian Hypothesis

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Abstract

By introducing Jaimovich-Rebelo (JR) consumption-labor nonseparable preferences into an otherwise standard New Keynesian model, we show that the occurrence of positive comovement between inflation and the nominal interest rate conditional on a nominal shock - the so-called neo-Fisherian hypothesis - depends on the extent of wealth effects in households’ labor supply decisions. Neo-Fisherianism appears more prominent in economic environments with i) weaker wealth effects on labor supply (in particular for Greenwood-Hercowitz-Huffmann preferences where wealth effects are absent), and ii) smaller price-to-wage markups (for which the steady state is less distorted). The stabilizing properties of Taylor rules under JR preferences are scrutinized.

Keywords: Monetary Policy, Neo-Fisherianism, Wealth Effects, Markups

JEL Classifications: E4, E5
1 Introduction

The prolonged episode of close-to-zero nominal interest rates and low inflation observed in the US and other developed economies since 2008 has posed a serious challenge to the conventional wisdom that in order to lift (respectively, curb) short-term inflation central banks should cut (respectively, raise) policy rates.\(^1\) Starting with the work of Schmitt-Grohé and Uribe (2010), the lengthy period of low interest rates and low inflation has lead other prominent economists such as James Bullard, John H. Cochrane, and Stephen Williamson to promote the neo-Fisherian view of “raise interest rates to raise inflation.”\(^2\) While most economists agree on a Fisher equation regulating the long-run positive correlation between inflation and nominal interest rates, a null or even positive short-run relationship between these two variables appears prima facie at odds with the monetary policy transmission channel embedded in the baseline three-equation New Keynesian model.\(^3\) Nevertheless, insightful works by Garin et al. (2018) and Rupert and Sustek (2019) show that, following a transitory nominal shock (either to the inflation target or the policy rate), the baseline New Keynesian model can in fact generate positive comovement between inflation and short-term nominal interest rates when the shock is sufficiently persistent, prices are sufficiently flexible, and the intertemporal elasticity of substitution in consumption is sufficiently large.\(^4\)

Motivated by these findings, we put the New Keynesian paradigm under further scrutiny and assess whether the occurrence of neo-Fisherian behavior depends on the extent of wealth effects in households’ labor supply decisions. For this purpose, we embed a tractable form of the generalized Jaimovich-Rebelo preferences (as in Jaimovich (2008); Jaimovich and Rebelo (2009); henceforth JR) into an otherwise standard New Keynesian model. Our specification nests the diametrically opposed cases of a Greenwood-Hercowitz-Huffmann utility (from Greenwood et al. (1988), henceforth GHH), for which wealth effects are absent, and of a King-Plosser-Rebelo utility (from King et al. (1988), henceforth KPR), for which wealth effects are in full swing, as well as all intermediate cases. Assuming monetary policy takes the form of

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\(^1\)In the US for instance, despite the fact that the federal funds rate was kept within the zero to 25 basis points range between 2008 and 2015, inflation rarely overshot its 2 percent target, and indeed there has been no sign of runaway inflation dynamics following the massive unconventional QE interventions by the Fed. While this is reminiscent of what has been witnessed in Japan since the mid-1990s, Canada has not met a different fate. As documented by Williamson (2019a), the correlation between headline CPI inflation and the overnight interest rate (both in levels and detrended) has been consistently positive since the 1970s.

\(^2\)See Bullard (2010), Cochrane (2018b) and Williamson (2018) for non-technical descriptions of the neo-Fisherian view.

\(^3\)Both Williamson (2019a) and Uribe (2020) document a positive correlation between averages of inflation and nominal interest rates in a large cross-section of countries.

\(^4\)Rupert and Sustek (2019) go further, and provide an in-depth analysis of the mechanics of monetary policy transmission in New Keynesian models with and without capital accumulation.
an instrumental Taylor rule, we unveil analytical conditions yielding a positive equilibrium correlation between inflation and the nominal interest rate following a nominal interest rate shock.

The main takeaway from our work is that neo-Fisherianism is more prominent in environments featuring i) weaker wealth effects on labor supply (hence, closer to a GHH utility specification), and ii) smaller price-to-wage markups (hence, a less distorted steady-state equilibrium). Both channels amplify the intertemporal elasticity of substitution in consumption decisions, and therefore contribute to a larger (more negative) temporary elasticity of inflation to the initial nominal shock, which, through the Taylor rule’s endogenous feedback mechanism, facilitates nominal interest rates moving in the same direction as inflation along the equilibrium path. Relatedly, the effect of a contractionary monetary shock on nominal interest rates is attributed to two opposing channels: there is a i) negative effect operating through a decrease in expectations about future inflation (Fisherian channel), and a ii) positive effect through an increase in the endogenous real rate (real rate channel). A sufficiently high elasticity of intertemporal substitution dampens the real rate channel enough for the Fisherian channel to dominate and therefore for the neo-Fisherian hypothesis to hold. Importantly, we find that the occurrence of neo-Fisherian dynamics is independent of monetary policy parameters and of whether the Taylor rule is contemporaneous or forward-looking.

More specifically, we find that, with GHH preferences, the neo-Fisherian hypothesis always holds, provided labor supply is sufficiently wage-elastic. In this case, marginal costs are procyclical enough to induce a larger deflation following the contractionary nominal shock, even if the intertemporal substitution channel is weak. When that is not the case, a strong incentive to substitute intertemporally becomes crucial, and neo-Fisherianism occurs when market power is sufficiently low. Compared to a KPR utility specification - which behaves very similarly to the MaCurdy-type consumption-labor separable utility commonly used in New Keynesian models, as in Garin et al. (2018) and Rupert and Sustek (2019), for instance - GHH preferences are capable of inducing neo-Fisherian effects even under the assumption of rather rigid prices, low shock persistence, larger (Frisch) elasticity of labor, and stronger risk aversion. As a matter of fact, our analysis identifies a threshold value for the JR-utility parameter indexing the extent of wealth effects on labor supply below which (respectively, above which) the neo-Fisherian hypothesis (respectively, the conventional wisdom) holds. Such a threshold is a strictly decreasing function of the price-to-wage markup: namely, the less distorted the steady-state is, the higher the threshold.
While we remain focused on the relationship between nominal rates and inflation following a monetary shock throughout the main text, in Appendix A.6 we offer, in addition, two alternative narratives that lead to the exact same condition for neo-Fisherianism: namely, one where instead of a monetary shock there is an inflation target shock, and another where - instead of using a Taylor rule - the central bank commits to inflation being at the target every period. If neo-Fisherianism holds, in both cases the central bank would have to increase (respectively, decrease) nominal rates to accommodate a higher (respectively, lower) inflation target. The positive comovement between short-term nominal rates and inflation, in response to a shock to the inflation target, has been found in other insightful works. For instance, Khan and Knotek (2015) find that in a standard New Keynesian model with separable utility preferences that embed habit in consumption and money balances, both inflation and nominal rates fall, following a negative inflation target shock. This result is robust to whether the private sector has full knowledge about the inflation target or is learning about it.\footnote{Relatedly, Schorfheide (2005) shows that an estimated shift to a lower inflation target leads to the nominal rates and inflation comoving under full-information, whereas under learning such dynamics occur with some delay. Furthermore, the full information version of the DSGE model with staggered price and wage contracts in Erceg and Levin (2003) generates a positive relationship between the two variables as well. Under imperfect observability of the inflation target, neo-Fisherian dynamics take over with a lag. Similarly, Lukmanova and Rabitsch (2021) find delayed neo-Fisherian behavior in response to a (persistent) inflation target shock in an estimated DSGE model with imperfect information.}

As an interesting by-product, our study finds that parameterizations yielding neo-Fisherianism may expand the region of policy parameters that guarantee equilibrium determinacy when the monetary authority relies on a contemporaneous Taylor rule. Overall, neo-Fisherian dynamics do not pose an issue with respect to determinacy in a contemporaneous Taylor rule. On the contrary, parameterizations generating neo-Fisherian dynamics induce tighter policy parameter constraints in a forward-looking Taylor rule for the purpose of equilibrium determinacy. We show that the area of the determinacy region drops significantly as we move toward a GHH specification and approach an undistorted steady-state, and even more so as we consider a less elastic labor supply. In the extreme case of a fully undistorted GHH specification, a forward-looking Taylor rule is always conducive to equilibrium indeterminacy, irrespective of labor elasticity.

Related Literature

In addition to the previously mentioned contributions of Garin et al. (2018) and Rupert and Sustek (2019), other works have provided alternative justifications for the neo-Fisherian
hypothesis. Cochrane (2018a) argues that a positive short-run correlation between inflation and nominal interest rates may obtain via a government debt devaluation channel following the fiscal theory of the price level logic (FTPL). At the zero lower bound or under an interest rate peg (a passive Taylor rule), a positive (temporary or permanent) shock to the nominal interest rate reduces the present discounted value of expected fiscal surpluses. Assuming no fiscal adjustment on behalf of the government, inflation jumps up on impact to deflate outstanding public liabilities, and therefore guarantees intertemporal debt sustainability. A similar fiscalist argument appears in Bilbiie (2021) who shows that neo-Fisherian effects are a general feature of a fundamental rational expectations equilibrium under an interest rate peg, and of a passive monetary/active fiscal regime in the baseline New Keynesian model. He further shows that a neo-Fisherian monetary policy is optimal as long as the economy is in a liquidity trap. As argued by Williamson (2019a,b), neo-Fisherianism is not unique to models where money non-neutrality arises from price rigidities but instead applies also to frameworks where limited participation/segmented markets are the source of monetary distortions. Focusing on perfect foresight equilibria, Schmitt-Grohé and Uribe (2010, 2014) propose a neo-Fisherian interest-rate-based policy in order to lift the economy from a confidence-driven liquidity trap. García-Schmidt and Woodford (2019) show that departing from rational expectations and instead considering “reflective equilibria” can eliminate neo-Fisherianism because the latter mitigate the forward-looking nature of the New Keynesian model. Using both a Bayesian VAR and a fully fledged DSGE model estimated on US data, Uribe (2020) provides convincing evidence on the existence and quantitative importance of neo-Fisherianism, which he defines as the short-run positive correlation between inflation and nominal interest rates following a permanent shock.

Our work is also related to the literature that studies the macroeconomic implications of the extent of wealth effects on labor supply. In their seminal contribution, Jaimovich and Rebelo (2009) show that weak wealth effects help reconcile the comovement between aggregate quantities predicted by frictionless real business cycle models with that documented by the empirical literature, particularly that concerning the positive correlation between consumption and labor following positive news about future total factor productivity. A JR utility specification is assumed in Schmitt-Grohé and Uribe (2012) in their attempt to quantify the role of news shocks in business cycle fluctuations through the lens of an estimated RBC-type model.

\footnote{As argued by the same author, by a similar logic, the inflation response would be tamed (and could even become negative) if part of the outstanding public liabilities had longer maturity, as the interest rate hike would put downward pressure on long-term bond prices.}
Their analysis finds empirical support for a GHH utility specification. Works by Monacelli and Perotti (2008) and Bilbiie (2011) show that a GHH utility can yield a positive consumption multiplier for government spending. Focusing on a fully fledged New Keynesian model, Gali and Wouters (2012) and Dey (2014) find instead support for intermediate wealth effects.

Relatedly, Galí (2008) shows that if the persistence of a contractionary (respectively, expansionary) monetary shock is sufficiently large, then nominal rates will decrease (respectively, increase), i.e., move in the same (respectively, opposite) direction with inflation. In the present paper, we provide a detailed analysis on the occurrence of the neo-Fisherian hypothesis under GHH preferences, which depends not only on the shock’s persistence but also on the price-to-wage markup and Frisch elasticity of labor supply. Importantly, the main objective of our work is to study the occurrence of such a view for the whole spectrum of wealth effects on labor supply. Insightful work by Auclert et al. (2021) shows that GHH preferences can generate unrealistically large fiscal multipliers when price markups are sufficiently small. This result combined with our finding that lower price markups increase the likelihood of neo-Fisherian effects implies that such dynamics might be associated with improbable fiscal multipliers when wealth effects on labor supply are absent. However, as we show in the paper, for relatively small Frisch elasticities of labor supply (consistent with micro evidence) neo-Fisherianism occurs in the presence of price markups that are large enough to generate more realistic fiscal multipliers.

The extent of wealth effects is also important for the local determinacy of a rational expectations equilibrium in RBC models. Jaimovich (2008) shows that a one-sector RBC model can yield equilibrium indeterminacy for levels of aggregate returns to scale within empirical estimates provided the wealth effects on labor supply are intermediate between those implied by a GHH and a KPR utility specification. Abad et al. (2017) find that weaker wealth effects tend to rule out sunspot-driven fluctuations due to balanced budget fiscal rules.

The rest of the paper is organized as follows. Section 2 lays out a New Keynesian model with generalized JR preferences. Section 3 describes the log-linearized three-equation New Keynesian model and provides analytical conditions for the neo-Fisherian hypothesis to hold. Section 4 presents an extensive quantitative analysis of the model for the two extreme cases of GHH and KPR utility preferences as well as for intermediate JR utility specifications. Section 5 discusses equilibrium determinacy issues when neo-Fisherian dynamics materialize. Section 6 concludes.
2 The Model

2.1 Households

Consider an infinite horizon economy populated by a continuum of identical households. The representative household seeks to maximize his expected discounted lifetime utility, \( E_0 \sum_{t=0}^{\infty} \beta^t u_t \), for \( \beta \in (0, 1) \). The period utility function \( u_t \) is of the JR type, as proposed by Jaimovich (2008) and Jaimovich and Rebelo (2009), and defines preferences over consumption \( c_t \) and labor \( h_t \) as follows:

\[
   u_t = \frac{x_t^{1-\sigma} - 1}{1-\sigma}, \quad x_t \equiv c_t - \psi \frac{h_t^{1+\chi}}{1+\chi} c_t^\gamma. \tag{1}
\]

for \( \sigma, \psi \geq 0 \) and \( \gamma \in [0, 1] \). This specification nests both the GHH consumption-labor non-separable utility (for \( \gamma = 0 \)), and the KPR utility commonly used in the RBC literature (for \( \gamma = 1 \)). As is well known, a distinguishing feature of a GHH utility is that it yields a labor supply equation that is independent of the marginal utility of consumption, or, equivalently, that does not feature wealth effects on labor choices. On the contrary, such effects are in full swing under a KPR specification. For what concerns the neo-Fisherian hypothesis studied in this paper, the fully separable specification, \( u_t = \frac{x_t^{1-\sigma} - 1}{1-\sigma} - \psi \frac{h_t^{1+\chi}}{1+\chi} \) (often referred to as MaCurdy utility), commonly used in the New Keynesian literature, behaves very similarly to a KPR utility.\(^7\)

The representative household is subject to a standard budget constraint, which, in real terms, is:

\[
   c_t + b_t = \frac{R_{t-1}}{\pi_t} b_{t-1} + w_t h_t + d_t \tag{2}
\]

Namely, the household finances consumption \( c_t \) and new bond holdings \( b_t \) out of the gross return from previous holdings, labor income (at the hourly wage \( w_t \)), and lump-sum dividends \( d_t \) (distributed by firms). Letting \( \lambda_t \) denote the Lagrange multiplier, while defining the auxiliary variables

\[
   K_t \equiv 1 - \gamma \psi \frac{h_t^{1+\chi}}{1+\chi} c_t^{\gamma-1} \quad \text{and} \quad J_t \equiv 1 - \psi \frac{h_t^{1+\chi}}{1+\chi} c_t^{\gamma-1}, \tag{3}
\]

\(^7\)Our specification has the term \( c_t^\gamma \) entering \( x_t \) in the period utility function (1) instead of the stock variable \( n_t \equiv c_t^{1-\gamma} \) used in JR. By introducing some endogenous inertia into the model, the stock specification allows us to account better for business cycle dynamics and sectoral commovements observed in the data. Good examples are works by Dey (2014), Gali and Wouters (2012), Holden et al. (2018), Monacelli and Perotti (2008), and Schmitt-Grohé and Uribe (2012). Abad et al. (2017) use instead one similar to ours in order to attain analytical results on the role of wealth effects in equilibrium (in)determinacy under balanced budget fiscal rules. Boppart and Krusell (2019) provide a full taxonomy of consumption-labor separable and nonseparable utility specifications, and a thorough discussion of their properties.
first-order conditions with respect to $c_t$, $b_t$, and $h_t$ give the following relationships:

\[ x_t^{-\sigma} K_t = \lambda_t, \quad x_t = c_t J_t, \quad (4) \]
\[ \lambda_t = \beta R_t E_t \left( \frac{\lambda_{t+1}}{\pi_{t+1}} \right), \quad (5) \]
\[ w_t K_t = \psi_h c_t^\gamma. \quad (6) \]

These equations correspond, respectively, to the marginal utility of consumption (now a function of both consumption $c_t$ and labor $h_t$, due to the nonseparability of preferences), the Euler equation for optimal consumption-saving decisions equating the intertemporal marginal rate of substitution to the \textit{ex ante} real interest rate, and the consumption-labor intratemporal trade-off.

2.2 Productive Sector

As the supply side of the economy is standard, we keep its description short. A flexible-price perfectly competitive retail sector produces the final consumption good $y_t$ out of a continuum of intermediate goods via the Dixit-Stiglitz aggregator $y_t = \left[ \int_0^1 y_t(i) (\epsilon-1)/\epsilon \, di \right]^{1/(\epsilon-1)}$, with elasticity of substitution $\epsilon > 1$. The optimal demand for the $i$-th intermediate good is $y_t(i) = \left( \frac{P_t(i)}{\bar{P}_t} \right)^{-\epsilon} y_t$, with $\bar{P}_t = \left[ \int_0^1 P_t(i) (1-\epsilon) \, di \right]^{1/(1-\epsilon)}$ denoting the final good’s price. A continuum of monopolistically competitive wholesale firms, indexed by $i \in [0, 1]$, hire labor from a competitive labor market to produce the $i$-th intermediate good (then sold to retailers) under the linear technology $y_t(i) = h_t(i)$. Each firm is subject to Calvo-type nominal rigidities, with $\theta \in [0, 1)$ denoting the probability of no price change per period. Profit maximization makes the optimal price $P_t^*$ (relative to the aggregate $P_t$) for a price-setting firm equal to a markup over a nonlinear combination of appropriately discounted current and expected future marginal costs. Aggregation across firms will yield a standard New Keynesian Phillips curve (to be described later).

2.3 Monetary Policy

Monetary policy takes the form of a Taylor-type interest rate rule whereby the short-term nominal interest rate is set in response to deviations of inflation from its target, $\pi^*$, and output from its steady-state level, $\bar{y}$.

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8Given the purpose of the analysis, without loss of generality, total factor productivity is assumed to be constant and normalized to unity.
\[ R_t = R^* \mathbb{E}_t \left[ \left( \frac{\pi_{t+j}}{\pi^*} \right)^{\phi_\pi} \left( \frac{y_{t+j}}{\bar{y}} \right)^{\phi_y} \right] v_t, \text{ for } j \in \{0, 1\} \]  

(7)

where the policy coefficients \( \phi_\pi \) and \( \phi_y \) are assumed to be non-negative. We consider two cases of the policy rule: one where the central bank reacts to contemporaneous deviations of inflation and output from their respective steady-states \((j = 0)\), and another where it reacts to anticipated future deviations of inflation and output from their respective steady-state values \((j = 1)\). The variable \( v_t \) represents a shock to the policy rate with unconditional mean equal to unity. Letting \( \hat{v}_t \equiv \ln(v_t) \), we assume that \( \hat{v}_t = \rho \hat{v}_{t-1} + \varepsilon_t \), where \( \rho \in [0, 1) \) and \( \varepsilon_t \) is a mean-zero i.i.d. disturbance with variance \( \sigma^2_\varepsilon \). The term \( R^* \) is the interest rate target, which is attained if \( \pi_t = \pi^* \), \( y_t = \bar{y} \), and \( v_t = 1 \).

2.4 Equilibrium

In a rational expectations equilibrium (henceforth, REE): i) households and firms optimize, taking as given prices and aggregate quantities; ii) all markets clear; iii) individuals form expectations about aggregate quantities in a way that is consistent with their equilibrium law of motion; and iv) aggregate and individual quantities coincide. In particular, output equals consumption, \( y_t = c_t \), and aggregate dividends are \( d_t = \int_0^1 d_t(i) di = y_t (1 - mc_t \Xi_t) \), with \( \Xi_t \equiv \int_0^1 \left[ \frac{P_t(i)}{P_i} \right]^{-\epsilon} di \) denoting price dispersion across firms. As in the baseline New Keynesian model, market clearing in the labor market requires \( h_t = \int_0^1 h_t(i) di = y_t \Xi_t \). Since all households are identical, bonds are in zero net supply: \( b_t = 0 \) in every period. Given the Calvo-style price rigidity, the aggregate price level \( P_t \) evolves according to \( P_t = (1 - \theta) P_t^* + \theta P_{t-1} \).

2.4.1 Steady-state Equilibrium

In a zero inflation deterministic steady-state equilibrium, where \( v_t = 1 \) and \( \pi^* = \bar{\pi} = 1 \), all endogenous variables are constant. From the Euler equation (5), we obtain the steady-state (nominal and real) interest rate: \( \bar{R} = \beta^{-1} \). Letting \( \mu \equiv \frac{\epsilon}{1 - \epsilon} \) denote the (steady-state) gross price-to-marginal cost markup of firms, optimal pricing by firms implies that net real marginal costs \( \overline{mc} \) equal \( \mu^{-1} \). At the steady-state, the real wage is therefore described by \( \bar{w} = \mu^{-1} \). The steady-state expressions for the utility composite \( \bar{x} \) and output \( \bar{y} \) (hence, also consumption \( \bar{c} \)
and labor $\bar{h}$), respectively, are:

\begin{align*}
\bar{x} &= \frac{(1 + \chi) \mu + \gamma - 1}{(1 + \chi) \mu + \gamma} \bar{y} \\
\bar{y} &= \bar{c} = \bar{h} = \frac{(1 + \chi)}{\psi((1 + \chi) \mu + \gamma)} \frac{1}{\gamma + \chi}.
\end{align*}

(8)  
(9)

For utility to be well-defined, $x_t$ has to be strictly positive. Note that $\mu > 1$ guarantees that to be the case in a sufficiently small neighborhood of the steady-state.

3 Neo-Fisherian Dynamics

We define neo-Fisherian dynamics to occur when, following a monetary shock, nominal interest rates and inflation comove.

**Definition 1** Along an REE, the neo-Fisherian hypothesis holds when, following a nominal shock, inflation and the nominal interest rate display positive comovement.

We characterize the model’s equilibrium dynamics around its unique zero-inflation steady-state equilibrium. For a generic variable $n_t$, we let $\hat{n}_t \equiv \ln (n_t/\bar{n})$ denote its log-deviation from the steady-state. Log-linearization of equations (4) and (5) gives

\begin{align*}
\hat{x}_t &= E_t \hat{x}_{t+1} - \sigma^{-1} \left( \hat{R}_t - E_t \hat{\pi}_{t+1} \right) - \sigma^{-1} E_t \left( \hat{K}_{t+1} - \hat{K}_t \right), \quad \text{and} \quad \hat{x}_t = \hat{c}_t + \hat{J}_t. 
\end{align*}

(10)

From the latter, the expressions for $K_t$ and $J_t$ in (3), and market clearing, after simple manipulation of terms, we obtain the aggregate Euler equation for output:

\begin{align*}
\hat{y}_t &= E_t \hat{y}_{t+1} - \xi \left( \hat{R}_t - E_t \hat{\pi}_{t+1} \right).
\end{align*}

(11)

The composite coefficient $\xi$ - i.e., the (negative) temporary elasticity of output to the *ex ante* real interest rate - captures the strength of the intertemporal substitution channel of monetary policy transmission, and is defined as:

\begin{align*}
\xi &\equiv \left[ \sigma \left( 1 + J_y - \sigma^{-1} K_y \right) \right]^{-1} 
\end{align*}

(12)

where $J_y$ and $K_y$ denote the steady-state elasticities of auxiliary variables $J_t$ and $K_t$ (defined

\[9\]See Appendix A.1 for a detailed derivation of the aggregate Euler equation (11) and the Phillips curve (14) below.
in (3)) with respect to output, and are given by

\[ J_y = -\frac{\gamma + \chi}{(1 + \chi)\mu + \gamma - 1}, \quad K_y = -\frac{\gamma (\gamma + \chi)}{\mu(1 + \chi)} \]  

(13)

Simple algebra shows that \( \xi > 0 \) if \( \mu > 1 \). Moreover, it is important to notice that \( \xi \) is not just the inverse of risk aversion, as for the standard consumption-labor separable MacCurdy utility used in the literature, but a rather convoluted expression of key structural parameters.\(^{10}\)

Letting \( \kappa \equiv \frac{(1 - \beta\theta)(1 - \theta)}{\beta} \) denote the temporary elasticity of inflation to marginal costs, log-linearization of the optimal price setting rule of firms and aggregation give rise to a New Keynesian Phillips curve:

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \eta \hat{y}_t \\
\eta = \frac{(1 + \chi)\mu + \gamma}{(1 + \chi)\mu} (\chi + \gamma)
\]  

(14)  

(15)

The curve’s slope, \( \kappa \eta \), depends negatively on the degree of price rigidity \( \theta \) (entering \( \kappa \)) and positively on the extent of wages’ procyclicality, as indexed by \( \eta \).

Finally, the linearized Taylor rule for monetary policy is

\[
\hat{R}_t = \phi_x E_t \hat{\pi}_{t+j} + \phi_y E_t \hat{y}_{t+j} + \hat{\nu}_t, \quad \text{for } j \in \{0, 1\} \\
\hat{\nu}_t = \rho \hat{\nu}_{t-1} + \varepsilon_t.
\]  

(16)  

(17)

Our objective is to assess the possibility of neo-Fisherianism in a (locally) determinate REE, i.e., where parameters of the Taylor policy rule are set such that they guarantee equilibrium determinacy.\(^{11}\) This is in contrast to recent contributions by Bilbiie (2021) and Cochrane (2018a) showing that a baseline New Keynesian model with separable preferences may be consistent with the neo-Fisherian hypothesis when monetary policy follows a passive rule such that the REE is indeterminate unless the model includes an active fiscal policy (and the fiscal

\(^{10}\)In Appendix A.7, we enrich the model with fiscal policy in the form of a fixed labor subsidy extended to the monopolistically competitive firms. In that case, \( \xi \) could, in principle, be negative, yielding an inverted aggregate demand logic (IADL) of monetary policy transmission: namely, an increase in the ex ante real interest rate leading to an increase (rather than a decrease) in current output via the Euler equation. Importantly, in the presence of IADL, neo-Fisherianism will always prevail. A similar channel appears in the New Keynesian model with limited asset market participation laid out in Bilbiie (2008) when the share of non-Ricardian (hand-to-mouth) households is sufficiently large. As shown in Appendix A.7, there exist parameterizations for which the IADL occurs despite all agents being Ricardian and therefore capable of intertemporally smoothing consumption through financial markets.

\(^{11}\)The conditions for equilibrium determinacy are analogous to those spelled out in Bullard and Mitra (2002): a (locally) unique REE requires \( \kappa \eta (\phi_x - 1) > -(1 - \beta)\phi_y \) when the rule is contemporaneous, and \( -(1 - \beta)\phi_y < \kappa \eta (\phi - 1) < 2(1 + \beta) - (1 + \beta)\phi_y \) as well as \( \phi_y < \frac{1 + \beta}{\phi} \) when forward-looking. In Section 5, we discuss in detail how the determinacy region of policy parameters is affected.
theory of the price level logic applies).

**Proposition 1** Consider the reduced-form model described by equations (11)-(17). Along a determinate REE, the neo-Fisherian hypothesis holds if and only if

\[
\xi \eta > \frac{(1 - \beta \rho) (1 - \rho)}{\rho \kappa} \frac{f^*(\gamma)}{f(\gamma)}
\]  

(18)

**Proof.** See Appendix A.4.1.

While papers such as Garin et al. (2018) and Rupert and Sustek (2019) focus primarily on the right-hand side of inequality (18), we pay special attention to what affects the left-hand side of the neo-Fisherianism condition. In particular, the neo-Fisherian inequality (18) is more likely to hold when the monetary policy transmission channel is stronger (higher \(\xi\)) and/or wages are more procyclical (higher \(\eta\)). In the next sections, we provide analytical conditions for this to happen, highlighting the role played by three key structural parameters of our model: the wealth effect \(\gamma\), the labor elasticity parameter \(\chi\), and the price-to-wage markup \(\mu\).

It is evident that (18) tends to hold when the nominal shock is more persistent (higher \(\rho\)), and/or prices are more flexible (lower \(\theta\), hence higher \(\kappa\)), and/or the private sector is more forward-looking (higher \(\beta\)). Moreover, when the monetary shock is an i.i.d. innovation (\(\rho = 0\)) we have that \(\lim_{\rho \to 0} f^* = \infty\), and (18) will never hold. On the contrary, in the other extreme when the monetary shock approaches a random walk, we have \(\lim_{\rho \to 1} f^* = 0\), so neo-Fisherianism is always guaranteed to occur. These results are reminiscent of empirical findings in Uribe (2020) that first, sufficiently transitory monetary shocks (the author estimates persistence to be around 0.2 at the posterior mean) cannot generate a positive comovement between nominal interest rates and inflation in the short term, and second, a forever-lasting monetary shock yields neo-Fisherianism.

Note that the central bank’s aggressiveness toward (contemporaneous or anticipated future) inflation or output deviations from their respective steady-state values plays no role in the occurrence of the neo-Fisherian hypothesis along a determinate REE. Moreover, in Appendix A.6 we show two alternative narratives that yield the same neo-Fisherian condition as in Proposition 1: i) instead of a monetary shock there is an inflation target shock; and ii) the central bank commits to inflation being exactly equal to the inflation target in each period.
3.1 Intuition

To gather some intuition on inequality (18), and specifically why a sufficiently large \((\xi \eta)\) is needed for neo-Fisherian dynamics, assume the central bank follows a simpler Taylor rule: 
\[
\hat{R}_t = \phi_\pi \hat{\pi}_t + \hat{v}_t,
\]
with \(\phi_\pi > 1\) to guarantee REE determinacy.\(^{12}\) Let \(\xi > 0\) and suppose \(\hat{v}_t > 0\), i.e., there is a contractionary monetary shock. Combining the Euler equation in (11) with the Taylor rule, iterating forward, and setting \(\lim_{h \to \infty} \left( \frac{1}{\phi_\pi} \mathbb{E}_t \hat{\pi}_{t+h} \right) = 0\), we have the following expression for inflation

\[
\hat{\pi}_t = \frac{1}{\phi_\pi} \left( \frac{1}{\phi_\pi} \sum_{h=0}^{\infty} \left( \frac{1}{\phi_\pi} \right)^h v_{t+h} + \frac{1}{\xi} \sum_{h=0}^{\infty} \left( \frac{1}{\phi_\pi} \right)^h \left( \hat{y}_{t+h+1} - \hat{y}_{t+h} \right) \right)
\]

(19)

The effects of a contractionary monetary shock on inflation are attributed to two opposing channels: on the one hand, a positive monetary shock has a direct negative effect on inflation (Fisherian principle), but on the other hand, it has an indirect positive effect on inflation through the real rate channel. The Fisherian channel is the most prominent when prices are fully flexible and the real rate is exogenous. As prices become stickier, the real rate channel gains momentum, but in equilibrium it can never dominate the Fisherian one, i.e., in equilibrium a contractionary monetary shock will always lead to disinflation.\(^{13}\)

Consequently, for neo-Fisherianism to occur following a contractionary monetary shock, the drop in inflation has to be sufficiently large such that the negative change in \(\phi_\pi \hat{\pi}_t\) exceeds the positive change in \(\hat{v}_t\). For this to happen, the real rate channel in (21) needs to be sufficiently dampened. From the Phillips curve in (14),

\[
\mathbb{E}_t(\hat{y}_{t+h+1} - \hat{y}_{t+h}) = \frac{1}{\kappa \eta} \mathbb{E}_t(\hat{\pi}_{t+h+1} - \beta \hat{\pi}_{t+h+2})
\]

(20)

Plugging (20) into (21), we have

\[
\hat{\pi}_t = \frac{1}{\phi_\pi} \left( \frac{1}{\phi_\pi} \sum_{h=0}^{\infty} \left( \frac{1}{\phi_\pi} \right)^h v_{t+h} + \frac{1}{\xi} \sum_{h=0}^{\infty} \left( \frac{1}{\phi_\pi} \right)^h \left( \hat{\pi}_{t+h+1} - \beta \hat{\pi}_{t+h+2} \right) \right)
\]

(21)

\(^{12}\)We assume \(\phi_y = 0\) and \(j = 0\) here for the sole purpose of expository simplicity.

\(^{13}\)As Rupert and Sustek (2019) put it, “... the reason why inflation declines in response to the positive monetary shock is the [Fisherian term, not] the increase in the real rate... The increase in the real rate, in fact, works against the decline in inflation.”
Therefore, the larger \((\xi \eta)\) is, the more the real rate channel is dampened, increasing the likelihood of neo-Fisherianism.

To see why the type of Taylor rule (contemporaneous or forward-looking) and its policy parameters have no effect on the occurrence of the neo-Fisherian hypothesis, let’s isolate the nominal interest rate from the Euler equation in (11),

\[
\hat{R}_t = \underbrace{E_t \hat{\pi}_{t+1}}_{\text{Fisherian channel (-)}} + \frac{1}{\xi} \underbrace{(E_t \hat{y}_{t+1} - \hat{y}_t)}_{\text{real rate channel (+)}} = \underbrace{E_t \hat{\pi}_{t+1}}_{\text{Fisherian channel (-)}} + \frac{1}{\xi \eta \kappa} \underbrace{E_t (\Delta \hat{\pi}_{t+1} - \beta \Delta \hat{\pi}_{t+2})}_{\text{real rate channel (+)}}
\]  

(22)

where the second equality follows from the Phillips curve in (20). Similarly to (21), two opposing channels contribute to the effects of a positive monetary shock on nominal rates: a negative impact through a decline in inflation expectations (Fisherian channel), but a positive impact through an increase in the endogenous real rate (real rate channel). Equation (22) shows once again that, following a monetary shock, a sufficiently high \((\xi \eta)\) mitigates the real rate channel, reinforcing the positive relationship between inflation and nominal rates.

Since the economy is subject to only a monetary shock with persistence \(\rho\),

\[
E_t (\Delta \hat{\pi}_{t+1} - \beta \Delta \hat{\pi}_{t+2}) = -(1 - \beta \rho)(1 - \rho) \hat{\pi}_t, \quad \text{and} \quad E_t \hat{\pi}_{t+1} = \rho \hat{\pi}_t.
\]

Then, we can rewrite the nominal interest rate in (22) as

\[
\hat{R}_t = \left( \rho - \frac{(1 - \rho)(1 - \beta \rho)}{\xi \eta \kappa} \right) \hat{\pi}_t
\]  

(23)

Therefore, the condition for the occurrence of neo-Fisherianism is robust to policy parameters as well as the type of Taylor rule.

### 3.2 GHH Preferences

Setting \(\gamma = 0\) in (13), we have that \(K_y = 0\) and \(J_y = -\frac{\chi}{(1 + \chi) \mu - 1}\). Simple algebra gives the following expressions for the two key elasticities entering the reduced form model described in (11) and (14):

\[
\xi \equiv \frac{\mu(1 + \chi) - 1}{\sigma(1 + \chi)(\mu - 1)}, \quad \text{and} \quad \eta = \chi
\]  

(24)

Notice that with \(K_t = 1\) in (6), the structural parameter \(\chi\) corresponds to the inverse Frisch elasticity of labor supply. We therefore assume \(\chi \geq 0\). A similar expression for the intertemporal elasticity of substitution under GHH preferences appears in Auclert et al. (2021). More specifically, \(\xi = -\frac{1}{\text{labor wedge} u_{c,H}(\bar{c}, \bar{h})} \), where \(\text{labor wedge} = 1 + \frac{u_{c,H}(\bar{c}, \bar{h})}{\frac{1}{m(H)} f'(h)} \frac{1}{\mu} = \frac{\mu - 1}{\mu}\), with \(f(h) = h\) being the production technology employed by the monopolistically competitive firms.

The functional form of \(\xi\) in (24) combined with the neo-Fisherian condition in (18) under-
lines the critical role played by the degree of imperfect competition in the goods market, as indexed by \( \mu \), for the neo-Fisherian hypothesis to hold.

**Proposition 2** Consider a GHH utility, i.e., \( \gamma = 0 \). Define \( \chi^* \equiv \frac{\sigma(1-\beta\rho)(1-\rho)}{\rho\kappa} \). In a determinate REE, the neo-Fisherian hypothesis holds if and only if

I. \( \mu \geq 1 \) when \( \chi \geq \chi^* \);

II. \( 1 \leq \mu < \mu^* \equiv (1+\chi)(\chi^* - \chi) - \frac{\chi(1+\chi)(\chi^* - \chi)}{\chi^*} \) when \( \chi \in (0, \chi^*) \).

**Proof.** See Appendix A.4.2. ■

When the labor supply is *sufficiently wage-inelastic* (i.e., \( \chi \geq \chi^* \) in Case I), the neo-Fisherian hypothesis always holds, irrespective of the degree of imperfect competition in the goods market (as indexed by \( \mu \)). For a more elastic labor supply (i.e., \( \chi < \chi^* \) in Case II), neo-Fisherianism occurs provided the price markup is below a certain threshold.

**Inspecting the Mechanism.** The objective of this paragraph is to provide some intuition for why the intertemporal elasticity of substitution, \( \xi \), is larger under GHH preferences relative to standard separable preferences, whose intertemporal elasticity of substitution is \( \sigma^{-1} \). Consider the general Euler equation (5), which, other things equal, implies that a 1 percent increase in the *ex ante* real interest rate, \( r_t \equiv E_t \frac{R_t}{\pi_{t+1}} \), requires a 1 percent increase in the marginal utility of current consumption, \( \lambda_t \). To a first order approximation, we have \( \dot{\lambda}_t = \frac{u_{cc}}{u_{cc}} \dot{c}_t + \frac{u_{ch}}{u_{ch}} \dot{h}_t \). Using steady-state definitions, total differentiation gives

\[
\frac{\partial \dot{\lambda}_t}{\partial t} = -\frac{\sigma \mu (1 + \chi)}{\mu (1 + \chi) - 1} \left[ \frac{\partial \dot{c}_t}{\partial t} - \frac{1}{\mu} \partial \dot{h}_t \right] \Rightarrow \frac{\partial \dot{c}_t}{\partial t} = -\frac{1}{\sigma} \frac{\mu (1 + \chi) - 1}{\mu (1 + \chi)} \partial \dot{\lambda}_t + \frac{1}{\mu} \partial \dot{h}_t
\]

for GHH preferences, and \( \frac{\partial \dot{\lambda}_t}{\partial t} = -\sigma \dot{c}_t \) for MaCurdy separable preferences. Therefore, at the partial equilibrium level, to attain an increase in \( \dot{\lambda}_t \), the household can cut consumption by less relative to the MaCurdy preferences.

At the general equilibrium level, labor hours also respond to changes in the *ex ante* real rate, with a 1 percent increase (decrease) in \( \dot{h}_t \) implying an additional \( \frac{1}{\mu} \) percent increase (decrease) in \( \dot{c}_t \). Since in equilibrium \( \dot{c}_t = \dot{y}_t = \dot{h}_t \),

\[
\frac{\partial \dot{c}_t}{\partial \dot{y}_t} = -\frac{1}{\sigma} \frac{\mu (1 + \chi) - 1}{(\mu - 1)(1 + \chi)} \partial \dot{\lambda}_t
\]

for GHH preferences, and \( \frac{\partial \dot{\lambda}_t}{\partial \dot{y}_t} = -\sigma \frac{\partial \dot{c}_t}{\partial \dot{y}_t} \) for MaCurdy separable preferences. Therefore, at the partial equilibrium level, to attain an increase in \( \dot{\lambda}_t \), the household can cut consumption by less relative to the MaCurdy preferences.
So, $\hat{c}_t$ will drop by more w.r.t. the separable utility preferences case for a given implied increase in $\hat{\lambda}_t$. General equilibrium effects increase $\xi$ as $\mu$ declines. Consider then an economy with $\xi > 0$ that is hit by a positive and persistent nominal interest rate shock: $\hat{v}_t > 0$. By the Euler equation (11), output drops on impact, $\hat{y}_t < 0$; the larger the magnitude, the higher is the interest rate elasticity $\xi$ (the closer $\mu$ is to unity) and the more persistent the shock is (bigger $\rho$, as mean reversion implies $0 > \mathbb{E}_t \hat{\pi}_{t+1} > \hat{\pi}_t$). The induced recession yields a contraction in wages, the more so the less elastic is the labor supply (higher $\chi$), as that makes wages more procyclical. Through the Phillips curve (14), the resulting downward pressure on inflation, amplified by the persistence-driven mean reversion effect giving $0 > \mathbb{E}_t \hat{\pi}_{t+1} > \hat{\pi}_t$, yields $\hat{\pi}_t < 0$. The final impact on the nominal interest rate $\hat{R}_t$ is the net effect of the exogenous positive impulse, $\hat{v}_t > 0$, and the endogenous negative policy response, $\phi_\pi \hat{\pi}_t < 0$. The latter prevails (hence, the neo-Fisherian hypothesis materializes) when the contraction in inflation is sufficiently large. This occurs when either

(a) the marginal cost channel in the Phillips curve is strong enough (i.e., large $\chi$), irrespective of the strength of the monetary policy transmission channel (i.e., for any positive $\xi$) in the Euler equation (Case I in Proposition 2); or

(b) the marginal cost channel is weaker (i.e. smaller $\chi$), but monetary policy is sufficiently contractionary (i.e., larger $\xi$, or, equivalently, smaller $\mu$ (Case II in Proposition 2)).

### 3.3 KPR Preferences

Imposing $\gamma = 1$ in (13), we obtain:

$$J_y = K_y = \frac{-1}{\mu} \tag{27}$$

Using (27) in (12) and (15), simple algebra gives:

$$\xi = \frac{\mu}{\sigma(\mu - 1) + 1}, \quad \text{and} \quad \eta = \frac{1 + (1 + \chi)\mu}{\mu} \tag{28}$$

Recalling that, in the KPR case, $\sigma$ corresponds to the coefficient of relative risk aversion, we follow common practice in the macro literature and assume $\sigma \geq 1$.\(^{14}\)

**Proposition 3** Consider a KPR utility, i.e., $\gamma = 1$, with $\sigma \geq 1$. Define $\chi^H \equiv \sigma \frac{1-\beta(1-\rho)}{\rho_\pi} - 1$ and $\chi^L \equiv \frac{(1-\beta\rho)(1-\rho)}{\rho_\pi} - 2$, where $\chi^L < \chi^H$.\(^{15}\) In a determinate REE, the neo-Fisherian hypothesis

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\(^{14}\)This assumption reduces the number of parametric cases without altering the main message. We will consider $\sigma \in (0, 1)$ in the numerical analysis in the next section.

\(^{15}\)Since $\chi$ no longer represents the inverse Frisch elasticity of labor, it does not necessarily have to be positive.
holds if and only if

I. \( \mu \geq 1 \) when \( \chi \geq \chi^H \);

II. \( 1 \leq \mu < \mu^* \equiv \frac{\sigma + (\sigma - 1)(1 + \chi^*)}{\sigma (\chi^H - \chi)} \) when \( \chi \in (\chi^L, \chi^H) \).

When \( \chi \leq \chi^L \), neo-Fisherianism never occurs.

Proof. See Appendix 3. ■

Once we realize that consumption and hours are Edgeworth complements (respectively, neutral) for \( \sigma \) larger than (respectively, equal to) unity, the intuition for the results is similar to that for the GHH case. Following the same logic, around the steady-state, the (log) marginal utility of consumption is:

\[
\hat{\lambda}_t = -\sigma \left( \hat{c}_t - \frac{\sigma - 1}{\sigma \mu} \hat{h}_t \right) \Rightarrow \partial \hat{c}_t = -\frac{1}{\sigma} \left( \partial \hat{\lambda}_t + \frac{\sigma - 1}{\sigma \mu} \partial \hat{h}_t \right)
\] (29)

At the partial equilibrium level, KPR preferences behave like a McCurdy utility function, i.e., \( \partial \hat{c}_t = -\frac{1}{\sigma} \partial \hat{\lambda}_t \). At the general equilibrium level, we need to account for \( \partial \hat{h}_t \). Since in equilibrium \( \hat{c}_t = \hat{y}_t = \hat{h}_t \),

\[
\partial \hat{c}_t = -\frac{\mu}{\sigma (\mu - 1) + 1} \partial \hat{\lambda}_t \leq 1
\] (30)

As the GE elasticity of output to marginal utility (hence, to the real interest rate), \( \xi = \frac{\mu}{\sigma (\mu - 1) + 1} \), can be at most equal to unity, the monetary policy transmission channel appears less powerful than under GHH. The neo-Fisherian hypothesis holds then when either the Phillips curve is sufficiently steep (Case I in Proposition 3), or when that is not the case but \( \mu \) is low enough to induce a sufficiently strong demand-side transmission. Under both circumstances the downward pressure on inflation is sizable enough to generate a Taylor-rule-driven contraction in the nominal interest rate \( \hat{R}_t \), counteracting the initial positive monetary impulse. When the Phillips curve is instead excessively flat, \( \chi \leq \chi^L \), since \( \xi \) is strictly bounded above by unity, neo-Fisherianism never occurs.

4 Quantitative Analysis

Although the chances for the neo-Fisherian hypothesis to hold appear slimmer under KPR, one has to be careful before drawing conclusions, since the model’s parameters \( \sigma \) and \( \chi \) have different meanings across the two extreme preference specifications. To facilitate the comparison,
we append a subscript to both $\sigma$ and $\chi$ to index their model specificity: namely, $K$ for KPR and $G$ for GHH. For what concerns the coefficient of relative risk aversion (henceforth, simply RA), using the standard consumption-based definition, $RA \equiv -\frac{u_{cc}}{u_c}$, simple calculus gives:\footnote{Swanson (2012) provides an alternative definition accounting for endogenous labor choice.}

$$RA = \begin{cases} \sigma_K (KPR) \\ \sigma_G \frac{(1+\chi_G)\mu}{(1+\chi_G)\mu-1} (GHH) \end{cases}$$

The Frisch elasticity of labor supply (henceforth, simply FE) is instead defined as the elasticity of the labor supply to the real wage, keeping the marginal utility of consumption constant, namely, $\partial\hat{h}_t/\partial\hat{w}_t$ for $\hat{\lambda}_t = 0$. As shown in Appendix A.2, FE is given by the following expressions:

$$FE = \begin{cases} \frac{(\chi_K + \frac{2\sigma_K-1}{\sigma_K\mu})}{\sigma_K(1+\chi_G)\mu} (KPR) \\ \frac{1}{\chi_G} (GHH) \end{cases}$$

To assess the model’s propensity to generate neo-Fisherian effects, it seems appropriate to ensure that the GHH and the KPR specifications share the same calibration of all structural parameters, including RA and FE.\footnote{Obviously, the CRRA is endogenous in the GHH case (it varies along the equilibrium path as consumption and labor vary), and the Frisch elasticity is endogenous in the KPR case. As the model is approximated around the nonstochastic steady-state, we require the same RA and FE in steady state.} We therefore impose two consistency requirements:

$$\sigma_K = \sigma_G \frac{(1+\chi_G)\mu}{(1+\chi_G)\mu-1}$$

$$\chi_G = \chi_K + \frac{2\sigma_K-1}{\sigma_K\mu}$$

Our calibration strategy is the following: i) we parameterize $\sigma_K$ and $\chi_G$ to match, respectively, the RA and FE from the empirical literature (in what follows we will consider baseline values, as well as wider ranges); ii) for any given value assigned to the price-to-wage markup $\mu$, we solve (33)-(34) for $\sigma_G$ and $\chi_K$.

Using the appropriate definitions of $\xi$ and $\eta$, imposing (33)-(34), and recalling that $f^* \equiv (1-\beta\rho)(1-\rho)/\rho\kappa$, the necessary and sufficient neo-Fisherianism condition (18) for GHH and KPR preferences, respectively, can be written as follows:

$$\frac{\mu \chi_G}{\sigma_K(\mu-1)} > f^* \quad \text{and} \quad \frac{\sigma_K(1+\chi_G)\mu + 1 - \sigma_K}{\sigma_K(\mu\sigma_K + 1 - \sigma_K)} > f^*$$

As the following proposition shows, other things equal, GHH preferences are more prone to
yield neo-Fisherian effects as long as the price-to-wage markup $\mu$ remains below a certain threshold.

**Proposition 4** Let $f_G(\mu) = \frac{\mu \chi_G}{\sigma_K(\mu - 1)}$ and $f_K(\mu) = \frac{\sigma_K(1 + \chi_G)\mu + 1 - \sigma_K}{\sigma_K(\mu \sigma_K + 1 - \sigma_K)}$. Then, there exists a threshold value $\mu^c > 1$ such that $f_G(\mu) \gtrless f_K(\mu)$ for $\mu \gtrless \mu^c$.

**Proof.** See Appendix A.4.4. ■

By plotting the functions $f_G(\mu)$ and $f_K(\mu)$ for $\mu \in [1, 2]$, Figure 1 allows a visual inspection of the analytical conditions stated in (35) and Proposition 4, accounting for the consistency in calibration given by (33)-(34). Our parameterization is standard: $\beta = 0.99$, $\psi = 1$, $\theta = 0.75$, $\rho = 0.5$, and a unitary RA, $\sigma_K = 1$. In each panel - where a different value for the FE, $\chi_G^{-1}$, is assumed - the flat red and the downward-sloping black lines correspond, respectively, to $f_K(\mu) = 1 + \chi_G$ for $\sigma_K = 1$ and $f_G(\mu)$, with the dashed-line indicating $f^*$. Recall from the conditions in (35) that the neo-Fisherian hypothesis holds for values of $\mu$ where $f_i(\mu) > f^*$, for $i = G$ or $K$.

The first thing to notice is that, under the baseline calibration, KPR preferences do not deliver neo-Fisherian effects (the bold red lines are below the dashed threshold $f^*$ in all panels) even if we impose a rather low labor elasticity, e.g., $1/3$ in Panel c. In this respect, GHH preferences are clearly superior to KPR: the function $f_G(\mu)$ is larger than $f_K(\mu)$ for any $\mu \gtrless \mu^c$.

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18 A unitary RA gives a log-separable KPR specification and is indeed the most common parameterization used in the macro literature. Available estimates for the Frisch elasticity vary between low (micro-based) to very high (macro-based) values (see Kean and Rogerson (2012), for an extensive review). We use a unitary elasticity as baseline.

19 Under KPR utility, neo-Fisherianism kicks in for an FE equal to 0.2 or smaller.
Figure 2: GHH versus KPR: Sensitivity analysis. For GHH preferences neo-Fisherianism holds in the white and grey regions; for KPR, it holds only in the grey region. Black region: conventional wisdom. Baseline parameterization: $\beta = 0.99$, $\epsilon = 11$, $\theta = 0.75$, $\rho = 0.5$, with both risk aversion and the Frisch elasticity set equal to unity.

reasonable value of $\mu$. The threshold $\mu^c$ identified in Proposition 4 - the point where the bold black and red lines intersect each other - is 2 for a unitary FE, and higher as FE is raised. Focusing on the GHH specification, neo-Fisherian effects arise for $\mu$ smaller than (about) $\mu^* \approx 1.2$ for $\chi_G^{-1} = 1$ (Panel a.) and 1.5 for $\chi_G^{-1} = 0.5$ (Panel b.), but for any $\mu$ smaller than 2 for $\chi_G^{-1} = 1/3$ (or lower) (Panel c.). Making use of the definition of $\mu \equiv \frac{\epsilon}{\epsilon - 1}$, this is equivalent to requiring an intertemporal elasticity of substitution across goods varieties $\epsilon$ larger than a threshold $\bar{\epsilon} \equiv \frac{\mu^*}{\mu^* - 1}$. Then, $\bar{\epsilon}$ is equal to 6 (for $\chi_G^{-1} = 1$), 3 (for $\chi_G^{-1} = 0.5$), and 2 (for $\chi_G^{-1} = 1/3$).

Auclert et al. (2021) show that GHH preferences can give rise to unrealistically high fiscal multipliers - in the data fiscal multipliers range between 0.6 and 2. In particular, the multiplier for GHH preferences in our model is given by the inverse of the labor wedge, i.e., $\frac{1}{\text{labor wedge}} = \frac{\mu}{\mu - 1}$. The left panel in Figure 1 shows that if the Frisch elasticity is equal to unity, neo-Fisherianism occurs for $\mu \leq 1.2$. Such values of the price-to-wage markup generate fiscal multipliers higher than or equal to 6. However, as the value of the Frisch elasticity decreases, neo-Fisherianism can occur without implying controversial values of the fiscal multiplier. For instance, when the Frisch elasticity is equal to 1/3, GHH preferences yield neo-Fisherianism for any $\mu < 2.02$, which corresponds to the fiscal multiplier being higher than 1.98. Note that for more persistent monetary shocks, neo-Fisherian dynamics can happen for larger $\mu$ under GHH preferences and are associated with smaller fiscal multipliers. For example, if $\rho = 0.55$ and the Frisch elasticity is 1/3, neo-Fisherianism occurs for any $\mu < 4.44$ and the fiscal multiplier will be higher than 1.29 whenever the neo-Fisherian hypothesis holds.

In the left panel of Figure 2, we let the degree of price stickiness $\theta$ and persistence of the
interest rate shock $\rho$ vary, while keeping $\epsilon = 11$ (such that $\mu = 1.1$, or 10 percent net markup) and setting $RA = FE = 1$. KPR preferences yield neo-Fisherianism for combinations of $\theta$ and $\rho$ falling inside the grey area, whereas GHH preferences generate neo-Fisherian effects for $\theta$ and $\rho$ pairs inside the grey and the white region. Within the black region, common wisdom applies: following the nominal shock, inflation and the nominal interest rate display negative comovement. Under both specifications, monetary policy is always (respectively, never) neo-Fisherian when prices are fully flexible, $\theta = 0$ (respectively, fully rigid, $\theta = 1$), and/or the nominal shock is extremely persistent, $\rho \approx 1$ (respectively, i.i.d., $\rho = 0$). For instance, under a standard parameterization of price stickiness (Calvo probability equal to 0.75), the neo-Fisherian hypothesis holds, provided the nominal shock’s autocorrelation is at least 0.7 under KPR, but only 0.4 under GHH.

In the right panel, we instead fix price stickiness and persistence at baseline values ($\theta = 0.75$ and $\rho = 0.5$, respectively), and have both risk aversion, $\sigma_K$, and the Frisch elasticity, $\chi^{-1}_G$, range between 0 and 2. Under both specifications - but more so for the KPR case - the neo-Fisherian frontiers are downward-sloping: the higher risk aversion is, the less elastic the labor supply should be for the neo-Fisherian hypothesis to hold. The intuition is the same as discussed in Section 3.2: as higher risk aversion weakens the negative impact of interest rates on real activity (weakening the intertemporal substitution channel of policy transmission), the labor supply needs to be sufficiently inelastic to make wages sufficiently procyclical, a necessary condition to induce an endogenous contraction in the policy rate. Notice that, keeping the Frisch elasticity at its baseline unitary value, i) GHH preferences remain neo-Fisherian for any risk aversion between 0 and, roughly, 2, while ii) KPR preferences can become neo-Fisherian for risk aversion lower than 0.2.

4.1 General JR Preferences

For the case of $\gamma \in (0, 1)$, the reduced-form model is still described by the Euler equation (11), the Phillips curve (14), and the Taylor rule (16)-(17). We follow the same calibration procedure: for a given parameterization of all structural parameters, including now $\gamma \in (0, 1)$, we choose $\chi$ and $\sigma$ to match the empirical values of RA and FE.\textsuperscript{20}

Keeping all other parameters at baseline values and setting $\mu = 1.1$ (which occurs, for instance, for $\epsilon = 11$), as displayed by the bold black lines in the left panel of Figure 3, moving away from the GHH specification ($\gamma = 0$), a positive wealth effect brings in two opposite...

\textsuperscript{20}See Appendix A.3.
Figure 3: **Neo-Fisherianism under general JR preferences.** Left panel: interest rate elasticity $\xi$, and wage procyclicality $\eta$. Right panel: neo-Fisherian condition, $f(\gamma) = \xi \eta > f^*$, as a function of $\gamma \in [0,1]$. Neo-Fisherian thresholds are $\gamma^* \approx 1.1$ for $\mu = 1.1$, and $\gamma^* \approx 0.28$ for $\mu = 1.01$. Baseline calibration: $\beta = 0.99$, $RA = FE = 1$, $\theta = 0.75$, $\rho = 0.5$.

Figure 4: **Neo-Fisherianism under general JR preferences: Sensitivity.** Baseline parameterization ($\beta = 0.99$, $RA = FE = 1$, $\theta = 0.75$, $\rho = 0.5$): neo-Fisherianism (white area), conventional wisdom (black area). Grey area corresponds to the change in the neo-Fisherian region (directional arrows indicate the shift) following a change in: higher risk aversion, $RA = 2$ (Panel a); lower Frisch elasticity, $FE = 0.5$ (Panel b); lower price stickiness, $\theta = 2/3$ (Panel c); more persistent monetary shock, $\rho = 0.6$ (Panel d).

**forces.** On the one hand, it weakens the intertemporal substitution channel of monetary shock transmission (lower $\xi$). On the other hand, it increases the responsiveness of wages (hence
marginal costs) to real activity (higher $\eta$). The net effect is what determines whether neo-Fisherianism arises or not: from condition (18), using previous notation, it requires $\xi \eta > f^*$. Moving to the right panel, we can see that $f(\gamma) \equiv \xi \eta$ is strictly declining in $\gamma$, indicating that the negative impact of wealth effects on $\xi$ overcomes their positive impact on $\eta$. For $\mu = 1.1$, the model turns from neo-Fisherian to the conventional wisdom as $\gamma$ becomes larger than $\gamma^* \approx 0.16$. By strengthening the demand-side channel (while having a neutral impact on wage procyclicality), a smaller price markup, $\mu = 1.01$ (bold red lines) widens the range of wealth effects yielding neo-Fisherianism: $\gamma^* \approx 0.28$.

Finally, Figure 4 inspects the model’s overall propensity to generate neo-Fisherianism for various combinations of wealth effects and markups. In all four panels, the white (neo-Fisherian) versus grey (conventional wisdom) areas are for the baseline calibration. It is noticeable that the range of neo-Fisherian markups $\mu$ quickly shrinks as the wealth effect gets stronger. The dashed lines plotted in each panel show that the range of wealth effects yielding neo-Fisherianism expands as we make the labor supply less wage-elastic (Panel b), prices more flexible (Panel c) and the shock more persistent (Panel d), but contracts as we raise risk aversion (Panel a).

5 Equilibrium Determinacy and Impulse Responses

Following Bullard and Mitra (2002), under a contemporaneous Taylor rule, determinacy of REE is guaranteed if and only if the following inequality holds true,

$$\kappa \eta (\phi_\pi - 1) + (1 - \beta) \phi_y > 0 \quad (36)$$

Whether neo-Fisherian dynamics expand the determinacy region of policy parameters depends on the intersection of line $\phi_\pi = 1 - \frac{(1-\beta)}{\kappa \eta} \phi_y$ with the $\phi_y$ axis. In particular, the region of determinacy expands as the intersection with the $\phi_y$ axis, namely, $\kappa \eta / (1 - \beta)$ decreases. Given the functional form of $\eta$ in (15), $\eta$ decreases with wealth effects decreasing but increases with price markups decreasing. Decreases in wealth effects and price markups both reinforce neo-Fisherianism. Importantly, all else equal, a one-unit change in $\gamma$ causes a significantly larger change in $\eta$ relative to the same one-unit change in $\mu$.

To see this, consider

$$\frac{\partial \eta}{\partial \gamma} = 1 + \frac{\chi + 2\gamma}{\mu (1 + \chi)} \quad \text{and} \quad \frac{\partial \eta}{\partial \mu} = -\frac{\gamma (\chi + \gamma)}{\mu^2 (1 + \chi)}$$

where $\frac{\partial \eta}{\partial \gamma} > -\frac{\partial \eta}{\partial \mu}$.

21To see this, consider
Figure 5: **Intersection of** $\phi_\pi = -\frac{1-\beta}{\kappa \eta} \phi_y + 1$ **with the** $\phi_y$ **axis.** Baseline parameterization: $\beta = 0.99$, $RA = FE = 1$, $\theta = 0.75$, $\rho = 0.5$.

effects and price markups reinforce neo-Fisherian dynamics, one should anticipate that $\eta$ will decline and the determinacy region will expand.\(^{22}\) As shown in Figure 5, the intersection gets smaller as wealth effects and price markups decrease, i.e., as neo-Fisherianism becomes more prominent.

With respect to the adoption of a forward-looking Taylor rule, there is an important caveat. For such a rule, Bullard and Mitra (2002) show that determinacy of REE is guaranteed if and only if (36) holds simultaneously with

$$\kappa \eta (\phi_\pi - 1) + (1 + \beta) \phi_y < \frac{2(1 + \beta)}{\xi} \quad \text{and} \quad \phi_y < \frac{1 + \beta}{\beta \xi}$$

With a forward-looking rule, the determinacy region of policy parameters is bounded from above. Therefore, we analyze how the neo-Fisherian hypothesis affects the policy parameters’ region for equilibrium determinacy by computing the area of the determinacy region, $A$. In Appendix A.5 we show that

$$A = \begin{cases} 
\frac{(1+\beta)^2}{\beta \xi \kappa} & \text{if } 1 - \beta^2 \leq \beta \kappa \eta \\
\frac{2(1-\beta^2)(1+\xi \kappa) - (\xi \kappa)^2}{\xi^2 \eta \kappa (1-\beta^2)} & \text{otherwise} 
\end{cases}$$

\(^{22}\)Note that if $\phi_y = 0$, we only need $\phi_\pi > 1$ for equilibrium determinacy, regardless of whether neo-Fisherianism or the conventional wisdom materializes.
Figure 6: **Computed area of policy parameters’ region that guarantees REE determinacy under a forward-looking Taylor rule.** Baseline parameterization: $\beta = 0.99$, $RA = FE = 1$, $\theta = 0.75$, $\rho = 0.5$.

Neo-Fisherianism is associated with higher $(\xi \eta \kappa)$, or higher $(\xi \eta)$. From (38), it is straightforward to see that higher values of $(\xi \eta \kappa)$ diminish the area value, thus shrinking the region of policy parameters that guarantee determinacy of REE.

The constraints on policy parameters $\phi_{\pi}$ and $\phi_{y}$ get tighter as utility approaches the GHH specification ($\gamma \to 0$) and market power disappears ($\mu \to 1$), leading to an almost nonexistent determinacy region. This is displayed in the 3D panel of Figure 6, with the surface to the left of the plane contoured with dashed lines denoting the area of the determinacy region for parameterization generating neo-Fisherian effects. Under KPR utility, the area ranges between 25 and 30 square units depending on the markup $\mu$. As wealth effects and markups diminish, the area moves toward zero, threatening the implementability of a forward-looking rule. These effects become even more pronounced as we assume a lower Frisch elasticity of labor (see Figure 7). However, irrespective of the latter, a forward-looking rule is always conducive to indeterminacy in a model featuring GHH utility and no market power on the part of the monopolistically competitive firms (i.e., $\mu \to 1$). Relatedly, Figure 8 visualizes the determinacy regions in the $(\phi_{y}, \phi_{\pi})$ plane when the Taylor rule is forward-looking for different wealth effects on labor supply. As the wealth effects on labor supply increase, the determinacy

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23A similarly large upper bound is obtained under the common consumption-labor separable MaCurdy utility.
24This is contrast with what happens in RBC models whereby equilibrium indeterminacy is less likely when the wealth effects are weak (see Jaimovich (2008); Abad et al. (2017)).
Figure 7: Computed area of policy parameters’ region that guarantees REE determinacy under a forward-looking Taylor rule for different values of Frisch elasticity. Baseline parameterization: $\beta = 0.99$, $RA = 1$, $\theta = 0.75$, $\rho = 0.5$.

The model’s impulse responses to the nominal shock are in line with the theoretical predictions, and give a better sense of the quantitative implications of wealth effects and markups. Figure 9 presents the case of 1 percent interest rate shock under a contemporaneous and forward-looking Taylor rule in the left and right panel, respectively. As displayed in both 3D panels, the initial response of the nominal interest rate is strictly increasing in both $\gamma$ and $\mu$, going from negative to positive as either one of the two (or both) parameters becomes suffi-
Figure 9: Impact of a 1 percent positive monetary shock on nominal interest rates and inflation. Left panel (a): contemporaneous Taylor rule; right panel (b): forward-looking Taylor rule. Parameterization: $\beta = 0.99$, $RA = FE = 1$, $\theta = 0.75$, $\rho = 0.5$, $\phi_\pi = 1.1$, $\phi_y = 0$.

1. Concluding Remarks

Since the Great Recession, in the majority of developed economies, inflation has been rarely above its policy target, despite the fact that short-term nominal interest rates have been intentionally kept at (close to) zero to stimulate economic activity. This apparently puzzling outcome has become fertile ground for the so-called neo-Fisherian view, according to which central banks should start thinking about raising interest rates to raise inflation. As shown by previous contributions, the baseline New Keynesian model - the backbone of larger scale DSGE models used by central banks for monetary policy analysis - can deliver neo-Fisherian
effects when either a) monetary policy is active while prices are sufficiently flexible and/or the underlying nominal shock is sufficiently persistent (Garin et al. (2018); Rupert and Sustek (2019)), or b) monetary policy is passive (e.g., a pure peg, like during ZLB episodes) and fiscal forces pin down inflation to guarantee intertemporal debt sustainability (Cochrane (2018b); Bilbiie (2021)).

In this paper, by introducing Jaimovich-Rebelo-type preferences into an otherwise standard New Keynesian framework, we have assessed the extent to which neo-Fisherianism may depend on the strength of the wealth effects entering households’ labor supply decisions, spanning from the zero wealth effects implied by a GHH utility to the full wealth effects implied by a KPR utility. Our key finding is that neo-Fisherianism is more prominent in environments where wealth effects are weaker and the price-to-wage markups (indexing how distorted the steady-state equilibrium is) are smaller. Both channels contribute in fact to generate a larger (more negative) temporary elasticity of inflation to the initial nominal shock, which, through the Taylor rule’s endogenous feedback mechanism, facilitates a drop in the nominal interest rate along the equilibrium path. As a result, in comparison with previous contributions, we no longer need to assume low nominal rigidities, strong exogenous persistence, or a passive monetary/active fiscal policy regime to retrieve a positive short-run correlation between inflation and the policy rate. Importantly, we show that the occurrence of neo-Fisherian dynamics is robust to policy parameters and to whether the Taylor rule is contemporaneous or forward-looking. Interestingly, we also find that, for those parameterizations yielding neo-Fisherian effects, the equilibrium might be indeterminate if monetary policy takes the form of a forward-looking Taylor rule. This is in contrast with previous works showing that, in the context of neoclassical growth models, weaker wealth effects are beneficial to equilibrium determinacy.
A Appendix

A.1 Derivation of Aggregate Euler Equation and Phillips Curve

Consider the Euler equation (10), where $\hat{x}_t = \hat{c}_t + \hat{J}_t$. Log-linearization of the expressions for $K_t$ and $J_t$ in (3) yields:

$$
\hat{K}_t = \frac{\gamma(1-\gamma)}{\mu(1+\chi)} \hat{c}_t - \frac{\gamma}{\mu} \hat{h}_t
$$  \hspace{1cm} (A.1)

$$
\hat{J}_t = \frac{1-\gamma}{(1+\chi)\mu+\gamma-1} \hat{c}_t - \frac{1+\chi}{(1+\chi)\mu+\gamma-1} \hat{h}_t
$$  \hspace{1cm} (A.2)

Substituting these expressions into (10), together with the market clearing conditions $\hat{c}_t = \hat{y}_t$ and $\hat{h}_t = \hat{y}_t$, after simple manipulation of terms, we obtain the aggregate Euler equation (11), where $K_y = K_c - K_h$ and $J_y = J_c - J_h$.

From the optimal pricing rule of firms, we obtain the standard relationship between inflation and real marginal costs:

$$
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{m}c_t
$$  \hspace{1cm} (A.3)

for $\kappa \equiv \left(1-\beta(1-\theta)\right)^{-1}$. From the definition of marginal costs, $mc_t = w_t$, the household’s labor supply condition (6), the expression for $\hat{K}_t$ in (A.1) above, and market clearing, simple algebra gives $\hat{m}c_t = (\chi + \gamma - K_y)\hat{y}_t$. Substituting the latter into (A.3), we obtain the Phillips curve in (14).

A.2 Derivation of the Frisch Elasticity of Labor Supply

For the GHH case - where $\gamma = 0$ and $K_t = 1$ - log-linearizing the first-order conditions (4) and (6), we obtain:

$$
-\sigma_G \hat{x}_t = \hat{\lambda}_t
$$

$$
-\sigma_G \hat{x}_t + \chi_G \hat{h}_t = \hat{w}_t + \hat{\lambda}_t
$$
Combining these two equations and imposing $\hat{\lambda}_t = 0$ (a constant marginal utility $\lambda_t$), the Frisch elasticity of labor supply is simply the inverse of $\chi_G$:

$$\left.\frac{\partial \hat{h}_t}{\partial \hat{w}_t}\right|_{\hat{\lambda}_t=0} = \chi_G^{-1} \quad (A.4)$$

For the KPR case, the derivation is slightly more involved. Recalling that $\gamma = 1$ implies $J_t = K_t$, from the same first-order conditions, we find the following expressions:

$$\begin{align*}
-\sigma K \hat{x}_t + \hat{K}_t &= \hat{\lambda}_t \\
\hat{w}_t + \hat{\lambda}_t &= -\sigma K \hat{x}_t + \chi K \hat{h}_t + \hat{c}_t \\
\hat{x}_t &= \hat{c}_t + \hat{K}_t
\end{align*}$$

By simple substitution, after setting $\hat{\lambda}_t = 0$, the Frisch elasticity under KPR preferences is:

$$\left.\frac{\partial \hat{h}_t}{\partial \hat{w}_t}\right|_{\hat{\lambda}_t=0} = \left(\chi_K + \frac{2\sigma_K - 1}{\sigma K \mu}\right)^{-1} \quad (A.5)$$

Expressions (A.4) and (A.5) are entered in (32) in the main text.

For the more general case of $\gamma \in (0, 1)$, log-linearization of the first-order conditions (4) and (6), with $\hat{\lambda}_t = 0$, gives:

$$\begin{align*}
\hat{K}_t &= \sigma \hat{x}_t, \\
\hat{c}_t &= \hat{x}_t - \hat{J}_t, \\
\hat{w}_t &= -\sigma \hat{x}_t + \hat{c}_t + \chi \hat{h}_t.
\end{align*}$$

After combining the first two equations with the expressions for $\hat{K}_t$ and $\hat{J}_t$ in (A.1)-(A.2), simple algebra gives consumption as a function of labor:

$$\begin{align*}
\hat{c}_t &= C_h \hat{h}_t, \\
J_c &= \frac{1 - \gamma}{(1 + \chi)\mu + \gamma - 1}, \\
K_c &= \frac{\gamma (1 - \gamma)}{\mu(1 + \chi)}, \\
C_h &= \frac{\sigma J_h - K_h}{\sigma (1 + J_c) - K_c},
\end{align*}$$

We can then substitute the latter together with $\hat{x}_t = \sigma^{-1} \hat{K}_t = K_c \hat{c}_t - K_h \hat{h}_t$ back into the expression for $\hat{w}_t$ above:

$$\hat{w}_t = [\chi + K_h + C_h (\gamma - K_c)] \hat{h}_t. \quad (A.6)$$
The coefficient in front of $\hat{h}_t$ is the inverse Frisch elasticity of labor.

### A.3 Parameterization for the General Case

Let $IF = FE^{-1}$ denote the inverse Frisch elasticity. Recalling the definitions of $K_t$ and $J_t$ in (3), and letting $N_i$ denote the steady-state elasticity of variable $N$ with respect to $i$, for $N = K, J$ and $i = c, h$, as shown in Appendix A.2, the FE takes the following form:

$$FE = \left[ \chi + K_h + \frac{(\sigma J_h - K_h) (\gamma - K_c)}{\sigma (1 + J_c) - K_c} \right]^{-1} \tag{A.7}$$

For that concerning RA, simple calculus and algebra give:

$$RA = \sigma J_h + 1 + \chi \frac{K_h + 1 + \chi}{K_h + 1 + \chi - K_c} - K_c \tag{A.8}$$

From equation (A.7) and explicit expressions for $C_h, K_c$ and $K_h$ in Appendix A.1, we have:

$$IF = \chi + \frac{K_h}{A_0} + \frac{(\gamma - K_c) \sigma J_h - K_h}{A_1 \sigma (1 + J_c) - K_c},$$

where the terms $A_0$, $A_1$, $J_i$ and $K_i$ (for $i = h, c$) do not depend on $\sigma$. This equation can be solved explicitly for $\sigma_F$, the value of $\sigma$ that, for given $\chi$, allows us to match the empirical value for $IF$, namely,

$$\sigma_F = \frac{K_c (IF - A_0) - A_1 K_h}{(1 + J_c) (IF - A_0) - A_1 J_h} \tag{A.9}$$

Following a similar logic, we can use the expression for relative risk aversion RA in (A.8) - where neither $J_h$ nor $K_h$ nor $K_c$ depends on $\sigma$ - to solve for $\sigma_R$, the value of $\sigma$ that, for given $\chi$, allows us instead to match the value we assign to RA:

$$\sigma_R = \frac{K_h + 1 + \chi}{J_h + 1 + \chi} (RA + K_c) \tag{A.10}$$

Setting $\sigma_F = \sigma_R$, we obtain a non-linear equation that we can solve numerically for $\chi$. Plugging the latter into either (A.9) or (A.10) we find $\sigma$. 

30
A.4 Proofs

A.4.1 Proposition 1

Assume the conditions for equilibrium determinacy are met, under either a contemporaneous or a forward-looking rule. Using the method of undetermined coefficients, the unique REE has the following minimal state variable representations for inflation, output, and the nominal and real interest rates:

\[
\hat{\pi}_t = \frac{-\kappa \eta \xi}{D} \hat{v}_t, \\
\hat{y}_t = \frac{-\xi (1-\beta \rho)}{D} \hat{v}_t, \\
\hat{R}_t = \frac{(1-\beta \rho)(1-\rho) - \rho \xi \kappa \eta}{D} \hat{v}_t, \\
\hat{R}_t - E_t \hat{\pi}_{t+1} = \frac{(1-\beta \rho)(1-\rho)}{D} \hat{v}_t,
\]

(A.11)

\[
D \equiv \begin{cases} 
\xi (\kappa \eta (\phi_x - \rho) + \phi_y (1-\beta \rho)) + (1-\beta \rho)(1-\rho) > 0 & \text{(contemporaneous rule)} \\
\xi \rho (\kappa \eta (\phi_x - 1) + \phi_y (1-\beta \rho)) + (1-\beta \rho)(1-\rho) > 0 & \text{(forward-looking rule)}
\end{cases}
\]

(A.12)

Note that under both a contemporaneous and forward-looking Taylor rule, \( \kappa \eta \phi_x > \kappa \eta - (1-\beta \rho) \phi_y \) should hold in order for the equilibrium to be determinate. It is then straightforward to see that the determinacy condition also guarantees that \( D > 0 \).

Letting \( \sigma^2_v \equiv \frac{\sigma^2_z}{1-\rho^2} \) denote the unconditional variance of the nominal shock \( \hat{v}_t \), we have \( \text{Cov}(\hat{\pi}_t, \hat{R}_t) = \sigma^2_v A_{\pi} A_{R} \). Following Definition 1, the neo-Fisherian hypothesis holds when \( \sigma^2_v A_{\pi} A_{R} > 0 \). Then,

1. \( \xi < 0 \): as both \( A_{\pi} \) and \( A_{R} \) are strictly positive, in this case, the neo-Fisherian hypothesis always holds.

2. \( \xi > 0 \): with \( A_{\pi} < 0 \), the neo-Fisherian hypothesis holds if and only if \( (1-\beta \rho)(1-\rho) < \rho \xi \kappa \eta \).

A.4.2 Proposition 2

From Proposition 1 and the expression for \( \xi \) in (24), the neo-Fisherian condition in (18) becomes:

\[
\frac{\mu (1+\chi) - 1}{\sigma (1+\chi) (\mu - 1)} f(\mu) > \frac{(1-\beta \rho)(1-\rho)}{\rho \kappa f^*.}
\]

(A.13)
Let \( f(\mu) \) and \( f^* \) denote, respectively, the left- and right-hand side of (A.13). Since \( \mu > 1 \), \( f(\mu) \) is a strictly decreasing convex function of \( \mu \), with \( \lim_{\mu \to 1} f(\mu) = +\infty \) and \( f^{\lim} \equiv \lim_{\mu \to \infty} f(\mu) = \frac{\chi}{\sigma} \).

We have two separate cases.

**Case I:** \( \chi \geq \chi^* \equiv \sigma f^* \). Since in this case the lower bound \( f^{\lim} \) is never smaller than \( f^* \), we have \( f(\mu) > f^* \) for any \( \mu > 1 \), and the inequality (A.13) always holds. We can therefore conclude that if the (inverse) Frisch elasticity of labor supply is sufficiently large, \( \chi \geq \chi^* \), neo-Fisherian effects arise for any \( \mu > 1 \).

**Case II:** \( 0 \leq \chi < \chi^* \). This implies that \( f^{\lim} < f^* \). Hence, there exists a threshold value \( \mu^* \equiv \frac{(1+\chi)\chi^*-\chi}{(1+\chi)(\chi^*+\chi)} > 1 \) such that \( f(\mu) \geq f^* \) for \( \mu \geq \mu^* \). In this case, the strict inequality (A.13) holds - i.e., the model displays neo-Fisherianism - if and only if \( 1 < \mu < \mu^* \).

**A.4.3 Proposition 3**

The proof follows the same structure of the proof of Proposition 2. From Proposition 1 and the expression for \( \xi \) in (28), the neo-Fisherian condition in (18) becomes:

\[
\frac{1+(1+\chi)\mu}{\sigma(\mu)-1+1} \frac{f(\mu)}{f^*} < (1-\beta\rho) \frac{1-\rho}{\rho} \frac{\sigma}{f^*}.
\] (A.14)

Let \( f(\mu) \) and \( f^* \) denote, respectively, the left- and right-hand side of (A.14), and assume that \( \sigma \geq 1 \). Simple calculus shows that \( f(\mu) \) is a strictly decreasing convex function of \( \mu \), with \( f^{\max} \equiv f(1) = 2 + \chi \) and \( f^{\lim} \equiv \lim_{\mu \to \infty} f(\mu) = \frac{1+\chi}{\sigma} \). Letting \( \chi^H \equiv \sigma f^* - 1 \) and \( \chi^L \equiv f^* - 2 \), we have three separate cases.

**Case I:** \( \chi \geq \chi^H \). Since in this case the lower bound \( f^{\lim} \) is never smaller than \( f^* \), we have \( f(\mu) > f^* \) for any \( \mu > 1 \), and the inequality (A.14) always holds. We can therefore conclude that if \( \chi > \chi^H \) neo-Fisherian effects arise for any \( \mu \geq 1 \).

**Case II:** \( \chi^L < \chi < \chi^H \). In this case, \( f^{\lim} < f^* < f^{\max} \). Hence, there exists a threshold value \( \mu^* \equiv \frac{\sigma(1+\chi)(\chi^*+\chi)}{(1+\chi)(\chi^*+\chi)} > 1 \) such that \( f(\mu) \geq f^* \) for \( \mu \geq \mu^* \). In this case, the strict inequality (A.14) holds if and only if \( 1 < \mu < \mu^* \).

**Case III:** \( \chi \leq \chi^L \). In this case, since \( f^{\max} < f^* \), we have \( f(\mu) < f^* \) for any \( \mu > 1 \). Hence, the strict inequality (A.14) never holds: the model never displays neo-Fisherian features.
A.4.4 Proof of Proposition 4

Simple calculus shows that i) $f_G(\mu)$ is a strictly decreasing and convex function of $\mu$ with $\lim_{\mu \to 1} f_G(\mu) = \infty$ and $\lim_{\mu \to +\infty} f_G(\mu) = \frac{\chi G}{\sigma K} > 0$; and ii) $f_K(\mu)$ is a strictly decreasing and convex function of $\mu$ with $\lim_{\mu \to 1} f_K(\mu) = \frac{1+\sigma K\chi K}{\sigma K}$ and $\lim_{\mu \to +\infty} f_G(\mu) = \frac{\chi G}{\sigma K}$. It immediately follows that there exists a unique value, $\mu^c > 1$, such that $f_G(\mu) \geq f_K(\mu)$ for $\mu \leq \mu^c$.

A.5 Area of Determinacy Region Under a Forward-looking Taylor Rule

Bullard and Mitra (2002) identify the following three conditions that guarantee a determinate REE under a forward-looking Taylor rule:

$$\kappa \eta (\phi_\pi - 1) + (1 - \beta) \phi_y > 0 \quad \text{(A.15)}$$
$$\kappa \eta (\phi_\pi - 1) + (1 + \beta) \phi_y < \frac{2(1 + \beta)}{\xi} \quad \text{(A.16)}$$
$$\phi_y < \frac{1 + \beta}{\beta \xi} = \bar{\phi}_y \quad \text{(A.17)}$$

Consider then the following two lines in the $(\phi_y, \phi_\pi)$-plane,

$$\mathcal{L}_1 : \kappa \eta (\phi_\pi - 1) + (1 - \beta) \phi_y = 0 \quad \text{(A.18)}$$
$$\mathcal{L}_2 : \kappa \eta (\phi_\pi - 1) + (1 + \beta) \phi_y - \frac{2(1 + \beta)}{\xi} = 0 \quad \text{(A.19)}$$

Let $\tilde{\phi}_y = \frac{\kappa \eta}{1 - \beta}$ be the intersection of $\mathcal{L}_1$ with the $\phi_y$-axis, and $\bar{\phi}_y = \frac{2}{\xi} + \frac{\kappa \eta}{1 + \beta}$ be the intersection of $\mathcal{L}_2$ with the $\phi_y$-axis. The intersection of $\mathcal{L}_1$ with the $\phi_\pi$-axis happens at $\phi_\pi = 1$, whereas the intersection of $\mathcal{L}_2$ with the same axis occurs at $\phi_\pi = 1 + \frac{2(1+\beta)}{\xi \kappa \eta}$. Moreover, $\mathcal{L}_1$ and $\mathcal{L}_2$ intersect at $\phi_y = \bar{\phi}_y$.

One can show that if $1 - \beta^2 \geq \beta \xi \kappa \eta$, then $\phi_y \leq \bar{\phi}_y \leq \tilde{\phi}_y$. Therefore, the determinacy region lies between lines $\mathcal{L}_1$ and $\mathcal{L}_2$, which do not intersect for any $\phi_y \in [0, \tilde{\phi}_y]$. Hence, the area of the determinacy region in this case is
\[ A = \int_{0}^{\text{\bar{\phi}}_y} \left( 1 + \frac{2(1+\beta)}{\xi \kappa \eta} - \frac{1+\beta}{\kappa \eta} \phi_y \right) d\phi_y - \int_{0}^{\text{\bar{\phi}}_y} \left( 1 - \frac{1-\beta}{\kappa \eta} \phi_y \right) d\phi_y = \left( 1 + \frac{2(1+\beta)}{\xi \kappa \eta} \right) \frac{\text{\bar{\phi}}_y - \phi_y}{2} \]  

(A.20)

Substituting for \( \text{\bar{\phi}}_y \) and \( \phi_y \), we get the expression for \( A \) as in (38) for \( 1 - \beta^2 \geq \beta \xi \kappa \eta \).

On the other hand, if \( 1 - \beta^2 < \beta \xi \kappa \eta \), then \( \text{\bar{\phi}}_y < \phi_y < \phi_y \). Therefore, the determinacy region lies between lines \( \mathcal{L}_1 \) and \( \mathcal{L}_2 \), which now intersect at \( \phi_y = \text{\bar{\phi}}_y \). Hence, the area of the determinacy region in this case is given by

\[ A = 2 \int_{0}^{\text{\bar{\phi}}_y} \left( \frac{1+\beta}{\xi \eta \kappa} - \frac{\beta}{\kappa \eta} \phi_y \right) d\phi_y = \frac{(1+\beta)^2}{\xi^2 \beta \kappa \eta} \]  

(A.21)

A.6 Alternative Narratives

First, suppose that instead of the monetary shock, \( \hat{v}_t \), there is an inflation target shock, \( \hat{\pi}_t^* = \rho \hat{\pi}_{t-1}^* + \epsilon_t \), with \( \rho \in [0,1) \) and \( \epsilon_t \sim N(0, \sigma^2_\epsilon) \). The Taylor rule becomes

\[ \hat{R}_t = \phi_x \mathbb{E}_t(\hat{\pi}_{t+j} - \hat{\pi}_t^*) + \phi_y \mathbb{E}_t \hat{y}_{t+j}, \text{ for } j \in \{0,1\} \]  

(A.22)

One can then easily show that the minimum state variable solution for output, inflation, and nominal rates in the case of a contemporaneous Taylor rule is

\[ \hat{y}_t = \frac{\xi (1 - \beta \rho)}{D} \phi_x \hat{\pi}_t^* \quad \text{and} \quad \hat{\pi}_t = \frac{\xi \eta \kappa}{D} \phi_x \hat{\pi}_t^* \]  

(A.23)

\[ \hat{R}_t = \frac{\xi \eta \kappa \rho - (1 - \rho)(1 - \beta \rho)}{D} \phi_x \hat{\pi}_t^* \]  

(A.24)

where \( D = \xi (\kappa \eta (\phi_\phi - \rho) + (1 - \beta \rho) \phi_y) + (1 - \rho)(1 - \beta \rho) > 0 \).

In the case of a forward-looking Taylor rule, the solution is

\[ \hat{y}_t = \frac{\xi \rho (1 - \beta \rho)}{D} \phi_x \hat{\pi}_t^* \quad \text{and} \quad \hat{\pi}_t = \frac{\xi \eta \kappa \rho}{D} \phi_x \hat{\pi}_t^* \]  

(A.25)

\[ \hat{R}_t = \frac{\xi \eta \kappa \rho - (1 - \rho)(1 - \beta \rho)}{D} \phi_x \hat{\pi}_t^* \]  

(A.26)

where \( D = \xi \rho (\kappa \eta (\phi_\phi - \rho) + (1 - \beta \rho) \phi_y) + (1 - \rho)(1 - \beta \rho) > 0 \). Evidently, following an inflation
target shock, nominal rates and inflation move in the same (opposite) direction if and only if the condition in Proposition 1 holds (does not hold).

Second, suppose that instead of using a Taylor rule, the central bank fully commits to an exogenous inflation target, \( \hat{\pi}^*_t = \rho \hat{\pi}^*_{t-1} + \varepsilon_t \), with \( \rho \in [0, 1) \) and \( \varepsilon_t \sim \mathcal{N}(0, \sigma^2_\varepsilon) \). The Euler equation is

\[
\hat{y}_t = E_t \hat{y}_{t+1} - \xi (R^*_t - E_t \hat{\pi}_{t+1})
\]

where \( R^*_t \) is set by the monetary authority such that in equilibrium, inflation exactly equals the target. The Phillips curve is the same as in the main text, i.e., \( \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \eta \hat{y}_t \). In equilibrium, \( \hat{\pi}_t = \hat{\pi}^*_t \) and \( E_t \hat{\pi}_{t+1} = \rho \hat{\pi}^*_t \). From the Phillips curve we have \( \hat{y}_t = (1 - \beta \rho) \kappa \eta \hat{\pi}^*_t \), and from the Euler equation in (A.27) we have \( \hat{R}^*_t = \frac{\xi \eta \rho - (1 - \beta \rho)(1 - \beta \rho)}{\xi \eta \rho} \hat{\pi}^*_t \). Therefore, in the event of a positive shock to the inflation target, in order to ensure that inflation is exactly equal to the target, the central bank will have to increase (decrease) nominal rates if and only if the neo-Fisherian condition in Proposition 1 holds (does not hold).

A.7 Labor Subsidy and the Inverted Aggregate Demand Logic (IADL)

Suppose the fiscal authority collects lump-sum taxes from households, \( \tau_t \), and redistributes collections to monopolistically competitive firms in the form of a labor subsidy. Assume that the fiscal government runs a balanced budget such that in any period \( \tau_t = sw_t \). Then, the marginal cost changes to

\[
m_{ct} = w_t (1 - s)
\]

Let \( \tilde{\mu} = \mu (1 - s) \). Then, at the steady-state equilibrium we have

\[
m\tilde{c} = \mu^{-1} \quad \text{and} \quad \tilde{w} = \tilde{\mu}^{-1}
\]

Combining (A.29) with the labor supply first-order condition in (6), we have that

\[
x = \frac{(1 + \chi) \mu + \gamma - 1}{(1 + \chi) \mu + \gamma} \tilde{y} \quad \text{(A.30)}
\]

\[
\tilde{y} = \left[ \frac{1 + \chi}{\psi((1 + \chi) \mu + \gamma)} \right]^{\frac{1}{\psi}} \quad \text{(A.31)}
\]
For the utility function to be well-defined, \( x_t \) and \( \bar{x} \) have to be strictly positive. Assumption 1 guarantees that to be the case.

**Assumption 1.** Let \( \tilde{\mu}^L = \frac{1 - \gamma}{\chi + 1} \in [0, 1) \). Assume \( \tilde{\mu} > \tilde{\mu}^L \).

Note that different from the case when \( s = 0 \), \( \tilde{\mu} \) can be less than unity. Closely following derivations in Appendix A.1, one can show that

\[
\xi = [\sigma(1 + J_y - \sigma^{-1}K_y)]^{-1} \tag{A.32}
\]

where

\[
J_y = -\frac{\gamma + \chi}{(1 + \chi)\mu + \gamma - 1} \quad \text{and} \quad K_y = -\frac{\gamma(\gamma + \chi)}{(1 + \chi)\mu} \tag{A.33}
\]

Let

\[
g(\tilde{\mu}) = 1 + J_y - \frac{1}{\sigma} K_y = \frac{(1 + \chi)(\tilde{\mu} - 1)}{(1 + \chi)\tilde{\mu} + \gamma - 1} + \frac{\gamma(\chi + \gamma)}{\sigma\tilde{\mu}(1 + \chi)}
\]

It is straightforward to see that for \( \tilde{\mu} > 1 \) - which would always be the case with no labor subsidy - \( \xi > 0 \). Moreover, \( \lim_{\tilde{\mu} \to (\tilde{\mu}^L)^+} g(\tilde{\mu}) = -\infty < 0 \), whereas \( \lim_{\tilde{\mu} \to 1} g(\tilde{\mu}) = \frac{\gamma(\chi + \gamma)}{\sigma(1 + \chi)} > 0 \). Therefore, there exists \( \tilde{\mu}^* < 1 \), such that for any \( \tilde{\mu}^L < \tilde{\mu} < \tilde{\mu}^* \), \( \xi < 0 \) or equivalently the IADL prevails.

Similarly, the wage procyclicality, \( \eta \), is described by

\[
\eta = \frac{(1 + \chi)\tilde{\mu} + \gamma}{(1 + \chi)\tilde{\mu}} (\gamma + \chi) \tag{A.34}
\]

Finally, the minimum state variable solutions for output, inflation, and nominal rates are given by equation (A.11). In the case of an IADL with \( \xi < 0 \), inflation and nominal rates will both respond positively (respectively, negatively) to a contractionary (respectively, expansionary) monetary shock, yielding neo-Fisherian dynamics.
References


