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Liquidity, Capital Pledgeability and Inflation Redistribution[*]

Paola Boel[†]   Julián P. Díaz[‡]   Daria Finocchiaro[§]

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Abstract

We study the redistributive effects of expected inflation in a microfounded monetary model with heterogeneous discount factors and collateral constraints. In equilibrium, this heterogeneity leads to borrowing and lending. Model assumptions also guarantee a tractable distribution of money and capital holdings. Several results emerge from our analysis. First, in this framework expected inflation is detrimental to capital accumulation. Second, expected inflation affects borrowing and lending when collateral constraints are present, thus also inducing redistributive effects through credit. Third, we find this channel to be regressive when we calibrate our model using US data. This is because the drop in borrowers' capital caused by inflation is larger when capital is used as collateral.

Keywords: Money, Heterogeneity, Collateral Constraint, Welfare Cost of Inflation

JEL codes: E40, E50

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1   Introduction

This paper contributes to the literature on the redistributive effects of long-run inflation. Work on this topic, however extensive, is still divided on whether expected inflation acts as a regressive or progressive tax. On the one hand, an expansionary monetary policy can be beneficial because it provides partial insurance to cash-poor agents, as shown in a variety of settings by Levine (1991), İmrohoroğlu (1992), Bhattacharya, Haslag, and Martin (2005), Molico (2006), Manuelli and Sargent (2010), Rocheteau, Weill, and Wong (2018), Chiu and Molico (2010), and Chiu and Molico (2011), among others. Also, Chatterjee and Corbae (1992) show that inflation can be viewed as a tax on savings since it lowers the real interest rate, thus reducing wefare for lenders and increasing it for borrowers. On the other hand, in models like Erosa and Ventura (2002), Albanesi (2007), Boel and Camera (2009), and Camera and Chien (2014), expected inflation acts as a regressive tax on transactions that are particularly costly for low-income households that rely more on cash as a means of payment.

Our paper contributes to this debate by building a model where expected inflation acts both as a tax on consumption *and* investment. In our framework, an increase in the steady-state growth of the money supply endogenously reduces monetary transactions. This drop in demand also has a negative impact on capital formation and in turn on collateralized debt, thereby redistributing wealth between lenders and borrowers. We thus uncover a novel redistribution mechanism linked to the credit channel of monetary policy.[1]

Our explored mechanism depends on the link between inflation and capital accumulation, which is a classic issue in monetary economics (Tobin 1965; Sidrauski 1967). An often cited channel through which inflation could have a positive effect on capital formation is the so-called Mundell-Tobin effect; that is, a reduction in the return on the nominal asset—due to an increase in inflation—induces a portfolio reallocation toward capital, thereby stimulating investment. A more recent branch of the literature has tackled this issue in microfounded models of money (Aruoba and Wright 2003; Lagos and Rocheteau 2008; Aruoba, Waller, and Wright 2011). In the first paper, inflation acts as a tax on investment and reduces capital accumulation, thus generating a reverse Mundell-Tobin effect. Aruoba, Waller, and Wright (2011) also show that this is in line with a negative semi-elasticity of investment with respect to inflation estimated on US data.

We follow the latter literature and formalize our idea in a microfounded monetary econ-

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1. Our proposed channel differs from Doepke and Schneider (2006), where *unexpected* inflation lowers the real value of assets and liabilities and redistributes wealth from lenders to borrowers. Our channel also differs from Loenser and Schabert (2020), because their focus is on the effects of expected inflation on *unsecured* debt.

omy in the spirit of Aruoba, Waller, and Wright (2011). Our paper builds on their framework, but incorporates heterogeneous rates of time preference and collateralized lending. Our work is therefore also related to the seminal article of Kiyotaki and Moore (1997) and the related papers such as Iacoviello (2005), Iacoviello and Neri (2010) and Kiyotaki and Moore (2019), which have shown the importance of collateral constraints in the amplification of aggregate fluctuations. We follow that literature in the formalization of our borrowing constraint, but focus on the long-run distributional effects of inflation. Related to our work, Ferraris and Watanabe (2008), Venkateswaran and Wright (2014), and Finocchiaro et al. (2018) explore the interplay between collateralized credit and steady-state inflation. However, our focus is on the redistributive channel that such interplay generates.

Our economy is characterized by sequential markets in each period. In the first one, agents are subject to stochastic trading opportunities, capital is a factor of production, and money is the only means of payment. In the second market, a final good is produced using only labor inputs and all portfolio decisions are made. Due to heterogeneity in the rates of time preference, borrowing and lending arise in equilibrium. As in Aruoba, Waller, and Wright (2011), expected inflation is a tax on trading in the first market, and via this channel, it also discourages capital accumulation in the second market. Since we assume borrowing and lending to be collateralized, expected inflation also reduces debt.

Quantitatively, inflation acts as a progressive tax overall when we calibrate our model using US data. However, when we isolate the redistributive effect generated by inflation via collateralized borrowing, we find the channel to be regressive; so, the overall effect is less progressive than it would be otherwise. That is, when collateral constraints are present, the welfare cost of expected inflation is higher for borrowers and lower for lenders compared to an economy with a fixed borrowing limit. This happens because the drop in borrowers' capital caused by inflation is larger when capital is used as collateral. Quantitatively, this regressive effect is important—the welfare cost of 10 percent inflation, relative to zero inflation, increases by 22 percent for borrowers and decreases by 15 percent for lenders compared to the case of a fixed borrowing limit.

The remainder of the paper is organized as follows. Section 2 describes the model, Section 3 builds and discusses the stationary monetary equilibrium, Section 4 discusses the calibration procedure and reports the quantitative results, and Section 5 concludes. Proofs and figures are in the appendix.

2 The model

The model builds on Lagos and Wright (2005), Kiyotaki and Moore (1997) and Aruoba, Waller, and Wright (2011). Time is discrete, the horizon is infinite and there is a large population of infinitely-lived agents who consume perishable goods and discount only across periods. In each period, agents may visit two sequential rounds of trade—we refer to the first as the decentralized market (DM) and the second as the centralized market (CM). In the CM, everybody can consume and work to produce a general good that can be used for either consumption or investment. Goods in the CM are produced using labor only, but capital accumulated in the CM becomes productive in the following DM. In the DM, agents draw i.i.d. trade shocks determining whether they can trade, i.e., agents can either produce, consume, or do neither (idle). Agents trade with probability σ and are idle with the remaining probability $1 - \sigma$, with consumption and production being equally likely. We refer to consumers as buyers and producers as sellers. Agents are heterogeneous in their degree of patience. For convenience, we divide the population into two types $j = H, L$ in proportions ρ and $1 - \rho$ respectively, with $1 > \beta_H > \beta_L > 0$. We refer to agents L as impatient and agents H as patient.

Notation is as follows. In the CM, an agent of type j consumes $x_j \geq 0$ goods and supplies $n_j \geq 0$ labor. As in Lagos and Wright (2005), preferences are quasi-linear with $U(x_j) - n_j$. We assume $U'(x) > 0$, $U''(x) \leq 0$, $U'(0) = +\infty$ and $U'(\infty) = 0$. In the DM, consumers of type j derive utility $u(q_j)$ from $q_j \geq 0$ consumption. We assume $u'(q) > 0$, $u''(q) \leq 0$, $u'(0) = +\infty$ and $u'(\infty) = 0$. An agent's capital k_j accumulated in the CM can be used in the following DM to produce a different good y_j with technology $f(k_j) = y_j$. We assume $f'(k) > 0$, $f''(k) \leq 0$, $f'(0) = +\infty$ and $f'(\infty) = 0$. Capital depreciates at a rate δ with $\delta \in (0, 1)$. Agents are price takers.

We assume a government exists that is in charge of monetary policy and is the only supplier of fiat money, of which an initial stock $M_0 > 0$ exists. We denote the gross growth rate of the money supply by $\pi = M_t/M_{t-1}$, where M_t indicates the money stock in the CM in period t. The central bank implements its long-term inflation goal by providing deterministic lump-sum injections of money $\tau = (\pi - 1)M_{t-1}$, which are given to private agents at the beginning of the CM. If $\pi > 1$, agents receive lump-sum transfers of money; if $\pi < 1$, the central bank must be able to extract money via lump-sum taxes from the economy.

2.1 Information frictions, money, and credit

The preference structure we selected generates a single-coincidence problem in the DM since consumers cannot produce. Moreover, three additional frictions characterize the DM. First,

agents are anonymous as in Kocherlakota (1998), since their trading histories in the goods markets are private information. This rules out trade credit between individual buyers and sellers. Second, there is no public communication of single trading outcomes, which in turn eliminates the use of social punishments to support gift-giving equilibria. Third, as in Aruoba, Waller, and Wright (2011), we assume buyers cannot carry capital to a seller's location in the DM and thus capital cannot be used as a means of payment. The combination of these frictions, together with the single coincidence problem, implies that sellers require immediate compensation from buyers. So, buyers must use money to acquire goods in the DM.

Money is not essential for trade in the CM, and indeed agents can finance their consumption by working, using money balances acquired earlier and borrowing. To model credit, we assume agents are allowed to borrow in the CM, subject to a collateral constraint. Specifically, in each period t agents borrow $p_{at}a_{t+1}$ (or lend $-p_{at}a_{t+1}$), where p_{at} is the price of a bond that delivers one unit of money in $t + 1$, and pay back a_t. Borrowing is subject to a collateral constraint à la Kiyotaki and Moore (1997): $a_{t+1} \leq \theta k_{t+1}$ with $0 < \theta \leq 1$. That is, agents can borrow as long as the repayment does not exceed the market value of a fraction θ of their capital in the next period. We also assume that any funds borrowed or lent in the CM are repaid in the following CM. We show that, even with quasi-linearity of preferences in the CM, there are gains from multi-period contracts due to time-preference heterogeneity. Of course, default is a serious issue in all models with credit. We simplify the analysis by assuming a mechanism exists that ensures repayment of loans in the CM.

3 Stationary monetary allocations

In what follows, we focus on stationary monetary outcomes such that end-of-period real money, capital, and bond balances are time invariant. Due to stationarity, we simplify notation omitting t subscripts and use a $'$ superscript to identify next-period variables, when necessary. Accordingly, we let p_1 denote the nominal price of DM goods and p_2 the price in the CM. In addition, we choose to work with real variables normalizing all nominal variables by the unit of account p_2 so that $p = p_1/p_2$ denotes the relative price.

An agent of type $j = H, L$ starts each period with a portfolio $\omega_j = m_j, k_j, a_j$, listing real holdings of money m_j, capital k_j, and nominal bonds a_j, all accumulated in the preceding period. Then, the idiosyncratic trading shock $z = b, s, o$ is realized, where b, s, and o denote a buyer, a seller, and an idle agent respectively. Subsequently, trade occurs and after the DM closes, the agent enters the CM with a portfolio $\omega_j^z = m_j^z, k_j, a_j$ where m_j^z denotes money

balances at the beginning of the CM. Real balances evolve within each period according to:

$$m_j^b = m_j - pq_j, \ \ m_j^s = m_j + py_j = m_j + pf(k_j) \ \text{ and } \ m_j^o = m_j \tag{1}$$

That is, when agents trade in the DM, real balances of a buyer decrease by pq_j whereas those of a seller increase by $pf(k_j)$. Cash left over is used to trade in the CM. Note that bonds a_j and capital k_j are not affected by trade in the DM, given that they cannot be used as a means of payment there. Therefore, $a_j^z = a_j$ and $k_j^z = k_j$ for $j = H, L$ and $z = b, s, o$.

In a stationary economy real asset holdings must be constant, i.e., $m_j' = m_j$, $k_j' = k_j$ and $a_j' = a_j$. If M is cash at the start of a period and $M' = \pi M$ is cash available in the CM, then:

$$\frac{p_2'}{p_2} = \frac{M'}{M} = \pi \tag{2}$$

Money-market clearing also implies that the stationary real money stock $\bar{m} = M/p_2$ is:

$$\bar{m} = \rho m_H + (1 - \rho)m_L \tag{3}$$

3.1 The CM problem

Given the recursive structure of the model, a dynamic programming approach is used to describe the problem faced by an agent of type j. Let $V_j(\omega_j)$ be the agent's expected lifetime utility when she starts a period with a portfolio ω_j before trade shocks are realized. Let instead $W_j(\omega_j^z)$ be the expected lifetime utility when entering a CM with a portfolio ω_j^z. The problem of a type j agent at the start of a CM then is:

$$W_j(\omega_j^z) = \max_{x_j^z, n_j^z, m_j', k_j'} U(x_j^z) - n_j^z + \beta_j V_j'(\omega_j') \tag{4}$$

$$\text{s.t. } x_j^z = n_j^z - k_j' + (1 - \delta)k_j + m_j^z - \pi m_j' + \tau + p_a \pi a_j' - a_j$$

$$a_j' \leq \theta k_j'$$

$$k_j' \geq 0$$

where $a_j' \leq \theta k_j'$ is the collateral constraint. The resources available to the agent in the CM thus depend on the realization of the DM trading shock $z = b, s, o$ since she carries over m_j^z money balances. Other resources are labor income n_j^z, net capital holdings $(1 - \delta)k_j$, lump-sum transfers τ, and $\pi a_j'$ borrowing at price p_a (lending if $\pi a_j' < 0$). These resources can be used to finance current consumption x_j^z, to carry over real money holdings $\pi m_j'$ and capital k_j' into the next period, and to pay back loans a_j. Note that the factor $\pi = p_2'/p_2$ multiplies m_j' and a_j' because the budget constraint is expressed in terms of real variables.

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Substituting for n_j^z into (4) it follows that:[2]

$$W_j(\omega_j^z) = \max_{x_j^z, m_j', k_j', a_j'} U(x_j^z) - x_j^z - k_j' + (1-\delta)k_j + m_j^z - \pi m_j' + \tau + p_a \pi a_j' - a_j + \beta_j V_j'(\omega_j')$$

$$\text{s.t. } a_j' \le \theta k_j'$$
$$k_j' \ge 0$$

We get the following first-order conditions:

$$1 = U'(x_j) \tag{5}$$

$$\pi = \beta_j \frac{\partial V_j'}{\partial m_j'} \tag{6}$$

$$1 = \beta_j \frac{\partial V_j'}{\partial k_j'} + \mu_j^k + \theta \lambda_j^a \tag{7}$$

$$-p_a = \frac{\beta_j}{\pi} \frac{\partial V_j'}{\partial a_j'} - \frac{\lambda_j^a}{\pi} \tag{8}$$

where μ_j^k is the multiplier on the non-negativity constraint for capital and λ_j^a is the multiplier on the collateral constraint. The first-order conditions above have several implications. First, (5) implies that all agents consume the same amount in the CM, i.e., $x_H = x_L = x^*$. Second, (6), (7), and (8) imply that the savings choices m_j', k_j' and a_j' are independent of trading histories but might be type dependent. To see exactly how, we will examine DM trades later. Also, note that the marginal valuations of money and capital are type independent as shown by the envelope conditions below:

$$\frac{\partial W_j}{\partial m_j^z} = 1, \ \frac{\partial W_j}{\partial a_j} = -1 \text{ and } \frac{\partial W_j}{\partial k_j} = 1 - \delta \tag{9}$$

Last, note that if we let $\omega_j^z = (0, 0, 0) \equiv 0$, then we have:

$$W_j(0) = U(x^*) - (x^* + k_j' + \pi m_j' - p_a \pi a_j' - \tau) + \beta_j V_j'(\omega_j') \tag{10}$$

Therefore, from (4) and (10) we have:

$$W_j(\omega_j^z) = W_j(0) + m_j^z + k_j(1 - \delta) - a_j \tag{11}$$

CM goods market clearing implies:

$$X + I = N \tag{12}$$

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2. Conditions for $n_j^z > 0$ are in the appendix.

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where $X = x^*$ and aggregate labor N satisfies:

$$N = \rho\left\{\frac{\sigma}{2}(n_H^s + n_H^b) + (1 - \sigma)n_H^o\right\} + (1 - \rho)\left\{\frac{\sigma}{2}(n_L^s + n_L^b) + (1 - \sigma)n_L^o\right\} \quad (13)$$

Moreover, aggregate capital satisfies:

$$K = \rho k_H + (1 - \rho)k_L \quad (14)$$

and investment is:

$$I = [\rho k_H' + (1 - \rho)k_L'] - (1 - \delta)[\rho k_H + (1 - \rho)k_L] \quad (15)$$

Last, the government budget constraint is:

$$\tau = \bar{m}(\pi - 1) \quad (16)$$

where $\bar{m}$ is defined in (3).

3.2   The DM problem

The problem of an agent of type j starting a generic period t with a portfolio ω_j is:

$$V_j(\omega_j) = \frac{\sigma}{2}V_j^b(\omega_j) + \frac{\sigma}{2}V_j^s(\omega_j) + (1 - \sigma)V_j^o(\omega_j) \quad (17)$$

$$\text{s.t. } pq_j \le m_j$$

First, we determine DM consumption q_j. The buyer's problem can be written as:

$$V_j^b(\omega_j) = \max_{q_j} \quad u(q_j) + W_j(\omega_j^b) \quad (18)$$

$$\text{s.t. } pq_j \le m_j$$

Given (1) and (11), we have the following first-order condition:

$$u'(q_j) + \frac{\partial W_j}{\partial m_j^b}\frac{\partial m_j^b}{\partial q_j} - \lambda_j^b p = 0 \quad \Rightarrow \quad u'(q_j) = p(1 + \lambda_j^b) \quad (19)$$

where λ_j^b is the multiplier on the buyer's budget constraint. If the buyer is constrained and $pq_j = m_j$, then $u'(q_j) > p$. If the buyer is not constrained, then $pq_j < m_j$ and $u'(q_j) = p$.

The seller's problem is trivial since capital has already been chosen in the previous CM and it is the only factor of production in the DM. Thus, the seller's problem is just the

continuation value:

$$V_j^s(\omega_j) = W_j(\omega_j^s) = m_j + pf(k_j) + (1 - \delta)k_j - a_j + W_j(0)$$

The DM market-clearing condition is:

$$\rho q_H + (1 - \rho)q_L = \rho f(k_H) + (1 - \rho)f(k_L) \tag{20}$$

3.3 Savings decisions

To find the optimal portfolio of an agent, we must calculate the expected marginal values $\partial V_j/\partial m_j$, $\partial V_j/\partial k_j$ and $\partial V_j/\partial a_j$. To do so, we combine (10) and (17) to get:

$$\begin{aligned} V_j(\omega_j) =& m_j + (1 - \delta)k_j + \frac{\sigma}{2}u(q_j) + U(x^*) - x^* + \beta_j V_j'(\omega_j') \\ &+ \frac{\sigma p}{2}(f(k_j) - q_j) + \tau - \pi m_j' - k_j' + p_a \pi a_j' - a_j \end{aligned} \tag{21}$$

Therefore:

$$\frac{\partial V_j}{\partial m_j} = 1 + \frac{\sigma}{2p}\left[u'(q_j) - p\right] \tag{22}$$

$$\frac{\partial V_j}{\partial k_j} = 1 - \delta + \frac{\sigma}{2}pf'(k_j) \tag{23}$$

$$\frac{\partial V_j}{\partial a_j} = -1 \tag{24}$$

From (6), (7), (8), (22), (23), and (24), the agents' optimal portfolio choices must satisfy:

$$\pi = \beta_j\left[1 + \frac{\sigma}{2p}(u'(q_j) - p)\right] \tag{25}$$

$$\pi p_a = \beta_j + \lambda_j^a \tag{26}$$

$$1 = \beta_j\left[(1 - \delta) + \frac{\sigma}{2}pf'(k_j)\right] + \mu_j^k + \theta\lambda_j^a \tag{27}$$

The central observation we derive from the Euler equations above is that money, bonds, and capital are valued differently in the economy by patient and impatient agents, since the expressions in (25), (26), and (27) all depend on the discount factor β_j. Of course, agents' decisions are also affected by asset-specific characteristics. First, the choice of real money balances described in (25) depends on the real yield on cash $1/\pi$ and on $u'(q_j)/p$, which can be interpreted as the expected liquidity premium from having cash in the DM, given that money is necessary to trade in that market. This premium grows with the severity of

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the cash constraint, which is reflected in $u'(q_j)$, and the likelihood of a consumption shock $\sigma/2$. Second, the choice of bonds described in (26) is influenced by the real yield $1/(\pi p_a)$. Whether the choice of bonds is affected by inflation will depend on the equilibrium price p_a. Note also that (26) reflects the fact that bonds have no liquidity premium, since they cannot be used to buy consumption in the DM. Last, the choice of capital described in (27) depends on the depreciation rate δ, the rate of return on capital $(\sigma/2)pf'(k_j)$, the fraction of capital that can be collateralized θ, and, importantly, by inflation, given that p enters both (25) and (27). Inflation matters for capital choices because money is needed to trade in the DM, and therefore inflation affects consumption there. This in turn influences capital accumulation, since capital is a factor of production in that market.

At this point, we can provide a definition of equilibrium.

Definition 1. *A symmetric stationary monetary equilibrium consists of (m_j, a_j, k_j) satisfying (25), (26), and (27) for $j = H, L$.*

The reason is that once the equilibrium stocks of money, bonds, and capital are determined, all other endogenous variables can be derived. We want to understand now if an equilibrium with collateralized borrowing indeed exists in this economy. We find that the following result holds:

Lemma 1. *A stationary monetary equilibrium exists with impatient agents borrowing and patient agents lending at the price $p_a = \beta_H/\pi$. Specifically, $a_L = \theta k_L$ and $a_H = -(1 - \rho)\theta k_L/\rho$.*

Why are agents interested in borrowing and lending even if they are always consuming the same quantity x^* in the CM? The reason lies in the heterogeneity in discount factors. For impatient agents, borrowing at the rate π/β_H is cheaper than carrying money across periods at the cost π/β_L. Thus, with quasi-linear preferences, they would like to borrow an infinite amount of the general good if at all possible. Because of the collateral constraint, their borrowing is bounded by θk_L and therefore a stationary equilibrium exists.

Once we know the price $p_a = \beta_H/\pi$ at which these bonds circulate in equilibrium, we can pin down the net borrowing rate:

$$i_a = \pi/\beta_H - 1 \tag{28}$$

which is affected directly by monetary policy via the inflation rate π.

We now want to determine the returns on money and bonds that are consistent with equilibrium.

Lemma 2. *Any stationary monetary equilibrium must be such that $\pi \geq \beta_H$.*

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This result derives from a simple no-arbitrage condition—in a monetary equilibrium, the value of money cannot grow too fast with $\pi < \beta_H$ or else type H agents will accumulate an infinite amount of it, which cannot be a stationary equilibrium. This, together with (28), implies that the borrowing rate $i_a = 0$ if $\pi \to \beta_H$. This is what we will refer to as the Friedman rule in the remainder of the paper.

Summing up, both policy π and discount factors β_H and β_L affect the expected returns on money, bonds, and capital, the latter of which influences agents' ability to borrow due to the collateral constraint. Since patient and impatient agents price future consumption differently, we expect their portfolio choices to differ, with effects on their consumption choices, labor effort and welfare. We investigate this next and we first show that the existence of borrowing and lending in equilibrium hinges on the concavity of the DM production function.

Lemma 3. *If $\pi \geq \beta_H$ and $f''(k_L) = 0$, $k_L = 0$ in a stationary monetary equilibrium.*

Thus, when the production function is linear, there cannot be any collateralized borrowing in equilibrium because in this case the return on capital is too low for the impatient agents to hold any. Collateralized borrowing, however, exists when the production function is concave, as we show next.

Proposition 1. *Let $\pi \geq \beta_H$ and $f''(k_L) < 0$. There exists a stationary monetary equilibrium with $m_H > m_L > 0$, $k_H > k_L > 0$, $a_L = \theta k_L$ and $a_H = -\theta \rho k_L / (1 - \rho)$.*

The equilibrium described in Proposition 1 is one in which patient agents are savers and hold more assets—both money and capital—than impatient ones, who are also borrowers. This must be an equilibrium because patient and impatient agents value future consumption at different rates and thus assets have different discounted returns for them.

Our ultimate goal here is to understand if expected inflation can have redistributive effects via borrowing and lending. We already know from Lemma 1 that the return on bonds adjusts perfectly for inflation. So, if any redistribution exists through the credit channel, it has to operate through the collateral constraint's principal θk_L. We proceed as follows. We first identify how equilibrium allocations are affected by collateral constraints and we then pin down how collateral constraints are affected by inflation.

Proposition 2. *Let $\pi \geq \beta_H$, $a_L = \theta k_L$ and $a_H = -(1 - \rho)k_L / k_H$. Then, $dp/d\theta < 0$, $dq_H/d\theta > 0$, $dq_L/d\theta > 0$, $dk_H/d\theta < 0$, and $dk_L/d\theta > 0$.*

From Proposition 2 we learn that collateral constraints lead to higher capital holdings for borrowers, since they can use it as collateral. At the same time, agents of type H need to accumulate less capital, since they can lend out more now. Interestingly, this implies that

collateral constraints reduce wealth inequality. Next, we investigate how agents' portfolio choices are affected by inflation.

Proposition 3. *Let* $\pi \geq \beta_H$, $a_L = \theta k_L$ *and* $a_H = -(1 - \rho)k_L/k_H$. *Then,* $dk_j/d\pi < 0$ *for* $j = H, L$ *and* $dq_H/d\pi < 0$.

Inflation here reduces investment, just as in Aruoba, Waller, and Wright (2011). As inflation increases, the real value of money decreases together with DM consumption, since money is necessary to buy in that market. This implies that q_H decreases as well. As demand for consumption is reduced, less capital is necessary to produce a lower output. In the following proposition, we show that the effect of inflation on impatient agents' consumption depends on the coefficient of relative risk aversion $\mu = -\frac{u''(q)}{u'(q)}q$ and the elasticity of the production function with respect to capital $\varepsilon = \frac{f'(k)k}{f(k)}$.

Proposition 4. *Assume* $f(k)$ *is isoelastic. Let* $\pi \geq \beta_H$, $a_L = \theta k_L$, $a_H = -(1 - \rho)k_L/k_H$ *and* $\bar{\mu} = \frac{1-\alpha}{\alpha} \frac{\beta_H - \beta_L}{\beta_H} \left(\frac{2-\sigma}{\sigma}\right)$. *Then,* $dq_L/d\pi < 0$ *if* $\mu > \bar{\mu}$.

Summing up, expected inflation affects prices, consumption, capital accumulation, and the collateral constraint in return. On the one hand, an increase in long-run inflation will reduce the amount of labor necessary to pay for borrowing and investment, but on the other hand, inflation will also reduce consumption—at least for lenders—and returns from capital. The balance of these different effects is ambiguous and must be determined quantitatively, as we do next.

4 Quantitative analysis

In this section, we calibrate the model for the United States. Data are annual for the sample period 1929–2019. The nominal interest rate i is the annualized yield on short-term commercial paper, the nominal price level P is the CPI, aggregate nominal output PY is nominal GDP, and the nominal money supply M is M1.[3] For ease of comparison with other studies based on Lagos and Wright (2005), we consider the following functional forms: $U(x) = B\ln(x)$, $f(k) = k^\alpha$, $u(q) = \ln(q)$. Therefore, the vector of parameters to identify is $(B, \sigma, \rho, \theta, \beta_H, \beta_L, \delta, \alpha)$.

We proceed as follows. We set the proportion of patient agents ρ to 0.40, in line with

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3. For 1929–1975, the yield on commercial paper is from Friedman and Schwartz (1982) (Table 4.8, col. 6). For 1976–1996, it is from the Economic Report of the President (1996, Table B-69). For 1997–2019, it is from the St. Louis Fed FRED Database. M1 is in billions of dollars, December of each year, not seasonally adjusted. For 1929–1958, it is from Friedman and Schwartz (1982) (pp. 708–718, col. 7). For 1959-2019, it is from the St. Louis Fed FRED Database. For 1929-2019, nominal GDP and CPI are from the St. Louis Fed FRED Database.

Boel and Camera (2009). We consider discount factors $\beta_H = 0.97$ and $\beta_L = 0.89$, which are consistent with Iacoviello and Neri (2010) and with the empirical estimates of distributions of discount factors in Lawrance (1991), Carroll and Samwick (1997), and Samwick (1998). We use a depreciation rate $\delta = 0.07$ as in Aruoba, Waller, and Wright (2011). We set $\theta = 0.85$, which is consistent with the loan-to-value ratio in Iacoviello and Neri (2010), among others.

We calibrate the remaining parameters α, σ and B simultaneously. We proceed as follows. First, we fit the ratio $L = M/PY$. As explained in Lagos and Wright (2005), this relationship can be interpreted as money demand in the sense that the desired real balances M/P are proportional to Y, with a factor of proportionality L that depends on the opportunity cost of holding cash, i. The theoretical expression for money demand in the model is $L = \bar{m}/(\frac{\sigma}{2}p(\rho q_H + (1-\rho)q_L) + B + \delta(\rho k_H + (1-\rho)k_L))$, where the numerator denotes average money holdings and the denominator is the sum of output in the DM and the CM. Since average money holdings are $\bar{m} = p(\rho q_H + (1-\rho)q_L)$, the theoretical money demand in equilibrium becomes:

$$L = \cfrac{1}{\cfrac{\sigma}{2} + \cfrac{B + \delta(\rho k_H + (1-\rho)k_L)}{p(\rho q_H + (1-\rho)q_L)}} \tag{29}$$

Second, we match the theoretical share of DM consumption Λ with its empirical counterpart. The latter is calculated as the share of cash consumption transactions using data from the Diary of Consumer Payment Choice for the period 2015–2019.[4] The theoretical expression for DM consumption is $\Lambda = (\frac{\sigma}{2}p(\rho q_H + (1-\rho)q_L))/(\frac{\sigma}{2}p(\rho q_H + (1-\rho)q_L) + B)$, where the numerator is DM consumption and the denominator is the sum of DM and CM consumption. Simplifying, we get:

$$\Lambda = \cfrac{1}{1 + \cfrac{2B}{\sigma p(\rho q_H + (1-\rho)q_L)}} \tag{30}$$

Last, we match the theoretical capital share of income Φ with its empirical counterpart. The latter is calculated as $1 -$ Compensation of Employees/GDP, using data for 1929-2019 from the Bureau of Economic Analysis. The theoretical share Φ in equilibrium is:

$$\Phi = \cfrac{\cfrac{\alpha\sigma}{2}p(\rho f'(k_H) + (1-\rho)f'(k_L))(\rho k_H + (1-\rho)k_L)}{\cfrac{\sigma}{2}p(\rho f(k_H) + (1-\rho)f(k_L)) + B + \delta(\rho k_H + (1-\rho)k_L)} \tag{31}$$

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4. In equilibrium, consumers use all their money holdings in the DM.

Following this approach, we pin down $B = 2.84$, $\sigma = 0.61$ and $\alpha = 0.70$. Table 1 summarizes our calibration and Figure 1 shows how the calibrated money demand (solid line) defined in (29) fits the data for M/PY in the sample period (circles).

Table 1: Calibrated parameters

Parameter	Value
β_H	0.97
β_L	0.89
θ	0.85
ρ	0.40
α	0.70
δ	0.07
B	2.84
σ	0.61

With the calibrated parameters, we quantify the welfare cost of inflation for both type L and type H agents, i.e., borrowers and lenders, respectively. Our measure of the welfare cost of inflation follows Lucas (2000) and asks how much an agent of type $j = H, L$ would be willing to give up in terms of total consumption to have $\gamma - 1$ instead of $\pi - 1$ inflation. Fixing π, we let $q_{j\pi}$, $k_{j\pi}$, p_π, $m_{j\pi}$ and $\bar{m}_\pi$ denote equilibrium quantities. Then, we use (4) and (17) to define equilibrium ex-ante welfare $\bar{V}_j$ for an agent of type j:

$$
\begin{aligned}
(1 - \beta_j)\bar{V}_j(\omega_{j\pi}) = & \frac{\sigma}{2}u(q_{j\pi}) + U(x^*) - x^* - \delta k_{j\pi} \\
& + \frac{\sigma}{2}p_\pi(f(k_{j\pi}) - q_{j\pi}) + (\pi - 1)(\bar{m}_\pi - m_{j\pi}) - a_{j\pi}(1 - \beta_H)
\end{aligned}
\tag{32}
$$

Inflation π affects ex-ante welfare in several ways. It impacts DM consumption $\frac{\sigma}{2}u(q_{j\pi})$ and also capital accumulation in the CM, thus affecting the labor effort necessary to make up for the depreciated capital $\delta k_{j\pi}$. These are the only distortions induced by inflation in a representative-agent setting, since in that case the second line in (32) vanishes because $m_{j\pi} = m_\pi$, $k_{j\pi} = k_\pi$, $q_{j\pi} = q_\pi$ for $j = H, L$, $f(k_\pi) = q_\pi$ and there is no credit. Instead, with heterogeneity, inflation generally affects $\bar{V}_j$ through three additional channels. First, it impacts expected net earnings in the DM $\frac{\sigma}{2}p_\pi(f(k_{j\pi}) - q_{j\pi})$, which can be nonzero since agents may produce and consume different amounts. Second, given that money balances are heterogeneous, inflation also redistributes monetary wealth because of inequalities in the net inflation tax $(\pi - 1)(\bar{m}_\pi - m_{j\pi})$. Third, inflation affects the collateral constraint and thus the amount impatient agents borrow in equilibrium $a_{j\pi} = \theta k_{j\pi}$. Note that the cost of borrowing adjusts perfectly for inflation, given that $p_a = \beta_H/\pi$. Thus, inflation would not affect credit in an economy with an exogenous borrowing limit, since in that case neither the principal

nor the cost of borrowing would change because of inflation.

If we reduce π to γ, but also change the consumption of both CM and DM goods by a factor $\Delta_{j\gamma}$, ex-ante welfare for an agent j becomes:

$$(1 - \beta_j)\bar{V}_j(\omega_{j\gamma}) = \frac{\sigma}{2}u(\Delta_{j\gamma}q_{j\gamma}) + U(\Delta_{j\gamma}x^*) - x^* - \delta k_{j\gamma} \tag{33}$$
$$+ \frac{\sigma}{2}[p_\gamma(f(k_{j\gamma}) - q_{j\gamma})] + (\gamma - 1)(\bar{m}_\gamma - m_{j\gamma}) - a_{j\gamma}(1 - \beta_H)$$

We measure the cost of $\pi - 1$ inflation as opposed to $\gamma - 1$ inflation for a type j agent as the value $\Delta_{j\gamma}$ that solves $\bar{V}_{j\pi}(\omega_{j\pi}) = \bar{V}_{j\gamma}(\omega_{j\gamma})$. That is, agents would be willing to give up $1 - \Delta_{j\gamma}$ percent of consumption to have $\gamma - 1$ rather than $\pi - 1$ inflation.

Table 2 reports the welfare costs of 10 percent expected inflation as opposed to both zero inflation and the Friedman rule for both borrowers (type L) and lenders (type H) as well as the average welfare cost of inflation for that economy. Note that, given Lemma 1, the Friedman rule in our framework corresponds to $\pi = \beta_H$ and therefore, using the calibrated values in Table 1, to a net inflation rate $\pi - 1 = -3$ percent. The first row in Table 2 reports the results for the case when borrowing is subject to a collateral constraint, i.e., $a'_j \leq \theta k'_L$. The second row focuses on the case of a fixed borrowing limit, i.e., $a'_j \leq A$.[5] The last row displays results for the case where credit is shut down, i.e. $\theta = 0$. Under all scenarios, we use the calibrated parameters listed in Table 1 to quantify welfare costs.

Table 2: Percentage welfare cost of 10% inflation relative to zero inflation and the Friedman rule.

Model	Zero Inflation			Friedman Rule		
	Type L	Type H	Average	Type L	Type H	Average
Collateralized borrowing	0.55	1.43	0.90	0.68	1.92	1.17
Fixed borrowing limit	0.45	1.68	0.94	0.53	2.27	1.23
No credit	0.45	1.68	0.94	0.53	2.27	1.23

Notes: Welfare costs are expressed in consumption percentage points. At the Friedman rule, the net inflation rate is $\beta_H - 1 = -3\%$. In the models with borrowing—both collateralized and uncollateralized—type H agents are lenders and type L agents are borrowers. The average welfare cost of inflation is the weighted average of the welfare cost for type H and type L agents.

Several results emerge. First, expected inflation does affect welfare via collateralized borrowing, as is evident when comparing the first and second rows of Table 2. This happens because, as per Proposition 3, inflation reduces capital accumulation for borrowers and thus has an effect on the loan's principal. This effect is regressive. We know from Proposition 2 that collateral constraints increase capital holdings for borrowers and decrease them for

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5. For the calculations reported in Table 2, we set A to the level of the collateral constraint at 10 percent inflation. We show later that this specific level chosen for A is irrelevant for our calculations of the welfare cost of inflation.

lenders, thus reducing wealth inequality, as Figures 2 and 3 illustrate. However, we also know from Figure 4 that inflation reduces capital by a larger amount when k_L is used as collateral compared to the case of a fixed borrowing limit A. Quantitatively, this regressive effect is substantial. When a loan is collateralized, the welfare cost of 10 percent inflation relative to zero inflation increases by 22 percent for borrowers and decreases by 15 percent for lenders compared to the case of a fixed borrowing limit. When comparing 10 percent inflation to the Friedman rule, the welfare cost for borrowers increases by 28 percent and decreases by 15 percent for lenders, again using the case of a fixed borrowing limit as a benchmark.

Second, Table 2 shows that the welfare cost of inflation is the same in a model with an uncollateralized borrowing limit A and in a model with no credit. That is, expected inflation does not generate redistributive effects via the credit channel if borrowing is uncollateralized. When borrowing depends on an exogenous limit A, the borrowing constraint can be written as $a'_j \leq A$. Following the proof of Lemma 1, it is straightforward to show that type L agents borrow up to the limit A. Since in this case $p_a = \beta_H/\pi$, $a_L = A$ and $a_H = A(1-\rho)/\rho$, neither the cost of borrowing nor the loan principal depends on inflation. Indeed, Figure 5 shows that the welfare cost of 10 percent versus 0 percent inflation does not depend on the specific value taken by the borrowing limit A for either type H or type L agents, although, of course, it differs between the two.

Third, overall inflation remains a progressive tax. An increase in inflation reduces the real value of money balances and has a negative impact on capital formation. That is, inflation acts as a tax on savings here, and since lenders are saving more, they are facing a heavier tax burden. The regressive effect induced by collateral constraints, however significant, only mitigates the progressivity of the inflation tax.

5 Conclusion

Our study identifies and measures three effects of expected inflation in an economy with collateral constraints. First, inflation is detrimental to capital accumulation. Second, inflation affects borrowing and lending when collateral constraints are present, and thus it also induces redistributive effects through credit. The redistributive channel we identify is novel. Third, when we calibrate our model using US data, we find inflation acts as a progressive tax overall but the redistributive effects of inflation generated solely by collateralized borrowing are regressive and important in size, thus reducing the overall progressivity of the tax from expected inflation. This is because inflation causes a larger drop in borrowers' capital when debt is collateralized compared to the case of a fixed borrowing limit.

References

Albanesi, Stefania. 2007. "Inflation and Inequality." *Journal of Monetary Economics* 54 (4): 1088–1114. https://doi.org/10.1016/j.jmoneco.2006.02.009.

Aruoba, Boragan S., Christopher J. Waller, and Randall D. Wright. 2011. "Money and Capital." *Journal of Monetary Economics* 58 (2): 98–116. https://doi.org/10.1016/j.jmoneco.2011.03.003.

Aruoba, Boragan S., and Randall D. Wright. 2003. "Search, Money, and Capital: A Neo-classical Dichotomy." *Journal of Money, Credit and Banking* 35 (6): 1085–1105. http://www.jstor.org/stable/3649876.

Bhattacharya, Joydeep, Joseph H. Haslag, and Antoine Martin. 2005. "Heterogeneity, Re-distribution, and the Friedman Rule." *International Economic Review* 46 (2): 437–454. https://doi.org/10.1111/j.1468-2354.2005.00327.x.

Boel, Paola, and Gabriele Camera. 2009. "Financial Sophistication and the Distribution of the Welfare Cost of Inflation." *Journal of Monetary Economics* 56 (7): 968–978. https://doi.org/10.1016/j.jmoneco.2009.09.001.

Camera, Gabriele, and Yili Chien. 2014. "Understanding the Distributional Impact of Long-Run Inflation." *Journal of Money, Credit and Banking* 46 (6): 1137–1170. https://doi.org/10.1111/jmcb.12136.

Carroll, Christopher D., and Andrew A. Samwick. 1997. "The Nature of Precautionary Wealth." *Journal of Monetary Economics* 40 (1): 41–71. https://doi.org/10.1016/S0304-3932(97)00036-6.

Chatterjee, Satyajit, and Dean Corbae. 1992. "Endogenous Market Participation and the General Equilibrium Value of Money." *Journal of Political Economy* 100 (3): 615–646. https://doi.org/10.1086/261832.

Chiu, Jonathan, and Miguel Molico. 2010. "Liquidity, Redistribution, and the Welfare Cost of Inflation." *Journal of Monetary Economics* 57 (4): 428–438. https://doi.org/10.1016/j.jmoneco.2010.03.004.

———. 2011. "Uncertainty, Inflation, and Welfare." *Journal of Money, Credit and Banking* 43:487–512. https://doi.org/10.1111/j.1538-4616.2011.00448.x.

Doepke, Matthias, and Martin Schneider. 2006. "Inflation and the Redistribution of Nominal Wealth." *Journal of Political Economy* 114 (6): 1069–1097. https://doi.org/10.1086/508379.

Erosa, Andrés, and Gustavo Ventura. 2002. "On Inflation as a Regressive Consumption Tax." *Journal of Monetary Economics* 49 (4): 761–795. https://doi.org/10.1016/S0304-3932(02)00115-0.

Ferraris, Leo, and Makoto Watanabe. 2008. "Collateral Secured Loans in a Monetary Economy." *Journal of Economic Theory* 143 (1): 405–424. https://doi.org/10.1016/j.jet.2008.02.002.

Finocchiaro, Daria, Giovanni Lombardo, Caterina Mendicino, and Philippe Weil. 2018. "Optimal Inflation with Corporate Taxation and Financial Constraints." *Journal of Monetary Economics* 95 (C): 18–31. https://doi.org/10.1016/j.jmoneco.2018.02.003.

Friedman, Milton, and Anna J. Schwartz. 1982. *Monetary Trends in the United States and the United Kingdom, 1867–1975.* University of Chicago Press, for the National Bureau of Economic Research.

Iacoviello, Matteo. 2005. "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle." *American Economic Review* 95 (3): 739–764. https://doi.org/10.1257/0002828054201477.

Iacoviello, Matteo, and Stefano Neri. 2010. "Housing Market Spillovers: Evidence from an Estimated DSGE Model." *American Economic Journal: Macroeconomics* 2 (2): 125–64. https://doi.org/10.1257/mac.2.2.125.

İmrohoroğlu, Ayşe. 1992. "The Welfare Cost of Inflation under Imperfect Insurance." *Journal of Economic Dynamics and Control* 16 (1): 79–91. https://doi.org/10.1016/0165-1889(92)90006-Z.

Kiyotaki, Nobuhiro, and John Moore. 1997. "Credit Cycles." *Journal of Political Economy* 105 (2): 211–248. https://doi.org/10.1086/262072.

———. 2019. "Liquidity, Business Cycles, and Monetary Policy." *Journal of Political Economy* 127 (6): 2926–2966. https://doi.org/10.1086/701891.

Kocherlakota, Narayana R. 1998. "Money is Memory." *Journal of Economic Theory* 81 (2): 232–251. https://doi.org/10.1006/jeth.1997.2357.

Lagos, Ricardo, and Guillaume Rocheteau. 2008. "Money and Capital as Competing Media of Exchange." *Journal of Economic Theory* 142 (1): 247–258. https://doi.org/10.1016/j.jet.2006.07.005.

Lagos, Ricardo, and Randall D. Wright. 2005. "A Unified Framework for Monetary Theory and Policy Analysis." *Journal of Political Economy* 113 (3): 463–484. https://doi.org/10.1086/429804.

Lawrance, Emily C. 1991. "Poverty and the Rate of Time Preference: Evidence from Panel Data." *Journal of Political Economy* 99 (1): 54–77. https://doi.org/10.1086/261740.

Levine, David K. 1991. "Asset Trading Mechanisms and Expansionary Policy." *Journal of Economic Theory* 54 (1): 148–164. https://doi.org/10.1016/0022-0531(91)90110-P.

Loenser, Christian, and Andreas Schabert. 2020. "Monetary Policy, Financial Constraints and Redistribution." *International Economic Review* 61 (4): 1501–1529. https://doi.org/10.1111/iere.12468.

Lucas, Robert. 2000. "Inflation and Welfare." *Econometrica* 68 (2): 247–274. https://doi.org/10.1111/1468-0262.00109.

Manuelli, Rodolfo, and Thomas J. Sargent. 2010. "Alternative Monetary Policies in a Turnpike Economy." *Macroeconomic Dynamics* 14:727–762. https://doi.org/10.1017/S1365100509990940.

Molico, Miguel. 2006. "The Distribution of Money and Prices in Search Equilibrium." *International Economic Review* 47 (3): 701–722. https://doi.org/10.1111/j.1468-2354.2006.00393.x.

Rocheteau, Guillaume, Pierre-Olivier Weill, and Tsz-Nga Wong. 2018. "A Tractable Model of Monetary Exchange with Ex Post Heterogeneity." *Theoretical Economics* 13:1369–1423. https://doi.org/10.3982/TE2821.

Samwick, Andrew A. 1998. "Discount Rate Heterogeneity and Social Security Reform." *Journal of Development Economics* 57 (1): 117–146. https://doi.org/10.1016/S0304-3878(98)00080-7.

Sidrauski, Miguel. 1967. "Inflation and Economic Growth." *Journal of Political Economy* 75 (6): 796–810. https://doi.org/10.1086/259360.

Tobin, James. 1965. "Money and Economic Growth." *Econometrica* 33 (4): 671–684. https://doi.org/10.2307/1910352.

Venkateswaran, Venky, and Randall D. Wright. 2014. "Pledgability and Liquidity: A New Monetarist Model of Financial and Macroeconomic Activity." *NBER Macroeconomics Annual* 28 (1): 227–270. https://doi.org/10.1086/674600.

Appendix

A. Proofs

Conditions for $n_j^z \geq 0$

We now want to provide conditions that guarantee $n_j^z \geq 0$ in the equilibrium described in Proposition 1. Note that if $n_j^s \geq 0$ then $n_j^b > 0$. We check the conditions for both $n_H^s \geq 0$ and $n_L^s \geq 0$—type L agents pay a lower inflation tax when inflation is positive and their depreciated capital δk_L is also lower, so it is theoretically possible that they could work less.

From (4) and Proposition 1, in steady state we have:

$$n_H^s = x^* + \delta k_H - pf(k_H) + (\pi - 1)(m_H - \bar{m}) - \theta k_L \frac{1 - \rho}{\rho}(1 - \beta_H)$$

Since $\pi(m_H - \bar{m}) > 0$ and $\bar{m} = \rho m_H + (1 - \rho)m_L$ from (3), we have:

$$n_H^s > x^* - pf(k_H) - \theta k_L \frac{1 - \rho}{\rho}(1 - \beta_H) - (1 - \rho)m_H$$

Let p^*, q^* solve (25) and (27) for $j = H$ with $\pi = \beta_H$. Since $m_H \leq p^* q^*$ from (18), $pf(k_H) < p^* q^*$ from (20), $u'(q^*) = p^*$ from (19) and $k_H > k_L$ from Proposition 1, we have:

$$n_H^s > x^* - \theta k_H \frac{1 - \rho}{\rho}(1 - \beta_H) - 2u'(q^*)q^*$$

Let k^* solve $\frac{1}{\beta_H} = (1 - \delta) + \frac{\sigma}{2}u'(q^*)f'(k^*)$. Then we have:

$$n_H^s > x^* - \theta k^* \frac{1 - \rho}{\rho}(1 - \beta_H) - 2u'(q^*)q^* \tag{34}$$

Now we check under what conditions $n_L^s > 0$. From (4) and Proposition 1, in steady state we have:

$$n_L^s = x^* + \delta k_L - pf(k_L) + (\pi - 1)(m_L - \bar{m}) + \theta k_L(1 - \beta_H)$$

Since $m_L < \bar{m}$ where $\bar{m}$ is defined in (3), then:

$$n_L^s > x^* - pf(k_L) + \pi(m_L - \bar{m})$$

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Since $\pi m_H \leq \sigma p^* q^*$, $p f(k_L) < p^* q^*$ from (20) and $u'(q^*) = p^*$ from (19), we have:

$$n_L^s > x^* - (1 + \sigma \rho) u'(q^*) q^* \tag{35}$$

The condition for $n_H^s \geq 0$ in (34) is more stringent than the one for $n_L^s \geq 0$ in (35), and so from now on we focus on (34). This implies that a sufficient condition for $n_j^z \geq 0$ is:

$$x^* - \theta k^* \frac{1 - \rho}{\rho} (1 - \beta_H) - 2 u'(q^*) q^* \geq 0$$

That is, for $n_j^z \geq 0$ to hold we need x^* sufficiently bigger than q^* and k^*.

Proof of Lemma 1

From the Euler equation in (26) we have that the following condition must hold:

$$\beta_L + \lambda_L^a = \beta_H + \lambda_H^a$$

Since $\beta_H > \beta_L$, it must be that $\lambda_L^a > \lambda_H^a \geq 0$. If $\lambda_L^a > \lambda_H^a > 0$, then there is no borrowing or lending. If instead $\lambda_L^a > \lambda_H^a = 0$, then $a_L = \theta k_L$ and given the bonds market-clearing condition:

$$\rho a_H + (1 - \rho) a_L = 0 \tag{36}$$

we have that $a_H = -\theta k_L (1 - \rho)/\rho$. From (26) and $\lambda_H^a = 0$ we have that $\pi p_a = \beta_H$, and therefore $p_a = \beta_H / \pi$. $\square$

Proof of Lemma 2

We know from (19) that $u'(q_j) \geq p$ for $j = H, L$. This, together with (25), implies that $\pi \geq \beta_H$. $\square$

Proof of Lemma 3

If $f(k)$ is linear and $f'(k) = \gamma$ with $\gamma > 0$, then the Euler equations for capital in (27) for $j = H, L$ become:

$$\frac{1}{\beta_L} = (1 - \delta) + \frac{\sigma}{2} p \gamma + \frac{\mu_L^k}{\beta_L} + \frac{\theta(\beta_H - \beta_L)}{\beta_L} \tag{37}$$

$$\frac{1}{\beta_H} = (1 - \delta) + \frac{\sigma}{2} p \gamma + \frac{\mu_H^k}{\beta_H} \tag{38}$$

From (37) and (38), we get that the following condition has to hold:

$$\frac{(\beta_H - \beta_L)(1 - \theta\beta_H)}{\beta_L\beta_H} = \frac{\mu_L^k}{\beta_L} - \frac{\mu_H^k}{\beta_H}$$

which implies $\mu_L^k > 0$ and therefore $k_L = 0$. $\square$

Proof of Proposition 1

Let $\pi \geq \beta_H$. Since $\beta_H > \beta_L$, from (25) we know that $u'(q_H) < u'(q_L)$ and therefore $q_H > q_L$ and $m_H > m_L$. Now combine (26), (27) and $0 = \lambda_H^a < \lambda_L^a = \beta_H - \beta_L$ from Lemma 1 so that:

$$\frac{(\beta_H - \beta_L)(1 - \theta\beta_H)}{\beta_H\beta_L} = \frac{\sigma}{2}p(f'(k_L) - f'(k_H))$$

which implies $f'(k_L) > f'(k_H)$ and $k_L < k_H$.

Now we want to make sure that $k_L > 0$. In order to do that, type L agents must work less when $k_L > 0$ than when $k_L = 0$. If $k_L > 0$, we have:

$$n_{L,k_L>0}^s = x^* + \delta k_L - pf(k_L) + (\pi - 1)(m_L - \bar{m}) + (1 - \beta_H)\theta k_L$$

If instead $k_L = 0$, then we have:

$$n_{L,k_L=0}^s = x^* + (\pi - 1)m_L - (\pi - 1)\bar{m}$$

So, in order for $k_L > 0$ to hold we need $n_{L,k_L>0}^s < n_{L,k_L=0}^s$, and therefore:

$$p\frac{f(k_L)}{k_L} > \delta + (1 - \beta_H)\theta$$

Since $f(k)$ is a concave function and $f(0) = 0$, then it is always the case that $\frac{f(k_L)-f(0)}{k_L-0} > f'(k_L)$. Therefore, the following is a sufficient condition for $k_L > 0$:

$$pf'(k_L) > \delta + (1 - \beta_H)\theta \tag{39}$$

The intuition for (39) is that $k_L > 0$ if the marginal benefit of having more capital $pf'(k_L)$ is greater than its marginal cost $\delta + (1 - \beta_H)\theta$. If we combine (27) and (39), we have:

$$\frac{1}{\beta_L} - 1 + \delta\left(1 - \frac{\sigma}{2}\right) > \theta\left[\frac{\sigma}{2}(1 - \beta_H) + \frac{\beta_H - \beta_L}{\beta_L}\right] \tag{40}$$

With some algebra, $\frac{1}{\beta_L} - 1 + \delta\left(1 - \frac{\sigma}{2}\right) > \frac{\sigma}{2}(1 - \beta_H) + \frac{\beta_H - \beta_L}{\beta_L}$ can be rewritten as:

$$1 - \frac{\beta_L \sigma}{2} + \beta_L \delta\left(1 - \frac{\sigma}{2}\right) > \beta_H\left(1 - \frac{\beta_L \sigma}{2}\right)$$

The condition above always holds, since $1 - \frac{\beta_L \sigma}{2} > \beta_H(1 - \beta_L \frac{\sigma}{2})$ and $\beta_L \delta\left(1 - \frac{\sigma}{2}\right) > 0$. This implies the condition in (40) is always verified for $0 \leq \theta \leq 1$. $\square$

Proof of Proposition 2

If we differentiate (20), (25) and (27) with respect to θ for $j = H, L$ we get:

$$0 = \frac{\sigma}{2}\left[\frac{u''(q_j)}{p}\frac{dq_j}{d\theta} - \frac{u'(q_j)}{p^2}\frac{dp}{d\theta}\right] \quad \text{for } j = H, L \tag{41}$$

$$\frac{dk_H}{d\theta} = -\frac{dp}{d\theta}\frac{f'(k_H)}{pf''(k_H)} \tag{42}$$

$$\frac{dk_L}{d\theta} = -\frac{dp}{d\theta}\frac{f'(k_L)}{pf''(k_L)} - \frac{2(\beta_H - \beta_L)}{\sigma p \beta_L f''(k_L)} \tag{43}$$

$$\rho\frac{dq_H}{d\theta} + (1 - \rho)\frac{dq_L}{d\theta} = \rho f'(k_H)\frac{dk_H}{d\theta} + (1 - \rho)f'(k_L)\frac{dk_L}{d\theta} \tag{44}$$

If we plug (42) and (43) into (44), we find that the following conditions have to hold:

$$\frac{\sigma}{2}\left[\frac{u''(q_H)}{p}\frac{dq_H}{d\theta} - \frac{u'(q_H)}{p^2}\frac{dp}{d\theta}\right] = 0$$

$$\frac{\sigma}{2}\left[\frac{u''(q_L)}{p}\frac{dq_L}{d\theta} - \frac{u'(q_L)}{p^2}\frac{dp}{d\theta}\right] = 0$$

$$\rho\frac{dq_H}{d\theta} + (1 - \rho)\frac{dq_L}{d\theta} + \left[\rho\frac{f'(k_H)^2}{pf''(k_H)} + (1 - \rho)\frac{f'(k_L)^2}{pf''(k_L)}\right]\frac{dp}{d\theta} = 0$$

The system of equations can be rewritten as:

$$\begin{bmatrix} \frac{\sigma}{2}\frac{u''(q_H)}{p} & 0 & -\frac{\sigma}{2}\frac{u'(q_H)}{p^2} \\ 0 & \frac{\sigma}{2}\frac{u''(q_L)}{p} & -\frac{\sigma}{2}\frac{u'(q_L)}{p^2} \\ \rho & 1 - \rho & \Gamma(k_H, k_L) \end{bmatrix}\begin{bmatrix} \frac{dq_H}{d\theta} \\ \frac{dq_L}{d\theta} \\ \frac{dp}{d\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \Upsilon(k_L) \end{bmatrix}$$

where

$$\Gamma(k_H, k_L) = \rho\frac{f'(k_H)^2}{pf''(k_H)} + (1 - \rho)\frac{f'(k_L)^2}{pf''(k_L)} \tag{45}$$

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and

$$\Upsilon(k_L) = \frac{2(\beta_L - \beta_H)}{\sigma p \beta_L f''(k_L)}$$

so that $\Gamma(k_H, k_L) < 0$ and $\Upsilon(k_L) > 0$. We can calculate the following determinants:

$$|D^\theta| = \frac{\sigma^2}{4p^2} \left[u''(q_H)u''(q_L)\Gamma(k_H, k_L) + \rho \frac{u''(q_L)u'(q_H)}{p} + (1-\rho)\frac{u''(q_H)u'(q_L)}{p} \right] \tag{46}$$

so that $|D^\theta| < 0$;

$$|D_H^\theta| = \Upsilon \left(\frac{\sigma}{2p} \right)^2 [pu''(q_L)u'(q_H)] \tag{47}$$

so that $|D_H^\theta| < 0$;

$$|D_L^\theta| = \Upsilon \left(\frac{\sigma}{2p} \right)^2 [pu''(q_H)u'(q_L)] \tag{48}$$

so that $|D_L^\theta| < 0$;

$$|D_p^\theta| = \Upsilon \left(\frac{\sigma}{2p} \right)^2 u''(q_H)u''(q_L) \tag{49}$$

so that $|D_p^\theta| > 0$.

Using Cramer's rule, we know that:

$$\frac{dp}{d\theta} = \frac{|D_p^\theta|}{|D^\theta|}, \quad \frac{dk_H}{d\theta} = -\frac{dp}{d\theta}\frac{f'(k_H)}{pf''(k_H)}, \quad \frac{dq_H}{d\theta} = \frac{|D_H^\theta|}{|D^\theta|}, \quad \frac{dq_L}{d\theta} = \frac{|D_L^\theta|}{|D^\theta|}$$

Therefore, we have that $\dfrac{dp}{d\theta} < 0$, $\dfrac{dk_H}{d\theta} < 0$, $\dfrac{dq_H}{d\theta} > 0$ and $\dfrac{dq_L}{d\theta} > 0$. Since $\dfrac{dq_j}{d\theta} > 0$ for $j = H, L$ but $\dfrac{dk_H}{d\theta} < 0$, it must be that $\dfrac{dk_L}{d\theta} > 0$. $\square$

Proof of Proposition 3

If we differentiate (20), (25) and (27) for $j = H, L$ with respect to π we get:

$$\frac{1}{\beta_j} = \frac{\sigma}{2} \left[\frac{u''(q_j)}{p}\frac{dq_j}{d\pi} - \frac{u'(q_j)}{p^2}\frac{dp}{d\pi} \right] \tag{50}$$

$$\frac{dk_j}{d\pi} = -\frac{dp}{d\pi}\frac{f'(k_j)}{pf''(k_j)} \tag{51}$$

$$\rho\frac{dq_H}{d\pi} + (1-\rho)\frac{dq_L}{d\pi} = \rho f'(k_H)\frac{dk_H}{d\pi} + (1-\rho)f'(k_L)\frac{dk_L}{d\pi} \tag{52}$$

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If we plug (51) for $j = H, L$ into (52), we find the following system of equations has to hold:

$$
\begin{bmatrix}
\dfrac{\sigma}{2}\dfrac{u''(q_H)}{p} & 0 & -\dfrac{\sigma}{2}\dfrac{u'(q_H)}{p^2} \\[2ex]
0 & \dfrac{\sigma}{2}\dfrac{u''(q_L)}{p} & -\dfrac{\sigma}{2}\dfrac{u'(q_L)}{p^2} \\[2ex]
\rho & 1-\rho & \Gamma(k_H, k_L)
\end{bmatrix}
\begin{bmatrix}
\dfrac{dq_H}{d\pi} \\[2ex]
\dfrac{dq_L}{d\pi} \\[2ex]
\dfrac{dp}{d\pi}
\end{bmatrix}
=
\begin{bmatrix}
\dfrac{1}{\beta_H} \\[2ex]
\dfrac{1}{\beta_L} \\[2ex]
0
\end{bmatrix}
$$

for which the determinant is:

$$
|D| = \frac{\sigma^2}{4p^2}\left[u''(q_H)u''(q_L)\Gamma(k_H, k_L) + \rho\frac{u''(q_L)u'(q_H)}{p} + (1-\rho)\frac{u''(q_H)u'(q_L)}{p} \right] \tag{53}
$$

and $|D| < 0$. We also define the following determinants:

$$
|D_H| = \frac{\sigma}{2p}\left[\frac{1}{\beta_H}u''(q_L)\Gamma(k_H, k_L) - (1-\rho)\frac{1}{\beta_L}\frac{u'(q_H)}{p} + (1-\rho)\frac{1}{\beta_H}\frac{u'(q_L)}{p} \right] \tag{54}
$$

From (25) for $j = H, L$ we have that $(\beta_H - \beta_L)(1 - \sigma/2)2p/\sigma = \beta_L u'(q_L) - \beta_H u'(q_H)$ so that $u'(q_L)/\beta_H > u'(q_H)/\beta_L$ and $|D_H| > 0$.

$$
|D_p| = -\rho\frac{\sigma}{2\beta_H}\frac{u''(q_L)}{p} - (1-\rho)\frac{\sigma}{2\beta_L}\frac{u''(q_H)}{p} \tag{55}
$$

so that $|D_p| > 0$.

$$
|D_L| = \frac{1}{\beta_L}\frac{\sigma u''(q_H)}{2p^2}\left(\rho\frac{f'(k_H)^2}{f''(k_H)} + (1-\rho)\frac{f'(k_L)^2}{f''(k_L)} \right) - \rho\frac{\sigma}{2p^2}\left(\frac{u'(q_L)}{\beta_H} - \frac{u'(q_H)}{\beta_L} \right) \tag{56}
$$

Using Cramer's rule, we know that:

$$
\frac{dp}{d\pi} = \frac{|D_p|}{|D|}, \quad \frac{dk_j}{d\pi} = -\frac{dp}{d\pi}\frac{f'(k_j)}{pf''(k_j)}, \quad \frac{dq_H}{d\pi} = \frac{|D_H|}{|D|}, \quad \frac{dq_L}{d\pi} = \frac{|D_L|}{|D|}
$$

Therefore, we know that $\dfrac{dp}{d\pi} < 0$, $\dfrac{dk_H}{d\pi} < 0$, $\dfrac{dk_L}{d\pi} < 0$ and $\dfrac{dq_H}{d\pi} < 0$. $\square$

Proof of Proposition 4

From (53) and (56), we know that $dq_L/d\pi = |D_L|/|D| < 0$ if $|D_L| > 0$. Assume that the production function is isoelastic, so that $\varepsilon f(k) = f'(k)k$. By differentiating this condition,

we have that $\frac{f''(k)k}{f'(k)} = \varepsilon - 1$ and therefore:

$$\frac{f'(k)^2}{f''(k)} = \frac{f'(k)}{f''(k)} \frac{k}{k} \frac{f(k)}{f(k)} f'(k) = \frac{f'(k)}{f''(k)k} \frac{kf'(k)}{f(k)} f(k) = \frac{\varepsilon}{\varepsilon - 1} f(k) \qquad (57)$$

Then, from (56) and (57), $|D_L|$ can be written as:

$$|D_L| = \frac{\sigma}{2} \frac{1}{p^2} \frac{1}{\beta_L} \left(u''(q_H) \frac{\varepsilon}{\varepsilon - 1} \left(\rho f(k_H) + (1-\rho)f(k_L) \right) - \rho \frac{1}{\beta_H} \left(u'(q_L)\beta_L - u'(q_H)\beta_H \right) \right)$$

Therefore:

$$|D_L| > \frac{\sigma}{2} \frac{1}{p^2} \frac{1}{\beta_L} \left(u''(q_H) \frac{\varepsilon}{\varepsilon - 1} \rho q_H - \rho \frac{1}{\beta_H} \left(u'(q_L)\beta_L - u'(q_H)\beta_H \right) \right)$$

The condition above can be simplified to:

$$|D_L| > -\frac{\sigma}{2} \frac{1}{p} \frac{1}{\beta_L} \left(\frac{1}{p} u''(q_H) \frac{\varepsilon}{1 - \varepsilon} \rho q_H + \rho \frac{\beta_H - \beta_L}{\beta_H} \left(1 - \frac{\sigma}{2} \right) \frac{2}{\sigma} \right)$$

Therefore, $|D_L| > 0$ if:

$$\frac{1}{p} u''(q_H) \frac{\varepsilon}{1 - \varepsilon} \rho q_H + \rho \frac{\beta_H - \beta_L}{\beta_H} \left(1 - \frac{\sigma}{2} \right) \frac{2}{\sigma} < 0$$

and

$$-\frac{u''(q_H)q}{p} > \frac{1 - \varepsilon}{\varepsilon} \rho \frac{\beta_H - \beta_L}{\beta_H} \left(\frac{2 - \sigma}{\sigma} \right)$$

Note that since $u'(q_H) > p$ from (19), then:

$$\frac{-u''(q_H)\rho q_H}{p} > \frac{-u''(q_H)\rho q_H}{u'(q_H)} = \rho\mu$$

where μ denotes risk aversion. It follows that if:

$$\mu > \frac{1 - \varepsilon}{\varepsilon} \frac{\beta_H - \beta_L}{\beta_H} \left(\frac{2 - \sigma}{\sigma} \right)$$

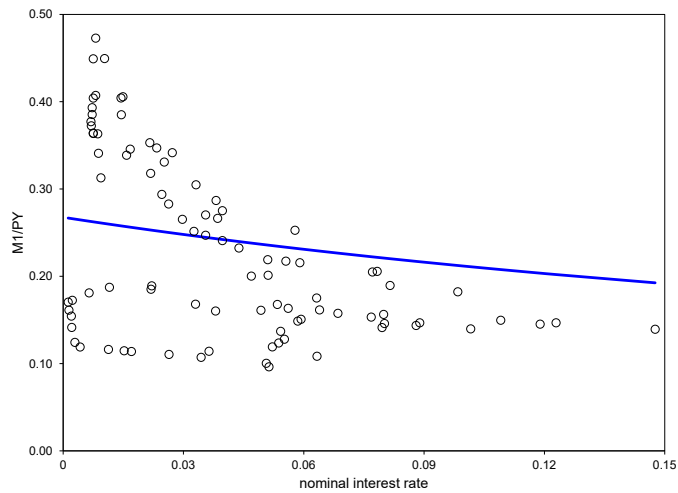then $|D_L| > 0$. $\square$

B. Figures

Figure 1: U.S. money demand with fitted model for the sample period 1929–2019. Circles identify empirical money demand M/PY against the nominal interest rate i for each year in the sample period. The solid line identifies the calibrated money demand L in the benchmark model with a collateral constraint.
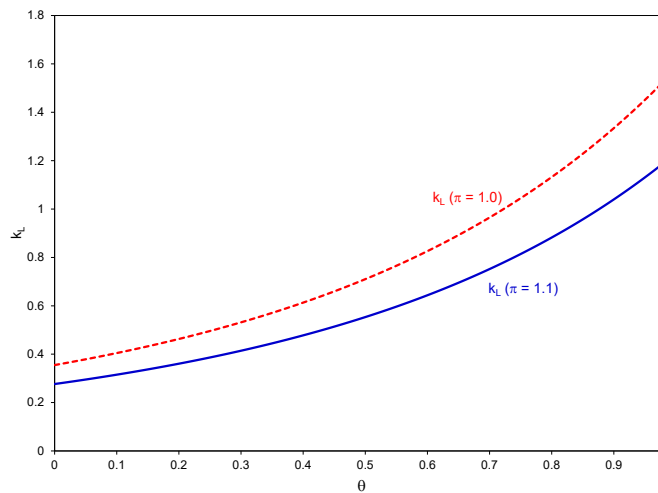

Figure 2: k_L against θ in the benchmark model with a collateral constraint. The solid and dotted lines identify k_L with $\pi = 1.1$ and with $\pi = 1$ respectively.
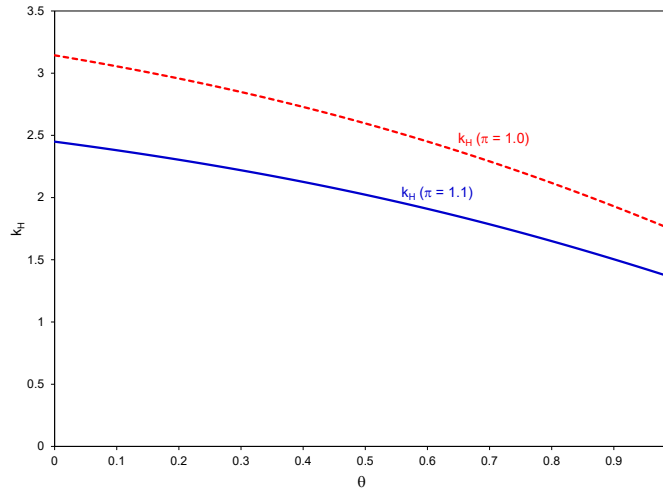
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Figure 3: k_H against θ in the benchmark model with a collateral constraint. The solid and dotted lines identify k_H with $\pi = 1.1$ and with $\pi = 1$ respectively.
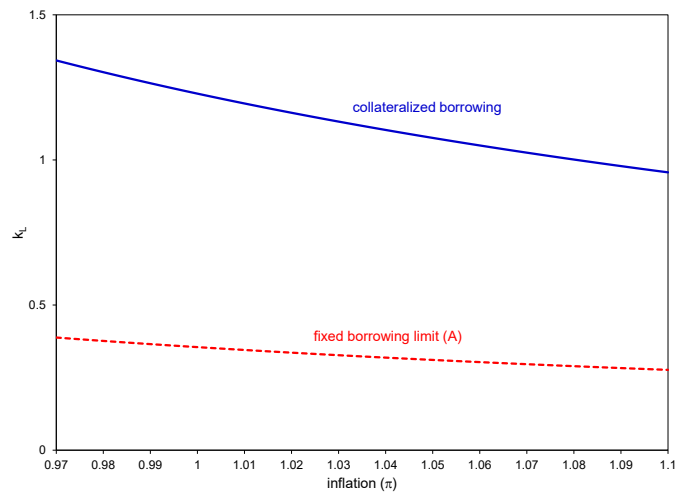

Figure 4: k_L against gross inflation π. The solid line identifies k_L in the benchmark model with a collateral constraint. The dotted line identifies k_L in the model with a fixed borrowing limit A.
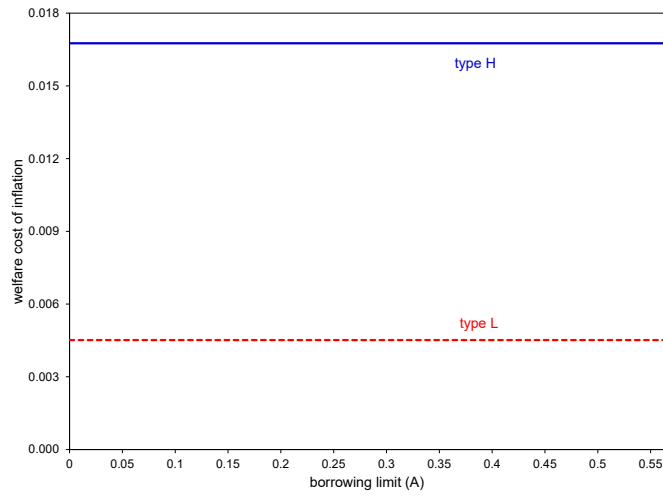
Figure 5: Percentage welfare cost of 10% inflation, relative to zero inflation, against the borrowing limit. The figure is drawn for the model with an exogenous borrowing limit A. The solid and dotted lines identify the welfare cost of inflation for type H and type L agents respectively.