SUPPLEMENTARY APPENDICES
Addressing COVID-19 Outliers in BVARs with Stochastic Volatility*

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Abstract

This online appendix provides additional descriptions and results for our paper. Among others, we provide a detailed description of our data set, the calculation of predictive likelihoods, various results for individual variables not shown in the paper, as well as various robustness checks. The robustness checks include an AR(1) specification for the SV processes in the standard and outlier-augmented SV models and VARs with variables in levels. In addition, we document the the stability of VAR-SV parameters (in contrast to a CONST model) over the COVID-19 sample and investigate the sensitivity of predictive densities from VAR-SVO to variable orderings.

*The views expressed herein are solely those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Cleveland, the Federal Reserve System, the Eurosystem or the Deutsche Bundesbank. Replication codes are available at `https://github.com/elmarmertens/CCMMoutlierVAR-code`. Corresponding author: Todd E. Clark, todd.clark@researchfed.org.

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Part A

Additional details (models and data)

I Data set

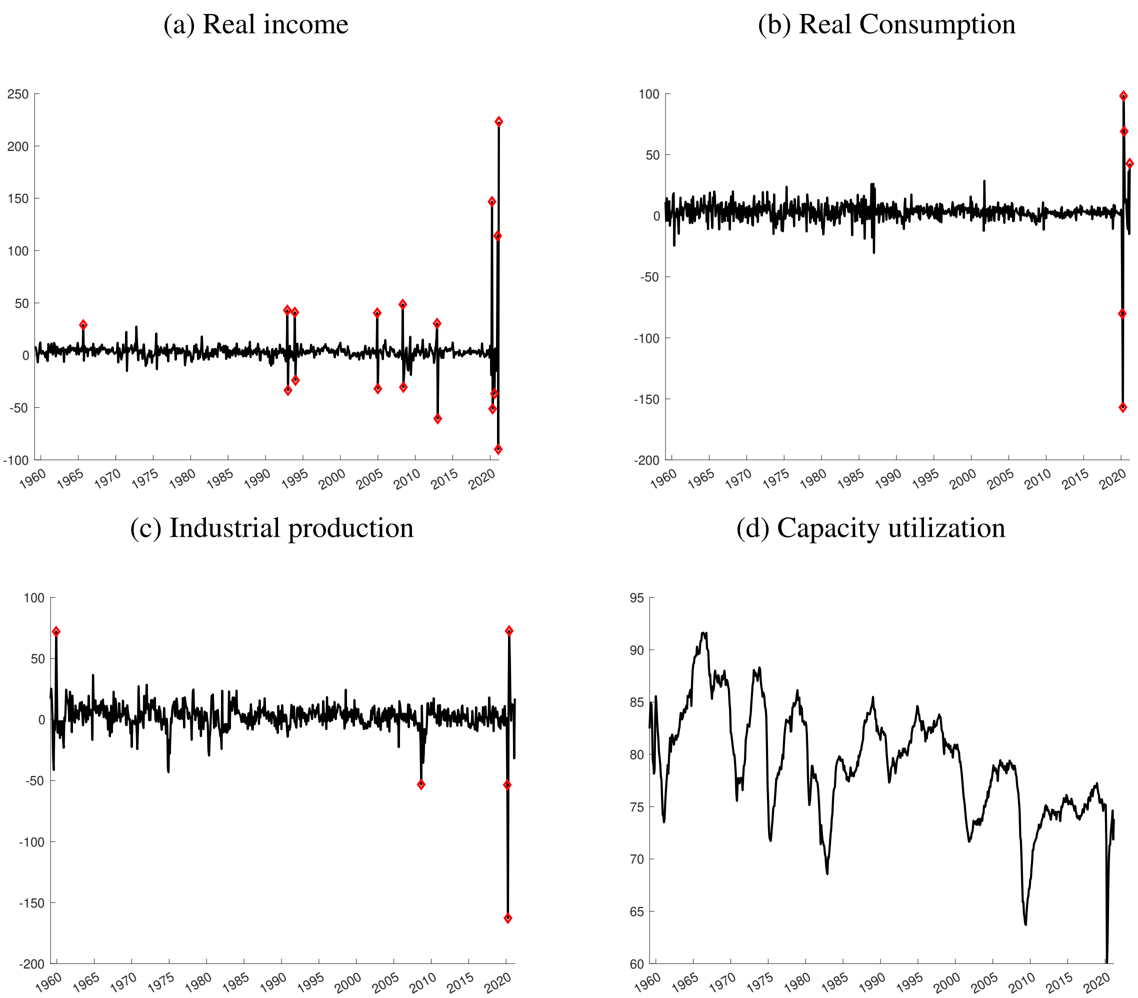
Table S.1 lists the 16 variables used in our baseline VAR specification. All data has been obtained from the April 2021 vintage of the FRED-MD data set compiled by McCracken and Ng (2016) and continuously updated by the Federal Reserve Bank of St. Louis. The table also lists FRED-MD codes and data transformations for each variables, and whether the mean of the Minnesota prior for each variable's first own-lag coefficients has been set to zero or unity. Figures S.1–S.4 provide plots of each transformed data series as used in our estimation. These figures also indicate which observations were deemed to be outliers based on their distance being more than five times the inter-quartile range away from the median of each variable's time series (based on a full-sample assessment). Figure S.5 indicates which observations were marked as outliers (based on the aforementioned criterion) in earlier quasi-real-time samples.

Table S.1: List of variables

Variable	FRED-MD code	transformation	Minnesota prior
Real Income	RPI	$\Delta \log(x_t) \cdot 1200$	0
Real Consumption	DPCERA3M086SBEA	$\Delta \log(x_t) \cdot 1200$	0
IP	INDPRO	$\Delta \log(x_t) \cdot 1200$	0
Capacity Utilization	CUMFNS		1
Unemployment Rate	UNRATE		1
Nonfarm Payrolls	PAYEMS	$\Delta \log(x_t) \cdot 1200$	0
Hours	CES0600000007		0
Hourly Earnings	CES0600000008	$\Delta \log(x_t) \cdot 1200$	0
PPI (Fin. Goods)	WPSFD49207	$\Delta \log(x_t) \cdot 1200$	1
PCE Prices	PCEPI	$\Delta \log(x_t) \cdot 1200$	1
Housing Starts	HOUST	$\log(x_t)$	1
S&P 500	SP500	$\Delta \log(x_t) \cdot 1200$	0
USD / GBP FX Rate	EXUSUKx	$\Delta \log(x_t) \cdot 1200$	0
5-Year Yield	GS5		1
10-Year Yield	GS10		1
Baa Spread	BAAFFM		1

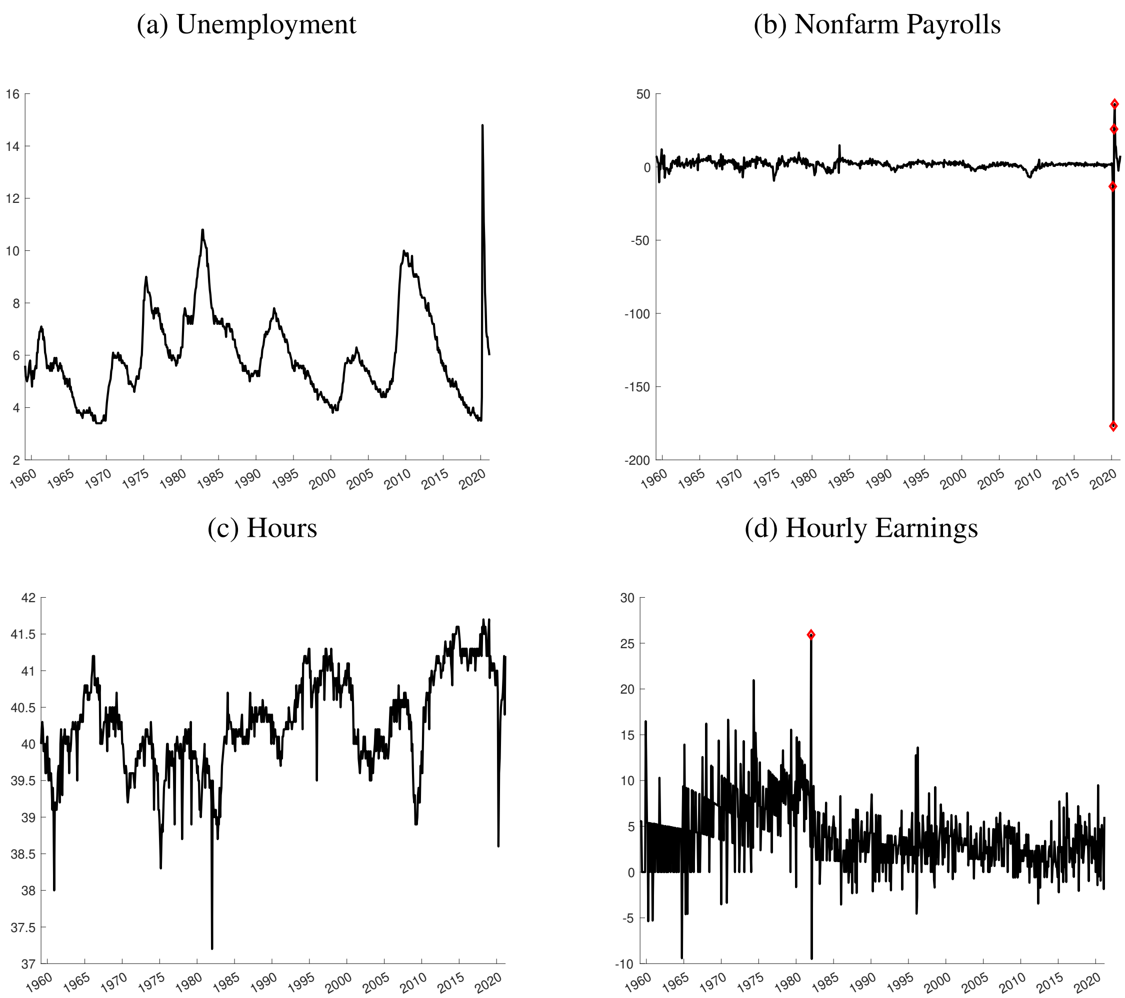
Note: Data obtained from the 2021-04 vintage of FRED-MD. Monthly observations from 1959:M03 to 2021:M03. Entries in the column “Minnesota prior” report the prior mean on the first own-lag coefficient of the corresponding variable in each BVAR. Prior means on all other VAR coefficients are set to zero.

Figure S.1: Input data series



Note: Data for selected time series, with data transformations as listed in Table S.1. Red dots denote observations that are more than five times the inter-quartile range away from the series median.

Figure S.2: Input data series (ctd.)



(a) Unemployment

(b) Nonfarm Payrolls

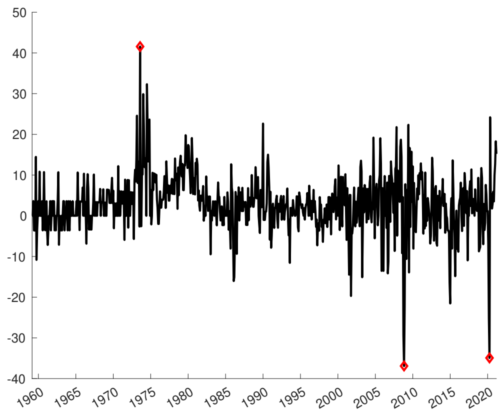
(c) Hours

(d) Hourly Earnings

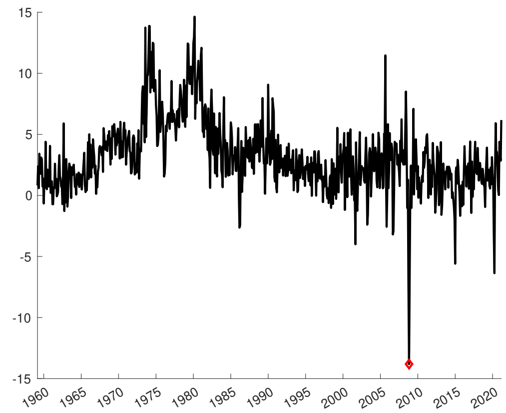
Note: Data for selected time series, with data transformations as listed in Table S.1. Red dots denote observations that are more than five times the inter-quartile range away from the series median.

Figure S.3: Input data series (ctd.)

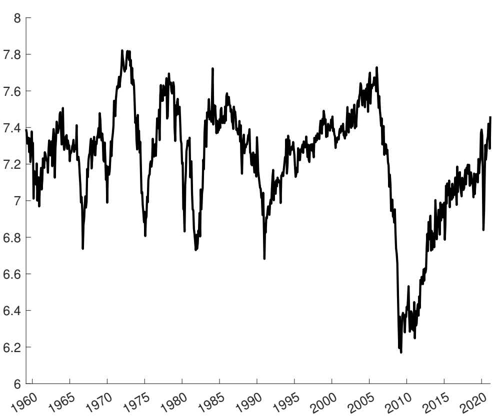
(a) PPI (fin. goods)



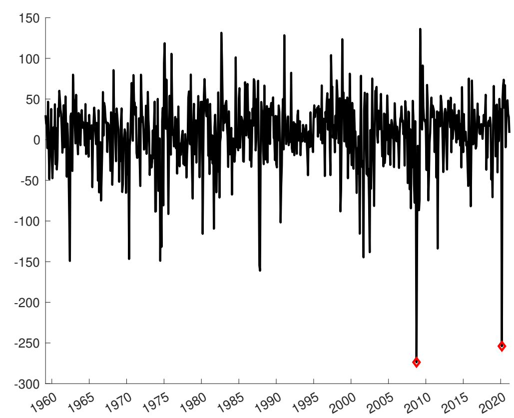
(b) PCE prices



(c) Housing Starts

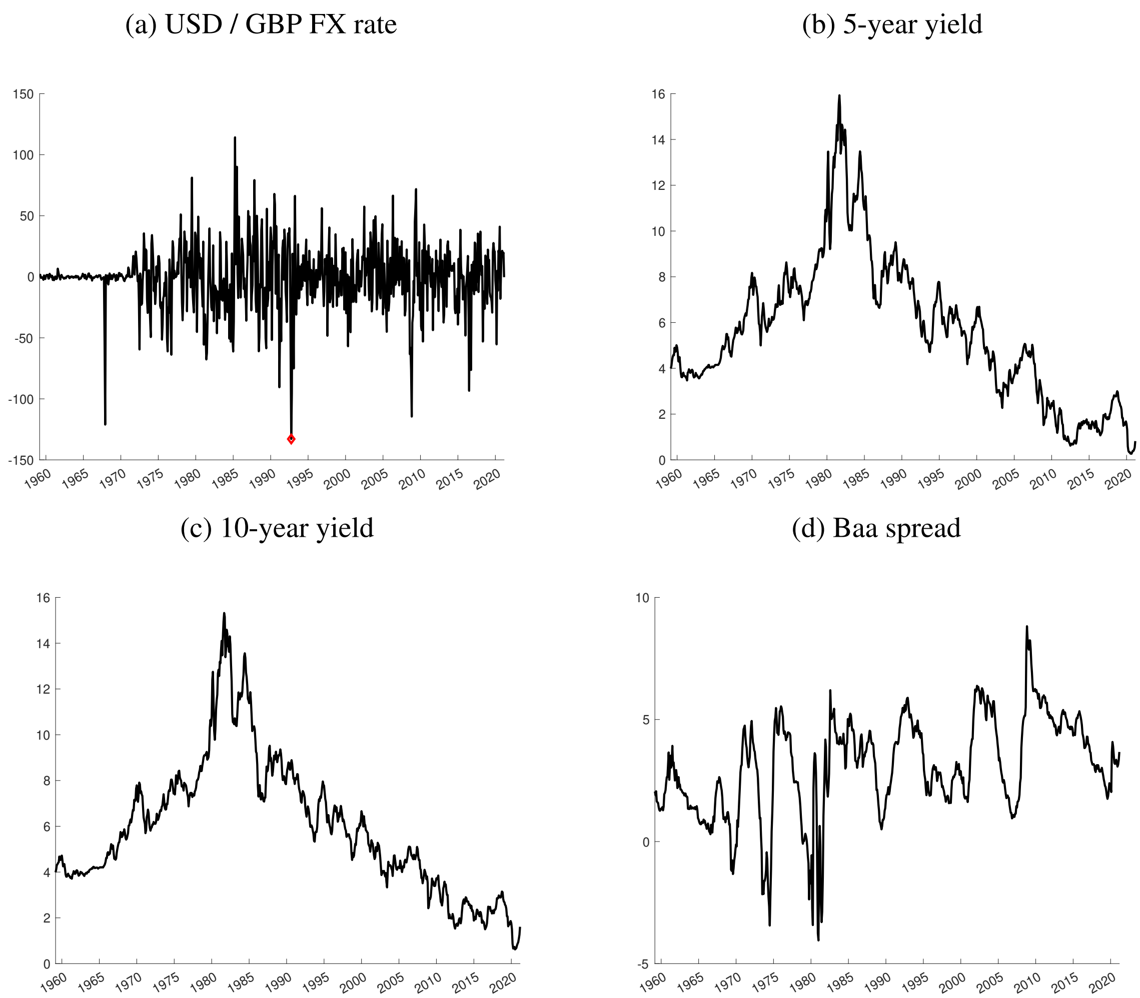


(d) S&P 500



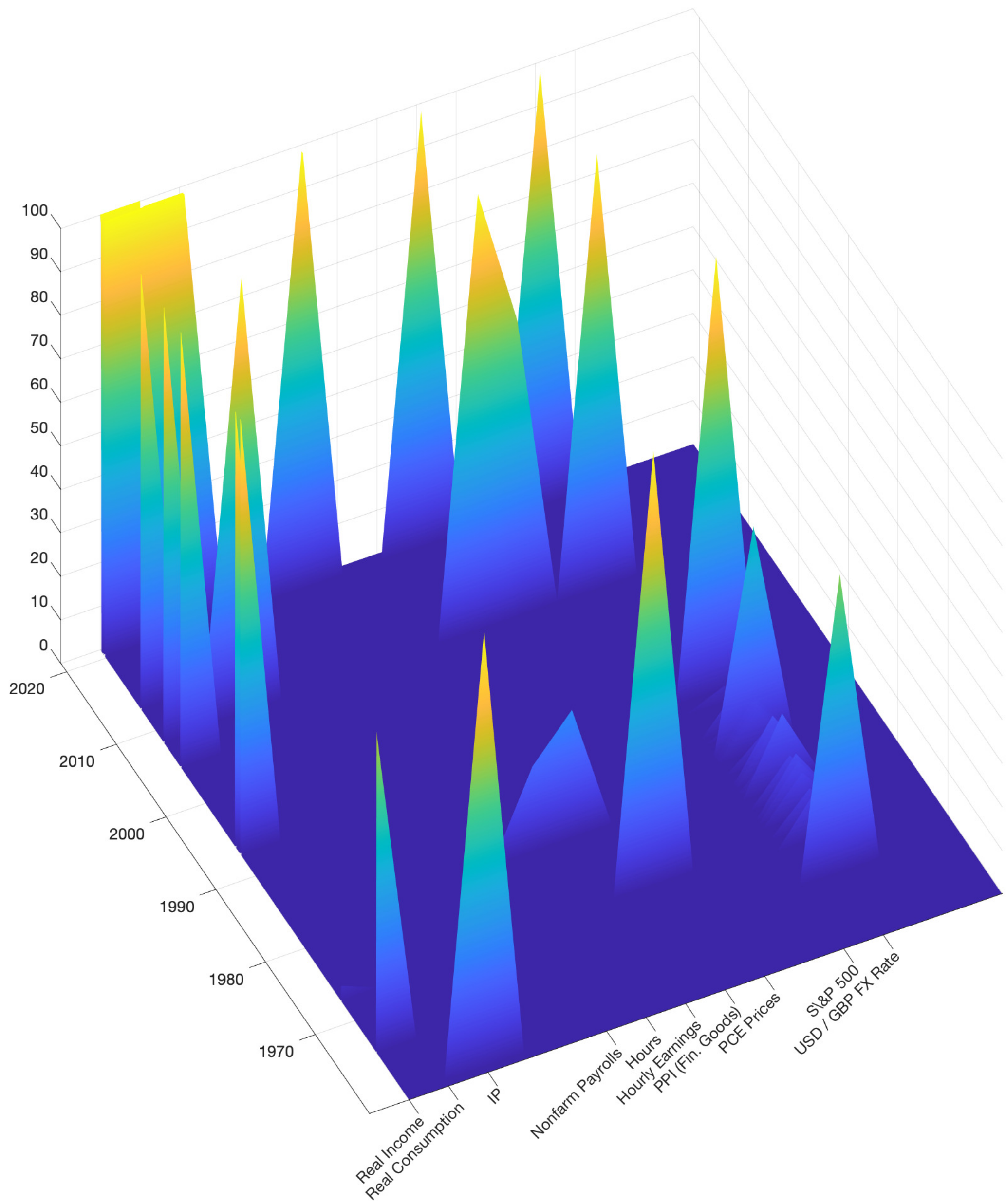
Note: Data for selected time series, with data transformations as listed in Table S.1. Red dots denote observations that are more than five times the inter-quartile range away from the series median.

Figure S.4: Input data series (ctd.)



Note: Data for selected time series, with data transformations as listed in Table S.1. Red dots denote observations that are more than five times the inter-quartile range away from the series median.

Figure S.5: Potential outliers in the data



Note: Occurrence of potential outliers in our 16-variable data set (as described in Table S.1). Potential outliers are identified as observations that are more than five times the inter-quartile range away from the series median in a given sample. In quasi-real time, the assessment may change, and the graph above indicates the average occurrence (in percentage points) of an observation being designated as an outlier over all quasi-real-time samples that include a given observation. We consider growing quasi-real-time samples, all starting in 1959:M3 with the first sample ending in 1984:M12.

II Predictive likelihoods from MCMC output

This appendix describes how we calculate predictive likelihoods from the MCMC outputs of the various VAR models used in the paper. Denote the 1-step-ahead predictive likelihood from model $\mathcal{M}_i$ of the variable vector y_{t+1}, conditional on past data y^t, by $p(y_{t+1}|y^t, \mathcal{M}_i)$. Following Geweke and Amisano (2010), model fit is then assessed based on the sum of the logs of predictive likelihoods over a given set of observations,

$$\log L(\mathcal{M}_i) = \sum_{t=T_0}^{T_1} \log p(y_{t+1}|y^t, \mathcal{M}_i)\,, \tag{S.1}$$

where T_0 and T_1 denote the start and end points of the evaluation window. The cumulative difference in predictive log-likelihoods of two models, $\log L(\mathcal{M}_i) - \log L(\mathcal{M}_j)$ $(i \neq j)$, gives the log-Bayes factor for comparison of $\mathcal{M}_i$ over $\mathcal{M}_j$ (assuming uniform prior weights on all models).

The remainder of this appendix considers the computation of the predictive likelihood for a given model, and dependence on $\mathcal{M}_i$ will henceforth be suppressed from our notation. For each forecast origin t, the predictive likelihood is computed from the output of an MCMC sampler used to estimate the model based on data $y^t = \{1, 2, \ldots, t\}$ in quasi-real-time fashion, reestimating all parameters and latent state variables based on y^t but without considering real-time revisions in vintage data for the variables in y_t.[1]

To construct $p(y_{t+1}|y^t)$ from MCMC output, we follow Geweke and Amisano (2010) and Krüger, et al. (forthcoming), who integrate over predictive likelihoods that condition on parameters and latent states drawn at each MCMC replication. Given the structure of our VAR models, conditional likelihoods are straightforward to compute analytically. In case of the VAR-SV model, analytic tractability requires a 1-step out-of-sample simulation of the SV processes, λ_{t+1}. The same applies to the outlier-augmented models (SVO, SV-t, SVO-t). Importantly, for the outlier-augmented SV models, only the persistent SV components, λ_{t+1}, need to be simulated forward.

[1]In our application, each estimation window begins with data for 1959:M03, and the (earliest) evaluation window starts at T_0=1975:M1.

The effects from the transitory outlier states, in Q_t and O_t, can instead be integrated out. This application of Rao-Blackwellization turns out to vastly improve the accuracy of the likelihood computations, in particular in case of extreme observations.

The remainder of this appendix describes the formal computations for each model in more detail.

i CONST model

To begin, consider the CONST case of a constant-parameter VAR:

$$y_{t+1} = \Pi_0 + \Pi(L)y_t + v_{t+1}\,, \qquad v_{t+1} \sim N(0,\Sigma)\,, \qquad \Theta = \{\Pi_0, \Pi(L), \Sigma\}\,. \tag{S.2}$$

Let there be M MCMC draws, each indexed by $m = 1, 2, \dots, M$, and $\Theta^{(m)}$ denotes the mth MCMC draw of all model parameters. Conditional on a parameter draw, $\Theta^{(m)}$, the predictive likelihood is

$$\log p\left(y_{t+1}|y^t, \Theta^{(m)}\right) = -\frac{1}{2}\left(N \cdot \log\left(2\cdot\pi\right) + \log|\Sigma| + v'_{t+1}\Sigma^{-1}_{t+1}v_{t+1}\right)\,. \tag{S.3}$$

with v_{t+1} as given by the VAR equation (S.2). The marginal likelihood can be approximated by numerical integration over all MCMC nodes:[2]

$$\log p\left(y_{t+1}|y^t\right) \approx \log\left(\frac{1}{M}\sum_m p\left(y_{t+1}|y^t, \Theta^{(m)}\right)\right)\,. \tag{S.4}$$

The VAR-SV model, and its outlier-augmented variants, are not only characterized by param-

[2]For numerical stability, it is advisable to implement the computations in terms of log-likelihoods $l_t^{(m)} \equiv \log p(y_{t+1}|\Theta^{(m)}, y^t)$. Numerical overflow may arise in computing, $p\left(y_{t+1}|\Theta^{(m)}, y^t\right) = \exp l_t^{(m)}$ when $l_t^{(m)}$ is large. To avoid such numerical issues, it is useful to normalize the log-likelihoods for each m by their maximal value as follows:

$$\log p(y_{t+1}|y^t) \approx \log\left(\frac{1}{M}\sum_{m=1}^{M}\exp\left(l_t^{(m)} - \max\left(l_t^{(m)}\right)\right)\right) + \max\left(l_t^{(m)}\right)\,.$$

eters, Θ, but also latent volatility states, in particular the vector of persistent SV processes, λ_t, which requires adapting the expression for $\log p\left(y_{t+1}|y^t, \Theta^{(m)}\right)$ as described next.

ii SV model

The VAR-SV model differs from CONST by adding the following stochastic volatility specification to the description of VAR residuals v_{t+1}:

$$v_{t+1} = A^{-1}\Lambda_{t+1}^{0.5}\, \varepsilon_{t+1}\,, \qquad \varepsilon_{t+1} \sim N(0, I)\,, \tag{S.5}$$

$$\lambda_{t+1} = \lambda_t + e_{t+1}\,, \qquad e_{t+1} \sim N(0, \Phi). \tag{S.6}$$

where Λ_{t+1} is a diagonal matrix, whose diagonal elements are stacked in the vector λ_{t+1}. A is a unit-lower triangular matrix. A and Φ are part of the collection of parameters, Θ.

Conditional on parameters and Λ_{t+1}, the 1-step-ahead predictive likelihood of y_{t+1} is straightforward to compute. While draws from Λ_{t+1} are not directly generated by MCMC estimation over data until t, it is straightforward to simulate Λ_{t+1} conditional on $\Lambda_t^{(m)}$ and $\Phi^{(m)}$. Building on ideas of Waggoner and Zha (1999), at every MCMC node m, we employ J draws of Λ_{t+1}, to better balance computational costs against a high degree of accuracy in the Monte Carlo approximation of the predictive likelihood.[3] We denote the jth draw of Λ_{t+1} at the mth MCMC node by $\Lambda_{t+1}^{(j,m)}$

[3]Typically, additional draws of Λ_{t+1} at node m are less costly than drawing additional MCMC nodes. See also the related discussion in Krüger, et al. (forthcoming).

and obtain the predictive likelihood as follows:[4]

$$\Lambda_{t+1}^{(j,m)} \sim p\left(\Lambda_{t+1} \middle| \Lambda_t^{(m)}, \Theta^{(m)}\right) = p\left(\Lambda_{t+1} \middle| \Lambda_t^{(m)}, \Theta^{(m)}, y^t\right) , \tag{S.7}$$

$$\begin{aligned} l_t^{(j,m)} &\equiv \log p\left(y_{t+1} \middle| \Theta^{(m)}, \Lambda_{t+1}^{(j,m)}, y^t\right) \\ &= -\frac{1}{2}\left(N \cdot \log\left(2 \cdot \pi\right) + \log\left|\Lambda_{t+1}^{(j,m)}\right| + \left(\eta_{t+1}^{(j,m)}\right)' \eta_{t+1}^{(j,m)}\right) , \end{aligned} \tag{S.8}$$

$$\text{with} \quad \eta_{t+1}^{(j,m)} = \left(\Lambda_{t+1}^{(j,m)}\right)^{-0.5} A^{(m)} \left(y_{t+1} - \Pi_0^{(m)} - \Pi(L)^{(m)} y_t\right) ,$$

$$\log p(y_{t+1}|y^t) \approx \log\left(\frac{1}{M}\frac{1}{J}\sum_{m=1}^{M}\sum_{j=1}^{J}\exp\left(l_t^{(j,m)}\right)\right) . \tag{S.9}$$

Since A is unit-lower-triangular, we have $|A^{-1}| = 1$, and the expression for the log-likelihood in (S.8) uses $|\operatorname{Var}(v_{t+1} \mid A, \Lambda_{t+1})| = |\Lambda_{t+1}|$.[5] Formulating the conditional log-likelihood, $l_t^{(j,m)}$ in terms of standardized VAR residuals,

$$\eta_{t+1} = \Lambda_{t+1}^{-0.5} A v_{t+1} , \tag{S.10}$$

is convenient for generalizing the SV calculations to the SV-t, SVO and SVO-t models. Moreover, in all models considered, η_{t+1} is a vector of mutually and serially independent variables, with typical element η_{t+1}^i.[6] As a result, in all our SV variants, the conditional log-likelihood takes the form

$$l_t^{(j,m)} = -\frac{1}{2}\log\left|\Lambda_{t+1}^{(j,m)}\right| + \sum_{i=1}^{N}\log p\left(\eta_{t+1}^{i,(j,m)} \middle| \Theta^{(m)}\right) . \tag{S.11}$$

The extensions to SV-t, SVO and SVO-t, discussed below, are solely concerned with variations in specifying the conditional density of (quasi-)standardized residuals, $p\left(\eta_{t+1}^i \middle| \Theta\right)$. Application to the SV-OutMiss model is identical to the standard SV case, since both models do not conceptually

[4] In these calculations, dependence on the history of SV states prior to $t+1$ can be ignored, due to the Markov nature of the SV process for Λ_t, unless explicitly noted.

[5] Since Λ_{t+1} is diagonal, we also have $\log|\Lambda_{t+1}| = \sum_i \log \lambda_{t+1}^i$ where λ_{t+1}^i denotes the typical diagonal element of Λ_{t+1}.

[6] Through A and Λ_{t+1}, η_{t+1} depends on VAR parameters and SV states, and thus on draws indexed by j and m.

differ in their characterization of future events.[7]

iii SV-t model

As noted above, likelihood calculations for the SV-t model differ from the SV case merely in the density for the quasi-standardized residuals, η_{t+1}, defined in (S.10). We continue to use the numerical integration in (S.9) over M MCMC nodes, with J draws of $\Lambda_{t+1}^{(j,m)}$ as in (S.7). In the SV-t case, η_{t+1} consists of N mutually and serially independent t-distributed random variables, with variable-specific degrees of freedom. Denoting the typical element of η_{t+1} by η_{t+1}^i we have

$$\eta_{t+1}^i = i.i.d.\ t(\nu_i)\,. \tag{S.12}$$

The degree-of-freedom parameters $\{\nu_i\}_{i=1}^N$ are collected in Θ, so that

$$\log p(\eta_{t+1}^i|\Theta) = \log\Gamma\left(\frac{v_i+1}{2}\right) - \frac{1}{2}\log\left(v_i\cdot\pi\right) - \log\Gamma\left(\frac{v_i}{2}\right) - \frac{v_i+1}{2}\left(1+\frac{\left(\eta_{t+1}^i\right)^2}{2}\right), \tag{S.13}$$

which, together with (S.9), is used to calculate the 1-step-ahead predictive likelihood.

iv SVO and SVO-t model

In case of the SVO model, a typical element of the vector of quasi-standardized residuals, η_{t+1} defined in (S.10), is conditionally normally distributed, with variance depending on the $i.i.d.$ outlier state o_{t+1}^i:

$$\eta_{t+1}^i \sim i.i.d.\ N\left(0, \left(o_{t+1}^i\right)^2\right). \tag{S.14}$$

[7] SV-OutMiss differs from SV in that MCMC draws of parameters and past states neglect signals contained in past data points that were marked as outliers as part of the models pre-screening step.

The distribution of the outlier state is given by a discrete mixture with K possible realizations. Generically, denote each of the possible realizations for o^i_{t+1} by s_k with $k = 1, \ldots K$ and the associated probability by w^i_k, where the outlier probability may be variable specific).[8] Given the conditionally Gaussian distribution of η^i_{t+1}, the likelihood of $p\left(\eta^i_{t+1}|\Theta\right)$ is given by

$$p\left(\eta^i_{t+1}|\,\Theta\right) = \sum_k w_k \cdot p\left(\eta^i_{t+1}|s_k\right) \tag{S.15}$$

$$\text{with} \quad p\left(\eta^i_{t+1}|s_k\right) = \frac{1}{\sqrt{2 \cdot \pi \cdot s^2_k}} \cdot \exp\left(-\frac{1}{2} \cdot \frac{(\eta^i_{t+1})^2}{s^2_k}\right) \tag{S.16}$$

which, together with (S.9), is used to calculate the 1-step-ahead predictive likelihood.

For our purpose, it is sufficient to characterize only the marginal distributions of η^i_{t+1} and thus only of o^i_{t+1}, which is straightforward to Rao-Blackwellize even in our multivariate setting. In contrast, characterizing the probability of an entire draw of O_{t+1} is daunting, since with $K = 20$ possible realizations for each diagonal element of O_t, and $N = 16$ elements of y_t, we would have to consider a mixture over $20^{16} \approx 6.55 \cdot 10^{20}$ possible states.

Predictive likelihoods for the SVO-t model are straightforward to compute by combining the approaches used for SVO and SV-t as described above.

v Censored predictions of nominal yields

In our application, the data vector y_t contains measures of longer-term nominal interest rates. In our simulations of predictive densities we censor draws for nominal interest rates that fall below an assumed value for the effective lower bound (ELB) on nominal interest rates of 25 basis points. For the characterization of 1-step-ahead densities, the assumed censoring of predictive densities converts the tails of the forecast distribution that fall below the ELB into a point mass at the ELB, leaving the density for realizations above the ELB unchanged. In our data, longer-term nominal rates have never been observed at the ELB, which also applies to the realized data used in

[8] In our specific application we have $s_k = k$, $w^i_1 = 1 - p_i$ and $w^i_k = p_i/(K-1)$ for $k > 1$, and p_i is the variable-specific probability of drawing a value of o^i_t larger than one, which is also included in Θ for this model.

evaluating interest rate predictions. The upshot of these conditions is that the predictive likelihood calculations described above need not be adjusted for the ELB, provided they are applied only to realized values y_{t+1} that are above the ELB.[9]

III Comparison of SVO/SV-t/SVO-t specifications

Figure S.6 illustrates the differences in densities implied for $o_{j,t}$, $q_{j,t}$, and $o_{j,t} \cdot q_{j,t}$ in, respectively, the SVO, SV-t and SVO-t models. The densities for the outlier states, $o_{j,t}$ and $q_{j,t}$, depend on the outlier probability p_j and the t-error degrees of freedom d_j.

For the sake of illustration, the densities are calibrated to generate roughly the same variance of the (combined) outlier states in the SVO, SV-t and SVO models, respectively. For the SVO model, the outlier probability p_j has been set to correspond to one outlier every four years in monthly data, $p_j = 1/(4 \cdot 12)$ (which is also the prior mean for p_j in our SVO estimates). The degrees of freedom for the SV-t model have been set equal to five. For the SVO-t model, the outlier probability has been lowered to correspond to one outlier every 10 years, and the degrees of freedom of the t-component have been set to 9.

For the SVO model, with $p_j = 1/(4 \cdot 12)$ we have $\text{Var}\left(o_{j,t}\right) \approx 1.54$. In the SV-t case with $d_j = 5$ we get $\text{Var}\left(q_{j,t}\right) \approx 1.67$ and in the SVO-t case, with $p_j = 1/120$ and $d_j = 9$ we obtain $\text{Var}\left(o_{j,t} \cdot q_{j,t}\right) \approx 1.56$. The variances can be computed from

$$\text{Var}\left(o_{j,t}\right) = (1 - p_j) + p_j \cdot \frac{(20-2)^2}{12}, \qquad \text{Var}\left(q_{j,t}\right) = \frac{d_j}{d_j - 2},$$

$$\text{and} \quad \text{Var}\left(o_{j,t} \cdot q_{j,t}\right) = (1 - p_j) + p_j \cdot \frac{(20-2)^2}{12} \cdot \frac{d_j}{d_j - 2}.$$

For the SV-t case, we obtain similar estimates for our paper with $d_j = 6$, which implies $\text{Var}\left(q_{j,t}\right) = 1.5$.

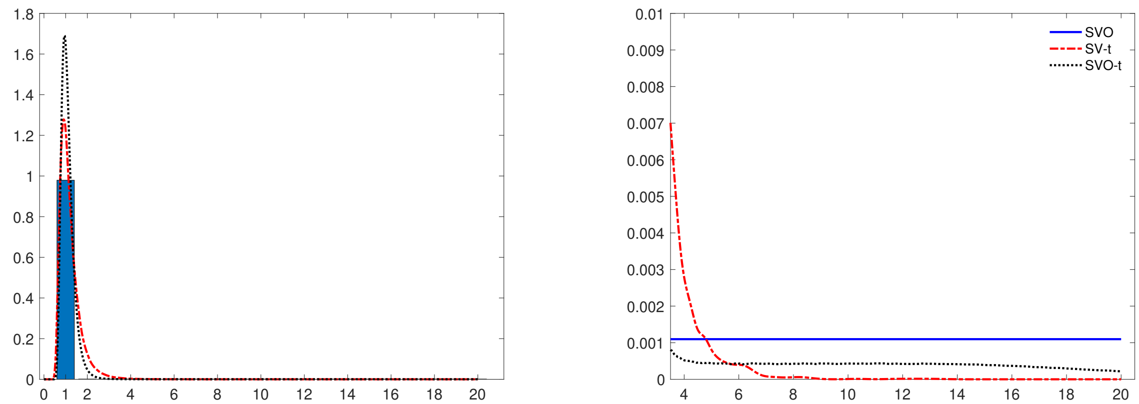
The density for the outlier states peaks at (SVO) or near (SVO-t, SV-t) the value of 1 with a

[9]For the purpose of this discussion, we refer to y_t being at or above the ELB when any or all of the nominal interest rate measures contained in y_t are at or above the ELB, respectively.

Figure S.6: Prior densities of outlier states in different models

(a) Full figure

(b) Zoomed into right tail



Note: Densities for the outlier state $o_{j,t}$ in the SVO model, $q_{j,t}$ in the SV-t model of Jacquier, et al. (2004), and $o_{j,t} \cdot q_{j,t}$ in the combined SVO-t model. As described in the text, the densities are calibrated to generate roughly the same variance of the outlier states.

fat right-hand tail. In the SVO case, there is equal probability on outlier states between 2 and 20, whereas the SV-t case assigns most probability on values close to 1, albeit with some minimal measure also placed on values far above 20. SVO-t is in between, with more probability to values higher than 6 than SV-t but less than SVO. Also, while the outlier states in the SVO case cannot take values below 1, the SV-t and SVO-t cases also assign some mass to values below 1.

Part B

Additional results (baseline models)

IV Forecast evaluations for different sub-samples

The tables in this appendix complement the comparisons of RMSE and CRPS measures of forecast performance for individual variables shown in the paper for an evaluation window from 1975–2017. Here we report alternative comparisons over the following subsamples: the Global Financial Crisis (GFC) and 1985–2017.

Table S.2: Relative RMSE around the GFC

					Relative to SV …							
	SV				CONST				SVO-t			
Variable / Horizons	1	3	12	24	1	3	12	24	1	3	12	24
Real Income	10.85	10.95	11.00	9.97	1.02	1.00	1.00	0.92**	1.00	1.00	1.00	0.93***
Real Consumption	4.16	3.96	4.15	4.19	1.01	0.98	1.05	1.05	1.01	1.01	0.96*	1.01
IP	7.84	8.85	10.69	9.13	1.01	0.98	0.94**	1.01	0.99	0.99	0.99	0.96*
Capacity Utilization	0.48	1.04	3.79	5.59	1.02	1.00	0.96	0.88*	1.00	0.98	0.98	0.89**
Unemployment Rate	0.15	0.29	1.13	2.14	0.99	0.95	0.99	1.05	0.99	0.98	0.97	0.96
Nonfarm Payrolls	1.17	1.44	2.25	2.49	1.14***	1.07	1.04	1.05	1.02	1.03	0.98	0.95*
Hours	0.19	0.26	0.53	0.51	1.03	0.95	0.98	0.99	1.01	0.99	1.00	1.03
Hourly Earnings	2.10	2.16	2.21	2.48	1.03	0.98	1.10	1.23**	0.99	1.01	1.00	1.01
PPI (Fin. Goods)	10.05	10.41	10.94	9.30	0.99	1.02	1.05	1.13	1.00	1.00	1.00	1.01
PCE Prices	2.70	3.35	3.82	3.78	0.99	1.04	1.15	1.30**	1.01	1.00	1.01	1.05*
Housing Starts	0.09	0.15	0.39	0.59	0.95	0.96	1.03	1.09***	0.98**	0.98*	0.99	1.02**
S&P 500	52.05	52.32	53.19	39.38	0.96	1.00	0.99	0.99	1.01	1.01	1.01	1.01
USD / GBP FX Rate	26.82	28.75	28.32	26.77	1.04	1.04	1.05	1.03	0.99	1.00	1.01	1.00
5-Year yield	0.24	0.56	1.03	1.43	1.15***	1.14**	1.21***	0.96	1.00	1.01	1.03	0.94
10-Year yield	0.24	0.55	1.07	1.20	1.09**	1.13**	1.27**	1.25	1.01	1.02	1.02	1.01
Baa Spread	0.33	0.81	1.80	1.42	1.21**	1.33	1.17	1.28	0.98*	0.99	1.02	0.95*

Note: Comparison of "SV" (baseline, in denominator of relative comparisons) against "CONST" and "SVO-t." Values below 1 indicate improvement over baseline. Evaluation window from 2007:M01 through 2014:M12. Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags.

Table S.3: Relative Avg CRPS around the GFC

	SV				Relative to SV … CONST				SVO-t			
Variable / Horizons	1	3	12	24	1	3	12	24	1	3	12	24
Real Income	4.52	4.96	5.51	5.23	1.15***	0.97	0.89**	0.77**	0.99	0.94*	0.92**	0.81***
Real Consumption	2.22	2.15	2.55	3.06	1.09**	1.10***	1.02	0.87***	1.00	1.00	0.94***	0.89***
IP	3.94	4.30	5.72	5.76	1.05	1.02	0.88***	0.87***	1.00	1.00	0.96**	0.89***
Capacity Utilization	0.27	0.54	1.98	3.40	1.04*	1.06	0.97	0.84**	1.00	1.01	0.99	0.88***
Unemployment Rate	0.09	0.16	0.57	1.21	0.98	0.97	1.06	1.13***	1.00	0.99	0.99	0.97*
Nonfarm Payrolls	0.65	0.78	1.26	1.59	1.18***	1.11***	1.02	0.92	1.01	1.03	0.97**	0.91***
Hours	0.11	0.14	0.28	0.33	1.04	0.96	1.00	0.89***	1.00	0.99	1.01	0.95*
Hourly Earnings	1.19	1.23	1.38	1.78	1.08*	1.05	1.06**	0.99	0.98	0.99	0.97*	0.92***
PPI (Fin. Goods)	5.32	5.70	6.13	5.76	1.04	1.02	1.06	1.01	1.02***	1.00	0.99	0.96**
PCE Prices	1.34	1.71	2.07	2.21	1.04	1.06	1.16	1.18**	1.03**	1.01	0.99	0.98
Housing Starts	0.05	0.08	0.21	0.34	0.97	1.00	1.12	1.17***	0.99*	1.00	1.00	1.02*
S&P 500	26.72	26.73	29.44	27.59	0.95	0.99	0.92*	0.79***	1.00	0.99	0.96*	0.90***
USD / GBP FX Rate	14.63	15.62	16.10	17.02	1.05	1.03	1.01	0.90***	0.99	1.00	0.99	0.94***
5-Year yield	0.13	0.30	0.58	0.69	1.15***	1.17***	1.26***	1.12	1.00	1.02	1.02	0.99
10-Year yield	0.13	0.29	0.63	0.74	1.08***	1.13**	1.27**	1.23*	1.01	1.02	1.01	1.01
Baa Spread	0.15	0.40	0.98	1.00	1.35***	1.36**	1.18	1.02	0.98*	0.99	1.00	0.94**

Note: Comparison of "SV" (baseline, in denominator of relative comparisons) against "CONST" and "SVO-t." Values below 1 indicate improvement over baseline. Evaluation window from 2007:M01 through 2014:M12. Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags.

Table S.4: Relative RMSE around the GFC (alternative models)

	SVO				SV-t				SV-OutMiss			
Variable / Horizon	1	3	12	24	1	3	12	24	1	3	12	24
Real Income	1.00	1.00*	1.00	1.00	1.00	1.00**	1.00	0.99	1.00	1.00	1.00	0.99
Real Consumption	0.99	0.97	1.01	0.97	1.00	1.00	1.00	1.01	0.96	0.94	1.04***	0.99
IP	1.00	0.99	1.00	1.01	1.00	1.00	1.00	1.01	1.03	1.03	1.00	1.02*
Capacity Utilization	0.99	0.98	0.99	1.06*	1.01	1.00	1.00	1.00	1.01	1.07	1.00	1.06*
Unemployment Rate	1.01	1.02	1.02	1.03*	1.00	1.00	0.99	0.99***	1.02	1.02	1.02	1.04*
Nonfarm Payrolls	1.00	0.98	1.01	1.02	1.01	1.01	0.99	1.01	0.98	1.01	1.02	1.03***
Hours	1.01	1.01	0.98**	0.96*	1.00	1.00	1.00	1.00	1.00	1.02	1.00	0.97
Hourly Earnings	1.00	0.99	0.98	0.96**	1.01	1.01	1.01	1.01	0.97	0.97	0.99	0.98**
PPI (Fin. Goods)	1.00	1.00	0.99	0.98**	1.00	1.00	1.00	1.00	1.00	1.01	1.00	1.00
PCE Prices	1.00	1.00	0.98	0.95	1.00	1.00	1.01	1.01	1.01	1.00	1.01	0.99
Housing Starts	1.01*	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.02	1.02	1.00	0.98
S&P 500	0.98*	0.99	1.00	1.00	1.00	1.00	1.00	0.99*	1.02	0.99	0.99	0.99
USD / GBP FX Rate	1.00	0.99*	0.99	1.00	1.01	1.02**	1.00	1.00	1.00	0.98	0.99	0.99*
5-Year Yield	0.99	0.98	0.99	1.08***	0.99	1.00	0.98	0.94**	1.01	1.00	0.96	0.97
10-Year Yield	1.00	0.98	0.97	1.02	0.99	0.99	0.99	0.96	1.00	1.01	0.98	0.96
Baa Spread	0.99	0.98*	0.93	1.03	1.01	1.01	1.02	1.01	1.04	1.03	0.98	1.07*

Note: Comparison of "SVO-t" (baseline, in denominator) against "SVO," "SV-t," and "SV-OutMiss." Values below 1 indicate improvement over baseline. Evaluation window from 2007:M01 through 2014:M12. Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Due to the close behavior of some of the models compared, and rounding of the report values, a few comparisons show significant relative RMSE of 1.00. These cases arise from persistent differences in performance that are, however, too small to be relevant after rounding.

Table S.5: Relative Avg CRPS around the GFC (alternative models)

	SVO				SV-t				SV-OutMiss			
Variable / Horizon	1	3	12	24	1	3	12	24	1	3	12	24
Real Income	0.99	1.00	1.02	1.05^{**}	1.00	1.00	1.00	1.00	1.00	0.99	1.00	0.99
Real Consumption	0.99	1.00	1.05^{***}	1.07^{***}	1.01	1.00	0.99^{**}	0.99^{***}	0.96	0.95^{*}	1.03^{***}	1.01
IP	0.99	1.00	1.03^{*}	1.09^{***}	1.00	1.00	1.00	1.00	1.01	1.03	1.02^{*}	1.05^{***}
Capacity Utilization	1.00	0.99	1.01	1.08^{***}	1.01	1.00	0.99^{**}	0.99	1.00	1.04	1.00	1.05
Unemployment Rate	1.00	1.01	1.02	1.03^{***}	1.00	1.00	0.99^{***}	0.99	1.01	1.01	1.02	1.05^{*}
Nonfarm Payrolls	1.00	0.99	1.05^{***}	1.08^{***}	1.00	1.00	0.99^{***}	0.99	0.97	0.99	1.03^{*}	1.04^{***}
Hours	1.02^{*}	1.01	0.99	1.03	1.00	1.00	1.00	0.99^{***}	1.00	1.02	1.00	1.01
Hourly Earnings	1.02	1.01	1.04^{***}	1.07^{***}	1.01	1.00	0.99^{**}	0.98^{***}	0.99	0.98	1.01	1.04^{***}
PPI (Fin. Goods)	0.98^{**}	1.00	1.01	1.03^{**}	1.00	1.00	1.01^{**}	1.00	0.99	1.00	1.01	1.02^{*}
PCE Prices	0.99^{*}	0.99	0.99	1.00	1.00	1.00	1.01^{**}	1.00	0.99	1.00	1.01	0.99
Housing Starts	1.01	1.00	0.99	0.98	1.00	1.00	1.00	1.00	1.01	1.02	1.01	0.99
S&P 500	1.00	1.01	1.03^{*}	1.08^{***}	1.00	1.00	0.99	0.99^{**}	1.02	1.00	1.01	1.05^{**}
USD / GBP FX Rate	1.00	1.00	1.01	1.03^{**}	1.02^{**}	1.02^{**}	1.00	0.99	1.01	0.99	1.01	1.04^{***}
5-Year Yield	1.00	0.98	0.98	1.03^{***}	0.99	0.99	0.99	0.97^{**}	1.02	1.01	0.98	0.99
10-Year Yield	1.00	0.99	0.99	1.01	0.99	0.99	0.99	0.98	1.00	1.01	0.99	0.98
Baa Spread	0.99	0.98^{*}	0.97	1.08^{***}	1.00	1.01	1.02	0.98	1.02	1.01	0.99	1.04

Note: Comparison of “SVO-t” (baseline, in denominator) against “SVO,” “SV-t,” and “SV-OutMiss.” Values below 1 indicate improvement over baseline. Evaluation window from 2007:M01 through 2014:M12. Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags.

Table S.6: Relative RMSE since 1985

					Relative to SV …							
	SV				CONST				SVO-t			
Variable / Horizons	1	3	12	24	1	3	12	24	1	3	12	24
Real Income	7.60	7.65	7.72	8.44	1.00	1.01	1.00	0.92*	1.01	1.00	1.01**	0.93*
Real Consumption	5.38	5.60	5.36	5.05	1.00	1.00	1.00	1.00	1.01	1.01*	1.00	1.00
IP	6.68	7.08	7.80	8.39	1.01	1.00	0.99	0.98	1.00	0.99	1.00	0.96***
Capacity Utilization	0.45	0.85	2.79	4.28	1.02*	1.00	0.99	0.96	1.00	0.99	1.00	0.96
Unemployment Rate	0.14	0.22	0.73	1.33	0.99	0.97	1.00	1.05	1.00	0.99	0.99	0.99
Nonfarm Payrolls	1.21	1.33	1.86	2.11	1.04*	1.02	1.02	1.01	1.00	1.01	1.01	0.97
Hours	0.19	0.23	0.40	0.43	1.04***	1.00	1.04	1.07*	1.00	1.00	0.99	1.00
Hourly Earnings	2.48	2.46	2.54	2.73	1.04**	1.01	1.03	1.06	1.00	1.00	1.01**	1.03
PPI (Fin. Goods)	7.16	7.35	7.58	7.43	1.00	1.02	1.05	1.07	1.00	1.00	1.00	1.00
PCE Prices	2.11	2.42	2.64	2.81	1.01	1.03	1.13*	1.19**	1.01	1.00	1.01	1.03
Housing Starts	0.07	0.10	0.23	0.35	0.99	0.99	1.03	1.08**	0.99	0.99	1.00	1.03***
S&P 500	43.71	44.17	44.03	43.28	0.99	1.01	1.00	1.00	1.01	1.00	1.01*	1.01**
USD / GBP FX Rate	28.35	29.79	28.26	33.35	1.03**	1.02	1.02	0.86	0.99	1.00	1.00	0.85
5-Year yield	0.25	0.55	1.14	1.43	1.05***	1.06***	1.04	0.92	1.01*	1.00	1.02*	0.97
10-Year yield	0.23	0.51	1.08	1.22	1.03	1.06**	1.08*	1.03	1.01**	1.01	1.01	0.99
Baa Spread	0.26	0.61	1.33	1.51	1.20***	1.25**	1.12*	1.15	0.99	0.99**	1.00	0.98

Note: Comparison of "SV" (baseline, in denominator of relative comparisons) against "CONST" and "SVO-t." Values below 1 indicate improvement over baseline. Evaluation window from 1985:M01 through 2017:M12. Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags.

Table S.7: Relative Avg CRPS since 1985

	SV				Relative to SV … CONST				SVO-t			
Variable / Horizons	1	3	12	24	1	3	12	24	1	3	12	24
Real Income	3.17	3.32	3.63	4.19	1.09***	1.00	0.94*	0.82***	0.99	0.96***	0.94***	0.86***
Real Consumption	2.76	2.88	3.05	3.56	1.04**	1.04***	0.98	0.82***	1.00	1.00	0.97***	0.91***
IP	3.65	3.83	4.50	5.37	1.03**	1.02	0.95***	0.84***	1.00	0.99*	0.96***	0.90***
Capacity Utilization	0.26	0.48	1.54	2.57	1.03**	1.04*	0.99	0.90**	1.00	1.00	1.00	0.96
Unemployment Rate	0.08	0.12	0.38	0.72	0.99	1.00	1.03	1.05	1.00	1.00	1.01	1.00
Nonfarm Payrolls	0.69	0.77	1.12	1.48	1.10***	1.07***	0.99	0.84***	0.99	1.00	0.98*	0.93***
Hours	0.10	0.13	0.23	0.29	1.07***	1.02	1.02	0.91**	0.99	0.99*	0.98*	0.92***
Hourly Earnings	1.39	1.41	1.58	2.04	1.07***	1.05***	0.98	0.84***	1.00	0.99**	0.98***	0.93***
PPI (Fin. Goods)	3.75	3.90	4.15	4.58	1.02	1.02	1.02	0.93**	1.00	0.99	0.98***	0.95***
PCE Prices	1.10	1.26	1.47	1.81	1.03**	1.04*	1.07	0.96	1.01**	1.00	1.00	0.98**
Housing Starts	0.04	0.05	0.12	0.19	0.99	1.00	1.05	1.03	1.00	1.00	1.01**	1.02**
S&P 500	22.98	23.44	25.00	28.39	1.00	1.01	0.94***	0.82***	1.00	0.99*	0.97***	0.92***
USD / GBP FX Rate	15.59	16.14	16.53	18.39	1.03**	1.02	0.96**	0.86***	0.99*	0.99	0.97***	0.91***
5-Year yield	0.14	0.30	0.63	0.80	1.06***	1.08***	1.06	0.97	1.01**	1.01	1.01*	1.01
10-Year yield	0.13	0.28	0.60	0.75	1.04***	1.06***	1.08*	1.00	1.01***	1.01	1.02**	1.02**
Baa Spread	0.14	0.32	0.75	1.01	1.34***	1.30***	1.11*	0.99	1.00	0.99	0.99	0.97**

Note: Comparison of "SV" (baseline, in denominator of relative comparisons) against "CONST" and "SVO-t." Values below 1 indicate improvement over baseline. Evaluation window from 1985:M01 through 2017:M12. Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags.

Table S.8: Relative RMSE since 1985 (alternative models)

	SVO				SV-t				SV-OutMiss			
Variable / Horizon	1	3	12	24	1	3	12	24	1	3	12	24
Real Income	0.99**	0.99***	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.02
Real Consumption	0.99*	0.99**	1.00	0.99**	1.00	1.00	1.00	1.00	0.98	0.98	1.00	1.00
IP	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.00	1.02**
Capacity Utilization	0.99**	0.99	0.99	1.01	1.00	1.00	0.99	1.00	1.00	1.02	0.99	1.01
Unemployment Rate	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99**	0.99	1.00	1.00	1.01
Nonfarm Payrolls	1.01	0.99	0.99	0.99	1.00	1.00	0.99	1.00	0.99	0.99	0.98	1.01
Hours	1.00	1.00	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.01	1.00	1.00
Hourly Earnings	1.00	1.00	0.98***	0.95***	1.00	1.00	1.00	1.00	0.99	1.00	0.99**	0.97*
PPI (Fin. Goods)	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
PCE Prices	1.00	1.00	0.99	0.97*	1.00	1.00	1.00	1.00	0.99	0.99	1.01	0.99
Housing Starts	1.00	1.00	0.99**	0.98*	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98**
S&P 500	1.00	1.00	0.99	0.99***	1.00	1.00	1.00	1.00	1.00	0.99	0.99*	1.00
USD / GBP FX Rate	1.00	0.99*	1.00	1.00	1.00	1.00	1.00	1.01**	1.02	0.99	1.00	0.99**
5-Year Yield	1.00	0.99	0.99*	1.02	0.99	1.00	1.00	0.99	0.99	0.98*	0.98	0.99
10-Year Yield	0.99	0.99	0.98**	0.99	0.99*	1.00	1.00	1.00	0.99	0.99	0.99	0.99
Baa Spread	1.00	0.99	0.98	1.02	1.01	1.01*	1.01	1.00	1.01	1.00	0.99	1.04

Note: Comparison of "SVO-t" (baseline, in denominator) against "SVO," "SV-t," and "SV-OutMiss." Values below 1 indicate improvement over baseline. Evaluation window from 1985:M01 through 2017:M12. Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags.

Table S.9: Relative Avg CRPS since 1985 (alternative models)

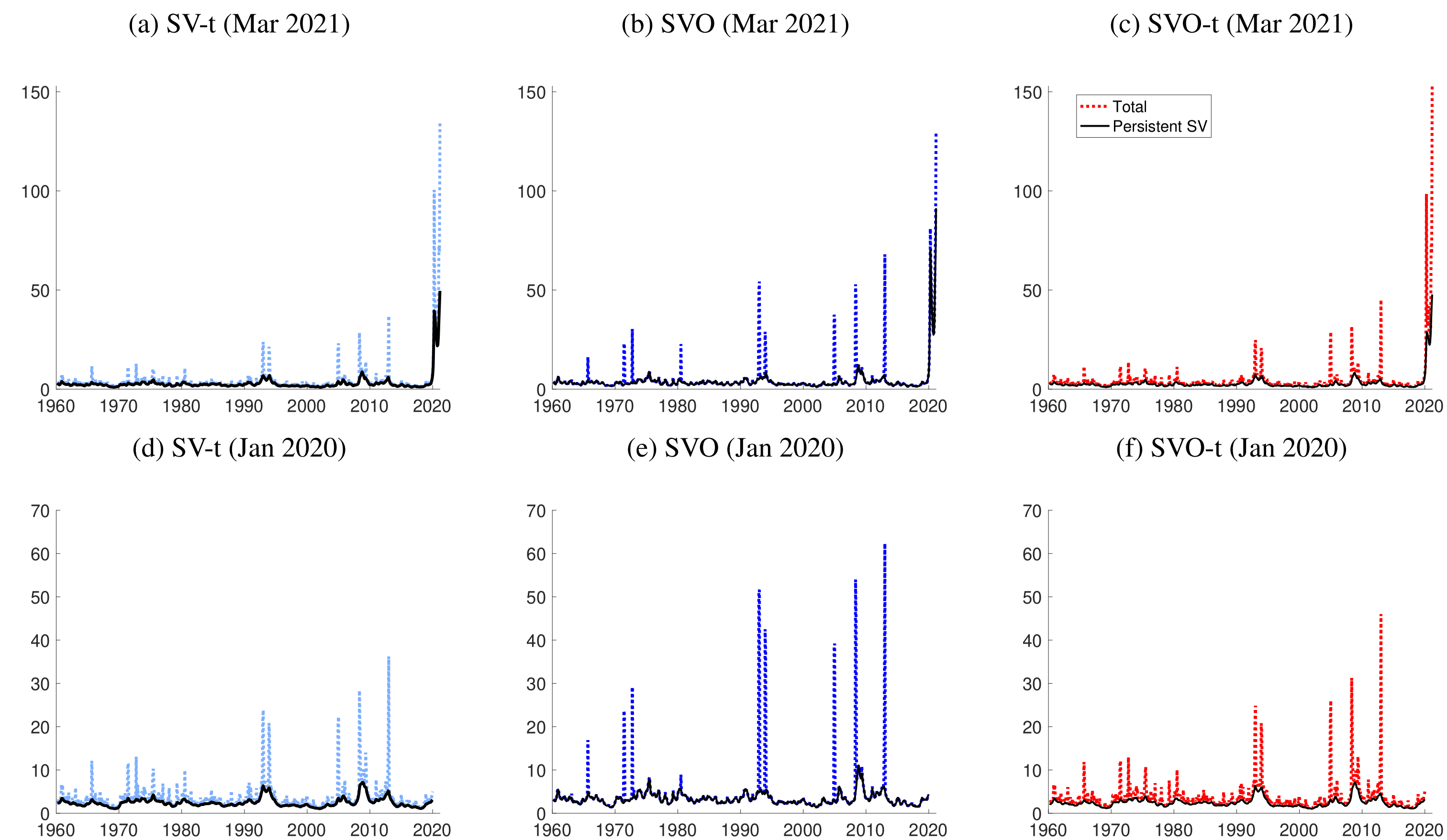
	SVO				SV-t				SV-OutMiss			
Variable / Horizon	1	3	12	24	1	3	12	24	1	3	12	24
Real Income	1.00	1.01	1.03***	1.07***	1.00	1.00	1.00*	0.99***	0.99	0.99*	1.00	1.01
Real Consumption	1.00	1.01	1.05***	1.09***	1.00*	1.00	0.99***	0.98***	0.98	0.99	1.01	1.04***
IP	0.99	1.02***	1.05***	1.10***	1.00	1.00	0.99***	0.99***	0.99	1.02	1.02***	1.06***
Capacity Utilization	0.99	1.00	1.01	1.05*	1.00	1.00	0.99***	0.98***	0.99	1.01	0.99	1.00
Unemployment Rate	1.00	1.00	1.01	1.02	1.00	1.00*	0.99***	0.99***	0.99	0.99	0.98	1.00
Nonfarm Payrolls	1.01**	1.01	1.04***	1.08***	1.00	1.00	0.98***	0.98***	0.99	1.00	1.00	1.03**
Hours	1.01***	1.01***	1.02**	1.07***	1.00	1.00*	0.99***	0.98***	1.01	1.02**	1.02	1.05***
Hourly Earnings	1.00	1.02***	1.05***	1.08***	1.00	1.00*	0.98***	0.98***	0.99	1.01	1.01	1.04***
PPI (Fin. Goods)	1.00	1.01**	1.03***	1.06***	1.00	1.00	1.00	0.99***	0.99	1.00	1.01**	1.02***
PCE Prices	1.00	1.00	1.02***	1.05***	1.00**	0.99**	0.99***	0.98***	0.98**	0.99***	1.00	0.99
Housing Starts	1.00	1.01*	1.01	1.01	1.00	1.00**	0.99**	0.99	0.99	1.00	0.98*	0.98**
S&P 500	1.00	1.01**	1.04***	1.07***	1.00	1.00	0.99***	0.99***	1.00	1.00	1.01*	1.05***
USD / GBP FX Rate	1.01*	1.00	1.01***	1.03***	1.01	1.00	1.00	1.00	1.01	1.00	1.00	1.02
5-Year Yield	1.00	1.00	1.00	1.03**	1.00	1.00	1.00	0.98***	0.99	0.99	0.99	0.98**
10-Year Yield	1.00	1.00	1.00	1.03***	0.99*	0.99	0.99	0.98***	0.99	0.99	0.99	0.98***
Baa Spread	1.00	1.00	1.02	1.07***	1.00	1.00	0.99	0.97***	1.00	0.99	0.99	1.01

Note: Comparison of "SVO-t" (baseline, in denominator) against "SVO," "SV-t," and "SV-OutMiss." Values below 1 indicate improvement over baseline. Evaluation window from 1985:M01 through 2017:M12. Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Due to the close behavior of some of the models compared, and rounding of the report values, a few comparisons show significant relative CRPS of 1.00. These cases arise from persistent differences in performance that are, however, too small to be relevant after rounding.

V Outlier-augmented volatility estimates

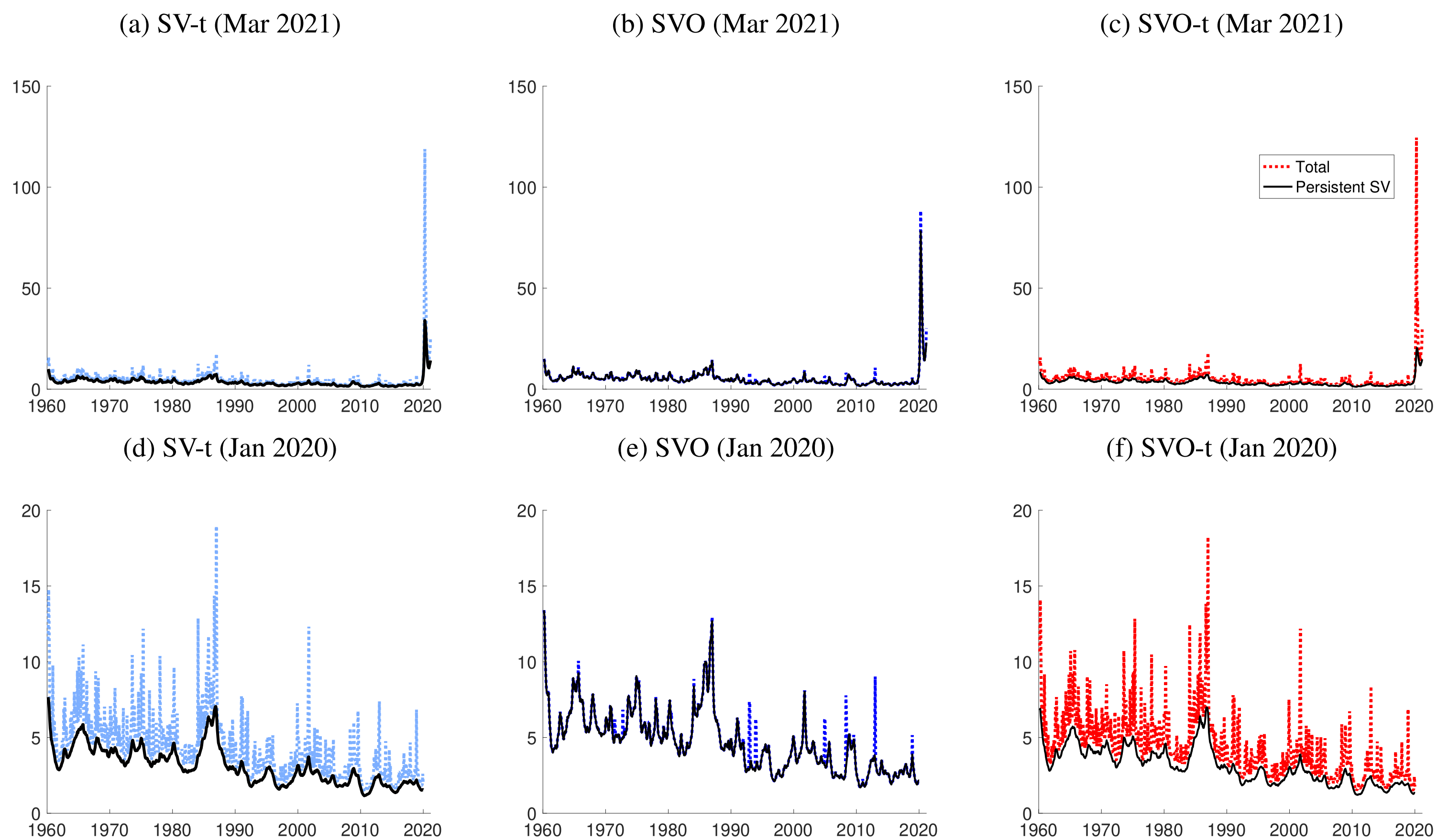
This part of the appendix provides estimates of time-varying volatilities for all variables to complement the selection for individual variables shown in the paper. Figures S.7–S.22 provide a decomposition of full-sample estimates of time-varying forecast error volatility into the total and its persistent component. Each figure provides decompositions from different models (SVO, SV-t, SVO-t) for a given variable. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1}\, O_t\, Q_t\, \Lambda_t\, Q_t'\, O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1}\, \Lambda_t\, A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure. Figures S.23–S.38 document the evolution of estimated paths for forecast error volatilities in quasi-real time for different variables.

Figure S.7: Posteriors of outlier states for Real Income



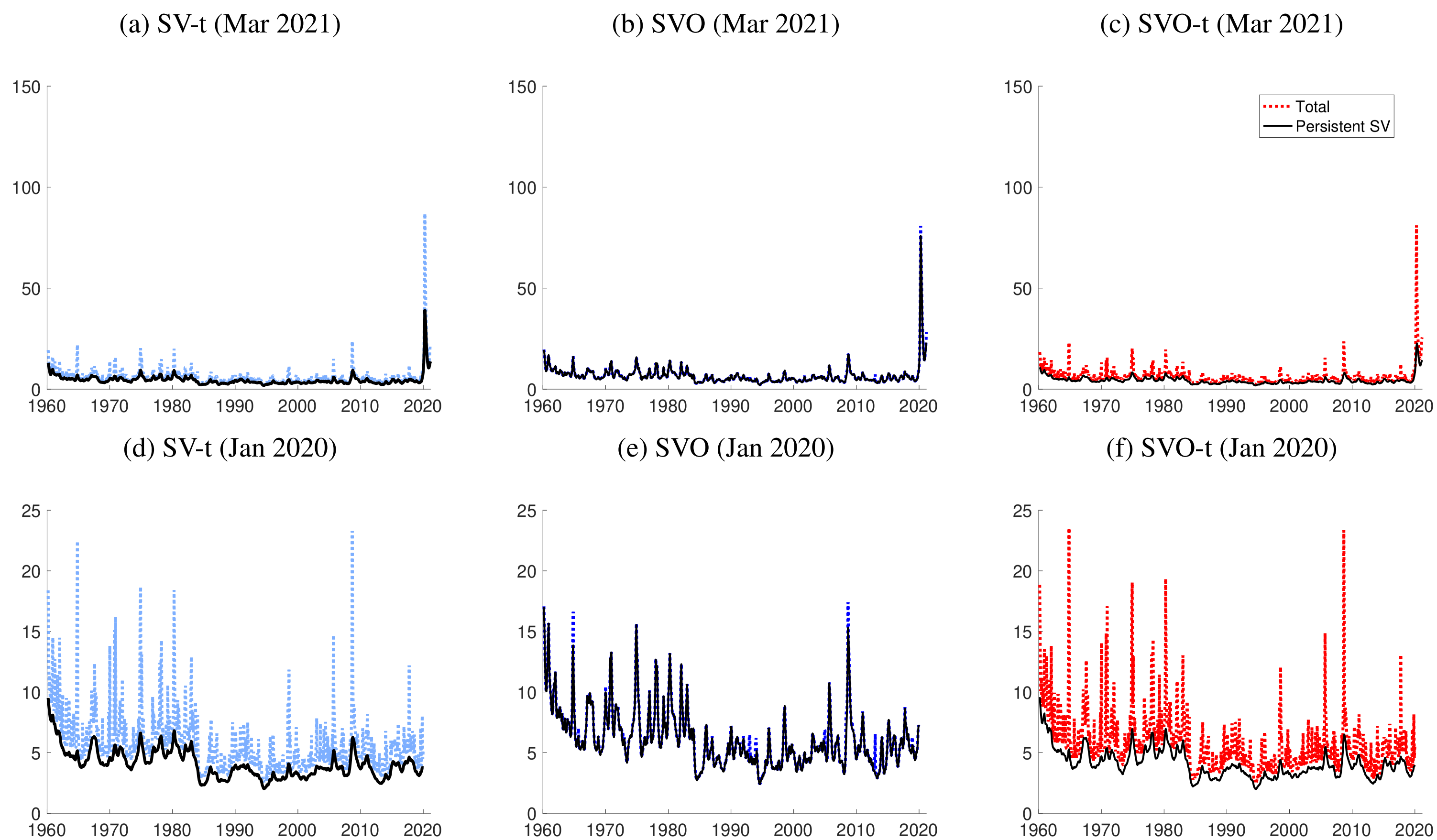
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.8: Posteriors of outlier states for Real Consumption



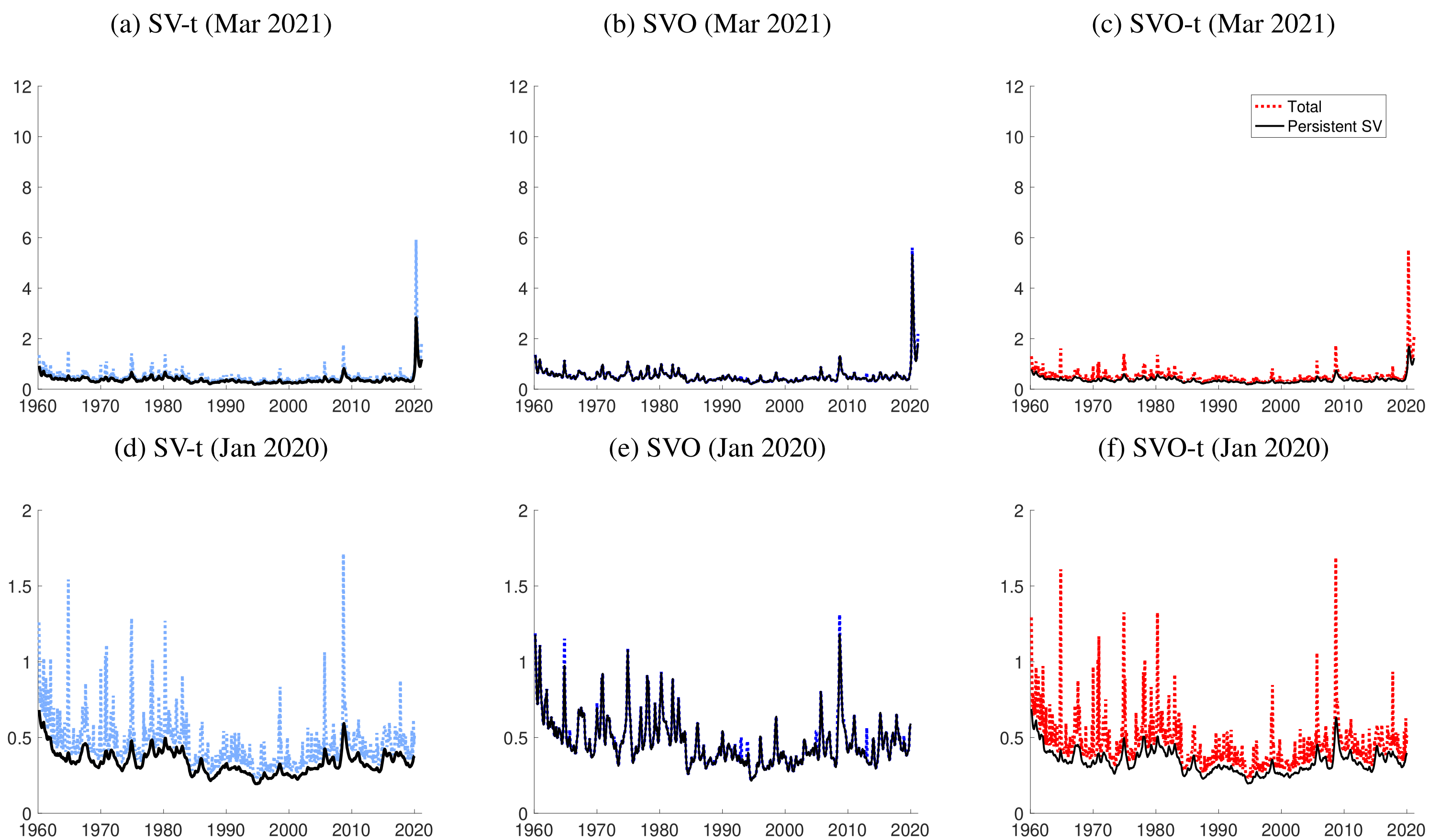
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.9: Posteriors of outlier states for IP



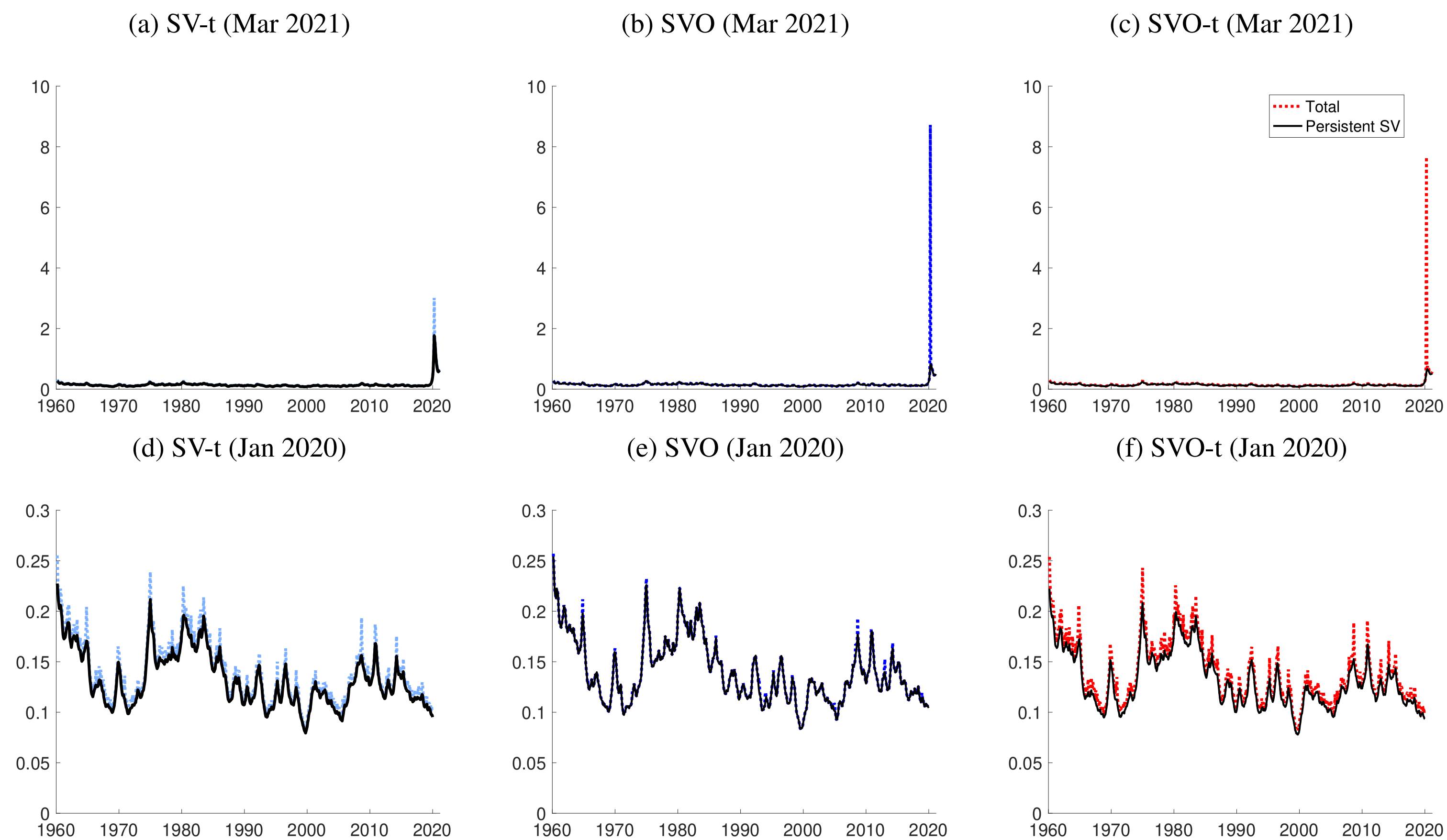
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.10: Posteriors of outlier states for Capacity Utilization



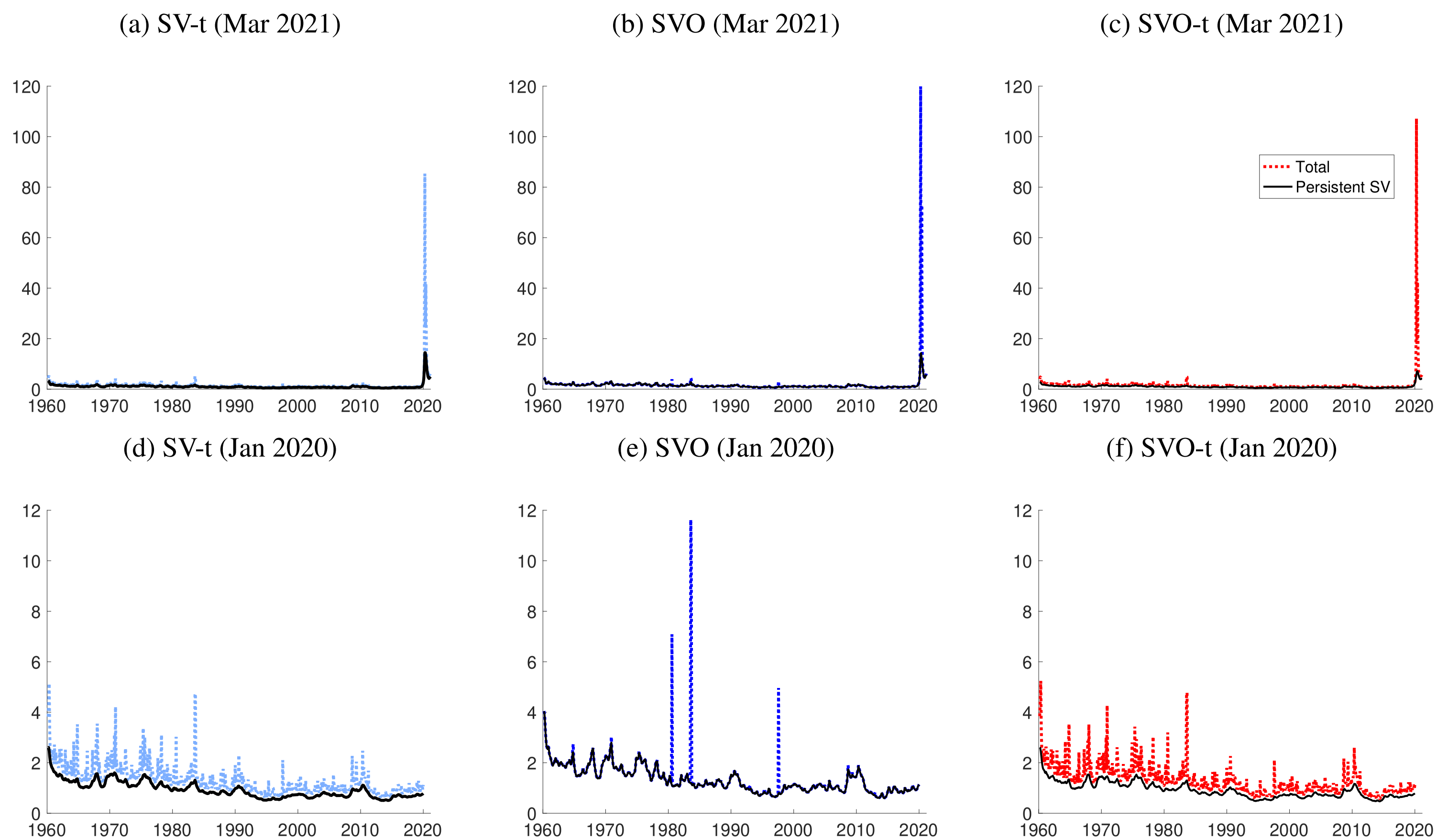
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.11: Posteriors of outlier states for Unemployment



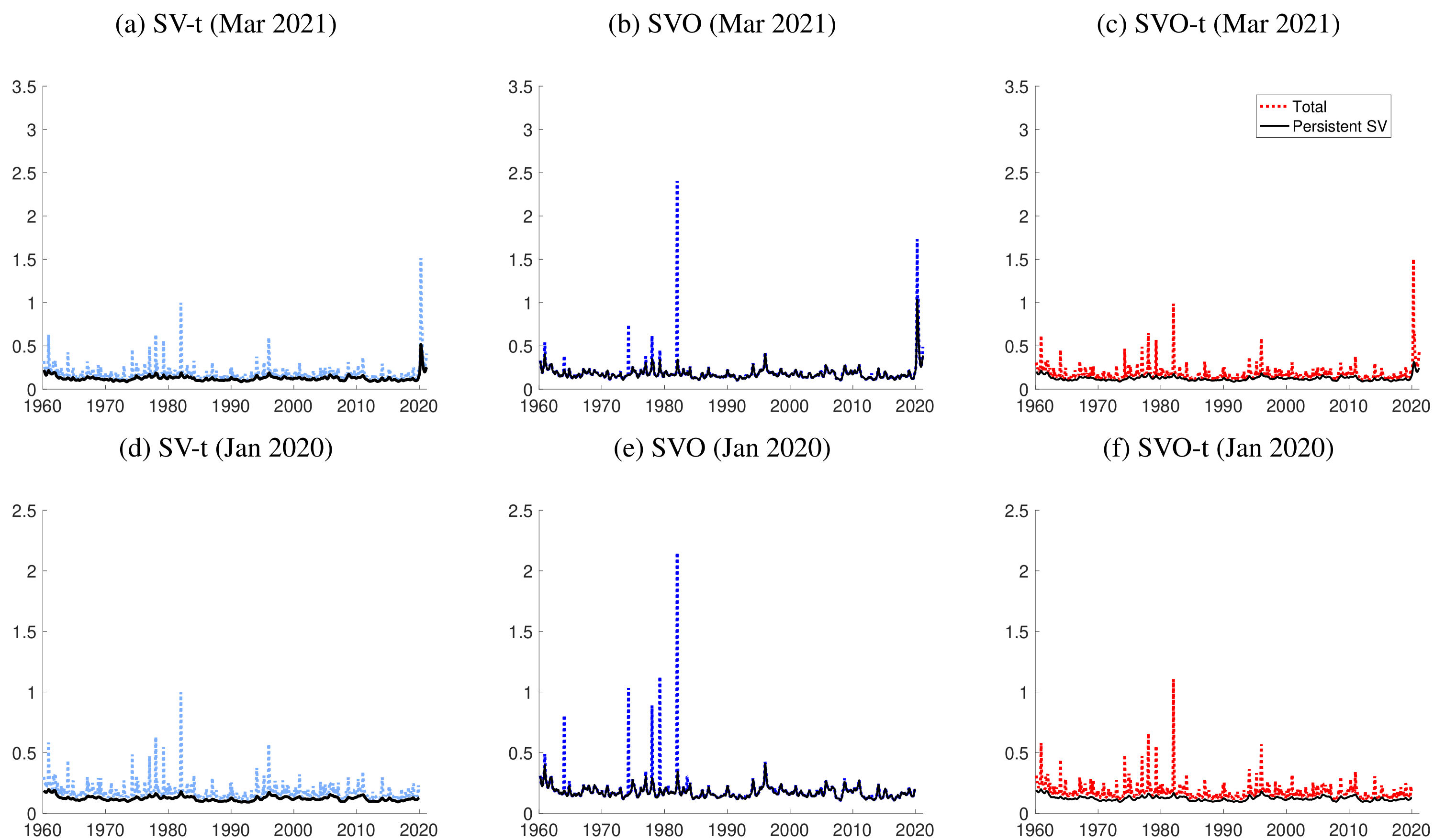
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.12: Posteriors of outlier states for Nonfarm Payrolls



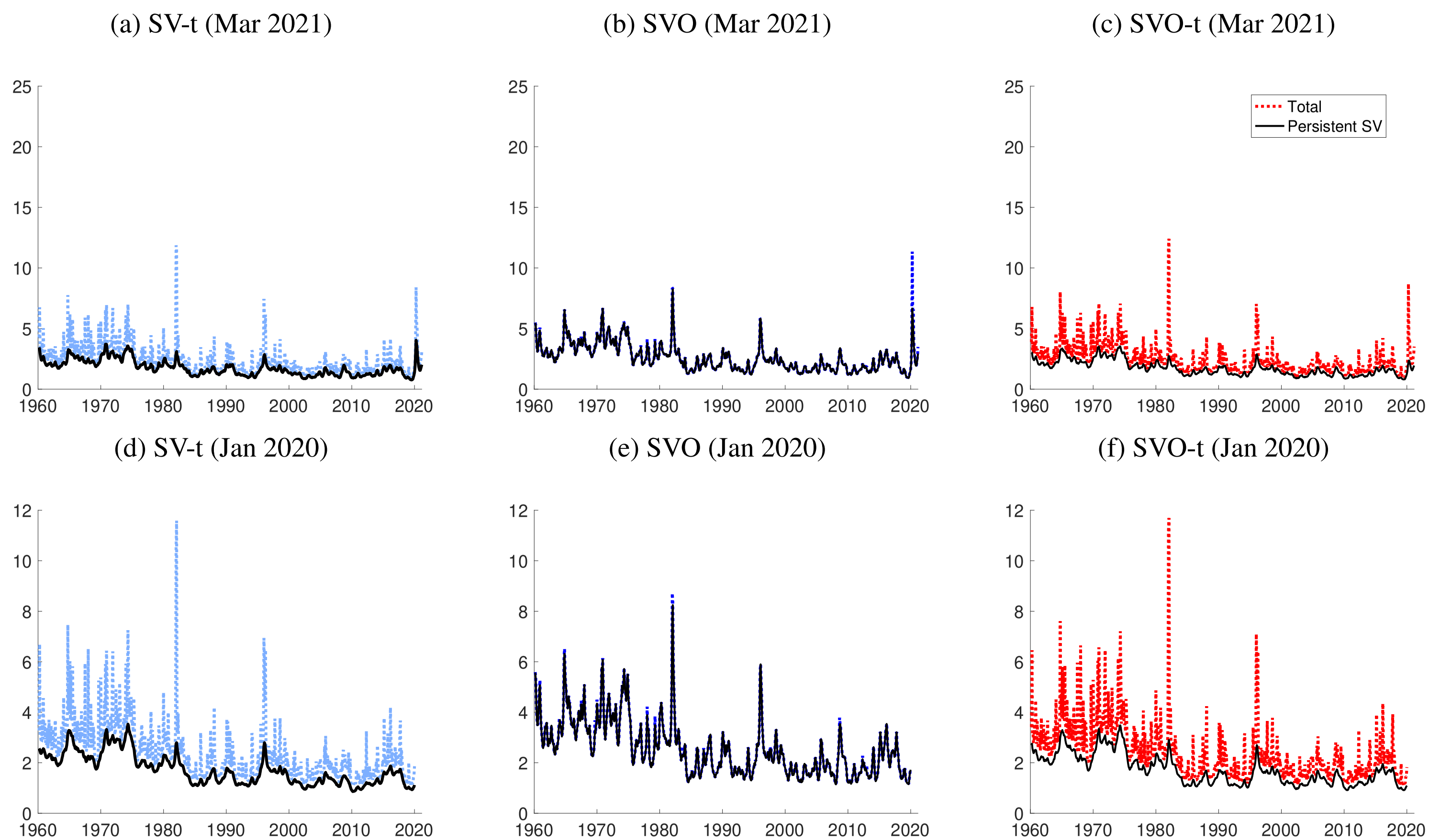
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.13: Posteriors of outlier states for Hours



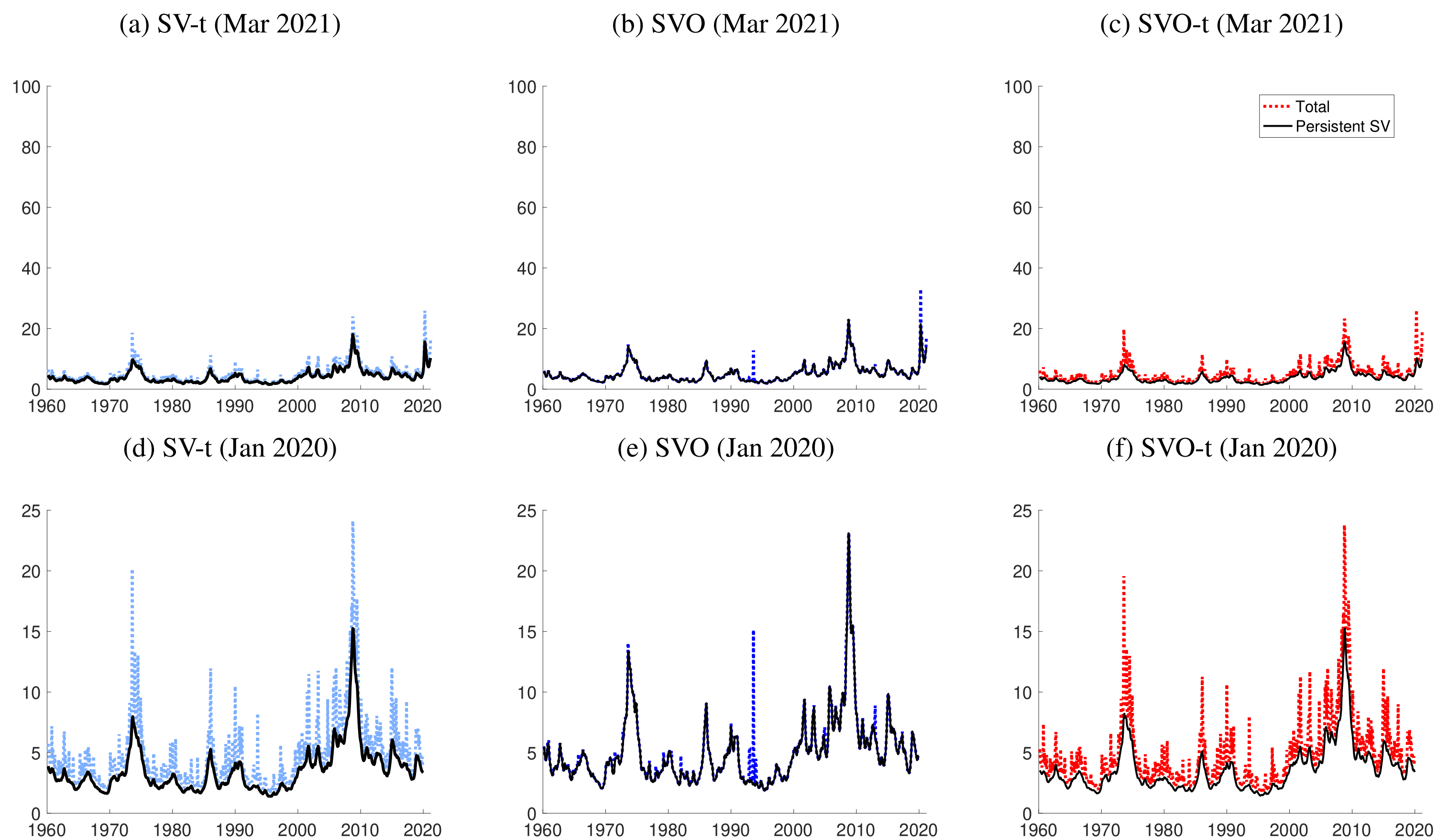
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.14: Posteriors of outlier states for Hourly Earnings



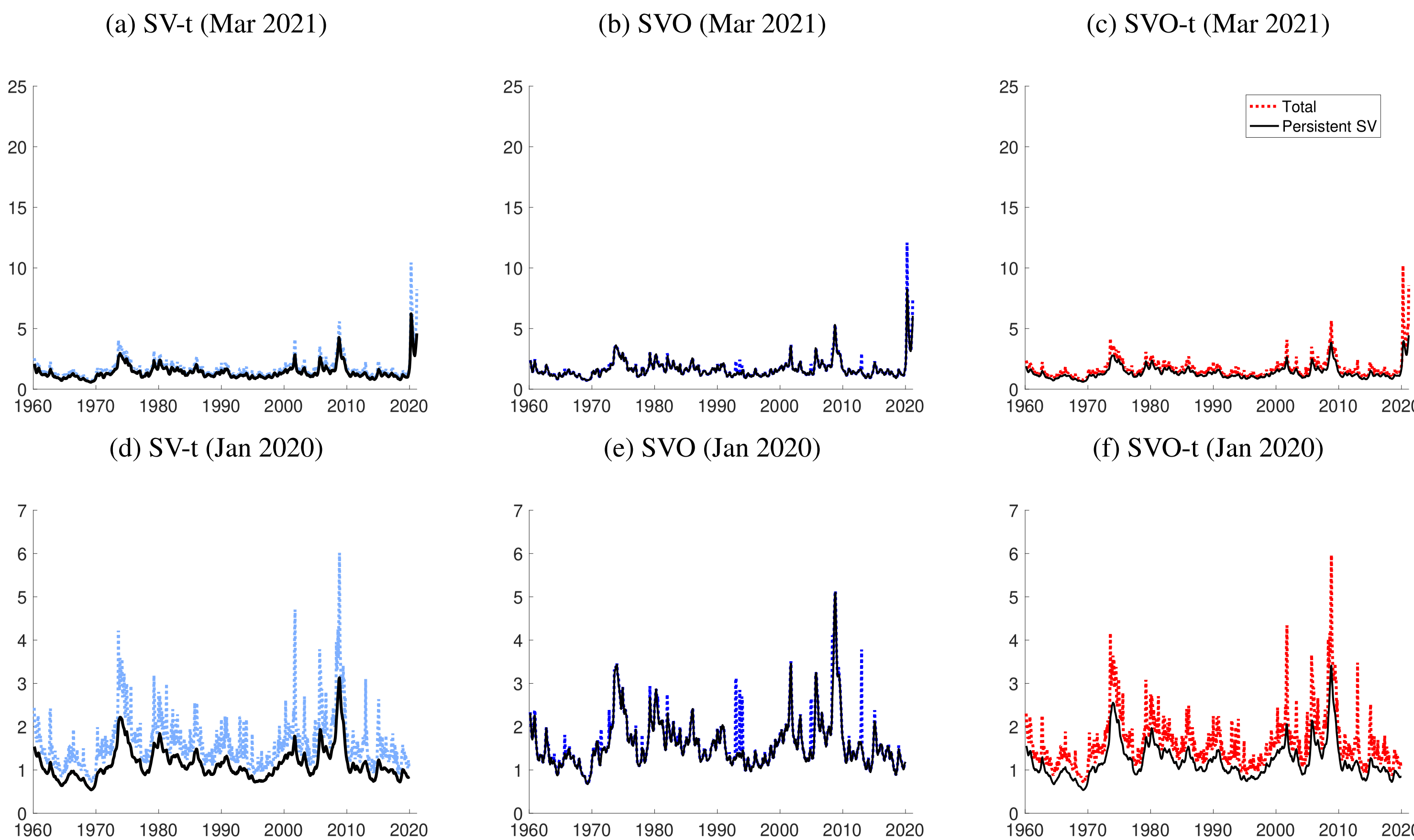
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.15: Posteriors of outlier states for PPI (fin. goods)



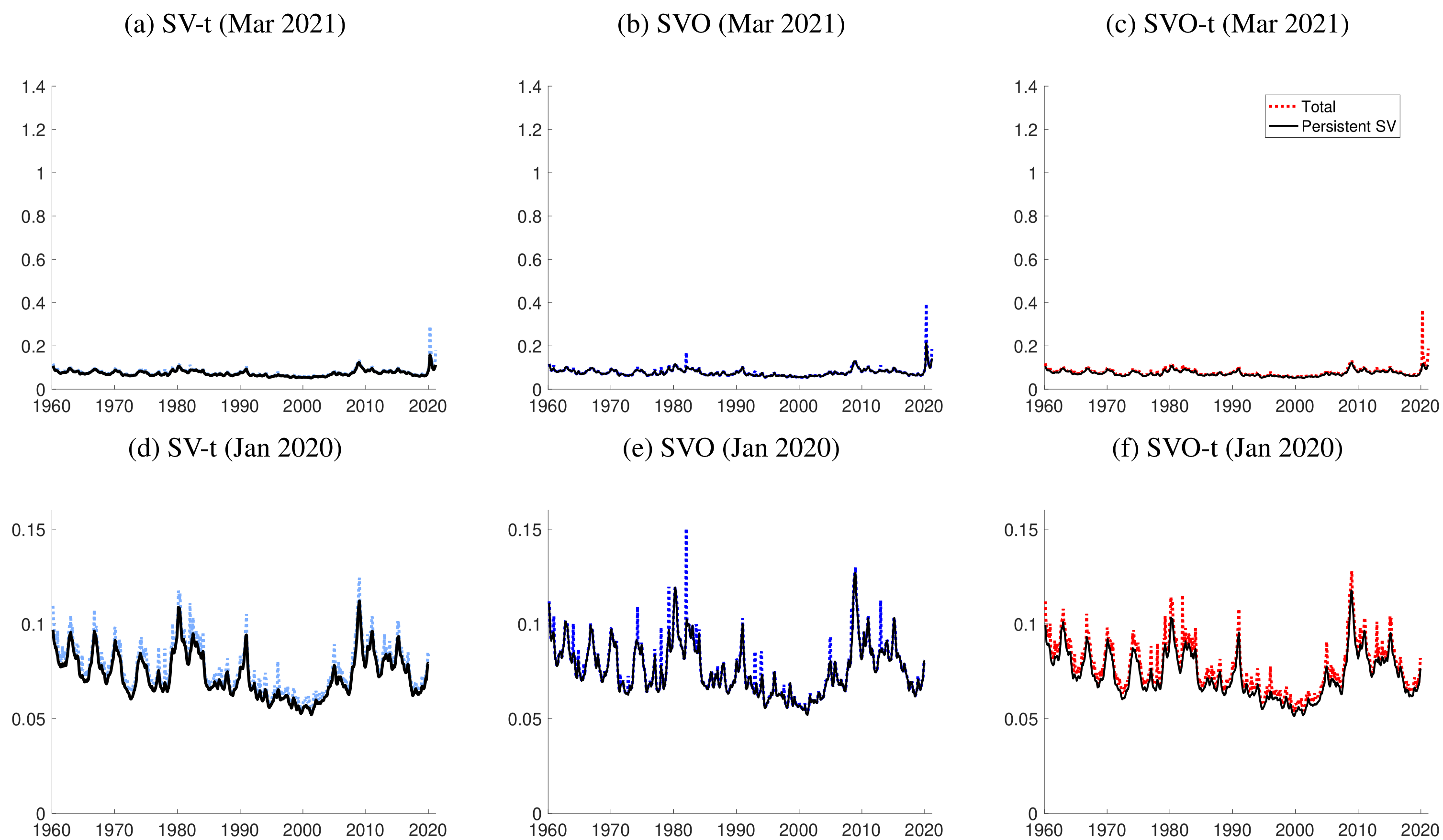
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1}\, O_t\, Q_t\, \Lambda_t\, Q_t'\, O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1}\, \Lambda_t\, A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.16: Posteriors of outlier states for PCE prices



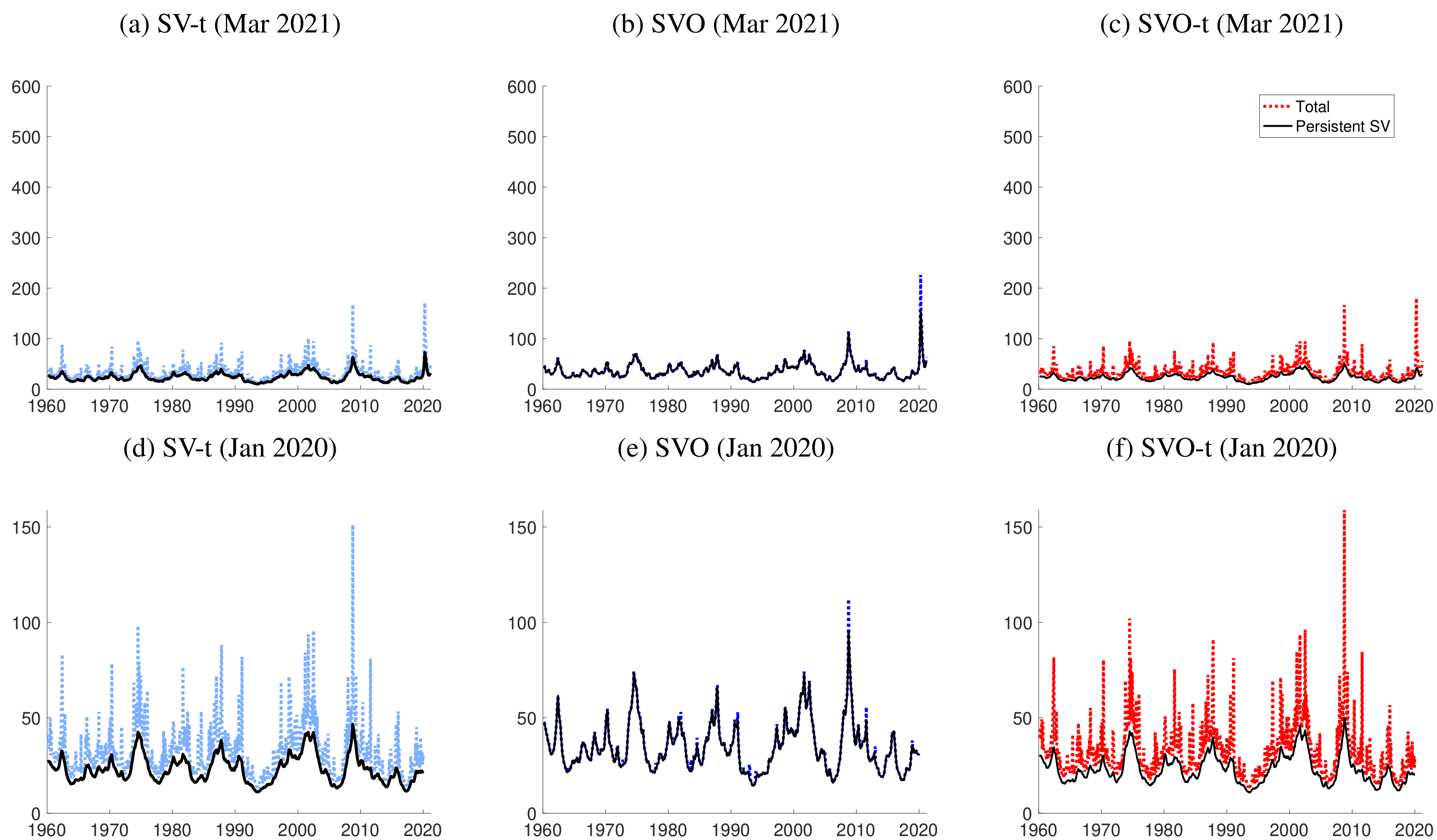
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.17: Posteriors of outlier states for Housing Starts



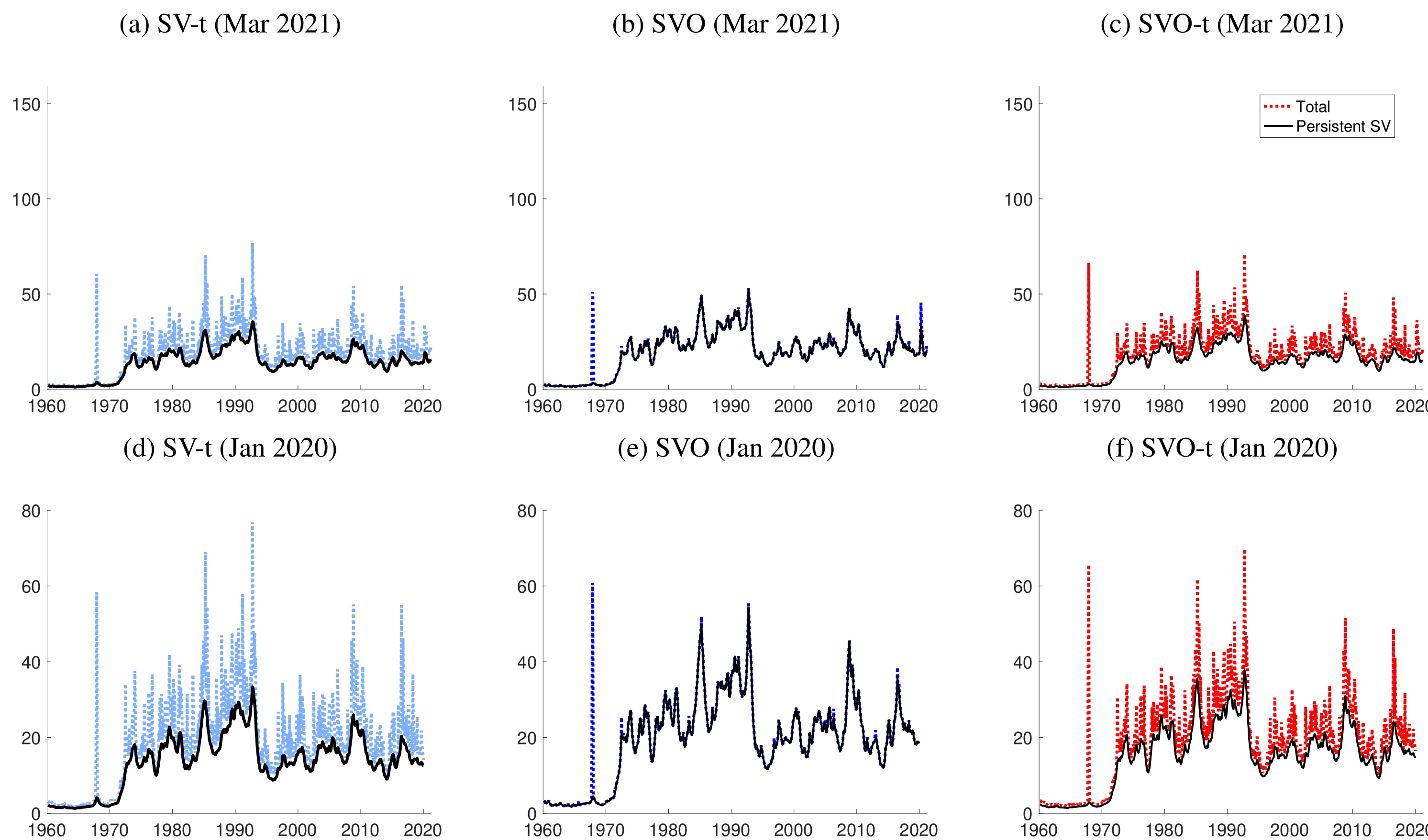
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.18: Posteriors of outlier states for S&P 500



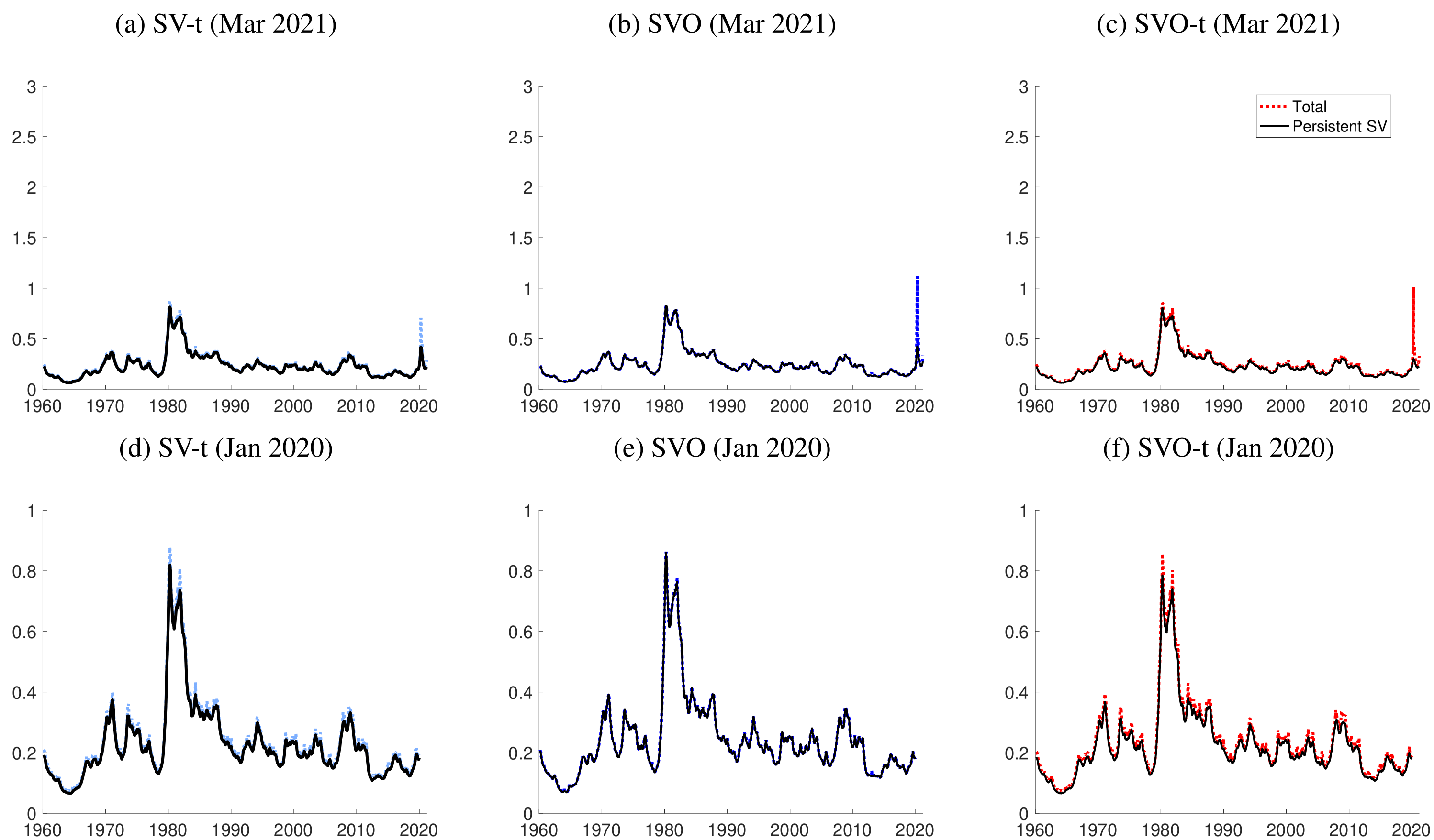
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.19: Posteriors of outlier states for U.S. / U.K. Forex



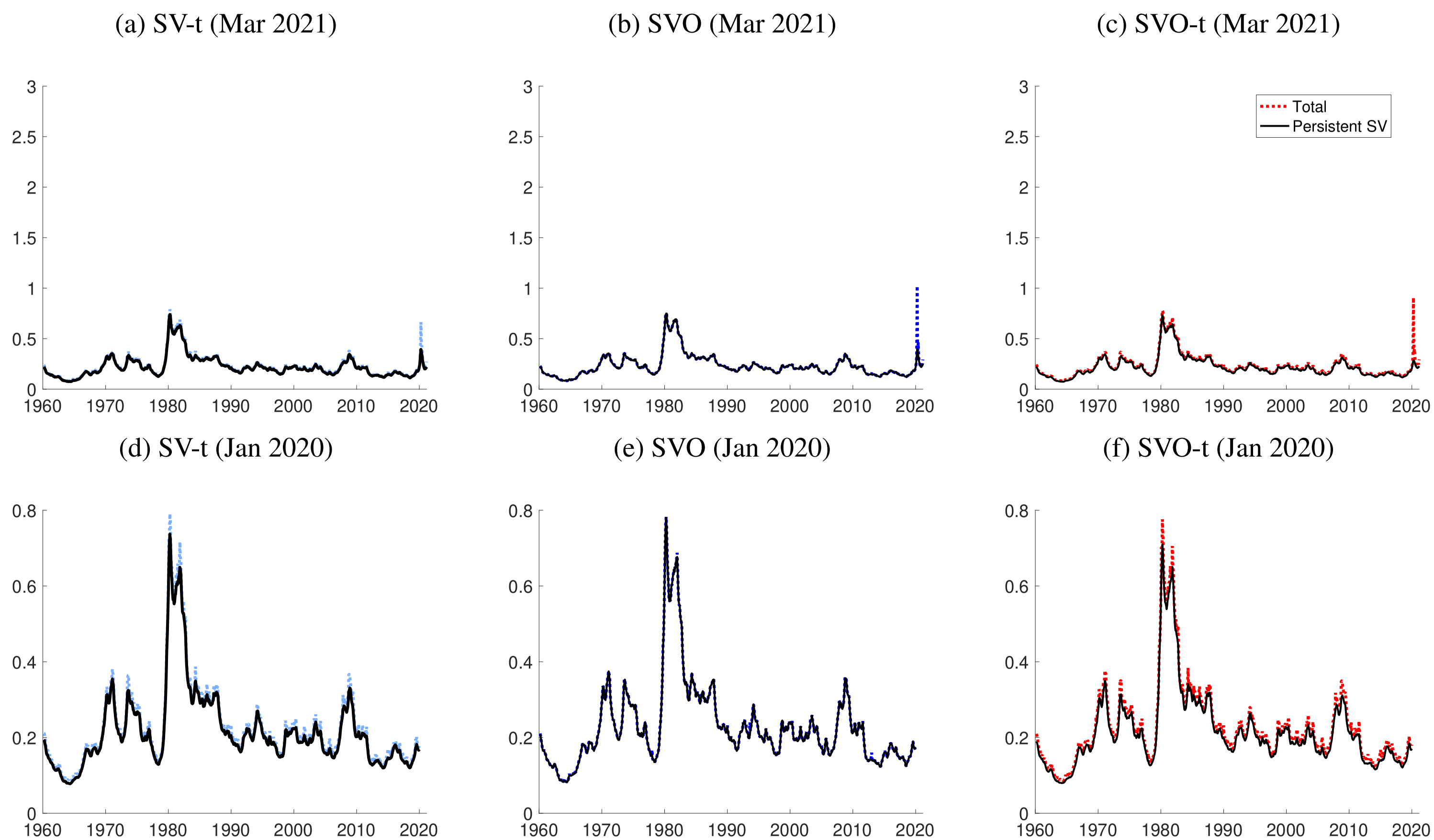
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.20: Posteriors of outlier states for 5-year yield



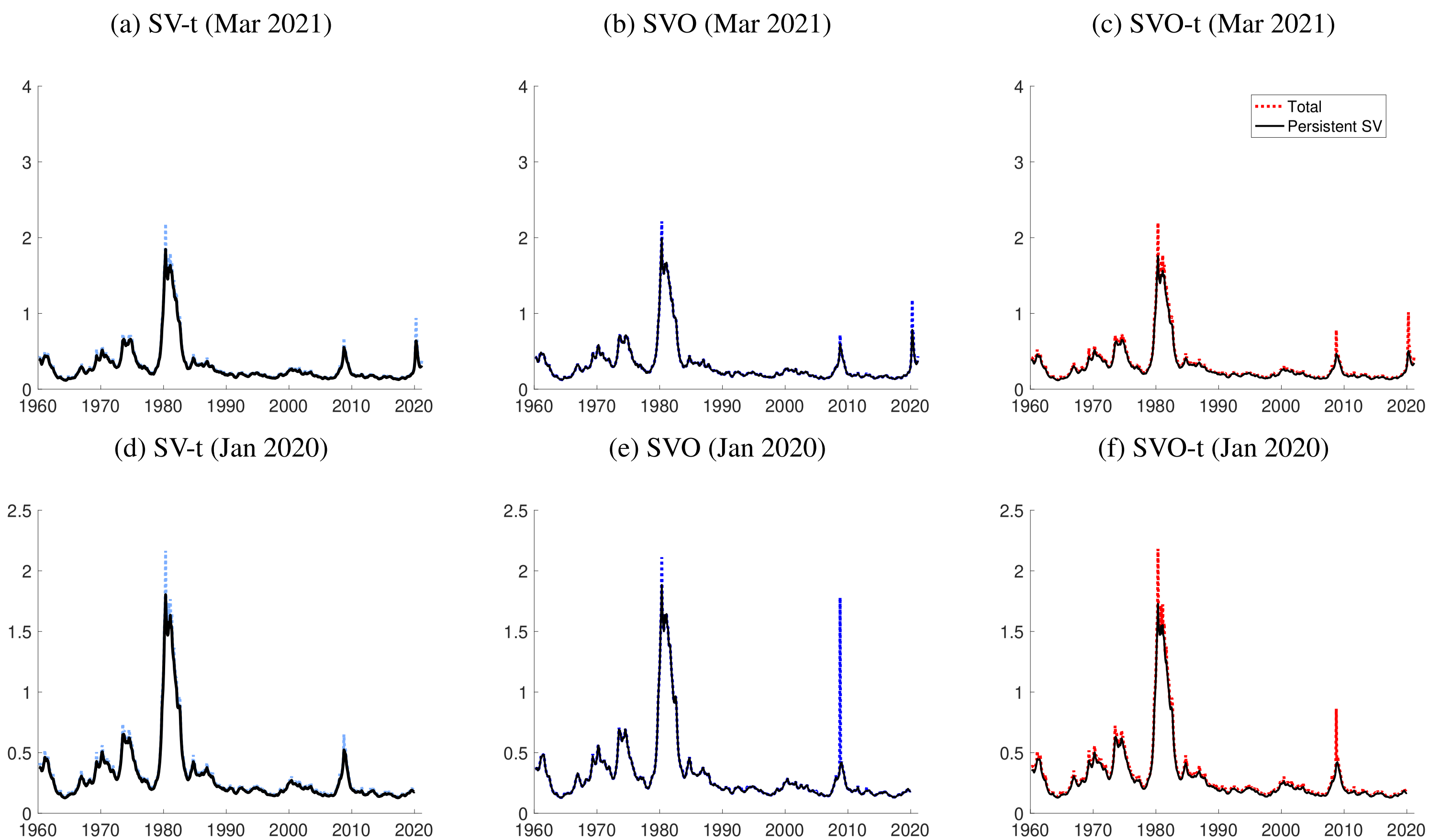
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.21: Posteriors of outlier states for 10-year yield



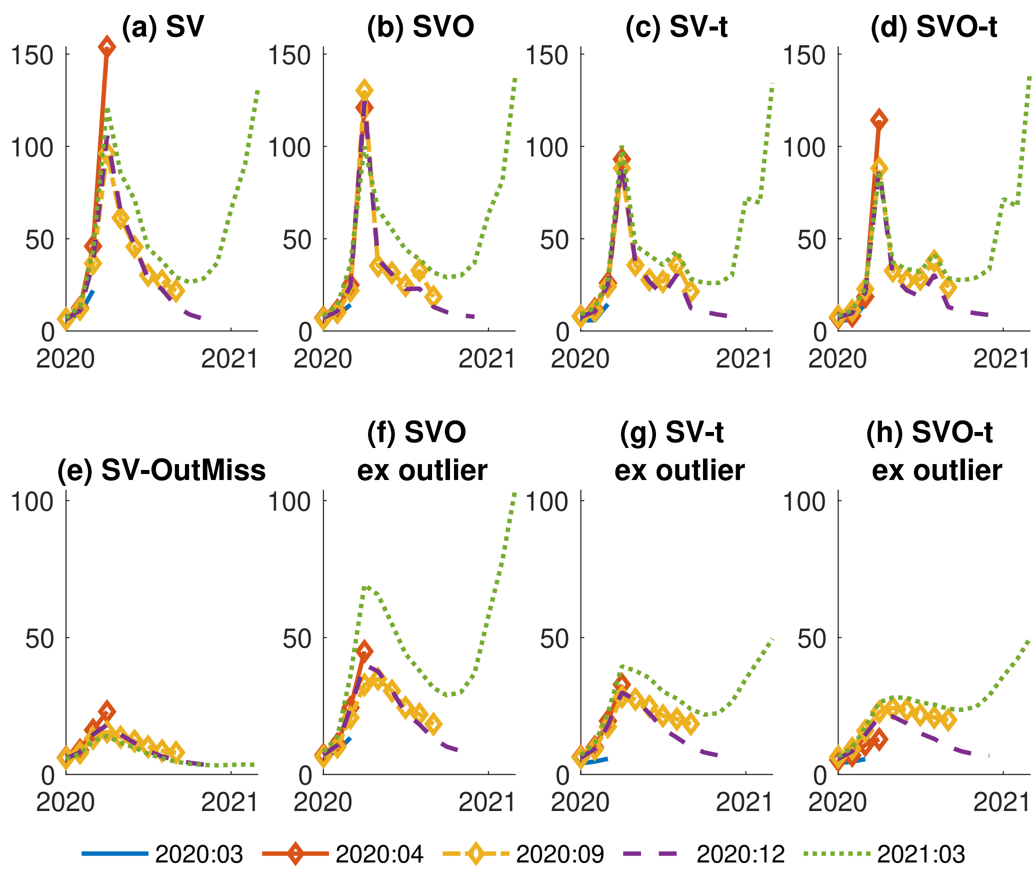
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.22: Posteriors of outlier states for Baa spread



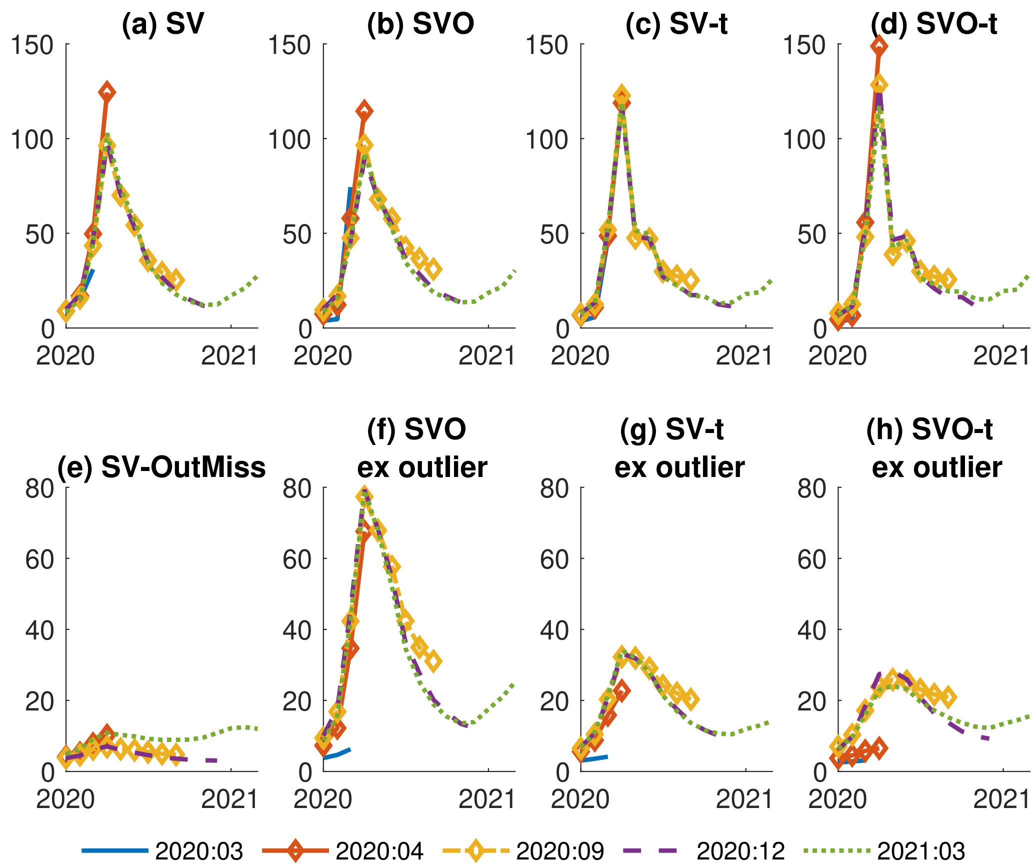
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1}\, O_t\, Q_t\, \Lambda_t\, Q_t'\, O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1}\, \Lambda_t\, A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.23: Time-varying volatilities since 2020 of Real Income



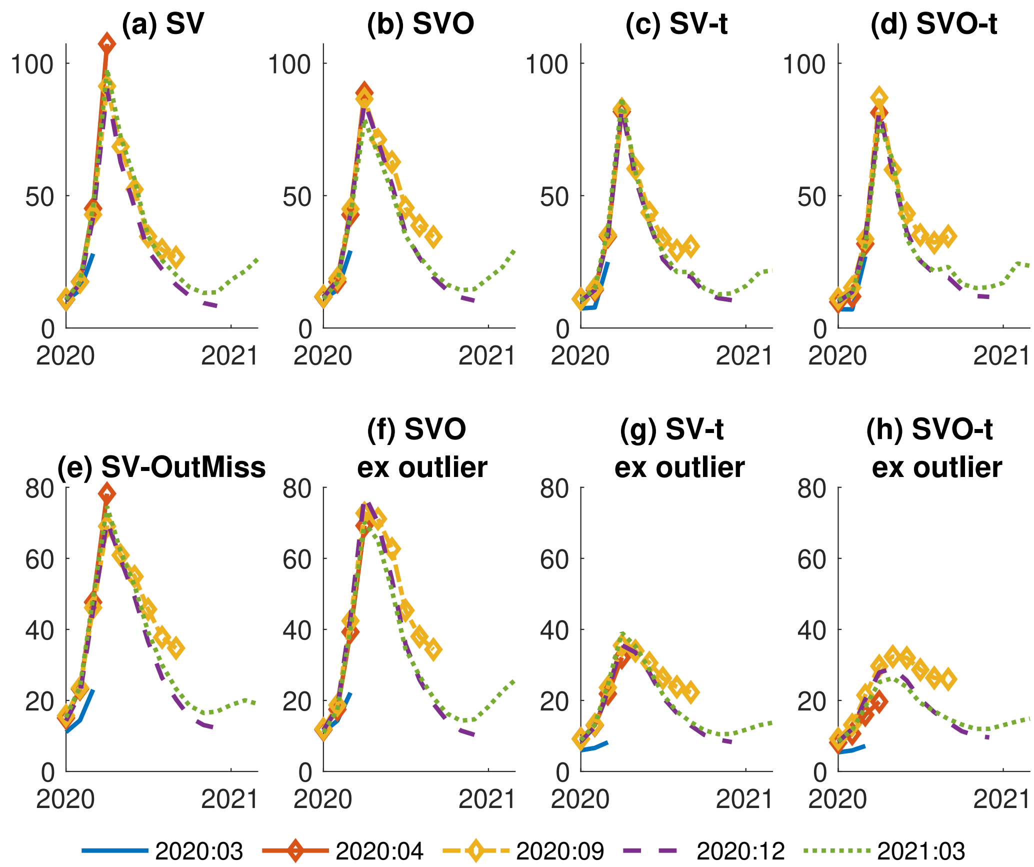
Note: Quasi-real-time trajectories of time-varying volatility in VAR residuals, measured by the diagonal elements of $\mathrm{Var}_t(v_t) = \Sigma_t$ implied by different models. Medians of (smoothed) posterior obtained from different data samples ending at forecast origins as indicated in the figure legend. Panels (b) and (d) display estimates of stochastic volatility for SVO-t that ignore the contributions from outliers and that are computed from $\tilde{\Sigma}_t = A^{-1}\,\Lambda_t\,A^{-T}$ (i.e., neglecting the O_t and Q_t components in the computation of the uncertainty measures shown here, while including these outliers in estimation of A^{-1}, Λ_t, etc.). Reflecting the sizable differences in the size of estimates resulting with and without outlier treatment, different scales are used in upper- and lower-row panels.

Figure S.24: Time-varying volatilities since 2020 of Real Consumption



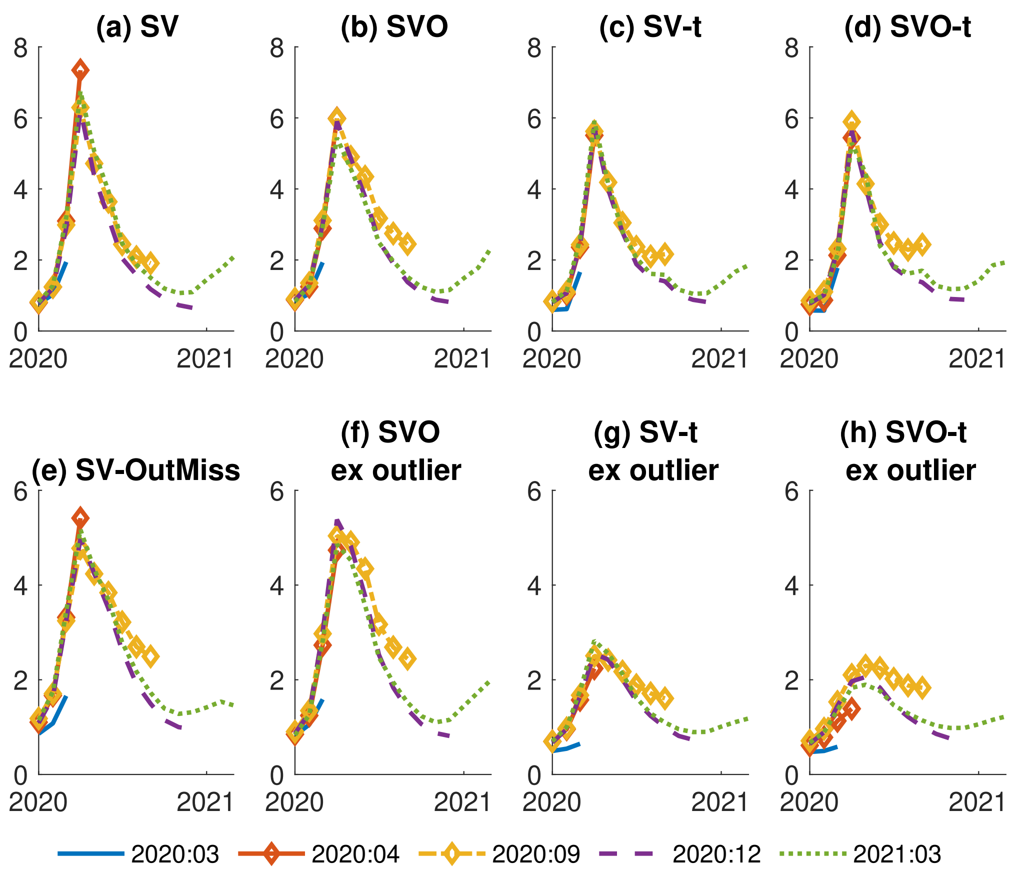
Note: Quasi-real-time trajectories of time-varying volatility in VAR residuals, measured by the diagonal elements of $\mathrm{Var}_t(v_t) = \Sigma_t$ implied by different models. Medians of (smoothed) posterior obtained from different data samples ending at forecast origins as indicated in the figure legend. Panels (b) and (d) display estimates of stochastic volatility for SVO-t that ignore the contributions from outliers and that are computed from $\tilde{\Sigma}_t = A^{-1}\,\Lambda_t\,A^{-T}$ (i.e., neglecting the O_t and Q_t components in the computation of the uncertainty measures shown here, while including these outliers in estimation of A^{-1}, Λ_t, etc.). Reflecting the sizable differences in the size of estimates resulting with and without outlier treatment, different scales are used in upper- and lower-row panels.

Figure S.25: Time-varying volatilities since 2020 of IP



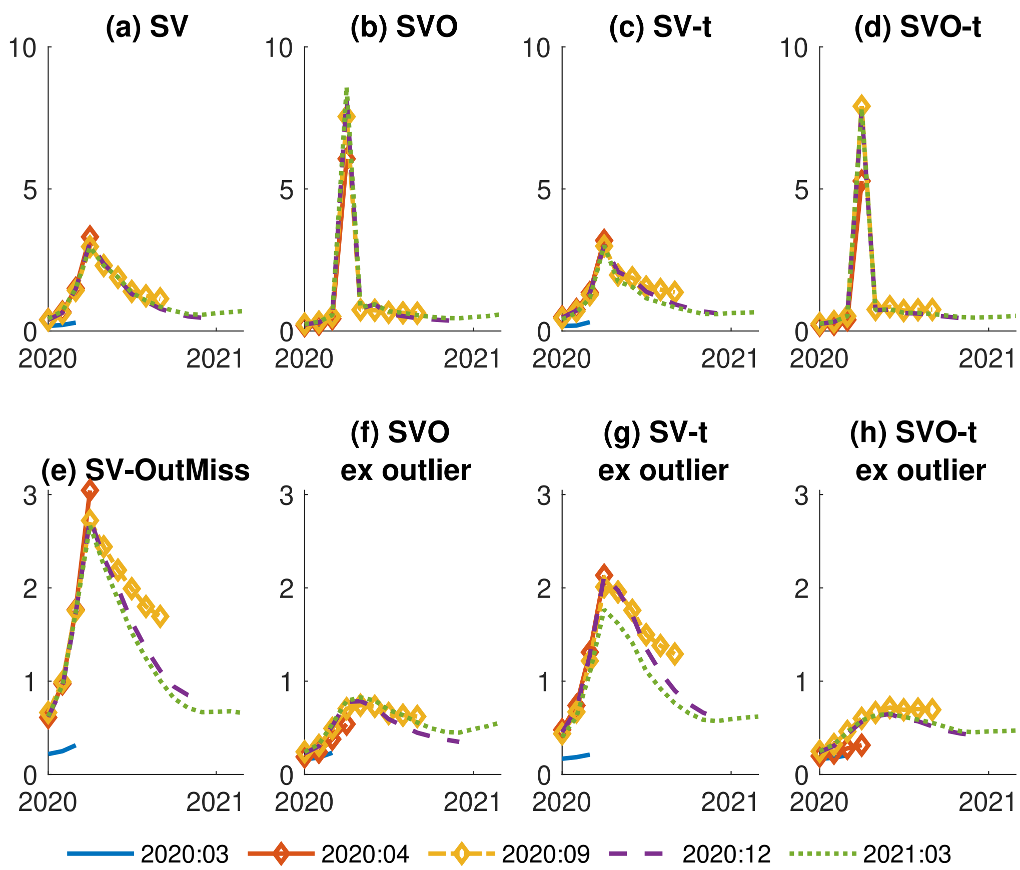
Note: Quasi-real-time trajectories of time-varying volatility in VAR residuals, measured by the diagonal elements of $\mathrm{Var}_t\,(v_t) = \Sigma_t$ implied by different models. Medians of (smoothed) posterior obtained from different data samples ending at forecast origins as indicated in the figure legend. Panels (b) and (d) display estimates of stochastic volatility for SVO-t that ignore the contributions from outliers and that are computed from $\tilde{\Sigma}_t = A^{-1}\,\Lambda_t\,A^{-T}$ (i.e., neglecting the O_t and Q_t components in the computation of the uncertainty measures shown here, while including these outliers in estimation of A^{-1}, Λ_t, etc.). Reflecting the sizable differences in the size of estimates resulting with and without outlier treatment, different scales are used in upper- and lower-row panels.

Figure S.26: Time-varying volatilities since 2020 of Capacity Utilization



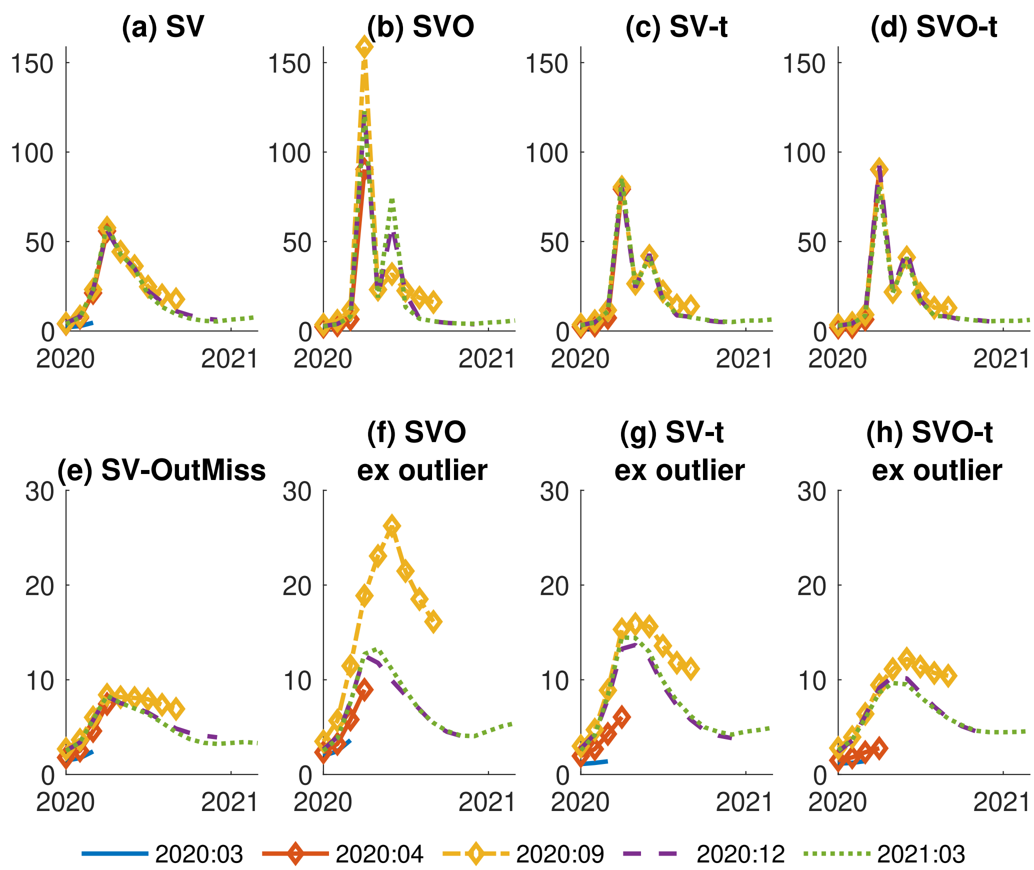
Note: Quasi-real-time trajectories of time-varying volatility in VAR residuals, measured by the diagonal elements of $\mathrm{Var}_t(v_t) = \Sigma_t$ implied by different models. Medians of (smoothed) posterior obtained from different data samples ending at forecast origins as indicated in the figure legend. Panels (b) and (d) display estimates of stochastic volatility for SVO-t that ignore the contributions from outliers and that are computed from $\tilde{\Sigma}_t = A^{-1}\,\Lambda_t\,A^{-T}$ (i.e., neglecting the O_t and Q_t components in the computation of the uncertainty measures shown here, while including these outliers in estimation of A^{-1}, Λ_t, etc.). Reflecting the sizable differences in the size of estimates resulting with and without outlier treatment, different scales are used in upper- and lower-row panels.

Figure S.27: Time-varying volatilities since 2020 of Unemployment



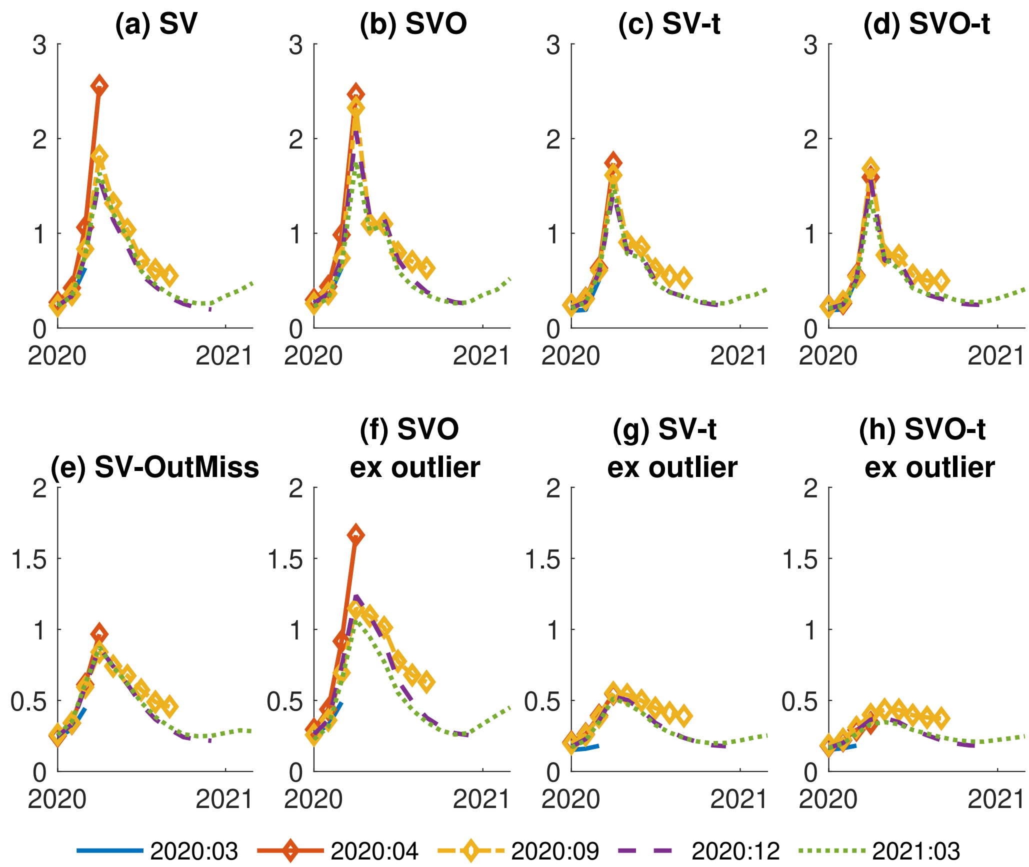
Note: Quasi-real-time trajectories of time-varying volatility in VAR residuals, measured by the diagonal elements of $\mathrm{Var}_t(v_t) = \Sigma_t$ implied by different models. Medians of (smoothed) posterior obtained from different data samples ending at forecast origins as indicated in the figure legend. Panels (b) and (d) display estimates of stochastic volatility for SVO-t that ignore the contributions from outliers and that are computed from $\tilde{\Sigma}_t = A^{-1}\,\Lambda_t\,A^{-T}$ (i.e., neglecting the O_t and Q_t components in the computation of the uncertainty measures shown here, while including these outliers in estimation of A^{-1}, Λ_t, etc.). Reflecting the sizable differences in the size of estimates resulting with and without outlier treatment, different scales are used in upper- and lower-row panels.

Figure S.28: Time-varying volatilities since 2020 of Nonfarm Payrolls



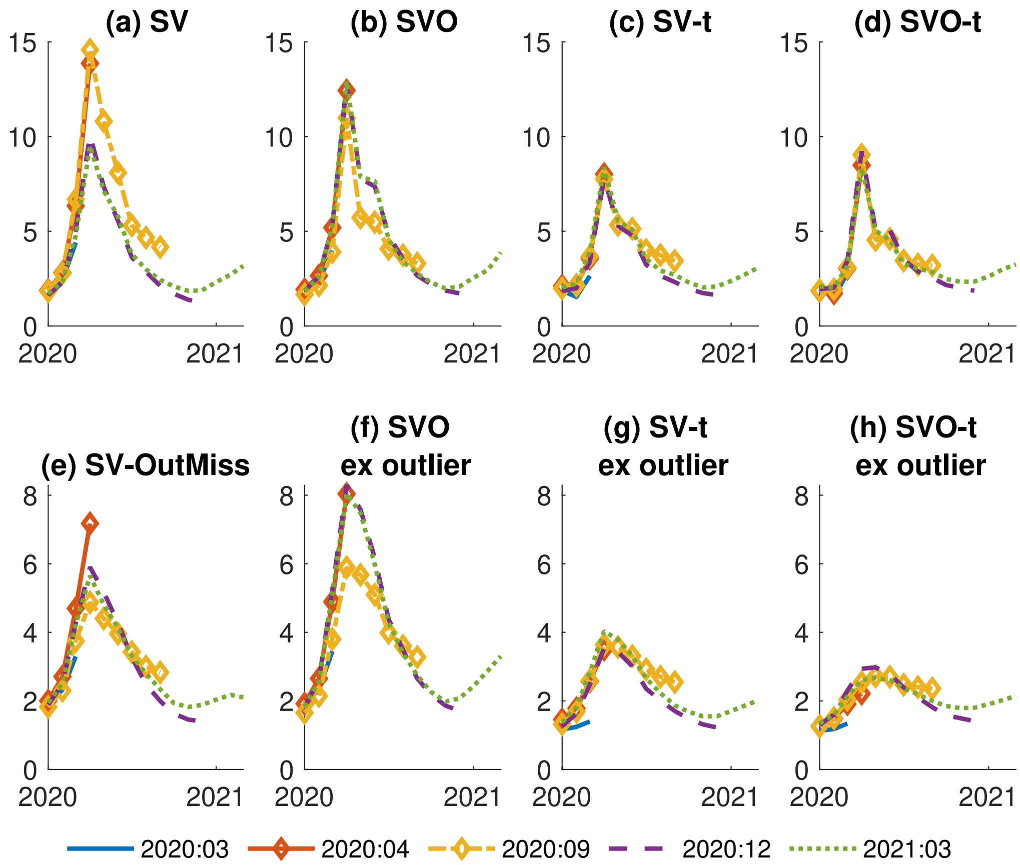
Note: Quasi-real-time trajectories of time-varying volatility in VAR residuals, measured by the diagonal elements of $\mathrm{Var}_t(v_t) = \Sigma_t$ implied by different models. Medians of (smoothed) posterior obtained from different data samples ending at forecast origins as indicated in the figure legend. Panels (b) and (d) display estimates of stochastic volatility for SVO-t that ignore the contributions from outliers and that are computed from $\tilde{\Sigma}_t = A^{-1}\,\Lambda_t\,A^{-T}$ (i.e., neglecting the O_t and Q_t components in the computation of the uncertainty measures shown here, while including these outliers in estimation of A^{-1}, Λ_t, etc.). Reflecting the sizable differences in the size of estimates resulting with and without outlier treatment, different scales are used in upper- and lower-row panels.

Figure S.29: Time-varying volatilities since 2020 of Hours



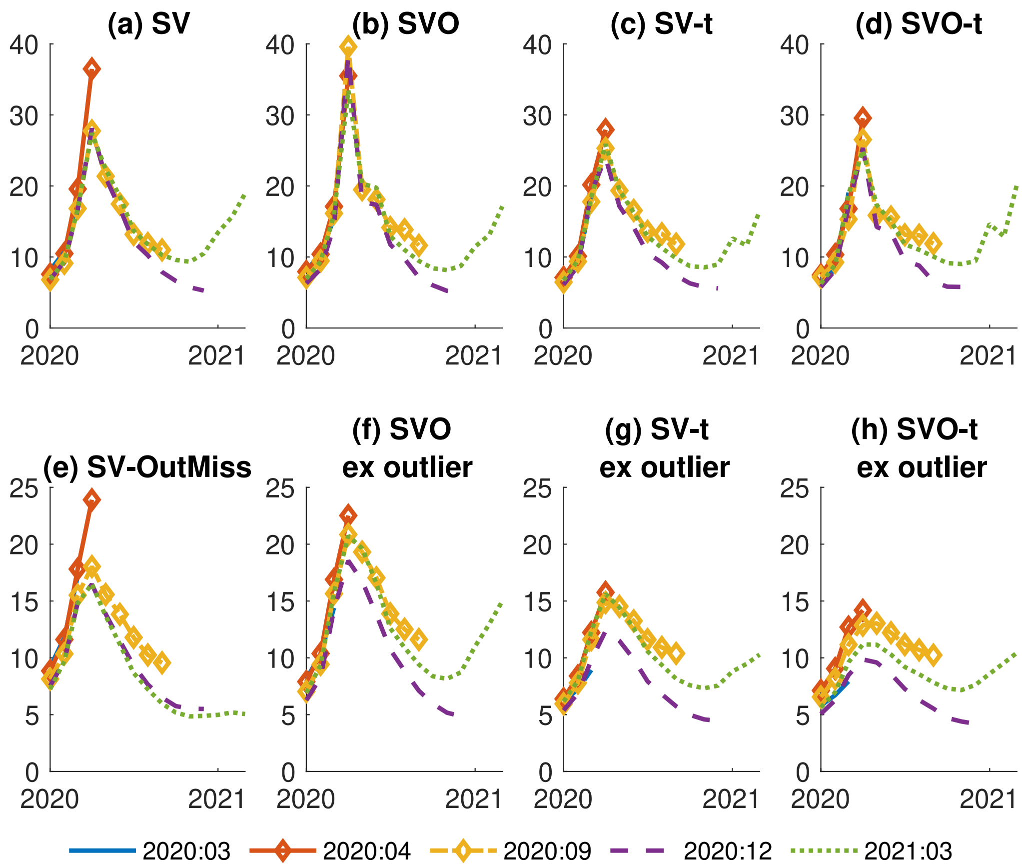
Note: Quasi-real-time trajectories of time-varying volatility in VAR residuals, measured by the diagonal elements of $\mathrm{Var}_t(v_t) = \Sigma_t$ implied by different models. Medians of (smoothed) posterior obtained from different data samples ending at forecast origins as indicated in the figure legend. Panels (b) and (d) display estimates of stochastic volatility for SVO-t that ignore the contributions from outliers and that are computed from $\tilde{\Sigma}_t = A^{-1}\,\Lambda_t\,A^{-T}$ (i.e., neglecting the O_t and Q_t components in the computation of the uncertainty measures shown here, while including these outliers in estimation of A^{-1}, Λ_t, etc.). Reflecting the sizable differences in the size of estimates resulting with and without outlier treatment, different scales are used in upper- and lower-row panels.

Figure S.30: Time-varying volatilities since 2020 of Hourly Earnings



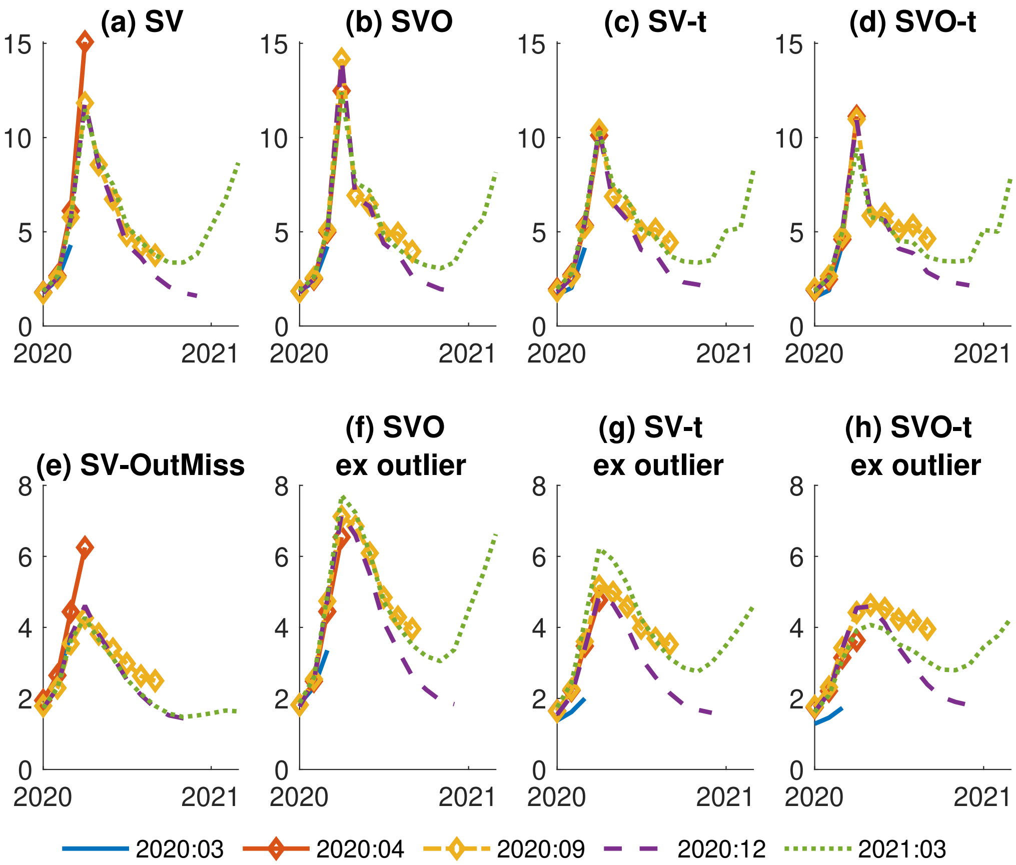
Note: Quasi-real-time trajectories of time-varying volatility in VAR residuals, measured by the diagonal elements of $\mathrm{Var}_t\left(v_t\right) = \Sigma_t$ implied by different models. Medians of (smoothed) posterior obtained from different data samples ending at forecast origins as indicated in the figure legend. Panels (b) and (d) display estimates of stochastic volatility for SVO-t that ignore the contributions from outliers and that are computed from $\tilde{\Sigma}_t = A^{-1}\,\Lambda_t\,A^{-T}$ (i.e., neglecting the O_t and Q_t components in the computation of the uncertainty measures shown here, while including these outliers in estimation of A^{-1}, Λ_t, etc.). Reflecting the sizable differences in the size of estimates resulting with and without outlier treatment, different scales are used in upper- and lower-row panels.

Figure S.31: Time-varying volatilities since 2020 of PPI (fin. goods)



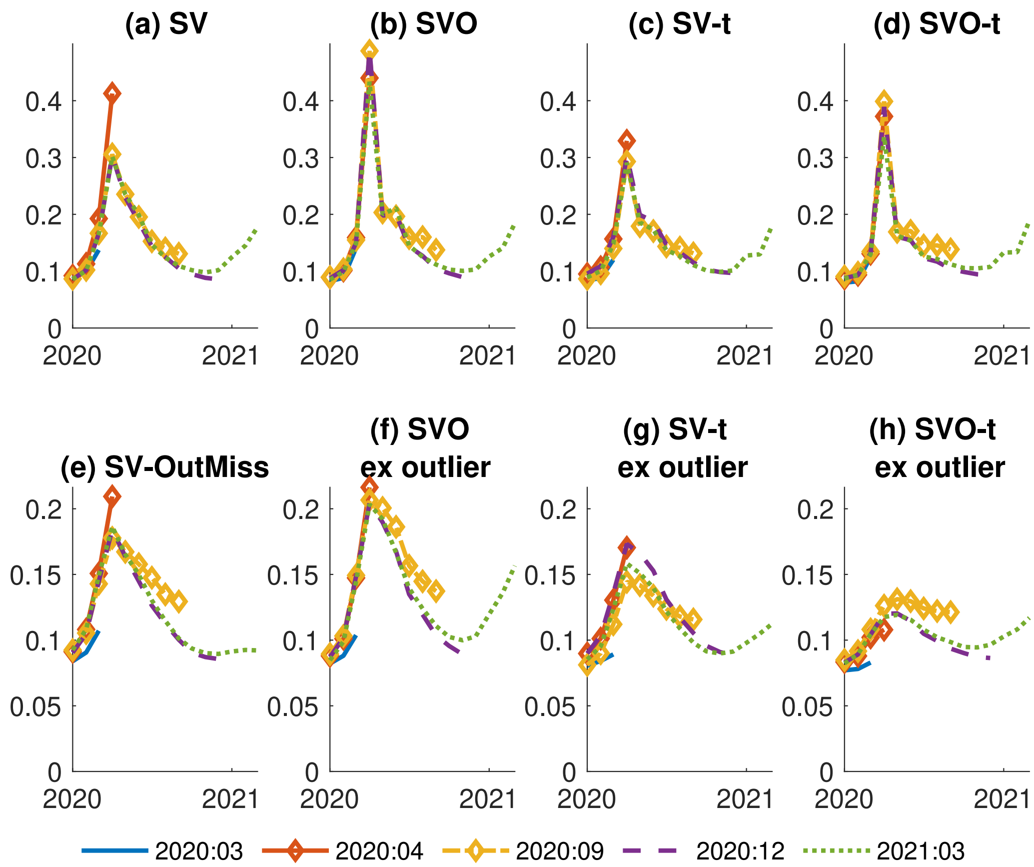
Note: Quasi-real-time trajectories of time-varying volatility in VAR residuals, measured by the diagonal elements of $\mathrm{Var}_t(v_t) = \Sigma_t$ implied by different models. Medians of (smoothed) posterior obtained from different data samples ending at forecast origins as indicated in the figure legend. Panels (b) and (d) display estimates of stochastic volatility for SVO-t that ignore the contributions from outliers and that are computed from $\tilde{\Sigma}_t = A^{-1}\,\Lambda_t\,A^{-T}$ (i.e., neglecting the O_t and Q_t components in the computation of the uncertainty measures shown here, while including these outliers in estimation of A^{-1}, Λ_t, etc.). Reflecting the sizable differences in the size of estimates resulting with and without outlier treatment, different scales are used in upper- and lower-row panels.

Figure S.32: Time-varying volatilities since 2020 of PCE prices



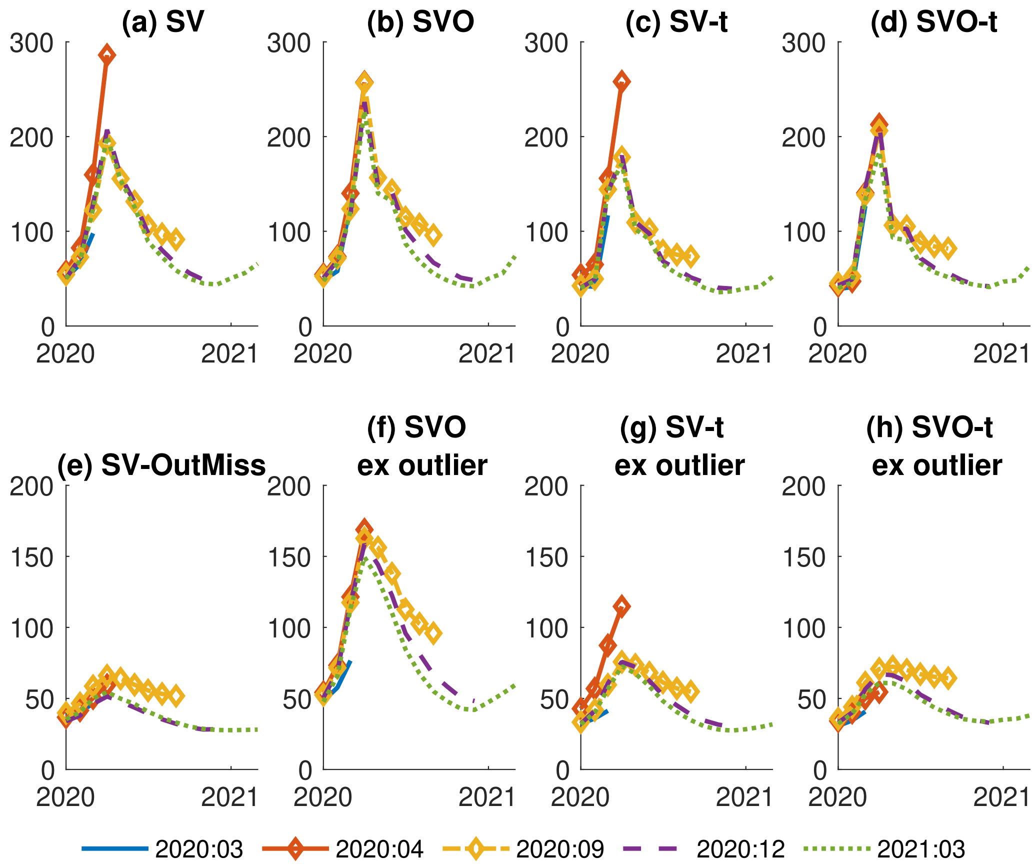
Note: Quasi-real-time trajectories of time-varying volatility in VAR residuals, measured by the diagonal elements of $\mathrm{Var}_t\left(v_t\right) = \Sigma_t$ implied by different models. Medians of (smoothed) posterior obtained from different data samples ending at forecast origins as indicated in the figure legend. Panels (b) and (d) display estimates of stochastic volatility for SVO-t that ignore the contributions from outliers and that are computed from $\tilde{\Sigma}_t = A^{-1}\,\Lambda_t\,A^{-T}$ (i.e., neglecting the O_t and Q_t components in the computation of the uncertainty measures shown here, while including these outliers in estimation of A^{-1}, Λ_t, etc.). Reflecting the sizable differences in the size of estimates resulting with and without outlier treatment, different scales are used in upper- and lower-row panels.

Figure S.33: Time-varying volatilities since 2020 of Housing Starts



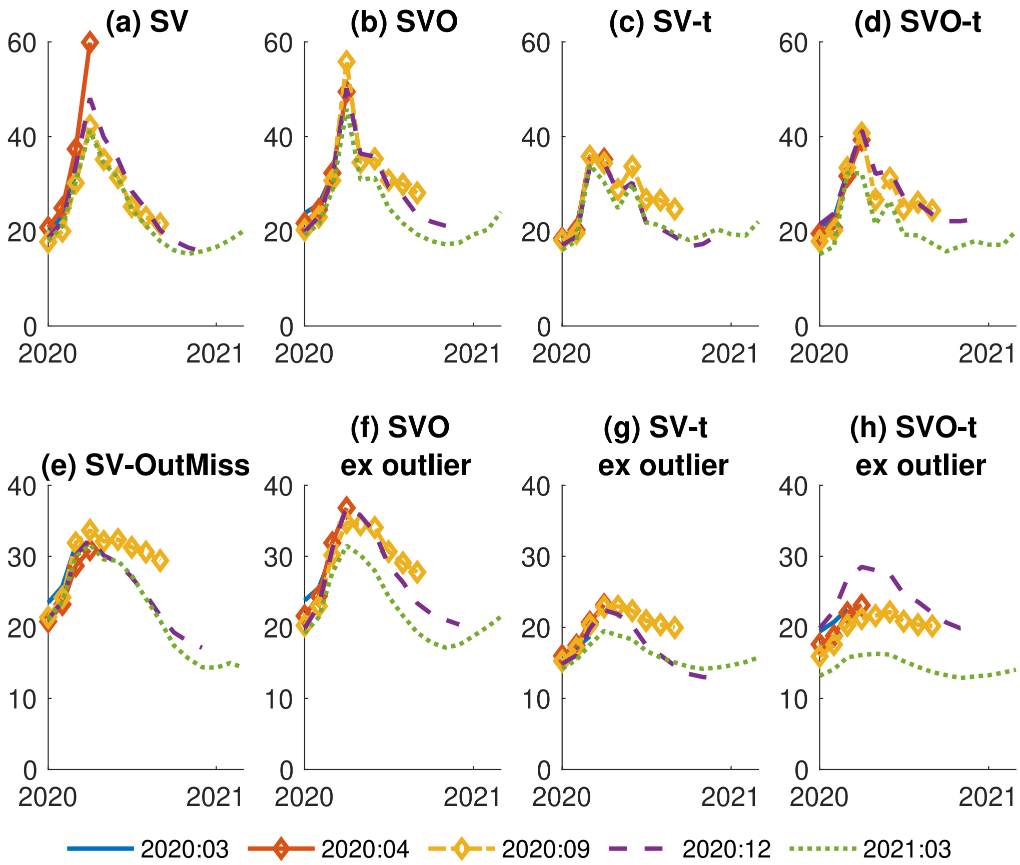
Note: Quasi-real-time trajectories of time-varying volatility in VAR residuals, measured by the diagonal elements of $\mathrm{Var}_t(v_t) = \Sigma_t$ implied by different models. Medians of (smoothed) posterior obtained from different data samples ending at forecast origins as indicated in the figure legend. Panels (b) and (d) display estimates of stochastic volatility for SVO-t that ignore the contributions from outliers and that are computed from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (i.e., neglecting the O_t and Q_t components in the computation of the uncertainty measures shown here, while including these outliers in estimation of A^{-1}, Λ_t, etc.). Reflecting the sizable differences in the size of estimates resulting with and without outlier treatment, different scales are used in upper- and lower-row panels.

Figure S.34: Time-varying volatilities since 2020 of S&P 500



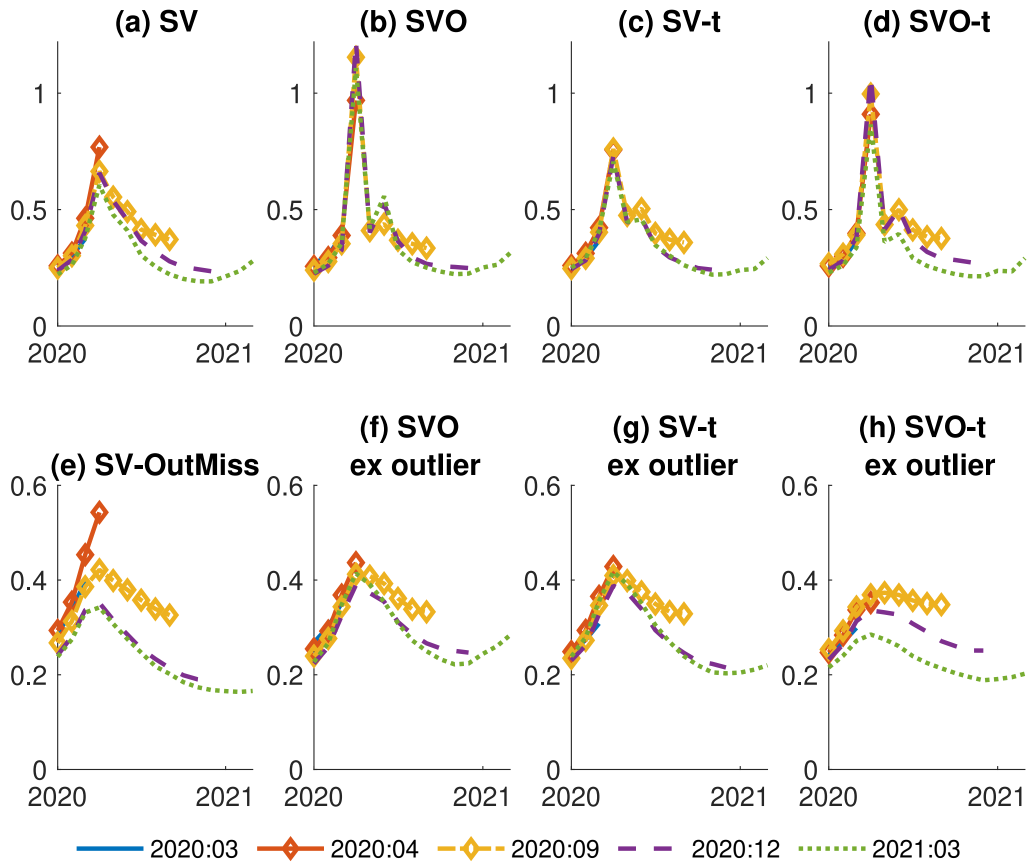
Note: Quasi-real-time trajectories of time-varying volatility in VAR residuals, measured by the diagonal elements of $\mathrm{Var}_t(v_t) = \Sigma_t$ implied by different models. Medians of (smoothed) posterior obtained from different data samples ending at forecast origins as indicated in the figure legend. Panels (b) and (d) display estimates of stochastic volatility for SVO-t that ignore the contributions from outliers and that are computed from $\tilde{\Sigma}_t = A^{-1}\Lambda_t A^{-T}$ (i.e., neglecting the O_t and Q_t components in the computation of the uncertainty measures shown here, while including these outliers in estimation of A^{-1}, Λ_t, etc.). Reflecting the sizable differences in the size of estimates resulting with and without outlier treatment, different scales are used in upper- and lower-row panels.

Figure S.35: Time-varying volatilities since 2020 of USD / GBP FX rate



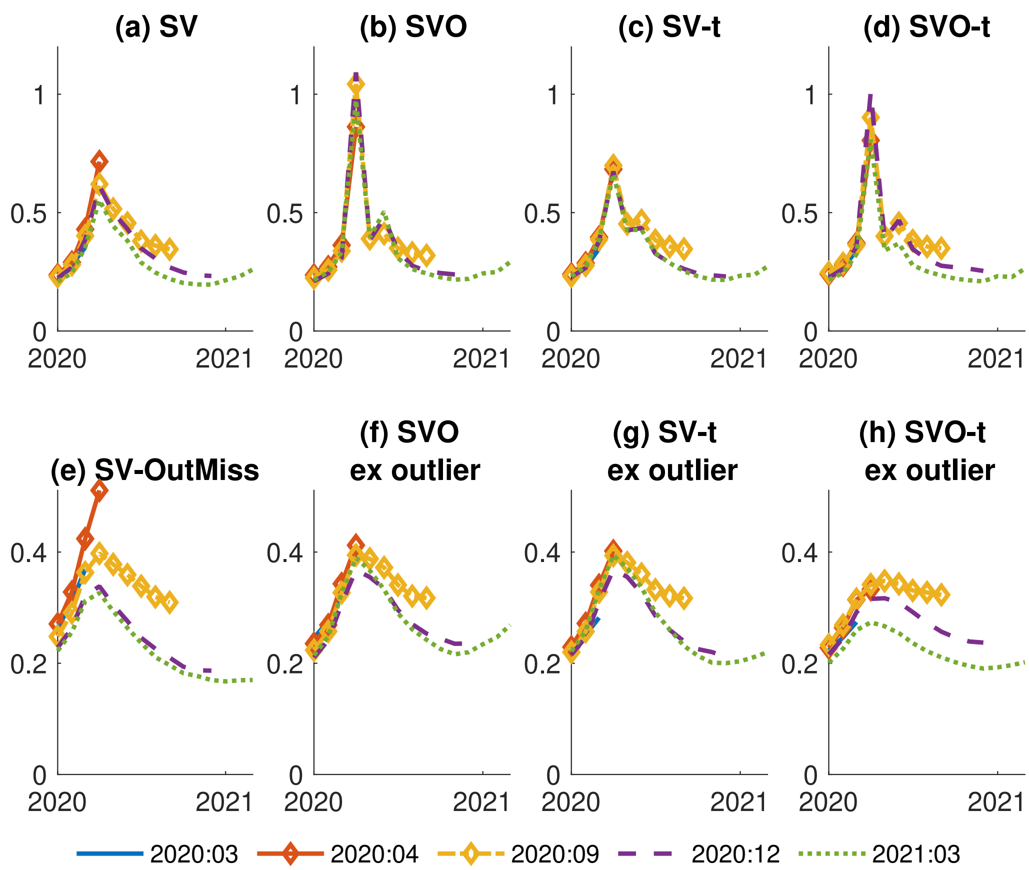
Note: Quasi-real-time trajectories of time-varying volatility in VAR residuals, measured by the diagonal elements of $\mathrm{Var}_t(v_t) = \Sigma_t$ implied by different models. Medians of (smoothed) posterior obtained from different data samples ending at forecast origins as indicated in the figure legend. Panels (b) and (d) display estimates of stochastic volatility for SVO-t that ignore the contributions from outliers and that are computed from $\tilde{\Sigma}_t = A^{-1}\,\Lambda_t\,A^{-T}$ (i.e., neglecting the O_t and Q_t components in the computation of the uncertainty measures shown here, while including these outliers in estimation of A^{-1}, Λ_t, etc.). Reflecting the sizable differences in the size of estimates resulting with and without outlier treatment, different scales are used in upper- and lower-row panels.

Figure S.36: Time-varying volatilities since 2020 of 5-year yield



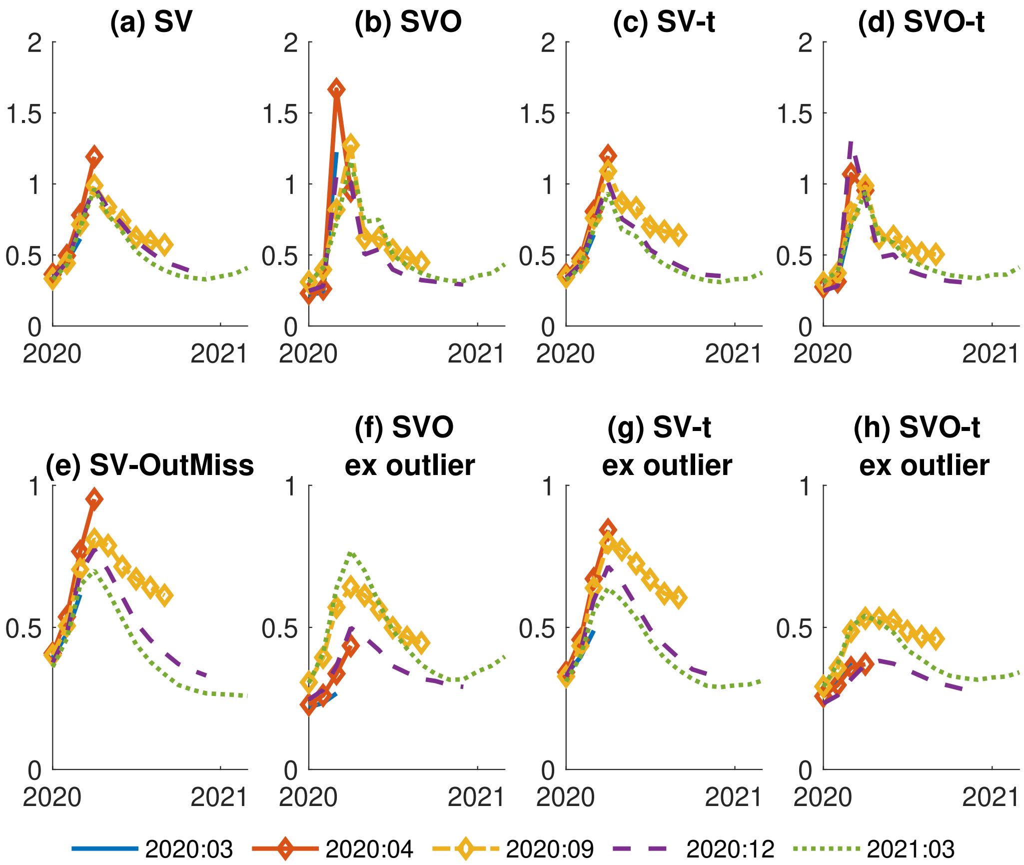
Note: Quasi-real-time trajectories of time-varying volatility in VAR residuals, measured by the diagonal elements of $\mathrm{Var}_t(v_t) = \Sigma_t$ implied by different models. Medians of (smoothed) posterior obtained from different data samples ending at forecast origins as indicated in the figure legend. Panels (b) and (d) display estimates of stochastic volatility for SVO-t that ignore the contributions from outliers and that are computed from $\tilde{\Sigma}_t = A^{-1}\Lambda_t A^{-T}$ (i.e., neglecting the O_t and Q_t components in the computation of the uncertainty measures shown here, while including these outliers in estimation of A^{-1}, Λ_t, etc.). Reflecting the sizable differences in the size of estimates resulting with and without outlier treatment, different scales are used in upper- and lower-row panels.

Figure S.37: Time-varying volatilities since 2020 of 10-year yield



Note: Quasi-real-time trajectories of time-varying volatility in VAR residuals, measured by the diagonal elements of $\mathrm{Var}_t(v_t) = \Sigma_t$ implied by different models. Medians of (smoothed) posterior obtained from different data samples ending at forecast origins as indicated in the figure legend. Panels (b) and (d) display estimates of stochastic volatility for SVO-t that ignore the contributions from outliers and that are computed from $\tilde{\Sigma}_t = A^{-1}\,\Lambda_t\,A^{-T}$ (i.e., neglecting the O_t and Q_t components in the computation of the uncertainty measures shown here, while including these outliers in estimation of A^{-1}, Λ_t, etc.). Reflecting the sizable differences in the size of estimates resulting with and without outlier treatment, different scales are used in upper- and lower-row panels.

Figure S.38: Time-varying volatilities since 2020 of Baa spread

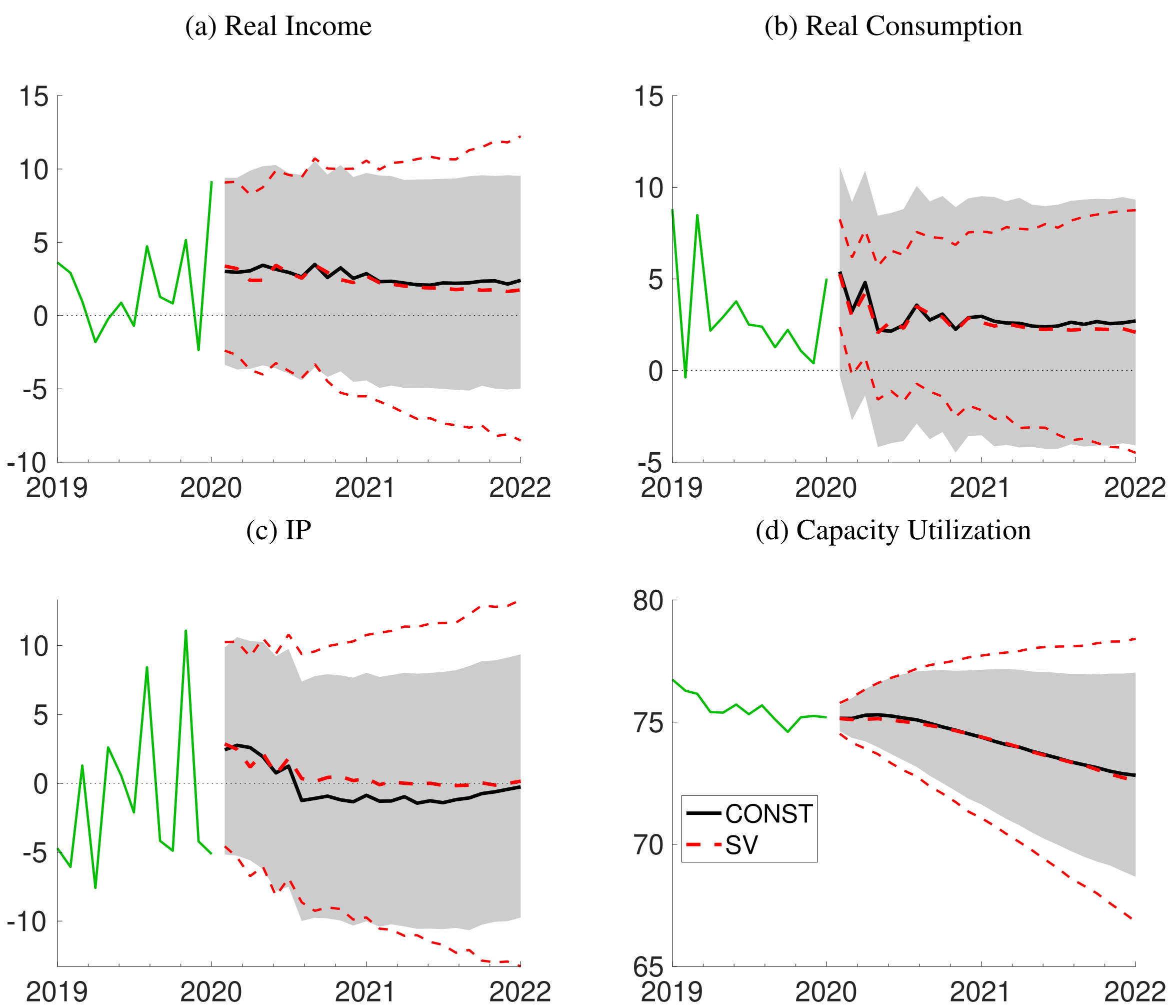


Note: Quasi-real-time trajectories of time-varying volatility in VAR residuals, measured by the diagonal elements of $\mathrm{Var}_t(v_t) = \Sigma_t$ implied by different models. Medians of (smoothed) posterior obtained from different data samples ending at forecast origins as indicated in the figure legend. Panels (b) and (d) display estimates of stochastic volatility for SVO-t that ignore the contributions from outliers and that are computed from $\tilde{\Sigma}_t = A^{-1}\,\Lambda_t\,A^{-T}$ (i.e., neglecting the O_t and Q_t components in the computation of the uncertainty measures shown here, while including these outliers in estimation of A^{-1}, Λ_t, etc.). Reflecting the sizable differences in the size of estimates resulting with and without outlier treatment, different scales are used in upper- and lower-row panels.

VI Predictive densities since 2020 for all variables

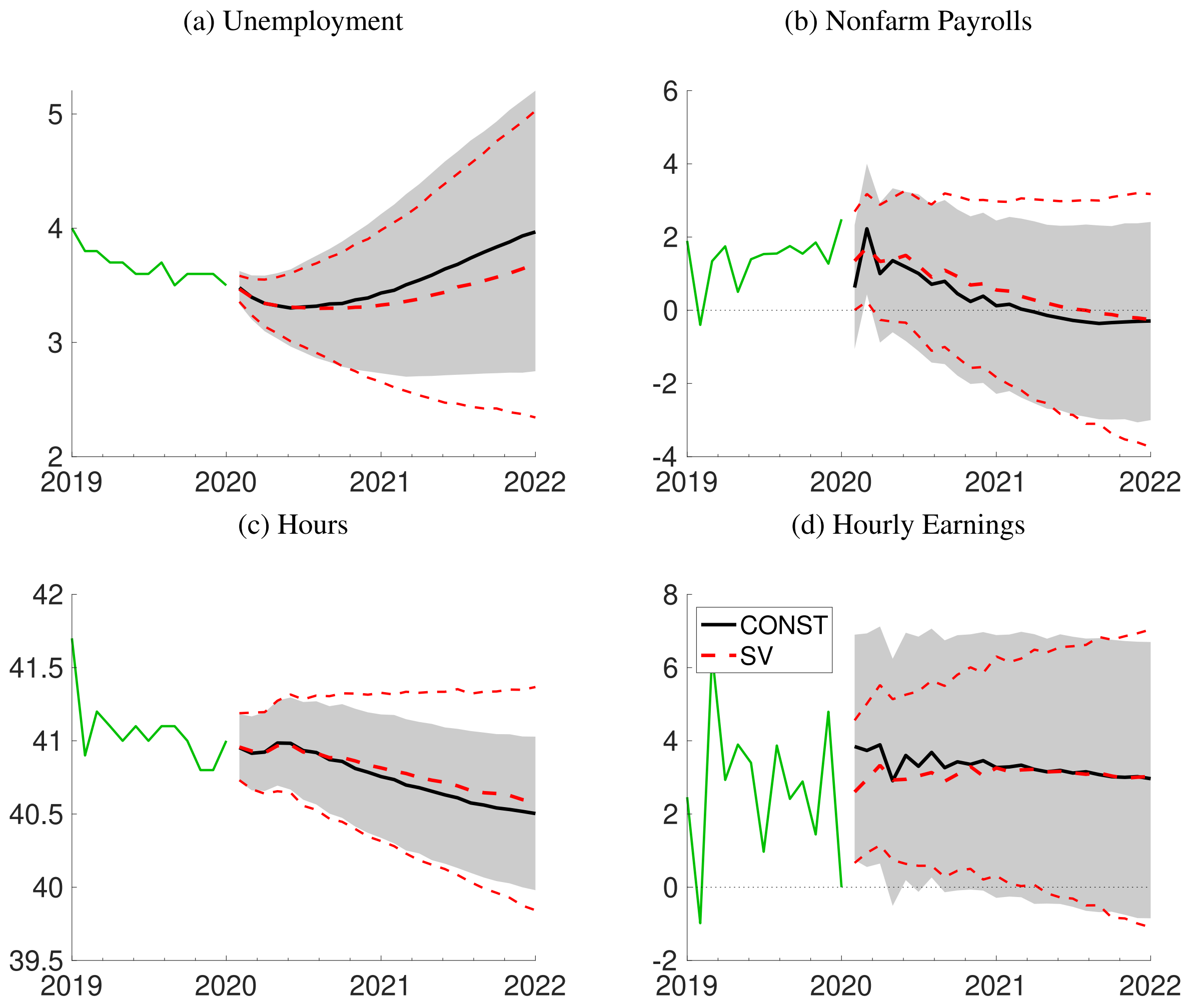
This part of the appendix provides additional figures of predictive densities generated since early 2020 from different models, not shown in the paper. Figures S.39–S.42 depict forecasts made from CONST and SV models in January 2020. Densities from conventional CONST and SV models for April 2020 are shown in Figures S.43–S.46. These figures also report densities from a CONST model with parameters estimated with data only through February 2020. Figures S.47–S.62 complement the densities since March 2020 shown in the paper for selected variables with an overview of all variables. The same applies to Figures S.63–S.66 for densities since later 2020.

Figure S.39: Predictive densities in January 2020



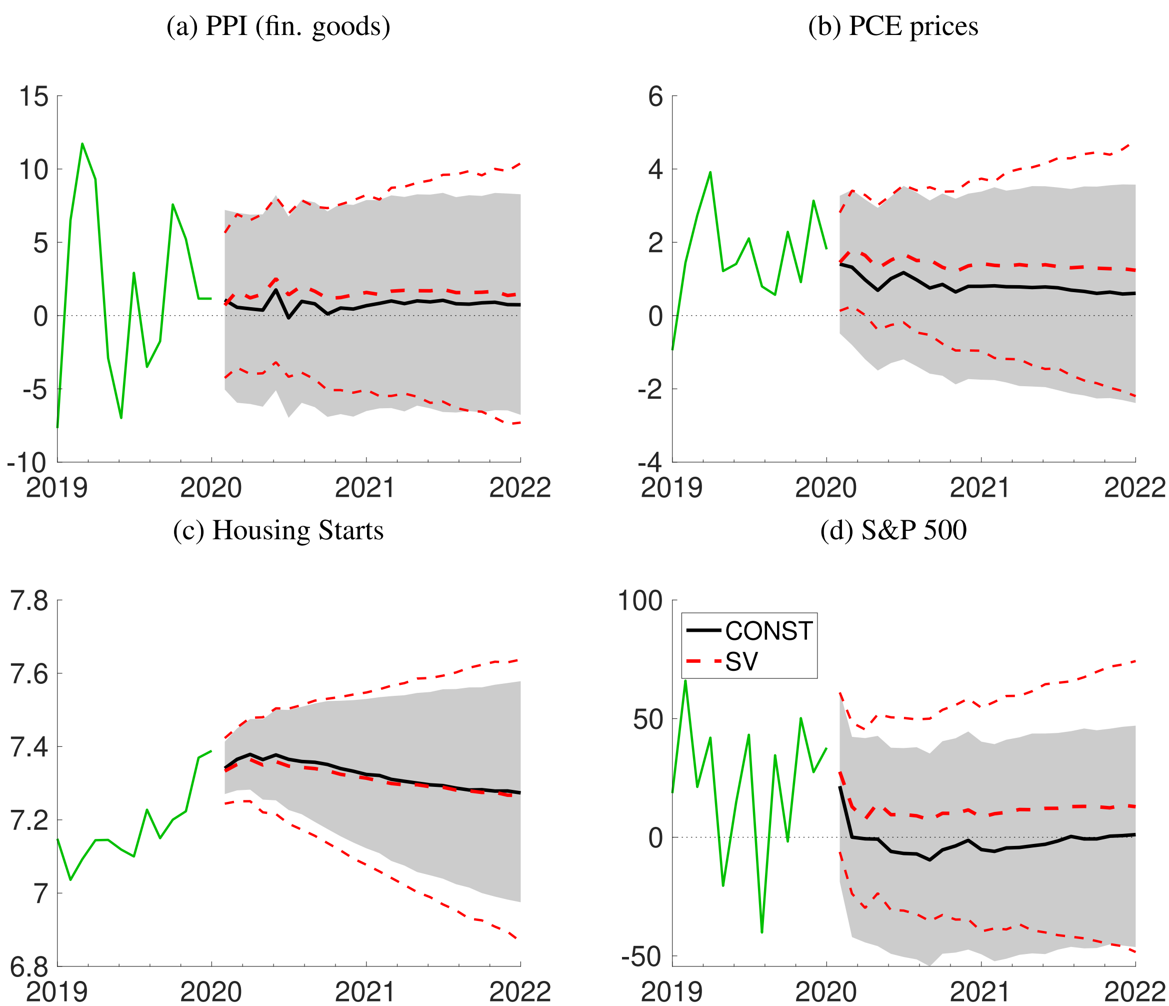
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. The solid green line denotes realized data prior to the forecast origin.

Figure S.40: Predictive densities in January 2020 (ctd.)



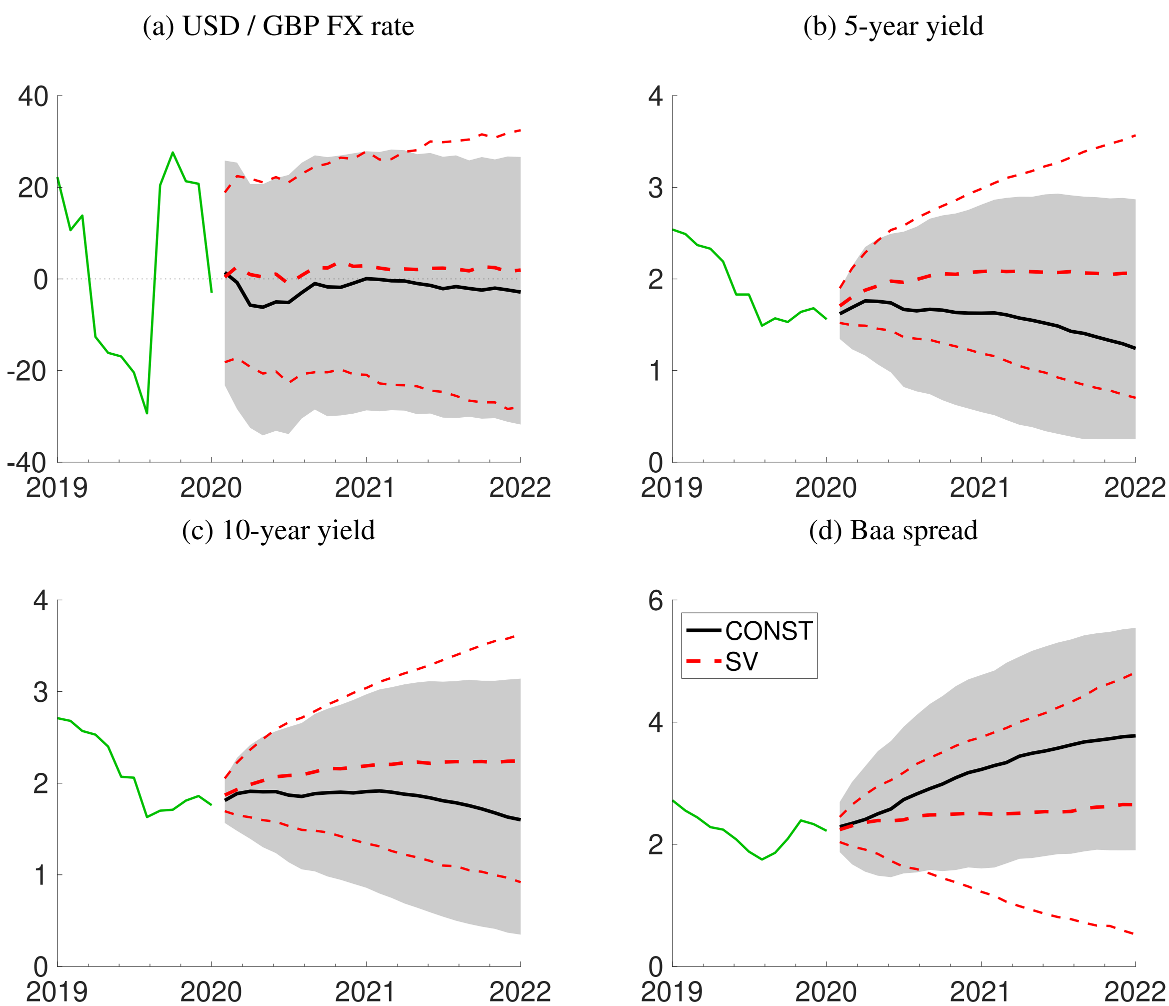
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. The solid green line denotes realized data prior to the forecast origin.

Figure S.41: Predictive densities in January 2020 (ctd.)



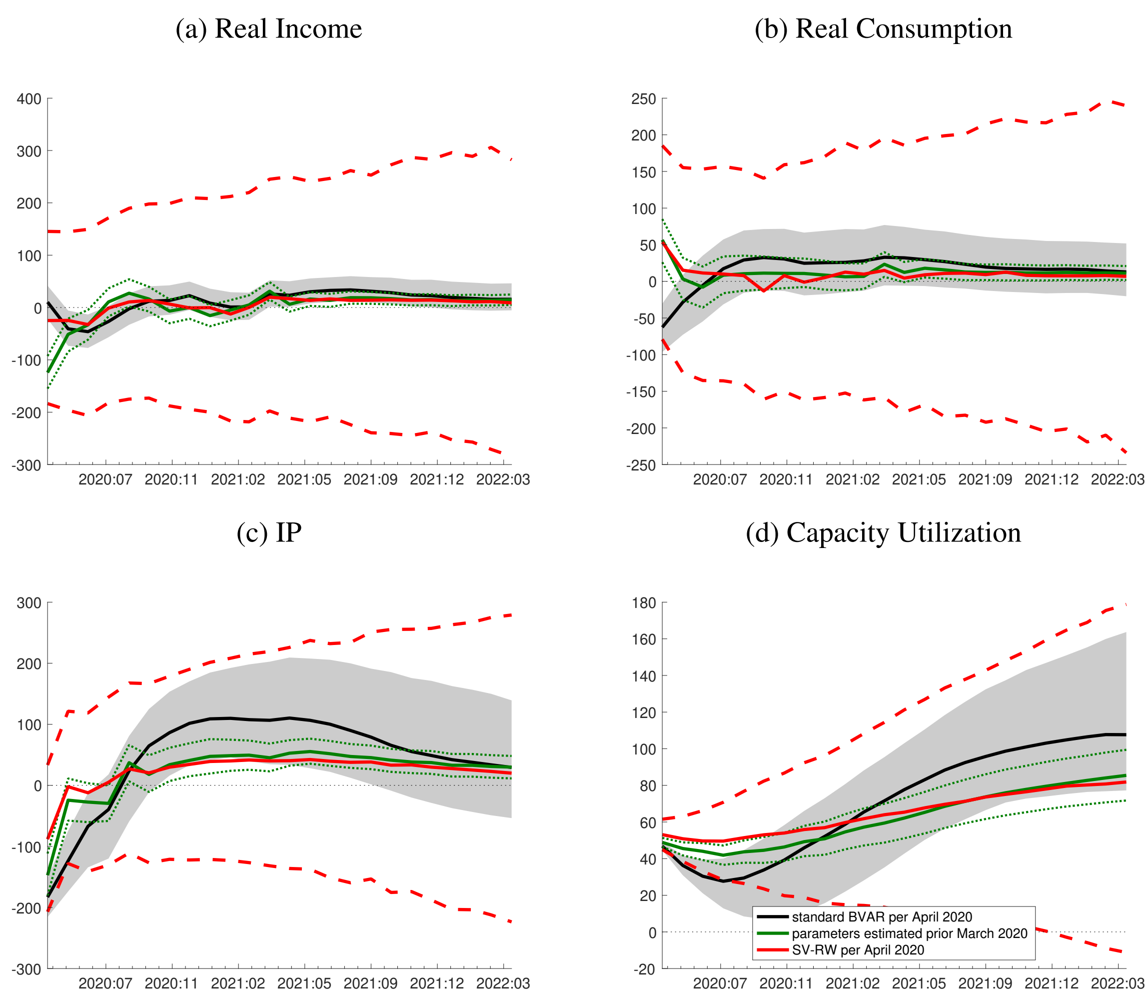
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. The solid green line denotes realized data prior to the forecast origin.

Figure S.42: Predictive densities in January 2020 (ctd.)



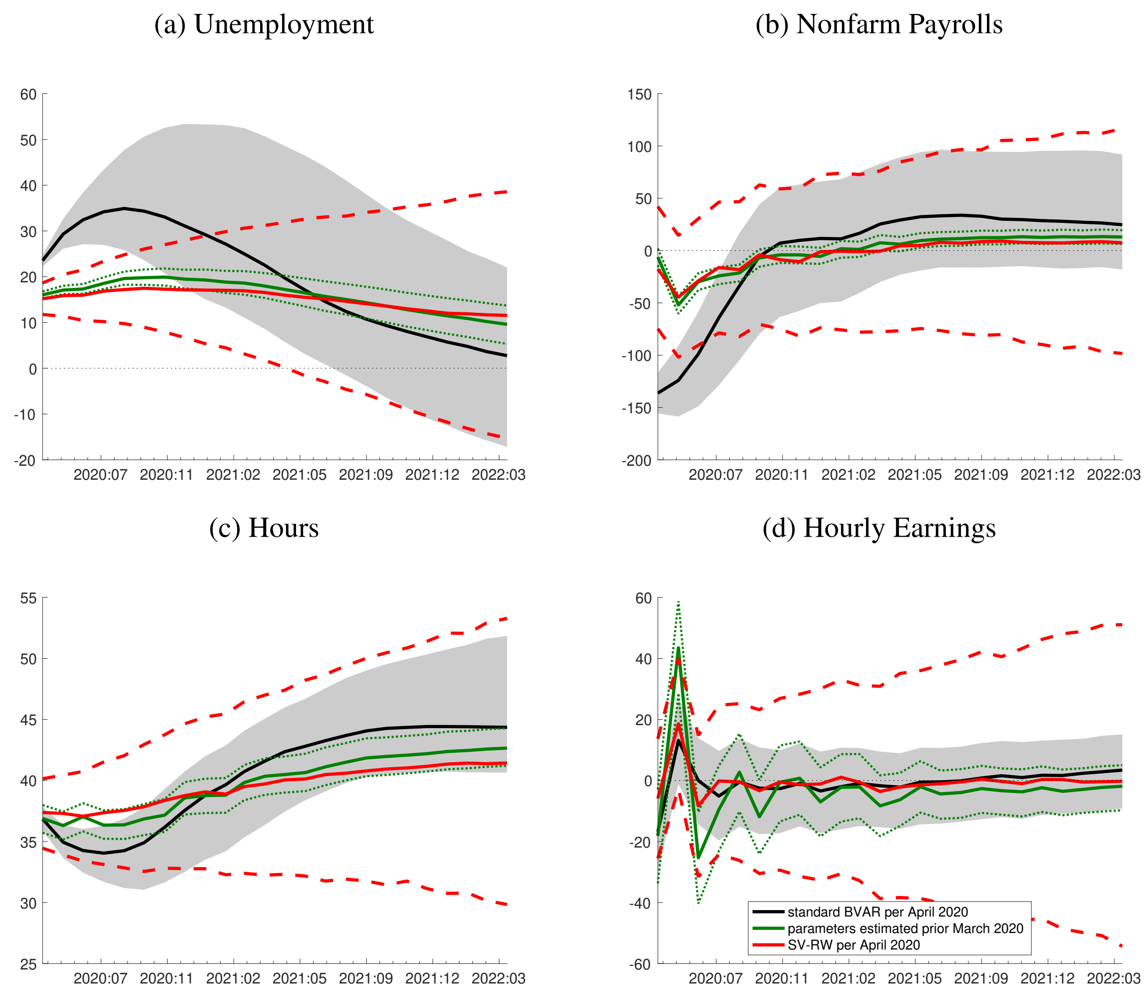
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. The solid green line denotes realized data prior to the forecast origin.

Figure S.43: Predictive densities in April 2020



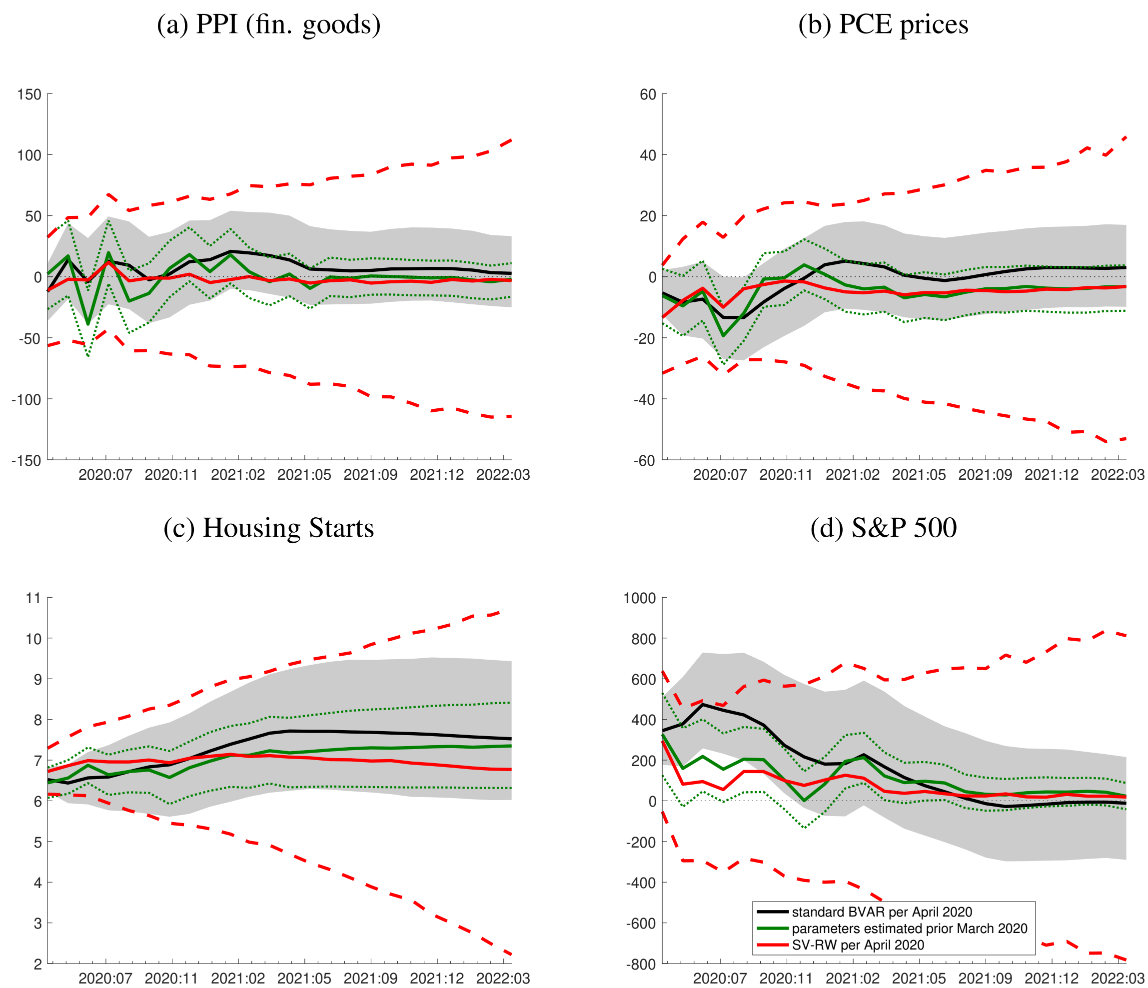
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. Black and red densities generated from CONST and SV models per April 2020, respectively. The green density is simulated from the CONST model per April 2020, but using parameter estimates obtained from data only through February 2020.

Figure S.44: Predictive densities in April 2020 (ctd)



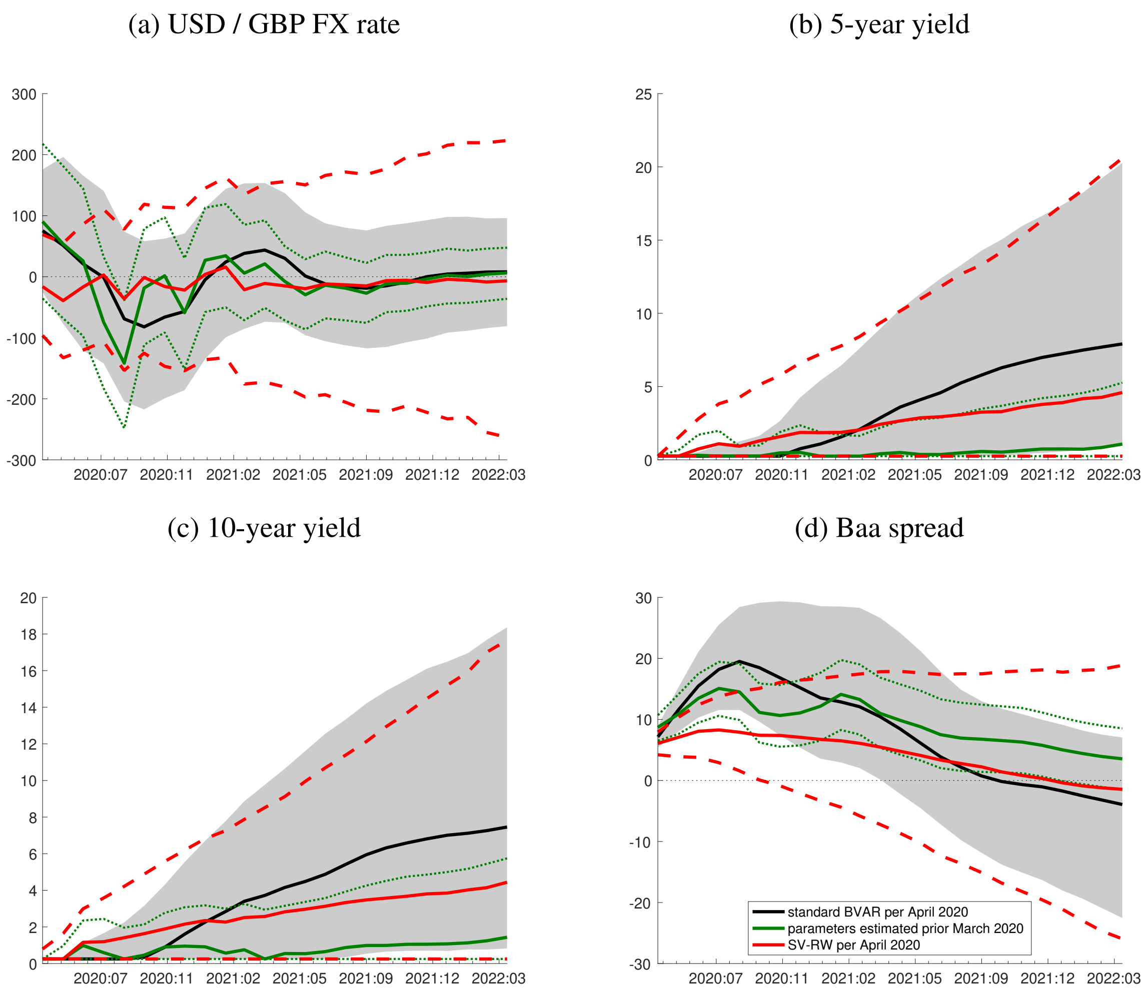
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. Black and red densities generated from CONST and SV models per April 2020, respectively. The green density is simulated from the CONST model per April 2020, but using parameter estimates obtained from data only through February 2020.

Figure S.45: Predictive densities in April 2020 (ctd.)



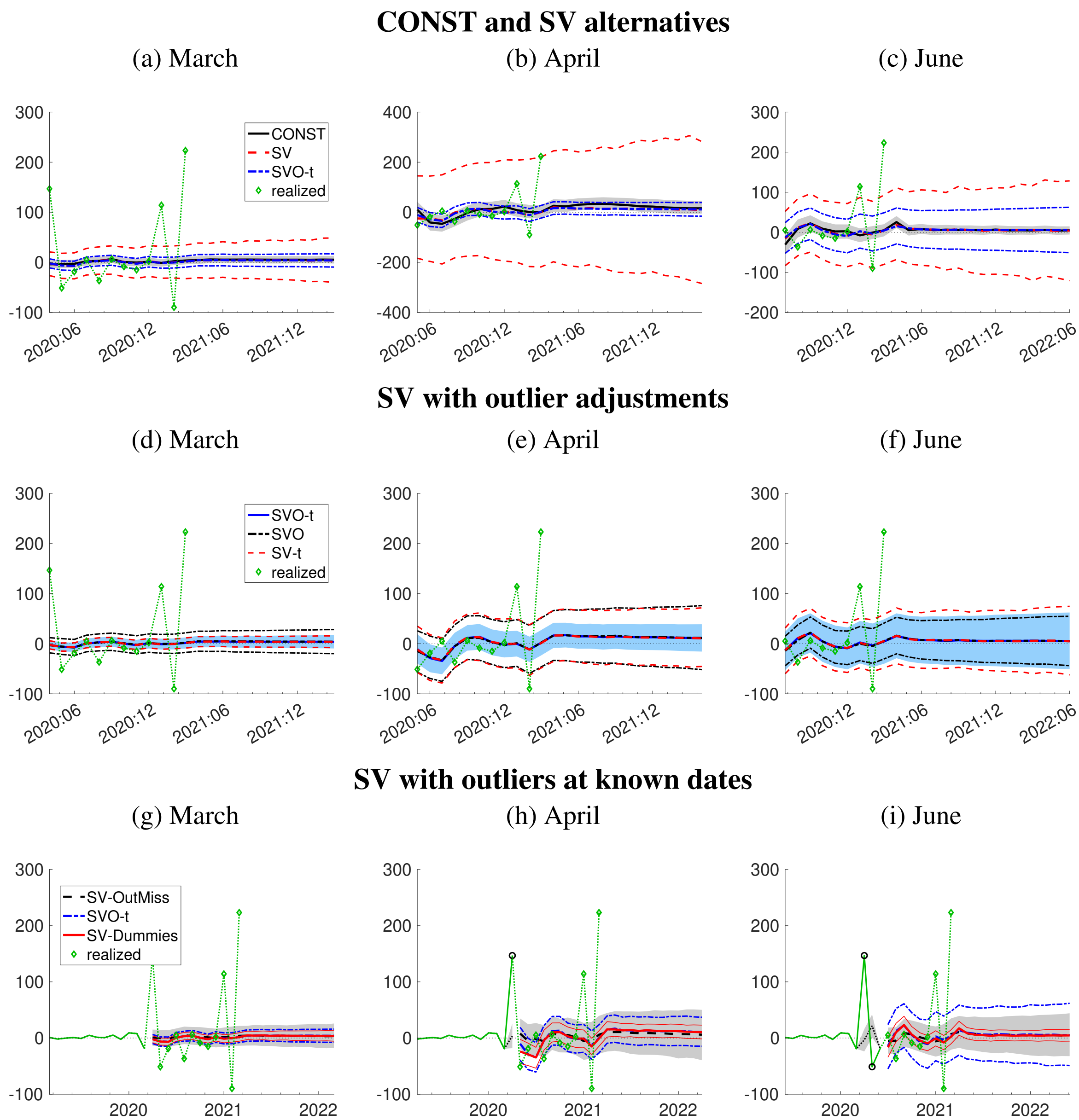
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. Black and red densities generated from CONST and SV models per April 2020, respectively. The green density is simulated from the CONST model per April 2020, but using parameter estimates obtained from data only through February 2020.

Figure S.46: Predictive densities in April 2020 (ctd.)



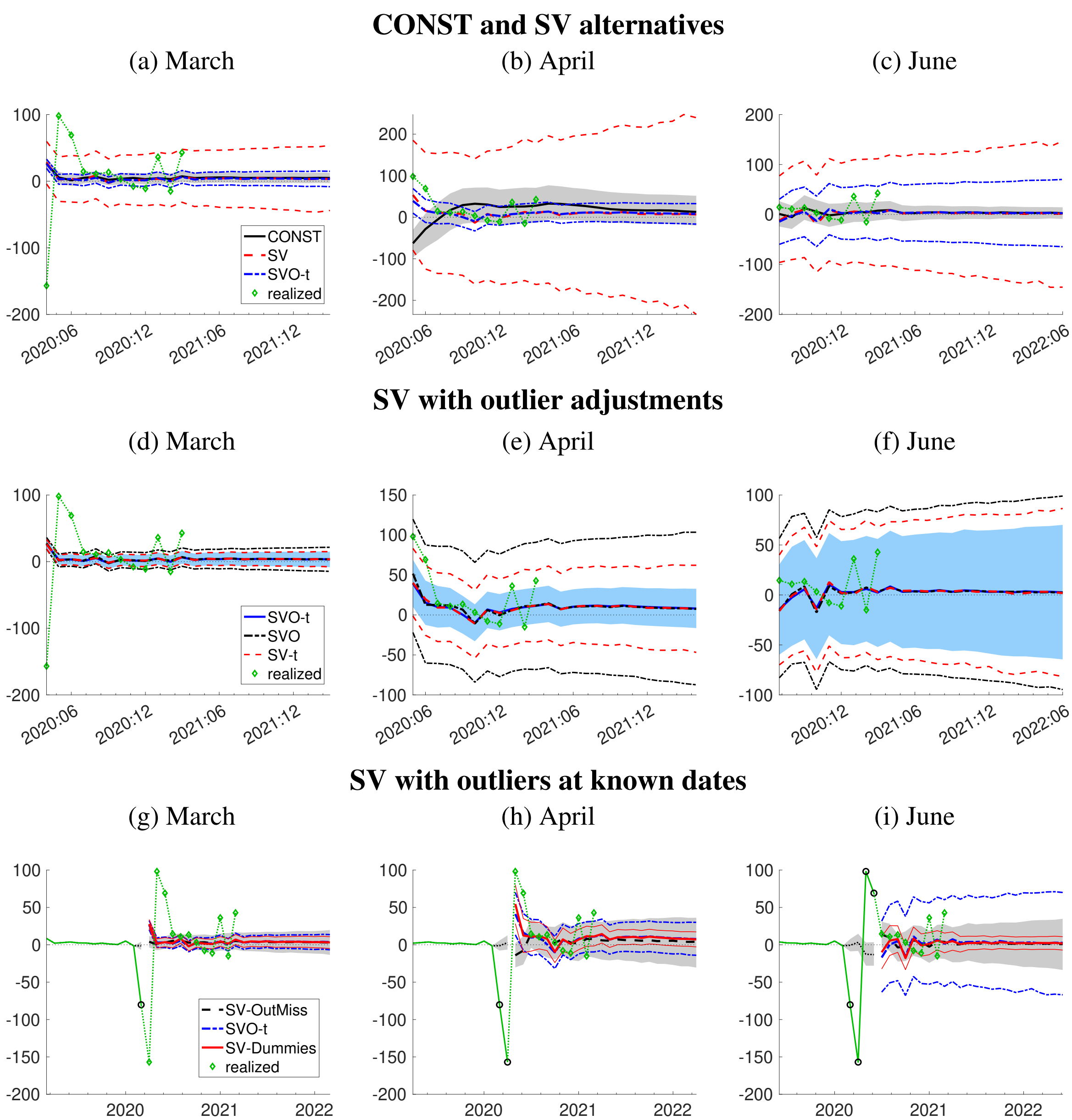
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. Black and red densities generated from CONST and SV models per April 2020, respectively. The green density is simulated from the CONST model per April 2020, but using parameter estimates obtained from data only through February 2020.

Figure S.47: Predictive densities since March 2020 for Real Income



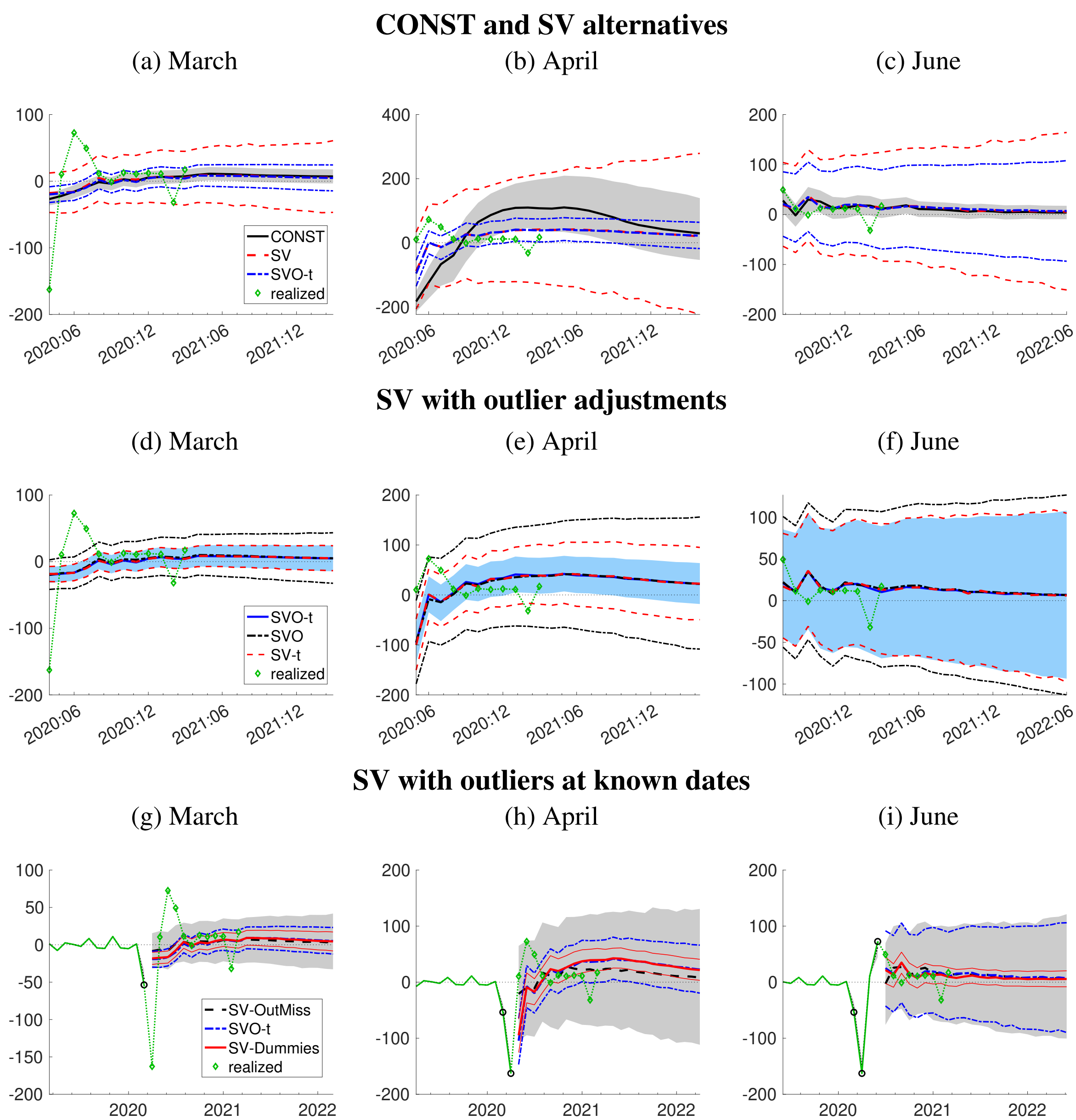
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. The solid green line denotes realized data prior to the forecast origin. In panels (g) – (i), observations identified ex-ante as outliers, based on being more than 5 times the inter-quartile range away from the median, are indicated with a circle, and the corresponding backcast densities from the SV-OutMiss model are superimposed.

Figure S.48: Predictive densities since March 2020 for Real Consumption



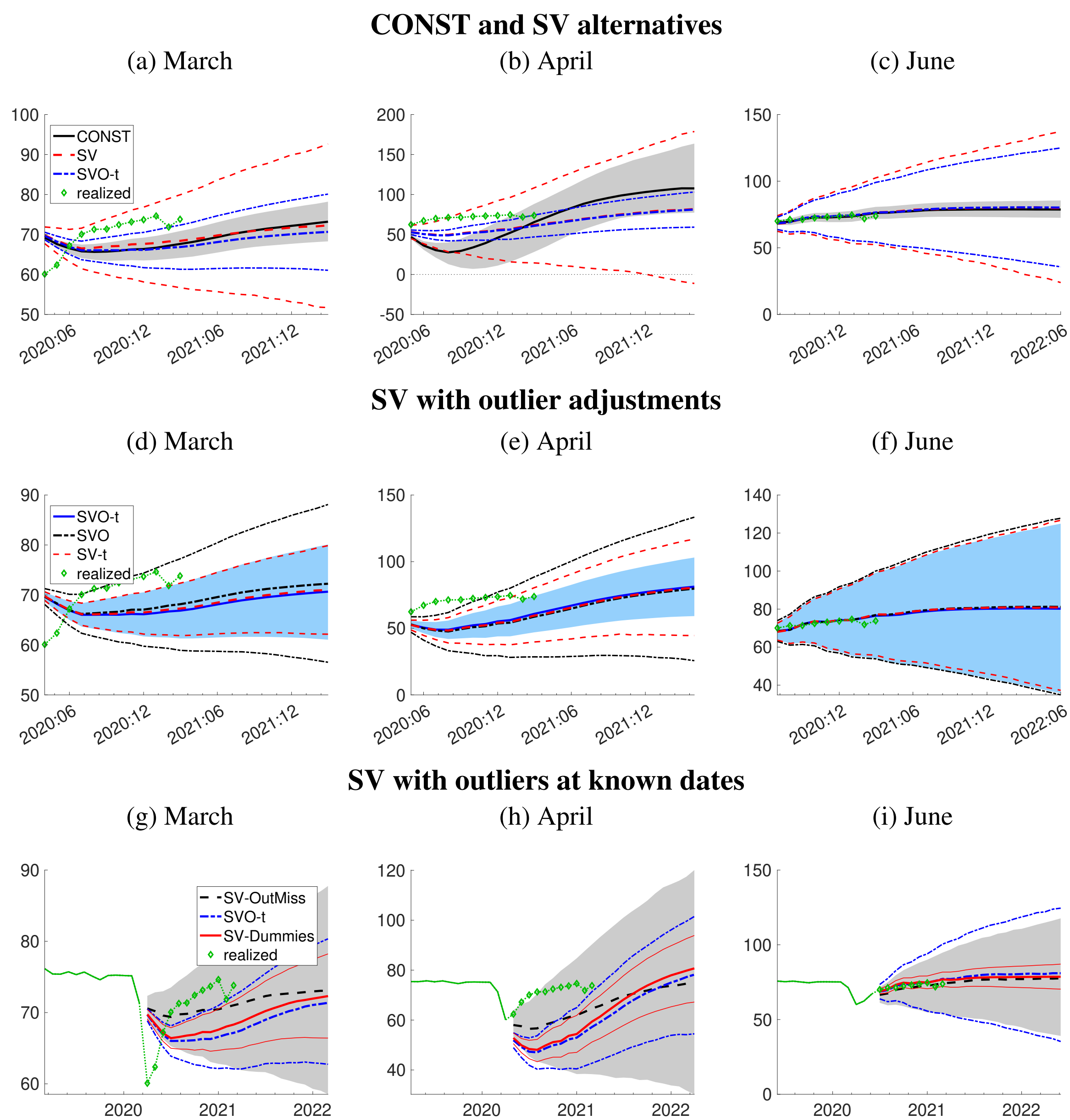
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. The solid green line denotes realized data prior to the forecast origin. In panels (g) – (i), observations identified ex-ante as outliers, based on being more than 5 times the inter-quartile range away from the median, are indicated with a circle, and the corresponding backcast densities from the SV-OutMiss model are superimposed.

Figure S.49: Predictive densities since March 2020 for IP



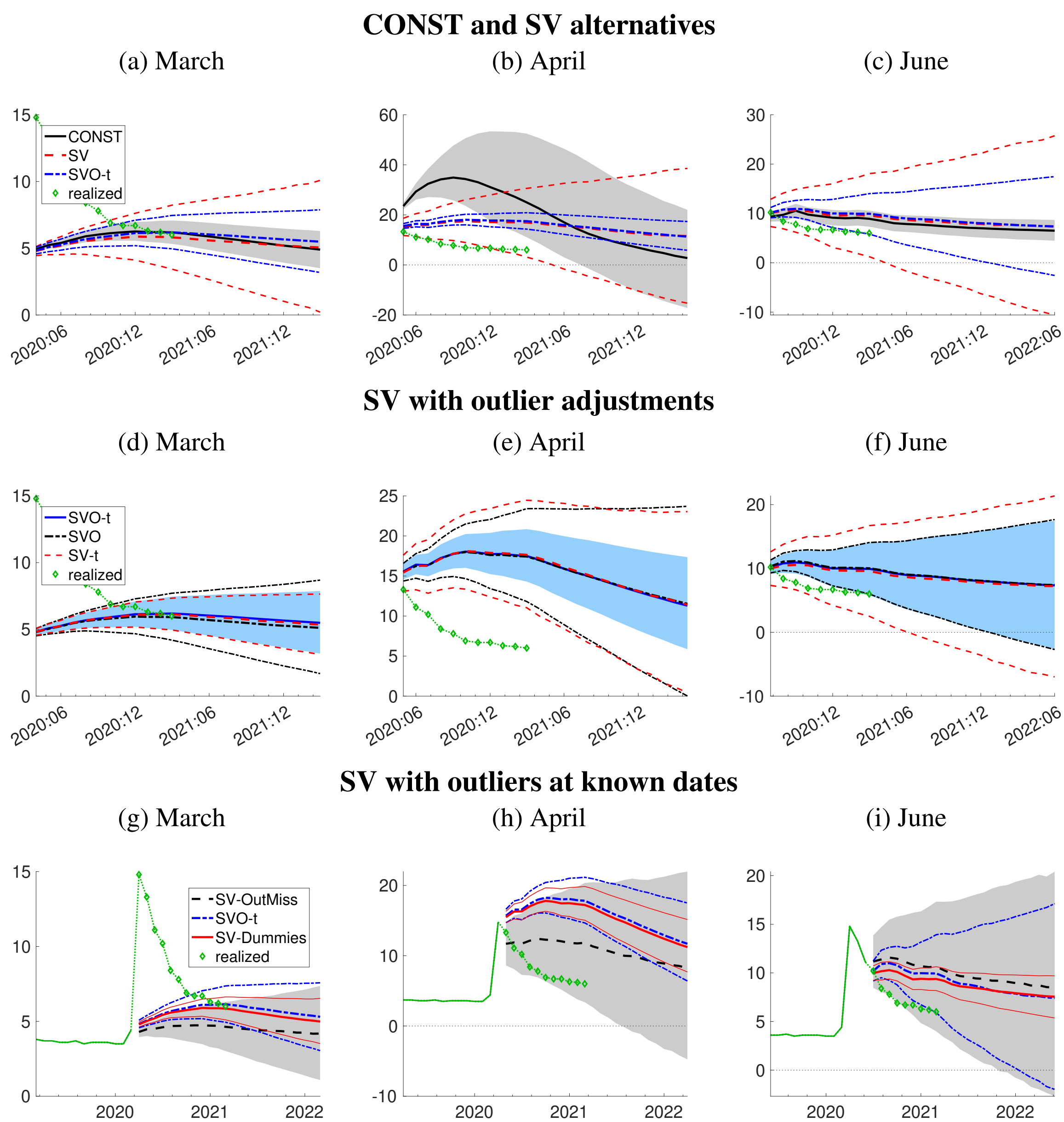
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. The solid green line denotes realized data prior to the forecast origin. In panels (g) – (i), observations identified ex-ante as outliers, based on being more than 5 times the inter-quartile range away from the median, are indicated with a circle, and the corresponding backcast densities from the SV-OutMiss model are superimposed.

Figure S.50: Predictive densities since March 2020 for Capacity Utilization



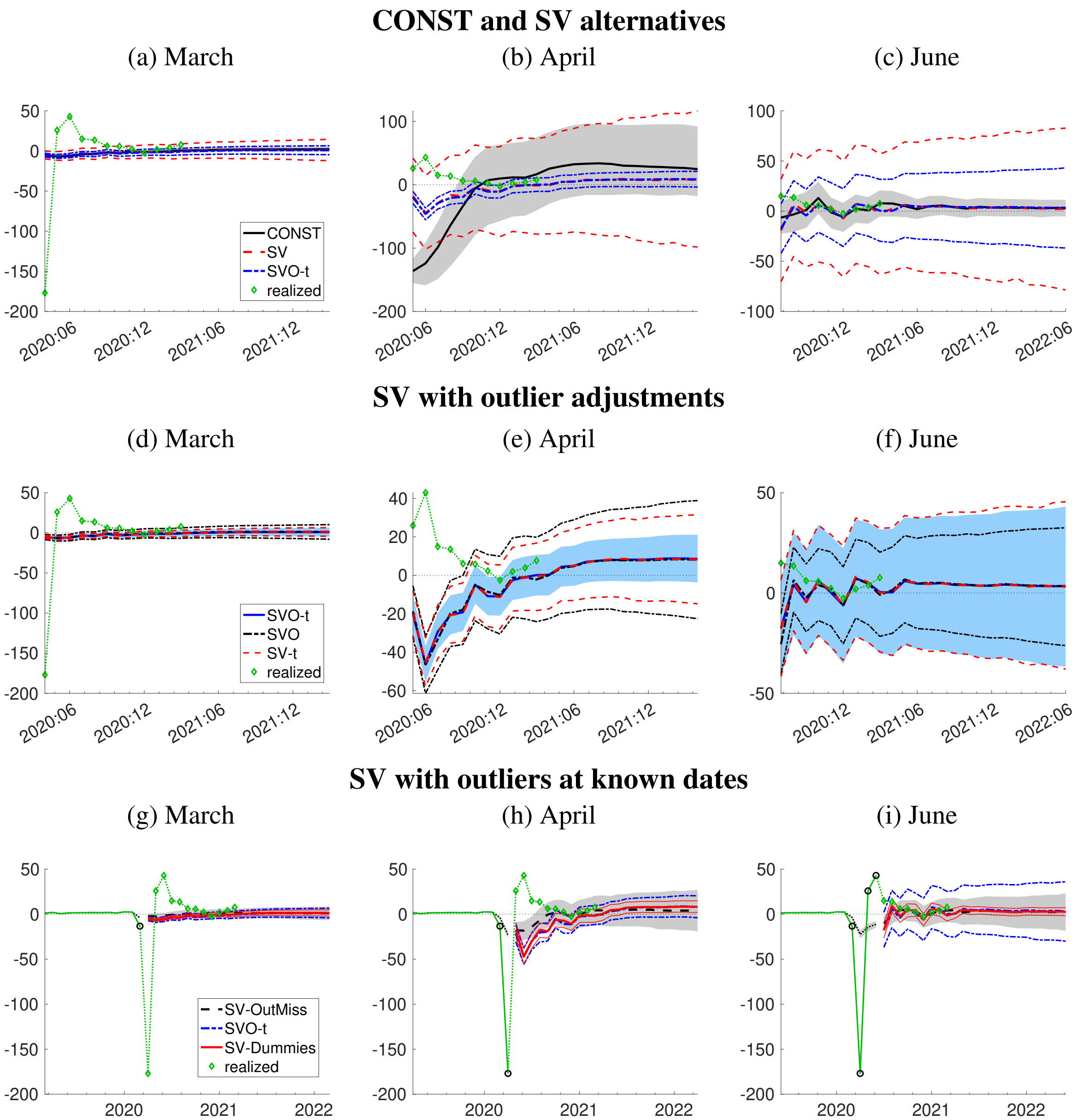
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. The solid green line denotes realized data prior to the forecast origin. In panels (g) – (i), observations identified ex-ante as outliers, based on being more than 5 times the inter-quartile range away from the median, are indicated with a circle, and the corresponding backcast densities from the SV-OutMiss model are superimposed.

Figure S.51: Predictive densities since March 2020 for Unemployment



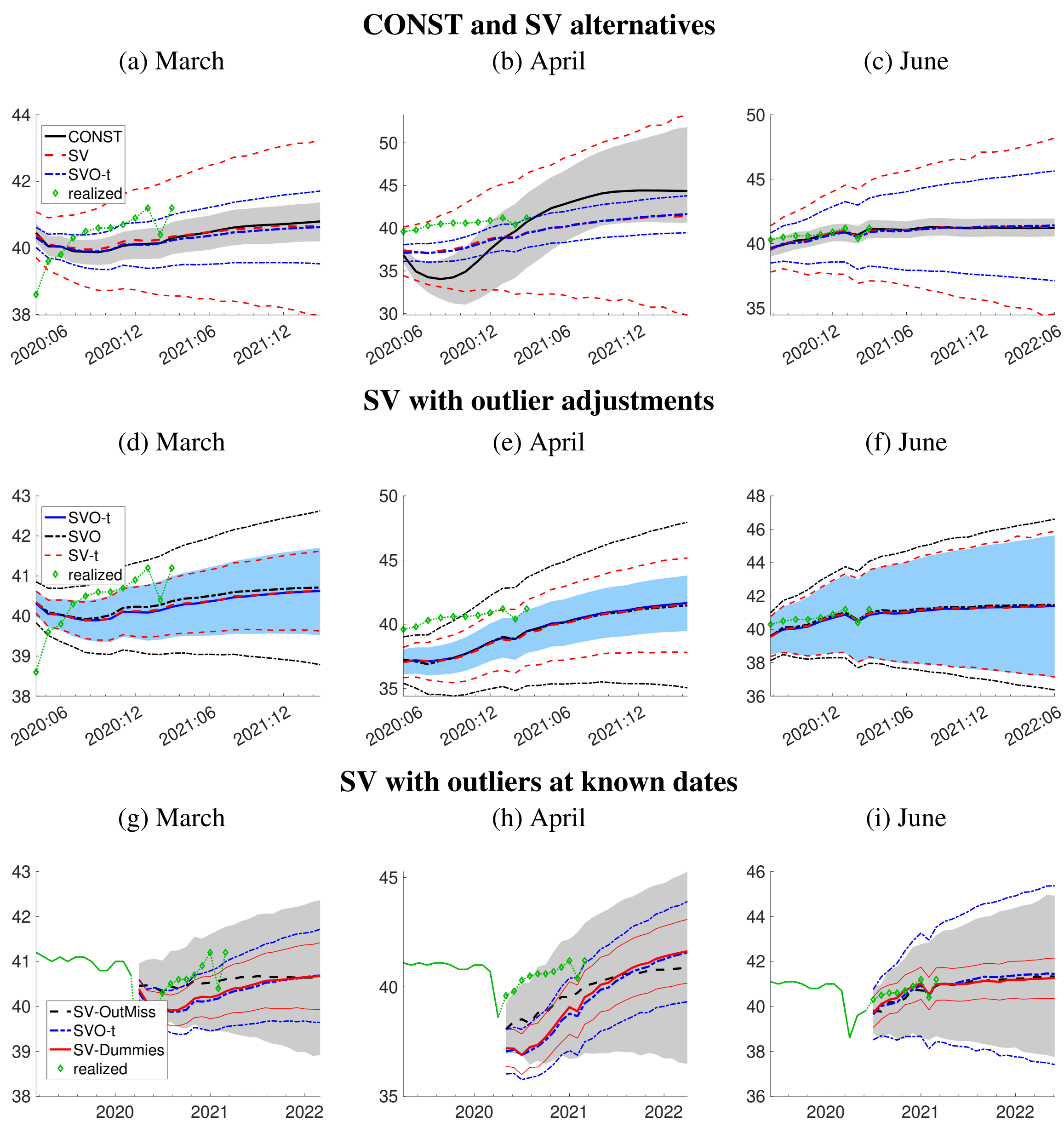
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. The solid green line denotes realized data prior to the forecast origin. In panels (g) – (i), observations identified ex-ante as outliers, based on being more than 5 times the inter-quartile range away from the median, are indicated with a circle, and the corresponding backcast densities from the SV-OutMiss model are superimposed.

Figure S.52: Predictive densities since March 2020 for Nonfarm Payrolls



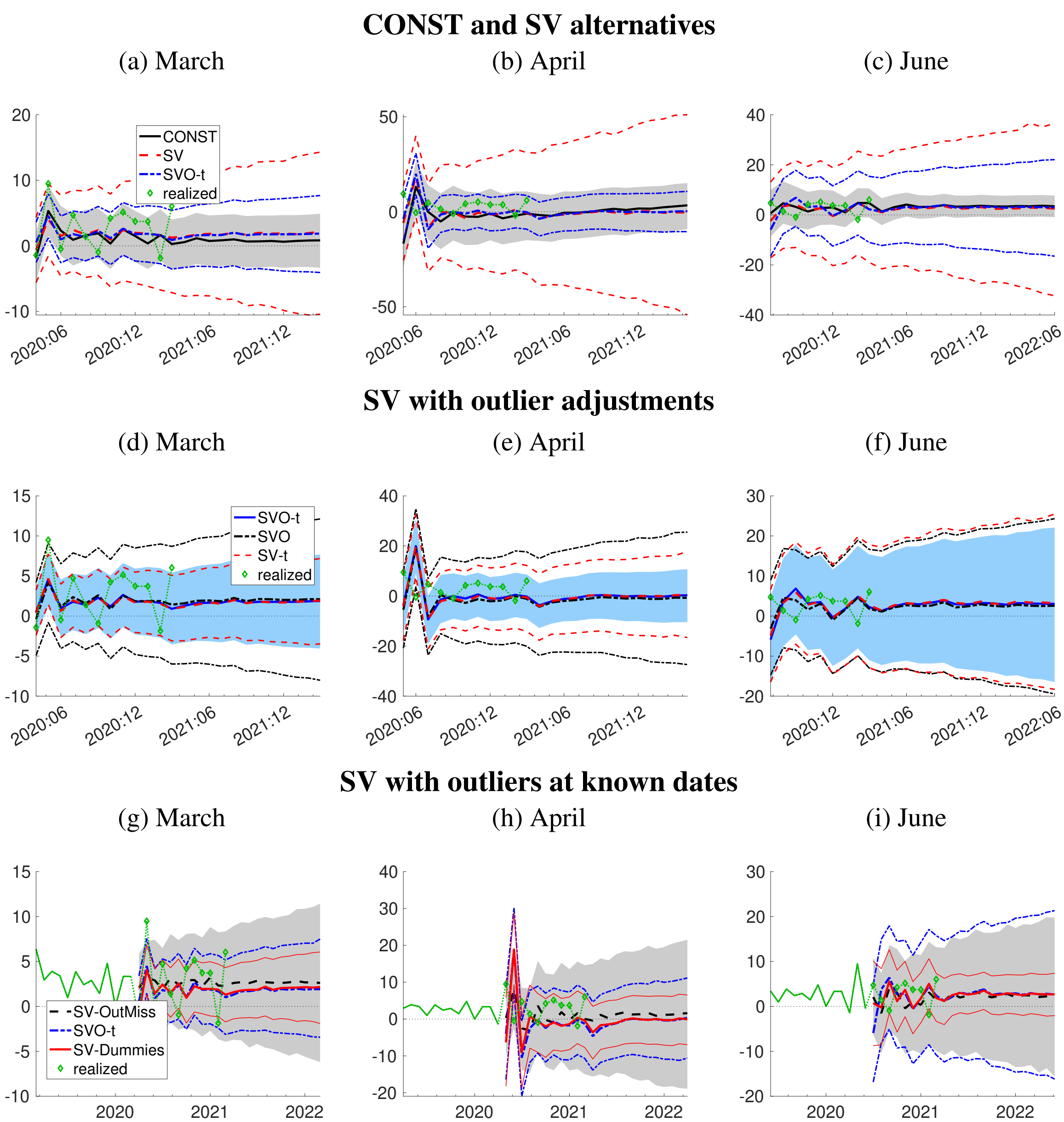
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. The solid green line denotes realized data prior to the forecast origin. In panels (g) – (i), observations identified ex-ante as outliers, based on being more than 5 times the inter-quartile range away from the median, are indicated with a circle, and the corresponding backcast densities from the SV-OutMiss model are superimposed.

Figure S.53: Predictive densities since March 2020 for Hours



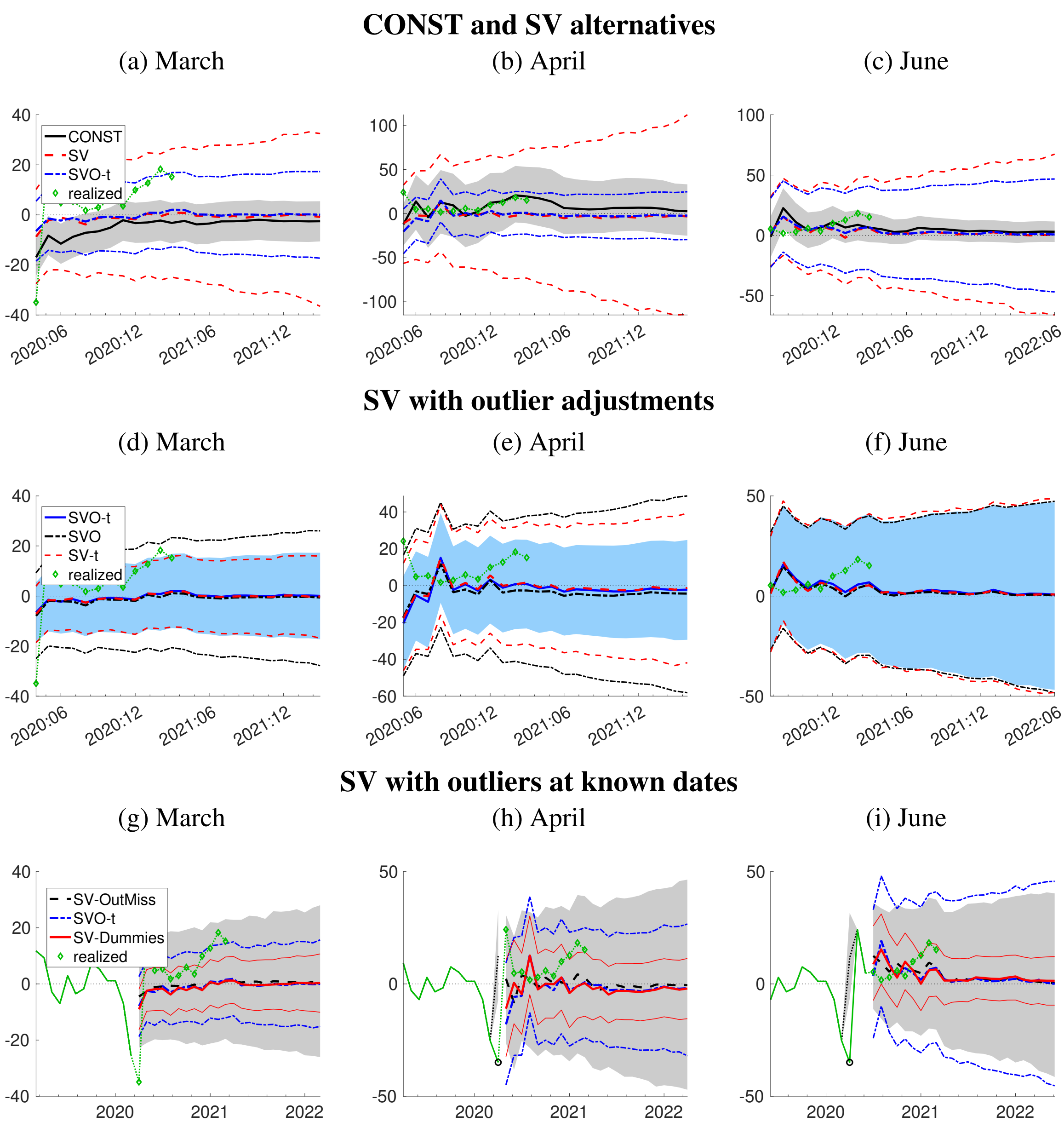
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. The solid green line denotes realized data prior to the forecast origin. In panels (g) – (i), observations identified ex-ante as outliers, based on being more than 5 times the inter-quartile range away from the median, are indicated with a circle, and the corresponding backcast densities from the SV-OutMiss model are superimposed.

Figure S.54: Predictive densities since March 2020 for Hourly Earnings



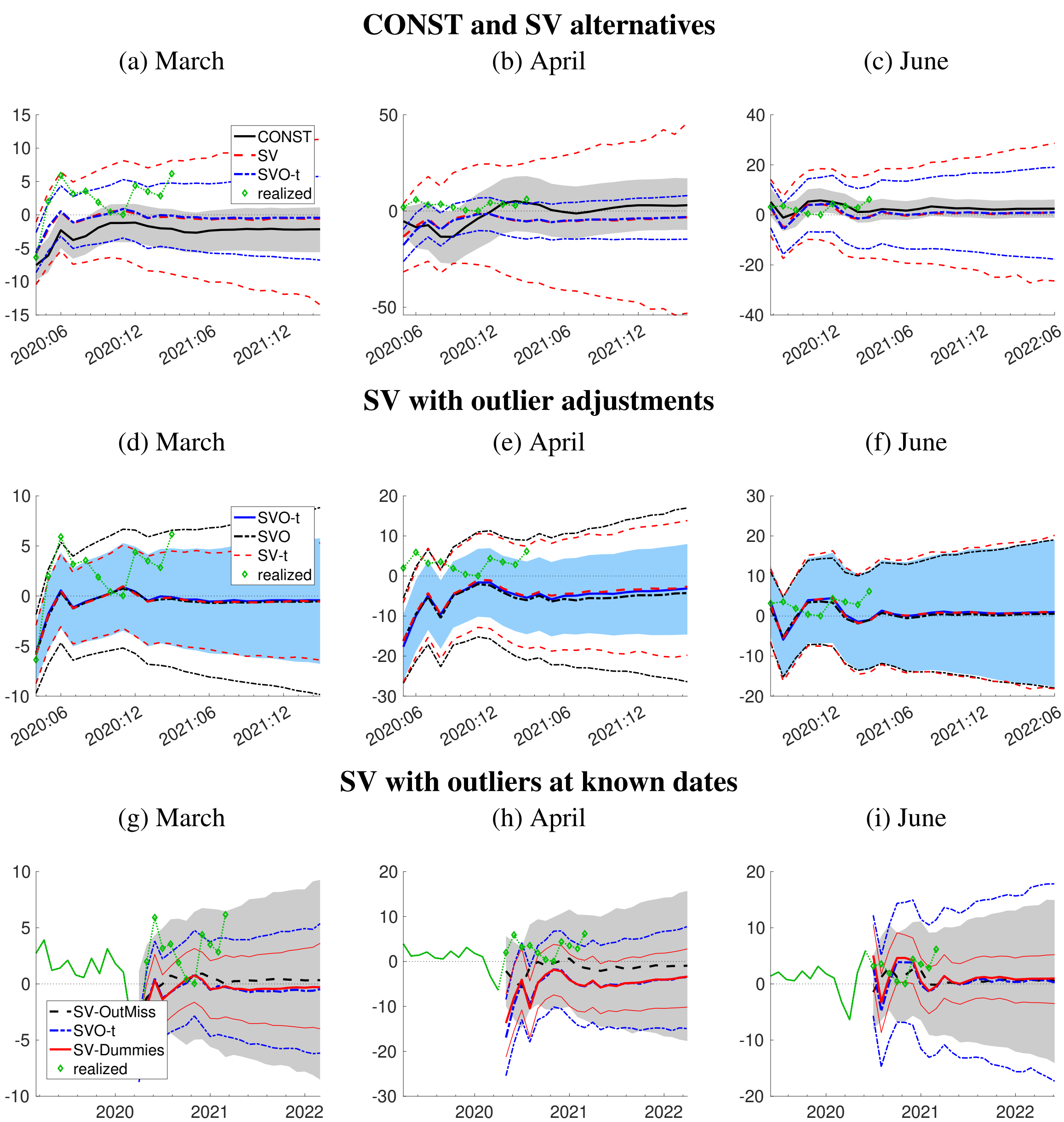
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. The solid green line denotes realized data prior to the forecast origin. In panels (g) – (i), observations identified ex-ante as outliers, based on being more than 5 times the inter-quartile range away from the median, are indicated with a circle, and the corresponding backcast densities from the SV-OutMiss model are superimposed.

Figure S.55: Predictive densities since March 2020 for PPI (fin. goods)



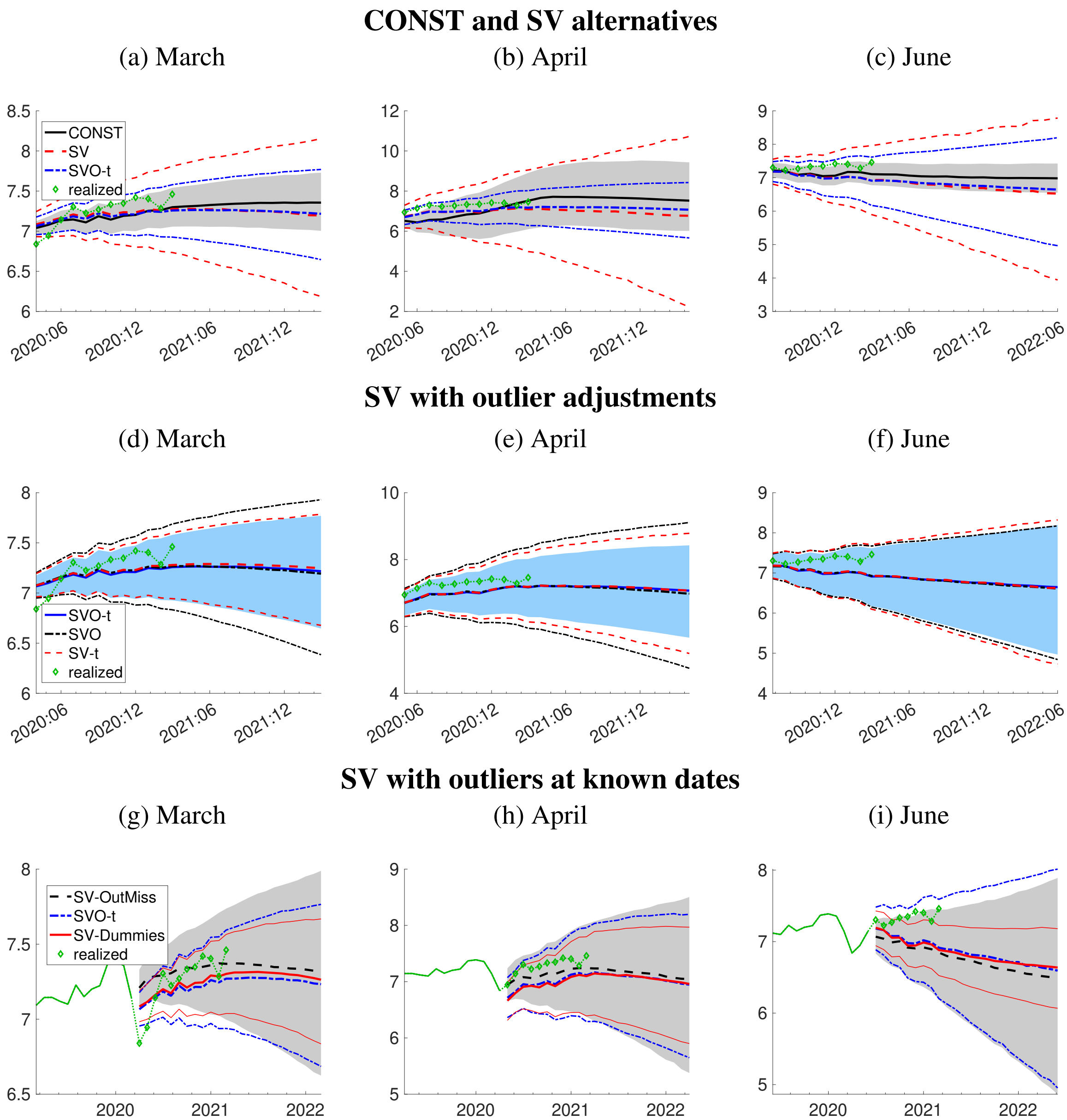
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. The solid green line denotes realized data prior to the forecast origin. In panels (g) – (i), observations identified ex-ante as outliers, based on being more than 5 times the inter-quartile range away from the median, are indicated with a circle, and the corresponding backcast densities from the SV-OutMiss model are superimposed.

Figure S.56: Predictive densities since March 2020 for PCE prices



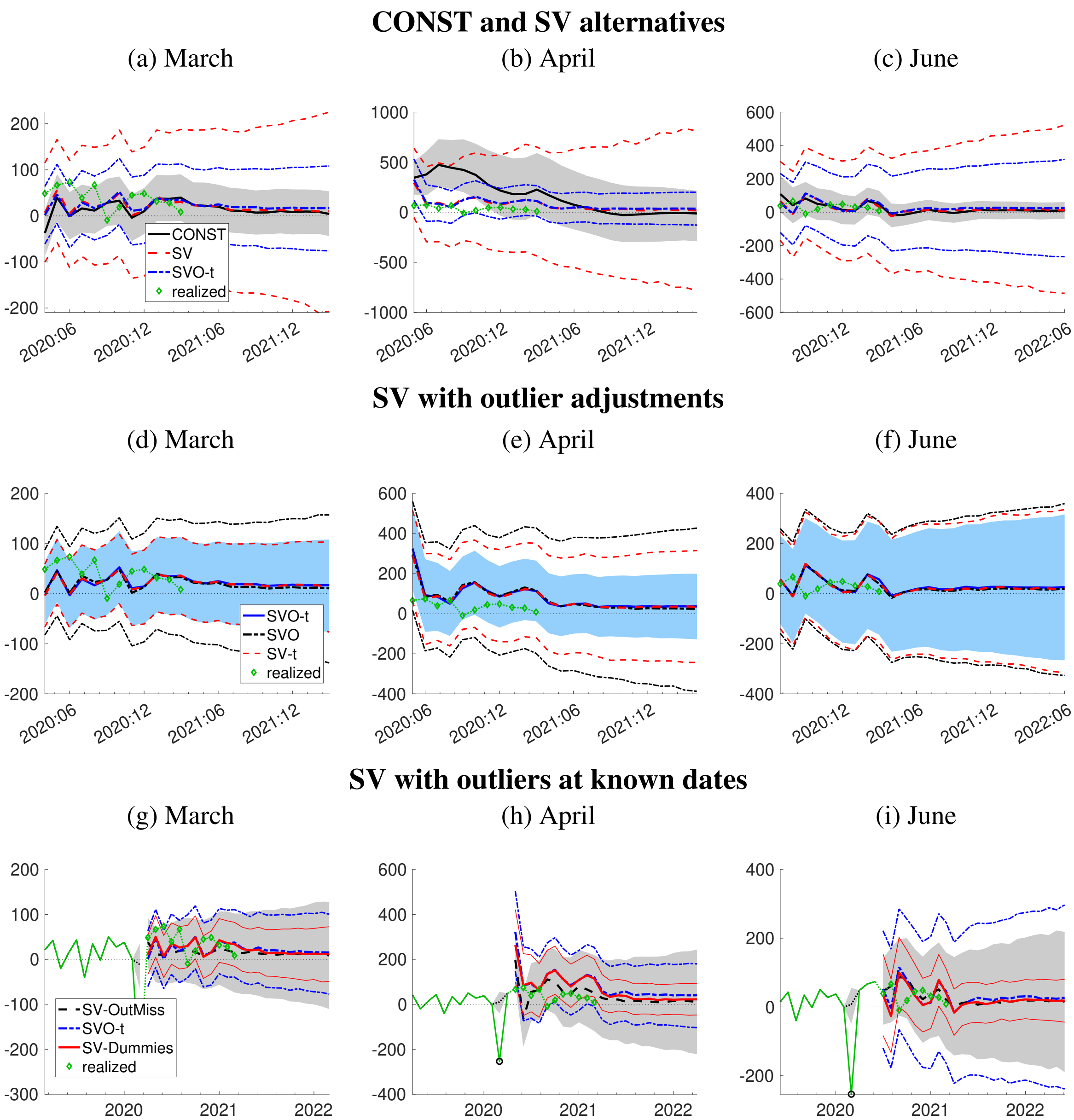
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. The solid green line denotes realized data prior to the forecast origin. In panels (g) – (i), observations identified ex-ante as outliers, based on being more than 5 times the inter-quartile range away from the median, are indicated with a circle, and the corresponding backcast densities from the SV-OutMiss model are superimposed.

Figure S.57: Predictive densities since March 2020 for Housing Starts



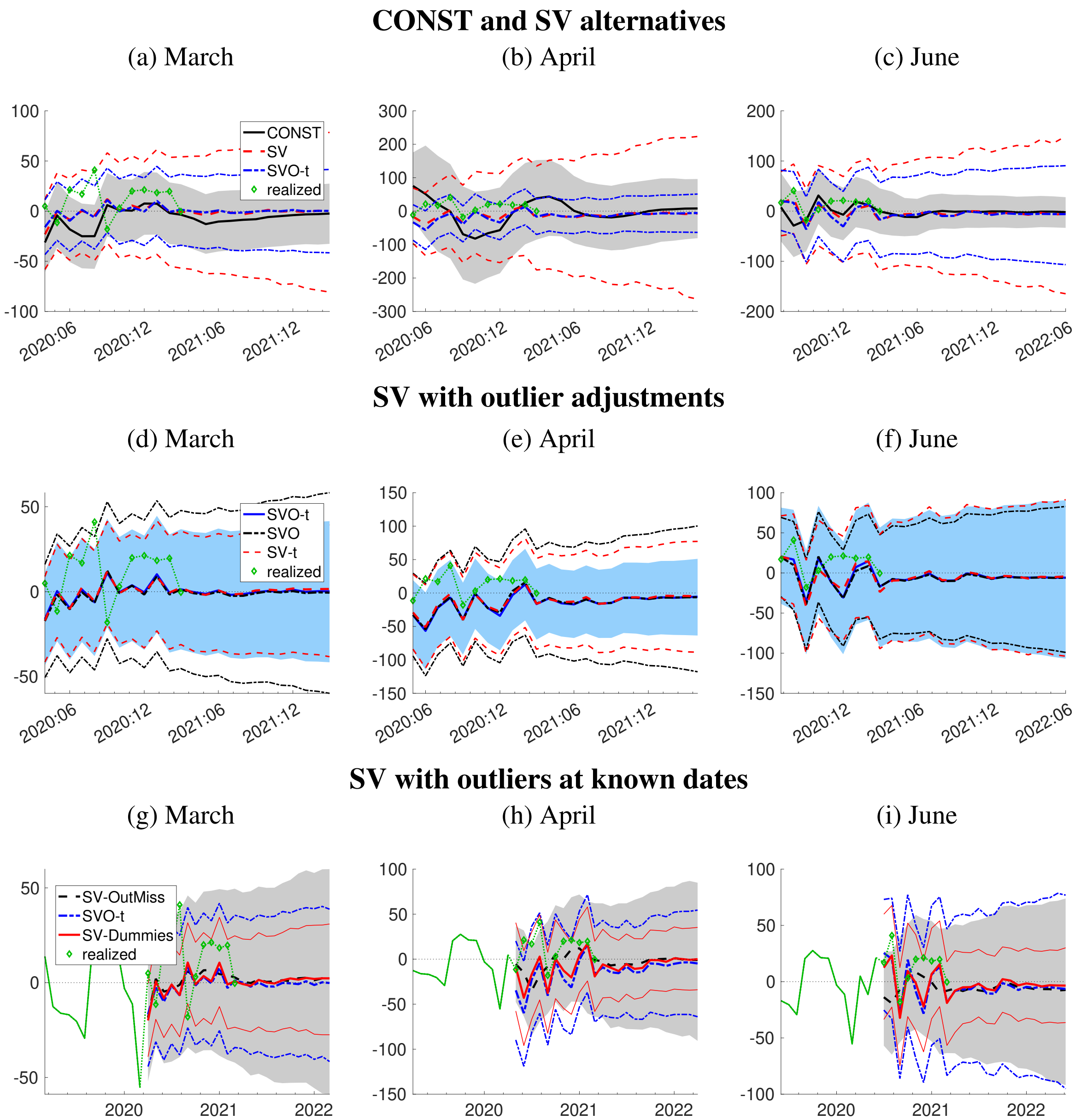
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. The solid green line denotes realized data prior to the forecast origin. In panels (g) – (i), observations identified ex-ante as outliers, based on being more than 5 times the inter-quartile range away from the median, are indicated with a circle, and the corresponding backcast densities from the SV-OutMiss model are superimposed.

Figure S.58: Predictive densities since March 2020 for S&P 500



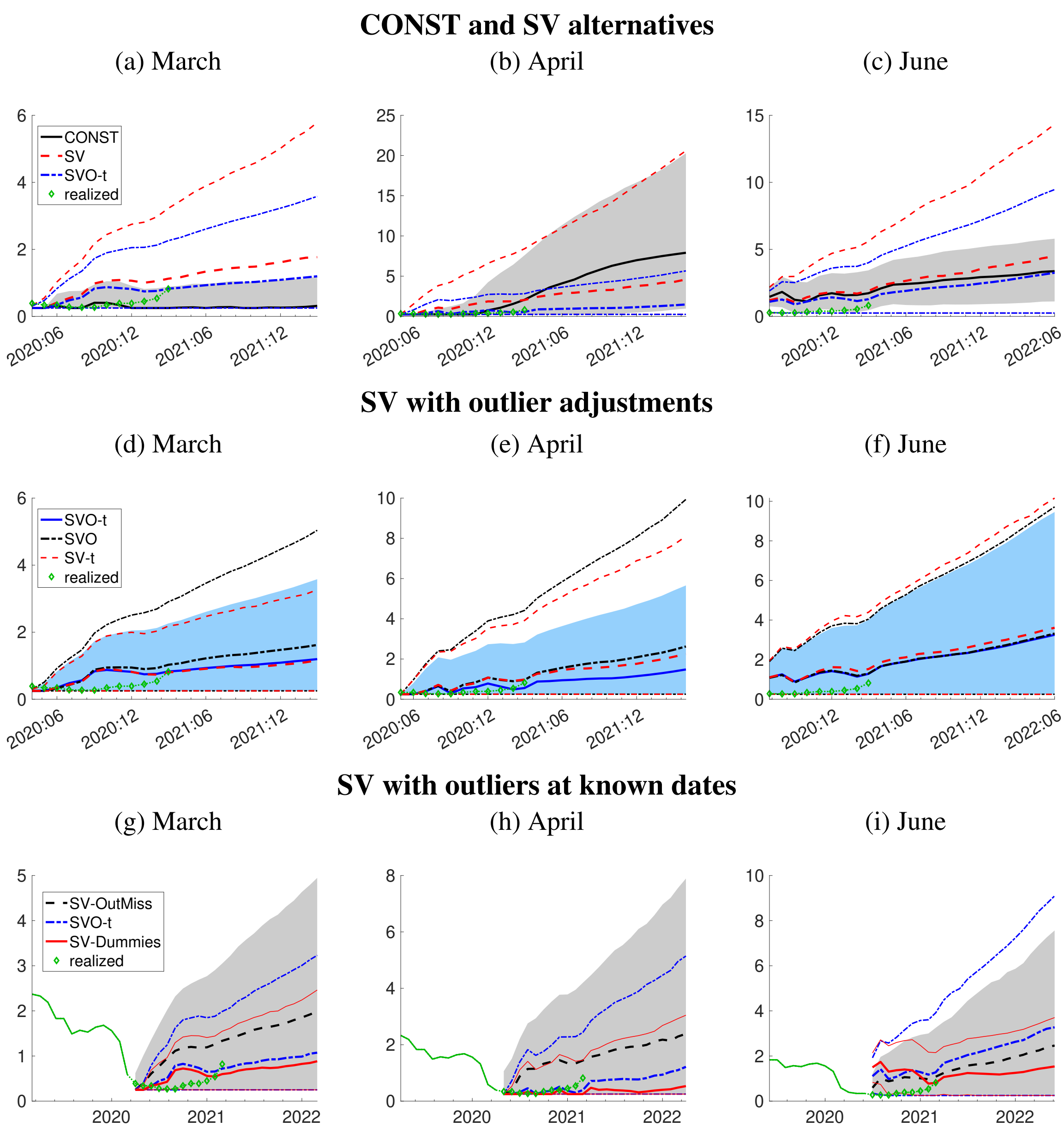
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. The solid green line denotes realized data prior to the forecast origin. In panels (g) – (i), observations identified ex-ante as outliers, based on being more than 5 times the inter-quartile range away from the median, are indicated with a circle, and the corresponding backcast densities from the SV-OutMiss model are superimposed.

Figure S.59: Predictive densities since March 2020 for USD / GBP FX rate



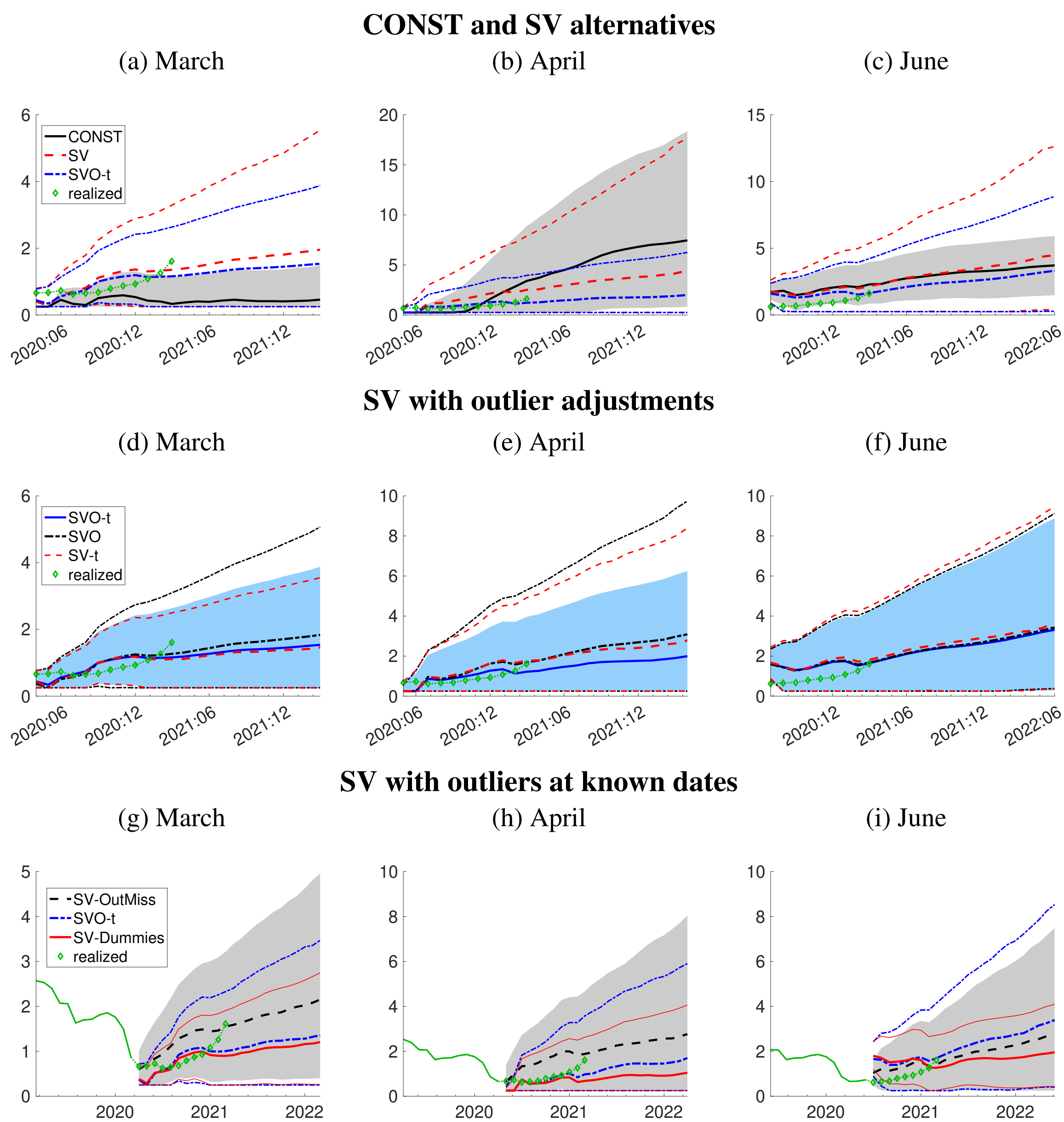
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. The solid green line denotes realized data prior to the forecast origin. In panels (g) – (i), observations identified ex-ante as outliers, based on being more than 5 times the inter-quartile range away from the median, are indicated with a circle, and the corresponding backcast densities from the SV-OutMiss model are superimposed.

Figure S.60: Predictive densities since March 2020 for 5-year yield



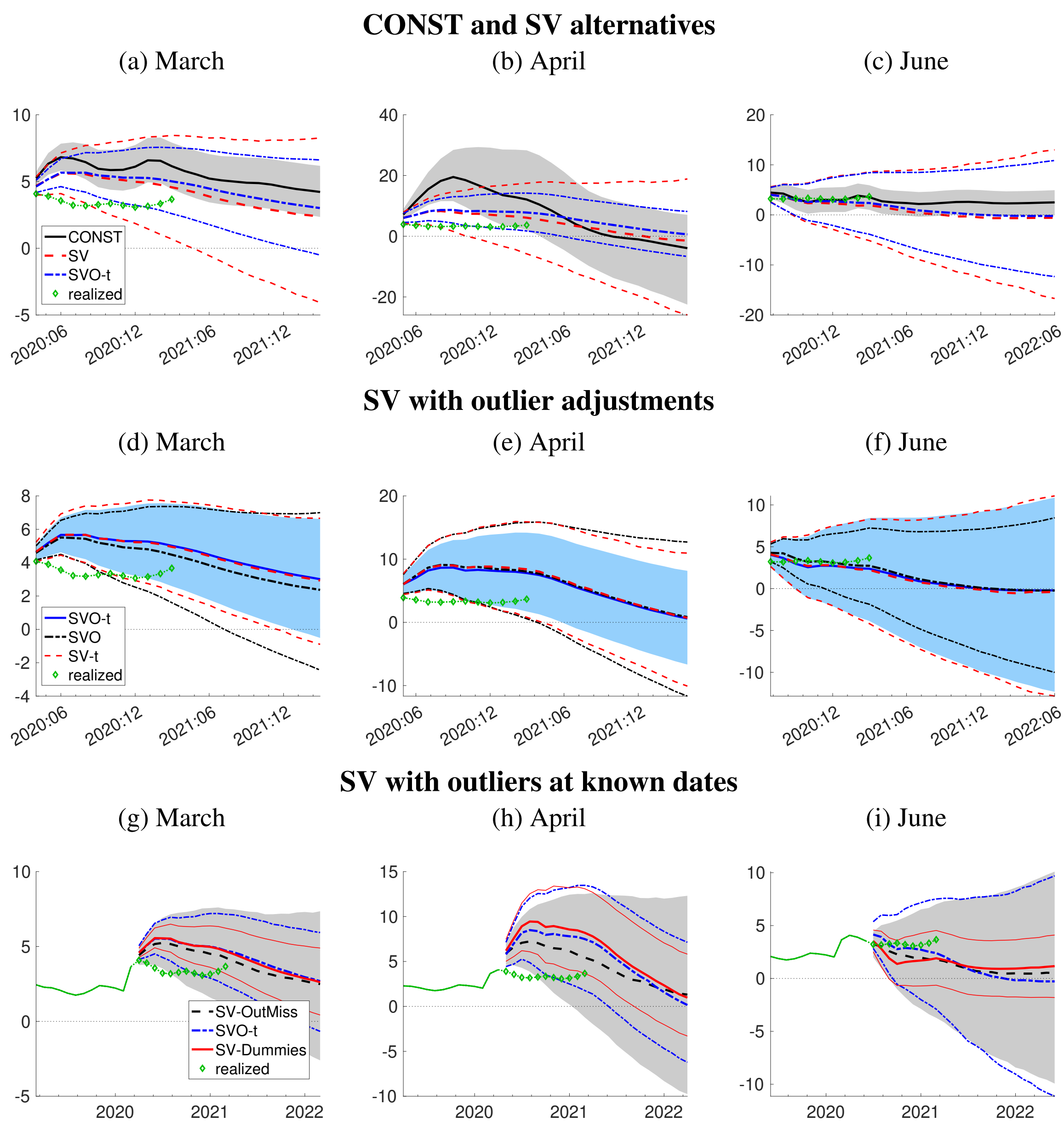
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. The solid green line denotes realized data prior to the forecast origin. In panels (g) – (i), observations identified ex-ante as outliers, based on being more than 5 times the inter-quartile range away from the median, are indicated with a circle, and the corresponding backcast densities from the SV-OutMiss model are superimposed.

Figure S.61: Predictive densities since March 2020 for 10-year yield



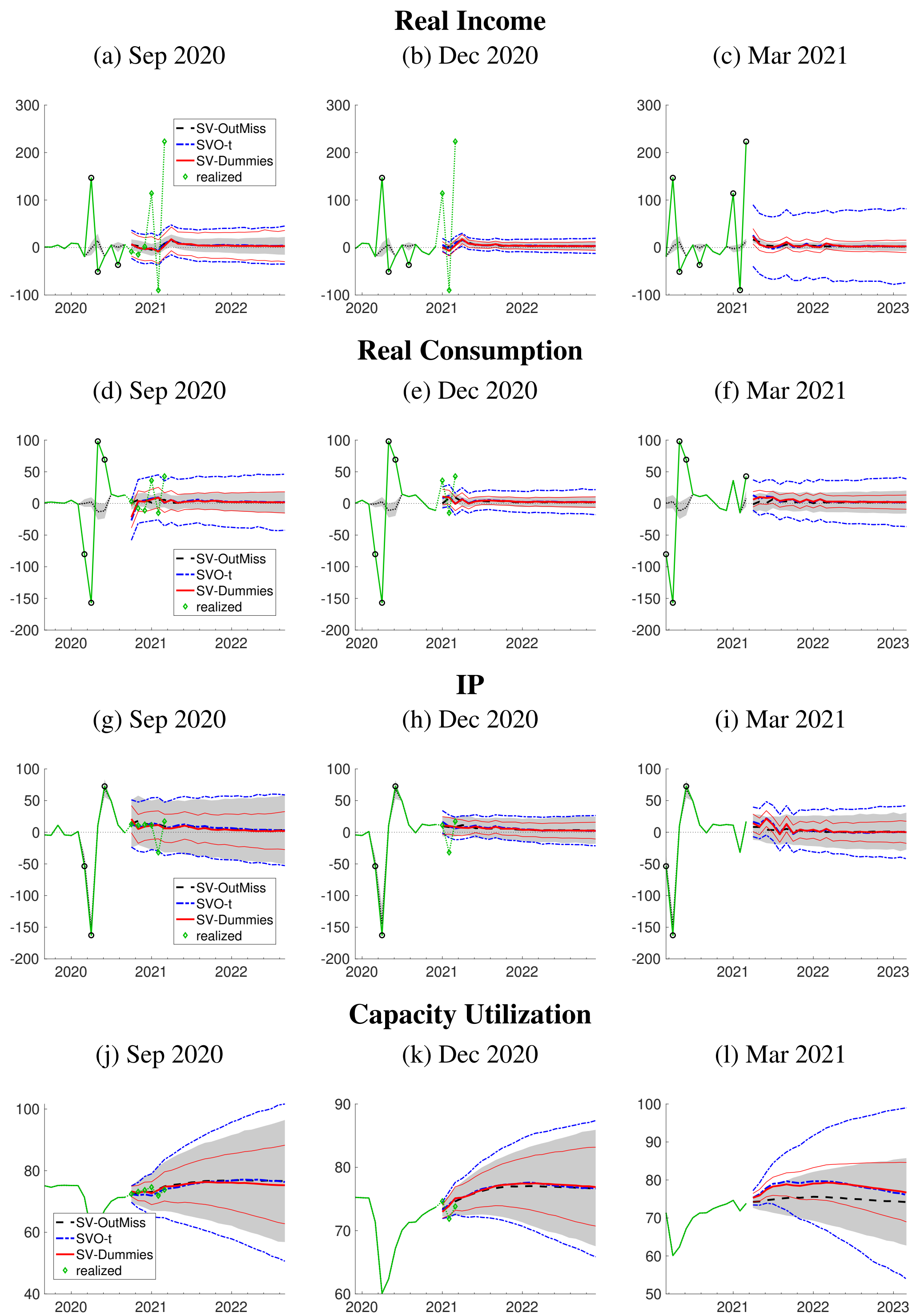
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. The solid green line denotes realized data prior to the forecast origin. In panels (g) – (i), observations identified ex-ante as outliers, based on being more than 5 times the inter-quartile range away from the median, are indicated with a circle, and the corresponding backcast densities from the SV-OutMiss model are superimposed.

Figure S.62: Predictive densities since March 2020 for Baa spread



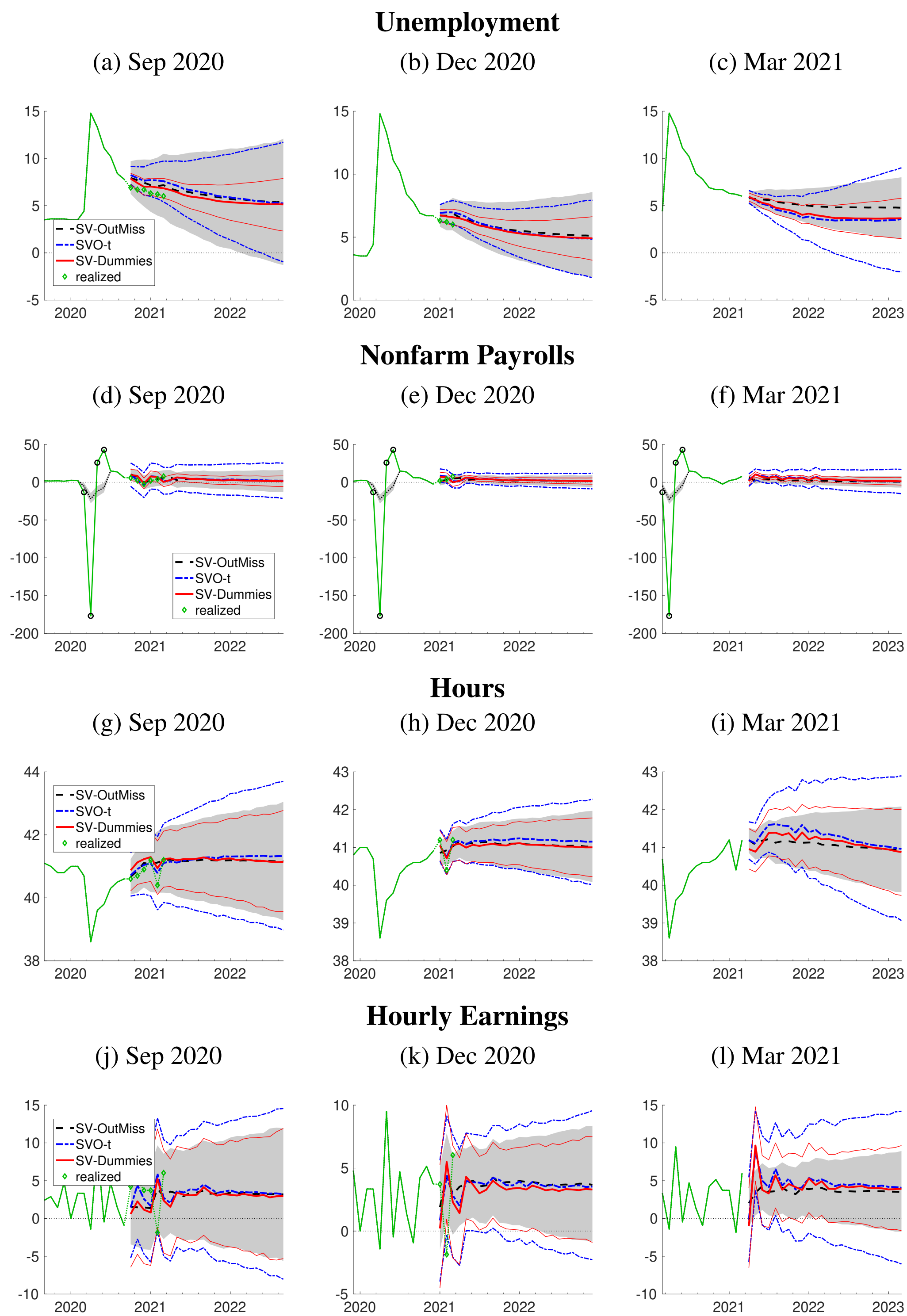
Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. The solid green line denotes realized data prior to the forecast origin. In panels (g) – (i), observations identified ex-ante as outliers, based on being more than 5 times the inter-quartile range away from the median, are indicated with a circle, and the corresponding backcast densities from the SV-OutMiss model are superimposed.

Figure S.63: Predictive densities since late 2020



Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. Forecasts generated from the SV-OutMiss approach identify observations ex-ante as outliers, based on being more than 5 times the inter-quartile range away from the median; these outliers are indicated with a circle, and the corresponding backcast densities from the SV-OutMiss model are superimposed.

Figure S.64: Predictive densities since late 2020 (ctd.)

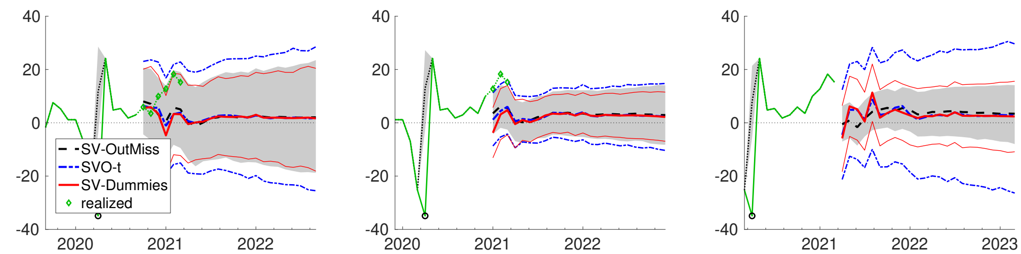


Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. Forecasts generated from the SV-OutMiss approach identify observations ex-ante as outliers, based on being more than 5 times the inter-quartile range away from the median; these outliers are indicated with a circle, and the corresponding backcast densities from the SV-OutMiss model are superimposed.

Figure S.65: Predictive densities since late 2020 (ctd.)

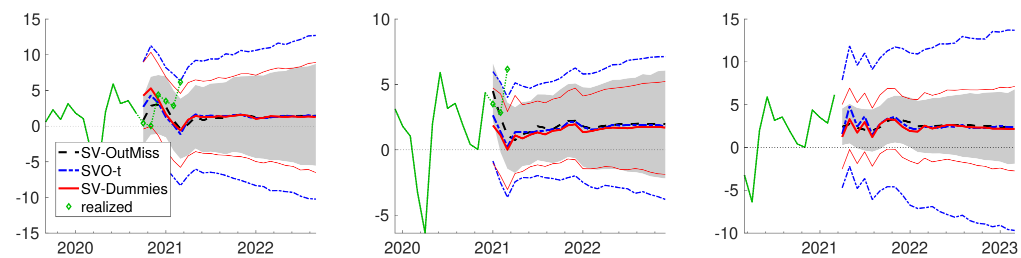
PPI (fin. goods)

(a) Sep 2020 (b) Dec 2020 (c) Mar 2021



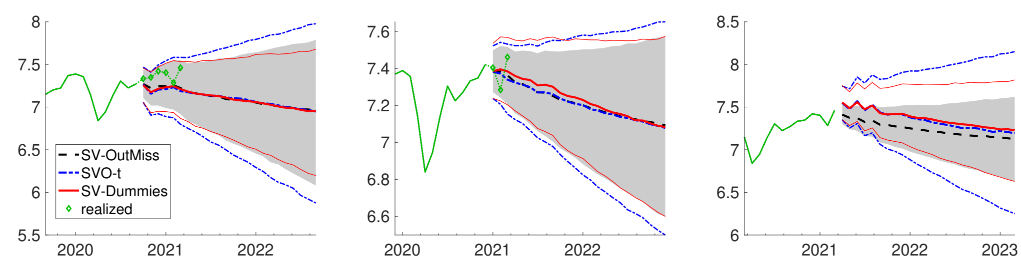
PCE prices

(d) Sep 2020 (e) Dec 2020 (f) Mar 2021



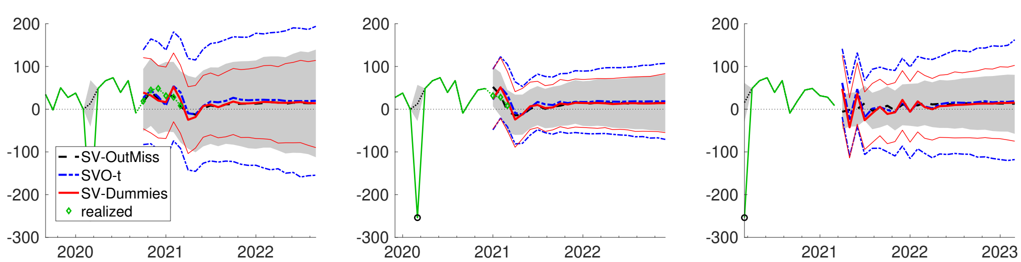
Housing Starts

(g) Sep 2020 (h) Dec 2020 (i) Mar 2021



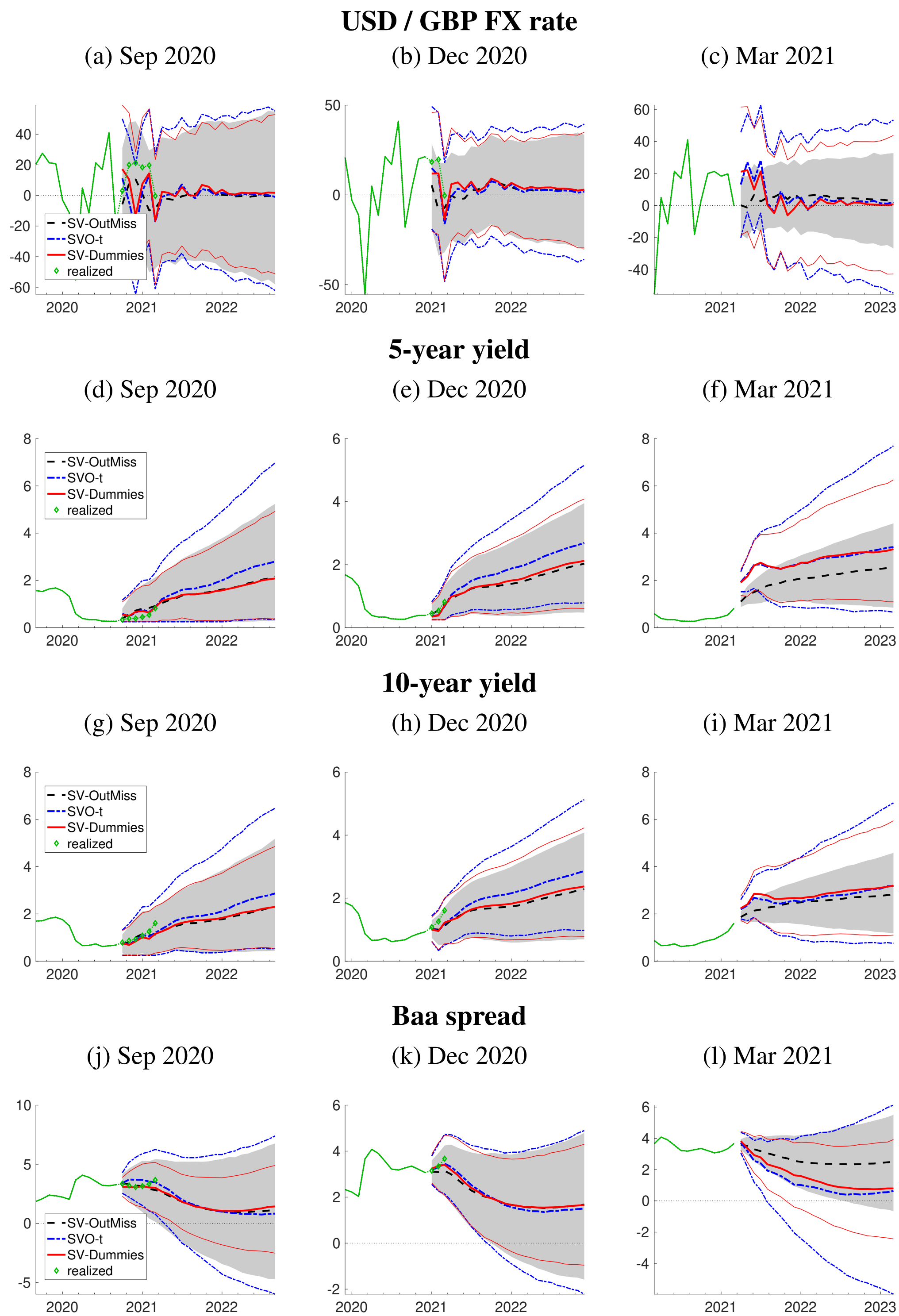
S&P 500

(j) Sep 2020 (k) Dec 2020 (l) Mar 2021



Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. Forecasts generated from the SV-OutMiss approach identify observations ex-ante as outliers, based on being more than 5 times the inter-quartile range away from the median; these outliers are indicated with a circle, and the corresponding backcast densities from the SV-OutMiss model are superimposed.

Figure S.66: Predictive densities since late 2020 (ctd.)



Note: Medians and 68% uncertainty bands of predictive densities, simulated out-of-sample at various forecast origins as indicated in each panel. Forecasts generated from the SV-OutMiss approach identify observations ex-ante as outliers, based on being more than 5 times the inter-quartile range away from the median; these outliers are indicated with a circle, and the corresponding backcast densities from the SV-OutMiss model are superimposed.

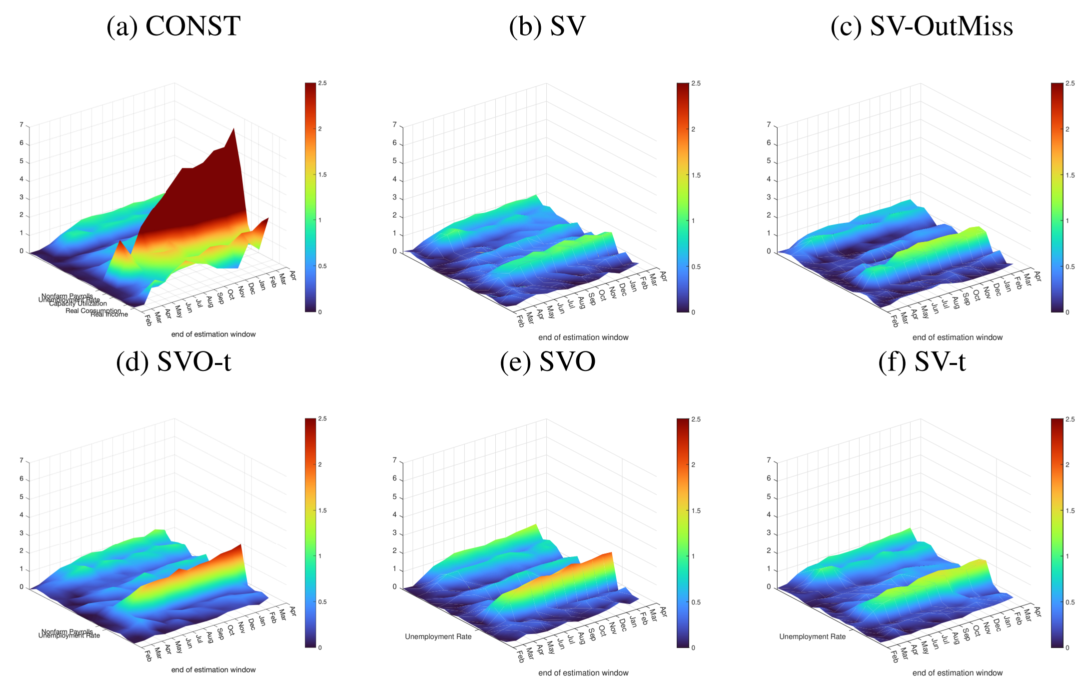
VII Stability in VAR parameters since Jan 2020

The unusual developments of the pandemic inevitably raise a question as to whether it represents a break in conventional business cycle dynamics and time series models. Our results treat the VARs as stable, taking various steps to limit the influence that extreme observations can have on model estimates. Of course, although it would be ideal to be able to formally test model stability, the sample since March 2020 is too short to permit formal inference with conventional tests or metrics.

As a simple and feasible alternative, we examine the stability of recursive estimates of the VAR from January 2020 through the end of our sample in 2021. To assess the significance of change in each coefficient, we take the January 2020 posterior for each coefficient as reference point. For sake of comparability, we standardize the change in the posterior means obtained at subsequent forecast origins, by dividing the changes by January's posterior standard deviation.[10] Figures S.67– S.69 provide charts of these statistics over time, for each variable and month, for the intercepts and lag coefficients. We consider stability for all of the paper's specifications — CONST, SV, SV-OutMiss, SVO-t, SVO, and SV-t specifications. Broadly, these results indicate that — except for CONST case — there are at most only fairly limited changes in some coefficients, while the vast majority of coefficients show little change. By our simple measures of significance, the CONST specification is quite prone to some coefficient change, most sizably for some economic activity indicators. In the SV specifications, coefficient change appears much less significant. The SVO and SVO-t models show changes in intercepts for some variables, but otherwise, estimates look to be broadly stable over the period.

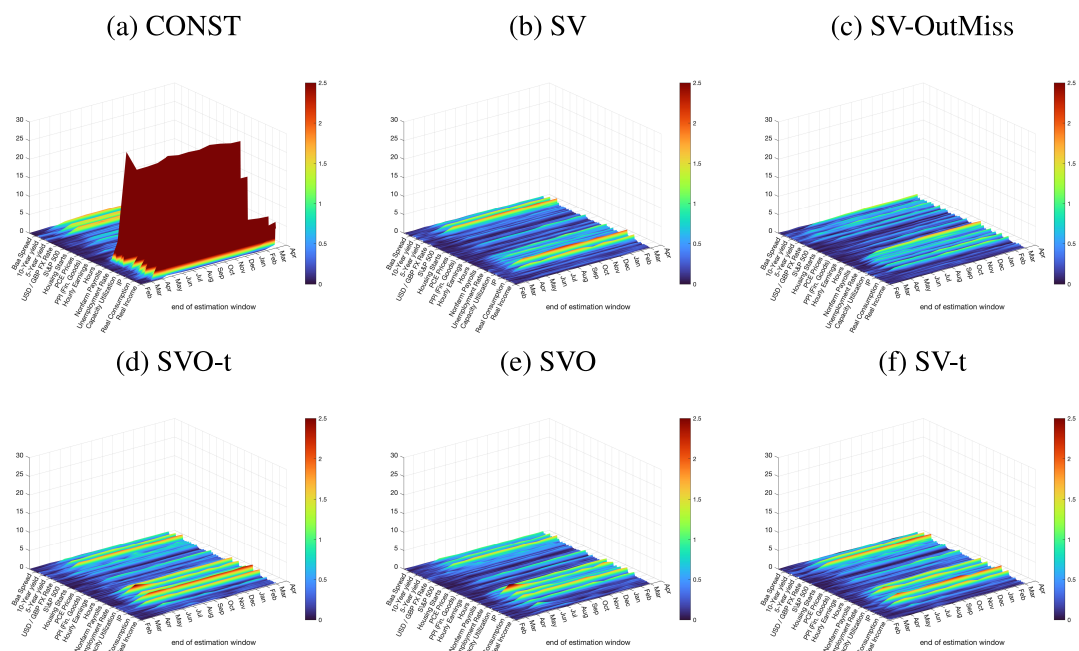
[10] The resulting statistics are similar in spirit to a frequentist t-statistic, though without necessarily identical interpretation in the context of our Bayesian application.

Figure S.67: VAR intercept changes



Note: Absolute values of standardized mean estimates of intercepts in VAR equations of different models, estimated at different sample ends since January 2020 (data for all estimation windows begin in 1959:M03). The estimates are standardized based on the posterior mean and standard deviation of each coefficient estimated per January 2020.

Figure S.68: VAR lag 1 coefficient changes

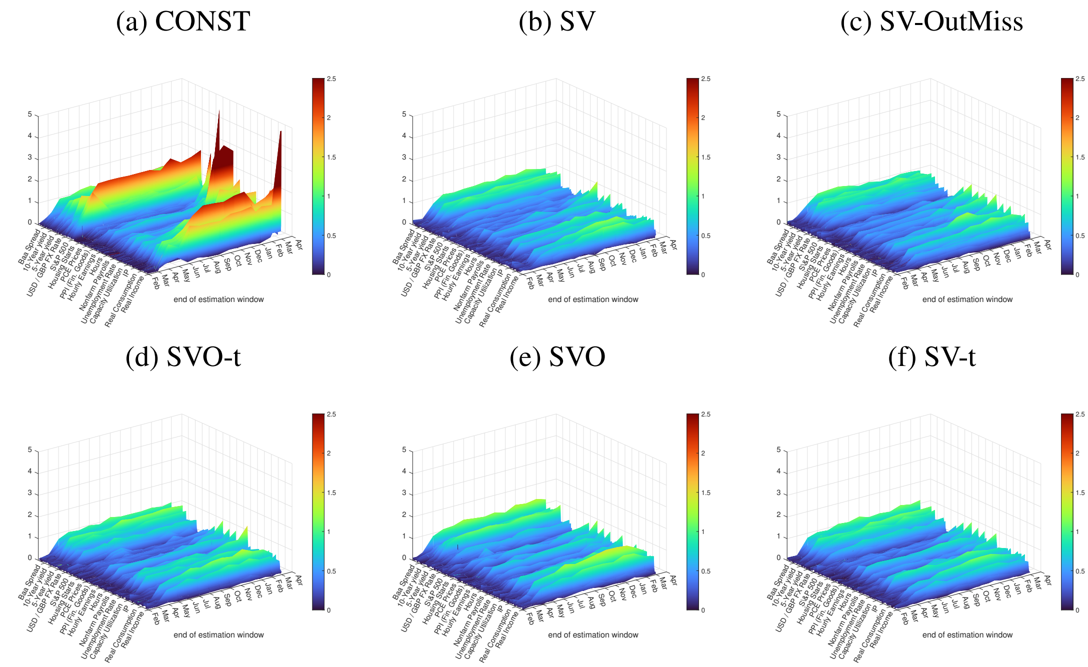


(a) CONST (b) SV (c) SV-OutMiss

(d) SVO-t (e) SVO (f) SV-t

Note: Absolute values of standardized mean estimates of first own-lag coefficients in VAR equations of different models, estimated at different sample ends since January 2020 (data for all estimation windows begin in 1959:M03). The estimates are standardized based on the posterior mean and standard deviation of each coefficient estimated per January 2020.

Figure S.69: VAR other-lag coefficient changes



Note: Absolute values of standardized mean estimates of other-lag (non intercept and non first own-lag) coefficients in VAR equations of different models, estimated at different sample ends since January 2020 (data for all estimation windows begin in 1959:M03). The estimates are standardized based on the posterior mean and standard deviation of each coefficient estimated per January 2020.

VIII Effects of Pruning the Jump-off in SV-OutMiss

As discussed in the paper, the SV-OutMiss model pre-screens the data for outliers, based on their distance from the historical median value of each series. In our application, an observation is marked as outlier if its distance from the median exceed's five times the inter-quartile range of the variable's time series. The VAR-SV estimation then treats outliers as missing data.[11] Treating outliers as missing data not only insulates parameter estimates from extreme observations but also prunes the jump-off vector used in the VAR's forecast simulations. Specifically, denote the history of observed data ex-outliers as z^t, and continue the AR(1) example introduced in the paper: Forecasts are then generated by $y_{t+1|t} = \pi^h E(y_t|z^t)$ where $E(y_t|z^t)$ is identical to y_t in the no-outlier case. Similarly, forecast uncertainty is generated based on estimates of SV that condition only on z^t, not potential outliers in the history of y_t.

To better parse the effects of pruning outliers from the forecast jump-off, we conducted forecast simulations from the SV-OutMiss model that (i) use parameter (and SV) estimates drawn from the model's posterior while (ii) using the actual data (including outliers) as the jump-off vector, and refer to this alternative approach as "SV-OutMiss ALT." The tables provided in this section compare RMSE, MAE (median absolute errors), and CRPS measures for forecasts of individual variables from either version of the SV-OutMiss approach. Results are reported for the 1975-2017 evaluation window, as well as the GFC. In both cases, relative RMSE, MAE, or CRPS are very close to one, suggesting only little effect from the pruned jump-off vector (compared to the benefits from insulated parameter and SV estimates as documented above and in the paper).

[11] In the limit, the missing data approach corresponds to an extreme version of attaching additive measurement error to specific observations, but with infinite variance, whereas the remaining observations are observed without error.

Table S.10: Effects of Pruning Jump-off Vector in SV-OutMiss

	RMSE				MAE				CRPS			
Variable / Horizon	1	3	12	24	1	3	12	24	1	3	12	24
Real Income	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Real Consumption	1.02**	1.02*	1.00	1.00	1.03***	1.03**	1.00	1.00	1.03**	1.02**	1.00	1.00
IP	1.00	0.99	1.00**	1.00**	1.01	1.00	1.00**	1.00**	1.01	1.00	1.00*	1.00**
Capacity Utilization	1.00	0.99	1.00	1.00	1.01	1.00	1.00	1.00	1.01	1.00	1.00	1.00
Unemployment Rate	1.01**	1.00	1.00	1.00	1.01**	1.00	1.00	1.00	1.01**	1.00	1.00	1.00
Nonfarm Payrolls	1.01*	1.00	1.00	1.00	1.01	1.01	1.00	1.00	1.01**	1.01	1.00	1.00
Hours	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Hourly Earnings	1.00	1.00	1.00	1.00	1.01**	1.01	1.00	1.00	1.01*	1.01*	1.00	1.00
PPI (Fin. Goods)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
PCE Prices	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00**	1.00	1.00
Housing Starts	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
S&P 500	1.01	1.01**	1.00	1.00	1.02*	1.01*	1.00	1.00	1.01*	1.01**	1.00	1.00
USD / GBP FX Rate	0.98**	1.00	1.00	1.00	0.98**	1.00	1.00*	1.00	0.99**	1.00	1.00	1.00
5-Year yield	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.00	1.01	1.00	1.00	1.00
10-Year yield	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Baa Spread	1.00	1.00	1.00*	1.00	0.99	1.00	1.00*	1.00	1.00	1.00	1.00**	1.00

Note: Comparison of “SV-OutMiss” (baseline, in denominator) against “SV-OutMiss ALT.” Values below 1 indicate improvement over baseline. Evaluation window from 1975:01 through 2017:12. Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h+1$ lags. Due to the close behavior of some of the models compared, and rounding of the reported values, a few comparisons show significant ratios of 1.00. These cases arise from persistent differences in performance that are, however, too small to be relevant after rounding.

Table S.11: Effects of Pruning Jump-off Vector in SV-OutMiss (GFC)

	RMSE				MAE				CRPS			
Variable / Horizon	1	3	12	24	1	3	12	24	1	3	12	24
Real Income	1.01	1.00	1.00	1.00	1.01	0.99	1.00	1.00	1.00	1.00	0.99	1.00
Real Consumption	1.00	1.05*	1.00	1.00	1.01	1.05*	1.00	1.00	0.99	1.04*	1.00	1.00
IP	0.96	0.94	1.00*	1.01**	0.99	0.97	1.00*	1.01**	0.99	0.97	1.00*	1.00**
Capacity Utilization	0.99	0.94	1.00	1.00	1.02	0.97	0.99	0.99**	1.00	0.97	0.99	1.00*
Unemployment Rate	1.01	0.99	0.99	1.00	0.99	1.00	0.99	1.00	1.00	0.99	1.00	1.00
Nonfarm Payrolls	1.01	0.98	1.00	1.00	1.02	1.02	1.00	1.00	1.02	1.00	1.00	1.00
Hours	0.99	0.97	0.99	1.00**	0.99	0.99	0.99	0.99**	0.99	0.98	0.99	1.00*
Hourly Earnings	1.03	1.04**	1.00	1.01**	1.03	1.05**	1.00	1.01**	1.02	1.04**	1.00	1.00**
PPI (Fin. Goods)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
PCE Prices	0.97	1.00	1.00	1.01	0.98	0.99	1.00	1.00	0.98	1.00	1.00	1.00
Housing Starts	0.98	0.98	1.00	1.00	0.99	0.98	1.00	1.00	0.99	0.99	1.00	1.00
S&P 500	1.00	1.01	1.00	1.00	1.00	1.01	1.00	1.00	1.01	1.01*	1.00	1.00
USD / GBP FX Rate	1.00	1.01	1.00	1.00	0.99	1.01	1.00	1.00	1.00	1.01	1.00	1.00
5-Year yield	0.99	1.00	1.00	1.00	0.98	1.00	1.01	1.01*	0.99	1.00	1.01	1.00
10-Year yield	1.00	0.99	1.00	1.00	1.00	0.99	1.01	1.00	1.00	0.99	1.00	1.00
Baa Spread	0.99	1.00	1.01	1.00	0.98	1.00	1.01	1.00	0.99	1.00	1.01	1.00

Note: Comparison of "SV-OutMiss" (baseline, in denominator) against "SV-OutMiss ALT." Values below 1 indicate improvement over baseline. Evaluation window from 2007:01 through 2014:12. Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Due to the close behavior of some of the models compared, and rounding of the reported values, a few comparisons show significant ratios of 1.00. These cases arise from persistent differences in performance that are, however, too small to be relevant after rounding.

Part C

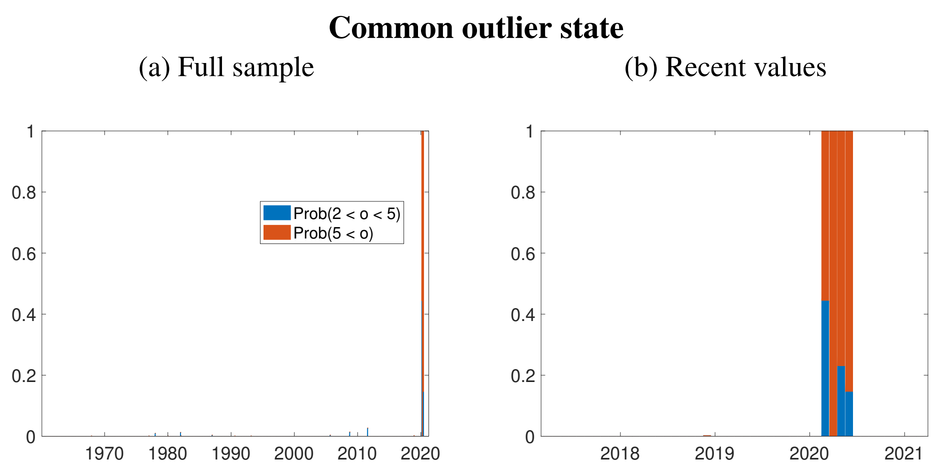
Robustness

IX Common outlier model

With the COVID-19 pandemic inducing extreme volatility in a number of variables, some may view it as plausible that the outlier is common to all variables, rather than independent across variables as in the SVO specification. Some other work, such as Lenza and Primiceri (2020), has developed models in which the pandemic induces a common shift in volatility in an otherwise homoskedastic VAR. Accordingly, we have also considered a specification in which the outlier state is common to all variables, in which case the time-varying variance-covariance matrix of the VAR residuals is given by $\Sigma_t = \bar{o}_t^2 A^{-1}\, \Lambda_t\, (A^{-1})'$, where $\bar{o}_t$ denotes a scalar outlier state.

The results shown in Figure S.70 and the following forecast comparison tables indicate that the common outlier specification seems to have no advantages. In historical estimates, the common-outlier specification registers virtually no outliers prior to the COVID-19 pandemic. Instead, the common-outlier specification sees outliers only in the early stages of the pandemic period, from March through June 2020, when a good number of variables experienced enormous realizations at the same time, but none in late 2020 or early 2021. Imposing the same outlier on all variables during COVID-19 leads to some marked differences in the width of predictive densities compared to the SVO(-t) models that feature variable-specific outliers, but fairly identical performance in point forecasts over the pandemic period. In general, making outliers common in some cases makes forecasts slightly less accurate compared to the SVO specification that models outliers as independent across variables.

Figure S.70: Estimates of a common outlier state



Note: Full-sample estimates per March 2021 of posterior probabilities for realizations of the common outlier state in a VAR-SV model that features only a common and scalar outlier factor, $\bar{o}_t$, in place of the diagonal matrix O_t as used in the SVO model described in the paper. The prior for the common outlier state is identical to the (common) prior used for each outlier state in the SVO model.

Table S.12: RMSE (individual vs common outlier models)

					Relative to SVO …							
	SVO				SVO-t				SV-common outlier			
Variable / Horizons	1	3	12	24	1	3	12	24	1	3	12	24
Real Income	7.11	7.10	7.13	7.27	1.01**	1.01**	1.00	1.00	1.00	1.00	1.00	1.05**
Real Consumption	5.77	5.89	5.82	5.90	1.01*	1.01*	1.00	1.01**	1.01	1.00	1.00	1.01
IP	7.25	7.91	8.17	8.77	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.01
Capacity Utilization	0.52	1.06	2.87	4.30	1.00	0.99	0.99	0.98	0.98*	0.98	1.00	1.01
Unemployment Rate	0.16	0.27	0.81	1.25	1.00	1.00	0.99	0.99	1.00	1.00	1.00	0.99
Nonfarm Payrolls	1.67	1.85	2.12	2.48	0.99	1.00	1.00	1.00	0.99	1.00	1.01	1.00
Hours	0.24	0.28	0.44	0.48	1.00	0.99	0.98	0.98	1.00	0.99	1.00	1.00
Hourly Earnings	3.00	3.01	3.24	3.74	1.00	1.00	1.02***	1.04***	1.00	1.00	1.00	1.02*
PPI (Fin. Goods)	6.63	6.78	7.14	7.60	1.00	1.00	1.00	1.01**	1.00	1.00	1.01**	1.00
PCE Prices	2.11	2.49	2.80	3.41	1.01*	1.01	1.02**	1.04***	1.00	1.00	1.01	1.00
Housing Starts	0.08	0.11	0.24	0.37	1.00	1.00	1.01**	1.02*	1.00	1.00	1.00	0.99*
S&P 500	43.55	44.53	43.70	43.12	1.00	1.00	1.00	1.01***	1.00	1.00	1.00	1.00
USD / GBP FX Rate	27.87	29.59	29.38	29.26	1.00*	1.00	1.00	1.00	1.00	1.00	1.00	1.10*
5-Year yield	0.33	0.71	1.44	1.97	1.01**	1.01***	1.01	0.99	1.00	1.00	0.99	1.01
10-Year yield	0.29	0.62	1.32	1.77	1.01*	1.01**	1.01	0.99	1.00	1.00	0.99	1.00
Baa Spread	0.52	1.11	1.77	1.89	1.00	1.00	1.01	0.99	1.00	1.01**	1.00	0.99

Note: Comparison of "SVO" (baseline, in denominator of relative comparisons) against "SVO-t" and "SV-common outlier." Values below 1 indicate improvement over baseline. Evaluation window from 1975:M01 through 2017:M12. Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h+1$ lags. Due to the close behavior of some of the models compared, and rounding of the report values, one of the comparisons shows a significant relative RMSE (SVO vs SVobar) of 1.00. This case arises from persistent differences in performance that are, however, too small to be relevant after rounding.

Table S.13: Avg CRPS (individual vs common outlier models)

					Relative to SVO ...							
	SVO				SVO-t				SV-common outlier			
Variable / Horizons	1	3	12	24	1	3	12	24	1	3	12	24
Real Income	3.06	3.10	3.49	4.12	1.00	0.99**	0.95***	0.91***	1.01	1.03**	1.03***	1.11***
Real Consumption	3.02	3.14	3.54	4.35	1.00	0.99**	0.95***	0.91***	1.01	0.99**	1.00	1.05***
IP	3.93	4.28	4.98	6.25	1.00	0.98***	0.94***	0.89***	0.99	1.00	1.00	1.05***
Capacity Utilization	0.29	0.57	1.67	2.90	1.00	0.99	0.97**	0.93***	0.98**	0.98**	0.98***	1.00
Unemployment Rate	0.09	0.15	0.43	0.78	1.00	1.00	0.98**	0.95***	1.00	1.00	0.99**	0.99***
Nonfarm Payrolls	0.85	0.96	1.35	1.90	0.99**	0.99	0.94***	0.90***	0.99	1.00	0.99**	1.03***
Hours	0.12	0.15	0.25	0.33	0.99***	0.98***	0.96***	0.91***	0.99	1.00	1.01	1.05***
Hourly Earnings	1.61	1.66	1.98	2.66	1.00	0.98***	0.95***	0.91***	1.00	1.00	0.99**	1.04***
PPI (Fin. Goods)	3.46	3.61	4.11	5.00	1.00	0.99***	0.97***	0.94***	1.00	1.00	1.00	1.03***
PCE Prices	1.13	1.33	1.61	2.19	1.01*	1.00	0.98***	0.95***	1.00	1.00*	0.99*	1.01**
Housing Starts	0.04	0.06	0.13	0.21	1.00	0.99*	0.99**	0.97**	1.00	0.99***	0.99***	0.99
S&P 500	23.29	24.15	26.51	31.24	1.00	0.99***	0.95***	0.91***	1.00	1.00	1.00	1.05***
USD / GBP FX Rate	15.42	16.32	17.05	18.50	0.99*	1.00	0.99***	0.96***	1.00	1.01**	1.03***	1.11***
5-Year yield	0.17	0.37	0.76	1.08	1.00	1.01**	1.00	0.97***	1.00	1.00	0.99**	0.99*
10-Year yield	0.15	0.33	0.71	1.00	1.00	1.01	1.00	0.97***	1.00	1.00	0.99***	0.98***
Baa Spread	0.20	0.46	0.97	1.35	1.00	1.00	0.98**	0.93***	1.00	1.00	0.99	0.99**

Note: Comparison of “SVO” (baseline, in denominator of relative comparisons) against “SVO-t” and “SV-common outlier.” Values below 1 indicate improvement over baseline. Evaluation window from 1975:M01 through 2017:M12. Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Due to the close behavior of some of the models compared, and rounding of the report values, one of the comparisons shows a significant relative CRPS (SVO vs SVobar) of 1.00. This case arises from persistent differences in performance that are, however, too small to be relevant after rounding.

X COVID-19 dummies

As another simple approach to conditioning on knowledge of when and which outliers occurred in the data, particularly the timing of the COVID-19 pandemic, we consider an otherwise standard BVAR-SV model with separate dummy variables added to represent each month of the sample since COVID's outbreak in March 2020. Wide priors are assigned to each dummy coefficient. Denote the dummy coefficient for each month $t \geq$ 2020:03 by δ_t. The prior for each δ_t is a mean-zero normal distribution, with a large variance set equal to $1/\varepsilon$, where ε is a small number chosen as a function of machine precision (identical to the output of the `eps` function in MATLAB). For $t \geq$ 2020:03, only the sum of δ_t and the residual v_t are identified. Predictive densities for selected forecast origins in 2020-21 are provided in Figures S.63–S.66 above. By soaking up all information contained in data since the onset of the pandemic, the dummy approach generates point forecasts comparable to our outlier-augmented SV models. But because the dummy approach is conditioned on ex-ante knowledge that all COVID-19 related data points are highly unusual, its forecast densities are much tighter than those derived from our more agnostic outlier-augmented SV models or the SV-OutMiss specification.

XI Models with AR(1) SV processes

Our SV specifications treat log volatility as a random walk, following studies such as Cogley and Sargent (2005), Stock and Watson (2007), Justiniano and Primiceri (2008), and Clark (2011). We have also considered models in which SV is a persistent AR(1) process rather than a random walk.[12] These model variants differ from our baseline models only by having the following specification of the SV process for the (orthogonalized) VAR residual of the jth variable:

$$\log \lambda_t^j = (1-\rho^j)\log \bar{\lambda}^j + \rho^j \lambda_{t-1}^j + e_t^j \tag{S.17}$$

As before, the vector of all SV shocks has a correlated multi-variate normal distrubtion, $e_t \sim N(0, \Phi)$.

Table S.14 compares log Bayes factors of outlier-augmented SV-AR(1) models against the SV-AR(1) model, and the following tables compare RMSE and CRPS for point and density forecasts of individual variables from different models over different sub-samples. These results are qualitatively similar to the random walk-based results presented above. For example, the SVO-t specification forecasts at least as well as SV in historical data and in the COVID period, and the predictive Bayes factors show overall similar patterns as reported for our random walk-based specifications.

[12] Clark and Ravazzolo (2015) find that random walk and AR(1) specifications yield relatively similar forecast performance in post-war US data.

Table S.14: Log Bayes Factors of outlier-agumented SV-AR(1) models

	Models			
Samples	**SVO-t-AR(1)**	**SVO-AR(1)**	**SV-t-AR(1)**	**SV-AR(1)-OutMiss**
Full sample				
1975:01–2021:02	262.66	365.15	255.40	-472.21
G Inflation				
1975–1984	-8.72	6.03	-12.79	-1.17
G Moderation				
1985–2007	-56.86	-17.99	-52.66	11.43
GFC				
2008–2014	-9.31	8.37	-11.63	-40.31
COVID-19				
2020:03–2021:02	332.14	364.88	334.22	-444.51

Note: Differences in cumulative log Bayes factors, $\log L(\mathcal{M}_i) - \log L(\mathcal{M}_0)$, where $\log L(\mathcal{M}_i) = \sum_{t=T_0}^{T_1} \log p(y_{t+1}|y^t, \mathcal{M}_i)$ between the different models listed above ($\mathcal{M}_i$) and the SV-AR(1) model ($\mathcal{M}_0$), measured over different subsamples of forecast origins, t. Unless stated otherwise, samples extend from January to December of the years given.

Table S.15: Relative RMSE since 1975 (SV-AR(1))

	SV-AR(1)-O				SV-AR(1)-t				SV-AR(1)-Ot			
Variable / Horizon	1	3	12	24	1	3	12	24	1	3	12	24
Real Income	1.00^{*}	1.00	1.00	1.00	1.00	1.00^{*}	1.00	1.00	1.01^{*}	1.00	1.00	1.00
Real Consumption	1.00	1.00	1.00	1.00	1.01^{*}	1.00	1.00	1.00	1.01^{*}	1.00	1.00	1.00
IP	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.00	1.01	0.99
Capacity Utilization	1.00	1.01	1.01	0.98	1.00	1.01	1.00	0.98	1.01	1.01	1.00	0.97^{*}
Unemployment Rate	1.00^{*}	1.00	1.01	1.00	1.00	1.00	1.00	0.99	1.00	1.00	0.99	0.99
Nonfarm Payrolls	1.01	1.01^{*}	1.01	0.99	1.00	1.02^{**}	1.00	1.01	1.00	1.02^{**}	1.01	1.00
Hours	1.00	1.00	1.01	0.98^{*}	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Hourly Earnings	1.00	1.00^{*}	1.01^{*}	1.00	1.01^{*}	1.00	1.02^{***}	1.03^{***}	1.01	1.00	1.02^{***}	1.03^{***}
PPI (Fin. Goods)	1.00	1.00	1.00^{*}	1.00	1.00	1.00	1.01^{*}	1.01^{**}	1.00	1.00	1.00	1.01^{*}
PCE Prices	0.99^{***}	1.00	1.00	1.01^{*}	1.00	1.01	1.02^{**}	1.03^{**}	1.00	1.01	1.02^{**}	1.03^{***}
Housing Starts	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.01	1.00	1.00	0.99	1.00
S&P 500	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00^{*}
USD / GBP FX Rate	1.00	1.00	1.00	0.93	1.00	1.00	1.00	0.92	1.00	0.99^{*}	1.00	0.93
5-Year Yield	1.00	1.01	1.01	1.01	1.00	1.00	1.01	1.01	1.00	1.00	1.00	1.01
10-Year Yield	1.00	1.01	1.00	1.01	1.00	1.00	1.01	1.01	1.00	1.00	1.00	1.01
Baa Spread	0.99^{*}	0.99^{*}	1.00	1.00	0.99^{**}	0.99	1.01	1.00	0.99	0.99	1.01	1.00

Note: Comparison of "SV-AR(1)" (baseline, in denominator) against "SV-AR(1)-O," "SV-AR(1)-t," and "SV-AR(1)-Ot." Values below 1 indicate improvement over baseline. Evaluation window from 1975:M01 through 2017:M12. Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Due to the close behavior of some of the models compared, and rounding of the report values, a few comparisons show significant relative RMSE of 1.00. These cases arise from persistent differences in performance that are, however, too small to be relevant after rounding.

Table S.16: Relative Avg CRPS since 1975 (SV-AR(1))

	SV-AR(1)-O				SV-AR(1)-t				SV-AR(1)-Ot			
Variable / Horizon	1	3	12	24	1	3	12	24	1	3	12	24
Real Income	1.00	1.00***	1.01***	1.02***	1.01*	1.00	1.00	1.00	1.01*	1.00	1.00	1.00
Real Consumption	1.00	1.01***	1.03***	1.03***	1.00	1.00	0.99***	0.99***	1.00	1.00	1.00	1.00
IP	1.01	1.01	1.02***	1.03***	1.00	1.00	1.00	0.99***	1.01*	1.00	1.01*	1.00
Capacity Utilization	1.01	1.01**	1.03***	1.02*	1.00	1.01	1.00	0.98	1.01	1.01*	1.01	0.99
Unemployment Rate	1.00	1.01**	1.02**	1.02**	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00
Nonfarm Payrolls	1.01**	1.02***	1.04***	1.04***	0.99	1.01	1.00	1.00	1.00	1.01	1.02***	1.02***
Hours	1.00	1.01***	1.02***	1.03***	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.01
Hourly Earnings	1.01**	1.01***	1.03***	1.04***	1.01	0.99**	1.01*	1.02***	1.01	1.00	1.02***	1.03***
PPI (Fin. Goods)	1.00	1.00*	1.01***	1.02***	1.01***	1.01	1.01	1.01	1.01***	1.01**	1.01	1.01***
PCE Prices	0.99**	1.00	1.02***	1.03***	1.00	1.01	1.02**	1.03***	1.01*	1.01**	1.02***	1.04***
Housing Starts	1.00*	1.01***	1.01	1.01	1.00	1.00	1.00	1.01*	1.00	1.01**	1.01	1.01
S&P 500	1.00	1.00	1.02***	1.02***	1.00	1.00	1.00	0.99*	1.00	1.00	1.00	1.00*
USD / GBP FX Rate	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99***	1.00	1.00	0.99**	0.99***
5-Year Yield	1.01	1.01	1.01	1.02***	1.00	1.00	1.01**	1.01*	1.00	1.00	1.01	1.01
10-Year Yield	1.00	1.01	1.01	1.03***	1.00	1.00	1.01*	1.01**	1.00	1.00	1.00	1.02**
Baa Spread	1.00	0.99	1.01	1.04***	1.00	1.00	1.00	1.00	0.99**	1.00	1.00	1.01

Note: Comparison of “SV-AR(1)” (baseline, in denominator) against “SV-AR(1)-O,” “SV-AR(1)-t,” and “SV-AR(1)-Ot.” Values below 1 indicate improvement over baseline. Evaluation window from 1975:M01 through 2017:M12. Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Due to the close behavior of some of the models compared, and rounding of the report values, a few comparisons show significant relative CRPS of 1.00. These cases arise from persistent differences in performance that are, however, too small to be relevant after rounding.

Table S.17: Relative RMSE around the GFC (SV-AR(1))

	SV-AR(1)-O				SV-AR(1)-t				SV-AR(1)-Ot			
Variable / Horizon	1	3	12	24	1	3	12	24	1	3	12	24
Real Income	1.00	1.00	1.00	1.00*	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Real Consumption	1.00	1.00	1.00	1.01	1.01	1.01	0.99	1.00	1.01	1.00	0.97*	1.00
IP	1.00	1.00	1.00	1.00**	0.99*	1.00	1.00	0.98*	0.98*	1.00	1.00	0.99
Capacity Utilization	1.01	1.00	1.00	1.00	0.99	0.98	0.98	0.95**	0.99	0.99	1.00	0.95**
Unemployment Rate	1.00	0.99	1.00	1.00	0.99	0.97*	0.97	0.97	1.00	0.98*	0.97	0.97
Nonfarm Payrolls	1.00	1.00	1.00	1.01	0.99	1.01	0.98	0.97*	0.99	1.01	0.98	0.97
Hours	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.01*	1.00	0.99	1.00	1.02
Hourly Earnings	1.01*	1.00	1.00	1.01*	1.02*	1.01	1.03	1.04***	1.01	1.01	1.02	1.02***
PPI (Fin. Goods)	1.00	1.00**	1.00*	1.00	1.01	1.01*	1.01*	1.02	1.01*	1.01	1.01	1.01
PCE Prices	0.99	1.00	1.01**	1.01	1.00	1.01	1.02**	1.04*	1.01	1.00	1.01	1.04*
Housing Starts	1.00	1.00	1.00	1.00	0.99	0.99	0.99	1.00	0.99	0.98	0.98*	0.99
S&P 500	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.01	1.01*	1.01	1.00	1.01***
USD / GBP FX Rate	1.01	1.01*	1.00	1.00	1.00	1.01	1.01	1.01***	1.00	1.00	1.01	1.01***
5-Year Yield	0.99	1.00	1.00	1.01	0.99	1.00	1.02*	1.00	0.99	0.99	1.00	0.99
10-Year Yield	0.99	1.00	0.99	0.98	0.99	1.00	1.01	1.00	0.99	0.99	1.00	0.98*
Baa Spread	1.00	1.00	0.99	0.98	1.00	1.00	1.02	0.99	1.00	1.00	1.01	0.99

Note: Comparison of "SV-AR(1)" (baseline, in denominator) against "SV-AR(1)-O," "SV-AR(1)-t," and "SV-AR(1)-Ot." Values below 1 indicate improvement over baseline. Evaluation window from 2007:M01 through 2014:M12. Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Due to the close behavior of some of the models compared, and rounding of the report values, a few comparisons show significant relative RMSE of 1.00. These cases arise from persistent differences in performance that are, however, too small to be relevant after rounding.

Table S.18: Relative Avg CRPS around the GFC (SV-AR(1))

Variable / Horizon	SV-AR(1)-O 1	3	12	24	SV-AR(1)-t 1	3	12	24	SV-AR(1)-Ot 1	3	12	24
Real Income	1.00	1.00	1.00	1.01	1.02	1.01	1.00	1.00	1.02	1.01	1.00	1.00
Real Consumption	1.00	1.01*	1.03***	1.03***	1.00	0.99	0.98***	0.98***	1.00	0.99	0.98***	0.99**
IP	1.00	1.01	1.01	1.01**	0.99	1.00	1.00	0.98*	0.99	1.00	1.00	0.99
Capacity Utilization	1.00	1.01	1.01	1.00	0.99	0.99	0.99	0.94*	1.00	1.00	1.00	0.95
Unemployment Rate	1.00	0.99	1.00	0.99	1.00	0.98*	0.97	0.96	1.00	0.98	0.97	0.96
Nonfarm Payrolls	1.00	1.01**	1.01	1.02	1.00	1.01	0.99	0.98	1.00	1.02	0.99	0.99
Hours	1.00	1.00	1.00	1.00	1.00	0.99	1.01	1.01	1.01	0.99	1.01	1.02
Hourly Earnings	1.01***	1.01*	1.02***	1.03***	1.00	1.00	1.01	1.02*	0.99	1.00	1.01	1.02***
PPI (Fin. Goods)	1.00	1.00	1.00	1.00	1.03***	1.02***	1.02**	1.02	1.03***	1.02***	1.01	1.01
PCE Prices	0.99	1.00	1.01	1.01	1.01	1.01**	1.02**	1.04**	1.02*	1.01	1.01	1.03***
Housing Starts	1.00	1.00	0.99	0.98	1.00	0.99	0.99*	1.00	0.99	0.99	0.98*	0.99
S&P 500	1.00	1.00	1.00	1.01**	1.01	1.00	1.00	1.00	1.02	1.00	1.00	1.01
USD / GBP FX Rate	1.01	1.00	1.00	1.00	1.01	1.01	1.00	1.00	1.00	1.01	1.00	1.00
5-Year Yield	0.99	1.01	0.99	1.00	0.99	1.00	1.02**	1.00	0.99	0.99	1.00	0.99
10-Year Yield	0.99	1.01	1.00	1.00	0.99	0.99	1.02	1.01	1.00	0.99	1.00	1.00
Baa Spread	1.00	0.99	0.99	1.02	1.00	1.00	1.01	0.99	0.99	1.00	1.00	1.01

Note: Comparison of "SV-AR(1)" (baseline, in denominator) against "SV-AR(1)-O," "SV-AR(1)-t," and "SV-AR(1)-Ot." Values below 1 indicate improvement over baseline. Evaluation window from 2007:M01 through 2014:M12. Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h+1$ lags.

XII VARs in levels

The model specifications used in our paper (and elsewhere in this document) use (log) growth rates of trending variables (income, consumption, employment, etc.). Some work in the forecasting literature instead uses log levels in VARs, including examples such as Bańbura, Giannone, and Reichlin (2010) and Lenza and Primiceri (2020). In a factor model setting, Antolín-Díaz, Drechsel, and Petrella (2021) model outliers as occurring in levels and not growth rates. The list of variables is identical to what has been used in our baseline, but the specification differs in the data transformations, and the Minnesota prior for the first own-lag coefficients, applied to each variable as listed in Table S.19. We omit the SV-OutMiss model from the comparison, since the ex-ante criterion based on a data point's distance from the time series median is ill-suited for trending variables.

Table S.19: List of variables for VARs in levels

Variable	FRED-MD code	transformation	Minnesota prior
Real Income	RPI	$\log(x_t)$	1
Real Consumption	DPCERA3M086SBEA	$\log(x_t)$	1
IP	INDPRO	$\log(x_t)$	1
Capacity Utilization	CUMFNS		1
Unemployment Rate	UNRATE		1
Nonfarm Payrolls	PAYEMS	$\log(x_t)$	1
Hours	CES0600000007		1
Hourly Earnings	CES0600000008	$\log(x_t)$	1
PPI (Fin. Goods)	WPSFD49207	$\log(x_t)$	1
PCE Prices	PCEPI	$\log(x_t)$	1
Housing Starts	HOUST	$\log(x_t)$	1
S&P 500	SP500	$\log(x_t)$	1
USD / GBP FX Rate	EXUSUKx	$\log(x_t)$	1
5-Year Yield	GS5		1
10-Year Yield	GS10		1
Baa Spread	BAAFFM		1

Note: Data obtained from the 2021-04 vintage of FRED-MD. Monthly observations from 1959:M01 to 2021:M03. Entries in the column "Minnesota prior" report the prior mean on the first own-lag coefficient of the corresponding variable in each BVAR. Prior means on all other VAR coefficients are set to zero.

Table S.20: Log Bayes Factors of VARs in levels with outlier-agumented SV

	Models			
Samples	**SVO-t**	**SVO**	**SV-t**	**CONST**
Full sample				
1975:01–2021:02	368.45	408.65	298.74	-9111.56
G Inflation				
1975–1984	12.68	2.93	8.02	-393.42
G Moderation				
1985–2007	-9.47	10.98	5.47	-401.98
GFC				
2008–2014	4.46	-2.47	-15.86	-254.10
COVID-19				
2020:03–2021:02	339.43	370.96	295.67	-7857.34

Note: Differences in cumulative log Bayes factors, $\log L(\mathcal{M}_i) - \log L(\mathcal{M}_0)$, where $\log L(\mathcal{M}_i) = \sum_{t=T_0}^{T_1} \log p(y_{t+1}|y^t, \mathcal{M}_i)$ between the different models listed above ($\mathcal{M}_i$) and the SV model ($\mathcal{M}_0$), measured over different subsamples of forecast origins, t. Unless stated otherwise, samples extend from January to December of the years given. Data in (log-)levels as listed in Table S.19.

Table S.20 compares log Bayes factors of outlier-augmented SV models against the SV model (for VARs in levels), and the following tables compare RMSE and CRPS for point and density forecasts of individual variables from different models over different sub-samples. These results are qualitatively similar to the results for VARs in growth rates presented above. For example, the SVO-t specification forecasts at least as well as SV in historical data and in the COVID period, and the predictive Bayes factors show very similar patterns as reported for VARs in growth rates above. Moreover, the figures in this appendix depict similar decompositions of time-varying forecast error volatilities into components due to transient outliers and persistent SV as what is shown in Appendix V for models with variables in growth rates.

Table S.21: Relative RMSE since 1975 (VARs in levels)

	SVO				SV-t				SVO-t			
Variable / Horizon	1	3	12	24	1	3	12	24	1	3	12	24
Real Income	1.00	1.01	1.00	1.00	1.00	1.00	0.99	0.99	1.00	1.00	1.00	1.00
Real Consumption	1.00	1.00	0.99	1.01	1.00	0.99	0.97	0.99	1.00	0.99	0.98	1.01
IP	1.00	1.00	1.00	1.01	0.99	0.99	0.99	0.98	1.00	0.98	0.99	0.99
Capacity Utilization	0.99	1.00	1.00	1.01	1.00	1.00	0.98	0.97*	1.00	0.99	0.98	0.98
Unemployment Rate	1.00*	1.00	1.00	1.00	1.00	0.99	0.99	0.98	1.00	1.00	0.99	0.98
Nonfarm Payrolls	0.99	1.00	1.00	1.00	1.00	0.99	0.98	0.97*	1.00	1.00	0.99	0.98
Hours	1.00	1.00	0.98	0.99	1.00	1.00	0.97	0.96	1.00	1.00	0.97	0.98
Hourly Earnings	0.99	0.98*	0.99	1.00	1.00	0.99	1.00	1.02	0.99	0.98	1.00	1.02
PPI (Fin. Goods)	1.00	1.00	1.01	1.02	1.00	1.00	1.01	1.03	1.00	1.01	1.01	1.04
PCE Prices	1.00	1.00	1.02**	1.05	1.00	1.00	1.02*	1.05**	1.00	1.00	1.02*	1.06*
Housing Starts	1.00	1.00	0.99	1.00	1.00	1.00	0.99	1.00	1.00	1.00	0.99	1.01
S&P 500	1.00	1.00	1.00	1.01	0.99	0.99	0.98	1.01	0.99	0.99	0.99	1.02
USD / GBP FX Rate	1.00	0.99	1.00	1.03	1.00	1.00	1.01	1.01	1.00	0.99	1.00	1.04
5-Year Yield	1.01	1.00	1.01	1.02***	1.00	1.01	1.01	0.99	1.00	1.00	1.01*	1.01
10-Year Yield	1.01	1.00	1.00	1.01**	1.00	1.00	1.01	1.00	1.00	1.00	1.01	1.01*
Baa Spread	0.99**	0.99	1.01	1.05	0.99	1.00	1.02*	1.01	0.99	1.00	1.02**	1.03

Note: Comparison of “SV” (baseline, in denominator) against “SVO,” “SV-t,” and “SVO-t.” Values below 1 indicate improvement over baseline. Evaluation window from 1975:M01 through 2017:M12. Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Due to the close behavior of some of the models compared, and rounding of the report values, one of the comparisons shows a significant relative RMSE of 1.00. This case arises from persistent differences in performance that are, however, too small to be relevant after rounding.

Table S.22: Relative Avg CRPS since 1975 (VARs in levels)

	SVO				SV-t				SVO-t			
Variable / Horizon	1	3	12	24	1	3	12	24	1	3	12	24
Real Income	1.00	1.00	1.00	1.00	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Real Consumption	1.00	1.00	1.00	1.01	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.01
IP	1.00	1.00	1.00	1.02*	1.00	0.99	0.99	0.98	1.00	0.99	0.99	1.00
Capacity Utilization	1.00	1.00	1.01	1.02**	1.00	1.00	0.98*	0.97**	1.00	0.99	0.98	0.98
Unemployment Rate	1.00	1.00	1.01	1.01	1.00	1.00	0.99	0.98	1.00	1.00	0.99	0.99
Nonfarm Payrolls	1.00	1.01	1.01	1.01	0.99**	0.99	0.99	0.98	1.00	1.00	0.99	0.99
Hours	1.00	1.00	0.99	0.99	0.99	0.99	0.96***	0.93***	0.99	0.99	0.97**	0.94***
Hourly Earnings	1.00	0.99	0.99	1.01	1.00	1.00	1.00	1.01	1.00	1.00	1.00	1.02
PPI (Fin. Goods)	1.00	1.00	1.01	1.02	1.00	1.00	1.01	1.01	1.00	1.00	1.01	1.02
PCE Prices	1.01**	1.00	1.02***	1.05***	1.00	1.00	1.02	1.03**	1.00	1.00	1.01	1.04*
Housing Starts	1.00	1.00	1.00	1.01	1.01	1.00	0.99	0.99	1.00	1.00	1.00	1.00
S&P 500	1.00	1.00	1.00	1.01*	0.99*	0.99	0.98*	1.00	0.99*	0.99	0.99	1.01
USD / GBP FX Rate	1.00	0.99	1.00	1.01	1.00	1.00	1.00	0.99	1.00	0.98**	0.99	1.00
5-Year Yield	1.00	1.00	1.01	1.03***	1.00	1.00	1.01	1.01	1.00	1.00	1.01*	1.02**
10-Year Yield	1.00	1.00	1.01	1.03***	1.00	1.00	1.01	1.01	1.00	1.00	1.01	1.02***
Baa Spread	0.99**	0.99	1.01	1.05***	0.99*	1.00	1.01	0.98	0.99**	1.00	1.02*	1.01

Note: Comparison of "SV" (baseline, in denominator) against "SVO," "SV-t," and "SVO-t." Values below 1 indicate improvement over baseline. Evaluation window from 1975:M01 through 2017:M12. Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags.

Table S.23: Relative RMSE around the GFC (VARs in levels)

	SVO				SV-t				SVO-t			
Variable / Horizon	1	3	12	24	1	3	12	24	1	3	12	24
Real Income	1.01	1.01	1.02	1.02	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.02
Real Consumption	0.99	0.96**	0.97	0.98	0.99	0.97	0.98	1.00	0.98	0.97	0.99	1.00
IP	1.00	0.99	0.99	1.01	1.00	0.99	1.02	1.01	0.98**	0.96	1.02	1.02**
Capacity Utilization	0.99	0.99	0.99	1.02*	1.00	0.99	1.01	0.97	0.98*	0.97	1.00	0.98
Unemployment Rate	1.00	1.00	1.00	1.01	1.00	0.99	0.99	0.98	1.00	0.99	0.99	0.99
Nonfarm Payrolls	1.00	1.00	0.99	1.00	0.99	1.00	1.00	0.98	0.98	0.98	0.99	0.98
Hours	1.02*	1.01	0.97*	1.00	1.02	0.99	0.96*	0.99	1.01	0.99	0.96*	0.99
Hourly Earnings	1.01	1.02	1.01	1.00	0.99	1.00	1.03	1.02	0.99	1.01	1.03	1.02
PPI (Fin. Goods)	1.00	1.00	1.01	1.00	1.00	1.01	1.04*	1.09*	1.00	1.01	1.02	1.06
PCE Prices	1.00	1.01	1.01	1.02	1.01	1.01**	1.04*	1.10**	1.00	1.00	1.02*	1.08***
Housing Starts	0.99	0.98	0.99	1.00	0.98	0.98	0.99	1.00	0.98	0.98	1.00	1.00
S&P 500	0.99	1.00	1.02	1.05*	0.99	0.99	1.01	1.06	0.99	0.99	1.02	1.07
USD / GBP FX Rate	1.00	0.98	1.01	1.01	1.00	1.01	1.04***	1.01	1.00	0.98**	1.02	1.01
5-Year Yield	1.00	0.99	1.02	1.06*	1.00	1.01	1.02	0.98	1.00	1.02	1.02	1.00
10-Year Yield	1.01	0.99	1.02	1.06	1.00	1.01	1.04	1.03	1.00	1.02	1.02	1.04
Baa Spread	0.94**	0.93*	0.96	1.00	0.98*	1.00	1.04	1.12	0.97*	0.98	1.02	1.09*

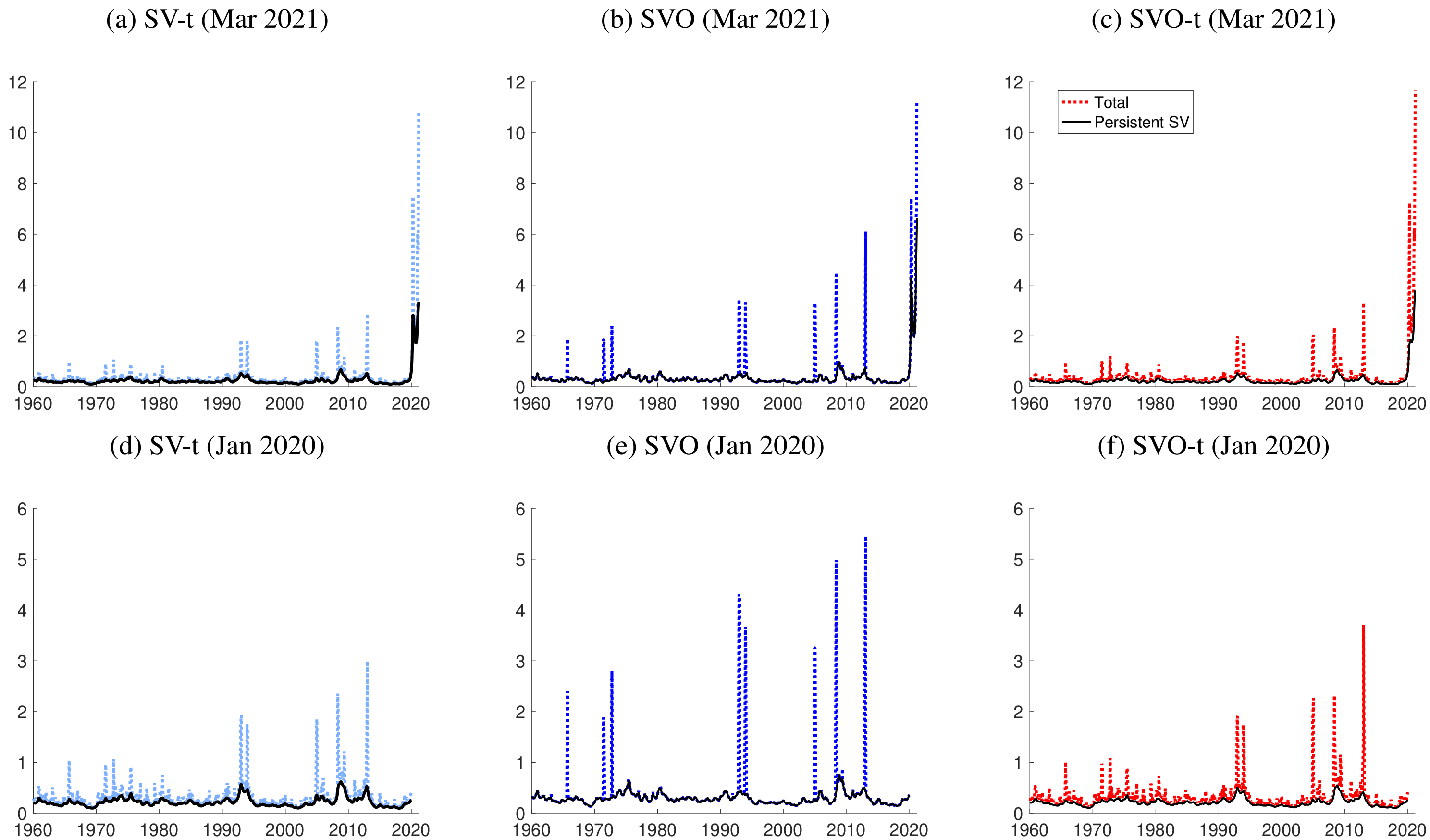
Note: Comparison of "SV" (baseline, in denominator) against "SVO," "SV-t," and "SVO-t." Values below 1 indicate improvement over baseline. Evaluation window from 2007:M01 through 2014:M12. Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags.

Table S.24: Relative Avg CRPS around the GFC (VARs in levels)

	SVO				SV-t				SVO-t			
Variable / Horizon	1	3	12	24	1	3	12	24	1	3	12	24
Real Income	1.00	1.00	0.99	0.99	0.98	0.99	0.97	0.96	0.98	0.99	0.97	0.97
Real Consumption	0.99	0.98*	0.98	0.98	1.00	0.99	0.99	1.00	0.99	0.99	0.99	1.00
IP	1.00	0.99	1.00	1.03*	1.01	1.01	1.02	0.99	0.98	0.99	1.02	1.00
Capacity Utilization	0.99**	1.00	1.01	1.02**	1.00	1.02	1.01	0.97	0.98*	1.00	1.01	0.98
Unemployment Rate	1.00	1.01	1.00	1.00	1.00	1.00	0.99	0.99	1.00	1.00	0.99	0.99
Nonfarm Payrolls	1.00	1.01	1.01	1.00	0.99	1.02	1.02	0.98	0.98*	1.00	1.01	0.98
Hours	1.02	1.02	0.98	0.99	1.01	1.00	0.96***	0.93***	1.00	0.99	0.96***	0.93***
Hourly Earnings	1.01	1.02**	1.01	1.00	1.00	1.01	1.03	1.01	1.00	1.02	1.03	1.01
PPI (Fin. Goods)	1.00	1.00	1.01	1.02	1.01	1.01	1.04*	1.07*	1.01	1.01	1.03*	1.06*
PCE Prices	1.01	1.01	1.02*	1.04***	1.01*	1.02**	1.04*	1.08**	1.01	1.01	1.03**	1.07***
Housing Starts	0.99	0.99	0.98	0.99	1.00	0.99	1.00	1.01	0.99	0.99	1.00	1.01
S&P 500	0.99	1.00	1.01	1.03	0.98	0.98	1.00	1.03	0.99	0.98	1.01	1.04
USD / GBP FX Rate	0.99	0.98	1.01	1.02	1.00	1.01	1.03**	1.00	0.99	0.98	1.01	1.00
5-Year Yield	1.00	0.99	1.00	1.03**	0.99	1.01	1.01	1.01	0.99	1.02	1.01	1.01
10-Year Yield	1.00	1.00	1.01	1.04***	1.00	1.00	1.01	1.03	1.00	1.02	1.01	1.03
Baa Spread	0.96*	0.96	0.97	1.02	0.99	1.00	1.04	1.03	0.99	1.00	1.02	1.03***

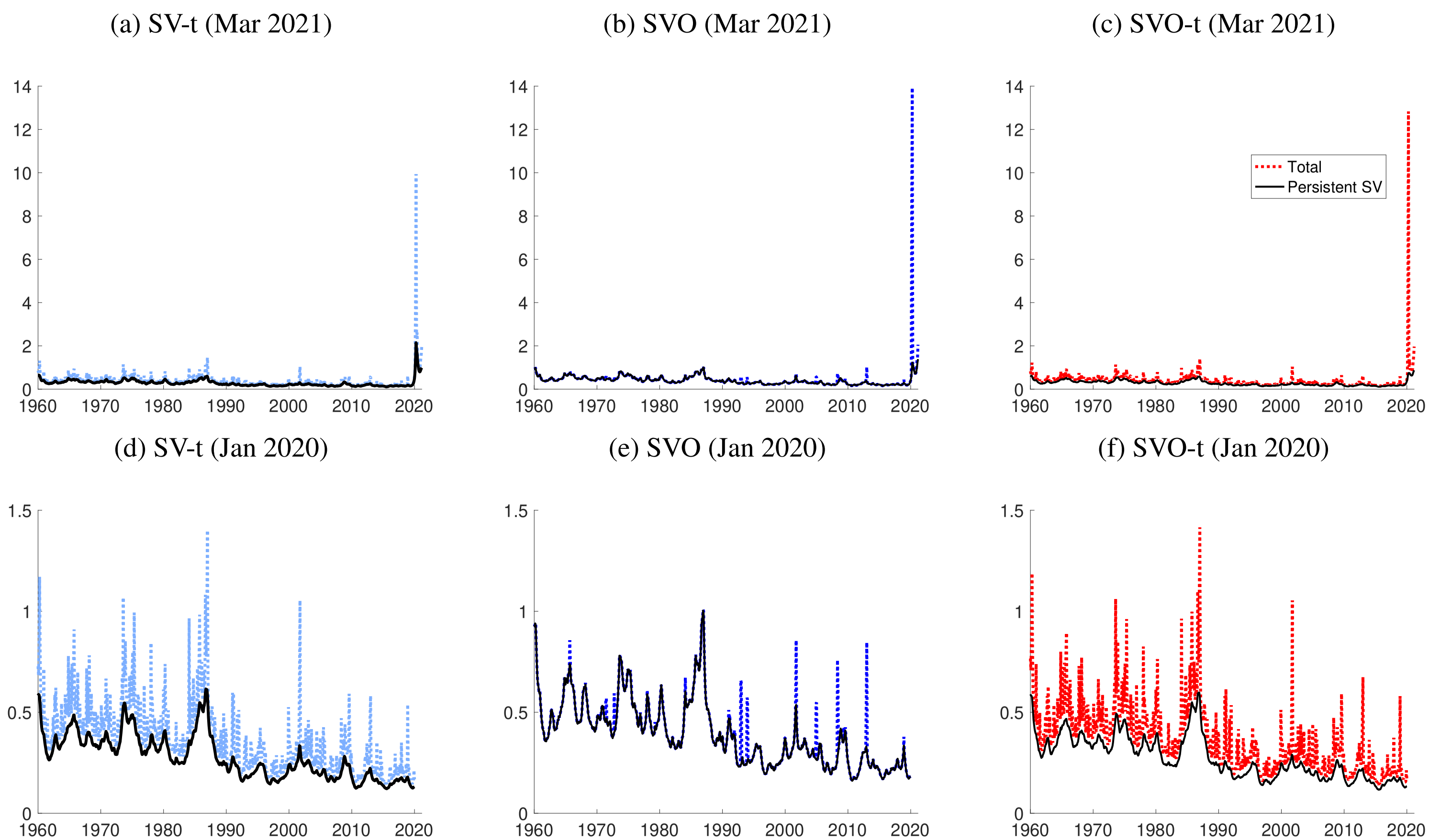
Note: Comparison of "SV" (baseline, in denominator) against "SVO," "SV-t," and "SVO-t." Values below 1 indicate improvement over baseline. Evaluation window from 2007:M01 through 2014:M12. Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags.

Figure S.71: Posteriors of outlier states for Real Income (VARs in levels)



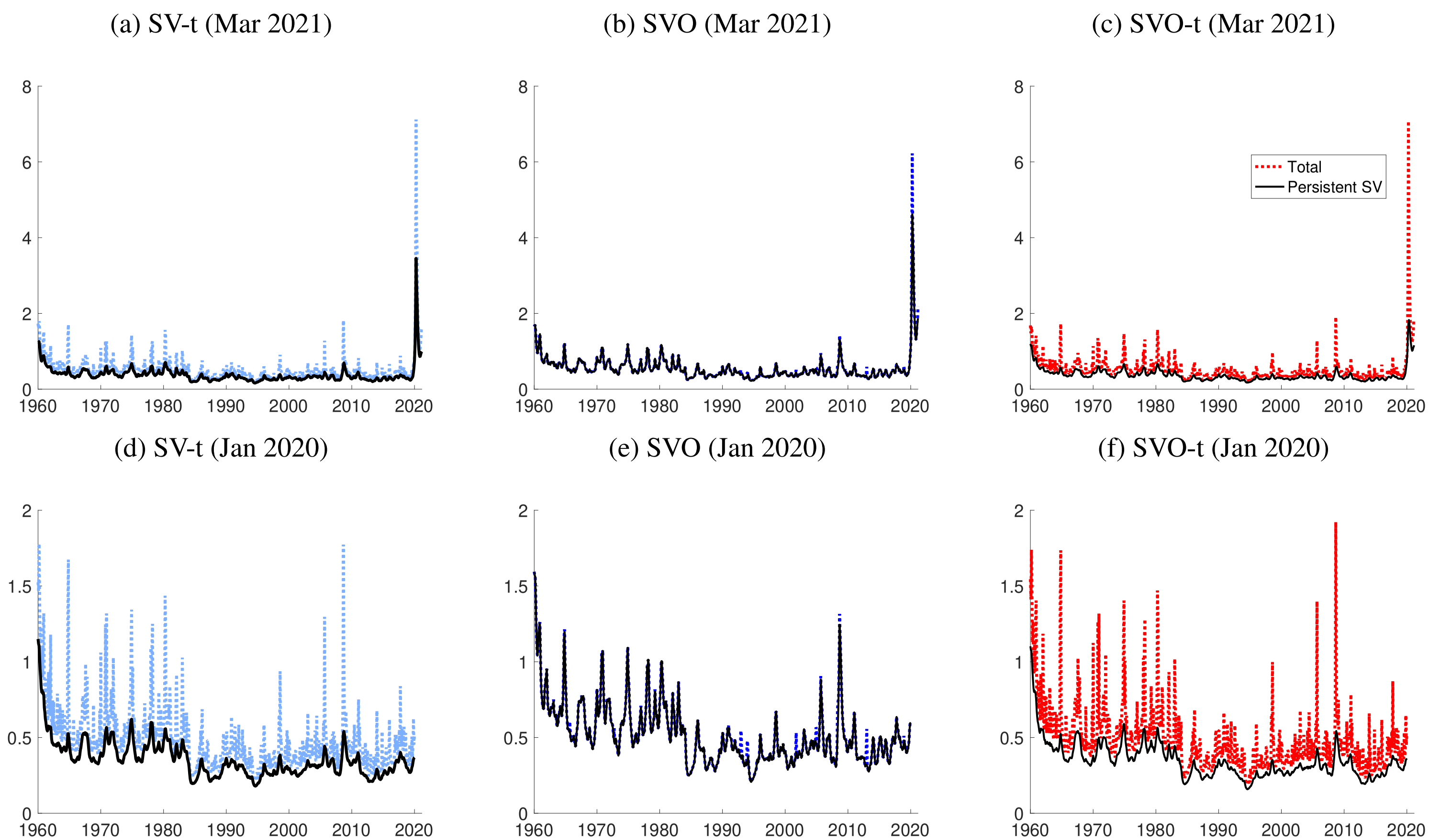
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.72: Posteriors of outlier states for Real Consumption (VARs in levels)



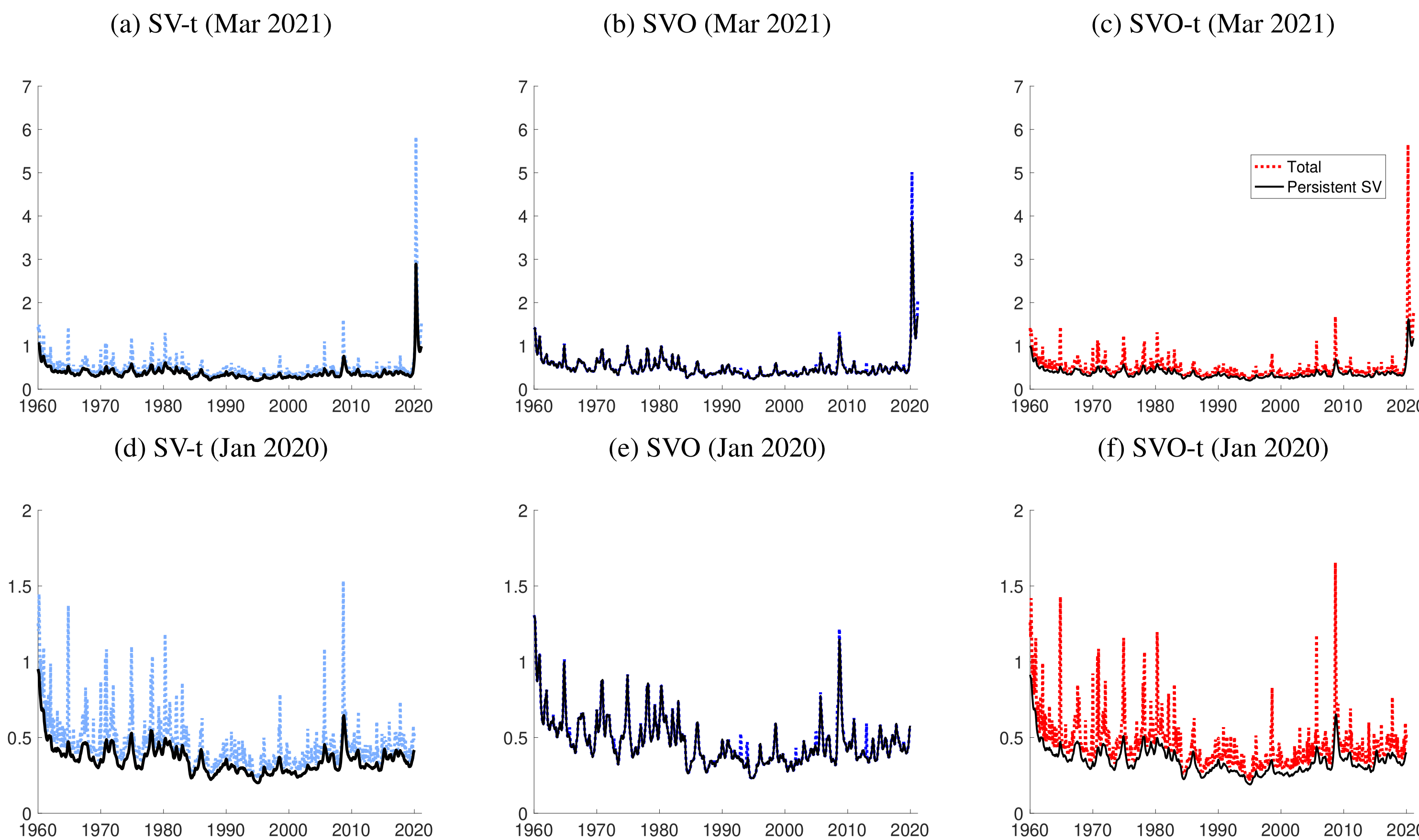
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.73: Posteriors of outlier states for IP (VARs in levels)



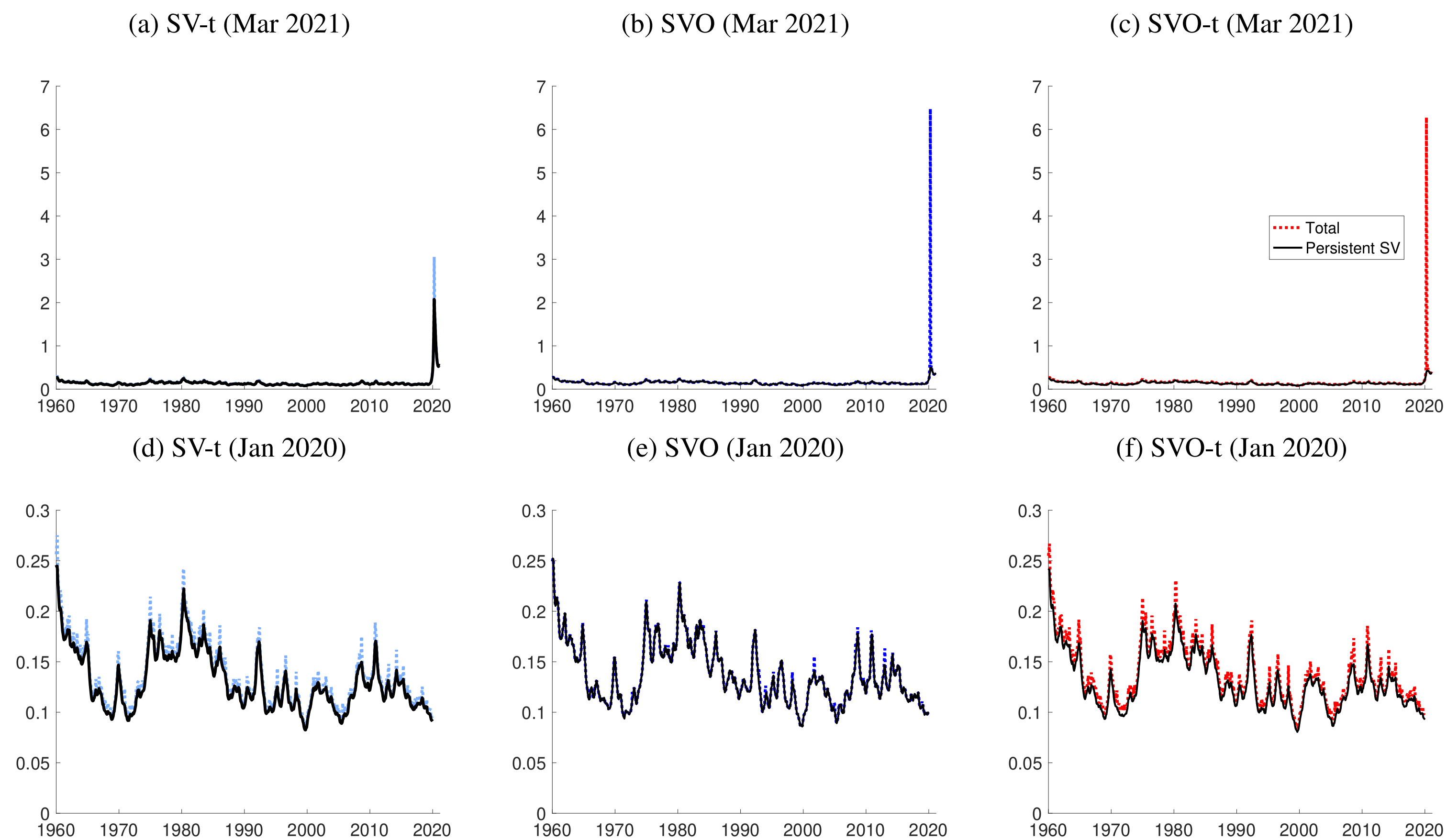
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.74: Posteriors of outlier states for Capacity Utilization (VARs in levels)



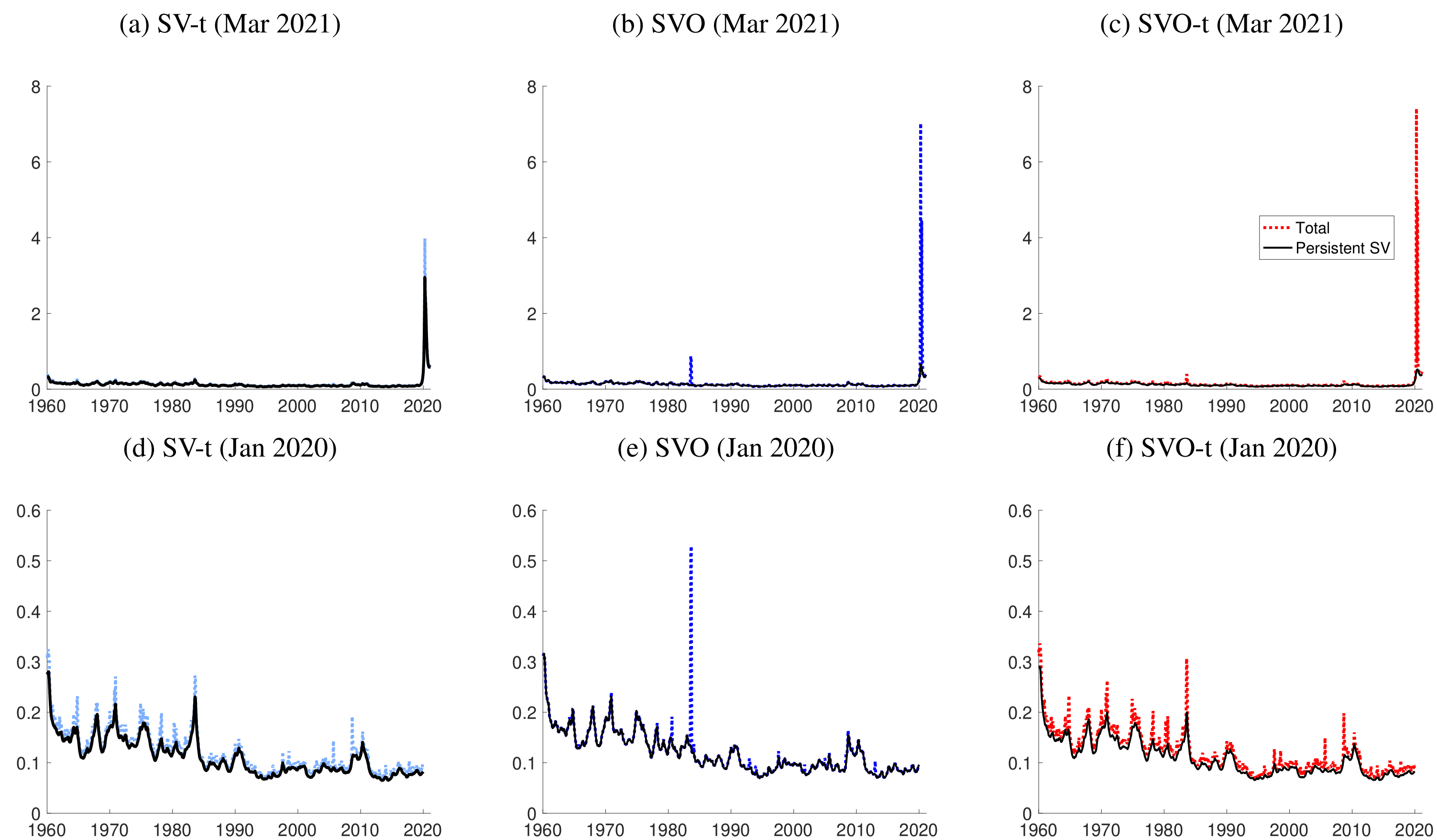
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.75: Posteriors of outlier states for Unemployment (VARs in levels)



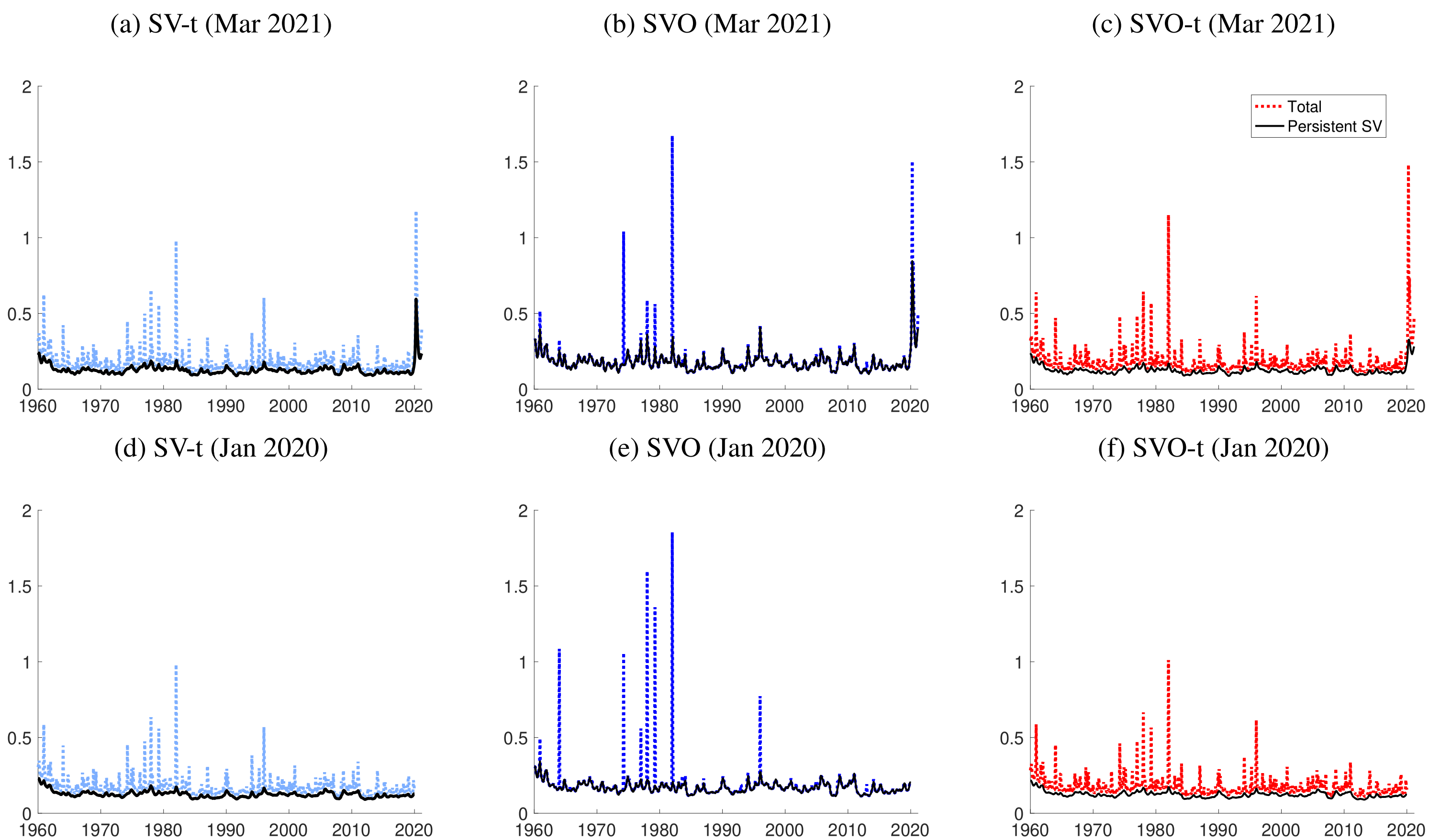
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.76: Posteriors of outlier states for Nonfarm Payrolls (VARs in levels)



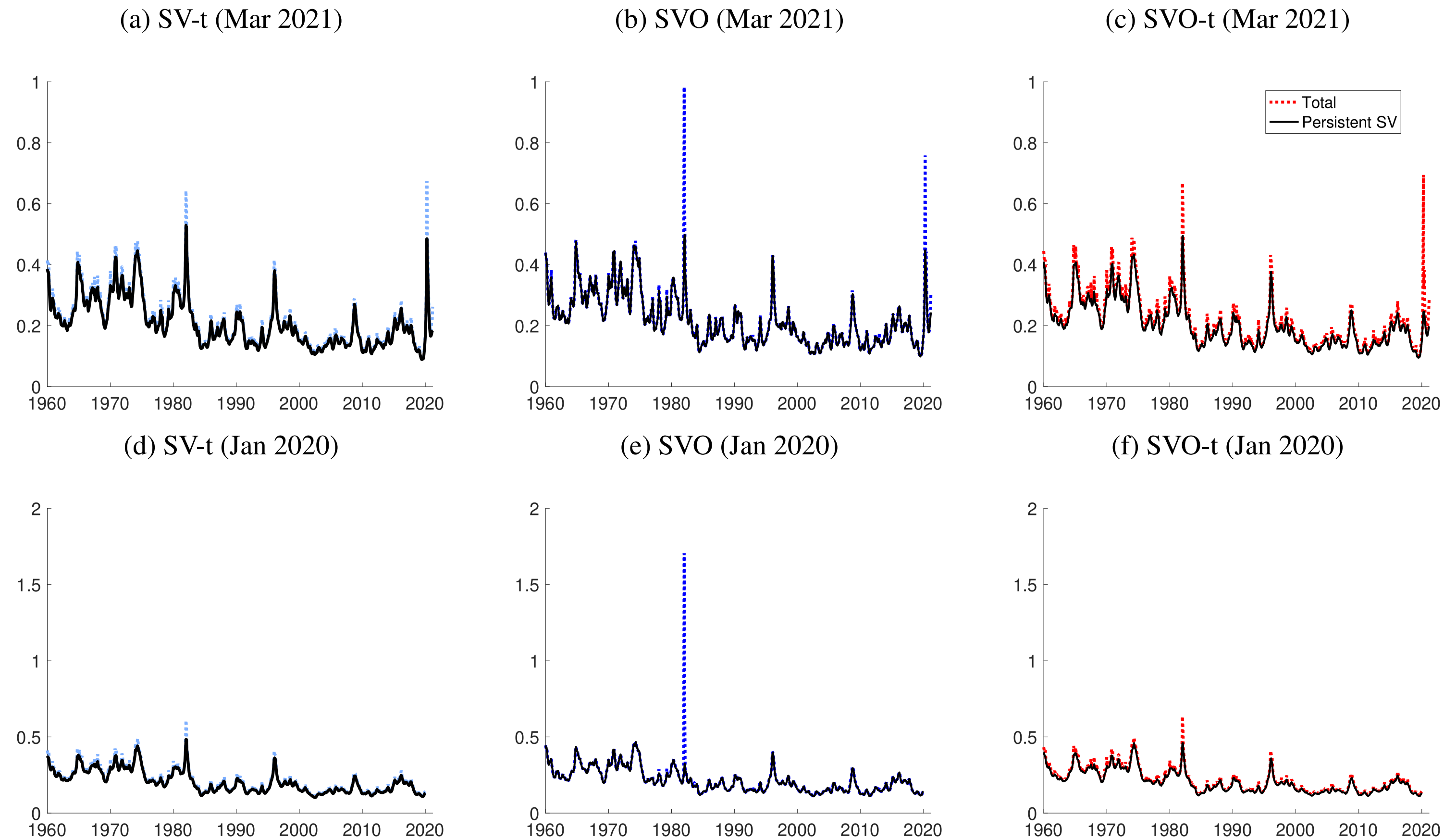
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.77: Posteriors of outlier states for Hours (VARs in levels)



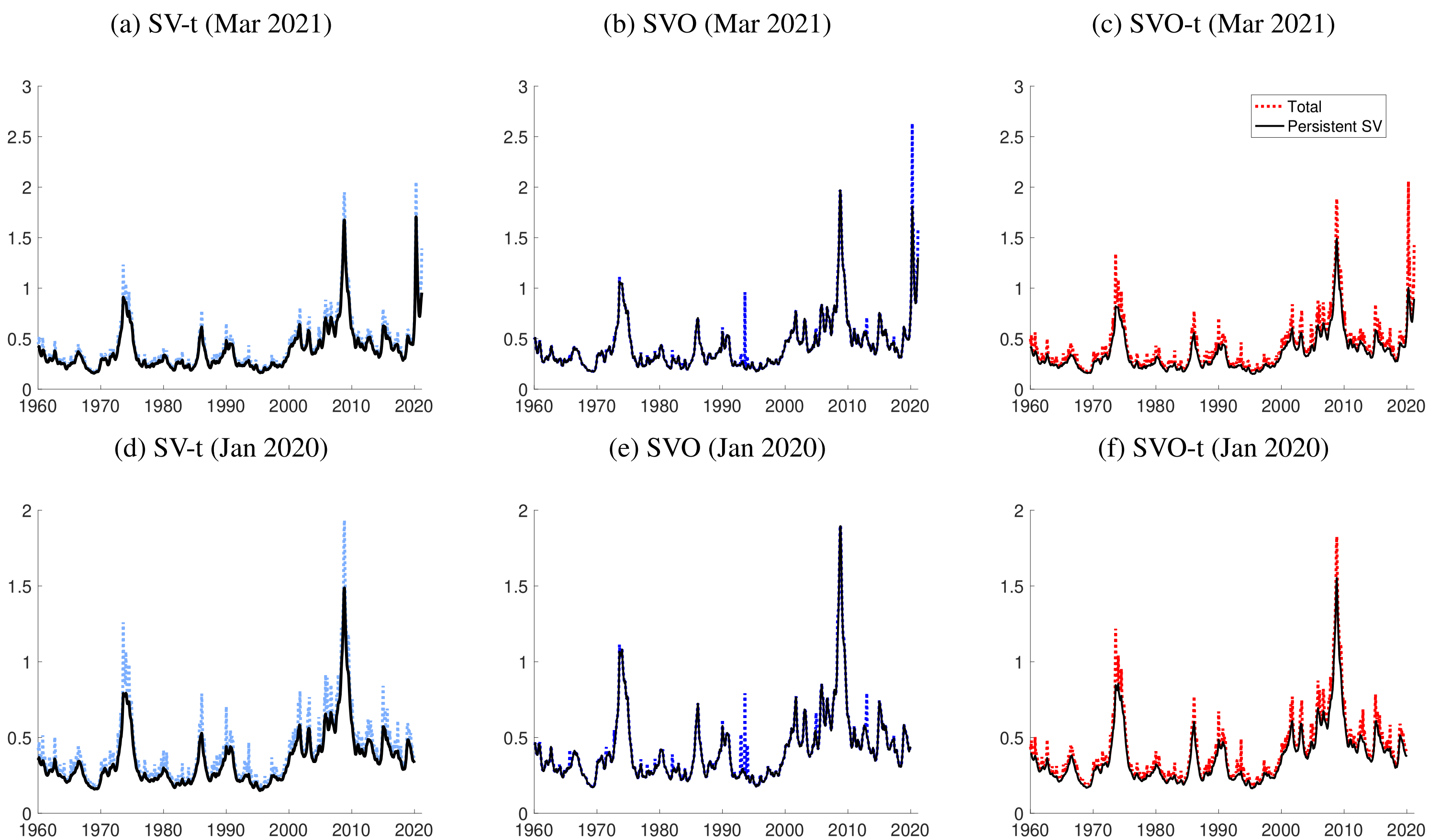
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.78: Posteriors of outlier states for Hourly Earnings (VARs in levels)



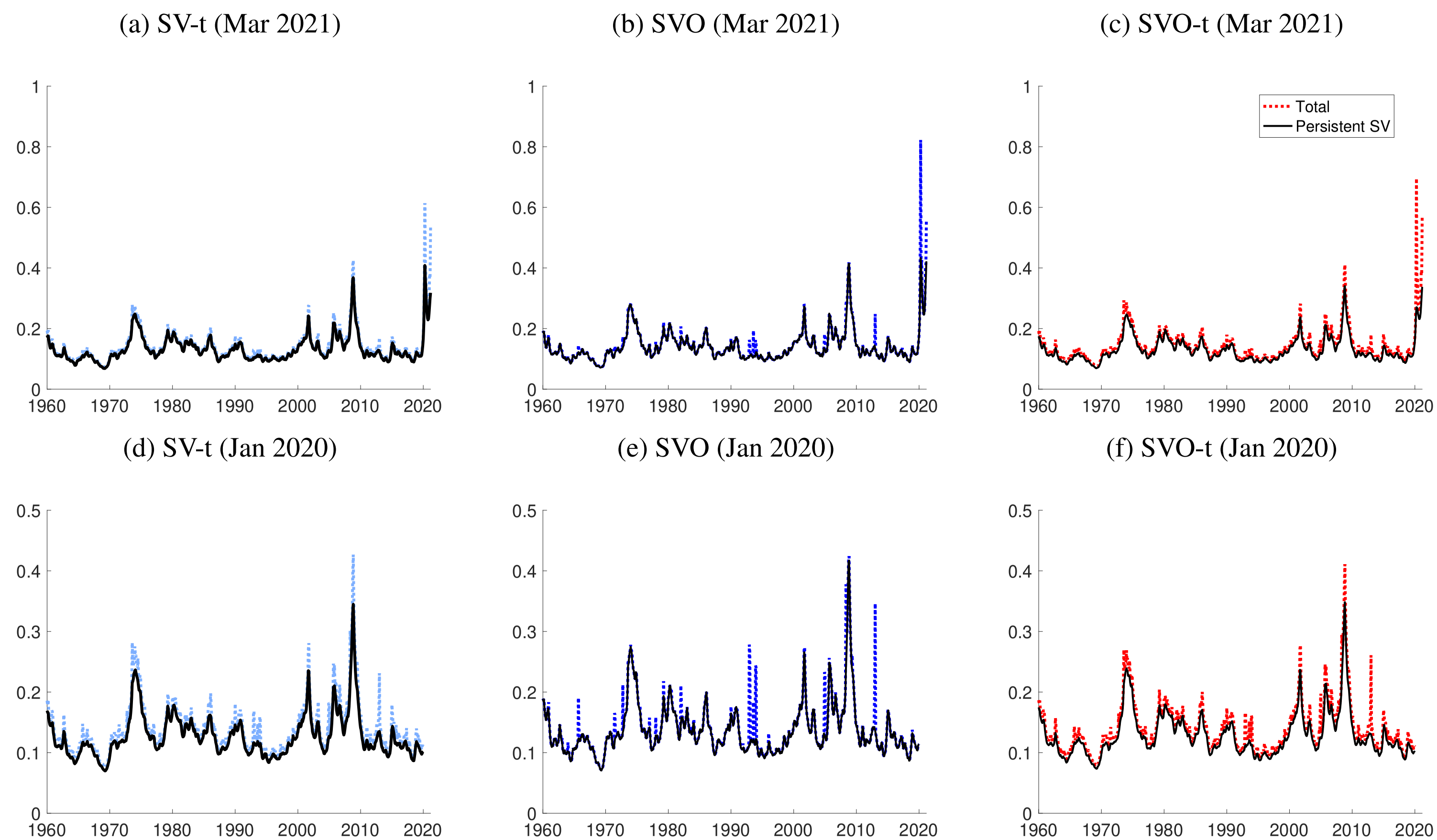
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.79: Posteriors of outlier states for PPI (fin. goods) (VARs in levels)



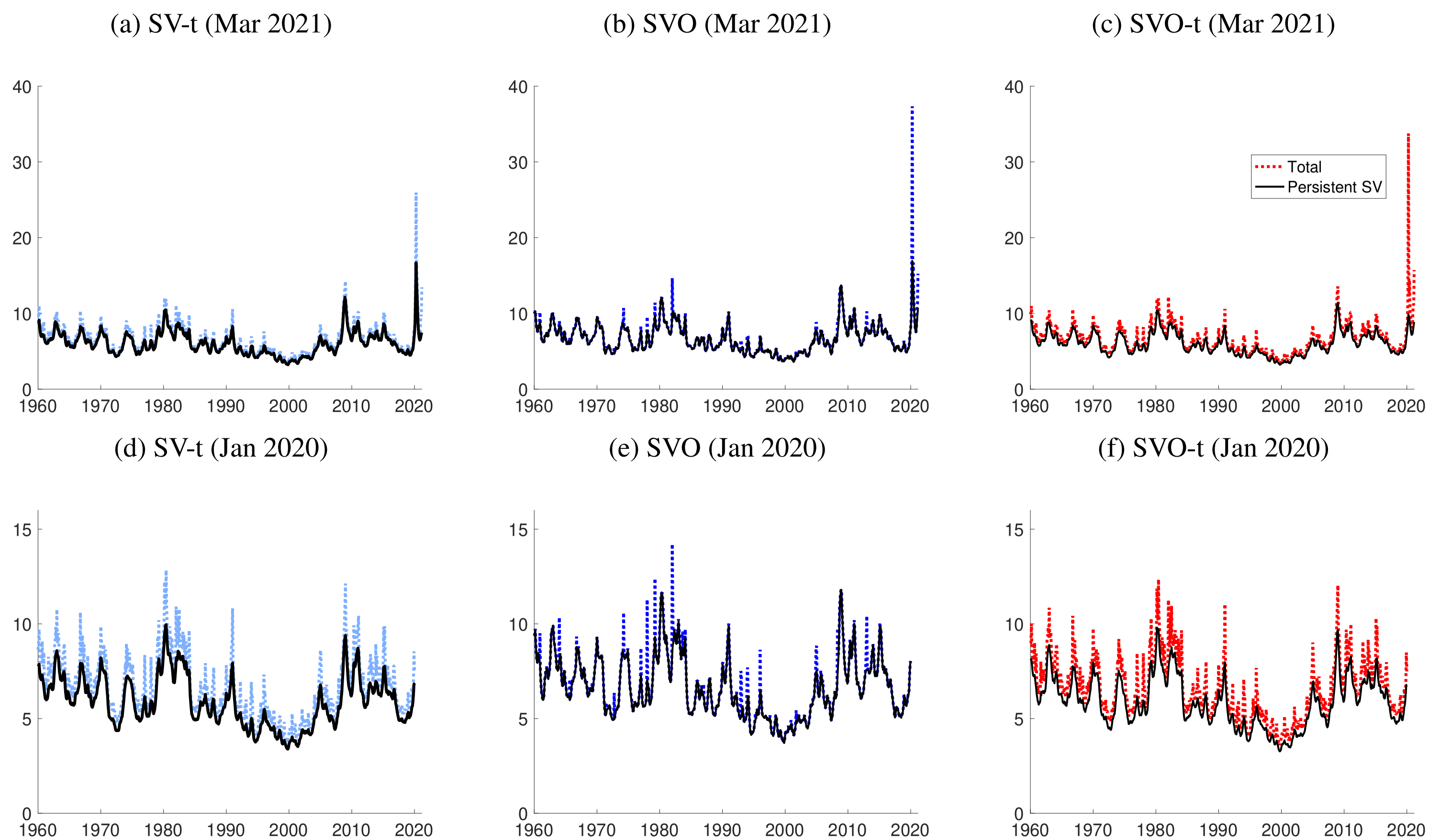
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.80: Posteriors of outlier states for PCE prices (VARs in levels)



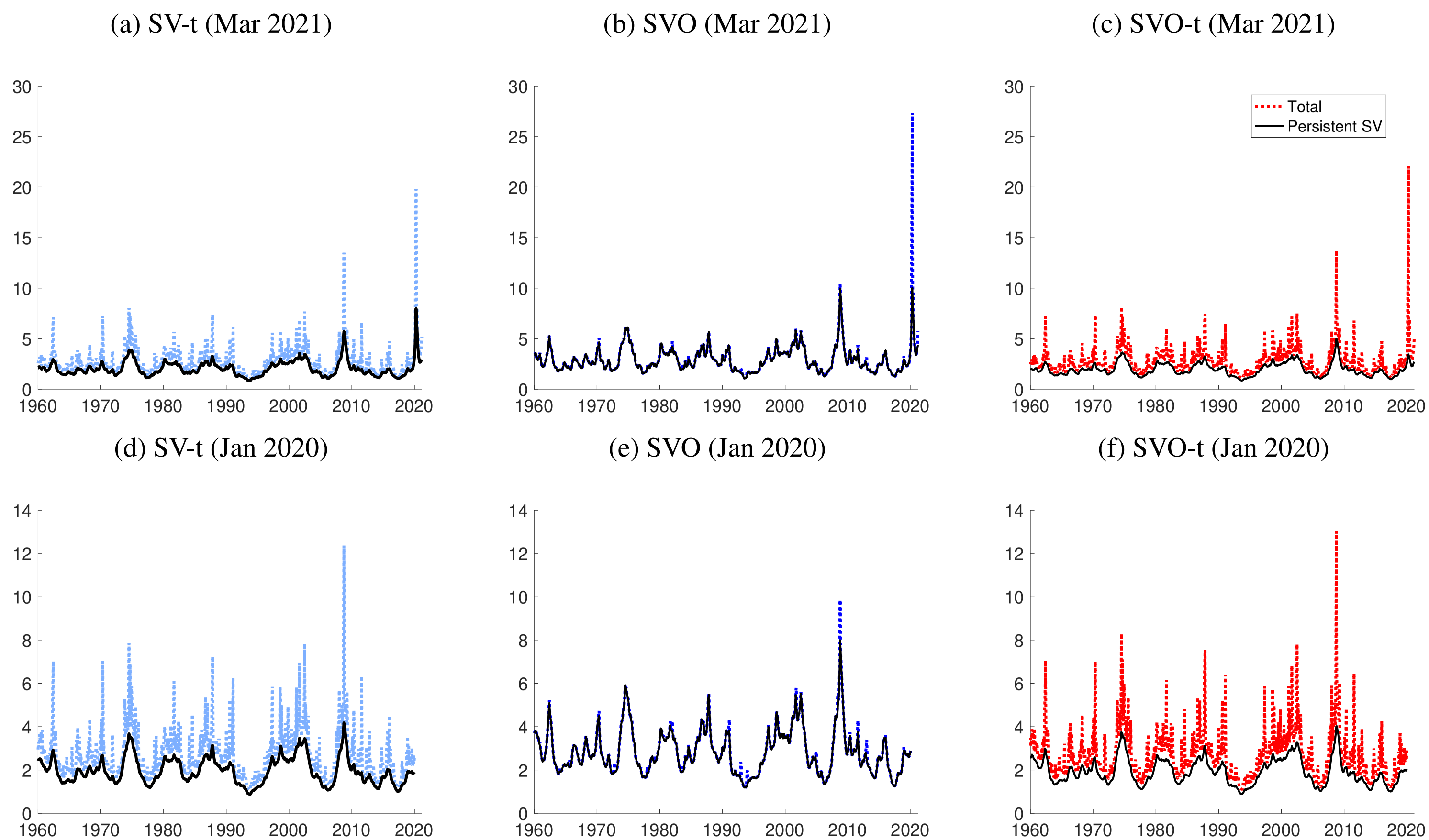
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.81: Posteriors of outlier states for Housing Starts (VARs in levels)



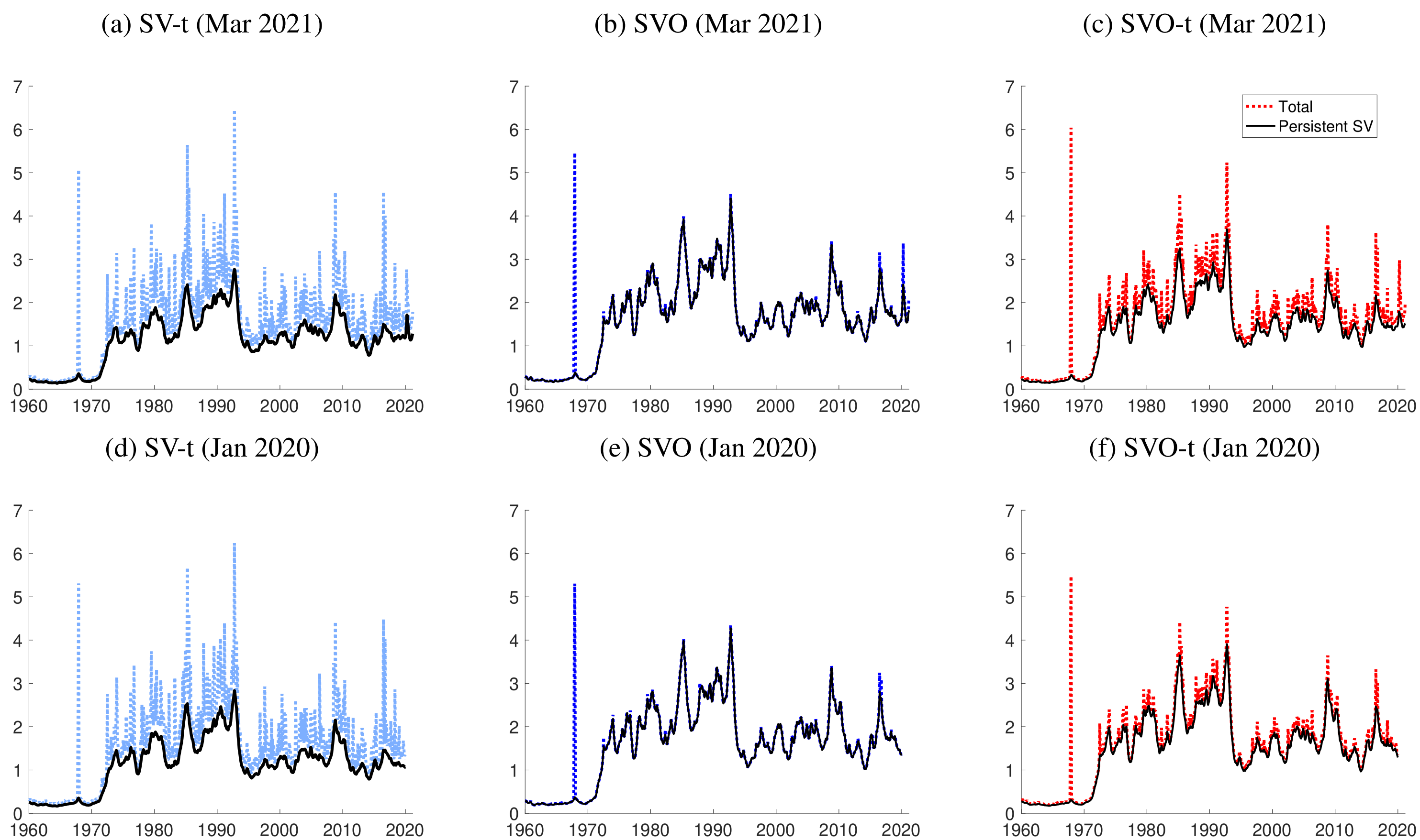
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.82: Posteriors of outlier states for S&P 500 (VARs in levels)



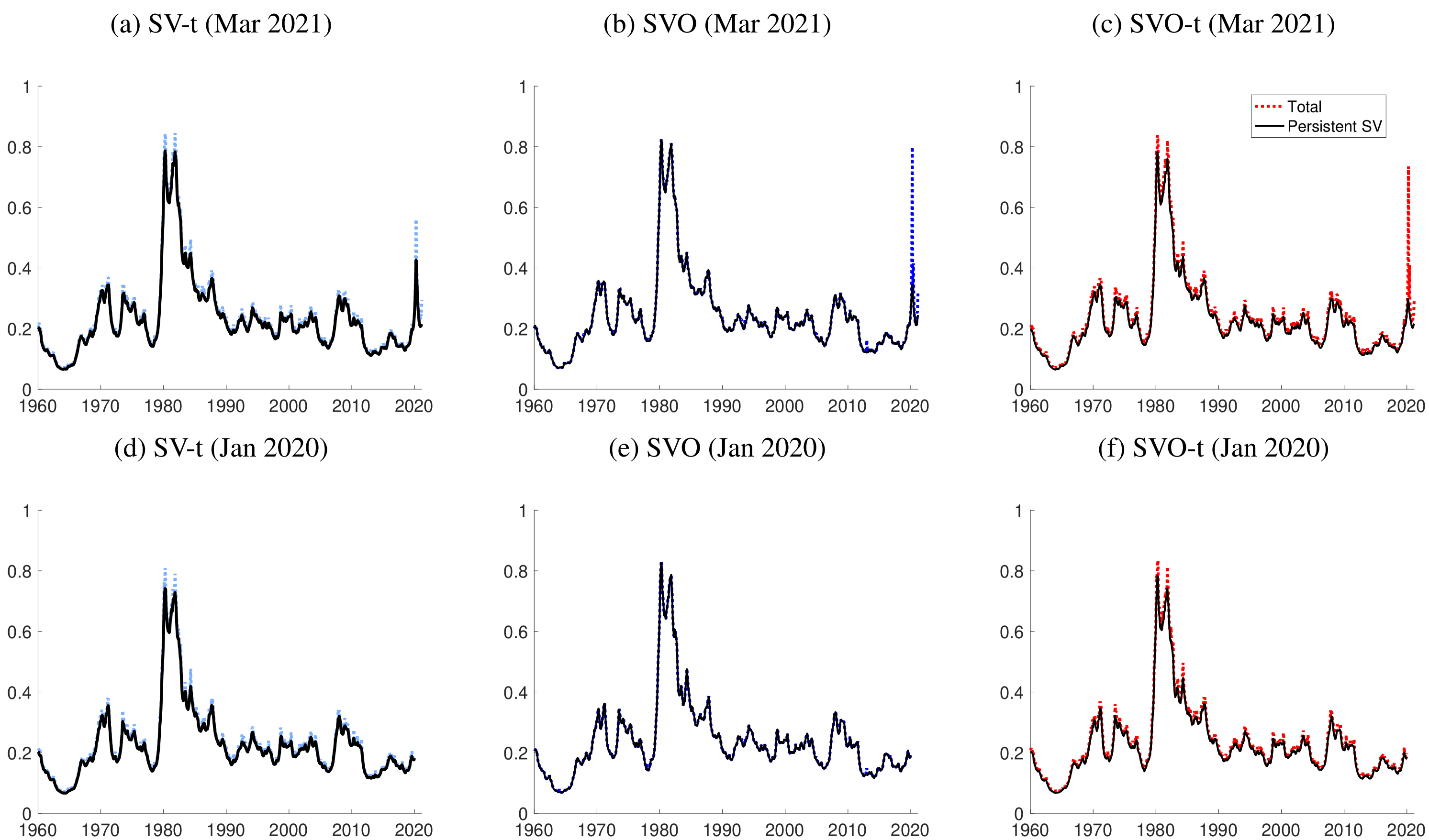
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.83: Posteriors of outlier states for U.S. / U.K. Forex (VARs in levels)



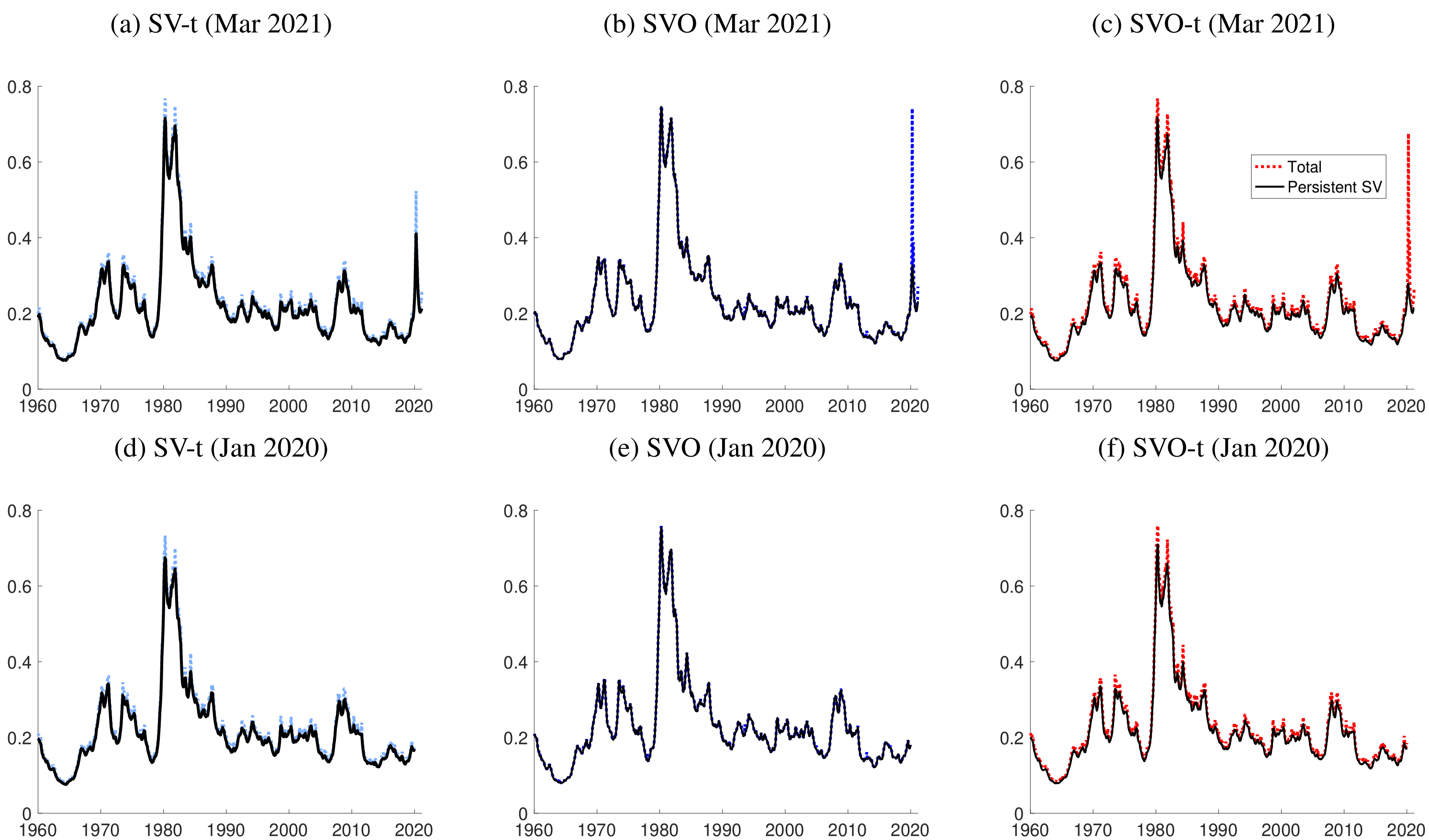
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.84: Posteriors of outlier states for 5-year yield (VARs in levels)



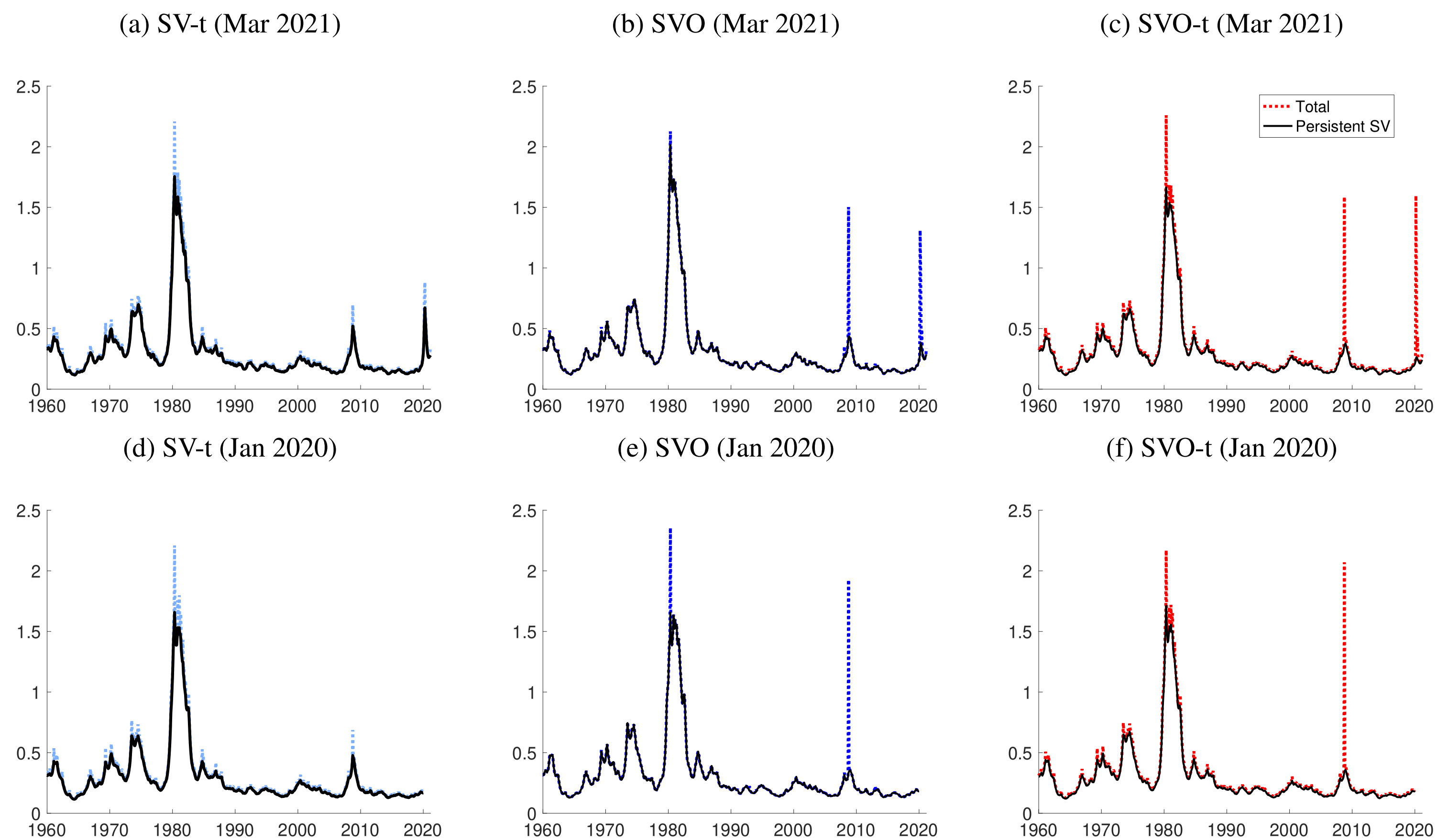
Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.85: Posteriors of outlier states for 10-year yield (VARs in levels)



Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

Figure S.86: Posteriors of outlier states for Baa spread (VARs in levels)



Note: Posterior median estimates per March 2021, of time-varying volatilities in forecast errors of the indicated variables in three outlier-augmented versions of the VAR-SV model. Dotted lines depict the actual forecast error volatility, including the effects of O_t and Q_t as applicable in each model. The solid lines depict the component of each variable's forecast error volatility due to the persistent SV component. Specifically, for the SVO-t model, the forecast error volatility is given by the square root of diagonal elements of $\Sigma_t = A^{-1} O_t Q_t \Lambda_t Q_t' O_t A^{-T}$ (labeled "Total"), whereas the contribution from the persistent SV component follows from $\tilde{\Sigma}_t = A^{-1} \Lambda_t A^{-T}$ (labeled "Persistent SV"). For SVO and SV-t, corresponding computations are performed using only O_t and Q_t, respectively. These calculations are performed for every MCMC draw, with the resulting medians reported in the figure.

XIII Reordering variables in VAR-SVO model

In VARs with stochastic volatility specified as in our paper, ordering affects estimates. In practice, some work, such as Cogley and Sargent (2005), has found results to not depend much on ordering. But recent work by Arias, Rubio-Ramirez, and Shin (2021) and Hartwig (2020) has shown that ordering choices in VARs with time varying parameters and SV can affect out-of-sample forecasts. In particular, in their results, ordering has little effect on point forecasts but measurable effects on density-related measures, including the standard deviation of the predictive density and the length of prediction intervals.

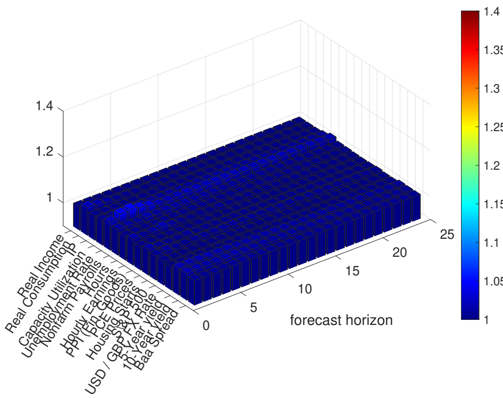
The relatively large number of variables in our model means a very large number of possible orderings. Accordingly, we have investigated sensitivity to variable ordering with an approach meant to be broad but streamlined to be computationally tractable (if still demanding). Our basic metric for sensitivity is the distance between predictive densities obtained in one ordering versus another. We assess the distance and its significance with the potential scale reduction factor (PSRF) of Gelman and Rubin (1992). In particular, we compare predictive densities generated from the VAR-SVO model at different forecast origins around and during the onset of the COVID-19 pandemic, December 2019, March and April 2020, and March 2021. For each of these origins, we randomly draw 640 different orderings of the model's 16 variables, estimate each model, form forecast densities (1 to 24 months ahead), and compute a Gelman-Rubin scale reduction test in each case.

The results presented in Figure S.87 suggest small ordering effects in our forecasts. The vast majority of Gelman-Rubin statistics are under 1.2. Only in April 2021 does a handful get as high as about 1.3, indicating some small to modest differences in densities, typically for economic activity variables at medium forecast horizons. Of course, we chose these forecast origins to reflect different conditions in the COVID period, with the economy near its depths in April 2020 and almost a year into recovery as of March 2021. Except for the limited differences detected for April 2020, the PSRF-Rubin statistics show no particular sensitivity to variable orderings.

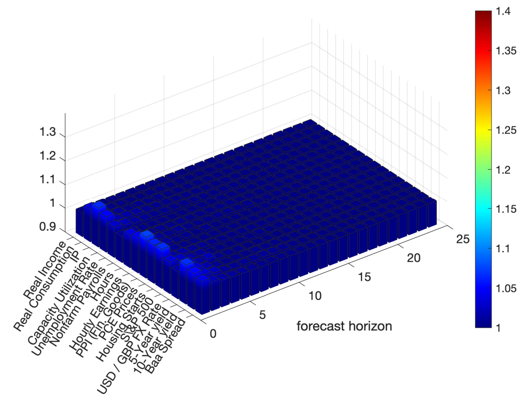
Figure S.87: Test for differences in forecasts from reordered VARs

Before/after Spring 2020

(a) December 2019

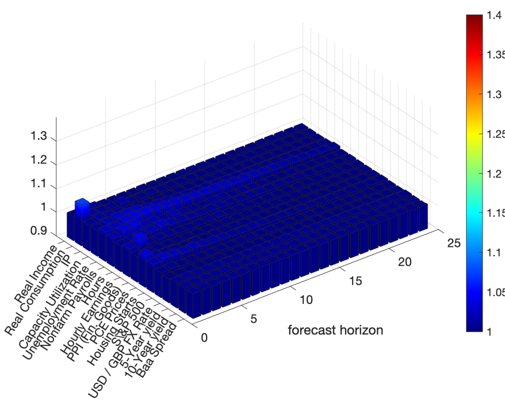


(b) March 2021

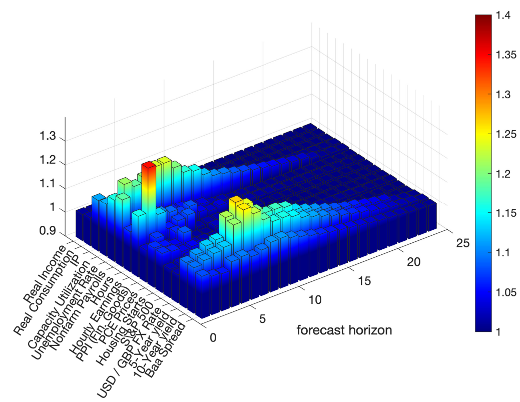


Spring 2020

(c) March 2020



(d) April 2020



Note: PSRF test statistics (Gelman and Rubin, 1992) for differences in predictive densities generated by 640 random reorderings of the VAR vector in the VAR-SVO model. Different panels consider densities generated at different forecast origins. Each panel reports PSRF statistics for forecasts of individual variables at various horizons. PSRF tests below 1.2 are typically considered to indicate lack of divergence in the densities generated by each reordering.

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