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This paper proposes a parsimonious model of network formation with introductions in the presence of intermediation rents. Introductions allow two nodes to form a new connection on favorable terms with the help of a common neighbor. The decision to form links via introductions is subject to a trade-off between the gains from having a direct connection at lower cost and the potential losses for the introducer from lower intermediation rents. When nodes take advantage of introductions, stable networks tend to exhibit a minimum of clustering. At the same time, intermediary nodes have incentives to protect their position, and stable networks can exhibit bridges across otherwise unconnected parts of the network earning intermediation rents.

JEL Classification: A14, D85.

Keywords: networks, network formation, clustering, intermediation, introductions.

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1 Introduction

When new connections are being formed in business or social networks, existing connections can play an important role in facilitating new links. A simple illustration of such network-based linking involves an introduction or referral, which creates a new connection between two previously unconnected nodes, thanks to a common acquaintance that brings them together. This paper proposes a model to study how incorporating introductions in a strategic network formation model affects the characteristics of the networks that result.

Link creation between nodes sharing a common neighbor is known as triadic closure, and it has been documented to be an important dynamic in the formation of networks in a number of different settings.[1] Uzzi (1996), in an in-depth study of the apparel industry in New York, describes how new relationships in business networks between two parties often result from referrals by common business partners:

> *In the firms I studied, third-party referral networks were often cited as sources of embeddedness. [. . .] One actor with an embedded tie to each of two unconnected actors acts as their go-between by using her common link* to establish trustworthiness *between them* (Uzzi, 1996, p. 679, emphasis added)

Also in the context of business networks, Kogut and Walker (2001) investigate the co-ownership network of German firms and identify a role for firms they call "brokers" that facilitate new acquisitions based on their existing connections and tend to be placed in central positions. In a different setting, Chaney (2014) shows that new connections in international trade networks can be explained with a dynamic model in which exporting firms create new trade relationships by exploiting their existing export contacts in the geographic vicinity of the desired new destination. More generally, business referrals can play an important role in establishing new relationships between market participants, thereby enabling trading to take place. Similar dynamics with network-based link creation have been observed in social networks. For example, Mayer and Puller (2008) study the dynamics of a network of relationships at a university. They show that an existing common neighbor within the network is a strong predictor of the formation of a new link between two nodes.

A key aspect of introductions as a way to create new relationships is the active role played by the go-between in the creation of the new connection. Without the agreement of the referring party, the new connection could not access the benefits of the introduction. This paper studies such introductions including the incentives for the parties involved and their implications for the formation of social and business networks.

[1] See, for example, Easley and Kleinberg (2010, Chapter 3) for an introduction.

Why would the presence of a common neighbor support the efforts of a pair of nodes to create a connection? As noted in the passage from Uzzi (1996) quoted above, a common neighbor can facilitate new links and help to build trust between two potential partners where this would be difficult without a go-between. In addition, links supported by common neighbors may be able to access greater benefits than connections without such support, for example to help enforce risk agreements among individuals (Jackson, Rodriguez-Barraquer, and Tan, 2012), or by offering redundancy advantages (Renou and Tomala, 2012). In sociology, the argument for benefits derived by individuals that are embedded in densely connected networks was made strongly by Coleman (1988) and labeled "network closure." The model in this paper captures the advantages of links created by introduction on the cost side, assigning them a lower cost than links created bilaterally.

Aside from these benefits, introductions also come with potential downsides to the nodes involved. The key one for the purposes of this paper concerns the redistribution of intermediation rents. Specifically, where a network node in a structurally important position between two other unconnected nodes earns rents for intermediating an indirect connection between them, such a go-between potentially risks losing these rents when new links are created among the intermediary's neighbors. Intermediation rents have been studied extensively in sociology and economics, in particular in the context of business and organizational networks. In his work on network advantage in such contexts, Burt (1992) labels such positions "structural holes" and provides evidence for the payoffs that individuals in these positions can earn. Since this earlier study, follow-up work has found additional evidence of the value of holding a central position in the network in various settings, including intrafirm organizational networks and interfirm R&D collaboration (Podolny and Baron, 1997; Ahuja, 2000; Mehra, Kilduff, and Brass, 2001; Owen-Smith and Powell, 2004). In economics, returns for intermediaries in networks have been studied by, among others, Blume et al. (2009), Condorelli, Galeotti, and Renou (2017), Farboodi, Jarosch, and Menzio (2017), and Manea (2018).

This paper presents a parsimonious model to study network formation with introductions in networks with intermediation rents, such as in the business networks studied in Burt (1992) and the related papers above. The model is a variant of the connections model of Jackson and Wolinsky (1996) with intermediation rents added, in the spirit of Goyal, van der Leij, and Moraga-González (2006). Network connections generate payoffs for the parties connected, directly or indirectly. Furthermore, as in Goyal and Vega-Redondo (2007) and Kleinberg et al. (2008), there are returns for intermediaries that connected otherwise disconnected nodes.

New links can be formed bilaterally or through introductions. Introductions can create a new link if the new link connects two nodes that share a common neighbor and all three parties agree. Introductions have a cost advantage over bilateral link formation, but they potentially threaten the rents for the central go-between.

The introduction mechanism thus combines a tendency for triadic closure with an explicit consideration of the incentives that arise from intermediation, generating a distinct trade-off for the introducing nodes. On the one hand, an introduction is an efficient way of creating connections as it reduces the costs involved in non-intermediated link formation. On the other hand, links created by introductions affect the distribution of payoffs and can expose the introducing player to circumvention, threatening intermediation payoffs received from being essential. The analysis of this trade-off and how it contributes to networks features such as high clustering, short distances, and network bridges forms the core of the paper.

The paper shows first that efficient network structures are either empty networks, star networks, or complete networks, reflecting the familiar patterns from the baseline connections model of Jackson and Wolinsky (1996). It then focuses on incentives for decentralized link formation by studying the set of networks that are stable to deviations by link creation or destruction. The analysis shows that the set of efficient network configurations is not necessarily stable for a given parameter configuration, revealing externalities in link creation and destruction. More generally, if benefits to linking are sufficiently high, stable networks tend to be connected and there exists a limit on the distance between any pair of nodes, given their degree. Next, the paper analyzes the use of introductions. It derives a lower bound on the clustering coefficient for any node in stable networks, with introductions providing the impetus for closing open triangles. The bound depends on the strength of returns to intermediation as well as the cost advantage of introductions over bilateral link formation. If parameters are such that nodes can take advantage of introductions, the predicted network exhibits features of a small world (Watts and Strogatz, 1998) with high clustering and short distances, a structure that has been identified in business networks (Kogut and Walker, 2001). What limits the closing of all open triangles through introductions is the incentive for essential intermediaries to protect the rents they receive from connecting otherwise disconnected parts of the network. These incentives permit the existence of bridges across densely connected subnetworks. Thus, under suitable conditions, links bridging otherwise disconnected parts of the network and earning substantial returns to intermediation as in Burt (1992) or Owen-Smith and Powell (2004) can coexist with high clustering in stable networks.

The paper contributes to the literature on network formation related to the study of the introduction mechanism and the compensation of intermediate nodes. While intuitive, studying link creation as a process conditional on the network in place at a given time is relatively novel in the literature on strategic network formation.[2] The two papers closest to this paper are Jackson and

[2] The literature on network formation can usefully be grouped into two main categories: (i) random network formation and (ii) strategic network formation. Whereas the first approach analyzes the outcome of an exogenous stochastic process of link creation and aims to explain the observed features of real-world networks, the second explicitly studies the incentives of nodes to form links among themselves. Network formation is then the outcome of individual payoff maximization by nodes. The present paper falls into the second category, although I will in places refer to work from the random-network-formation

Rogers (2007) and Jackson and Rogers (2005). In Jackson and Rogers (2007) the authors study a random network-formation setup with a growing network in which new nodes first connect to a set of randomly chosen existing nodes and in a second process may connect to neighbors of those nodes, that is, connect to friends of friends. Connections created by the second process are similar to those formed in an introduction as studied here in that they close open triangles. Using a combination of both processes, Jackson and Rogers (2007) are able to match a mean-field approximation of their model to the network properties of a number of applied settings, including high clustering coefficients. The present paper is complementary to Jackson and Rogers (2007) and different from it in at least two main ways. First, Jackson and Rogers (2007) model link creation via random meeting opportunities, which result in links if the pair of nodes to be linked finds it beneficial to connect. This ignores the strategic aspects of the key role that intermediary players perform in making the kind of introductions that facilitate the friends-of-friends meetings. In contrast, in this paper, I explicitly study intermediary incentives. Second, the underlying payoff model in Jackson and Rogers (2007) focuses on the returns of each connecting pair independent of the implications for returns from other parts of the network.[3] My model puts such externalities at the center of the analysis, studying how incentives for intermediaries to protect their returns shape the resulting network.

Beyond Jackson and Rogers (2007), an earlier contribution by Vázquez (2003) presents a mean-field analysis of graph dynamics with a nearest-friend link creation process. That contribution ignores strategic considerations and focuses instead on how local rules may motivate the preferential attachment hypothesis. More recently, local structures of the network such as triangles and the processes generating them have been exploited in empirical work on network data. For example, Chandrasekhar and Jackson (2018) show how local structures such as triangles and other subgraphs can be used to study network-formation processes empirically. Recent surveys on this active work stream can be found in Graham (2019) and de Paula (2019).

The second closely related paper is Jackson and Rogers (2005). In their paper the authors offer a strategic network-formation model that can predict small-world properties. The key ingredients in their analysis are the benefits from indirect links as in the connections model of Jackson and Wolinsky (1996) and an "islands" cost model, in which links connecting players on the same island are cheaper than links between islands. The present model similarly explains both clustering and short distances but, in addition, analyzes the effect of intermediation benefits and the resulting incentives for individuals to adopt and protect bridging positions. In Jackson and Rogers (2005) the grouping of players into easily connected islands is given exogenously; in my paper, the cost

tradition.

[3] In extensions in Section V, Jackson and Rogers (2007) discuss the implications of different payoff specifications and formally show the effect of making payoffs dependent on degree. However, they do not include an analysis of the externalities resulting from intermediation returns.

advantages arise endogenously through the introduction mechanism. This endogenous, network-based cost structure is one of the key differences between the present paper and Jackson and Rogers (2005).

In strategic network formation, Priazhkina and Page (2018) study referrals or market-sharing by sellers in buyer-seller networks. Although this setting is different from the connections model studied here, referrals are related to introductions in that they require an existing seller-buyer relationship in order for the seller to be able to execute a referral. In their paper, the authors study the incentives for referrals and the implications for market functioning.

In addition, this paper offers a contribution to the literature on incentives for intermediary nodes and rents from intermediation. The benefits accruing to nodes in specific positions crucial to the connectivity of the network are discussed extensively by Burt (1992) in the context of organizations. The author investigates the rents available to individuals who bridge "structural holes" and the dynamics of jockeying for the positions required to access these rents. Subsequent papers have studied the tension between triadic closure and structural holes. In the economics literature, Goyal and Vega-Redondo (2007) consider network formation in the presence of intermediation benefits and analyze the interplay of three motivations: (i) access to the network, (ii) benefits from intermediation, and (iii) avoidance of sharing benefits with intermediaries. They find that in the absence of capacity constraints, a star emerges, in which a single node acts as intermediary for all transactions, receiving significant intermediation rents. Contrary to the present paper, in their model, Goyal and Vega-Redondo (2007) focus on direct link creation and do not implement an introduction mechanism. Other related papers in this vein include Buskens and van de Rijt (2008), Kleinberg et al. (2008) and Kossinets and Watts (2006).

Ambrus and Elliott (2020) consider stable and efficient configurations of risk-sharing networks. They find a trade-off between stability and inequality: The most stable networks tend to be the most unequal as well, with central agents connecting more peripheral ones and capturing most of the benefits of the network. As in the present paper, Ambrus and Elliott (2020) find that stable networks tend to both exhibit short distances and include some nodes that bridge others and earn high returns. In their main model, closed triangles have no advantage over open triangles and indeed a closed triangle would be inefficient. In an extension Ambrus and Elliott (2020) study the implication of requiring agents to have a common friend for risk-sharing to work. This requirement implies that at least three nodes need to be involved for risk-sharing, and for this reason Ambrus and Elliott (2020) also modify their initially pairwise stability notion to allow for deviations by triplets. This concept of a tripletwise deviation mirrors the notion of deviations by introduction used in this paper. In that version of the model, network structures that are similar to the windmill graphs in Figure 5 arise as efficient and stable configurations.

Finally, my paper is related to the literature on strategic network formation with transfers. The introduction mechanism I study requires a mechanism to facilitate compensation of the intermediating node. As a result, transfer payments and their ability to overcome externalities play a key role. The seminal contribution in this area is Currarini and Morelli (2000), who study a sequential setup with transfers and find that with relatively few restrictions on payoffs, efficient networks are formed in equilibrium. A more general analysis of the various forms of transfers and their ability to implement efficiency in simultaneous network formation is found in Bloch and Jackson (2007). Neither paper studies the introduction process at the center of the present paper. My paper contributes to the literature by extending the standard game-theoretic tool of pairwise stability (Jackson and Wolinsky, 1996) to a setting with more than two players.

The remainder of the paper is structured as follows. Section 2 presents the main model, including network notions and payoff structures. Using this model, Section 3 presents the introduction mechanism and characterizes efficient networks. Section 4 studies the characteristics of networks that are stable to deviations, including introductions, and contains the key results regarding short distances, intermediation benefits, and minimum clustering. Finally, Section 5 concludes.

2 A Model of Network Formation with Intermediation and Introductions

This section introduces a model of network formation with introductions and intermediation rents. It builds on the symmetric connections model of Jackson and Wolinsky (1996): Nodes create payoffs from direct and indirect connections and there is a cost to maintaining links. On top of this, the model in this paper introduces rents for link intermediation as well as a new way of link creation by introduction.

2.1 A Connections Model with Intermediation Rents

To start, we set out some network notation. There is a finite set of *players* $N = \{1, 2, \ldots, n\}$ with $n > 3$. Players represent *nodes* that are linked in a *network* g. g is a set of *links*, that is, pairs of players that are connected to one another, with typical element ij representing a link between players i and j. Let g^N be the set of all pairs of players in N, describing the complete network. g^0 is the empty set and describes the empty network.

Payoffs are generated by connections between pairs of players. Direct connections generate a normalized surplus of 1; indirect connections that are intermediated by another node generate a surplus of $\delta \leq 1$. Following Kleinberg et al. (2008), connections requiring more than two steps generate a zero surplus. Burt (1992, 2007) shows that in organizational networks, benefits for nodes that are greater than two steps away are negligible, and argues in support of an analytical framework that ignores such higher-order intermediation.

The surplus generated by direct connections without an intermediary is split equally between the two nodes involved, that is, each receives a payoff of 1/2.

When indirect connections are facilitated by an intermediary, that node can capture a share of the surplus created by the connection, akin to the returns to brokerage identified in Burt (1992) and subsequent work. Such intermediation profits can also capture the benefits for intermediaries in trading networks as studied in Condorelli, Galeotti, and Renou (2017), Farboodi, Jarosch, and Shimer (2017), Choi, Galeotti, and Goyal (2017), and Manea (2018).

The value of intermediation rents depends on the extent to which the intermediary is indispensable for the connection: Where many alternatives exist, rents available to an intermediary will be lower. Formally, total rents for intermediation between i and j are denoted $\gamma(r_{ij})$, which is a function of r_{ij}, the number of intermediaries, that is, nodes that are connected to both i and j. Following Goyal and Vega-Redondo (2007) I assume that intermediary nodes capture a positive share of the total surplus if and only if they are *essential* to the connection, that is, if they are on every path connecting two nodes and thus $\gamma(r_{ij}) = \gamma$ if $r_{ij} = 1$ and $\gamma(r_{ij}) = 0$ otherwise. This assumption captures intermediation rents being competed away in the spirit of Bertrand competition as soon as there is more than one intermediary.[4]

In summary, in the connections model with intermediation rents, total payoffs for player i from network g are given by

$$\tilde{\pi}_i(g) = d_i(g)\frac{1-c}{2} + \sum_{\substack{j\neq i:\\ \ell_{ij}=2}} \delta\frac{1-I_{\{r_{ij}=1\}}\gamma}{2} + \sum_{\substack{j,k\neq i:\\ \ell_{jk}=2\wedge ij\in g\wedge ik\in g}} I_{\{r_{jk}=1\}}\delta\gamma \tag{1}$$

where $d_i(g)$ denotes the degree of player i in network g and ℓ_{ij} is the length of the shortest path between nodes i and j. Payoffs are the sum of the following components: (i) benefits from direct connections net of link maintenance costs, (ii) benefits from indirect connections net of intermediation rents paid, and (iii) intermediation rents received.

2.2 Link Costs and Link Formation with Introductions

This section describes the process for creating and deleting links. The model permits two different ways of creating new connections. First, under *bilateral link creation* two nodes agree to form a

[4] Aside from incorporating Bertrand competition between intermediaries, zero rents in the case of two or more intermediaries can be derived as a prediction of a model of bargaining in networks without replacement in the limit when bargaining frictions disappear. See Siedlarek (2015). Kleinberg et al. (2008) adopt a different approach in which intermediation benefits decay with the number of alternative paths in a more gradual way.

(a) Bilateral (b) Introduction

Figure 1: Forms of Link Formation

link between themselves. Second, under *link creation by introduction* a third-party intermediary can facilitate a new connection between two nodes that are not connected with each other but are both connected to the intermediary. Such connections have a cost advantage over connections created by regular bilateral link formation. Finally, links can be destroyed unilaterally.

Bilateral link creation allows any unconnected pair i, j to create a new link between themselves if both agree. See Figure 1a. Links created in this way incur the full cost $c/2 \geq 0$ for each node or c in total. This cost captures, for example, investments in efforts to screen a potential partner and develop and maintain a sufficient level of trust for a functional relationship.

In addition to bilateral link creation, I introduce link formation via introductions. An introduction occurs when a new connection is created between two unconnected nodes i, j that share a common neighbor k and that neighbor acts as intermediary. See Figure 1b for an example in a simple network of three nodes. Just like bilateral link formation, this process creates the link ij, but in contrast to bilateral link creation by i and j independently, it requires the agreement of the introducing node k.

To capture the benefits of leveraging a common partner in the new connection, introductions in the model have a cost advantage over bilateral link creation and create links that cost $(1-\epsilon)c$, with $\epsilon \in [0, 1]$. This creates a trade-off for the use of introductions: They are cheaper than regular bilateral link creation but require the agreement of the introducing node.

Allowing for a cost difference between the two types of link creation requires tracking the way in which a link was created. I therefore partition the set of links g as follows: g_B contains links created by bilateral link formation, g_I those created by introduction. As a partition the sets satisfy $g_B \cup g_I = g$ and $g_B \cap g_I = \emptyset$. Payoffs for player i for any given network $g = (g_B, g_I)$ can then be written as shown in Equation 2.

$$\pi_i(g) = d_i(g)\frac{1-c}{2} + \sum_{\substack{j\neq i:\\ \ell_{ij}=2}} \delta \frac{1 - I_{\{r_{ij}=1\}}\gamma}{2} + \sum_{\substack{j,k\neq i:\\ \ell_{jk}=2 \wedge ij\in g \wedge ik \in g}} I_{\{r_{jk}=1\}}\delta\gamma + d_i(g_I)\epsilon\frac{c}{2} \tag{2}$$

3 Efficient Networks with Introductions

Efficiency considerations in the model apply to both the process of network formation and the resulting structure of relationships. The definition of efficiency incorporates both by tracking the mode of link creation through the payoff equation (Eq. 2).

Definition 1. *A network structure $g^* = \left(g_B^*, g_I^*\right)$ is efficient among the set of feasible networks G if it maximizes the sum of total payoffs across nodes net of the minimum cost incurred in creating it. That is,*

$$g^* = \underset{g\in G}{\operatorname{argmax}} \sum_{i\in N} \pi_i(g) \tag{3}$$

We start by considering the cost of link creation. Generally speaking, introductions are more efficient than link creation without introductions due to their cost advantage ϵ. However, the requirement for nodes to share a common neighbor before introductions are feasible implies some role for non-mediated bilateral link creation. For example, creating a closed triangle from three isolated nodes requires at least two links created by bilateral link formation. The third link closing the triangle can then be formed by introduction. Without this restriction taking account of the dynamics of the introduction process, only networks with all links in g_I could ever be efficient.

Proposition 2 shows the interaction of the net benefits from connections and the costs from link creation. The structure of the proof follows Jackson and Wolinsky (1996) and is provided together with all other proofs in the Appendix.

Note that different from Jackson and Wolinsky (1996), the result requires a characterization not just of the links of the final network structure but also of the most efficient way of creating the network through a combination of bilateral link creation and introductions.

Proposition 2. *The unique efficient network structure g^* in the model with introductions is:*

1. *The* empty network (g^0, g^0) *if*

$$c > 1 + \frac{n-2}{2}\delta, \textit{ and}$$
$$c > \frac{1}{1 - \left(1 - \frac{2}{n}\right)\epsilon}$$

2. *The n-player* star network $(g^S, \emptyset)$ *if*

$$c < 1 + \frac{n-2}{2}\delta, \textit{ and}$$
$$c > \frac{1-\delta}{1-\epsilon}$$

The star network is formed by $n-1$ links created by bilateral link creation.

3. *The n-player* complete network $(g^S, g^C \setminus g^S)$ *if*

$$c < \frac{1}{1-\left(1-\frac{2}{n}\right)\epsilon}, \textit{ and}$$
$$c < \frac{1-\delta}{1-\epsilon}$$

The complete network consists of a star network g^S with $n-1$ links created by bilateral link creation with all remaining links created through introductions.

The efficient network structure is unique up to a permutation of players.

The characterization focuses on the relationship between the cost per link c and the efficiency of introductions ϵ. Figure 2 illustrates the parameter regions characterized in Proposition 2. As the cost efficiency of introductions increases, the efficient network is either complete or empty: Once link formation is productive at all, it pays to make maximum use of introductions and form the complete network.

At $\epsilon = 0$ introductions do not provide any advantage over bilateral link creation. As a result, the efficient structures and parameter ranges correspond to those in Jackson and Wolinsky (1996). However, as the cost advantage of introductions ϵ increases, differences between Jackson and Wolinsky (1996) and the setting with introductions emerge.

Initially, as ϵ increases from zero, it becomes more advantageous to close open triangles and the cost range for which the star network is efficient decreases, while the range for the complete network increases. Above a certain level of cost efficiency, the star network is not efficient for any level of linking cost anymore and only either the empty network or the complete network remains efficient. The result shows the extent to which the cost advantage of introductions pushes efficient structures toward high density and high levels of clustering. The efficient configuration is independent of the intermediation rent parameter γ as it only presents a transfer between players, without efficiency implications in the sense of Definition 1. The results given here provide a benchmark for subsequent analysis, which studies the incentives for players to create and remove links from the network.

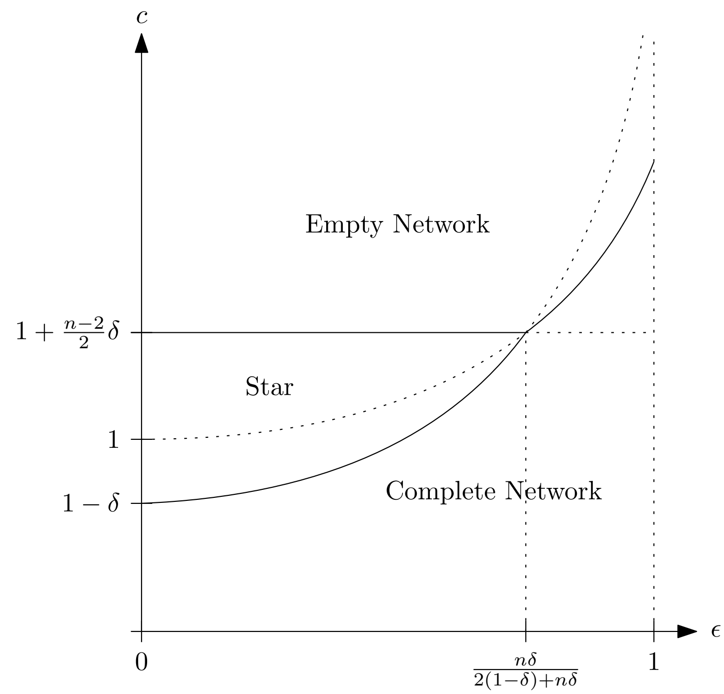


Figure 2: Efficient Network Configurations ($\epsilon = 1$)

4 Stable Networks with Introductions

This section presents results on networks that are stable when introductions are available. I build on the pairwise stability concept of Jackson and Wolinsky (1996) by incorporating a suitable extension that allows for introductions and transfers.

The analysis of introductions requires the inclusion of transfers to the introducing player in order to compensate that player for the potential loss of intermediation benefits. For illustration, consider the payoff implications of the new link that is created. The new link shortens the distance between the two players being introduced to one step, yielding an increase in connection benefits. However, the new link does not generate any additional benefits for the introducing player as he is already connected to both. Indeed, the new link results in the introducing player no longer being needed for the connection between the players introduced, and thus he will lose out. Introductions by themselves are at best payoff neutral for the introducing player, and transfer payments are necessary to make any introduction profitable for the introducer. As a result, the stability concept I use here is one that considers deviations with transfers as proposed in Bloch and Jackson (2006). Their stability concept recognizes unilateral link destruction and bilateral link creation. I extend their setting with an additional stability condition to account for link creation via the introduction process.

Definition 3 (Myopic stability under introductions)**.** *A network g is myopically stable with introductions if:*

a. (Destruction) $\forall\ ij \in g$,

$$\pi_i(g) + \pi_j(g) \geq \pi_i(g - ij) + \pi_j(g - ij)$$

b. (Bilateral Link Formation) $\forall\ ij \notin g$,

$$\pi_i(g) + \pi_j(g) \geq \pi_i(g + ij) + \pi_j(g + ij)$$

c. (Introduction) $\forall\ \{i, j, k : ik \in g \wedge jk \in g \wedge ij \notin g\}$,

$$\sum_{v \in \{i,j,k\}} \pi_v(g) \geq \sum_{v \in \{i,j,k\}} \pi_v(g + ij)$$

The first two conditions correspond to those used for networks that are *pairwise stable with transfers*, as analyzed in Bloch and Jackson (2006). The definition includes a third condition, which requires that in a stable network there be no opportunities for profitable introductions. That is, there cannot be a triplet of nodes that form an open triangle jointly benefiting from adding the missing link, with a lower fixed cost $(1 - \epsilon)f$.

Note that the third condition — introduction — allows a deviation by a coalition of three agents. This kind of deviation arguably requires an additional degree of coordination beyond the usual bilateral deviations familiar from Jackson and Wolinsky (1996) and other papers. In this paper, such deviations are permitted if they reflect introductions; that is, the three notes are already connected on a subnetwork induced by themselves and are forming the missing link. In the context of international trade, Chaney (2014) shows that such a process can successfully explain the dynamics of trade works over time.[5]

All three conditions allow for transfers between the players involved by considering the *sum of payoffs* rather than individual payoffs. This also applies to link destruction in order to maintain symmetry between link creation and link destruction.[6] Note that such transfers do not need to involve a monetary exchange at the time of the introduction. For example, a debt might be incurred in the form of owing a favor to another party in the future (Jackson, Rodriguez-Barraquer, and Tan, 2012).

[5] See also the discussion of tripletwise deviations in Section 6.2 of Ambrus and Elliott (2020). They allow deviations by three nodes as long as they create the new links among themselves and thus facilitate risk-sharing among themselves.

[6] See footnote 5 in Bloch and Jackson (2006) for a discussion of this issue.

Allowing for transfers in link destruction reduces the set of profitable deviations of this type: All link removals that are jointly profitable necessarily involve at least one player for whom it is unilaterally profitable; however, if one player loses out from the removal of the link, Definition 3 requires that the damage done to the other side involved in the link not be too high. In this sense, the solution concept with transfers is weaker than that without transfers as far as link destruction is concerned.

The following section analyzes networks that are stable when introductions are feasible by (i) considering the stability properties of the efficient configurations (Section 4.1) and (ii) characterizing the properties of stable networks in general (Sections 4.2 and 4.3).

4.1 Stability of Efficient Networks

This section derives the stability properties of the configurations that are possible efficient arrangements for some parameter ranges. The analysis starts with one of the possible efficient configurations — the empty network, the star network, and the complete network — and characterizes the parameter restrictions necessary for each configuration to be stable. The details of the derivation of the conditions have been relegated to the Appendix, Section A.2.

Proposition 4. *The three efficient network structures are myopically stable with introductions if the following conditions hold:*

1. *The* empty network *is stable if*

$$c \geq 1$$

2. *The* star network *is stable if*

$$c \geq \max\left\{1 - \delta(1-\gamma), \frac{1-\delta}{1-\epsilon}\right\}, \textit{ and}$$
$$c \leq 1 + (n-2)\delta\frac{1+\gamma}{2}$$

3. *The* complete network *is stable if*

$$c \leq 1 - \delta$$

The parameter thresholds are illustrated in Figure 3. Table 1 lists the parameter ranges for stability next to the corresponding thresholds for efficiency, which are derived in Section 3. In some areas, efficiency and stability are aligned. For example, whenever a complete network is

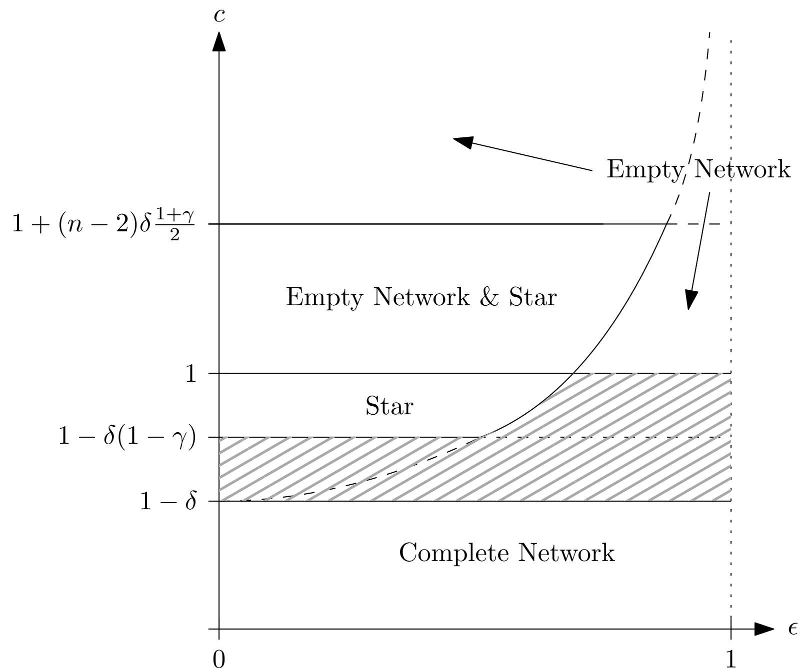


Figure 3: Myopic Stability of Empty, Star, and Complete Networks ($\epsilon = 1$)

efficient, it is also stable. However, in other areas stability and efficiency diverge reflecting the presence of externalities in payoffs. This misalignment between efficiency and stability is familiar from the literature on strategic network formation such as Jackson and Wolinsky (1996): When considering whether to create or destroy a link, the players involved assess the impact only on their own payoffs and disregard the effect on other players.

The shaded area in Figure 3 represents the parameter space in which none of the three configurations is stable for any $n > 3$. The complete network is not robust to deviations in which a pair of players delete a link created by bilateral link creation, because they prefer an indirect connection intermediated by at least two competing middlemen, and thus without intermediation rents, and recouping the linking cost. The star network is not stable either: Here, a deviation by two peripheral players creating a new direct connection via bilateral link formation would be profitable, as it not only gives them the benefit of a direct connection but also allows them to circumvent the intermediation rents earned by the center node in the star network.[7]

[7] Note that the fact that none of the networks identified as efficient in Proposition 2 is stable in the shaded area does not imply that no network at all would be stable in this parameter region. If $c < 1 - \delta(1 - \gamma)$ such that bilateral link creation to circumvent essential intermediaries is profitable, then the two-star network with two hub nodes with connections to all other nodes would be stable as long as $c > \frac{1-\delta}{1-\epsilon}$. If $c < \frac{1-\delta}{1-\epsilon}$ such that links created by introduction create more surplus than indirect connection, then there would be no incentive to ever remove such a link. This implies that under these conditions at least the complete network with all links created by introduction would be stable.

The derived conditions also imply that for certain parameter ranges, multiple configurations can be myopically stable. For example, if linking costs are in a medium range and the efficiency of introductions is not too high, then both the empty and the star network are stable. As shown in Section 3, only one of these would be efficient at the same time. The multiplicity of stable networks is a generic feature of the model and a function of the myopia inherent in the stability concept. Note that multiplicity also implies that even if an efficient structure is stable, it does not necessarily follow that this is the network outcome that is achieved. Under the stability rules proposed here, any stable network can be supported, irrespective of its efficiency properties.[8]

Network	Efficient	Myopically Stable
Empty	$c > 1 + \frac{n-2}{2}\delta$, and $c > \frac{1}{1-(1-\frac{2}{n})\epsilon}$	$c \geq 1$
Star	$c < 1 + \frac{n-2}{2}\delta$, and $c > \frac{1-\epsilon}{1-\delta}$	$c \leq 1 + (n-2)\delta\frac{1+\gamma}{2}$, $c \geq 1 - \delta(1-\gamma)$, and $c \geq \frac{1-\delta}{1-\epsilon}$
Complete	$c < \frac{1}{1-(1-\frac{2}{n})\epsilon}$, and $c < \frac{1-\epsilon}{1-\delta}$	$c \leq 1 - \delta$

Table 1: Parameter Ranges for Efficient and Myopically Stable Networks

4.2 Stable Networks' Maximum Distance and Connectedness

Next, I briefly study the maximum distance of networks that are pairwise stable with introductions. I identify an upper bound on the number of connections of the two highest degree nodes that are a distance of more than three steps apart.

Proposition 5. *Let g be a network that is pairwise stable with introductions. Let (i, j) be the pair of nodes with highest sum of degrees $d_i(g) + d_j(g)$ such that $\ell_{ij} > 3$. Then:*

$$d_i(g) + d_j(g) \leq -\frac{2\left[(1-c)\right]}{\delta(1+\gamma)} \tag{4}$$

The result shows that even in settings where an isolated link is not profitable ($1 - c < 0$), sufficiently high intermediation benefits can exert a force toward limiting the diameter of a network by encouraging higher degree nodes that are far away from each other to connect and benefit from intermediating a number of indirect connections.

[8] I thank an anonymous referee for making this point.

Note that the result leverages both the returns from indirect connections and the intermediation rents for nodes that bring together otherwise disconnected parts of the network. The threshold degree is falling as payoffs for indirect connections δ and returns for intermediation γ increase. In contrast, if $\delta \to 0$ the threshold tends to infinity if link costs exceed the direct benefits of a link ($c > 1$). Finally, once link costs are sufficiently small, the result implies that any stable network will be empty or fully connected.

Corollary 6. *If link costs are sufficiently small such that $c < 1 + \delta\frac{1+\gamma}{2}$, any non-empty network that is a pairwise stable network with introductions will be connected.*

4.3 Stable Networks Exhibit Minimum Level of Clustering

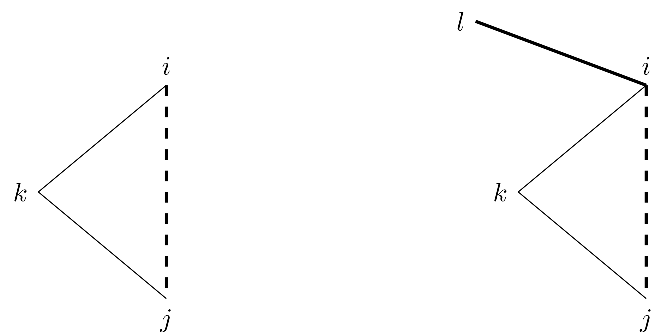
This section considers the impact of introductions on properties of stable networks more generally, focusing on clustering and connectedness. Beginning with clustering, I first ask which possible introductions will remain unused in stable networks. I then extend the analysis to derive a lower bound for local clustering coefficients in stable networks.

Proposition 7 shows that the payoff from an introduction of i and j by k depends on the local network environment only through nodes that are neighbors of either i or j but not both.

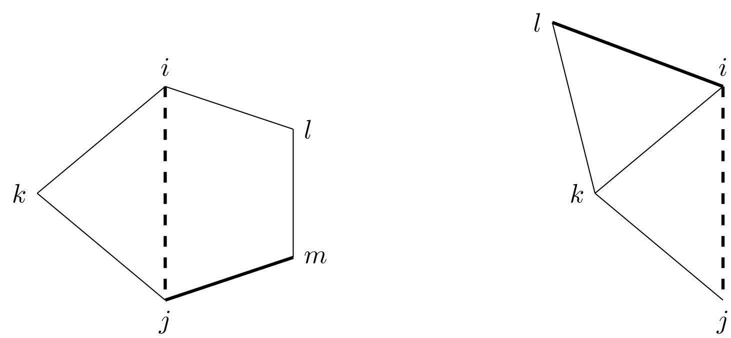
Proposition 7. *Consider a network g such that there is an opportunity for k to introduce i and j. The change in total payoffs to i, j, k from creating the introduction in the sense of Definition 3 is*

$$\begin{aligned}\Delta\pi_{ijk} =& 1-(1-\epsilon)c-\delta \\ &+\delta \sum_{\substack{u\neq v\neq w:\\ v,w\in\{i,j\},\, u\notin\{i,j,k\},\\ uv\in g,\, uw\notin g,\, uk\notin g}} \left[I_{\{\ell_{uw}>2\}}\frac{1+\gamma}{2}+I_{\{\ell_{uw}=2\,\wedge\, r_{uw}=1\}}\frac{\gamma}{2}\right] \\ &+\delta \sum_{\substack{u\neq v\neq w:\\ v,w\in\{i,j\},\, u\notin\{i,j,k\},\\ uv\in g,\, uw\notin g,\, uk\in g}} \left[I_{\{r_{uw}=1\}}\frac{-\gamma}{2}\right]\end{aligned}$$

The result decomposes the overall effect into several constituents based on the local network structure around the introduction: first, the payoff effect from the *new direct link* between i and j that replaces the indirect connection via k with a direct link (Figure 4a); second, the positive payoff effect from the *new indirect link* between any nodes not directly connected to k and connected to exactly one of the nodes to be introduced i, j, where no indirect link previously existed (Figure 4b); third, the positive payoff effect from such a newly created indirect link where that new link helps *avoid intermediation rents* previously paid to a third node (Figure 4c); and fourth, the negative payoff effect from such a newly created indirect link where that link means that the introducing node *loses*



(a) New Link Direct (b) New Link Indirect (+ve)



(c) Avoided Intermediation Rents (+ve) (d) Lost Intermediation Rents (-ve)

Figure 4: Payoff Implications of an Introduction

intermediation rents previously captured. Note that all but the first of these components relate to nodes that are connected to exactly one of the nodes to be introduced. Proposition 7 shows that these are the only configurations that have payoff implications for the three nodes involved in an introduction.

It follows immediately from Proposition 7 that we can bound the payoff from an introduction from below by focusing on the number of neighbors of i or j that can generate a negative payoff for $\{i, j, k\}$ jointly.

Proposition 8. *Consider a network g such that there is an opportunity for k to introduce i and j. Let μ be the number of nodes u such that (i) $uk \in g$, (ii) $ui \in g$ or $uj \in g$, but not both, and (iii) k is essential for the indirect connection between u and j (or u and i), respectively. If g is myopically stable with introductions then*

$$\mu \geq \mu^* \equiv \left\lfloor 2\frac{1-(1-\epsilon)c-\delta}{\delta\gamma} \right\rfloor$$

Note that for μ^* to be positive requires an introduction to be profitable in isolation, that is, in a setting without any neighbor to ijk such that $1-(1-\epsilon)c-\delta > 0$.We will focus on this case in the subsequent discussion. If introductions by themselves are not profitable to the three nodes involved even without any of the additional effects identified above, the results below have no bite. Conditional on isolated introductions being profitable, μ^* increases as intermediation profits γ become less important, because the loss of intermediation benefits to the introducing node is the friction that can prevent introductions from being profitable. In addition, μ^* is increasing in ϵ, the cost advantage of introductions relative to direct bilateral link creation.

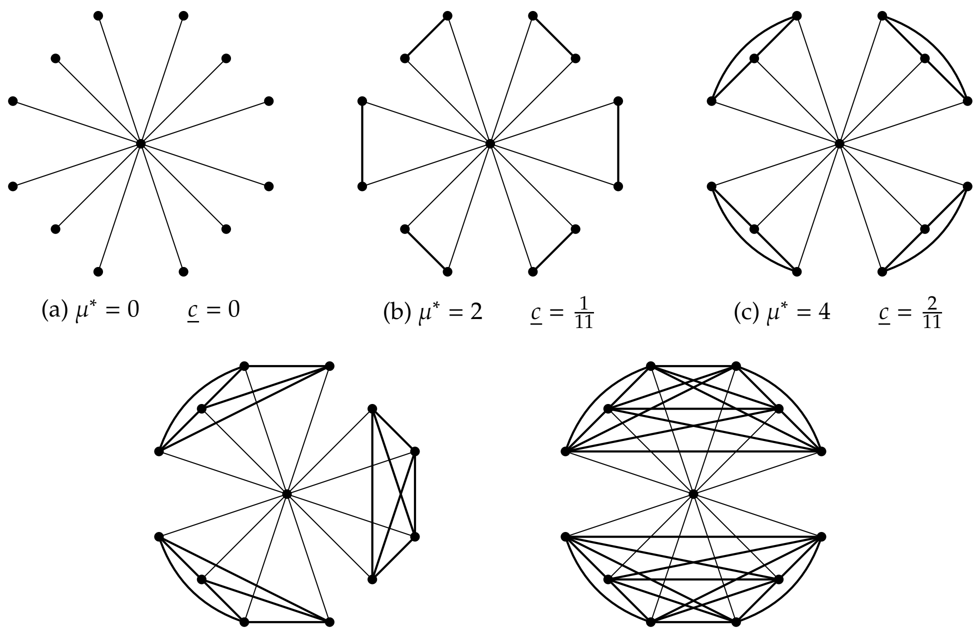
Proposition 8 connects any unused introduction opportunity to a minimum number of nodes that form a closed triangle. We can then establish the following result bounding the local clustering coefficient of any node in a myopically stable network based on the underlying parameters of the model.

Proposition 9. *Let g be a network that is myopically stable with introductions. Define $\lambda^* = \lfloor \frac{\mu^*}{2} \rfloor + 1$. Then the local clustering coefficient c_k of any node k with degree $d_k \geq 2$ in the network is bounded below by $\underline{c}(d_k)$ such that*

$$c_k \geq \underline{c}(d_k) \equiv \begin{cases} 1 & \text{if } d_k \leq \lambda^* \\ \left\lfloor \frac{d_k}{\lambda^*} \right\rfloor \frac{(\lambda^*-1)\lambda^*}{(d_k-1)d_k} & \text{if } d_k > \lambda^* \end{cases}$$

The bound is derived by assigning all neighbors of node k to cliques, that is, fully connected subnetworks (excluding links to k) of size λ^* nodes, which implies that each pair of unconnected nodes together have at least μ^* neighbors each that they share with k but that the nodes do not share with each other. This insures that any open triangles between cliques in the neighborhood of k are myopically stable by Proposition 8. The bound can be reached if μ^* is even and d_k is divisible by λ^* so that all neighbors of k are part of a clique of minimum size. If μ^* is odd, then stability requires that at most one clique can be of exactly size λ^* and all remaining ones have to be of size $\lambda^* + 1$. Furthermore, if d_k is not divisible by λ^*, the remaining nodes have to be added to existing cliques to ensure stability.

The higher μ^*, the higher the lower bound on local clustering. That is, as introductions become more profitable, minimum clustering increases. As the degree of k increases, the bound converges toward zero. Figure 5 illustrates Proposition 9 for a simple example of a central node with degree 12.[9] Note that depending on the underlying parameters, minimum clustering might be zero as in Figure 5.a or relatively high as in Figure 5.e.



(a) $\mu^* = 0$ $\quad \underline{c} = 0$ (b) $\mu^* = 2$ $\quad \underline{c} = \frac{1}{11}$ (c) $\mu^* = 4$ $\quad \underline{c} = \frac{2}{11}$

(d) $\mu^* = 6$ $\quad \underline{c} = \frac{3}{11}$ (e) $\mu^* = 10$ $\quad \underline{c} = \frac{5}{11}$

Figure 5: Example of Minimum Clustering with Cliques of Even Size $\frac{\mu^*}{2} + 1$

The driving force behind the minimum clustering in this result is the introduction mechanism and not the presence of intermediation rents alone. This is in contrast to the results in Goyal

[9] Möhlmeier, Rusinowska, and Tanimura (2016) generate similar windmill shaped networks or friendship graphs from very different mechanics in their model of network formation with both positive and negative externalities. In Ambrus and Elliott (2020), friendship graphs are efficient structures for risk-sharing if common friends are required to enforce risk-sharing agreements.

and Vega-Redondo (2007), who focus on the incentives of disconnected players to create a new link bilaterally in order to avoid paying intermediation rents. In my model, the incentive for triadic closure arises from the benefits that a direct link offers relative to an indirect link if the intermediating player can be sufficiently compensated for the loss of intermediation benefits. Indeed, in Goyal and Vega-Redondo (2007) a higher intermediation rent parameter would likely increase the tendency for links to be formed,[10] whereas in Proposition 9 higher intermediation rents per connection are associated with a lower μ^*. This implies a lower minimum clustering coefficient and consequently higher realized intermediation payoffs for the central node.

Proposition 9 explains how the combination of introductions and intermediation rents can result in networks with high degrees of clustering. This is not a unique prediction. Other models of network formation are also able to generate high degrees of clustering. Closely related to the present paper, Jackson and Rogers (2007) show in a random graph model of network formation how a process of linking to friends-of-friends can result in significant clustering. They find that a suitable combination of fully random and friend-of-friend linking can result in average clustering coefficients that match well those of a number of example networks from different domains, including the world wide web, academic citations, coauthorship, high school romance, and others.

The key conceptual difference between Jackson and Rogers (2007) and the present paper is that the former is a model of network formation based on a stochastic process of network growth, whereas the latter adopts a strategic approach that explicitly models the payoffs to individuals from any given network structure. Despite the differences between the two approaches, both provide useful insights and are complementary in nature. Relative to the random network formation approach, the strategic approach permits insights into the incentives for link formation resulting from an underlying structural description of costs and benefits. The strategic approach also permits a direct characterization of efficient network structures, whereas in random graph models, efficiency can be inferred only indirectly.

In addition, by definition random graph models of network formation generate a "random network," or a distribution of networks, which can be characterized to show certain stochastic properties, for example, the mean degree or the probability of a giant component, often using mean-field approximation. For example, Jackson and Rogers (2007) are able to derive their results on network clustering by characterizing the expected share of transitive triples or the clustering coefficient of the average node (Jackson and Rogers, 2007, Theorem 2). By contrast, the clustering characterization of Proposition 9 in this paper applies to each node and does not describe an average over the whole network.

In terms of predictions, a key distinction between this paper and Jackson and Rogers (2007)

[10] In their paper the authors hold the share of surplus captured by essential intermediaries fixed.

relates to network bridges and structural holes. The mechanism of introduction together with intermediation rents that this paper builds on explicitly connects both clustering and the existence of structural holes bridging otherwise unconnected parts of the network, such as those described in Burt (1992, 2007). In the present paper, as shown in Propositions 8 and 9, there are incentives for nodes to close open triangles up to the point where such new links would threaten their rents from intermediation. As a result, network bridges across densely connected subnetworks can be stable outcomes. The random linking process in Jackson and Rogers (2007) can create bridges between otherwise disconnected parts of the network as well, but in the absence of a model of node utility derived from network structure, the existence of each such link is a random variable and unrelated to whether or not it is profitable for the bridging nodes because of the parameters of the model and the specific network structure and intermediation opportunities surrounding the bridge.

4.4 Discussion

In summary, the myopic stability analysis of the model with introductions reveals how the possibility of introductions creates a lower bound on the level of clustering, providing a plausible mechanism for the observed high levels of clustering in many real-world networks. In addition, it finds that stable networks tend to be connected in a single component, which is consistent with the short distances often found in the data. Jointly, under certain parameters, networks with the small property (Watts and Strogatz, 1998) can be stable.

The combination of clustering with the existence of bridging nodes that confer substantial intermedation returns to the nodes operating them as illustrated in Figure 5 relies on having both intermediation rents and introductions in the model. If introductions do not offer a cost advantage ($\epsilon = 0$), then whether or not an open triangle is closed depends only on the bilateral net benefit $1 - c - \delta$ plus changes to intermediation returns. Likewise, if there are no intermediation rents ($\gamma = 0$), then by Proposition 7 there is no loss for introducing players. This implies that the lower bound of closed triangles required to support any open triangle in the sense of Proposition 8 goes to infinity, resulting in all feasible introductions taking place. There would be complete clustering and no structural holes remaining.

Note that while the tendency for connected components and increased clustering through introductions captures features of real-world organization and business networks (Kogut and Walker, 2001), the analysis also highlights the limits to this process. Specifically, the process of link creation by introduction stops with bridge agents that are essential and connect otherwise disconnected parts of the network. If these individuals earn sufficiently high intermediation rents, then their incentives will be to prevent connections forming between the two parts on either side of the bridge to protect these rents. The stability analysis thus provides a plausible explanation for network connectedness, clustering and the persistence of returns to bridging agents that connect across structural holes.

5 Conclusion

This paper analyzes network formation with introductions in a connections model with intermediation rents. Specifically, the paper considers a model of network formation in which players that are unconnected but share a direct neighbor can be "introduced" by that neighbor. Links created by introduction offer payoff advantages over links created bilaterally, but they can threaten intermediation rents for the introducing node. The trade-off between the costs and benefits limits the extent to which nodes take advantage of introductions to create highly clustered networks via triadic closure. The paper shows first that efficient network structures are either empty, star networks or complete networks, as in the baseline connections model of Jackson and Wolinsky (1996). The paper then conducts a stability analysis using a suitable adapted notion of pairwise stability with introductions, to study the incentives involved in introductions and the impact on network outcomes. As expected there can be a gap between efficiency and stability due to the externalities involved in network formation. More generally, if the benefits to linking are sufficiently high, stable networks tend to be connected and exhibit a minimum level of clustering which increases with the payoff advantages of introductions. Triadic closure is limited by intermediary nodes' incentive to protect the rents they receive from connecting others and thus minimum clustering declines with the share of rents captured by intermediaries.

The paper offers insights into how plausible local dynamics in link creation can lead to clustering, as groups of nodes that are already connected to some degree form additional beneficial connections among themselves. In applications, such network-based coordination may explain the high levels of clustering observed in real-world networks, even in settings where it may be difficult for two players to connect independently. More generally, the analysis underlines how the involvement of additional nearby players with suitable transfer in link creation may help to deal with externalities that tend to push equilibrium outcomes toward over- or under-connected networks, as in Bloch and Jackson (2007). Future research may consider how these insights can be applied to more general settings, such as coalitions of more than three players forming connected cliques.

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Appendix

A Proofs

A.1 Proof of Proposition 2 — Efficient Network Configurations

Proof. The proof adapts the efficiency results for the standard connections model described in Jackson (2008, Chapter 6.3). An important difference here is the distinction between link creation through search and introduction, which requires a discussion of link costs. For any component in a given network g, Lemma 10 provides a lower bound on the number of links that need to be created by search. The proof is immediate and omitted here.

Lemma 10. *The number of links created through search to create a component of k nodes is at least $k-1$. The minimum cost to form a component of k nodes with $m \geq k-1$ links is $mc - (m-(k-1))\,\epsilon c$.*

Having established the minimum-cost way to form any network g, we can proceed to the proof of efficient network structure.

First, consider the case with $c > \frac{1-\delta}{1-\epsilon}$. I will argue that the star is the efficient configuration to connect k nodes in this case. A star network of k nodes incurs link costs of exactly $(k-1)c$ and generates a net benefit including costs of link formation of:

$$(k-1)\,[1-c] + \frac{(k-1)(k-2)}{2}\delta \tag{5}$$

Now any other configuration connecting k nodes with $m \geq k-1$ links will generate at most

$$m\,[1-c] + \left(\frac{k(k-1)}{2} - m\right)\delta + [m-(k-1)]\,\epsilon c \tag{6}$$

where the first component represents the net benefits from direct links and the second component reflects the upper bound of the benefits that can be derived from any indirectly connected nodes. The third component reflects the cost savings from using the maximum feasible number of links created through introduction $m-(k-1)$. Subtracting the second equation from the first and rearranging yields the payoff advantage for the star relative to any other configuration of k nodes:

$$(k-1-m)\,[1-\delta-(1-\epsilon)c] \tag{7}$$

As $c > \frac{1-\delta}{1-\epsilon}$ and $m \geq k-1$, the payoff advantage of the star over any other $m \geq k-1$ link network is minimized at $m = k-1$. It is thus established that if $c < \frac{1-\delta}{1-\epsilon}$, efficient networks consist of star networks and isolated nodes.

Next, we further restrict the set of candidate efficient networks for $c > \frac{1-\delta}{1-\epsilon}$ by establishing that there is either a single star of all nodes or an empty network.

Assume a candidate network consisting of two stars with $k_1 \geq 1$ and $k_2 \geq 2$ nodes yielding positive utility each. Then total payoff is:

$$(k_1 - 1)\left[1 - c + (k_1 - 2)\frac{\delta}{2}\right] + (k_2 - 1)\left[1 - c + (k_2 - 2)\frac{\delta}{2}\right] \tag{8}$$

Reconfiguring the nodes into a single star yields:

$$(k_1 + k_2 - 1)\left[1 - c + (k_1 + k_2 - 2)\frac{\delta}{2}\right] \tag{9}$$

Now, subtracting the first equation from the second and simplifying yields:

$$[1 - c] + (2k_1 k_2 - 2)\frac{\delta}{2} \tag{10}$$

which is strictly positive if each separate star yields positive utility as $2k_1k_2 > k_1$ and $2k_1k_2 > k_2$. Thus, a network with more than one star (including the case where all but one star are single disconnected nodes) yields strictly less utility than a network in which the nodes involved are combined into a single star consisting of n nodes.

The treatment of the case $c > \frac{1-\delta}{1-\epsilon}$ is concluded by comparing payoffs of a single star involving n nodes and the empty network. As the latter derives zero utility, the star is the unique efficient structure if:

$$(n - 1)(1 - c) + \frac{(n - 1)(n - 2)}{2}\delta > 0 \tag{11}$$

which reduces to:

$$c < 1 + \frac{(n - 2)}{2}\delta \tag{12}$$

Next consider the case with $c \leq \frac{1-\delta}{1-\epsilon}$. In this case, adding a link by introduction weakly increases utility. For a component of k nodes, the minimum number of links created through search is $k - 1$. Given $k - 1$ links created by search, the structure that maximizes the number of introductions is the star, which allows a fully connected component of size k to be formed with $k - 1$ links created by search and all remaining links formed by introduction.

Now, for the case where there are more than one such fully connected components, an argument analogous to that for two stars used above shows that utility increases in a single connected component. Thus, if $c \leq \frac{1-\delta}{1-\epsilon}$, the efficient network is either complete or empty. Utility from the complete network with $n-1$ links created by search and all remaining links by introduction is strictly higher if:

$$(n-1)\frac{n}{2}[1-c] - (n-1)\frac{(n-2)}{2}(1-\epsilon)c > 0 \tag{13}$$

which reduces to

$$c < \frac{1}{1 - \frac{n-2}{2}\epsilon} \tag{14}$$

□

A.2 Proof of Proposition 4 — Stability of Efficient Network Configurations

This section derives the parameter restrictions for the stability of the three network configurations that can be efficient, including the empty network, the star network, and the complete network.

Empty Network In the empty network, no links exist implying $\pi_i(g) = 0 \; \forall \; i$ and thus the only condition to verify is bilateral link creation, yielding the stability condition:

$$c \geq 1 \tag{15}$$

Thus the empty network is stable if the benefits from a single new link do not outweigh the costs.

Star Network In the star network, the set of possible deviations to consider applies to two types of nodes (hub and spoke) and there are opportunities to destroy links as well as to create links through search and introductions:

(a) The star network is stable against *link destruction* by the hub and one peripheral node if:

$$c \leq 1 + (n-2)\frac{1+\gamma}{2}\delta \tag{16}$$

(b) The star network is stable against *bilateral link creation* of two peripheral nodes if:

$$c \geq 1 - \delta(1-\gamma) \tag{17}$$

(c) The star network is stable against *introduction* of a pair of peripheral nodes and the hub if:

$$c \geq \frac{1-\delta}{1-\epsilon} \tag{18}$$

Complete Network Total net payoffs from the complete network before search costs and transfers are given by:

$$\pi(g) = \frac{n-1}{2}(1-c) \tag{19}$$

As all possible links are in place, the only deviation to consider is whether a pair of players would find it profitable to destroy a link. In this case, a single direct link is replaced with a set of $n-2$ indirect connections intermediated by the remaining nodes connected to the two nodes breaking their link. For $n > 3$, this implies that there are at least two such intermediaries and the two disconnecting players jointly capture δ. The complete network is thus stable against link destruction if:

$$c \leq 1-\delta \tag{20}$$

A.3 Proof of Proposition 7

Proof. The introduction of i and j by k creates a new link ij. The direct effect on payoffs of i, j and k jointly results from replacing an indirect connection between i and j with a direct connection at the cost of the introduction. The direct effect on payoffs is thus given by

$$1-(1-\epsilon)c-\delta$$

In addition, the new link ij has implications for payoffs generating connections with other nodes. These effects only involve nodes that are neighbors of exactly one of i and j. This is because the new link ij affects payoffs arising from another node $u \notin \{i, j, k\}$ only if link ij either (i) shortens an existing path or (ii) provides a new path of length two (thereby creating connection benefits or cutting intermediation rents). Neither is possible if node u is at a distance of at least two from *both* i and j. Furthermore, if a node u is connected to both i and j, then the new link ij does not affect any connection where u acts as an end node. In addition, even though the link ij replaces intermediation via u with a direct link on the connection between i and j, this has no payoff implications as the existence of such u implies that there are at least two intermediaries for the i, j connection: u and k.

We thus focus on the changes to payoffs for ijk arising from nodes u that are neighbors of exactly one of i and j. There are then two cases to consider depending on whether or not u is connected to k as well.

1. $u \neq v \neq w : v, w \in \{i, j\}, u \notin \{i, j, k\}, uv \in g, uw \notin g, uk \notin g$
   In this case, u is not connected to k. The new link ij created by the introduction will create a new connection of length two between u and i or j. If prior to the introduction there was no such connection of length two, then the payoff to ijk will be $\delta\frac{1+\gamma}{2}$ as i and j will act as end node and unique intermediary for the link with u. If there already exists a connection of length two between u and one of i or j (intermediated by a neighbor of i or j, respectively) and this connection had only one intermediary, then the new link ij will create an alternative indirect connection and reduce the intermediation rents that have to be paid. Thus, payoffs to ijk will increase by $\frac{\gamma}{2}$.

2. $u \neq v \neq w : v, w \in \{i, j\}, u \notin \{i, j, k\}, uv \in g, uw \notin g, uk \notin g$
   In this case, u is connected to k and thus both i and j will already have a connection of length weakly less than two to u, one with a direct link and one via intermediation through k. Thus, the new link ij created by the introduction will not provide a new connection. However, if k is essential to the connection of u and i or j, then the new link ij will create an alternative connection of length two and thus the intermediation benefits captured by k will be lost. Only part of this will be captured by i and j and thus the overall loss to ijk is $\delta\frac{\gamma}{2}$.

Combining direct and indirect effects yields the expression in Proposition 7.

□

A.4 Proof of Proposition 8

Proof. The proof is by contradiction. Assume that network g is myopically stable with introductions and there exists an unused introduction opportunity for k to introduce i and j and μ, that is, the number of nodes u connected to k and either i (or j) such that k is essential for the connection between u and j (or i, respectively) is strictly less than $\mu^* = \left\lfloor 2\frac{1-(1-\epsilon)c-\delta}{\delta\gamma} \right\rfloor$.

Substituting the expression in Proposition 7 and dropping the non-negative middle term, the payoff change to ijk is then at least

$$
\begin{aligned}
\Delta\pi_{ijk} \geq& 1-(1-\epsilon)c-\delta-\mu\frac{\delta\gamma}{2} \\
>& 1-(1-\epsilon)c-\delta-\mu^*\frac{\delta\gamma}{2} \\
=& 1-(1-\epsilon)c-\delta-\left\lfloor 2\frac{1-(1-\epsilon)c-\delta}{\delta\gamma}\right\rfloor\frac{\delta\gamma}{2} \\
\geq& 1-(1-\epsilon)c-\delta-\left(2\frac{1-(1-\epsilon)c-\delta}{\delta\gamma}\right)\frac{\delta\gamma}{2} \\
=& 1-(1-\epsilon)c-\delta-(1-(1-\epsilon)c-\delta) \\
=& 0
\end{aligned}
$$

The introduction thereby is strictly jointly profitable to ijk, delivering the contradiction.

□

A.5 Proof of Proposition 9

Proof. The proof is by contradiction. Let g be myopically stable with introductions and with node k with degree d_k such that the clustering coefficient c_k is strictly below $\underline{c}(d_k) = \lfloor \frac{d_k}{\lambda^*} \rfloor \frac{(\lambda^*-1)\lambda^*}{(d_k-1)d_k} \leq \frac{\lambda^*-1}{d_k-1}$.

First some notation. Let the set of nodes that are neighbors of k in g be denoted by k. Define by g_K the subnetwork of g formed by the set of nodes k and the links that are links between nodes in k in g. The degree of node i within g_K is labeled d_i^K and the average degree across k is labeled d_K. Note that all the links in g_K are between neighbors of k and thus these are the links that are required for stability against introductions in the sense of Proposition 8.

We distinguish between two cases based on the number of nodes in the neighborhood of k:

Case 1: $d_k \leq \lambda^*$: If $d_k \leq \lambda^*$, then $\underline{c}(d_k) = 1$, that is, the neighbors of k form a fully connected subnetwork. Thus if $c_k < \underline{c}(d_k)$ there exists at least one pair i, j in the neighborhood of k with $ij \notin g_K$. Then the maximum number of links of both i and j within g_K is strictly less than $\lambda^* - 1 = \frac{\mu^*}{2}$ and thus the total number of neighbors that the pair i and j share with k but not with each other is strictly less than μ^*. By Proposition 8 this implies that g is not myopically stable with introductions, delivering the contradiction.

Case 2: $d_k > \lambda^*$: If $d_k > \lambda^*$, then $\underline{c}(d_k) = \left\lfloor \frac{d_k}{\lambda^*} \right\rfloor \frac{(\lambda^*-1)\lambda^*}{(d_k-1)d_k}$. Now, assume that this condition is violated and thus

$$\begin{aligned} c_k &< \underline{c}(d_k) \\ &= \left\lfloor \frac{d_k}{\lambda^*} \right\rfloor \frac{(\lambda^* - 1)\lambda^*}{(d_k - 1)d_k} \\ &\leq \frac{(\lambda^* - 1)}{(d_k - 1)} \end{aligned}$$

As the total number of triangles around k is given by $\frac{(d_k-1)d_k}{2}$, the condition on clustering c_k implies that the total number of closed triangles around k, and thus links, in network g_K is at most $(\lambda^* - 1)\,\frac{d_k}{2}$. With each link being shared by two nodes and a total of k nodes in g_K, it follows that the average degree d_K is strictly less than $\lambda^* - 1 = \frac{\mu^*}{2}$.

We can now show that $\min_{ij \notin g_K} \left[d_i^K + d_j^K \right] < \mu^*$, that is, there is at least one pair of nodes with no link between them whose degrees in g_K add up to less than μ^*. By Proposition 8 this implies that the network is not myopically stable with introductions.

$$\min_{ij:i,j\in K, ij\notin g_K}\left[d_i^K + d_j^K\right] \leq \frac{1}{|\{ij : i,j \in K, ij \notin g_K\}|}\sum_{ij:i,j\in K,ij\notin g_K}\left[d_i^K + d_j^K\right] \tag{21}$$

$$\leq \frac{1}{|\{ij : i,j\in K\}|}\sum_{ij:i,j\in K}\left[d_i^K + d_j^K\right] \tag{22}$$

$$= \frac{2}{(d_k - 1)}\sum_{ij:i,j\in K}\left[d_i^K + d_j^K\right] \tag{23}$$

$$= \frac{2}{d_k(d_k-1)}\sum_{i\in K}\sum_{j\in K:j>i}\left[d_i^K + d_j^K\right] \tag{24}$$

$$= \frac{1}{d_k(d_k-1)}\sum_{i\in K}\sum_{j\in K:j\leq i}\left[d_i^K + d_j^K\right] \tag{25}$$

$$= \frac{1}{d_k(d_k-1)}\left\{\sum_{i\in K}\sum_{j\in K:j\neq i} d_i^K + \sum_{j\in K}\sum_{i\in K:i\neq j} d_j^K\right\} \tag{26}$$

$$= \frac{1}{d_k(d_k-1)}\left\{\sum_{i\in K}(d_k-1)d_i^K + \sum_{j\in K}(d_k-1)d_j^K\right\} \tag{27}$$

$$= \frac{1}{d_k}\left\{\sum_{i\in K} d_i^K + \sum_{j\in K} d_j^K\right\} \tag{28}$$

$$= 2d_K < \mu^* \tag{29}$$

The first step uses the fact that the minimum is less than the average. In the second step we exploit the fact that the average of the sum of degrees across absent links is weakly less than the average across all links. This follows from the fact that in this expression conditioning on the absence of a link underweights high degree nodes relative to low degree nodes.[11]

□

A.6 Proof of Proposition 5

Proof. The proof is by contradiction. Assume that network g is myopically pairwise stable with introductions. Further, assume that there are i, j violating the condition in Equation 4 such that $\ell_{ij} > 3$ and $d_i(g) + d_j(g) > -\frac{2(1-c)}{\delta(1+\gamma)}$.

Consider a new connection being formed between i and j. The new link will create a direct

[11] See Vega-Redondo (2007, p.44f) for a discussion of this point.

connection with direct benefit net of costs of $1-c$. In addition there are $d_i(g)+d_j(g)$ new connections of length two creating a benefit of δ each. For each such connection, i and j form an end node and, as $\ell_{ij} > 3$, an essential intermediary, respectively, and thus the two nodes capture a benefit of $\frac{1+\gamma}{2}\delta$ for all these connections.

The total payoff from the connection to i and j is thus

$$
\begin{aligned}
\Delta\pi_{ij} &= \left[d_i(g) + d_j(g)\right] \delta \frac{1+\gamma}{2} + (1-c) && (30)\\
&> -(1-c) + (1-c) && (31)\\
&= 0 && (32)
\end{aligned}
$$

and thus the additional link is jointly profitable and the network g is not pairwise stable, delivering the contradiction.

□