Oligopsonies over the Business Cycle

Daniel Monte and Roberto Pinheiro

Oligopsonies over the Business Cycle
Daniel Monte and Roberto Pinheiro

With a duopsony model, we show how the degree of labor market slack relates to earnings inequality and firm size distribution across local labor markets and the business cycle. In booms, due to the high aggregate productivity, there is fierce competition with resulting high wages and full employment. During recessions, there is labor market slack and firms enjoy local market power. In periods in which the economy is moving in or out of a recession, there is an "accommodation" phase, with firms shrinking their labor forces and paying lower wages instead of competing for poached workers. We show that the impact of economic shocks on wage dispersion and inequality may vary not only due to the nature of the shock, but also based on which equilibrium the economy may have settled in.

Daniel Monte is at the São Paulo School of Economics–FGV (daniel.monte@fgv.br). Roberto Pinheiro is at the Federal Reserve Bank of Cleveland (roberto.pinheiro@clev.frb.org). The authors thank Klaus Desmet, Bruce Fallick, Fabian Lange, Jan-Peter Siedlarek, and Murat Tasci for their comments.

1 Introduction

The relationship between labor market slack and wages is still an open question. For example, even though the unemployment rate ended 2018 below 4 percent, substantially lower than most estimates of the natural rate of unemployment, wage growth has been moderate (Leduc et al. (2019)). Wage rigidities are often used to explain this mild response of wage growth to tight labor market conditions, depicted as a flattening of the wage Phillips curve (see Daly and Hobijn (2014)). However, recent empirical evidence has shown that downward nominal wage rigidities are not as binding as we initially thought (see Jardim et al. (2019) and Elsby and Solon (2018)).

Differently, a recent literature has highlighted the importance of market power for both product and labor markets. In particular, local labor markets have proven to be particularly concentrated (see Azar et al. (2018) and Benmelech et al. (2018)), giving local employers significant bargaining power. Moreover, the evidence shows that workers are likely to apply for jobs in a limited geography (Marinescu and Rathelot (2018)) and we have seen the decline in interstate migration (Molloy et al. (2011)). As a result, adjustments through entry and exit of workers in local labor markets that may reduce employers' market power are either muted or in decline. As a result, the importance of employers' labor market power is likely to have increased over time. Indeed, Dube et al. (2020) show that market power might be present even in thick markets, with low search frictions.

In this paper, we present a model that links labor market slack to the wage distribution, taking into account employers' market power. In particular, we extend the duopsony labor market model presented by Bhaskar et al. (2002) in order to take into account the impact of labor market slack on firm's optimal behavior. In the model, workers have heterogeneous preferences over the potential employers, but have the same productivity. Firms choose wages and workers choose which firm they want to work for, if any. Workers choose where to work based on posted wages and their intrinsic preferences. If they choose not to work, there is an exogenous outside option. We partition the parameter space and characterize the entire equilibrium set for each combination of parameters.

We show that there are three types of equilibria and relate them to the degree of slack in the labor market. First, when there is slack in the labor force, both firms behave as monopsonies. As the labor market tightens, firms move to an "accommodation" phase, in which both firms may prefer to shrink the size of their labor force and pay lower average wages instead of competing outright for poached workers. For the set of parameters in which firms accommodate, there are multiple equilibria, and in all of them both firms accommodate. Finally, once we move to a range of parameters in which "accommodation" becomes costly in comparison to the cost of fighting poaching, we observe the standard duopsony competition as the one described in Bhaskar et al. (2002).

The fact that there are multiple equilibria in the "accommodation" phase makes the changes in wage inequality as labor market slack declines ambiguous and possibly non-monotonic. The "accommodat-

1

ing" firm allows the competitor to poach its marginal employees. In exchange, the "accommodating" firm is able to reduce its wage bill by reducing the wage paid to intra-marginal workers. Initially, this benefit more than compensates for the loss of the marginal worker, taking into account the cost of fighting to avoid poaching. However, while "accommodation" may induce an increase in between-firm wage inequality, under these parameters, we have multiple equilibria with both firms concomitantly accommodating. As a result, the range of equilibrium wages across firms can vary significantly. Consequently, while we observe weak increases in average wages due to an economy-wide productivity shock in an equilibrium with accommodation, we may see either large increases or large declines in wage dispersion, depending on which "accommodation" equilibrium firms settle in. Eventually, as competitive pressures build up, wage dispersion becomes predictable as wage gaps shrink in order to reduce poaching. In summary, the fact that "accommodation" equilibria occur as the economy is moving in and out of periods with high labor market slack is in line with the ambiguous results found in the empirical literature in terms of the cyclical behavior of earnings and wage inequality over the business cycle (see Barlevy and Tsiddon (2006), Bonhomme and Hospido (2017), and Morin (2019), among others).

When parameters induce monopsonies or a competitive duopsony, the resulting equilibrium is shown to be unique, given any set of parameters. However, the behavior of wage inequality and wage growth may differ significantly depending on the nature of the economic shock. For example, while firm-specific idiosyncratic shocks have a significant effect on both firms' optimal wages in the case of duopsony, it has no effect on a rival's optimal compensation in the case of monopsonies. Similarly, while changes in the degree of workers' substitutability across firms (or industries) have a positive effect on firms' optimal wages in the case of duopsonies, it has no effect on the case of monopsonies, as long as they are not too large to trigger the "accommodation" phase.

The fact that wages in labor markets with different degrees of slack may react quite differently to a given economic shock is important as it contributes to the effects of composition on wage inequality and wage growth (see Daly and Hobijn (2017)). Evidence has shown that the degrees of local labor market slack as well as market power by local employers vary considerably across space as well as over time (Benmelech et al. (2018)). As a result, it is likely that at any given time we have a significant fraction of local labor markets in each type of equilibrium presented. As we move further into the expansion (recession), the fraction of markets in duopsony (monopsony) may increase.

The model presented in this paper is related to the literature on imperfect labor markets. In particular, it is related to the literature on oligopsonies (see Bhaskar and To (1999), Bhaskar and To (2003), and Kaas and Madden (2010), among others). As pointed out by Manning (2003), these models assume that workers have full information and no mobility costs but that jobs are differentiated in some way. In all these models, both workers and employers are located at some point in the characteristics space (modeled as a circle or a line). However, employers exist at only some points in the characteristics

space, henceforth enjoying some market power. Consequently, the extent of an employer's market power depends on how dense employers are in the characteristics space. Most of the literature has focused on the impact of a minimum wage on the wage distribution and on whether the establishment of a minimum wage can trigger an increase in average wages, as empirically shown by Card and Krueger (1994). As a result, the literature focuses on the cases in which there is no slack in the economy. Differently, we show that the presence of labor market slack can have a significant effect on the wage distribution and wage growth. Moreover, we show that labor market slack may also have a significant impact on how the wage distribution and the distribution of workers across firms react to different economic shocks.

Finally, our model is also related to the labor search literature (see Burdett and Mortensen (1998) and Delacroix and Shi (2006) among others). As pointed out by Manning (2003), these models assume that all jobs are identical, but that it takes time and effort for workers to find and switch jobs. These search frictions give employers some market power, even if employers are small compared to the market as a whole. Booth and Coles (2007) point out that oligopsony and search frameworks share several features – in particular, wage compression and the fact that wage and productivity may not grow one-to-one. However, these frameworks are quite distinct regarding the role of labor market slack. First, most search models focus on involuntary unemployment,[1] while unemployment is voluntary in the case of oligopsony models. Additionally, the impact of labor market slack on wage dispersion tends to be monotonic, with the increase in slack necessarily shrinking wage dispersion. Finally, search models with on-the-job search usually show no clear link between slack and the intensity of worker poaching – apart from the impact of employed workers' search effort on the creation of vacancies. Differently, our model highlights the link between labor market slack and worker poaching. In particular, it shows not only the conditions under which worker poaching is likely to be seen, but also how firms strategically set wages in response to the intensity of worker poaching. The presence of a stand-alone accommodation phase is unique to an oligopsony framework, even though one can argue that both accommodation and competition occur concomitantly in Burdett and Mortensen (1998). Even more, the fact that accommodation equilibria occur in the period of transition from a labor market with slack to a heated market generates the non-monotonic patterns in wage and compensation inequality that are unique to the model presented.

The rest of the paper is divided into four sections. Section 2 describes the model and equilibrium patterns. Section 3 introduces a minimum wage policy. Section 4 presents examples of two model extensions. First, we consider the case in which workers are not uniformly distributed across the characteristics space. Second, we consider the case in which workers' transportation cost function is convex. Our results show that the equilibrium patterns presented in Section 2 are robust to these

---

[1]Some search models do allow for households that optimally stay out of the labor force (see Tüzemen (2017)).

extensions. Finally, Section 5 concludes the paper, summarizing our findings. All proofs are in the Appendix.

2 Model

Consider a case in which we have two firms that are competing imperfectly in the labor market. The product market is a perfectly competitive model, while the labor market follows Bhaskar et al. (2002) which is an extension for labor markets of a linear Hotelling model. However, we allow for the fact that firms may not want to hire all available workers. In particular, we consider the case in which workers are distributed across a line of length $\overline{L}$. We will vary $\overline{L}$ as part of our analysis. Each firm is at one extreme of the linear space of length $\overline{L}$ and workers are uniformly distributed across the line. Consequently, firm 1 is in position $x = 0$, while firm 2 is in position $x = \overline{L}$. As in Bhaskar et al. (2002), we consider the worker's location as workers' heterogeneous preferences for different jobs; for example, a worker may have equal productivity in two jobs as measured by the marginal product, but the worker prefers the kind of work or working conditions in one job over the other. We assume here that workers have identical skills and abilities but have heterogeneous preferences over non-wage job characteristics (job specification, hours of work, distance of firm from the worker's home, and the social environment in the workplace). Figure 1 represents the characteristics space and the duopoly, with each firm in one of the two extreme locations.

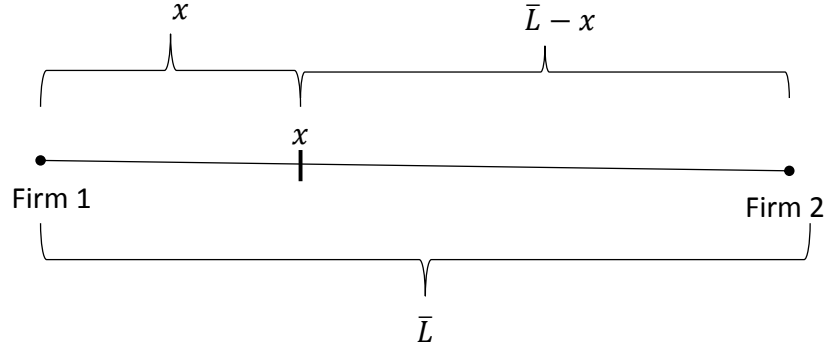Figure 1: The characteristics space

2.1 Worker's Problem

Consider a worker located in position x in the linear space. This worker must travel x to work for firm 1 and $\overline{L} - x$ to work for firm 2. We assume that the worker incurs a transportation cost of t for each mile she travels, so the worker would incur a cost tx to travel to firm 1 and a cost $t(\overline{L} - x)$ to travel to

firm 2. The worker has three options: a) work for firm 1, b) work for firm 2, or c) stay out of the labor force. We assume that workers live forever and are risk neutral (linear utility function). Therefore the benefit of working for each company is given by the received compensation minus traveling costs. The benefit of staying out of the labor force is normalized to zero. Consequently, it is optimal for a worker in position x to accept firm 1's offer if:

$$w_1 - tx > \max\{w_2 - t(\overline{L} - x), 0\} \tag{1}$$

If all workers are employed in equilibrium (no slack), the marginal worker (located at x^*) is indifferent between working for firm 1 and 2, i.e.:

$$w_1 - tx^* = w_2 - t(\overline{L} - x^*) \tag{2}$$

Rearranging it:

$$x^* = \frac{t\overline{L} + w_1 - w_2}{2t} \tag{3}$$

which is also the labor force for firm 1 in this case. Similarly, the labor force for firm 2 in the no-slack case is given by:

$$\overline{L} - x^* = \frac{t\overline{L} + w_2 - w_1}{2t} \tag{4}$$

Once $x^* \in [0, \overline{L}]$, we must have:

$$0 \leq \frac{t\overline{L} + w_1 - w_2}{2t} \leq \overline{L} \ \Rightarrow |w_1 - w_2| \leq t\overline{L} \tag{5}$$

The more interesting case is when the condition presented in equation (5) is satisfied in the case of competition; thus, we impose the following parameter restriction:

Assumption 1 $|A_1 - A_2| < \frac{3t\overline{L}}{p}$.

Differently, in the case in which there is labor market slack, the marginal worker at each firm is indifferent between working or staying out of the labor force; i.e.:

$$w_1 - \bar{x}_1 t = 0 \ \Rightarrow \ \bar{x}_1 = \frac{w_1}{t} \tag{6}$$

and

$$w_2 - t(\overline{L} - \bar{x}_1) < 0 \ \Rightarrow \ \bar{x}_1 < \overline{L} - \frac{w_2}{t} \tag{7}$$

Substituting (6) into (7), we have $w_1 + w_2 < t\overline{L}$. Similarly, for the marginal worker in firm 2 in a world with labor slack, we have:

$$w_2 - (\overline{L} - \bar{x}_2)t = 0 \ \Rightarrow \ \bar{x}_2 = \frac{t\overline{L} - w_2}{t} \tag{8}$$

5

Since again we have slack:

$$w_1 - t\bar{x}_2 < 0 \implies \bar{x}_2 > \frac{w_1}{t} \tag{9}$$

Again, substituting (8) into (9), we have:

$$\frac{t\overline{L} - w_2}{t} > \frac{w_1}{t} \implies w_1 + w_2 < t\overline{L} \tag{LS}$$

So (LS) is the constraint that must be satisfied in order to have labor market slack. Then, the labor supply faced by firms 1 and 2 are, respectively

$$L_1 = \begin{cases} \frac{t\overline{L}+w_1-w_2}{2t} & \text{if } w_1 + w_2 \geq t\overline{L} \\ \frac{w_1}{t} & \text{otherwise} \end{cases} \quad \text{and} \quad L_2 = \begin{cases} \frac{t\overline{L}+w_2-w_1}{2t} & \text{if } w_1 + w_2 \geq t\overline{L} \\ \frac{w_2}{t} & \text{otherwise} \end{cases}$$

2.2 Firm's Problem

We now consider the firm's problem of setting the wage offered to workers. In order to simplify the analysis, we assume that both firms have production functions that depend on only one input (labor) and that it has constant returns to scale, i.e., the production function is linear. Moreover, we assume that both firms have the same production function up to their total factor productivity (TFP) A_i. Finally, we assume that they produce the same homogeneous output that is the economy's numeraire.

In terms of the wage-setting mechanism, we assume that a firm pays the same wage for all workers, irrespective of their traveling costs. Consequently, by paying a higher wage, a firm would not only attract more workers, but it would also increase its wage bill for all currently employed workers at the firm. Moreover, if there is no slack in the labor market, a wage increase implies that the firm is poaching workers from its competitor, which may react by increasing its own wage offer. However, in order to simplify the problem, we assume that firms set their wages simultaneously. Therefore, firm 1's profit maximization is given by:

$$\max_{w_1}(A_1 * p - w_1)L_1(w_1, w_2; t, \overline{L}) \tag{10}$$

So, as we can see, firm 1's problem depends not only on technology parameters (firm 1's TFP A_1 and the overall sector productivity p), but also on the wage offered by firm 2 (w_2), the transportation cost (t), and the size of the potential labor force ($\overline{L}$). We present all the calculations for firm 1's profit maximization in the Appendix. Below, we present firm 1's reaction function, which is given by:

$$w_1^*(w_2) = \begin{cases} \frac{A_1 p}{2} & \text{if } w_2 < t\overline{L} - \frac{A_1 p}{2} \\ t\overline{L} - w_2 & \text{if } t\overline{L} - \frac{A_1 p}{2} \leq w_2 \leq t\overline{L} - \frac{A_1 p}{3} \\ \frac{A_1 p + w_2 - t\overline{L}}{2} & \text{if } w_2 > t\overline{L} - \frac{A_1 p}{3} \text{ and } w_2 < A_1 p + t\bar{L} \\ A_1 p & w_2 \geq A_1 p + t\bar{L} \end{cases} \tag{11}$$

6

Similarly, given the symmetry in the profit maximization problem for firm 2, we have:

$$w_2^*(w_1) = \begin{cases} \frac{A_2 p}{2} & \text{if } w_1 < t\overline{L} - \frac{A_2 p}{2} \\ t\overline{L} - w_1 & \text{if } t\overline{L} - \frac{A_2 p}{2} \leq w_1 \leq t\overline{L} - \frac{A_2 p}{3} \\ \frac{A_2 p + w_1 - t\overline{L}}{2} & \text{if } w_1 > t\overline{L} - \frac{A_2 p}{3} \text{ and } w_1 < A_2 p + t\overline{L} \\ A_2 p & w_1 \geq A_2 p + t\overline{L} \end{cases} \tag{12}$$
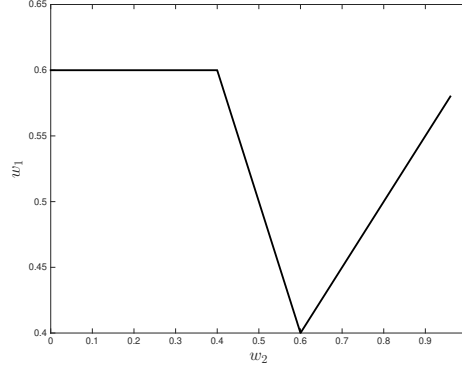

Figure 2: Reaction function for firm 1

Figure 2 presents firm 1' s reaction function. Notice that for low levels of w_2, the optimal wage for firm 1 does not change. The reason for that is that at these wage levels, there is slack in the labor market, and firm 1 is already hiring its optimal labor force; i.e., by increasing its wage, firm 2 attracts workers who preferred to be out of the labor force at the previously posted wages. As firm 2's wages keep increasing, we arrive at a point at which firm 1 must decide if it should accommodate firm 2's increasing demand for wages or compete for the workers. If firm 1 decides to accommodate, it allows firm 2 to poach some of its workers while it reduces the total wage bill by focusing on workers who would have a strong preference for working in firm 1 and consequently accept a lower wage. If firm 1 fights for workers, it must start to increase wages in order to poach back some of the workers that firm 2 is trying to attract. As we can see, firm 1's optimal strategy involves first trying to accommodate. During accommodation, the wage paid by firm 1 actually goes down, since by allowing firm 2 to poach some of its workers, firm 1 can actually reduce the wage paid for the remaining workers, since it does not need to convince marginal workers with high transportation costs to work for the firm. However, as firm 2 keeps increasing its wage, there is a threshold after which firm 1 eventually is better off competing with firm 2 for workers. Then, we observe that the wages paid by firm 1 strictly increase with firm 2's wages.

Given the presented reaction function, we can see that different patterns of equilibria can arise, depending on the parameters. Figure 3 shows three patterns. In the next section, we show that any equilibria in the model fits one of these three patterns.

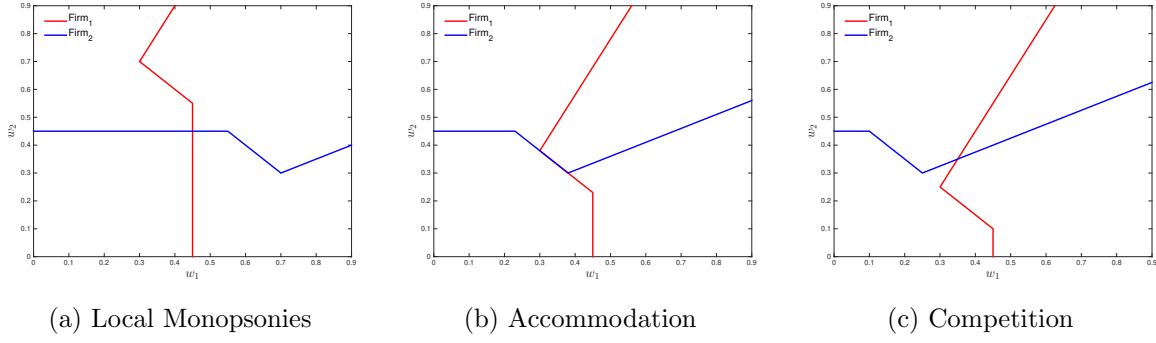|		
(a) Local Monopsonies	(b) Accommodation	(c) Competition

Figure 3: Equilibrium patterns

In Figure 3(a), we have an equilibrium in which both firms are local monopsonies and there is slack in the labor market; i.e., there are workers who in equilibrium prefer to stay out of the labor force at the posted wages. As we can see, in this case the equilibrium is unique. Differently, in the equilibria in which we observe accommodation – Figure 3(b) – all workers are employed, but we observe multiple equilibria. In particular, there is a range of wages posted by the competitors that would induce firms to try to accommodate the opponent's increased demand. As mentioned before, accommodation implies reducing demand and trying to boost profits by reducing the overall wage bill once giving up employees with the highest transportation costs allows the firm to pay less to all remaining employees. Finally, in Figure 3(c) we have the case of competition, in which all workers are either employees of firm 1 or firm 2 and both firms react to an increase in the competitor's wage by increasing their own wage. In this case, the equilibrium is again unique for a given set of parameters.

2.3 Equilibria

In this section, we show all the equilibrium patterns we can observe while ruling out other possible combinations. The following proposition summarizes the results, which we discuss afterward.

Proposition 1 *The following parameter restrictions pin down all three possible equilibrium patterns:*

1. *If $(A_1 + A_2)p < 2t\overline{L}$, both firms are monopsonies, there is labor market slack in equilibrium, and the equilibrium is unique;*

2. *If $(A_1 + A_2)p > 3t\overline{L}$, firms actively compete for workers, there is no labor market slack in equilibrium, and the equilibrium is unique;*

3. *If $\frac{A_1 p}{3} + \frac{A_2 p}{3} < t\overline{L} < \frac{A_1 p}{2} + \frac{A_2 p}{2}$, both firms accommodate each other's labor demand, there is no labor market slack in equilibrium, and there are multiple equilibria.*

8

Proposition 1 shows the parameter restrictions that would induce the different equilibrium patterns presented in Figure 3. In other words, there are no equilibria that follow a different pattern than the ones presented in Figure 3. Consequently, there are no equilibria in which firms are at different sections of their reaction functions.

A simple intuition is the following. Let $(A_1 + A_2)p$ represent aggregate productivity. When the productivity of a firm is low, its monopsony wage is also low, so workers who are further along the characteristics line will not be attracted to the firm. Thus, when aggregate productivity is very low (case 1), it results in a market with no competition and labor market slack, in which both firms act as monopsonies. At the other extreme, when aggregate productivity is high (case 2), firms are willing to pay higher wages in order to attract more workers. This leads to a competitive scenario. In the intermediate case where aggregate productivity is not so low to generate slack, but also not so high to induce competition, we have the accommodation scenario. If one firm is more aggressive and pays higher wages and hires more workers, the other firm's best response is to accommodate instead of competing, thus, the multiplicity of equilibria.

Corollary 1 gives us some interesting implications, showing how the different equilibrium patterns are affected by shocks to the model parameters:

Corollary 1 *Considering small changes in parameter values, i.e., changes that do not trigger a switch in the equilibrium pattern, we have:*

- *In an equilibrium with local monopsonies: Changes in t and $\overline{L}$ have no effect on equilibrium wages. Changes in p increase both firms' wages, while changes in A_i increase only firm i's equilibrium wage, while keeping its competitor's wages constant;*

- *In accommodation equilibria: An increase in t or $\overline{L}$ increases the overall wage bill of the economy, while its partition across firms is undetermined. Changes in A_i, $i \in \{1, 2\}$, and changes in p affect the set of possible equilibrium wages in the economy;*

- *In an equilibrium with competition: An increase in t and/or $\overline{L}$ decreases both firms' wages, while changes in A_i, $i \in \{1, 2\}$, and/or changes in p increase wages for both firms.*

2.4   Policy Implications

Although corollary 1 is a simple observation based on equations (11) and (12), it has some relevant implications for the labor market. If we consider that low-skill workers usually face a labor market in which there is lots of labor market slack due to a large contingent being out of the labor force, neither changes in the size of the potential labor force nor changes that make jobs more similar have any impact on equilibrium wages. Consequently, incremental policies that reduce the overall size of the potential

pool of low-skill workers, such as policies trying to induce workers to retire or to educate workers in order to move them out of the low-skill pool, or policies that would reduce the presence of low-skill immigrants, would have very little impact on the wage distribution of the workers who remain in this market. Similarly ineffective are incremental policies that make jobs more standardized, such as strict regulations on job conditions and other amenities. Overall, for this particular market, only incremental policies that may induce an increase in overall productivity would have an impact on the equilibrium wages.

Differently, in markets in which firms must fiercely compete for workers – for example, in markets with high-skill workers (STEM, abstract non-routine skills) – not only do increases in overall productivity increase wages, but there is also a spillover effect of firm-specific technological progress, i.e.; an increase in firm i's TFP (A_i) generates an increase in its competitor's posted wages. Moreover, a reduction in the size of the potential pool of high-skill workers or an increase in the degree of standardization of jobs (decreases in t) also generates an increase in overall salaries. Finally, policies that would move workers from the bottom of the skill distribution through attainment of education would at best reduce inequality by depressing wages at the top of the skill distribution, without any clear benefit to workers who stayed in the low-skill labor market, unless the measures are paired with a productivity boost.

Finally, the impact of incremental policies on the equilibria with accommodation is not as clear. Policies that boost productivity may not only not increase the wages at any given equilibrium, but may also affect the range of possible equilibria, increasing the possibility of higher overall inequality. Differently, either an increase in the size of the pool of potential workers or policies that make jobs less standardized, while increasing the overall wage bill in the economy may also increase wage inequality across firms. In the next section, we describe these changes in wage patterns in more detail.

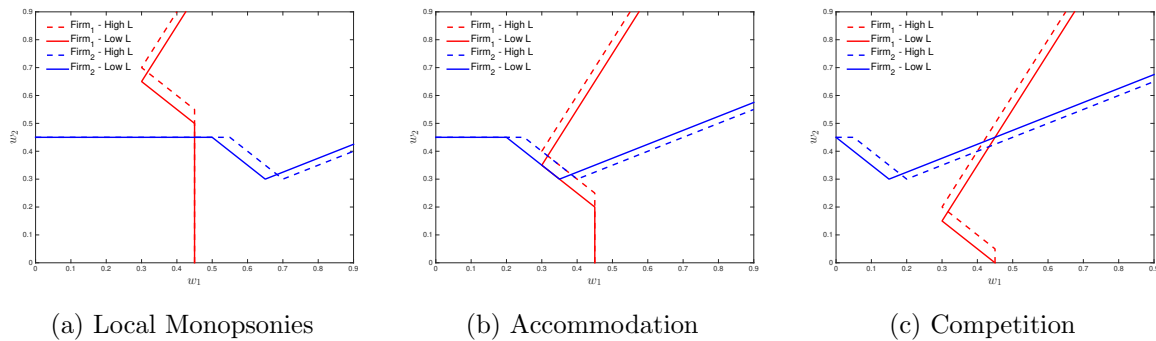Figures 4 to 6 illustrate examples of changes in the lines discussed above.

(a) Local Monopsonies   (b) Accommodation   (c) Competition

Figure 4: Changes in $\overline{L}$

(a) Local Monopsonies   (b) Accommodation   (c) Competition

Figure 5: Changes in p
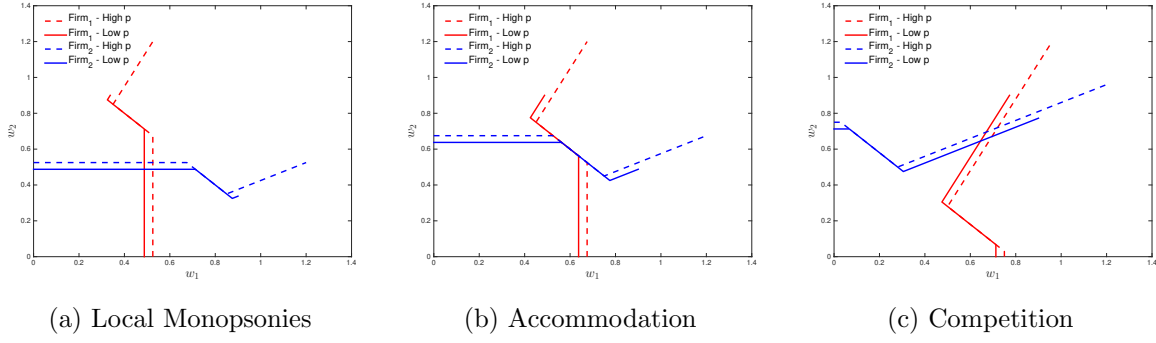

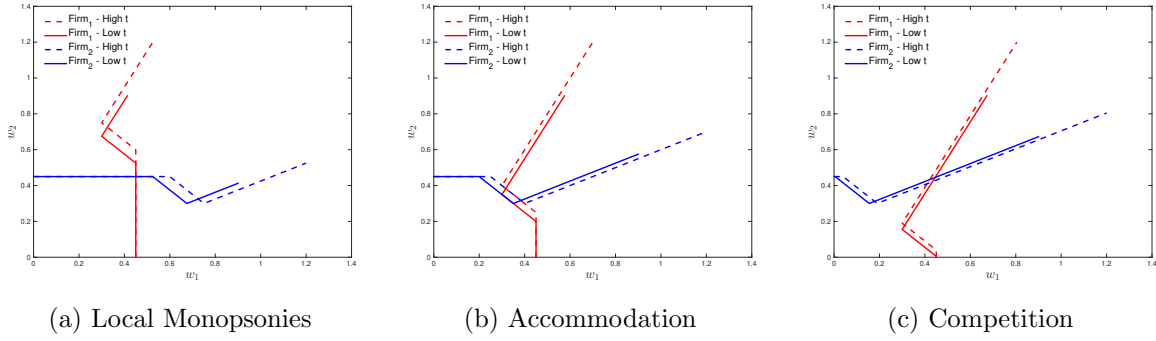(a) Local Monopsonies   (b) Accommodation   (c) Competition

Figure 6: Changes in t

2.5 Equilibrium Wage Patterns

In this section, we present a more in-depth discussion on how equilibrium wages change as the parameters in the model change. This discussion allows us to have a better understanding of the policy implications of measures that differently affect particular features of the labor market. Moreover, it provides us with a better view of the model implications that can be tested in the data, as well as possible weaknesses that can be dealt with by possible extensions.

From our discussions in Sections 2.2 and 2.3, we have that the boundaries of the three possible equilibria patterns – monopsony, accommodation, and competition – depend on the parameters $\overline{L}$, t, p, and $A_i, i \in \{1, 2\}$. Since accommodation not only depends on both thresholds, but it is also the only case in which equilibrium wages are non-unique, we focus our discussion on this case. In particular, from equations (11) and (12) and from the results of proposition 1, we have that if we are in an equilibrium with accommodation, both firms must be trying to accommodate the opponent's demand. In this case, we not only have that the parameters must satisfy condition 3 of proposition 1, but we also have that the equilibrium wages depend on the following additional constraints:

$$t\overline{L} \gtreqless \frac{A_1 p}{2} + \frac{A_2 p}{3} \quad \text{and} \quad t\overline{L} \gtreqless \frac{A_2 p}{2} + \frac{A_1 p}{3} \tag{13}$$

11

which are the restrictions that are imposed by equations (11) and (12) such that equilibrium wages respect the fact that both firms are accommodating.

In order to simplify the exposition, let's focus on the case in which $A_1 \geq A_2$.[2] In this case, we have that $\frac{A_1 p}{2} + \frac{A_2 p}{3} > \frac{A_2 p}{2} + \frac{A_1 p}{3}$. Consequently, the equilibrium wages in accommodation can be split into the following cases:

1. If $\frac{A_1 p}{3} + \frac{A_2 p}{3} < t\overline{L} < \frac{A_1 p}{3} + \frac{A_2 p}{2}$: In this case, the equilibrium wages for firms 1 and 2 must be within the following intervals:

$$\frac{A_1 p}{3} \leq w_1 \leq t\overline{L} - \frac{A_2 p}{3} \quad \text{and} \quad \frac{A_2 p}{3} \leq w_2 \leq t\overline{L} - \frac{A_1 p}{3} \tag{14}$$

2. If $\frac{A_1 p}{3} + \frac{A_2 p}{2} < t\overline{L} < \frac{A_1 p}{2} + \frac{A_2 p}{3}$: In this case, the equilibrium wages for firms 1 and 2 must be within the following intervals:

$$t\overline{L} - \frac{A_2 p}{2} \leq w_1 \leq t\overline{L} - \frac{A_2 p}{3} \quad \text{and} \quad \frac{A_2 p}{3} \leq w_2 \leq \frac{A_2 p}{2} \tag{15}$$

3. If $\frac{A_1 p}{2} + \frac{A_2 p}{3} < t\overline{L} < \frac{A_1 p}{2} + \frac{A_2 p}{2}$: In this case, the equilibrium wages for firms 1 and 2 must be within the following intervals:

$$t\overline{L} - \frac{A_2 p}{2} \leq w_1 \leq \frac{A_1 p}{2} \quad \text{and} \quad t\overline{L} - \frac{A_1 p}{2} \leq w_2 \leq \frac{A_2 p}{2} \tag{16}$$

Notice that the range of equilibrium wages allowed in the accommodation case may shrink or expand as the parameters change. In Figure 7 we show how the equilibrium wage for firm 1 changes as we vary different parameters. From Figure 7(a), let's consider that we initially start with a very high $\overline{L}$. So both firms are monopsonies. As we reduce $\overline{L}$, we are initially reducing the fraction of the labor force with the highest transportation cost, i.e., the ones who would have the highest reservation wage in order to accept working for either company. Then, once we cross the threshold presented in proposition 1's condition 1, the potential labor force is small enough that if both firms post their monopsony wages, all workers would like to work for at least one of the firms and some would accept working for both firms. Now, let's focus on the fraction of the labor force that at the monopsony wages would accept working for both firms at the current wages; i.e., both firms are posting wages above these workers' reservation wage. In this case, the worker picks whichever wage gives her the highest utility. But for the losing firm, there are two alternatives – either raise wages and try to outbid the competitor, or save money by reducing the posted wage, by saving in terms of the salaries of current employees. The latter is the optimal strategy initially. This is why we see a range of equilibria in which at least one of the firms is undercutting its monopsony wage. A very similar argument can be made about changes

---

[2]The alternative case in which $A_2 > A_1$ can be easily derived in a manner similar to the one presented here.

in transportation cost t in Figure 7(b). Finally, in Figure 7(c), we have changes in p. In this case, if we start at a low value of p, both firms are local monopsonies, since workers are not very productive; therefore, only workers with very low t are employed in equilibrium. As p increases, workers become more productive; therefore, firms expand their labor forces – as we can see, monopsony wages are increasing with p. Again, once we hit the threshold presented in proposition 1's condition 1, firms start accommodating. But now there are two main differences: not only are monopsony wages increasing in p, but the cost of accommodating is also increasing as p goes up. This is why the range of equilibrium wages in the accommodation equilibria is narrower and we quickly see firms that initially accommodated start reacting and ramping up wages.
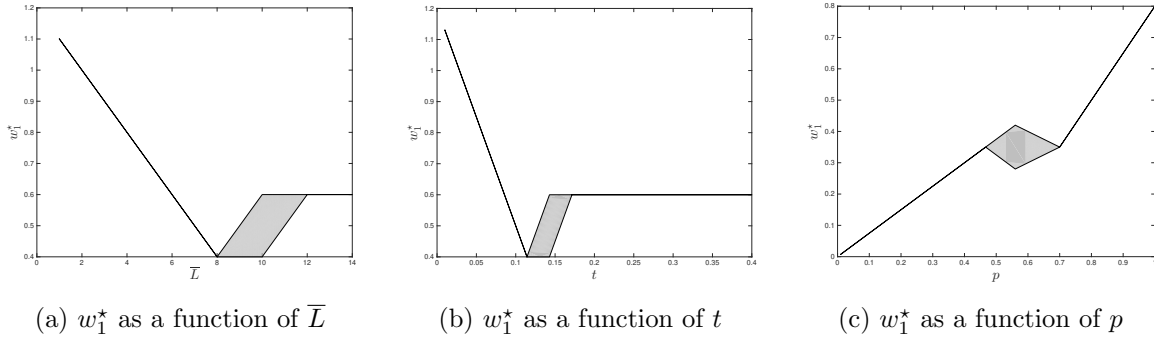

(a) $w_1^\star$ as a function of $\overline{L}$ (b) $w_1^\star$ as a function of t (c) $w_1^\star$ as a function of p

Figure 7: Equilibrium wages as a function of economy-wide parameters

Finally, in Figure 8 we look at changes in A_1 while keeping A_2 constant at $A_2 = 1.5$. As we see, results are different, now that we are changing the relative productivity of firms, as well as the boundaries of the different equilibrium patterns.

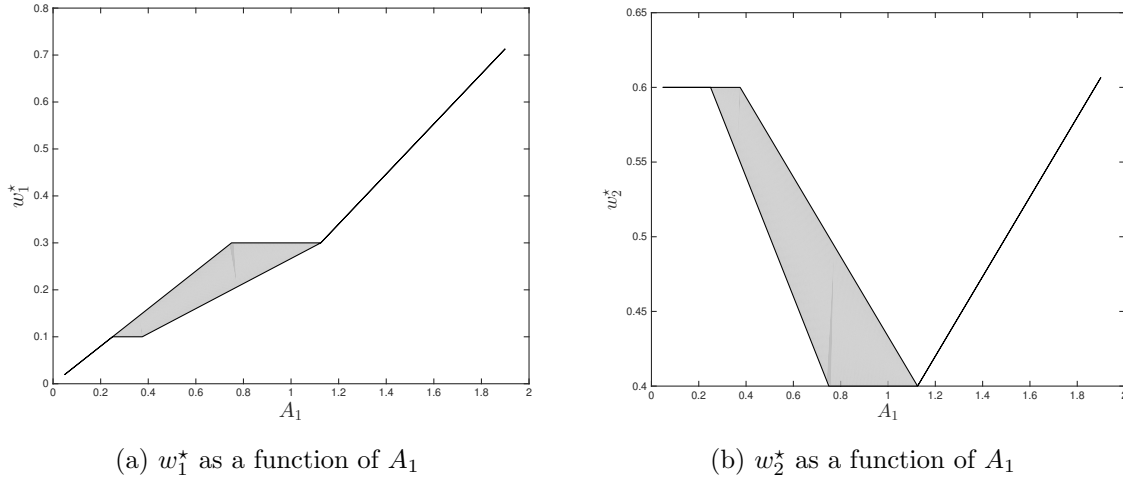(a) $w_1^\star$ as a function of A_1 (b) $w_2^\star$ as a function of A_1

Figure 8: Equilibrium wages as a function of firm 1 productivity

Overall, these results seem to rest on two simplifying assumptions in the model. First, workers

13

are homogeneously distributed across the linear city. Second, the transportation cost is linear. These two assumptions imply that: 1) Accommodation is not overly costly, since the loss of the firm's labor force is linear. Otherwise, if we have workers more concentrated at some particular values (so we assume workers are distributed across the line following a given distribution $F(\cdot)$ with support $[0, \overline{L}]$) we could have that by accommodating you lose a significant fraction of your labor force. 2) The cost-saving benefit of accommodating is reasonably large. Differently, if the transportation cost function was concave, the benefit of accommodating once the firm's labor force is at the somewhat flat portion of the cost function would have been small. We discuss the robustness of our results to these extensions in Section 4.

Finally, if we introduce a minimum wage, we can significantly reduce the ability of firms to accommodate, reducing the range of equilibrium wages in which we can see firms reducing wages below their monopsony values. We discuss the impacts of introducing a minimum wage policy in Section 3.

2.6 Average Wages and Wage Dispersion

In this section, we consider the average wages and wage dispersion in this economy. As before, we evaluate average wages and wage dispersion across the three different equilibrium patterns previously described.

In the case of an equilibrium with monopsonies, the average wage is given by:

$$E_w[w] = \frac{L_1}{L_1 + L_2} w_1^\star + \frac{L_2}{L_1 + L_2} w_2^\star \tag{17}$$

Substituting optimal wages and demands, we obtain:

$$\mu_w[w] = \frac{p\left(A_1^2 + A_2^2\right)}{2(A_1 + A_2)^2} \tag{18}$$

Similarly, the variance of wages is given by:

$$\sigma_w^2 = \frac{L_1}{L_1 + L_2}\left(w_1^\star - \mu_w[w]\right)^2 + \frac{L_2}{L_1 + L_2}\left(w_2^\star - \mu_w[w]\right)^2 \tag{19}$$

Substituting the values and rearranging, we obtain:

$$\sigma_w^2 = \frac{A_1 A_2 p^2 (A_1 - A_2)^2}{4(A_1 + A_2)^2} \tag{20}$$

As expected from corollary 1's condition 1, σ_w^2 does not depend on $\overline{L}$ and t in the case of monopsonies.

Let's now consider the average wage and variance in the case of competition. In this case, average wages are given by:

$$\mu_w[w] = \frac{(A_1 - A_2)^2 p^2 + 9\overline{L}t\left[A_1 p + A_2 p - 2\overline{L}t\right]}{18\overline{L}t} \tag{21}$$

while the variance of wages is given by:

$$\sigma_w^2 = -\frac{p^2 (A_1 - A_2)^2 \left[p^2(A_1 - A_2)^2 - 9\overline{L}^2 t^2\right]}{324\overline{L}^2 t^2} \tag{22}$$

Notice that assumption 1 warranties that $\sigma_w^2 \geq 0$. Moreover, we have that σ_w^2 is strictly decreasing in $\overline{L}$ and t, as expected based on corollary 1's condition 3.

Finally, in the case of accommodation, once the equilibrium is not unique, both mean wages and variance are functions of either w_1 or w_2. Without loss of generality, let's consider that they are both functions of w_1. Then, average wages are given by:

$$\mu_w[w] = \frac{\overline{L}^2 t^2 - 2\overline{L}tw_1 + 2w_1^2}{\overline{L}t} \tag{23}$$

while the variance is given by:

$$\sigma_w^2 = -\frac{w_1\left(2w_1 - \overline{L}t\right)^2 \left(w_1 - \overline{L}t\right)}{\overline{L}^2 t^2} \tag{24}$$

Notice that in this case, σ_w^2 is a function of w_1. First of all, notice that $\sigma_w^2 = 0$ if $w_1 = \left\{0, \overline{L}t, \frac{\overline{L}t}{2}\right\}$. However, the Cartesian points $(w_1, w_2) = (0, \overline{L}t)$ and $(w_1, w_2) = (\overline{L}t, 0)$ are necessarily outside the range of parameters for the "accommodation" equilibria.

In order to pin down the range of variance that can be reached by "accommodation" equilibria, we find the extreme points for σ_w^2. The first-order condition gives us the Cartesian points $(w_1, w_2) = \left(\frac{\overline{L}t}{2}, \frac{\overline{L}t}{2}\right)$, $(w_1, w_2) = \left(\overline{L}t\frac{(2-\sqrt{2})}{4}, \overline{L}t\frac{(2+\sqrt{2})}{4}\right)$, and $(w_1, w_2) = \left(\overline{L}t\frac{(2+\sqrt{2})}{4}, \overline{L}t\frac{(2-\sqrt{2})}{4}\right)$. From the second-order condition we obtain that $(w_1, w_2) = \left(\frac{\overline{L}t}{2}, \frac{\overline{L}t}{2}\right)$ is a minimum, while $(w_1, w_2) = \left(\overline{L}t\frac{(2-\sqrt{2})}{4}, \overline{L}t\frac{(2+\sqrt{2})}{4}\right)$, and $(w_1, w_2) = \left(\overline{L}t\frac{(2+\sqrt{2})}{4}, \overline{L}t\frac{(2-\sqrt{2})}{4}\right)$ pin down the maximum. As we have seen, $\sigma_w^2 = 0$ at $(w_1, w_2) = \left(\frac{\overline{L}t}{2}, \frac{\overline{L}t}{2}\right)$. Differently, at the two extreme points that we identified as maximum, we have $\sigma_w^2 = \frac{\overline{L}^2 t^2}{16}$. Consequently, the widest range of variances we could possibly observe is $\left[0, \frac{\overline{L}^2 t^2}{16}\right]$. As expected, the actual range depends on the parameter values and the range of equilibrium wages that can be observed in the "accommodation" equilibria, as discussed in Section 2.5.

Figure 9 shows the standard deviation as we vary different parameters and move across different equilibrium patterns. As expected, in all cases, the significant increase in the possibilities of wage dispersion occurs during "accommodation." The set of possible values for the standard deviation of wages in accommodation depends on the range of equilibrium wages that can be observed in accommodation, as described in Section 2.5. Differently, in the cases of monopsonies and competition, the standard deviation of wages is actually zero in the case in which both firms are equally productive ($A_1 = A_2$) – as we can see from equations (18) and (22). Moreover, in the case of monopsonies, average wages are
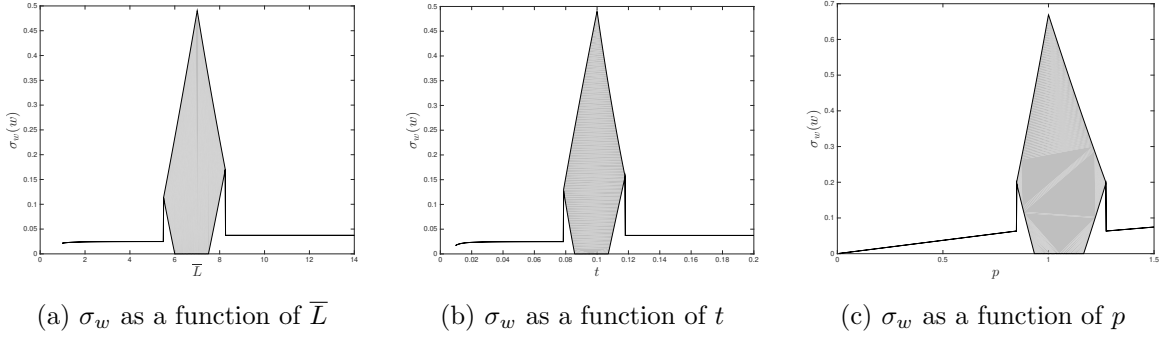
(a) σ_w as a function of $\overline{L}$ (b) σ_w as a function of t (c) σ_w as a function of p

Figure 9: Standard deviation of wages as a function of economy-wide parameters

unaffected by $\overline{L}$ and t, as we can see in Figures 9(a) and 9(b) for high values of $\overline{L}$ and t as well as in equation (18).

For completeness, we present in Figure 10 how average wages vary as we change different parameters and move across different equilibrium patterns. As in the case of the standard deviation of wages, average wages in the case of monopsonies are not affected by either $\overline{L}$ or t, as we can see in Figures 10(a) and 10(b) for high values of $\overline{L}$ and t as well as in equation (17). The decline in average wages as we enter in accommodation in the case of changes in $\overline{L}$ and t can be explained in part by the nature of the changes in these parameters. As we reduce $\overline{L}$ we are shrinking the labor force by moving out of the labor force the workers with the highest opportunity cost of working for either company. As a result, since the wage paid to all employees is affected by the opportunity cost of the marginal worker, it is natural to see a decline in wages. Similarly, a decline in t implies that it is less costly for workers to be employed at either company, again reducing the needed compensation.
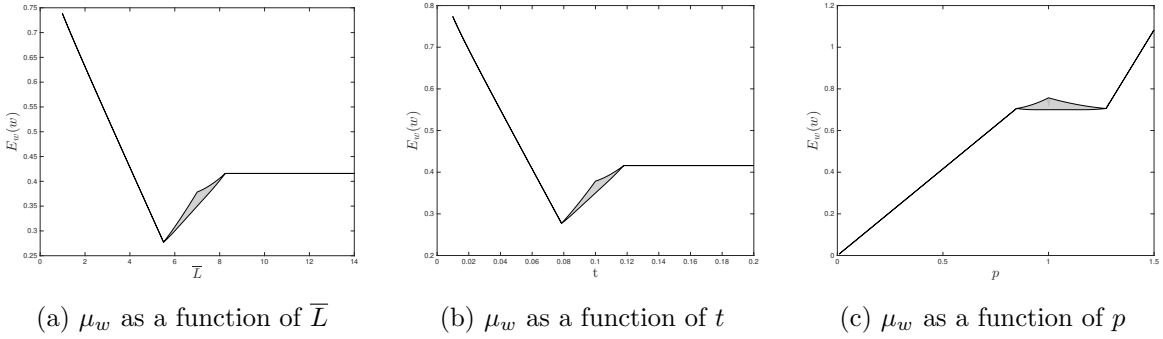

(a) μ_w as a function of $\overline{L}$ (b) μ_w as a function of t (c) μ_w as a function of p

Figure 10: Average wages as a function of economy-wide parameters

16

3 The Effects of Minimum Wage Policies

In this section we investigate the effects of introducing a minimum wage in our duopsony model. As we have seen before, the parameters of the model pin down the class of possible equilibria. Let us examine each of these possible cases.

3.1 Local Monopsonies

Recall that if this is the case, then the equilibrium wages and labor forces in each firm are determined by $w_i = \frac{A_i p}{2}$ and $x_i = \frac{A_i p}{2t}$, respectively. Let us consider the imposition of a minimum wage that is binding for both firms,[3] i.e., $w_{min} > \frac{\max\{A_1, A_2\} p}{2}$, but small enough so that both firms end up paying w^* while being local monopsonies.[4] In this case, the new policy induces higher wages being paid in equilibrium while decreasing labor market slack, and consequently increasing efficiency. This result might be counter intuitive at first, but it is a consequence of two features of the model. First, since firms cannot wage-discriminate among workers, a wage increase means that every worker must be paid a higher amount, which is the reason for slack in the first place (together with workers' heterogeneous preferences). The second reason is that the marginal productivity of each worker is constant; therefore, as long as the wage is lower than the marginal revenue of each worker, a firm that pays a wage w wishes to hire as many workers as possible at that wage.

Suppose now that the minimum wage is large enough that it moves at least one of the firms away from local monopsony. Then, the resulting equilibrium might be one in which both firms increase their wage or one in which one firm increases it, but the other decreases it. In the first case, both firms pay the minimum wage w_{min} and we move to full employment. The second case can only happen if firms have asymmetric marginal productivity and the minimum wage is initially binding only for the low-productivity firm. That is, a necessary condition for having such an asymmetric response to the minimum wage is $\frac{A_i p}{2} > w_{min} > \frac{A_j p}{2}$. In this case, it can happen that firm j raises its wage to w_{min} and firm i accommodates (or even competes) by paying a lower wage $w_i' < \frac{A_i p}{2}$ than it originally paid. Below, we present an example in which the response to the imposition of a minimum wage led to a wage decrease by the most productive firm. In such cases, the minimum wage leads to full employment and decreases the wage dispersion across firms. It might, however, lead to inefficiency, since some workers from the high-productivity firm move to the low-productivity one. The net impact on efficiency depends on how this productivity loss compares to the efficiency gains resulting from a mass of workers who were not employed moving to being employed at the low-productivity firm.

---

[3]Results are qualitatively similar if the minimum wage increase is binding for only one firm, as long as the new equilibrium still presents labor market slack.

[4]Formally, this means that the minimum wage $w_{min} < t\bar{L} - \frac{A_i p}{2}$, $i = 1, 2$.

Example 1 (Asymmetric Response to Minimum Wage) *We construct here an example in which after the introduction of a minimum wage, one firm increases its wage and the other firm decreases its wage, as a strategic response to the first firm's increase. Assume that $t\bar{L} > \frac{A_1p + A_2p}{2}$ and $A_1p > t\bar{L} > \frac{A_1p}{2}$. The first inequality implies that the equilibrium in the economy without a minimum wage is one with two local monopsonies. The second inequality implies that there is an asymmetry in productivity: firm 1 has higher marginal productivity than firm 2. Now consider a minimum wage $w_{min} = t\bar{L} - \frac{A_1p}{2} + \varepsilon$, with $\varepsilon > 0$ small (formally, $\varepsilon < \frac{A_1p - t\bar{L}}{2}$). In this economy, firm 2 must raise its wage, since $w_{min} > \frac{A_2p}{2}$. Moreover, $w_{min} > t\bar{L} - \frac{A_1p}{2}$, so that paying the monopsony wage is no longer a best response for firm 1. Indeed, firm 1's best response to such a minimum wage is $w_1^{BR} = t\bar{L} - w_{min} = \frac{A_1p}{2} - \varepsilon$. In this new scenario, note that $x_1 = \frac{A_1p}{2t} - \frac{\varepsilon}{t}$ and this marginal worker is indifferent between firm 1 and firm 2, and also indifferent between working and not working, consistent with the accommodation equilibrium. Total productivity before the minimum wage was: $A_1p * \frac{A_1p}{2t} + A_2p * \frac{A_2p}{2t} = \frac{(A_1p)^2}{2t} + \frac{(A_2p)^2}{2t}$. Once the minimum wage is imposed, the new productivity is $A_1p * \left(\frac{A_1p}{2} - \varepsilon\right)\frac{1}{t} + A_2p * \left(t\bar{L} - \frac{A_1p}{2} + \varepsilon\right)\frac{1}{t}$. Thus, we need to compare these two expressions. If ε is sufficiently small, the new equilibrium improves efficiency. Formally, as long as $\varepsilon < \frac{t\bar{L} - \frac{(A_1p + A_2p)}{2}}{\frac{A_1 - A_2}{A_2}}$ we have a policy that improves efficiency.*

3.2 Accommodation

Here, a minimum wage might force the economy to move from one equilibrium to another one, both in the accommodation area. It could also force the move so that both firms are in the competition area, implying that every worker receives a strictly positive surplus. Hence, while the introduction of a minimum wage does not alter employment in this case, it increases wages and might reduce wage inequality.[5] However, the effects on efficiency are ambiguous. If the new equilibrium increases the high-productivity firm's labor force at the expense of the low-productivity firm's labor force, then the policy is efficiency-enhancing. However, it is possible that the opposite happens and there is a loss of efficiency.

3.3 Competition

In this case, implications are more straightforward. Here, firms are competing, and if high minimum wages are imposed (higher than the current equilibrium wages), then both firms will pay the minimum wage and we remain at full employment. Here, the policy is either innocuous in terms of efficiency or inefficient. Specifically, if the economy is asymmetric, then the minimum wage might reduce wage inequality, forcing some workers out of the high-productivity firm and into the low-productivity one, which has increased its wage.

---

[5]In an asymmetric economy, both firms might end up paying the minimum wage.

We conclude this section with the following policy implications of a minimum wage.[6] The first implication is that, in a world with large local labor market power (local monopsonies), the imposition of a minimum wage improves efficiency by reducing or eliminating slack. Differently, in a world where competition is fierce, the policy is weakly inefficient. While employment initially is and remains at its highest level, there might be employment shifts from high-productivity firms to low-productivity firms. At intermediate levels of competition, the accommodation equilibria, the introduction of a minimum wage has ambiguous effects on efficiency: it may increase the employment level (if we start from the local monopsonies), but there might be shifts in the labor force from the high-productivity firm to the low-productivity one. Due to the multiplicity of equilibria, the opposite might also happen: a fraction of the labor force might shift from the less productive firm to the more productive one.

4   Extensions

In order to show that our results are robust to generalizing some of the model's assumptions, we present two extensions in this section. First, we consider the case in which transportation costs are convex. Second, we present the case in which workers are not uniformly distributed across the line.

4.1   Convex Transportation Cost

In this section, we consider the case in which transportation costs are given by $c(x) = tx^2$. Apart from that, all other assumptions are maintained. From the worker's problem, we have that, in the case in which there is no slack in equilibrium, the marginal worker's position is given by:

$$x^* = \frac{w_1 - w_2 + t\overline{L}^2}{2t\overline{L}} \tag{25}$$

In this case, $x^* \in [0, \overline{L}]$ implies:

$$|w_1 - w_2| \leq t\overline{L}^2 \tag{26}$$

In the case in which there is slack, the marginal workers' positions for firm 1 and 2 are given by:

$$\overline{x}_1 = \sqrt{\frac{w_1}{t}} \quad \text{and} \quad \overline{x}_2 = \overline{L} - \sqrt{\frac{w_2}{t}} \tag{27}$$

and in order to have slack, the following inequality must be satisfied:

$$\sqrt{w_1} + \sqrt{w_2} < \overline{L}\sqrt{t} \tag{28}$$

---

[6]We assume that the minimum wage is not large enough to drive monopsonies out of the market.

which is obtained in a manner similar to that in equation (LS). Therefore, labor supplies for firms 1 and 2 are given by:

$$L_1 = \begin{cases} \frac{w_1 - w_2 + t\overline{L}^2}{2t\overline{L}} & \text{if } \sqrt{w_1} + \sqrt{w_2} \geq \overline{L}\sqrt{t} \\ \sqrt{\frac{w_1}{t}} & \text{otherwise} \end{cases} \qquad L_2 = \begin{cases} \frac{w_2 - w_1 + t\overline{L}^2}{2t\overline{L}} & \text{if } \sqrt{w_1} + \sqrt{w_2} \geq \overline{L}\sqrt{t} \\ \sqrt{\frac{w_2}{t}} & \text{otherwise} \end{cases}$$

(29)

respectively. Taking the worker's problem into account and solving the firms' profit maximization problem, we have that firm 1's reaction function is given by:

$$w_1^*(w_2) = \begin{cases} \frac{A_1 p}{3} & \text{if } w_2 \leq \left[\overline{L}\sqrt{t} - \sqrt{\frac{A_1 p}{3}}\right]^2 \\ \left(\overline{L}\sqrt{t} - \sqrt{w_2}\right)^2 & \text{if } \overline{L}\sqrt{t} - \sqrt{\frac{A_1 p}{3}} < w_2 < \left[2\overline{L}\sqrt{t} - \sqrt{t\overline{L}^2 + A_1 p}\right]^2 \\ \frac{A_1 p + w_2 - t\overline{L}^2}{2} & \text{if } w_2 \geq \left[2\overline{L}\sqrt{t} - \sqrt{t\overline{L}^2 + A_1 p}\right]^2 \end{cases}$$
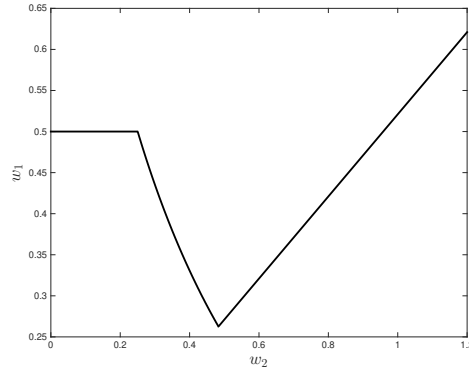
(30)

Figure 11: Reaction function for firm 1

Notice that the patterns observed in Figure 11 are exactly the same as the ones depicted by Figure 2 in the case of linear transportation costs, apart from some convexity in the accommodation interval. In fact, the equilibrium patterns are also quite similar, as we can see in Figure 12. Consequently, the introduction of convex transportation costs does not qualitatively affect the equilibrium outcomes.
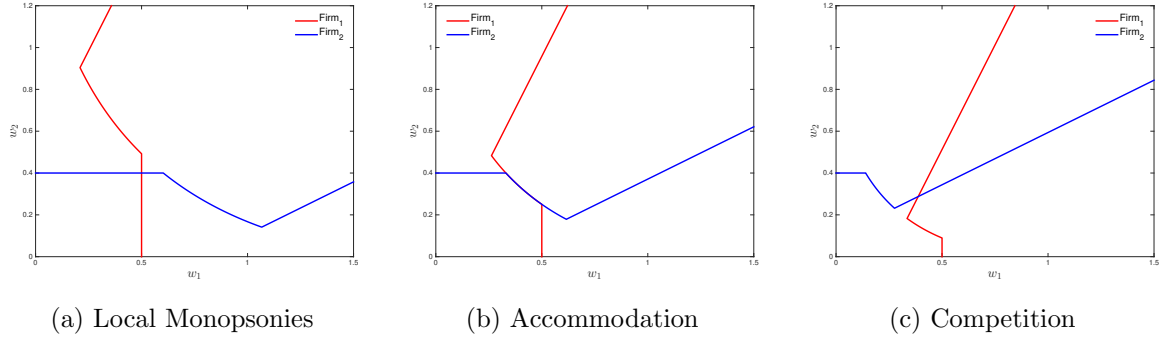
(a) Local Monopsonies   (b) Accommodation   (c) Competition

Figure 12: Equilibrium patterns

4.2 Beta Distribution

In this section, we ease the assumption that workers are distributed uniformly across the characteristics space, by assuming that workers' distribution follows a beta distribution with parameters α and β. As we can see in Figure 13, the beta distribution is quite flexible, allowing us to investigate several different configurations. In order to facilitate the comparison, and avoid the issues with a shrinking labor force, we keep the size of the labor force normalized to $\overline{L} = 1$. Since in this case we don't have
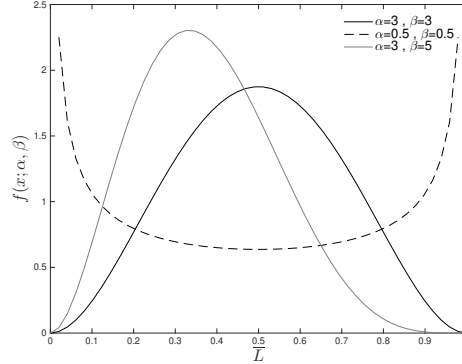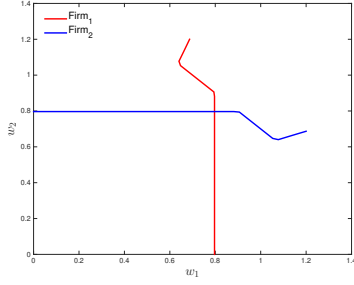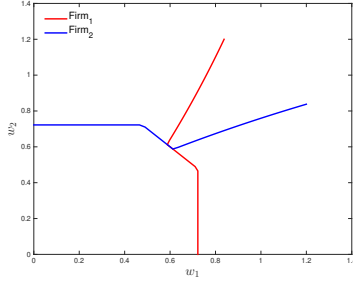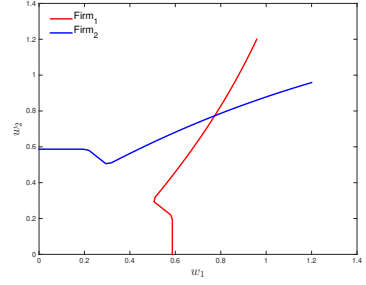

Figure 13: Beta distributions

closed-form solutions for wages, we focus on presenting some numerical solutions for the distributions presented in Figure 13. Results are shown in Figure 14 for all equilibrium patterns across the different beta distributions. As we can see in Figure 13, changing the distribution parameters α and β we change the distribution of workers across the line, from a distribution that is concentrated in the middle ($\alpha = \beta = 3$), to one in which the vast majority of workers are close to either firm 1 or firm 2, ($\alpha = \beta = 0.5$). We can also create a distribution that is asymmetric, in which the majority of workers are closer to firm 1 than firm 2 ($\alpha = 3$, $\beta = 5$). However, as we can see in Figure 14, equilibrium patterns are the same, regardless of the shape of the distribution of workers across the line.
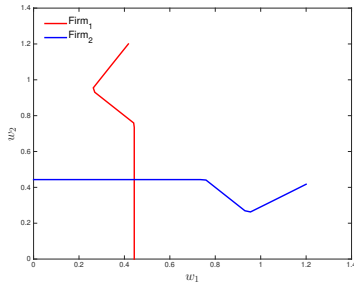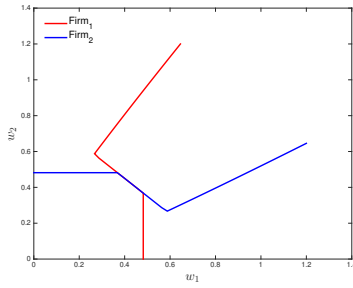
21

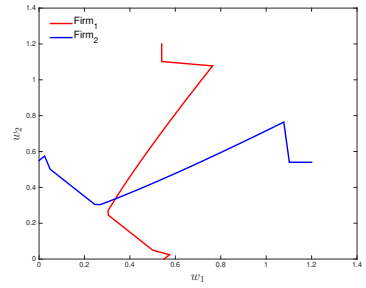(a) Monopsonies $\alpha = \beta = 3$ (b) Accommodation $\alpha = \beta = 3$ (c) Competition $\alpha = \beta = 3$
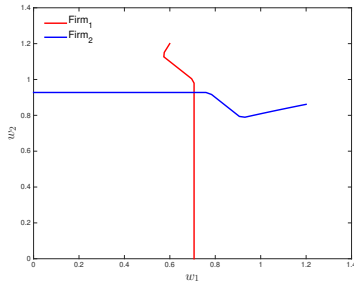
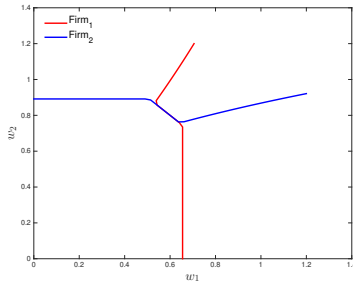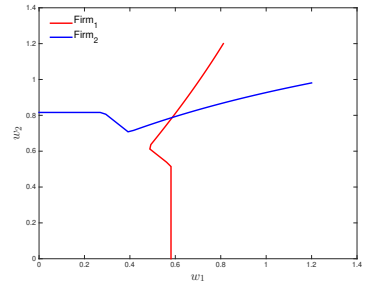(d) Monopsonies $\alpha = \beta = 0.5$ (e) Accommodation $\alpha = \beta = 0.5$ (f) Competition $\alpha = \beta = 0.5$

(g) Monopsonies $\alpha = 3$, $\beta = 5$ (h) Accommodation $\alpha = 3$, $\beta = 5$ (i) Competition $\alpha = 3$, $\beta = 5$

Figure 14: Equilibrium patterns across different beta distributions

5   Conclusion

Our aim is to provide a tractable model of wage and employment distribution that captures two features of the data that have been documented by a recent empirical literature. First, labor markets are concentrated and there are many employers with monopsony power in local labor markets. Thus, we depart not only from the standard competitive markets but also from models with atomistic monopsony. Instead, firms' strategic interactions play an important role in our analysis. Second, there is often slack in the labor market.

We incorporate these two elements and provide predictions on how wages and employment vary during the business cycle and across occupations. The important role that slack plays in our model allows us to explain empirical findings of wage and employment responses to different economic shocks. For example, non-employment often persists even in periods of boom and the labor market's response to shocks varies across sectors.

Finally, our model provides us with a tractable tool to explore important policy implications. While some of these implications have already been explored, others provide new insights into the labor literature. For example, it has already been shown that the imposition of a minimum wage might distort labor allocation, leading to a loss of efficiency.[7] However, when there is labor market slack, a minimum wage might be unambiguously efficiency-enhancing, provided that the minimum wage is not too high. The role of labor market slack within a strategic model of duopsonies might also prove helpful in analyzing other implications we might have overlooked in this paper.

References

Azar, José A., Ioana Marinescu, Marshall I. Steinbaum, and Bledi Taska (2018). "Concentration in US labor markets: Evidence from online vacancy data." Working Paper 24395, National Bureau of Economic Research. doi:10.3386/w24395.

Barlevy, Gadi and Daniel Tsiddon (2006). "Earnings inequality and the business cycle." *European Economic Review*, 50(1), pp. 55–89. doi:10.1016/j.euroecorev.2004.08.001.

Benmelech, Efraim, Nittai Bergman, and Hyunseob Kim (2018). "Strong employers and weak employees: How does employer concentration affect wages?" Working Paper 24307, National Bureau of Economic Research. doi:10.3386/w24307.

Bhaskar, Venkataraman, Alan Manning, and Ted To (2002). "Oligopsony and monopsonis-

---

[7]See, for example, Galenianos et al. (2011).

tic competition in labor markets." *Journal of Economic Perspectives*, 16(2), pp. 155–174. doi:10.1257/0895330027300.

Bhaskar, Venkataraman and Ted To (1999). "Minimum wages for ronald McDonald monopsonies: A theory of monopsonistic competition." *The Economic Journal*, 109(455), pp. 190–203. doi:10.1111/1468-0297.00427.

Bhaskar, Venkataraman and Ted To (2003). "Oligopsony and the distribution of wages." *European Economic Review*, 47(2), pp. 371–399. doi:10.1016/S0014-2921(01)00180-5.

Bonhomme, Stéphane and Laura Hospido (2017). "The cycle of earnings inequality: Evidence from Spanish social security data." *The Economic Journal*, 127(603), pp. 1244–1278. doi:10.1111/ecoj.12368.

Booth, Alison L. and Melvyn Coles (2007). "A microfoundation for increasing returns in human capital accumulation and the under-participation trap." *European Economic Review*, 51(7), pp. 1661–1681. doi:10.1016/j.euroecorev.2006.12.004.

Burdett, Kenneth and Dale T. Mortensen (1998). "Wage differentials, employer size, and unemployment." *International Economic Review*, 39(2), pp. 257–273. doi:10.2307/2527292.

Card, David and Alan B. Krueger (1994). "Minimum wages and employment: A case study of the fast-food industry in New Jersey and Pennsylvania." *American Economic Review*, 84(4), pp. 772–793. URL https://www.jstor.org/stable/2118030.

Daly, Mary C. and Bart Hobijn (2014). "Downward nominal wage rigidities bend the Phillips curve." *Journal of Money, Credit and Banking*, 46(S2), pp. 51–93. doi:10.1111/jmcb.12152.

Daly, Mary C. and Bart Hobijn (2017). "Composition and aggregate real wage growth." *American Economic Review*, 107(5), pp. 349–52. doi:10.1257/aer.p20171075.

Delacroix, Alain and Shouyong Shi (2006). "Directed search on the job and the wage ladder." *International Economic Review*, 47(2), pp. 651–699. doi:10.1111/j.1468-2354.2006.00392.x.

Dube, Arindrajit, Jeff Jacobs, Suresh Naidu, and Siddharth Suri (2020). "Monopsony in online labor markets." *AER: Insights*, 2(1), pp. 33–46. doi:10.1257/aeri.20180150.

Elsby, Michael W. and Gary Solon (2018). "How prevalent is downward rigidity in nominal wages? International evidence from payroll records and pay slips." Working Paper 25393, National Bureau of Economic Research. doi:10.3386/w25393.

Galenianos, Manolis, Philipp Kircher, and Gábor Virág (2011). "Market power and efficiency in a search model." *International Economic Review*, 52(1), pp. 85–103. doi:10.1111/j.1468-2354.2010.00620.x.

Jardim, Ekaterina S., Gary Solon, and Jacob L. Vigdor (2019). "How prevalent is downward rigidity in nominal wages? Evidence from payroll records in Washington state." Working Paper 25470, National Bureau of Economic Research. doi:10.3386/w25470. URL http://www.nber.org/papers/w25470.

Kaas, Leo and Paul Madden (2010). "Minimum wages and welfare in a hotelling duopsony." *Economic Theory*, 43(2), pp. 167–188. doi:10.1007/s00199-008-0412-2.

Leduc, Sylvain, Chitra Marti, and Daniel J. Wilson (2019). "Does ultra-low unemployment spur rapid wage growth?" *FRBSF Economic Letter*. URL https://ideas.repec.org/a/fip/fedfel/00182.html.

Manning, Alan (2003). "The real thin theory: Monopsony in modern labour markets." *Labour Economics*, 10(2), pp. 105–131. doi:10.1016/S0927-5371(03)00018-6.

Marinescu, Ioana and Roland Rathelot (2018). "Mismatch unemployment and the geography of job search." *American Economic Journal: Macroeconomics*, 10(3), pp. 42–70. doi:10.1257/mac.20160312.

Molloy, Raven, Christopher L. Smith, and Abigail Wozniak (2011). "Internal migration in the United States." *Journal of Economic Perspectives*, 25(3), pp. 173–96. doi:10.1257/jep.25.3.173.

Morin, Annaïg (2019). "Wage dispersion over the business cycle." *Economics Letters*, 181, pp. 17–21. doi:10.1016/j.econlet.2019.04.013.

Tüzemen, Didem (2017). "Labor market dynamics with endogenous labor force participation and on-the-job search." *Journal of Economic Dynamics and Control*, 75, pp. 28–51. doi:10.1016/j.jedc.2016.11.004.

Appendix

Firm's Problem in Duopsony Model

In this appendix, we focus on firm 1's profit maximization. Firm 2's profit maximization is quite similar.

$$\max_{w_1}(A_1 * p - w_1)L_1(w_1, w_2; t, \overline{L})$$

Then, let's consider the optimal profit in each segment and compare it against the overall profit.

$$\max_{w_1}(A_1 * p - w_1) * \frac{w_1}{t}$$

$$s.t. : w_1 + w_2 \leq t\overline{L}$$

Then the Lagrangian becomes:

$$\mathcal{L} = (A_1 * p - w_1)\frac{w_1}{t} - \lambda[w_1 + w_2 - t\overline{L}]$$

Then the F.O.C. becomes:

$$A_1 p * \frac{1}{t} - 2\frac{w_1}{t} - \lambda = 0$$

and complementary slackness is given by

$$\lambda[w_1 + w_2 - t\overline{L}] = 0, \lambda \geq 0$$

If the constraint is not binding, it means that $\lambda = 0$. Consequently, we have:

$$\frac{A_1 p}{t} = \frac{2w_1}{t} \Rightarrow w_1 = \frac{A_1 p}{2}$$

Differently, if the constraint binds, we must have:

$$w_1 = t\overline{L} - w_2$$

Therefore, we have:

$$w_1^* = \begin{cases} \frac{A_1 p}{2} & \text{if } \frac{A_1 p}{2} + w_2 - t\overline{L} \leq 0 \\ t\overline{L} - w_2 & \text{otherwise} \end{cases}$$

Rearranging it, we have:

$$w_1^* = \begin{cases} \frac{A_1 p}{2} & \text{if } w_2 \leq t\overline{L} - \frac{A_1 p}{2} \\ t\overline{L} - w_2 & \text{otherwise} \end{cases}$$

while profits are

$$\pi_1^* = \begin{cases} \frac{(A_1 p)^2}{4t} & \text{if } w_2 \leq t\overline{L} - \frac{A_1 p}{2} \\ (A_1 p - t\overline{L} + w_2)\frac{t\overline{L} - w_2}{t} & \text{otherwise} \end{cases}$$

Now let's consider the other segment of the labor supply:

$$\max_{w_1}(A_1 p - w_1)\left[\frac{t\overline{L} + w_1 - w_2}{2t}\right]$$

$$s.t. : w_1 + w_2 \geq t\overline{L}$$

26

Then the Lagrangian becomes:

$$\max_{w_1}(A_1 p - w_1)\frac{t\overline{L} + w_1 - w_2}{2t} - \lambda[t\overline{L} - w_1 - w_2]$$

Then the F.O.C. becomes:

$$-\left(\frac{t\overline{L} + w_1 - w_2}{2t}\right) + \left(\frac{A_1 p - w_1}{2t}\right) + \lambda = 0$$

and complementary slackness is given by:

$$\lambda[t\overline{L} - w_1 - w_2] = 0, \lambda \geq 0$$

If the constraint is not binding, $\lambda = 0$, then:

$$\frac{A_1 p - w_1}{2t} = \frac{t\overline{L} + w_1 - w_2}{2t} \Rightarrow w_1 = \frac{A_1 p + w_2 - t\overline{L}}{2}$$

In this case, profit is given by:

$$\pi_1 = \left(\frac{A_1 p}{2} - \frac{w_2}{2} + \frac{t\overline{L}}{2}\right) \times \left(\frac{t\overline{L} - w_2 + \left(\frac{A_1 p + w_2 - t\overline{L}}{2}\right)}{2t}\right) \Rightarrow \pi_1 = \frac{(A_1 p - w_2 + t\overline{L})^2}{8t}$$

while, if the constraint binds, we have

$$w_1 = t\overline{L} - w_2$$

and profits are given by:

$$\pi_1 = (A_1 p - t\overline{L} + w_2)\left(\overline{L} - \frac{w_2}{t}\right)$$

In summary, in this case we have:

$$w_1^* = \begin{cases} \frac{A_1 p + w_2 - t\overline{L}}{2} & \text{if } w_2 > t\overline{L} - \frac{1}{3}A_1 p \\ t\overline{L} - w_2 & \text{otherwise} \end{cases}$$

while profits are:

$$\pi_1^* = \begin{cases} \frac{(A_1 p - w_2 + t\overline{L})^2}{8t} & \text{if } w_2 > t\overline{L} - \frac{1}{3}A_1 p \\ (A_1 p - t\overline{L} + w_2)(\overline{L} - \frac{w_2}{t}) & \text{otherwise} \end{cases}$$

Notice that the profit for the case in which the constraint is non-binding is higher if:

$$\frac{(A_1 p - w_2 + t\overline{L})^2}{8t} > (A_1 p + w_2 - t\overline{L})\left(\frac{\overline{L}t - w_2}{t}\right)$$

Rearranging it:

$$[A_1 p + 3(w_2 - t\overline{L})]^2 > 0$$

27

And the inequality is satisfied if $A_1p + 3w_2 - 3t\overline{L} \neq 0 \Rightarrow w_2 \neq t\overline{L} - \frac{A_1p}{3}$. So if $w_2 > t\overline{L} - \frac{A_1p}{3}$, competition is the best option.

Similarly, for the first segment, we have that:

$$\frac{A_1^2p^2}{4t} > (A_1p - t\overline{L} + w_2)\frac{(t\overline{L} - w_2)}{t} \Rightarrow (A_1p - 2(t\overline{L} - w_2))^2 > 0$$

which is satisfied if $A_1p - 2t\overline{L} + 2w_2 \neq 0 \Rightarrow w_2 \neq t\overline{L} - \frac{A_1p}{2}$. So if $w_2 < t\overline{L} - \frac{A_1p}{2}$, firm 1 strictly prefers the "unconstrained solution."

Finally, the accommodation segment of the reaction function is non-empty if $t\overline{L} - \frac{A_1p}{3} > t\overline{L} - \frac{A_1p}{2}$. Simplifying it we have $\frac{A_1p}{6} > 0$, which is satisfied as long as $A_1p > 0$. Consequently, firm 1's reaction function is given by:

$$w_1^*(w_2) = \begin{cases} \frac{A_1p}{2} & \text{if } w_2 < t\overline{L} - \frac{A_1p}{2} \\ t\overline{L} - w_2 & \text{if } t\overline{L} - \frac{A_1p}{2} \leq w_2 \leq t\overline{L} - \frac{A_1p}{3} \\ \frac{A_1p + w_2 - t\overline{L}}{2} & \text{if } w_2 > t\overline{L} - \frac{A_1p}{3} \end{cases}$$

Possible Equilibria

Proof of Proposition 1. Let's start with item number 1. It is easy to see that if $\frac{A_2p}{2} + \frac{A_1p}{2} < t\overline{L}$, we have an equilibrium in which both firms are monopsonies and there is labor market slack. Notice that from the reaction functions in equations (11) and (12), $\frac{A_2p}{2} + \frac{A_1p}{2} < t\overline{L}$ is satisfied if both companies are in the first segment of their reaction functions and are local monopsonies. Toward a contradiction, let's consider the following alternative scenarios:

A. Firm 1 acts as if it is a local monopsony while firm 2 acts as if it is in competition (duopsony): In this case, based on the firms' reaction functions, we would have that:

$$w_1 = \frac{A_1p}{2}, \quad \text{and} \quad w_2 = \frac{A_2p + w_1 - t\overline{L}}{2}$$

Substituting w_1 into w_2's expression, we have:

$$w_2 = \frac{A_2p + \frac{A_1p}{2} - t\overline{L}}{2}$$

Based on the parameter restrictions from the optimal responses, we must have:

$$\frac{A_1p}{2} > t\overline{L} - \frac{A_2p}{3} \Rightarrow A_1p > 2t\overline{L} - \frac{2}{3}A_2p$$

and

$$\frac{A_1p}{2} + \frac{A_2p}{2} + \frac{A_1p}{4} - \frac{t\overline{L}}{2} - t\overline{L} < 0 \Rightarrow A_1p < 2t\overline{L} - \frac{2}{3}A_2p$$

28

So these constraints cannot be jointly satisfied. Consequently, we cannot have firm 1 acting as if it is a local monopsony and firm 2 competing as if it is in a duopsony. Due to symmetry, we can also discard firm 2 acting as if it is a local monopsony, firm 1 competing as if it is in a duopsony.

B. Firm 1 acts as if it is a local monopsony while firm 2 acts as if it is accommodating competition. In this case, based on the firms' reaction functions, we would have:

$$w_1 = \frac{A_1 p}{2}, \quad \text{and} \quad w_2 = t\overline{L} - \frac{A_1 p}{2}$$

But these wages are only optimal if they satisfy the constraints presented in the reaction functions, i.e.:

$$t\overline{L} - \frac{A_2 p}{2} < \frac{A_1 p}{2} < t\overline{L} - \frac{A_2 p}{3} \quad \text{and} \quad t\overline{L} - \frac{A_1 p}{2} < t\overline{L} - \frac{A_1 p}{2}$$

where the first inequality violates $\frac{A_2 p}{2} + \frac{A_1 p}{2} < t\overline{L}$ and the second is naturally violated. Due to symmetry, we can also discard firm 2 acting as if it is a local monopsony, firm 1 competing as if it is in a accommodation.

In order to show item 2, let's consider an equilibrium in which both firms are competing in a duopsony. Then, from equations (11) and (12), we must have:

$$w_2 = \frac{A_2 p + (\frac{A_1 p + w_2 - t\overline{L}}{2}) - t\overline{L}}{2}$$

Simplifying it:

$$w_2 = \frac{2}{3} A_2 p + \frac{1}{3} A_1 p - t\overline{L}$$

Due to symmetry, we also have:

$$w_1 = \frac{2}{3} A_1 p + \frac{1}{3} A_2 p - t\overline{L}$$

Then:

$$w_1 + w_2 = A_1 p + A_2 p - 2t\overline{L}$$

Then, if:

$$(A_1 + A_2)p > 3t\overline{L} \Rightarrow t\overline{L} < \frac{A_1 p}{3} + \frac{A_2 p}{3}$$

we have that both restrictions from the reaction functions are satisfied ($\frac{2}{3} A_2 p + \frac{1}{3} A_1 p - t\overline{L} > t\overline{L} - \frac{A_1 p}{3} \Rightarrow t\overline{L} < \frac{A_1 p}{3} + \frac{A_2 p}{3}$ and $\frac{2}{3} A_1 p + \frac{1}{3} A_2 p - t\overline{L} > t\overline{L} - \frac{A_2 p}{3} \Rightarrow t\overline{L} < \frac{A_1 p}{3} + \frac{A_2 p}{3}$) and consequently a duopsony is an equilibrium.

We now must show that the equilibrium is unique; i.e., there is no equilibrium in which one firm acts as if it is in a competitive duopsony, while the other acts as if it accommodates or is a local monopsony. The latter one we already ruled out while proving item 1. Therefore, we only need to rule out the case in which one of the firms accommodates. In order to do that, let's consider the case in which firm 1

acts as if it is in a competitive duopsony and firm 2 tries to accommodate competition. In this case, based on the firms' reaction functions (equations (11) and (12)), we have:

$$w_1 = \frac{A_1 p + w_2 - t\overline{L}}{2} \quad \text{and} \quad w_2 = t\overline{L} - w_1$$

solving the system of equations, we obtain:

$$w_1 = \frac{A_1 p}{3} \quad \text{and} \quad w_2 = t\overline{L} - \frac{A_1 p}{3}$$

But these wage offers are only optimal if the following constraints from the reaction functions are satisfied:

$$t\overline{L} - \frac{A_2 p}{2} < \frac{A_1 p}{3} < t\overline{L} - \frac{A_2 p}{3} \quad \text{and} \quad t\overline{L} - \frac{A_1 p}{3} > t\overline{L} - \frac{A_1 p}{3}$$

However, the first inequality violates the parameter restriction $(A_1 + A_2)p > 3t\overline{L}$ while the second inequality is naturally violated. Due to symmetry, we can also discard firm 1 tries to accommodate competition and firm 2 acts as if it is in a competitive duopsony.

Consequently, we have shown items 1 and 2, as well as corollary 2, i.e., we show that:

- If $(A_1 + A_2)p < 2t\overline{L}$: Both firms are monopsonies and the equilibrium is unique;

- If $(A_1 + A_2)p > 3t\overline{L}$: Both firms act as competitive duopsonists and the equilibrium is unique

- There is no equilibrium in which firms are in different segments of their reaction functions.

Now let's consider the case in which both firms accommodate competition. In terms of the reaction functions, in this case we have both firms in the decreasing part of their respective reaction functions. In the case of firm 2, for example, in equilibrium we would have $w_2^* = t\overline{L} - t\overline{L} - w_2^*$. So, as expected, in this case we must have multiple equilibria. Then, from the constraints, we have

$$t\overline{L} - \frac{A_1 p}{2} \leq w_2 \leq t\overline{L} - \frac{A_1 p}{3}$$

and

$$t\overline{L} - \frac{A_2 p}{2} \leq w_1 \leq t\overline{L} - \frac{A_2 p}{3}$$

Since $w_2 = t\overline{L} - w_1$, we have

$$t\overline{L} - \frac{A_1 p}{2} \leq t\overline{L} - w_1 \leq t\overline{L} - \frac{A_1 p}{3}$$

Therefore, constraints for w_1 are:

$$\frac{A_1 p}{3} \leq w_1 \leq \frac{A_1 p}{2}$$

and

$$t\overline{L} - \frac{A_2 p}{2} \leq w_1 \leq t\overline{L} - \frac{A_2 p}{3}$$

30

Both are satisfied if

$$\max\left\{\frac{A_1p}{3}, t\overline{L} - \frac{A_2p}{2}\right\} \le w_1 \le \min\left\{\frac{A_1p}{2}, t\overline{L} - \frac{A_2p}{3}\right\}$$

Similarly for firm 2, we have that both constraints are satisfied if:

$$\max\left\{\frac{A_2p}{3}, t\overline{L} - \frac{A_1p}{2}\right\} \le w_2 \le \min\left\{\frac{A_1p}{2}, t\overline{L} - \frac{A_2p}{3}\right\}$$

Finally, using the fact that in this case we must have $w_1 + w_2 = t\overline{L}$, we have:

$$\left[\begin{array}{c} \max\left\{\frac{A_2p}{3}, t\overline{L} - \frac{A_1p}{2}\right\} \\ + \max\left\{\frac{A_1p}{3}, t\overline{L} - \frac{A_2p}{2}\right\} \end{array}\right] \le \left[\begin{array}{c} t\overline{L} \le \min\left\{\frac{A_1p}{2}, t\overline{L} - \frac{A_2p}{3}\right\} \\ + \min\left\{\frac{A_2p}{2}, t\overline{L} - \frac{A_1p}{3}\right\} \end{array}\right]$$

Based on the constraints for the other equilibria, we know that:

$$\frac{A_1p}{2} + \frac{A_2p}{2} > t\overline{L} \quad \text{and} \quad \frac{A_1p}{3} + \frac{A_2p}{3} < t\overline{L}$$

i.e.:

$$\frac{A_1p}{3} + \frac{A_2p}{3} < t\overline{L} < \frac{A_1p}{2} + \frac{A_2p}{2}$$

But then, from the previous constraint, we have

$$\max\left\{\begin{array}{c} \frac{A_1p}{3} + \frac{A_2p}{3}, \\ t\overline{L} - \frac{A_2p}{6}, t\overline{L} - \frac{A_1p}{6}, \\ 2t\overline{L} - \frac{A_1p}{2} - \frac{A_2p}{2} \end{array}\right\} < t\overline{L} < \min\left\{\begin{array}{c} \frac{A_1p}{2} + \frac{A_2p}{2}, \\ t\overline{L} + \frac{A_1p}{6}, t\overline{L} + \frac{A_2p}{6}, \\ 2t\overline{L} - \frac{A_1p}{3} - \frac{A_2p}{3} \end{array}\right\}$$

Now let's consider the term on the left-hand side (LHS) of the inequality. If the maximum is given by either $t\overline{L} - \frac{A_2p}{6}$ or $t\overline{L} - \frac{A_1p}{6}$, the first inequality is trivially satisfied since $t\overline{L} > 0$. Moreover, if the maximum is given by $2t\overline{L} - \frac{A_1p}{2} - \frac{A_2p}{2}$, the fact that $t\overline{L} < \frac{A_1p}{2} + \frac{A_2p}{2}$ implies that $2t\overline{L} - \frac{A_1p}{2} - \frac{A_2p}{2} < t\overline{L}$ and consequently, trivially satisfied again. Consequently, the only real restriction on the LHS is $\frac{A_1p}{3} + \frac{A_2p}{3}$. Similarly, on the right-hand side (RHS) of the inequality, if the minimum is given by either $t\overline{L} + \frac{A_1p}{6}$ or $t\overline{L} + \frac{A_2p}{6}$, this restriction is trivially satisfied given that $A_1p > 0$ and $A_2p > 0$. Moreover, Given that $\frac{A_1p}{3} + \frac{A_2p}{3} < t\overline{L}$, if the minimum is given by $2t\overline{L} - \frac{A_1p}{3} - \frac{A_2p}{3}$, we know that $2t\overline{L} - \frac{A_1p}{3} - \frac{A_2p}{3} > t\overline{L}$ and the inequality is again trivially satisfied. Consequently, the only real restriction on the RHS is $\frac{A_1p}{2} + \frac{A_2p}{2}$. Therefore, the above expression can be simplified to:

$$\frac{A_1p}{3} + \frac{A_2p}{3} < t\overline{L} < \frac{A_1p}{2} + \frac{A_2p}{2}$$

concluding our proof. ∎