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State Contingent?**

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Forward Guidance under Imperfect Information: Instrument Based or State Contingent?

Chengcheng Jia

I study the optimal type of forward guidance in a flexible-price economy in which both the private sector and the central bank are subject to imperfect information about the aggregate state of the economy. In this case, forward guidance changes the private sector's expectations about both future monetary policy and the state of the economy. I study two types of forward guidance. The first type is instrument based, in which case the central bank commits to a value of the policy instrument. The second type is state contingent, in which case the central bank reveals its imperfect information and commits to a policy response rule. The key message is that forward guidance allows the central bank to reduce ex-ante price fluctuations by making the optimal trade-off between price deviations after the actual shock and after the noise shock. However, this benefit comes with a cost under the instrument-based forward guidance; that is, since firms perfectly know the change in monetary policy and prices are fully flexible, the real output level becomes independent of monetary policy. Consequently, while state-contingent forward guidance guarantees ex-ante welfare improvement, instrument-based forward guidance improves ex-ante welfare only if the central bank's information is sufficiently precise.

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1 Introduction

In recent years, central banks have increasingly used forward guidance as a monetary policy tool in addition to their traditional target, the current interest rate. The majority of the previous literature models the effects of forward guidance as changes in expected future interest rates, under the assumption that the central bank and the private sector have perfect information about the current state of the economy and hold same expectations about the future state of the economy. (Eggertsson and Woodford (2003), Del Negro, Giannoni, and Patterson (2012), Carlstrom, Fuerst, and Paustian (2015), McKay, Nakamura, and Steinsson (2016), among others). However, a practical problem faced by a central bank is that both the private sector and the central bank have imperfect information and may have different views on the economic fundamentals. As a result, forward guidance changes not only the private sector's expectations about future monetary policy actions but also its expectations about the future state of the economy. In this case, what type of forward guidance should the central bank provide? Specifically, is optimal forward guidance instrument based, in which case the central bank commits to the value of the policy instrument, or state contingent, in which case the central bank announces its imperfect information about the state of the economy and a policy response rule, but does not commit to a certain policy action?¹

In this paper, I study this question by modeling a flexible-price economy in which individual firms' pricing decisions are subject to imperfect information about the aggregate state of the economy. In addition, the central bank also has imperfect information at the stage when firms set prices, and chooses whether to reveal this information through forward guidance and how to reveal it. If forward guidance is provided, private agents get the central bank's imperfect information, which they use as a public signal and combine with their private signals to form expectations about the aggregate state of the economy. How forward guidance changes expectations about future monetary policy depends on the type of forward guidance. If forward guidance is instrument based, all firms have homogeneous expectations about future monetary policy, the same as what is communicated in the forward guidance. If forward guidance is state contingent, firms form expectations about future monetary policy conditional on their expectations about the state of the economy and thereby have heterogeneous expectations about future monetary policy.

The private sector is modeled as an island economy as in Lucas (1972) and Phelps (1970). The economy is monopolistic competitive, with each firm located on a separate island and producing an intermediate good that is an imperfect substitute for another. The technology

¹Since Campbell et al. (2012), the distinction between the instrument-based and the state-contingent forward guidance is also referred to as "Odyssean" versus "Delphic" forward guidance.

shock is firm specific, and is assumed to have an aggregate component and an idiosyncratic component. A firm can only observe its own technology shock and cannot distinguish between the aggregate component and the idiosyncratic component. This assumption makes the firm-specific technology shock a private signal of the aggregate technology shock.

Prices are perfectly flexible across periods. At the beginning of the period, firms make pricing decisions subject to imperfect information about the aggregate technology shock. At this stage, the central bank also has imperfect information about the aggregate technology shock. I start the analysis from the benchmark case in which the central bank does not provide forward guidance. At the end of the period, information becomes perfect for both private agents and the central bank. The central bank sets the aggregate nominal demand, which defines the total nominal consumption of the representative household. Since prices are already set at the beginning of the period, real consumption becomes demand determined. The representative household optimally allocates consumption over individual goods at given prices and all firms produce to meet the demand. In this benchmark case, the noise in the central bank's information never enters the equilibrium in the private sector.

Under rational expectations, if firms have perfect information about the aggregate technology shock, they can perfectly predict the response of monetary policy. Since prices are perfectly flexible, changes in monetary policy should be fully reflected in pricing decisions and thus do not affect the equilibrium real output. However, this real dichotomy breaks down under imperfect information. Firms form expectations about the change in monetary policy conditional on their expectations about the aggregate technology shock. After a positive aggregate technology shock, firms underestimate the realization of the technology shock on average and thereby underestimate the actual response of aggregate nominal demand. The difference between actual monetary policy and expected monetary policy becomes a monetary policy shock to the household, which causes monetary policy to have an effect on the real output level.

The imperfect information gives rise to the familiar “time inconsistency” problem à la [Kydland and Prescott \(1977\)](#) and [Barro and Gordon \(1983\)](#). If the central bank optimizes under discretion, it views prices to be unaffected by its choice of monetary policy, as prices are set in the previous stage. The optimal discretionary monetary policy is to completely close the output gap and let the price level fluctuate. If the central bank optimizes under credible commitment, it considers how the expectations of the forward-looking firms will be affected by its policy decisions. Bringing down price fluctuations requires the central bank's commitment to leaving the output gap open. Suppose that there is a positive aggregate technology shock and the central bank wants to stabilize the aggregate price level. As firms underestimate the aggregate technology shock under imperfect information, they underesti-

mate the actual monetary policy response. Therefore, to stabilize the price level, ex-ante, the central bank has to commit to a more accommodative policy than the optimal policy under discretion. Ex-post, when households make consumption decisions under perfect information, the committed monetary policy results in a demand for output that is higher than the efficient level of output. The optimal monetary policy rule under commitment targets a negative relationship between price levels and output gaps.

If the central bank provides instrument-based forward guidance, the central bank announces the value of aggregate nominal demand at the beginning of the period, before firms make pricing decisions. The central bank commits to implementing this policy action at the end of the period, regardless of whether its information turns out to be an actual shock or a noise shock. Since the central bank is subject to imperfect information when announcing the forward guidance, its choice of the value of aggregate nominal demand is conditional only on its imperfect signal. Under rational expectations, the private sector can perfectly infer the central bank's information about the aggregate technology shock.

This instrument-based forward guidance changes the private sector's expectations in two aspects. First, the private sector updates the expected monetary policy to be what is communicated in the forward guidance. Second, the private sector combines the public signal (the central bank's information about the aggregate technology shock) and the private signals to update expectations about the aggregate technology shock. The second aspect of the forward guidance reduces the conflict between price-level stabilization and output-gap stabilization caused by the technology shock under imperfect information. However, such reduction in the degree of information frictions does not guarantee welfare improvement, because by committing to the policy action communicated in the forward guidance, the central bank gives firms perfect information about monetary policy. Since prices are perfectly flexible, prices fully adjust to the change in monetary policy, and consequently, monetary policy has no impact on the real output level and is unable to make the optimal trade-off between the price level and the output gap. The optimal instrument-based forward guidance minimizes ex-ante price fluctuations by targeting a negative ratio between price deviations due to the actual shock and price deviations due to the noise shock of the central bank's imperfect information.

The second type of forward guidance is state contingent, in which case the central bank announces its imperfect information through forward guidance as well as the form of a policy response rule that responds to both the actual shock and the noise shock. The central bank does not commit to the value of aggregate nominal demand and sets actual monetary policy at the end of the period when perfect information becomes available. This state-contingent forward guidance changes the private sector's expectations about the aggregate technology

shock in the same way as the instrument-based forward guidance does. To form expectations about future monetary policy, firms need to form expectations about both the aggregate technology shock and the noise shock introduced by the forward guidance.

Under state-contingent forward guidance, firms cannot perfectly foresee the change in monetary policy, which means that monetary policy is able to affect both the price level and real output. I show that the optimal state-contingent forward guidance combines optimal commitment in two ways. First, the central bank commits to the optimal trade-off between the price level and the output gap, the same as the optimal policy rule without forward guidance. Second, the central bank commits to the optimal trade-off between the price deviation after the actual technology shock and the price deviation after the noise shock, the same as the optimal instrument-based forward guidance.

Lastly, I compare the ex-ante welfare in three cases: without forward guidance, with the optimal instrument-based forward guidance, and with the optimal state-contingent forward guidance. I show that ex-ante loss is minimized under the optimal state-contingent forward guidance. When the central bank's information is less precise, the ex-ante welfare under the instrument-based forward guidance is lower than in the benchmark case of no forward guidance. Providing instrument-based forward guidance is ex-ante welfare improving only if the central bank has sufficiently precise information.

Related Literature

In this paper, the benchmark case of no forward guidance builds on the literature on optimal monetary policy under information frictions, and I extend the model to study the question of optimal forward guidance policy.

In the benchmark case without forward guidance, the main argument is that optimal monetary policy targets a negative ratio between the price level and the output gap. The previous literature also reaches the same targeting rule under different assumptions on the information structure. (See [Ball, Mankiw, and Reis \(2005\)](#), [Adam \(2007\)](#) and [Angeletos and La'O \(2019\)](#) for examples.) Other papers in this field also show how optimal monetary policy depends on the balance between aggregate stabilization and cross-sectional efficiency ([Lorenzoni \(2010\)](#)), whether information is exogenous or endogenous ([Paciello and Wiederholt \(2013\)](#)) and whether monetary policy signals information about the state of the economy ([Baeriswyl and Cornand \(2010\)](#), [Tang \(2013\)](#) and [Jia \(2019\)](#))

Recent research on forward guidance is motivated by the discrepancy between the predicted explosive dynamics for inflation and output in a workhorse New Keynesian model and its limited effect in practice in the U.S. since the Great Recession, which is referred to

as the *forward guidance puzzle* by [Del Negro, Giannoni, and Patterson \(2012\)](#). Economists have shown how incomplete financial markets ([McKay, Nakamura, and Steinsson \(2016\)](#), [Kaplan, Moll, and Violante \(2018\)](#) and [Acharya and Dogra \(2018\)](#)), bounded rationality ([Gabaix \(2016\)](#) and [Farhi and Werning \(2017\)](#)), or imperfect information about either the state of the economy or monetary policy ([Angeletos and Lian \(2018\)](#), [Andrade et al. \(2019\)](#), and [Campbell et al. \(2019\)](#)) can reconcile the model predictions and the empirical evidence. [Bassetto \(2019\)](#) and [Bilbiie \(2019\)](#) study the question of the optimal form of forward guidance.

The paper most related to this one is [Angeletos and Sastry \(2018\)](#), who compare instrument-based forward guidance with target-based forward guidance when private agents have bounded rationality. In their paper, there is a trade-off of providing either type of forward guidance: that is, private agents either knows the policy instrument (if forward guidance is instrument based) or knows the outcome (if forward guidance is target based). Instead of assuming bounded rationality, in this paper, I assume that agents are fully rational but have imperfect information about the state of the economy. Another key difference is that I assume the central bank also has imperfect information when providing forward guidance, so perfectly knowing the policy instrument does not guarantee ex-ante welfare improvement. In fact, it is the central bank's ability to make the optimal trade-off between deviations after the actual shock and after the noise shock that improves the ex-ante welfare.

2 Private Sector: A Lucas-Phelps Island Economy

The private sector is modeled as a Lucas-Phelps island economy ([Lucas \(1972\)](#) and [Phelps \(1970\)](#)).² There is a continuum of firms, with each firm located in a separate island and producing an intermediate product that is an imperfect substitute for another. The market is monopolistic competitive. There is a representative household, which consists of a consumer and a continuum of workers.

The model is static in the sense that there is no consumption versus saving decisions, but there are multiple stages in each period. Forward guidance is defined as announcing the intention of monetary policy in the early stage and implementing monetary policy in the last stage. Specifically, I assume that each period has three stages. In the first stage, technology shocks are realized in all firms. Each firm i observes its own technology, A_i , but not the technology shocks in other firms, A_j , $j \neq i$. In the second stage, all firms set prices based

²Many previous papers use Lucas-Phelps island economy to formalize information frictions with a certain geographical segmentation. Examples include [Woodford \(2001\)](#), [Adam \(2007\)](#), [Nimark \(2008\)](#), [Angeletos and La'O \(2010\)](#), among others.

on their own information set ω_i . In the last stage, perfect information becomes available. All markets open. The household makes consumption decisions across products from all firms. The central bank chooses aggregate nominal demand, which defines aggregate nominal consumption. Since prices are set in the previous stage, real output becomes completely demand determined. Firms demand labor and produce output to meet the demand for their products.³

2.1 Household

There is a representative household that makes consumption and labor supply decisions in the last stage under perfect information. Denote Y as the Dixit-Stiglitz index of aggregate output and P as the corresponding price index. The optimal consumption decisions among intermediate goods yield the demand for each intermediate good to be

$$Y_i = \left(\frac{P_i}{P}\right)^{-\epsilon} Y, \quad (1)$$

where Y_i and P_i are the quantity and price of firm i .

The household's optimal labor supply decision yields the real wage to be the marginal substitution between consumption and leisure, which is given by:

$$\frac{W_i}{P} = \frac{N_i^\psi}{Y^{-\sigma}}, \quad (2)$$

where W_i and N_i denote the nominal wage and labor employed on island i .⁴

2.2 Firms

Prices are perfectly flexible across periods. Firms make pricing decisions at the beginning of the period while subject to imperfect information. The production function is a linear function of labor inputs, and technology is heterogeneous across firms. The production function of firm i is given by

$$Y_i = A_i N_i. \quad (3)$$

³When modeling an economy subject to information frictions, the existing literature differs on the assumption of whether firms make pricing or production decisions first. This assumption determines whether nominal variables or real variables are subject to imperfect information. Here, I follow the majority of papers and assume that pricing decisions are made prior to consumption decisions, and consumption decisions are made under perfect information. For economic implications under the alternative assumption that production decisions are made under imperfect information and prices clear the market under perfect information, see [Angeletos and La'O \(2010\)](#).

⁴Details of the household optimization problem is provided in Appendix A.

Each firm sets prices to maximize its expected profit conditional on its own information set, ω_i . The profit maximization problem is given by,

$$\max_{\{P_i\}} E \{P_i Y_i - W_i N_i | \omega_i\}. \quad (4)$$

Firms are forward looking and take into account how demand for their products will be affected by their pricing decisions at the end of the period. Specifically, firm i expects the quantity it will sell is $Y_i(P_i)$ given by equation (1), and will need to hire the amount of labor determined by the production function given by equation (3) at the wage specified in equation (2).

The first-order condition for optimal pricing is thus given by

$$E [\Pi_p(P_i; P, Y) | \omega_i] = 0. \quad (5)$$

Solving the optimal price, P_i , and taking log linearization yield the following result:

$$p_i = E_i [p + \alpha y] - \beta a_i \quad (6)$$

where the operator E_i represents the expectation of firm i conditional on its own information set, ω_i . α and β are constant and are functions of parameters in the model.⁵ Under normal parameter values, $\alpha > 0$ and $\beta > 0$, suggesting that a firm increases its price in response to a higher expected aggregate price level, a higher expected real output, and a lower actual firm-specific technology shock (i.e., a higher cost of production).

The central bank sets the aggregate nominal demand, n , which determines the relationship between the aggregate price level and aggregate real output, which in log form is given by

$$y = n - p. \quad (7)$$

Therefore, firms form expectations about y as the difference between n and p , which transforms equation (6) into

$$p_i = E_i [(1 - \alpha)p + \alpha n] - \beta a_i. \quad (8)$$

Higher-Order Beliefs

As suggested in equation (8), the optimal prices of individual firms depend on their expectations about the aggregate price level, and the aggregate price level in turn depends on the optimal prices of all individual firms. When firms form expectations about the

⁵See Appendix A for details.

aggregate price, they need to guess the expectations held by other firms. This feature leads to the higher-order beliefs problem. Following [Woodford \(2001\)](#), I solve the higher-order beliefs problem by successively substituting $p = \int p_i di$ and then applying E_i . Iterating this process yields

$$p_i = \alpha \sum_{j=1}^{\infty} (1 - \alpha)^{j-1} E_i \bar{E}^{j-1} n - \beta \sum_{j=1}^{\infty} (1 - \alpha)^j E_i \bar{E}^{j-1} \bar{a} - \beta a_i \quad (9)$$

where $\bar{E}[\cdot]$ denotes the average expectations operator, given by

$$\bar{E}^j[\cdot] = \int E_i \bar{E}^{j-1}[\cdot] di = \bar{E} \bar{E}^{j-1}[\cdot] \quad (10)$$

2.3 States and Signals

Aggregate States

The only aggregate state variable in the private sector is the aggregate technology shock, which I assume to be *i.i.d.* with log-normal distribution. Denote \bar{a} as the log of aggregate technology, which follows

$$\bar{a} \sim N(0, \sigma_a^2).$$

Signals

The firm-specific technology shock is assumed to be a linear sum of the aggregate technology shock and the idiosyncratic shock, which makes the firm-specific technology shock a private signal of the aggregate technology shock. The private signal relates to the aggregate shock as

$$a_i \equiv \log(A_i) = \bar{a} + s_i, \quad s_i \sim N(0, \sigma_s^2).$$

At the beginning of the period, the central bank also gets a noisy signal about the aggregate technology, which I denote as m . In the real world, this can be thought of as the central bank surveying a random sample of firms, and estimating the aggregate technology shock by the sample average. Private agents get this public signal from the central bank only under forward guidance. I assume that the noise component in the central bank's information is normally distributed with mean 0. The central bank's signal is given by:

$$m = \bar{a} + v, \quad v \sim N(0, \sigma_v^2)$$

3 The Benchmark Case: No Forward Guidance

I start the analysis from the benchmark case of no forward guidance. The central bank sets the aggregate nominal demand in the last period when perfect information becomes available. I study two types of central banks. A discretionary central bank optimizes the aggregate nominal demand decision at the end of the period, taking prices as given. A central bank under commitment chooses a monetary policy rule prior to the realization of shocks and takes into account how the expectations of the forward-looking firms will be affected by the choice of the policy rule. The central bank under commitment implements aggregate nominal demand according to the pre-determined rule at the end of the period. The sequence of events for the two types of central banks is summarized as follows.

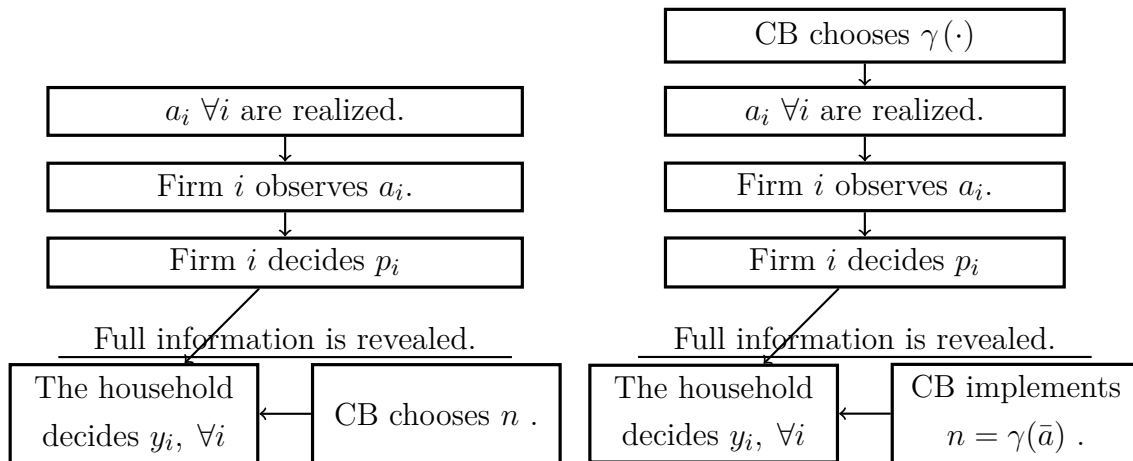


Figure 1: Sequence of Events (Left: Under Discretion, Right: Under Commitment)

A discretionary central bank (left) optimizes aggregate nominal demand ex-post, after it observes the realization of \bar{a} . A central bank under credible commitment (right) optimizes the policy rule for aggregate nominal demand ex-ante, and commits to implementing n according to the pre-determined rule. In both cases, pricing decisions are chosen under imperfect information and consumption decisions are chosen under full information.

3.1 Expectations in the Private Sector

As suggested in equation (9), optimal pricing decisions require firms to form expectations about two variables: the aggregate technology shock and the aggregate nominal demand. It is easy to see that without forward guidance, since the central bank's noisy signal never enters the private sector, optimal monetary policy responds only to the aggregate technology shock, not to the noise shock. Under rational expectations, firms expect aggregate nominal demand conditional on their expectations about the aggregate technology shock.

Without forward guidance, the information set of each firm consists only of its firm-specific technology, $\omega_i = \{a_i\}$. Firms weigh their private signals and their prior beliefs (the ex-ante mean of \bar{a}) to form expectations about the aggregate technology shock. The expected aggregate technology shock formed by firm i is given by

$$E_i \bar{a} = \frac{\kappa_s}{\kappa_s + \kappa_a} a_i + \frac{\kappa_a}{\kappa_s + \kappa_a} \mu_a = \frac{\kappa_s}{\kappa_s + \kappa_a} a_i, \quad (11)$$

where $\kappa_s = 1/\sigma_s^2$ denotes the precision of private signals, and $\kappa_a = 1/\sigma_a^2$ denotes the precision of the prior.

The optimal price is determined by higher-order beliefs, which are solved by first taking the average of individual beliefs,

$$\bar{E} \bar{a} = \frac{\kappa_s}{\kappa_s + \kappa_a} \bar{a}. \quad (12)$$

and then taking E_i over this first-order averaged expectation. Iterating this process yields higher-order expectations to be:

$$E_i \bar{E}^j \bar{a} = \left(\frac{\kappa_s}{\kappa_s + \kappa_a} \right)^{j+1} a_i. \quad (13)$$

For a discretionary central bank, I guess and verify that optimal discretionary policy is linear to the aggregate technology shock.⁶ For a central bank under commitment, I study the set of monetary policy rules that respond linearly to the aggregate technology shock. For both types of central banks, the aggregate nominal demand follows,

$$n = \gamma \bar{a}. \quad (14)$$

where γ is optimally chosen in both cases.

The expected aggregate nominal demand is thus given by

$$E_i n = \gamma E_i \bar{a} = \frac{\kappa_s}{\kappa_s + \kappa_a} \gamma a_i. \quad (15)$$

⁶Specifically, I first guess that firms expect aggregate nominal demand to be linear to the aggregate technology shock, and then solve for the optimal discretionary monetary policy, which turns out to be linear to the aggregate technology shock, consistent with the firms' expectations.

3.2 The Optimal Monetary Policy

The central bank's ex-ante loss function is the expected weighted sum of the squared output gap and the squared price level, which is given by

$$E[L] = E \left[(y - y^{eff})^2 + \tau p^2 \right], \quad (16)$$

where y^{eff} , the efficient level of output, is defined as the output level when information is perfect (specified in equation (19)).

Before diving into how optimal monetary policy depends on the information frictions in the private sector, let us first consider the following two extreme cases:

Extreme Case I - Perfect Information about the Aggregate Technology Shock

When the precision of private signals approaches infinity, the economy approaches the perfect information case. To find the equilibrium price and output level in this case, substitute $E_i \bar{a} = \bar{a}$ and $E_i n = n = \gamma \bar{a}$ into equation (9), and then apply $y = n - p$. The equilibrium price and the output level are shown to be:

$$p \rightarrow \frac{\alpha \gamma - \beta}{\alpha} \quad (17)$$

$$y \rightarrow \frac{\beta}{\alpha} \bar{a} \quad (18)$$

It shows that the real dichotomy holds, meaning that the output level is independent of the effect of monetary policy. Although monetary policy is set after pricing decisions are made, the effect of monetary policy is completely on the aggregate price level, not the output level. This is because firms are forward looking. Under rational expectations and full information, firms can perfectly forecast the aggregate nominal demand that will be set in the last stage. Therefore, prices fully adjust to the change in aggregate nominal demand, which leaves monetary policy with no effect on real output. If a central bank wants to minimize price deviations, it can achieve full price stabilization by setting $n = \frac{\beta}{\alpha} \bar{a}$.

I define the output level in this case under perfect information to be the efficient output level, which is given by

$$y^{eff} \equiv y \rightarrow \frac{\beta}{\alpha} \bar{a} \quad (19)$$

Extreme Case II - No Information on the Aggregate State

Consider the other extreme case in which the precision of private signals is zero. To find the equilibrium price and output level, substitute $E_i \bar{a} = 0$ and $n = \gamma \bar{a}$ into equation (9), and

then apply $y = n - p$. The equilibrium price and the output level are shown to be:

$$p \rightarrow -\beta\bar{a}, \quad (20)$$

$$y \rightarrow (\gamma + \beta)\bar{a}. \quad (21)$$

In this case, firms adjust prices only to their firm-specific technology shocks, not to any aggregate variables. Since monetary policy is only expected to respond to the aggregate technology shock and firms do not update expectations about the aggregate technology shock, they do not update expectations about the change in monetary policy. Consequently, monetary policy does not have any effect on the price level and all of the effect is on the output level. The real dichotomy breaks down owing to information frictions. If a central bank wants to minimize the output gap, $y - y^{eff}$, it can achieve complete output-gap stabilization by setting $n = \frac{\beta}{\alpha} - \beta\bar{a}$.

It is interesting to compare this with [Woodford \(2001\)](#), where firms have perfect information on the state of the economy, but they have imperfect information on the exogenous change in monetary policy. Consequently, changes in monetary policy affect real output. In my model, firms have perfect information on the endogenous response function of monetary policy, but they have imperfect information on the state of the economy. The gap between the actual and the expected state of the economy makes a fraction of the actual monetary policy an unanticipated shock to the private sector.

The Intermediate Case

We now turn to the intermediate case, in which both the variance of the actual aggregate technology shock and the variance of the idiosyncratic shock are non-zero and finite. After an aggregate technology shock, all firms update their expectations about the aggregate technology shock using their private signals. Under imperfect information, all firms underestimate the aggregate technology shock on average. Specifically, the first-order average of the expected aggregate technology shock is given by

$$\int E_i \bar{a} = \frac{\kappa_s}{\kappa_s + \kappa_a} \bar{a} < \bar{a}. \quad (22)$$

Firms expect monetary policy conditional on their expectations about the aggregate technology shock, which makes them also underestimate the actual change in aggregate nominal demand. Specifically,

$$\int E_i n = \int \frac{\kappa_s}{\kappa_s + \kappa_a} \gamma a_i di = \frac{\kappa_s}{\kappa_s + \kappa_a} n < n. \quad (23)$$

On average, a fraction of the change in monetary policy ($\frac{\kappa_s}{\kappa_s + \kappa_a} n$) is anticipated by firms,

and its effect is absorbed in pricing decisions. The rest of the change in monetary policy is unanticipated by firms, and this unanticipated fraction of the policy change affects the real output level. The equilibrium price level and the output level are summarized in the following proposition.

Proposition 1 *When $0 < \sigma_a < \infty$, $0 < \sigma_s < \infty$ and $n = \gamma\bar{a}$, the real dichotomy breaks down and monetary policy affects both the price level and the output level. The equilibrium aggregate price and the output level are given by:*

$$p = \frac{(\alpha\gamma - \beta)\kappa_s - \beta\kappa_a}{\kappa_a + \alpha\kappa_s} \bar{a} \quad (24)$$

$$y = \frac{\beta\kappa_s + (\gamma + \beta)\kappa_a}{\kappa_a + \alpha\kappa_s} \bar{a} \quad (25)$$

Proof: See Appendix B.

Corollary 1.1 *If the central bank optimizes under discretion, it minimizes the output gap (defined as the difference between the equilibrium output and the efficient output), and it can achieve complete output-gap stabilization by setting⁷*

$$n^{disc} = \left(\frac{\beta}{\alpha} - \beta \right) \bar{a}, \quad (26)$$

in which case the equilibrium price level and the equilibrium output level follow

$$p = -\beta\bar{a}, \quad (27)$$

$$y = \frac{\beta}{\alpha} \bar{a} = y^{eff}. \quad (28)$$

Corollary 1.2 *If the central bank sets monetary policy to minimize the price level, it can achieve complete price stabilization by setting*

$$n^{p \text{ stab}}(\bar{a}) = \frac{\beta}{\alpha} \frac{\kappa_a + \kappa_s}{\kappa_s} \bar{a}, \quad (29)$$

⁷Since the optimal discretionary monetary policy closes the output gap, I use the phrase “the optimal discretionary monetary policy” and “the output-gap stabilization policy” interchangeably in the rest of the paper.

in which case the equilibrium price level and the equilibrium output level follow

$$p = 0, \quad (30)$$

$$y = \frac{\beta(\kappa_a + \kappa_s)}{\alpha\kappa_s} \bar{a}. \quad (31)$$

Comparing (26) and (29), we find that the policy that stabilizes the price level leaves the output gap open. In addition, after an \bar{a} shock, $n^{p \text{ stab}}(\bar{a}) > n^{disc}(\bar{a})$, $\forall \kappa_s$. It suggests that if the central bank wants to stabilize the price level, it has to commit to leaving a positive output gap by making monetary policy more accommodative than what it would be under discretion. Information frictions result in the conflict between stabilizing the aggregate price level and closing the output gap.

If the central bank optimizes under credible commitment, it considers how its choice of policy rule affects the expectations of the forward-looking firms and thus affects both the price level and the output level. The central bank under commitment chooses the monetary policy rule, $n = \gamma \cdot \bar{a}$ to minimize the ex-ante loss of the central bank specified in equation (16). The first-order condition yields that

$$\frac{y - y^{eff}}{p} = -\tau \left(\frac{\partial p}{\partial \gamma} \right) \left(\frac{\partial y}{\partial \gamma} \right)^{-1}, \quad (32)$$

where

$$\frac{\partial p}{\partial \gamma} = \frac{\alpha\kappa_s}{\kappa_a + \alpha\kappa_s}, \quad (33)$$

$$\frac{\partial y}{\partial \gamma} = \frac{\kappa_a}{\kappa_a + \alpha\kappa_s}. \quad (34)$$

Solving the value of γ yields the following proposition.

Proposition 2 *When $0 < \sigma_a < \infty$, $0 < \sigma_s < \infty$, the optimal monetary policy rule that minimizes the ex-ante loss function of the central bank is*

$$\gamma^* = \left(\kappa_a^2 + \tau\alpha^2\kappa_s^2 \right)^{-1} \left(\tau\alpha\beta\kappa_s(\kappa_s + \kappa_a) + \left(\frac{\beta}{\alpha} - \beta \right) \kappa_a^2 \right). \quad (35)$$

To implement the optimal monetary policy rule, the central bank commits to leaving the output gap open. In addition, the optimal policy rule shifts from output-gap stabilization to price-level stabilization when the precision of private signals increases from zero to infinity.

Proof: γ as specified in equation (35) is a continuous function of κ_s , and $\frac{\partial \gamma}{\partial \kappa_s} > 0$. In

addition, $\gamma(\kappa_s = 0) = \gamma^{y \text{ stab}}$ and $\gamma(\kappa_s = \infty) = \gamma^{p \text{ stab}}$. So, as the precision of private information (κ_s) increases, monetary policy changes from the output-gap stabilization policy to the price-level stabilization policy.⁸ From equation (25), if $\gamma > \gamma^{y \text{ stab}}$, then $y > y^{eff}$ and $|p| < |-\beta\bar{a}|$, meaning that after a positive output gap, the central bank commits to a positive output gap to reduce price deviations from the equilibrium price under discretion.

In the following figure, I plot the optimal policy rule at varying precision of private signals and the equilibrium price level and the output level under the optimal policy rule.

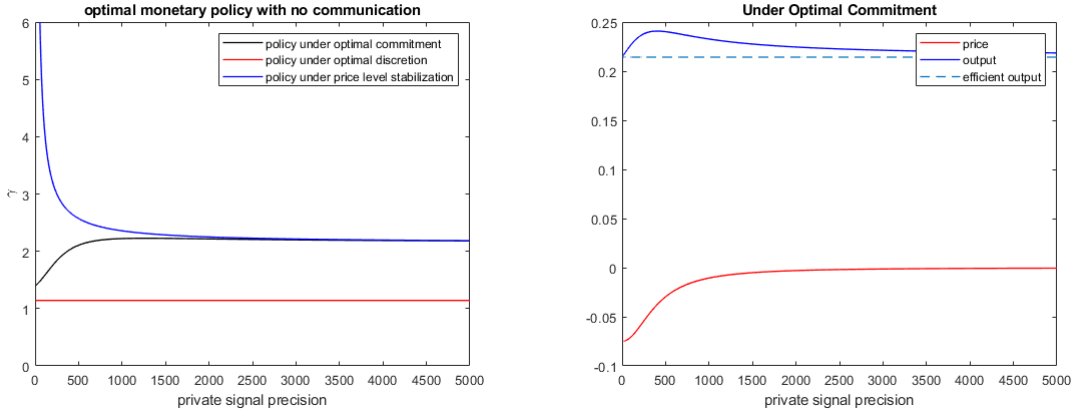


Figure 2: The Equilibrium Without Forward Guidance

Parameter values are chosen to be: $\sigma = 0.2$, $\psi = 0.5$, $\epsilon = 2$, which makes $\alpha = 0.35$ and $\beta = 0.75$. $\sigma_a = 0.1$.

The left panel compares the optimal policy rule under commitment with the optimal discretionary policy (Corollary 1.1) and the price-level stabilization policy (Corollary 1.2). It shows that the price-level stabilization policy responds more aggressively to the aggregate technology shock than the optimal discretionary policy. In addition, the more imprecise the private information is, the more aggressive response it requires to stabilize the aggregate price level. Therefore, when the central bank weighs output gaps and price deviations under optimal commitment, the policy rule weighs more towards the output-gap stabilization policy when private information is very imprecise. When the precision of the private information increases, the price-level stabilization policy requires a smaller output gap, which makes the

⁸One of the arguments made in Adam (2007) is also that under information frictions, optimal monetary policy shifts from output-gap stabilization in initial periods (when information is imprecise) to price-level stabilization in later periods (when information is precise), but the underlying assumptions are different from those in this paper. In Adam (2007), it is the coexistence of both the supply shock (shock to the efficient output level) and the real demand shock (time-varying price elasticity of intermediate products) that results in the conflict between price-level stabilization and output-gap stabilization. In my paper, the conflict is driven by the heterogeneity in firms' productivity. If all firms have the same production technology but have heterogeneous beliefs about this aggregate technology, there is no conflict between stabilizing the price level and closing the output gap. See Appendix E for derivations of this case.

optimal policy rule shift toward price-level stabilization policy. When the precision of private information approaches infinity, the real dichotomy holds asymptotically, and the real output level is independent of monetary policy. The right panel illustrates the equilibrium price level and the output gap under the optimal policy rule after a positive aggregate technology shock. As the central bank commits to a more accommodating monetary policy, the equilibrium output is higher than the efficient output and the price deviation is reduced from the equilibrium under optimal discretionary policy. As the precision of private information increases, the output level approaches the efficient level and the price level approaches zero.

4 Instrument-Based Forward Guidance

Instrument-based forward guidance is modeled as the central bank announces its decision about aggregate nominal demand before firms make pricing decisions. The central bank makes the policy decision based on its imperfect information about the aggregate technology shock. The central bank commits to implementing this policy decision at the last stage when perfect information becomes available. The sequence of events is summarized as follows.

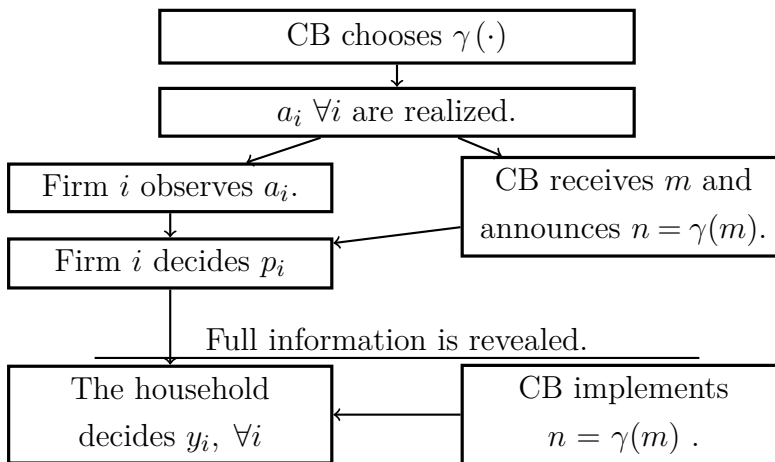


Figure 3: Sequence of Events under Instrument-Based Forward Guidance

Under instrument-based forward guidance, the central bank announces the value of aggregate nominal demand, n , which is conditional on its imperfect information, m . The central bank is committed to implementing the same policy action after full information becomes available.

4.1 Expectations in the Private Sector

In this case, the monetary policy decision is conditional only on one variable: the central bank's imperfect signal, m . I study the set of forward guidance policy rules that are linear

to the central bank's information, which is given by,

$$n = \gamma \cdot m \quad (36)$$

Under rational expectations, firms can perfectly infer m when observing n . Upon receiving the public signal provided by the central bank, the information set of individual firms becomes $\omega_i = \{m, a_i\}$. Firms form conditional expectations on the aggregate technology shock by weighing the public signal with their private signals,

$$E_i \bar{a} = \frac{\kappa_m}{\kappa_m + \kappa_s + \kappa_a} m + \frac{\kappa_s}{\kappa_m + \kappa_s + \kappa_a} a_i. \quad (37)$$

In the rest of the paper, I denote $K \equiv \kappa_m + \kappa_s + \kappa_a$. The first-order average expectation across all firms are:

$$\int E_i \bar{a} di = \frac{\kappa_m}{K} m + \frac{\kappa_s}{K} \bar{a} = \frac{\kappa_m + \kappa_s}{K} \bar{a} + \frac{\kappa_m}{K} v \quad (38)$$

Compared with equation (12), it is easy to see the ex-post trade-off of providing forward guidance: after a real technology shock, the gap between the averaged expected \bar{a} and the actual a is reduced by forward guidance. However, the noise shock also drives the expected technology shock away from the actual technology shock.

The higher-order beliefs on \bar{a} are solved by repeatedly taking the average of expectations across i and then applying E_i to the previous average. Specifically, to get the second-order expectations from the first order, apply E_i to equation (38) and get:

$$E_i \bar{E} \bar{a} = \frac{\kappa_m}{K} m + \frac{\kappa_s}{K} \left(\frac{\kappa_m}{K} m + \frac{\kappa_s}{K} a_i \right) = \left(\frac{\kappa_m}{K} + \frac{\kappa_s}{K} \frac{\kappa_m}{K} \right) m + \left(\frac{\kappa_s}{K} \right)^2 a_i \quad (39)$$

Iterate the process to the j -th order, which yields

$$E_i \bar{E}^{j-1} \bar{a} = \left(\frac{\kappa_m}{K} \right) \sum_{k=1}^{k=j} \left(\frac{\kappa_s}{K} \right)^{k-1} m + \left(\frac{\kappa_s}{K} \right)^j a_i. \quad (40)$$

When beliefs are taken to higher orders, the weight of the public signal gets amplified.

Firms have homogeneous expectations about monetary policy, as they expect the central bank to implement the same aggregate nominal demand as what is communicated in the forward guidance. Specifically,

$$E_i n = n = \gamma m \quad \forall i \quad (41)$$

and

$$E_i \bar{E}^{j-1} n = E_i n \quad \forall j \quad (42)$$

4.2 Optimal Monetary Policy

The following proposition describes the equilibrium price and output level under instrument-based forward guidance.

Proposition 3 *When instrument-based forward guidance is provided in the form of $n = \gamma m$, the equilibrium price and output are:⁹*

$$p = \left(\gamma - \frac{\beta}{\alpha} \frac{\alpha \kappa_a + \kappa_m + \alpha \kappa_s}{K'} \right) \bar{a} + \left(\gamma - \frac{\beta}{\alpha} \frac{(1 - \alpha) \kappa_m}{K'} \right) v \quad (43)$$

$$y = \frac{\beta}{\alpha} \frac{\alpha \kappa_a + \kappa_m + \alpha \kappa_s}{K'} \bar{a} + \frac{\beta}{\alpha} \frac{(1 - \alpha) \kappa_m}{K'} v \quad (44)$$

Proof: The equilibrium aggregate price level is solved by substituting $E_i \bar{E}^{j-1} \bar{a}$ and $E_i \bar{E}^{j-1} n$ in equation (9) with equation (40) and in equation (42). The equilibrium output is solved by taking the difference between the aggregate nominal demand and the equilibrium price level. See Appendix C for detailed derivations.

Corollary 3.1 *Under instrument-based forward guidance, the output level is independent of the effect of monetary policy.*

As shown in (44) the instrument-based forward guidance affects the real output level only by providing an extra source of information, measured by κ_m . The monetary policy rule, γ , does not affect real output. This is because since the central bank commits to implementing the monetary policy action provided by the forward guidance, firms have perfect information on the changes in monetary policy before making pricing decisions. Since prices are perfectly flexible, prices fully adjust to changes in monetary policy, leaving no effect on the real output.

Corollary 3.2 *Under instrument-based forward guidance, the output gap is negative after a positive technology shock and is positive after a positive noise shock.*

Since the output level is independent of the effect of monetary policy, there is no trade-off between price deviations and output gaps under instrument-based forward guidance. The output level is the result of information frictions. After a positive aggregate technology shock, firms underestimate the aggregate technology shock, and therefore the output level is lower than the efficient level. After a positive noise shock, firms overestimate the aggregate technology shock, and therefore the output level is higher than the efficient level.

⁹In the rest of the paper, I denote $K' = \kappa_a + \kappa_m + \alpha \kappa_s$.

Under instrument-based forward guidance, the equilibrium in the private sector changes in response to not only the aggregate technology shock but also to the noise in the central bank's information. The central bank's ex-ante loss function becomes

$$E[L] = E \left[(y - y^{eff})^2 + \tau p^2 \right] \equiv \int \int \left[(y - y^{eff})^2 + \tau p^2 \right] d\bar{a}d\epsilon \quad (45)$$

where y^{eff} is defined in equation (19). Since the real output level is independent of the choice of monetary policy, the optimization problem reduces to choosing the policy rule to minimize ex-ante price fluctuations, which is given by

$$\max_{n=\gamma m} E \left[p^2 \right]. \quad (46)$$

The first-order condition on γ yields

$$\frac{p(\bar{a})}{p(v)} = -\frac{\sigma_v^2}{\sigma_a^2}. \quad (47)$$

The following proposition describes the optimal instrument-based forward guidance.

Proposition 4 *The optimal instrument-based forward guidance minimizes ex-ante price deviations by targeting a negative ratio between price deviations due to technology shocks and price deviations due to noise shocks. The optimal policy rule that achieves this target is $n = \gamma^* m$ and*

$$\gamma^* = \left(\sigma_a^2 + \sigma_v^2 \right)^{-1} \left(\frac{\beta}{\alpha} \frac{\alpha \kappa_a + \kappa_m + \alpha \kappa_s}{K'} \sigma_a^2 + \frac{\beta}{\alpha} \frac{(1 - \alpha) \kappa_m}{K'} \sigma_\epsilon^2 \right). \quad (48)$$

In the following graph, I plot the monetary policy and the equilibrium price and output after a technology shock and a noise shock. It shows that after a positive technology shock, both the price level and the output gap are negative, whereas after a positive noise shock, both the price level and the output gap are positive. It suggests that the central bank makes the trade-off between equilibrium after technology shocks and equilibrium after noise shocks. In addition, as the precision of the public signal increases, the price approaches zero after the technology shock, whereas the price deviates further away from zero after the noise shock, because the targeted ratio of $\frac{p(\bar{a})}{p(v)}$ decreases in absolute value when the precision of the public signal increases. Intuitively, since the noise shock has a small variance, the deviations after noise shocks are less weighted by the central bank.

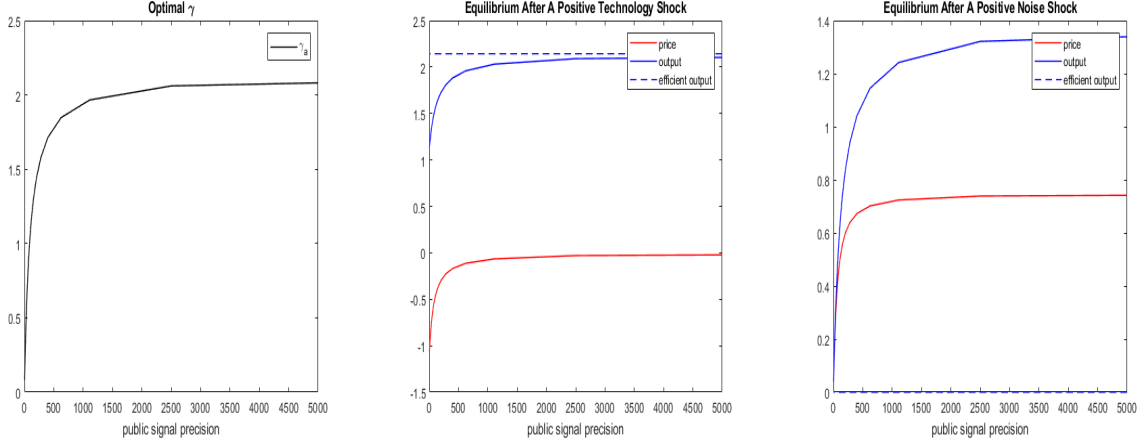


Figure 4: The Equilibrium under the Optimal Instrument Based Forward Guidance

Parameter values are chosen to be: $\sigma = 0.2$, $\psi = 0.5$, $\epsilon = 2$, which makes $\alpha = 0.35$ and $\beta = 0.75$. $\sigma_a = \sigma_s = 0.1$.

5 State-Contingent Forward Guidance

The state-contingent forward guidance is modeled as the central bank revealing its imperfect information to the private sector. It does not commit to the value of aggregate nominal demand, but rather it commits to a policy rule that responds to both the actual aggregate technology shock and the noise shock. In the last stage, the central bank sets aggregate nominal demand under perfect information. The sequence of events is summarized as follows.

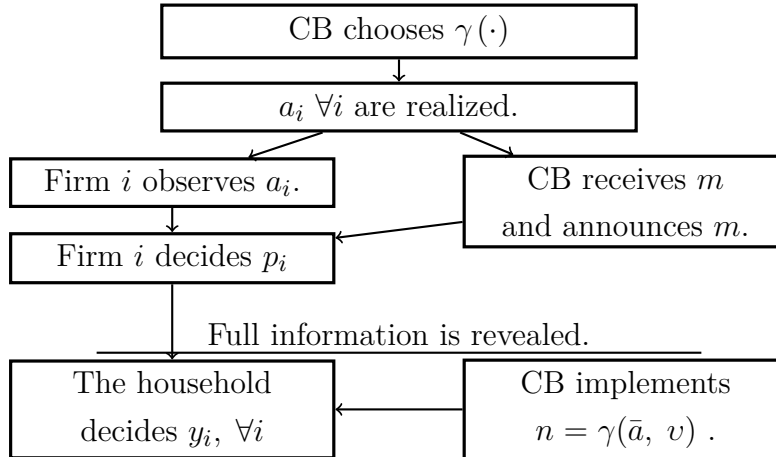


Figure 5: Sequence of Events under State-Contingent Forward Guidance

Under state-contingent forward guidance, the central bank announces its noisy signal, m , before firms make pricing decisions. The central bank waits until perfect information becomes available to set aggregate nominal demand, which allows it to condition the value of the aggregate nominal demand on both the actual shock and the noise shock.

5.1 Expectations in the Private Sector

In this case, the monetary policy decision is conditional on two aggregate variables: the actual aggregate technology shock and the noise shock. I study the set of policy rules that respond linearly to the two shocks, which is given by

$$n = \gamma^a \bar{a} + \gamma^v v. \quad (49)$$

Note that this function nests the case of instrument-based forward guidance, in which case $\gamma^a = \gamma^v$, and nests the case of no forward guidance, in which case $\gamma^v = 0$.

As long as $\gamma^a \neq \gamma^v$ and $\gamma^v \neq 0$, to form expectations about aggregate nominal demand, firms need to form expectations about both the actual shock and the noise shock as well.

Expectations about the Actual Shock

Under state-contingent forward guidance, the information set of individual firms is the same as the one under instrument-based forward guidance. Expectations about the aggregate technology shock are formed in the same way, which is given by equation (40).

Expectations about the Noise in the Public Signal

Firms form expectations about the noise shock by subtracting their expected \bar{a} from the public signal, which is given by

$$E_i v = E_i (m - \bar{a}) = m - E_i \bar{a}. \quad (50)$$

Substitute $E_i \bar{a}$ in the above equation with equation (37) to get the average expected noise shock, which is given by:

$$\int E_i v di = m - \frac{\kappa_m + \kappa_s}{\kappa_a + \kappa_m + \kappa_s} \bar{a} = \frac{\kappa_a}{\kappa_a + \kappa_m + \kappa_s} \bar{a} + \frac{\kappa_a + \kappa_s}{\kappa_a + \kappa_m + \kappa_s} v \quad (51)$$

It suggests that if the public signal provided by the central bank turns out to be a noise shock, the more precise the public signal is, the less private agents expect it to be a noise shock. Equivalently speaking, a noise shock misleads the private sector to expect an aggregate technology shock, and this effect is stronger the more precise the central bank's information is.

Expectations about Aggregate Nominal Demand

To form expectations about aggregate nominal demand, apply E_i to equation (49) and then substitute $E_i \bar{a}$ with equation (40) and $E_i v$ with equation (51).

$$E_i n = \left(\gamma^a \frac{\kappa_m}{\kappa_m + \kappa_s + \kappa_a} + \gamma^v \frac{\kappa_s + \kappa_a}{\kappa_m + \kappa_s + \kappa_a} \right) m + \left((\gamma^a - \gamma^v) \frac{\kappa_s}{\kappa_m + \kappa_s + \kappa_a} \right) a_i \quad (52)$$

In the rest of the paper, I simplify (52) as:

$$E_i n = \rho_m m + \rho_a a_i \quad (53)$$

where $\rho_m = \gamma^a \frac{\kappa_m}{\kappa_m + \kappa_s + \kappa_a} + \gamma^v \frac{\kappa_s + \kappa_a}{\kappa_m + \kappa_s + \kappa_a}$, and $\rho_a = (\gamma^a - \gamma^v) \frac{\kappa_s}{\kappa_m + \kappa_s + \kappa_a}$.

To calculate the higher-order beliefs on aggregate nominal demand, start by taking the first-order average of expectations, which is given by

$$\bar{E}n = (\rho_m + \rho_a)\bar{a} + \rho_m v. \quad (54)$$

and then applying E_i to the above equation as:

$$E_i \bar{E}n = \rho_m m + \rho_a E_i \bar{a} = \rho_m m + \rho_a \left(\frac{\kappa_m}{K} m + \frac{\kappa_s}{K} a_i \right). \quad (55)$$

It shows that the weight on the public signal is amplified when beliefs about aggregate nominal demand are taken to the higher order. Continuing this process to get the j -th order beliefs on the nominal aggregate demand:

$$E_i \bar{E}^j n = \rho_m m + \rho_a E_i \bar{E}^{j-1} \bar{a}. \quad (56)$$

5.2 Optimal Monetary Policy

The following proposition describes the equilibrium price and output level under state-contingent forward guidance.

Proposition 5 *When state-contingent forward guidance is provided in the form of $n = \gamma^a \bar{a} + \gamma^v v$, the equilibrium price and the output are:*

$$p = (\phi_m + \phi_a)\bar{a} + \phi_m v \quad (57)$$

$$y = (\gamma^a - \phi_m - \phi_a)\bar{a} + (\gamma^v - \phi_m)v \quad (58)$$

where

$$\phi_m = \rho_m + \left[\rho_a(1 - \alpha) - \beta \frac{1 - \alpha}{\alpha} \right] \frac{\kappa_m}{\alpha \kappa_s + \kappa_a + \kappa_m} \quad (59)$$

$$\phi_a = [\alpha \rho_a - \beta] \frac{(1 - \alpha) \kappa_s}{\kappa_m + \kappa_a + \alpha \kappa_s} + \alpha \rho_a - \beta \quad (60)$$

$$\rho_m = \gamma^a \frac{\kappa_m}{\kappa_m + \kappa_s + \kappa_a} + \gamma^v \frac{\kappa_s + \kappa_a}{\kappa_m + \kappa_s + \kappa_a} \quad (61)$$

$$\rho_a = (\gamma^a - \gamma^v) \frac{\kappa_s}{\kappa_m + \kappa_s + \kappa_a} \quad (62)$$

Proof: See Appendix D.

Before solving for the optimal monetary policy rule, first consider the case of a discretionary central bank that has no credible commitment to leaving the output gap open ex-post. Specifically, a discretionary central bank is expected to achieve $y = y^{eff}$ after the actual technology shock and $y = 0$ after the noise shock. The following corollary characterizes the output gap stabilization policy.

Corollary 5.1 *The condition for complete output-gap stabilization after both technology shocks and noise shocks is*

$$\gamma^a - \gamma^v = \frac{\beta}{\alpha} - \beta, \quad (63)$$

which does not yield a unique solution for $\{\gamma^a, \gamma^v\}$.

Proof: See Appendix C.

Since closing the output gap only requires the difference of γ^a and γ^v to be a constant, discretionary monetary policy has an extra degree of freedom, which allows the central bank to seek to minimize price deviations while keeping the output gap closed. Define the optimal discretionary monetary policy to be the one that minimizes ex-ante price deviations while keeping the output gap closed after both technology shocks and noise shocks. The optimal discretionary policy is solved by backward induction: in the last stage, the central bank chooses the set of output-gap stabilization policies that close the output gap (defined in Corollary 5.1). In the first stage, the central bank chooses among the output-gap stabilization policies to minimize the ex-ante price fluctuations. The following corollary summarizes the optimal discretionary policy with state-contingent forward guidance.

Corollary 5.2 *With state-contingent forward guidance, the central bank can reduce ex-ante price fluctuations while keeping the output gap closed ex-post. The optimization problem of the optimal discretionary policy under state-contingent forward guidance is given by*

$$\max_{n=\gamma^a \bar{a} + \gamma^v v} - E \left[p^2 \right] \quad (64)$$

subject to the optimization result in the second stage, which is given by equation (63). p follows equation (57) with parameters given by equations (59) to (62).

The optimization problem of the optimal discretionary monetary policy with state-contingent forward guidance becomes similar to the optimal policy rule with instrument-based forward guidance in the sense that the central bank minimizes ex-ante price deviations. The difference is that while under the optimal instrument-based forward guidance the central bank has no control over the output gap ex-post, the optimal discretionary monetary policy with state-contingent forward guidance can close the output gap ex-post thanks to the additional degree of freedom.

In the following graph, I plot the solution for the optimal discretionary policy with state-contingent forward guidance, together with the equilibrium price and output level after a positive technology shock and a positive noise shock.

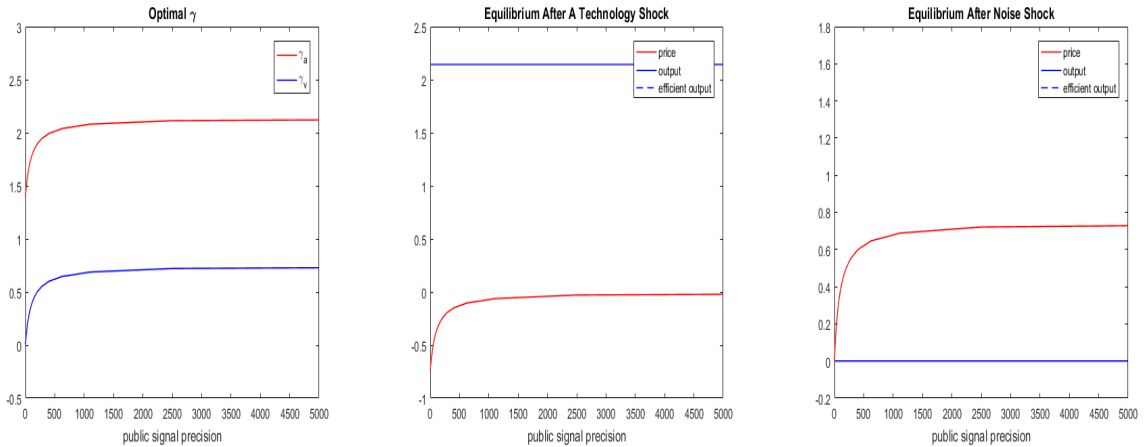


Figure 6: Optimal Discretionary Monetary Policy at Varying Degrees of Public Signal

Parameter values are chosen to be: $\sigma = 0.2$, $\psi = 0.5$, $\epsilon = 2$, which makes $\alpha = 0.35$ and $\beta = 0.75$. $\sigma_a = \sigma_s = 0.1$.

It shows that the equilibrium price is the same as the one in Figure 5, since in both cases the central bank seeks to minimize ex-ante price deviations by balancing between price fluctuation due to the actual shock and due to the noise shock. The output gap is closed after both the aggregate technology shock and the noise shock. This is due to the extra degree of flexibility in the monetary policy response function, which allows the central bank to target both price deviations and output gaps.

The ex-ante welfare can be further improved by optimal commitment. The optimization problem for a central bank with commitment is given by

$$\max_{\{n=\gamma^a \bar{a} + \gamma^v v\}} - E \left[\left(y - y^{eff} \right)^2 + \tau p^2 \right] \quad (65)$$

where p and y evolve according to equations (57) and (58) and parameters are specified in equations (59) to (62).

The first-order conditions are given by

$$\left[y(a) \frac{\partial y(a)}{\partial \gamma^a} + \tau p(a) \frac{\partial p(a)}{\partial \gamma^a} \right] \sigma_a^2 + \left[y(v) \frac{\partial y(v)}{\partial \gamma^a} + \tau p(v) \frac{\partial p(v)}{\partial \gamma^a} \right] \sigma_v^2 = 0 \quad (66)$$

$$\left[y(a) \frac{\partial y(a)}{\partial \gamma^v} + \tau p(a) \frac{\partial p(a)}{\partial \gamma^v} \right] \sigma_a^2 + \left[y(v) \frac{\partial y(v)}{\partial \gamma^v} + \tau p(v) \frac{\partial p(v)}{\partial \gamma^v} \right] \sigma_v^2 = 0 \quad (67)$$

where all derivatives to γ are functions of σ_a , σ_s and σ_v . Details are provided in Appendix D. The following proposition characterizes the optimal policy rule with state-contingent forward guidance.

Proposition 6 *The optimal state-contingent forward guidance minimizes the ex-ante loss by committing to a negative ratio between the weighted sum of deviations of the output gap and the price level after the actual aggregate technology shock and the weighted sum of deviations after the noise shock.*

Intuitively, the optimal policy rule with state-contingent forward guidance combines the optimal targeting in the benchmark case without forward guidance, and the optimal targeting in the case with instrument-based forward guidance. In the benchmark case without forward guidance, the deviations (both \hat{y} and p) after the noise shock are zero, and the central bank targets a negative ratio of the price level and the output gap. In the case with instrument-based forward guidance, the deviations of the output gap (both after the actual shock and the noise shock) are not included in the central bank's optimization problem. The central bank targets the price level after the actual shock and the noise shock. In the case with optimal state-contingent forward guidance, both \hat{y} and p and both the equilibrium after the actual shock and the equilibrium after the noise shock are considered and optimally traded off to minimize the ex-ante loss. In the following figure, I plot the solution for the optimal policy rule, together with the equilibrium price and output level after a positive technology shock and a positive noise shock.

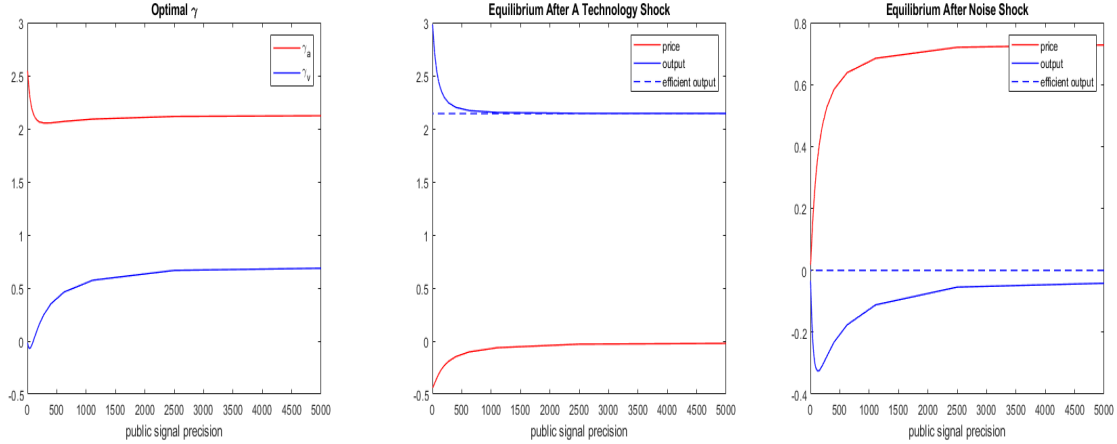


Figure 7: The Equilibrium Under the Optimal State-Contingent Forward Guidance

Parameter values are chosen to be: $\sigma = 0.2$, $\psi = 0.5$, $\epsilon = 2$, which makes $\alpha = 0.35$ and $\beta = 0.75$. $\sigma_a = \sigma_s = 0.1$.

It shows that after a positive aggregate technology shock, the output gap is positive while the price level is negative, similar to case without forward guidance, where the optimal policy rule targets a negative ratio between the output gap and the price level after the actual technology shock. After a positive noise shock, the price level is positive, similar to the case with instrument-based forward guidance, where the optimal monetary policy targets a negative ratio between price deviations after the actual shock and price deviations after the noise shock.

6 Ex-ante Welfare Comparison

There should be no surprise that the ex-ante welfare is maximized (equivalently speaking, the ex-ante loss is minimized) under the optimal state-contingent forward guidance, since both the benchmark case without forward guidance and the instrument-based forward guidance are the results of the same optimization problem but with restrictions on the set of policy choices. (The benchmark case restricts $\gamma^v = 0$ and the instrument based forward guidance restricts $\gamma^a = \gamma^v$.)

Whether providing the optimal instrument-based forward guidance improves ex-ante welfare from the benchmark case without forward guidance depends on the precision of the central bank's information. In the following figure, I plot the ex-ante loss for the three cases at varying precisions of the central bank's information. For the optimal instrument-based forward guidance, the ex-ante loss of losing control over the output level is higher when the central bank's information is less precise, and outweighs the benefits of being able to

optimally target price deviations due to the actual shock and due to the noise shock.

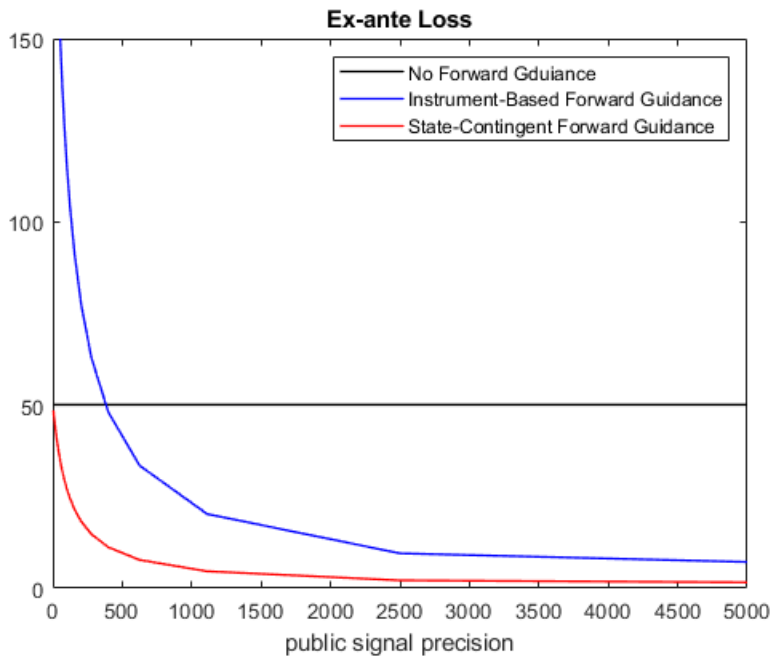


Figure 8: The Ex-ante Loss at Varying Degree of Central Bank's Information

Parameter values are chosen to be: $\sigma = 0.2$, $\psi = 0.5$, $\epsilon = 2$, $\sigma_a = \sigma_s = 0.1$

It is worth comparing this result with the trade-off of providing public information discussed in [Morris and Shin \(2002\)](#). In my paper, with the optimal policy state-contingent forward guidance, more precise public information is always ex-ante welfare-improving. There are two assumptions in my model, under which providing public information does not have the trade-off as discussed in [Morris and Shin \(2002\)](#). First, the central bank can make monetary policy conditional on the actual state of the economy. Second, the objective function of the central bank is the same as the social welfare function. In contrast, in [Morris and Shin \(2002\)](#), there is only individual optimization over imperfect information, and the individual's objective function is different from the social welfare function.

7 Conclusion

What is the optimal type of forward guidance? This paper argues that in an economy with flexible prices and imperfect information, the optimal type of forward guidance is state contingent, not instrument based. Imperfect information gives rise to the trade-off between price-level stabilization and output-gap stabilization after aggregate technology shocks. Forward guidance reduces the degree of information frictions, but since the central bank is also

subject to imperfect information, forward guidance also introduces a noise shock that comes from the central bank's own information.

The key message is that this noise shock gives the monetary policy function an extra degree of freedom. The central bank can reduce ex-ante price deviations by targeting the optimal trade-off between price deviations after the actual shock and price deviations after the noise shock. However, this benefit comes with a cost if forward guidance is instrument based. This is because since the central bank announces the value of aggregate nominal demand before firms set prices, prices will fully adjust to changes in monetary policy, leaving the output level independent of the effect of monetary policy. This cost is greater ex-ante when the central bank's information is less precise.

The optimal state-contingent forward guidance maximizes the ex-ante welfare by combining optimal commitment in two ways. First, it targets the optimal trade-off between the price level and the output gap - the same as the optimal policy rule without forward guidance. Second, it targets the optimal trade-off between deviations after the actual shock and deviations after the noise shock - the same as the optimal instrument-based forward guidance.

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Appendices

A Equilibrium in the Private Sector

A.1 Household Optimization Problem

There is a representative household, a “big family,” that consists of a continuum of workers to be sent to each island to supply labor. The household makes consumption and labor supply decisions under perfect information. The preferences of the household are defined over the aggregate consumption good, C , and the labor supplied to each firm, N_i . The decisions of the household are made when all information is revealed, and so the consumption and labor supply decisions are free from informational frictions. The household chooses consumption and labor to maximize its utility, which is given by

$$u(C, N_i) = \frac{C^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{N_i^{1+\psi}}{1+\psi} di, \quad (\text{A.1})$$

subject to the nominal budget constraint,

$$PC \leq \int_0^1 W_i N_i + \Pi, \quad (\text{A.2})$$

where Π stands for all lump-sum income including dividends of all firms and tax payments. $W(i)$ and $N(i)$ are the labor wage and labor supply of firm i , respectively.

The first-order conditions on C and N_i yield that

$$C^{-\sigma} = \lambda P \quad (\text{A.3})$$

$$N_i^\psi = \lambda W_i \quad (\text{A.4})$$

where λ is the Lagrangian multiplier of the budget constraint. Combining these two first-order conditions sets the real wage as the marginal rate of substitution between consumption and leisure:

$$\frac{W_i}{P} = \frac{N_i^\psi}{C^{-\sigma}}. \quad (\text{A.5})$$

The economy is monopolistic competitive and the final consumption, C , is a Dixit-Stiglitz composite of all intermediate goods C_i in the form of

$$C = \left(\int_0^1 C_i^{1-\frac{1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}. \quad (\text{A.6})$$

The household optimally allocates consumption among intermediate goods. Conditional on

the level of aggregate consumption, the household allocates intermediate goods consumption to minimize the cost of total expenditure, which is given by

$$\min_{\{C_i\}} \int_0^1 P_i C_i di - P \left(\int_0^1 C_i^{1-\frac{1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \quad (\text{A.7})$$

The first-order condition of the intermediate good i is:

$$C_i = \left(\frac{P_i}{P} \right)^{-\epsilon} C \quad (\text{A.8})$$

where P denotes the aggregate price level, taking the form of $P = \left(\int_0^1 p_j^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$.

A.2 Firms' Optimization Problem

Every firm produces intermediate outputs according to a constant returns to scale technology in labor, with a firm-specific productivity shock, A_i :

$$Y_i = A_i N_i. \quad (\text{A.9})$$

All firms set prices at the beginning of the period conditional on their information set, ω_i . The optimal price-setting decision for firm i is given by

$$E_{P_i} \{ P_i Y_i - W_i N_i | \omega_i \} \quad (\text{A.10})$$

All firms understand that the demand for their products is determined by the optimal consumption decisions by the household in the last stage, as

$$Y_i = \left(\frac{P_i}{P} \right)^{-\epsilon} Y. \quad (\text{A.11})$$

In addition, when firm i demands labor, it changes the household's marginal rate of substitution between consumption and leisure. So the equilibrium wage is given by

$$\frac{W_i}{P} = \frac{N_i^\psi}{Y^{1+\psi}} \quad (\text{A.12})$$

Plugging (A.9), (A.11) and (A.12) into (A.10), the expected profit of firm i conditional on its information set is given by

$$E \{ P_i Y_i - W_i N_i | \omega_i \} = E \left\{ \left(\frac{P_i}{P} \right)^{-\epsilon} Y P_i - P_i^{-\epsilon(1+\psi)} A_i^{-(1+\psi)} Y^{1+\psi+\sigma} P^{\epsilon(1+\psi)+1} | \omega_i \right\}. \quad (\text{A.13})$$

The first-order condition on P_i is calculated as:

$$E \left\{ (1 - \epsilon) P_i^\epsilon P^\epsilon Y - \epsilon (1 + \psi P_i^{-\epsilon - \epsilon \phi - 1} A_i^{-(1 + \psi)} Y^{1 + \psi + \sigma} P^{\epsilon + \epsilon \psi + 1} | \omega_i \right\} = 0 \quad (\text{A.14})$$

which yields that

$$P_i^{1 + \epsilon \varphi} = \frac{\epsilon(1 + \varphi)}{\epsilon - 1} E \left\{ P^{1 + \epsilon \varphi} Y^{\varphi + \sigma} A_i^{-(1 + \varphi)} | \omega_i \right\}. \quad (\text{A.15})$$

Take the log of the above equation to get

$$p_i = E_i [p + \alpha y] - \beta a_i \quad (\text{A.16})$$

where $\alpha = \frac{\psi + \sigma}{1 + \epsilon \psi}$ and $\beta = \frac{1 + \psi}{1 + \epsilon + \psi}$.

A.3 Price Setting with Higher Order Beliefs

The central bank chooses aggregate nominal demand, N , which sets the total nominal spending of the household, i.e.,

$$P \cdot Y = N \quad (\text{A.17})$$

To solve the optimal price under higher-order beliefs, first substitute y in equation (A.16) with $y = n - p$, which yields

$$p_i = E_i [p + \alpha(n - p)] - \beta a_i \quad (\text{A.18})$$

$$= (1 - \alpha) E_i p + \alpha E_i n - \beta a_i \quad (\text{A.19})$$

Next, to deal with the aggregate price level in log-linear form, take the log-linear approximation of the aggregate price, $P^{1 - \epsilon} = \int_0^1 P_i^{1 - \epsilon} di$, which yields $p = \int_0^1 p_i di$. Substitute the aggregate price level as the integral of individual prices, which is given by

$$p_i = (1 - \alpha) E_i \int_0^1 [(1 - \alpha) E_j p + \alpha E_j n - \beta a_j] dj + \alpha E_i n - \beta a_i, \quad (\text{A.20})$$

This can be simplified as:

$$p_i = (1 - \alpha)^2 E_i \bar{E} p + \alpha(1 - \alpha) E_i \bar{E} n + \alpha E_i n - (1 - \alpha) \beta E_i \bar{a} - \beta a_i, \quad (\text{A.21})$$

where $\bar{E}[\cdot]$ denotes the average expectation operator in the form of

$$\int_0^1 E_j(\cdot) dj = \bar{E}(\cdot), \quad (\text{A.22})$$

$$\bar{E}^j[\cdot] = \int E_i \bar{E}^{j-1}[\cdot] di = \bar{E} \bar{E}^{j-1}. \quad (\text{A.23})$$

Iterating this substitution process leads to the optimal individual price with higher-order beliefs:

$$p_i = (1 - \alpha)^\infty E_i \bar{E}^\infty p + \alpha \sum_{j=0}^\infty (1 - \alpha)^j E_i \bar{E}^j n - \beta \sum_{j=0}^\infty (1 - \alpha)^{j+1} E_i \bar{E}^j \bar{a} - \beta a_i \quad (\text{A.24})$$

B The Case Without Forward Guidance

This section derives the aggregate price and output level when the central bank does not provide forward guidance of any sort, i.e., the central bank does not reveal its imperfect information on the aggregate technology shock, nor does it provide its best estimate of monetary policy decisions.

B.1 Expectations in the Private Sector

In this case, firms use their private information on the aggregate technology shock to form expectations about the aggregate technology shock and about the response of monetary policy.

Firm i sees only its own technology a_i and uses it as the private signal. The conditional expectation of the aggregate technology shock becomes

$$E_i \bar{a} = \frac{\kappa_s}{\kappa_s + \kappa_a} a_i + \frac{\kappa_a}{\kappa_s + \kappa_a} \mu_a = \frac{\kappa_s}{\kappa_s + \kappa_a} a_i \quad (\text{B.1})$$

To solve for the higher-order beliefs, first take the average over i and get

$$\bar{E} \bar{a} = \frac{\kappa_s}{\kappa_s + \kappa_a} \bar{a}. \quad (\text{B.2})$$

Then apply E_i to this first-order averaged expectation and get

$$E_i \bar{E} \bar{a} = \frac{\kappa_s}{\kappa_s + \kappa_a} E_i \bar{a} = \left(\frac{\kappa_s}{\kappa_s + \kappa_a} \right)^2 a_i. \quad (\text{B.3})$$

Continuous iteration of this substitution process finally results in

$$E_i \bar{E}^j \bar{a} = \left(\frac{\kappa_s}{\kappa_s + \kappa_a} \right)^{j+1} a_i \quad (\text{B.4})$$

I consider the class of policy rule that is linear to the aggregate technology shock, $n = \gamma\alpha$, which makes $E_i n$ a linear function of $E_i \bar{a}$, given by $E_i n = \gamma E_i a$.

Apply (B.4) into (A.24) and get

$$p_i = (1-\alpha)^\infty E_i \bar{E}^\infty p + \alpha \gamma \sum_{j=0}^{\infty} (1-\alpha)^j \left(\frac{\kappa_s}{\kappa_s + \kappa_a} \right)^{j+1} a_i - \beta \sum_{j=0}^{\infty} (1-\alpha)^{j+1} \left(\frac{\kappa_s}{\kappa_s + \kappa_a} \right)^{j+1} a_i - \beta a_i \quad (\text{B.5})$$

I guess and verify that the higher-order expectations on p are less than $\frac{1}{1-\alpha}$, which makes $(1-\alpha)^\infty E_i \bar{E}^\infty p \rightarrow 0$. This leads to

$$p_i = \frac{(\alpha\gamma - \beta)\kappa_s - \beta\kappa_a}{\kappa_a + \alpha\kappa_s} a_i \quad (\text{B.6})$$

Integration over i results in the equilibrium aggregate price level and output, which are given by

$$p = \frac{(\alpha\gamma - \beta)\kappa_s - \beta\kappa_a}{\kappa_a + \alpha\kappa_s} \bar{a} \quad (\text{B.7})$$

$$y = \frac{\beta\kappa_s + (\gamma + \beta)\kappa_a}{\kappa_a + \alpha\kappa_s} \bar{a} \quad (\text{B.8})$$

B.2 Optimal Monetary Policy Without Forward Guidance

The central bank chooses the optimal linear policy rule prior to the realization of shocks, to minimize the weighted sum of variances of the price level and the output gap. The objective function of the central bank's optimization problem is:

$$\min_{\gamma} E \left[(y - y^{eff})^2 + \tau p^2 \right] \quad (\text{B.9})$$

where p and y follow (B.7) and (B.8).

The first-order condition on γ is

$$\frac{y - y^{eff}}{p} = -\tau \left(\frac{\partial p}{\partial \gamma} \right) \left(\frac{\partial y}{\partial \gamma} \right)^{-1}, \quad (\text{B.10})$$

which suggests that the optimal policy rule targets a negative ratio between the output gap and the price level.

In (B.10), the derivatives of p and y with respect to γ are given by

$$\frac{\partial p}{\partial \gamma} = \frac{\alpha \kappa_s}{\kappa_a + \alpha \kappa_s}, \quad (\text{B.11})$$

$$\frac{\partial y}{\partial \gamma} = \frac{\kappa_a}{\kappa_a + \alpha \kappa_s}. \quad (\text{B.12})$$

Substitute these derivatives into equation (B.10) and re-arrange the equation to get

$$\frac{\beta \kappa_s + (\gamma + \beta) \kappa_a}{\kappa_a + \alpha \kappa_s} = \frac{\beta}{\alpha} \quad (\text{B.13})$$

which yields

$$\gamma^* = \left(\kappa_a^2 + \tau \alpha^2 \kappa_s^2 \right)^{-1} \left(\tau \alpha \beta \kappa_s (\kappa_s + \kappa_a) + \left(\frac{\beta}{\alpha} - \beta \right) \kappa_a^2 \right) \quad (\text{B.14})$$

C Instrument-Based Forward Guidance

C.1 Expectations in the Private Sector

Firms form expectations about the aggregate technology shock by combining the public signal from the forward guidance and their private signals. $E_i \bar{a}$ is given by

$$E_i \bar{a} = \frac{\kappa_m}{\kappa_m + \kappa_s + \kappa_\xi} m + \frac{\kappa_s}{\kappa_m + \kappa_s + \kappa_\xi} a_i + \frac{\kappa_\xi}{\kappa_s + \kappa_\xi} \mu_a \quad (\text{C.1})$$

$$= \frac{\kappa_m}{\kappa_m + \kappa_s + \kappa_\xi} m + \frac{\kappa_s}{\kappa_m + \kappa_s + \kappa_\xi} a_i \quad (\text{C.2})$$

To get the higher-order beliefs, first integrate over i and get

$$\bar{E} \bar{a} = \frac{\kappa_m}{K} m + \frac{\kappa_s}{K} \bar{a} \quad (\text{C.3})$$

where I denote $K = \kappa_m + \kappa_s + \kappa_\xi$. Then apply E_i to the first-order averaged expectations to get:

$$E_i \bar{E} \bar{a} = \frac{\kappa_m}{K} m + \frac{\kappa_s}{K} E_i \bar{a} = \frac{\kappa_m}{K} m + \frac{\kappa_s}{K} \left[\frac{\kappa_m}{K} m + \frac{\kappa_s}{K} a_i \right] = \left[\frac{\kappa_m}{K} + \frac{\kappa_s}{K} \frac{\kappa_m}{K} \right] m + \left(\frac{\kappa_s}{K} \right)^2 a_i \quad (\text{C.4})$$

Successive iteration of this process leads to the higher-order beliefs on the aggregate technology:

$$E_i \bar{E}^{j-1} \bar{a} = \left(\frac{\kappa_m}{K} \right) \sum_{k=1}^{j-1} \left(\frac{\kappa_s}{K} \right)^{k-1} m + \left(\frac{\kappa_s}{K} \right)^j a_i \quad (\text{C.5})$$

Substitute $E_i n$ by n and $E_i \bar{a}$ by equation (C.2) into p_i , which yields

$$p_i = \left(\gamma^a - \beta \frac{1 - \alpha \kappa_m}{\alpha K'} \right) m - \beta \frac{K}{K'} a_i \quad (\text{C.6})$$

Integrating over i results in the aggregate price level. The real output level is the difference between n and p . i.e.,

$$p = \left(\gamma - \frac{\beta \alpha \kappa_a + \kappa_m + \alpha \kappa_s}{\alpha K'} \right) \bar{a} + \left(\gamma - \frac{\beta (1 - \alpha) \kappa_m}{\alpha K'} \right) v \quad (\text{C.7})$$

$$y = \frac{\beta \alpha \kappa_a + \kappa_m + \alpha \kappa_s}{\alpha K'} \bar{a} + \frac{\beta (1 - \alpha) \kappa_m}{\alpha K'} v \quad (\text{C.8})$$

C.2 Optimal Monetary Policy

The optimal monetary policy then reduces to choose γ to minimize the ex-ante variance of the price level, given by

$$E p^2 = \left(\gamma^a - \frac{\beta \alpha \kappa_a + \kappa_m + \alpha \kappa_s}{\alpha K'} \right) \sigma_a^2 + \left(\gamma^a - \frac{\beta (1 - \alpha) \kappa_m}{\alpha K'} \right) \sigma_v. \quad (\text{C.9})$$

The first-order condition yields that

$$\frac{\gamma - \frac{\beta \alpha \kappa_a + \kappa_m + \alpha \kappa_s}{\alpha K'}}{\gamma - \frac{\beta (1 - \alpha) \kappa_m}{\alpha K'}} = - \frac{\sigma_v^2}{\sigma_a^2}. \quad (\text{C.10})$$

Rearrange to get the solution for γ :

$$\gamma = \left(\sigma_a^2 + \sigma_v^2 \right)^{-1} \left(\frac{\beta \alpha \kappa_a + \kappa_m + \alpha \kappa_s}{\alpha K'} \sigma_a^2 + \frac{\beta (1 - \alpha) \kappa_m}{\alpha K'} \sigma_v^2 \right) \quad (\text{C.11})$$

D State-Contingent Forward Guidance

D.1 Expectations in the Private Sector

Firms form expectations about the aggregate technology shock by combining the public signal from the forward guidance and their private signals. $E_i \bar{a}$ is given by

$$E_i \bar{a} = \frac{\kappa_m}{\kappa_m + \kappa_s + \kappa_\xi} m + \frac{\kappa_s}{\kappa_m + \kappa_s + \kappa_\xi} a_i + \frac{\kappa_\xi}{\kappa_s + \kappa_\xi} \mu_a \quad (\text{D.1})$$

$$= \frac{\kappa_m}{\kappa_m + \kappa_s + \kappa_\xi} m + \frac{\kappa_s}{\kappa_m + \kappa_s + \kappa_\xi} a_i \quad (\text{D.2})$$

To get the higher-order beliefs, first integrate over i and get

$$\bar{E}\bar{a} = \frac{\kappa_m}{K}m + \frac{\kappa_s}{K}\bar{a} \quad (\text{D.3})$$

where I denote $K = \kappa_m + \kappa_s + \kappa_\xi$. Then apply E_i to the first-order averaged expectations to get:

$$E_i\bar{E}\bar{a} = \frac{\kappa_m}{K}m + \frac{\kappa_s}{K}E_i\bar{a} = \frac{\kappa_m}{K}m + \frac{\kappa_s}{K}\left[\frac{\kappa_m}{K}m + \frac{\kappa_s}{K}a_i\right] = \left[\frac{\kappa_m}{K} + \frac{\kappa_s}{K}\frac{\kappa_m}{K}\right]m + \left(\frac{\kappa_s}{K}\right)^2 a_i \quad (\text{D.4})$$

Successive iteration of this process leads to the higher-order beliefs on the aggregate technology:

$$E_i\bar{E}^{j-1}\bar{a} = \left(\frac{\kappa_m}{K}\right)\sum_{k=1}^{k=j}\left(\frac{\kappa_s}{K}\right)^{k-1}m + \left(\frac{\kappa_s}{K}\right)^j a_i \quad (\text{D.5})$$

In addition to forming expectations about the aggregate technology shock, firms also need to form expectations about the change in monetary policy. To do so, they also need to form expectations about the noise component in the central bank's information. Firms form expectations about the noise shock as the difference between the public signal provided by the central bank and their own expected \bar{a} . The expected noise shock is

$$E_i v = E_i(m - \bar{a}) = m - E_i\bar{a}. \quad (\text{D.6})$$

Substitute $E_i\bar{a}$ by (D.2) and take the average over i :

$$\int E_i v di = m - \frac{\kappa_m + \kappa_s}{\kappa_a + \kappa_m + \kappa_s}\bar{a} = \frac{\kappa_a}{\kappa_a + \kappa_m + \kappa_s}\bar{a} + \frac{\kappa_a + \kappa_s}{\kappa_a + \kappa_m + \kappa_s}v \quad (\text{D.7})$$

The expected monetary policy is given by

$$E_i n = \gamma^a E_i\bar{a} + \gamma^v E_i v \quad (\text{D.8})$$

Substitute $E_i\bar{a}$ and $E_i v$ and get

$$E_i n = \left(\gamma^a \frac{\kappa_m}{\kappa_m + \kappa_s + \kappa_a} + \gamma^v \frac{\kappa_s + \kappa_a}{\kappa_m + \kappa_s + \kappa_a}\right)m + \left((\gamma^a - \gamma^v) \frac{\kappa_s}{\kappa_m + \kappa_s + \kappa_a}\right)a_i \quad (\text{D.9})$$

In the rest of the paper, I simplify (D.9) as:

$$E_i n = \rho_m m + \rho_a a_i \quad (\text{D.10})$$

where $\rho_m = \gamma^a \frac{\kappa_m}{\kappa_m + \kappa_s + \kappa_a} + \gamma^v \frac{\kappa_s + \kappa_a}{\kappa_m + \kappa_s + \kappa_a}$, and $\rho_a = (\gamma^a - \gamma^v) \frac{\kappa_s}{\kappa_m + \kappa_s + \kappa_a}$.

To solve for the higher-order beliefs, first integrate individual expectations over i , which yields that

$$\int_0^i E_i n = \bar{E}n = \rho_m m + \rho_a \bar{a}, \quad (\text{D.11})$$

and then apply E_i to $\bar{E}n$. Successive iteration results in

$$E_i \bar{E}^j n = \rho_m m + \rho_a E_i \bar{E}^{j-1} \bar{a}. \quad (\text{D.12})$$

Having calculated the higher-order beliefs on \bar{a} and n , we are ready to calculate the equilibrium price. Re-write equation A.24 as:

$$p_i = \alpha \left[\sum_{j=1}^{\infty} (1-\alpha)^j E_i \bar{E}^j n + E_i n \right] - \beta \sum_{j=1}^{\infty} (1-\alpha)^j E_i \bar{E}^{j-1} \bar{a} - \beta a_i \quad (\text{D.13})$$

First, substitute $E_i \bar{E}^j n$ in the first term by (D.12). The first term becomes

$$\begin{aligned} \alpha \left[\sum_{j=1}^{\infty} (1-\alpha)^j E_i \bar{E}^j n + E_i n \right] &= \alpha \left\{ \sum_{j=1}^{\infty} (1-\alpha)^j \left[\rho_m m + \rho_a E_i \bar{E}^{j-1} \bar{a} \right] + \rho_m m + \rho_a a_i \right\} \\ &= \alpha \left\{ \sum_{j=0}^{\infty} (1-\alpha)^j \rho_m m + \rho_a \sum_{j=1}^{\infty} (1-\alpha)^j E_i \bar{E}^{j-1} \bar{a} + \rho_a a_i \right\} \end{aligned} \quad (\text{D.14})$$

Now, p_i is written in terms of public and private signals, which is given by

$$\begin{aligned} p_i &= \alpha \left\{ \rho_m m \frac{1}{\alpha} + \rho_a \sum_{j=1}^{\infty} (1-\alpha)^j \left[\left(\frac{\kappa_m}{K} \right) \sum_{k=1}^{k=j} \left(\frac{\kappa_s}{K} \right)^{k-1} m + \left(\frac{\kappa_s}{K} \right)^j a_i \right] + \rho_a a_i \right\} \\ &\quad - \beta \sum_{j=1}^{\infty} (1-\alpha)^j \left\{ \left(\frac{\kappa_m}{K} \right) \sum_{k=1}^{k=j} \left(\frac{\kappa_s}{K} \right)^{k-1} m + \left(\frac{\kappa_s}{K} \right)^j a_i \right\} - \beta a_i \end{aligned} \quad (\text{D.15})$$

To deal with $\sum_{j=1}^{\infty} (1-\alpha)^j \left\{ \left(\frac{\kappa_m}{K} \right) \sum_{k=1}^{k=j} \left(\frac{\kappa_s}{K} \right)^{j-1} \right\}$, first write out the infinite summation as

$$\begin{aligned} \sum_{j=1}^{\infty} (1-\alpha)^j \left\{ \left(\frac{\kappa_m}{K} \right) \sum_{k=1}^{k=j} \left(\frac{\kappa_s}{K} \right)^{j-1} \right\} &= \\ (1-\alpha) \frac{\kappa_m}{K} + (1-\alpha)^2 \frac{\kappa_m}{K} + (1-\alpha)^2 \frac{\kappa_m}{K} \frac{\kappa_s}{K} + (1-\alpha)^3 \frac{\kappa_m}{K} + (1-\alpha)^3 \frac{\kappa_m}{K} \left(\frac{\kappa_s}{K} \right)^2 + \dots \end{aligned} \quad (\text{D.16})$$

and then collect common terms, which yields that

$$(1-\alpha) \frac{\kappa_m}{K} \left\{ \Lambda_1 \cdot 1 + \Lambda_2 \cdot \frac{\kappa_s}{K} + \Lambda_3 \cdot \left(\frac{\kappa_s}{K} \right)^2 + \dots \right\} \quad (\text{D.17})$$

where $\Lambda_1 = \frac{1}{\alpha}$, $\Lambda_2 = \frac{1-\alpha}{\alpha}$, $\Lambda_3 = \frac{(1-\alpha)^2}{\alpha}$. This leads to

$$\sum_{j=1}^{\infty} (1-\alpha)^j \left\{ \left(\frac{\kappa_m}{K} \right) \sum_{k=1}^{k=j} \left(\frac{\kappa_s}{K} \right)^{j-1} \right\} = \frac{1-\alpha}{\alpha} \frac{\kappa_m}{\alpha\kappa_s + \kappa_\xi + \kappa_m} \quad (\text{D.18})$$

Substituting this into (D.15) results in

$$p_i = m \left\{ \rho_m + \left[\rho_a(1-\alpha) - \beta \frac{1-\alpha}{\alpha} \right] \frac{\kappa_m}{\alpha\kappa_s + \kappa_a + \kappa_m} \right\} + a_i \left\{ [\alpha\rho_a - \beta] \frac{(1-\alpha)\kappa_s}{\kappa_m + \kappa_a + \alpha\kappa_s} + \rho_a a_i - \beta \right\} \quad (\text{D.19})$$

Aggregate over i to get the aggregate price level. The aggregate real output is then calculated as the difference between aggregate nominal demand and the aggregate price level. The equilibrium p and y are thus given by

$$p = \phi_m m + \phi_a \bar{a} = (\phi_m + \phi_a) \bar{a} + \phi_m v \quad (\text{D.20})$$

$$y = n - p = (\gamma_a^{cc} - \phi_m - \phi_a) \bar{a} + (\gamma_v^{cc} - \phi_m) v \quad (\text{D.21})$$

where¹⁰

$$\phi_m = \rho_m + \left[\rho_a(1-\alpha) - \beta \frac{1-\alpha}{\alpha} \right] \frac{\kappa_m}{K'} \quad (\text{D.22})$$

$$\phi_a = (\alpha\rho_a - \beta) \frac{K}{K'} \quad (\text{D.23})$$

$$\rho_m = \gamma^a \frac{\kappa_m}{K} + \gamma^v \frac{\kappa_s + \kappa_a}{K} \quad (\text{D.24})$$

$$\rho_a = (\gamma^a - \gamma^v) \frac{\kappa_s}{K} \quad (\text{D.25})$$

D.2 Output-Gap Stabilization Policy

Proof of Corollary 5.1

To close the output gap in the last stage, the central bank chooses $\{\gamma^a, \gamma^v\}$ such that

$$y = \frac{\beta}{\alpha} \bar{a} + 0 \cdot v \quad (\text{D.26})$$

Comparing coefficients with equation (D.21), we have

$$\gamma^a - \phi_m - \phi_a = \frac{\beta}{\alpha} \quad (\text{D.27})$$

$$\gamma^v - \phi_m = 0 \quad (\text{D.28})$$

¹⁰In the rest of the paper, I denote $K = \kappa_a + \kappa_m + \kappa_s$ and $K' = \kappa_a + \kappa_m + \alpha\kappa_s$

Rearranging terms yields the expressions for $\{\phi_a, \phi_m\}$:

$$\phi_a = \gamma^a - \gamma^v - \frac{\beta}{\alpha} \quad (\text{D.29})$$

$$\phi_m = \gamma^v \quad (\text{D.30})$$

From equation (D.29), substitute ϕ_a by equation (D.23):

$$\gamma^a - \gamma^v - \frac{\beta}{\alpha} = [\alpha\rho_a - \beta] \frac{(1-\alpha)\kappa_s}{\kappa_m + \kappa_a + \alpha\kappa_s} + \alpha\rho_a - \beta \quad (\text{D.31})$$

which results in:

$$\rho_a = \frac{\left(\gamma_a - \gamma^v - \frac{\beta}{\alpha}\right) \frac{K'}{K} + \beta}{\alpha}. \quad (\text{D.32})$$

Substituting this result into (D.30) results in the expression for ρ_m given by

$$\rho_m = \gamma^v - \frac{1-\alpha}{\alpha} \left(\gamma^a - \gamma^v - \frac{\beta}{\alpha}\right) \frac{\kappa_m}{K} \quad (\text{D.33})$$

The solutions for (γ^a, γ^v) can be solved by equating the expressions of $\{\rho_a, \rho_m\}$ in equations (D.32) and (D.33) with their expressions in equations (D.24) and (D.25), which is given by

$$\frac{\left(\gamma^a - \gamma^v - \frac{\beta}{\alpha}\right) \frac{K'}{K} + \beta}{\alpha} = (\gamma^a - \gamma^v) \frac{\kappa_s}{K} \quad (\text{D.34})$$

$$\gamma^v - \frac{1-\alpha}{\alpha} \left(\gamma^a - \gamma^v - \frac{\beta}{\alpha}\right) \frac{\kappa_m}{K} = \gamma^a \frac{\kappa_m}{K} + \gamma^v \frac{\kappa_a + \kappa_a}{K} \quad (\text{D.35})$$

By rearranging the two equations we find that the two equations are co-linear in $(\gamma^a - \gamma^v)$. So the above two equations only yield the difference between γ^a and γ^v , which is given by

$$\gamma^a - \gamma^v = \frac{\beta}{\alpha} (1-\alpha) \quad (\text{D.36})$$

This defines the set of monetary policy that close the output gap in the last stage.

D.3 The Optimal Discretionary Monetary Policy

The optimal discretionary monetary policy is solved by backward induction. The result of optimization in the last stage is presented in Corollary 5.1. We now work on the first stage. First, substitute the solution in the last stage, $\gamma^a - \gamma^v = \frac{\beta}{\alpha} (1-\alpha)$ into equations ((D.22) -

(D.25)), which results in:

$$\phi_m = \gamma^v \quad (\text{D.37})$$

$$\phi_a = -\beta \quad (\text{D.38})$$

$$\rho_m = \gamma^a - \gamma^d \frac{\kappa_s + \kappa_a}{K} \quad (\text{D.39})$$

$$\rho_a = \gamma^d \frac{\kappa_s}{K} \quad (\text{D.40})$$

The equilibrium price and output level under discretionary monetary policy thus become:

$$p = (\gamma^v - \beta)\bar{a} - \gamma^v v \quad (\text{D.41})$$

$$y = \frac{\beta}{\alpha}\bar{a} + 0 \cdot v \quad (\text{D.42})$$

The extra degree of flexibility in the policy response function allows the central bank to minimize ex-ante price deviations while keeping the output gap closed ex-post. In this case, the central bank's objective function reduces to minimizing the ex-ante variance of the price level. The central bank's objective function then reduces to:

$$\min_{\gamma^v} E[p^2] = \min_{\gamma^v} \{(\gamma^v - \beta)^2 \sigma_a^2 + (\gamma^v)^2 \sigma_v^2\} \quad (\text{D.43})$$

The first-order condition on γ^v results in

$$\frac{\gamma^v - \beta}{\gamma^v} = -\frac{\sigma_v^2}{\sigma_a^2} \quad (\text{D.44})$$

which suggests that the central bank targets a negative ratio between the equilibrium price after \bar{a} shock and the equilibrium price after a noise shock. The absolute value of the ratio is the relative precision of the public signal and the prior of the technology shock.

Solving γ^v from the first-order condition results in:

$$\gamma^v = \beta \frac{\sigma_a^2}{\sigma_a^2 + \sigma_v^2} \quad (\text{D.45})$$

D.4 Optimal Policy Rule

The optimal policy rule minimizes the ex-ante loss of the central bank, given by

$$E[W] = (\gamma^a - \phi_m - \phi_a)^2 \sigma_a^2 + (\gamma^v - \phi_m)^2 \sigma_v^2 + \tau [(\phi_m + \phi_a)^2 \sigma_a^2 + \phi_m^2 \sigma_v^2] \quad (\text{D.46})$$

The first-order condition on γ^a results in:

$$\begin{aligned} & \sigma_a^2 \left[(\gamma^a - \phi_m - \phi_a) \frac{d(\gamma^a - \phi_m - \phi_a)}{d\gamma^a} + \tau(\phi_m + \phi_a) \frac{d(\phi_m + \phi_a)}{d\gamma_{fnc}^a} \right] \\ & + \sigma_v^2 \left[(\gamma^v - \phi_m) \frac{d(\gamma^v - \phi_m)}{d\gamma^a} + \tau\phi_m \frac{d\phi_m}{d\gamma_{fnc}^a} \right] = 0 \end{aligned} \quad (D.47)$$

Re-arrange the above equation to get:

$$\sigma_a^2 \left\{ (\gamma^a - \phi_m - \phi_a) + (-\gamma^a + (1 + \tau)(\phi_m + \phi_a)) \frac{d(\phi_m + \phi_a)}{d\gamma^a} \right\} \quad (D.48)$$

$$+ \sigma_v^2 \left\{ (\gamma^v - \phi_m) + (-\gamma^v + (1 + \tau)\phi_m) \frac{d\phi_m}{d\gamma_{fnc}^a} \right\} = 0 \quad (D.49)$$

where

$$\frac{d(\phi_m + \phi_a)}{d\gamma_{fnc}^a} = \frac{\partial\phi_m}{\partial\rho_m} \frac{\partial\rho_m}{\partial\gamma_{fnc}^a} + \frac{\partial\phi_m}{\partial\rho_a} \frac{\partial\rho_a}{\partial\gamma_{fnc}^a} + \frac{\partial\phi_a}{\partial\rho_m} \frac{\partial\rho_m}{\partial\gamma_{fnc}^a} + \frac{\partial\phi_a}{\partial\rho_a} \frac{\partial\rho_a}{\partial\gamma_{fnc}^a} \quad (D.50)$$

$$\frac{d\phi_m}{d\gamma_{fnc}^a} = \frac{\partial\phi_m}{\partial\rho_m} \frac{\partial\rho_m}{\partial\gamma_{fnc}^a} + \frac{\partial\phi_m}{\partial\rho_a} \frac{\partial\rho_a}{\partial\gamma_{fnc}^a} \quad (D.51)$$

and

$$\frac{\partial\phi_m}{\partial\rho_m} = 1 \quad (D.52)$$

$$\frac{\partial\phi_a}{\partial\rho_m} = 0 \quad (D.53)$$

$$\frac{\partial\phi_m}{\partial\rho_a} = (1 - \alpha) \frac{\kappa_m}{\alpha\kappa_s + \kappa_a + \kappa_m} \quad (D.54)$$

$$\frac{\partial\phi_a}{\partial\rho_a} = \frac{\alpha K}{K'} \quad (D.55)$$

$$\frac{\partial\rho_m}{\partial\gamma^a} = \frac{\kappa_m}{K} \quad (D.56)$$

$$\frac{\partial\rho_a}{\partial\gamma^a} = \frac{\kappa_s}{K} \quad (D.57)$$

$$\frac{\partial\rho_a}{\partial\gamma^v} = \frac{\kappa_s + \kappa_a}{K} \quad (D.58)$$

$$\frac{\partial\rho_a}{\partial\gamma^v} = -\frac{\kappa_s}{K} \quad (D.59)$$

The first-order condition on γ^v is calculated similarly.

E Alternative Assumption: Firms Have Common Technology Shocks

When firms have homogeneous technology but have imperfect information on this common shock, equation (8) becomes

$$p_i = E_i [p + \alpha y - \beta \bar{a}], \quad (\text{E.1})$$

as firms also need to guess their own cost of production.

Following the same procedure as described in Appendix A, substitute $y = n - p$ into p_i and get

$$p_i = (1 - \alpha)E_i p + \alpha E_i n - \beta E_i \bar{a} \quad (\text{E.2})$$

The higher-order beliefs are again solved by first taking the integral of p_i to get the aggregate price level,

$$p = (1 - \alpha)\bar{E}p + \alpha\bar{E}n - \beta\bar{E}\bar{a} \quad (\text{E.3})$$

and then substituting this expression for p into p_i . Iterating this process yields that

$$p_i = (1 - \alpha)E_i \left[(1 - \alpha)\bar{E}p + \alpha\bar{E}n - \beta\bar{E}\bar{a} \right] + \alpha E_i n - \beta E_i \bar{a} \quad (\text{E.4})$$

$$= (1 - \alpha)^2 E_i \bar{E}p + \alpha(1 - \alpha)E_i \bar{E}n + \alpha E_i n - (1 - \alpha)\beta E_i \bar{E}\bar{a} + \beta E_i \bar{a} \quad (\text{E.5})$$

Taking the expectation about the aggregate technology shock to the infinite order, p_i becomes

$$p_i = (1 - \alpha)^\infty E_i \bar{E}^\infty p + \alpha \sum_{j=0}^{\infty} (1 - \alpha)^j E_i \bar{E}^j n - \beta \sum_{j=0}^{\infty} (1 - \alpha)^j E_i \bar{E}^j \bar{a} \quad (\text{E.6})$$

where

$$E_i \bar{E}^j \bar{a} = \left(\frac{\kappa_s}{\kappa_s + \kappa_a} \right)^{j+1} a_i \quad (\text{E.7})$$

Substituting $n = \gamma \bar{a}$ and $E_i \bar{E}^j \bar{a}$ into p_i yields that

$$p_i = \frac{(\alpha\gamma - \beta)\kappa_s}{\kappa_a + \alpha\kappa_s} a_i. \quad (\text{E.8})$$

Take the average of p_i and apply $y = n - p$, which yields the equilibrium price level and the output level to be

$$p = \frac{(\alpha\gamma - \beta)\kappa_s}{\kappa_a + \alpha\kappa_s} \bar{a},$$

$$y = \frac{\gamma\kappa_a + \beta\kappa_s}{\kappa_a + \alpha\kappa_s} \bar{a}.$$

By choosing $\gamma = \frac{\beta}{\alpha}$, the central bank closes the output gap and stabilizes the price level at the same time, meaning that the price stabilization policy is time consistent, and implementing the optimal monetary policy does not require credible commitment.