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Decreasing Fixed Costs**

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Sandra Paterlini



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Recreating Banking Networks under Decreasing Fixed Costs

Dietmar Maringer, Ben R. Craig, and Sandra Paterlini

Theory emphasizes the central role of the structure of networks in the behavior of financial systems and their response to policy. Real-world networks, however, are rarely directly observable: Banks' assets and liabilities are typically known, but not who is lending how much and to whom. We first show how to simulate realistic networks that are based on balance-sheet information by minimizing costs where there is a fixed cost to forming a link. Second, we also show how to do this for a model with fixed costs that are decreasing in the number of links. To approach the optimization problem, we develop a new algorithm based on the transportation planning literature. Computational experiments find that the resulting networks are not only consistent with the balance sheets, but also resemble real-world financial networks in their density (which is sparse but not minimally dense) and in their core-periphery and disassortative structure.

Keywords: banking networks, network models, optimization.

JEL: E59, G21, C40.

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1 Introduction

It is well known that the structure of financial networks is important in assessing systemic risk ([Allen and Gale \(2000\)](#), [Freixas et al. \(2000\)](#), [Lux \(2017\)](#)). Network structure is also important in other areas of finance, such as market efficiency or payments processing (see [Glassermann and Young \(2015\)](#) for a review). In spite of its importance, the analysis of networks is made difficult by a lack of data for essential markets. For many crucial financial networks, a researcher only has the assets and liabilities of individual agents in a financial network. The bilateral arrangements between individuals are missing.

One strategy for dealing with the lack of network data is to back out a network structure from balance-sheet information and banks' optimizing decisions. Unfortunately, this is complicated by the lack of appropriate numerical tools. Network problems are complex. For example, consider the decision surrounding interbank loans where the links between nodes are whether to lend to a specific bank and how much to lend. This problem must consider many possibilities that are both discrete (the decision to open a costly link and make a loan between two particular banks) and continuous (the amount of lending through this link). If a network has more than 20 nodes, a numerical solution to a mixed continuous-discrete problem is well-known to be very hard to solve exactly, and with more than 100 nodes, it is hard to find even an approximate solution to the problem.

In this paper, we propose a set of heuristics for constrained network formation that are adapted from a rich literature in the analysis of transportation costs. We suitably modify these heuristics so that they can fit or simulate financial networks with characteristics that we observe in the real world. Our application incorporates cost measures that could be estimated structurally in our fairly simple framework.

We start from a basic fixed-cost of a link that is consistent with the observation that interbank activity is based on relationships ([Cocco et al. \(2009\)](#)). Establishing and maintaining a lending-borrowing relationship (that is, a link) is expensive. The monitoring costs associated with this link are paid by both borrower and lender, although usually the lender pays more up-front. Our model assigns to each bank a fixed cost that is independent of contract size but decreases for every additional link formed by a bank. The interpretation of this is that as a bank develops risk controls within its institution, these controls are subject to increasing returns to scale. This is also consistent with the observation of [Cocco et al. \(2009\)](#) and others that banks with large reserve imbalances typically engage in many relationships. It is in line with the idea that banks with more links will diversify their large portfolios. In other words, additional links are expensive due to information processing, risk management, and credit-worthiness checks, but such costs decrease as the bank establishes more links and its infrastructure

grows. Establishing more links promotes further diversification, and is helpful where it might not be possible to find a single counterparty to satisfy all their liquidity or borrowing needs. Moreover, our model allows us to distinguish costs that are borne by the lender, or the borrower, or split between the two parties.

The model determines the optimal network configuration, where optimality is defined with respect to a single parameter geometrically decreasing fixed cost. The optimization problem is NP-hard and the search space has frictions and multiple local and global optima. To solve it, borrowing from transportation theory, we introduce the North-West-Corner-Rule (NWCR) and one variation, relying on simulated annealing, which allows reliable solutions to be found within reasonable time and does not require function approximation by Markov-Chain Monte-Carlo as in [Anand et al. \(2015\)](#).

Our numerical experiments show that the proposed approach reproduces several stylized facts of interbank networks: sparsity, core-periphery and disassortative structures. By starting from a realistic marginal distribution of assets and liabilities, the core-periphery of [Craig and von Peter \(2014\)](#) emerges as a natural property of the system, and the distribution of contract sizes exhibits a smaller right tail. This indicates the less extreme lender-borrower relationships that we generally observe. Due to its simplicity, we can further explore what happens when the borrowers or the lenders pay the cost of a link, and we consider a shared-cost version.

The problem of reconstructing a network that satisfies a given set of assets and liabilities of the individual banks has prompted a stream of research (see [Squartini et al. \(2018\)](#), [Gao et al. \(2018\)](#), [Hueser \(2015\)](#) for reviews on these methods applied to financial networks). [Upper and Worm \(2004\)](#) suggested employing maximum entropy methods as they are easy to compute. However, maximum entropy, while satisfying the balance-sheet constraint, does not replicate characteristics such as sparsity and a core-periphery structure that are observed in networks where more complete data are available. In such a structure, core members are strongly connected to each other, whereas periphery members establish just a few links with core members but none with other periphery members ([Borgatti and Everett \(2000\)](#)). Alternative methods based on copulas, bootstrap, and iterative algorithms have been subsequently developed and compared. Among these alternatives, [Anand et al. \(2018\)](#) consider 25 different financial markets spread out among 13 regulatory jurisdictions for which the complete network data are available. The fixed-cost model developed by [Anand et al. \(2015\)](#) tends to simulate networks that outperform all other simulations with respect to the Hamming distance and accuracy score between the simulated and actual networks. This model constructed a sparse network that managed to avoid the inclusion of links that are not present in the actual network. The simulations very rarely exhibited a core-periphery structure, and used an algorithm that performed slowly, often

converging to a local optimum (see Figure 1 in [Anand et al. \(2015\)](#)).

Where bilateral data exist, a few works so far have focused on different financial markets. These include [Borgatti and Everett \(2000\)](#) for the Austrian interbank market; [Li et al. \(2018\)](#) for the Chinese loan network with core-periphery structure, [Silva et al. \(2016\)](#) for the Brazilian core-periphery market structure; [Finger et al. \(2013\)](#) for the e-MID overnight money market; [Iori et al. \(2015\)](#), [Temizsoy et al. \(2015\)](#) for the e-MID Interbank market; and [Van Lelyveld and In'T Veld \(2014\)](#) for the core-periphery structure in the Netherlands. These empirical investigations establish a few stylized facts of interbank lending, such as a typical core-periphery structure, network sparsity, and disassortativeness.

The agent-based modeling literature has focused on replicating some of these characteristics, as in the work of [Gurgone et al. \(2018\)](#) and [Liu et al. \(2018\)](#) and within dynamic modeling frameworks, as in [Zhang et al. \(2018\)](#), [Guleva et al. \(2017\)](#), [Xu et al. \(2016\)](#), and [Capponi and Chen \(2015\)](#). [Lux \(2015\)](#) introduces a simple dynamic agent-based model that, starting from a heterogeneous bank size distribution and relying on a reinforcement learning algorithm based on trust, allows the system to naturally evolve toward a core-periphery structure where core banks assume the role of mediators between the liquidity needs of many smaller banks. [Blasques et al. \(2018\)](#) propose a dynamic network model of interbank lending for the Dutch interbank market, pointing out that credit-risk uncertainty and peer monitoring are driving factors for the sparse core-periphery structure.

The paper is structured as follows. Section 2 introduces state-of-the-art methods and our proposed model with decreasing marginal fixed costs. Section 3 focuses on the optimization model and introduces the new NWCR obtained by combining heuristics, such as simulated annealing, with the classical NWCR from transportation theory. Section 4 describes the properties of networks under decreasing marginal fixed costs. Finally, Section 5 draws the main conclusions and the outlook for further research.

2 Reconstructing Networks

2.1 Existing Methods

The problem of reconstructing banking networks based on limited information has attracted some attention in the literature, in particular for the case where, for N banks, their (total) assets, A_i , and liabilities, L_j , with $i, j = 1, \dots, N$ are known, but not who is lending to whom, let alone the exact amounts, z_{ij} . All we do know is that the volumes are not negative and that budget constraints must hold; and if the banks' identities are known (and indices i and j correspond), self-lending

might be excluded:

$$\sum_{j=1}^N z_{ij} = A_i, \quad \sum_{i=1}^N z_{ij} = L_j, \quad z_{ij} \geq 0, \quad (z_{ii} = 0) . \quad (1)$$

The basic problem in this situation is that there are (potentially: infinitely) many networks $\mathbb{Z} = [z_{ij}]$ that meet these criteria. Network reconstruction methods can therefore not assume that there is a unique solution but require additional assumptions. These can simplify or exacerbate the computational requirements, and they often lead to different network properties more or less close to stylized facts for real networks.¹

A popular group of such reconstruction methods is based on the maximum entropy (ME) principle. In simple terms, the idea is to create a fully connected network (with or without the constraint $z_{ii} = 0$) where the contract size z_{ij} between lender i and borrower j is proportional to the size of A_i and L_j relative to their market shares. Initiated by [Upper and Worm \(2004\)](#), this method is simple and fast, but by construction misses some stylized facts of real-world networks. Among other things, real banking networks are far from fully connected, but tend to have a low density. Also, a lender to two borrowers could easily have the larger exposure to the smaller of the two, a portfolio of a real-world lender i twice the size of another lender l is not twice l 's portfolio. Subsequent methods try to incorporate this: [Drehmann and Tarashev \(2013\)](#), for example, suggest re-scaling ME networks based on stochastic principles while retaining 100 percent density.

Other approaches explicitly control for the density: [Cimini et al. \(2015\)](#), e.g., incorporate a density parameter in their fitness model, while [Musmeci et al. \(2013\)](#) combine the density-driven link selection with ME. [Hałaj and Kok \(2013\)](#), on the other hand, suggest an iterative sampling technique that starts with an empty network, $z_{ij}^{(0)}$, and where in each iteration, t , a random pair (i, j) of a lender i and a borrower j is drawn and their contract size is increased according to $z_{ij}^{(t)} = z_{ij}^{(t-1)} + \min\left(A_i^{(t-1)}, u \cdot L_j^{(t-1)}\right)$, where u is a uniform random number and $A_i^{(t-1)}$ and $L_j^{(t-1)}$ are not yet allocated assets and liabilities, respectively. This is repeated until all assets and liabilities are assigned. The resulting network typically exhibits low density, but not, e.g., a core-periphery structure.

[Anand et al. \(2015\)](#) consider sparsity as an explicit objective: links are costly and banks therefore have an inherent motive to keep the number of links as low as possible. They argue that any active link causes fixed costs irrespective of the actual contract size. If the variable costs, proportional to contract size, are the same between all parties (e.g., the interbank offered rate), it should not affect

¹[Anand et al. \(2018\)](#) provide a horse race between popular existing methods, and for some of them, their appendix provides some information and results.

the choice (and number) of partners (and, from an optimization perspective, can be neglected); additional or avoidable fixed costs, however, should, and keeping the number of links at a minimum is advantageous. Reconstructing a network can therefore be considered a minimization problem for the overall cost of the network, $F(\mathbf{Z})$:

$$\min_{\mathbf{Z}} F(\mathbf{Z}) \quad (2)$$

where

$$F(\mathbf{Z}) = c \cdot \sum_{i=1}^N \sum_{j=1}^N 1_{ij} \quad (3)$$

$$1_{ij} = \begin{cases} 1 & \text{if } z_{ij} > 0 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

under budget and non-negativity constraints (1). c is the constant fixed costs per link, and 1_{ij} is a binary indicator as to whether or not bank i lends to bank j . This is equivalent to finding the network with the lowest average degree, i.e., the lowest number of edges, under given constraints.

Since this is a directed network with edges (links) running from lenders (assets) to borrowers (liabilities), the out-degree, d_i^A , of bank i is the number of banks i is lending to, while j 's in-degree, d_j^L , is the number of banks j is borrowing from:

$$d_i^A = \sum_{j=1}^N 1_{ij}, \quad d_j^L = \sum_{i=1}^N 1_{ij}.$$

The cost function can be rewritten as a function of the banks' (and, ultimately, the network's) degrees—from the lenders' perspective as

$$F(\mathbf{Z}) = c \cdot \sum_{i=1}^N \left(\sum_{j=1}^N 1_{ij} \right) = c \cdot \sum_{i=1}^N d_i^A \quad (3^*)$$

or, from the borrowers' perspective as

$$F(\mathbf{Z}) = c \cdot \sum_{j=1}^N d_j^L,$$

or, if costs are incurred by either side and $c = c^A + c^L = c \cdot \alpha + c \cdot (1 - \alpha)$ with $0 \leq \alpha \leq 1$, then

$$F(\mathbf{Z}) = c \cdot \alpha \cdot \sum_{i=1}^N d_i^A + c \cdot (1 - \alpha) \cdot \sum_{j=1}^N d_j^L. \quad (3^{**})$$

The total number of links in the minimum density (MD) network \mathbb{Z}_{MD} is $\sum_{i,j} 1_{ij} = \sum_i d_i^A = \sum_j d_j^L$ and can be considered the lower bound for the actual number of links in the real (unobserved) network behind the A s and L s.

From an optimization point of view, c is just a scalar, and solving optimization problem (2) produces a minimum density network with the lowest possible degree. Anand et al. (2018) find that networks constructed using such methods compare favorably with those using other popular methods along many dimensions. Nonetheless, there are still aspects that can be improved. For one, real-world networks are usually less sparse than the MD solution, and MD networks do not have a pronounced core-periphery structure. Also, it might be a strong assumption that any link causes the same fixed costs. We suggest an extended model to remedy these issues.

2.2 A Model with Decreasing Marginal Fixed Costs

If the fixed costs of a link are caused by establishing and maintaining a link between two banks, then it seems reasonable to assume a learning curve: an already well-connected bank can draw from experience and can spread its overhead over more contracts, and any additional link will incur lower costs than the previous (or first) one. Assuming that there is a geometric decay in the additional fixed costs depending on the bank's already existing links and reflected by a factor γ with $0 \leq \gamma \leq 1$, then the first link will come with costs of c , the second with $c \cdot \gamma$, the third with $c \cdot \gamma^2$ and so on until the last with $c \cdot \gamma^{d-1}$ where d is this bank's degree, i.e., its number of links. In total, this bank's costs add up to

$$C(d, \gamma) = c \cdot (1 + \gamma + \dots + \gamma^{d-1}) = c \cdot \sum_{k=0}^{d-1} \gamma^k = c \cdot \begin{cases} d & \text{for } \gamma = 1 \\ \frac{1-\gamma^d}{1-\gamma} < d & \text{for } 0 \leq \gamma < 1 \end{cases} \quad (5)$$

Assume that, akin to (3*), all costs are from the lenders' perspective so that the new cost function now reads

$$F_A^*(\mathbb{Z}) = c \cdot \sum_{i=1}^N C(d_i^A, \gamma_A).$$

In this extended model, the costs of the network still depend on the degree, but no longer in a linear fashion when the marginal costs are shrinking, $\gamma_A < 1$. If there is no decay in fixed costs, $\gamma_A = 1$, then this model is identical to the minimum density (MD) model.

In the MD model, it makes no difference whether costs are incurred by lenders, borrowers, or both, and the solution for cost function (3**) is not affected by the choice of α . When marginal costs are decreasing, however, things are different:

lenders and borrowers can have different (distributions of) degrees, and they might face different learning curves with separate γ_A and γ_L .

The new optimization model with decreasing marginal fixed costs can then be summarized as follows:

$$\min_{\mathbf{Z}} F^*(\mathbf{Z}) = c \cdot \alpha \cdot \sum_{i=1}^N C(d_i^A, \gamma_A) + c \cdot (1 - \alpha) \cdot \sum_{j=1}^N C(d_j^L, \gamma_L) \quad (6)$$

with banks' cost function (5) and with constraints (1) on budget, non-negative volumes, and, where applicable, self-lending. The parameter α controls who incurs the costs: for $\alpha = 1$, it is exclusively the lenders, for $\alpha = 0$, it is the borrowers, and for values between these extremes, it is a combination.

3 Finding Cost-Optimized Networks

3.1 The Underlying Optimization Problem

Both the minimum density and the decreasing marginal fixed-cost models present challenging optimization problems: there are no closed-form analytical solutions, and numerically they are hard to tackle. They are discrete and non-convex with local optima, rendering hill-climbing methods unreliable. Non-deterministic methods might be suitable to overcome those local optima. For example, [Anand et al. \(2015\)](#) employ a Markov-Chain Monte-Carlo (MCMC) approach and find sparse solutions to their test problems. However, a closer look reveals that convergence is rather slow, and that even for their small test problem with just six lenders and five borrowers, their reported solution turns out to be suboptimal. Interestingly, they also point out that their problem is equivalent to the Fixed-Cost Transportation Problem (FCTP), a popular archetypal problem in logistics. There, the situation is that suppliers $i = 1, \dots, M$ can produce goods that are sold in outlets $j = 1, \dots, N$. The assignment problem then is to find the quantities x_{ij} that are shipped from producers i to outlets j such that the outlets' demands are met ($\sum_i x_{ij} = D_j$) without exceeding the suppliers' capacities ($\sum_j x_{ij} \leq S_i$) while minimizing the overall costs, $\sum_i \sum_j x_{ij} \cdot v_{ij} + \sum_i \sum_j 1_{ij} \cdot f$, where v_{ij} are proportional costs per unit of goods (e.g., costs per truck transporting one unit of goods over the distance between i and j) and f is the fixed costs for this active link (e.g., the set-up costs for the cooperation between i and j). Quantities must not be negative ($x_{ij} \geq 0$), and $1_{ij} = 1$ indicates an active link (if $x_{ij} > 0$; $1_{ij} = 0$ otherwise). For the special case of $\sum_i S_i = \sum_j D_j$, the problem is also known as Balanced FCTP.

In the banking network problem, the lenders correspond to the suppliers, the borrowers to the outlets, and contract sizes to the shipped quantities. v_{ij} could be seen as the interest rate; if this is the same for all combinations ($v_{ij} = v \forall (i, j)$),

e.g., in the absence of risk premia, time spreads, etc.), the variable part of the cost function can be dropped as it always adds up to the same value, leaving only the part with the fixed costs to be relevant. And it is exactly this part that makes it a challenging optimization problem; then for a relatively small number of producers and outlets (or, here, banks), optimal solutions cannot be found in a reasonable time frame. In a market with N banks, there are up to N^2 potential links each of which could be active or inactive; this amounts to as many as 2^{N^2} combinations even before considering how to distribute the quantities. With the constraint $z_{ii} = 0$ active (a situation hardly relevant in the traditional FCTP), these numbers are lowered to $N(N - 1) = N^2 - N$ and $2^{N^2 - N}$, respectively; either way, the search space is vast, even for very small markets: for example, for just $N = 10$ banks, there would be $2^{100} > 1.26e+30$ and $2^{90} > 1.23e+27$ alternatives, respectively, just for setting the links, and before assigning actual quantities z_{ij} . This is why a number of initialization methods have been developed that create solutions that are at least feasible and valid with respect to the constraints.

One such method is the North-West-Corner-Rule (NWCR; see, e.g., [Hillier and Liebermann \(2010\)](#)) for which we suggest a modification to approach our cost-minimization problem for network reconstruction.

3.2 The Modified North-West-Corner-Rule

The original North-West-Corner-Rule uses a tableau where the rows are the suppliers (here: lenders) and the columns are the outlets (here: borrowers) and where quantities are iteratively assigned. Using the notation from our problem, it is initialized by considering all assets and liabilities to be unassigned ($A_i^{(0)} = A_i$, $L_j^{(0)} = L_j$) and all links to be inactive ($z_{ij} = 0$). Then it follows an iterative procedure where in each iteration, t , one pair of a lender i and a borrower j is selected. In the first iteration, it is the combination of the top-left corner (“north-west” corner, hence the name) of the tableau with ($i = 1, j = 1$). Next, one checks how much of i 's supply is still unassigned, and how much open demand j has left. The lower of the two is the maximum contract size between the two and chosen for z_{ij} , while i 's available supply and j 's open demands are lowered accordingly: $z_{ij} = \min(A_i^{(t)}, L_j^{(t)})$, $A_i^{(t)} = A_i^{(t-1)} - z_{ij}$, and $L_j^{(t)} = L_j^{(t-1)} - z_{ij}$. If i 's assets are now exhausted ($A_i^{(t)} = 0$), one moves on to the next supplier ($i := i + 1$; move south in the tableau); if j has no further demand for liabilities ($L_j^{(t)} = 0$), one moves to the next borrower ($j := j + 1$; move east in the tableau). This finishes iteration t and starts iteration $t + 1$. These iteration steps are then repeated until no available assets and no demand for liabilities are left, and the south-east corner has been

reached. The network Z_{MD} in Table 1(b) has been created in exactly that fashion.²

The NWCR does produce a feasible solution as it is based on the constraints. However, it does not guarantee an optimal solution for two reasons: (a) costs, fixed and variable alike, are ignored in the creation process, and (b) the sequence in which the banks are listed drives the assignments. At the same time, it tends to produce very sparse networks, which, here, works to our advantage as it keeps the aggregate fixed costs low. The second aspect concerning the sequence is usually considered a downside, as the sorting and the resulting pairs (i, j) are often arbitrary. For the problem at hand, we suggest turning this into a crucial feature of the optimization: we use it to our advantage and restate the search process as finding a permutation of the lenders and borrowers such that the resulting network from the NWCR has minimum costs. This turns the optimization problem into a traveling salesman problem (TSP)—which is NP hard (i.e., with no known algorithm where the required computation time is no worse than polynomial in the number of instances), but for which non-deterministic search methods have been found to have favorable convergence properties. For the problem at hand, we adopt a Simulated Annealing algorithm, originally suggested by [Kirkpatrick et al. \(1983\)](#), and fit it to our problem: starting with a random solution, each iteration produces one new solution (here: a slightly varied permutation of lenders and borrowers) by mutating the current one. In the case of the TSP, that can be done by randomly switching two or more randomly chosen elements (e.g. aBcdEf becomes aEcdBf), or by randomly choosing a segment of the sequence and reversing its order (e.g., abcDEf becomes abEDCf). If the mutant solution is better than the current one, it replaces it; but a worse solution can also be accepted, with a certain probability, to overcome local optima: the larger the downhill step (and the longer the search process has been going on), the lower that probability. After its final iteration, the algorithm reports the best of all tested candidates. [B](#) provides pseudo-algorithms for the Simulated Annealing algorithm and the modified North-West Corner Rule.

Mimicking the real-world crystallization and annealing process, a “temperature” parameter gears the search process: if the temperature is too generous (high), any mutant is accepted and the search turns into a random walk; if it is converging to zero (i.e., even the slightest deterioration is unlikely to be accepted), then it turns into a hill-climber. With a well-chosen temperature and cooling plan, however, the process creates a trajectory through the search domain that can overcome local optima (thanks to accepting some of the downhill steps), but with a tendency to converge to the optimum (thanks to ignoring very damaging

²In some respects, it is similar to the method suggested by [Halaj and Kok \(2013\)](#), but without the random term in determining the actual quantity, always assigning the highest possible amount, and without randomness when selecting the pairs.

steps, but with a dominating preference for uphill steps). Typically, the temperature parameter is therefore calibrated with a reasonably generous value in the beginning, but gradually lowered toward zero, to shift the search process from an explorative to an exploitative behavior.³

4 Properties of Networks under Decreasing Marginal Fixed Costs

4.1 A Simple Illustrative Example

Assume there are six banks that are all lenders with assets $[A_i] = [50, 15, 12, 11, 8, 4]$ and another six banks that are all borrowers with $[L_j] = [51, 14, 13, 10, 9, 3]$. No bank is both lender and borrower; the self-lending constraint can be ignored. Banks are sorted by size in descending order to facilitate interpretation. Table 1 shows three different networks, all of which satisfy the budget and non-negativity constraints and produce the required assets and liabilities. The scalar c is assumed to be 1 here and in all subsequent experiments.

The network Z_{ME} in Subtable 1(a) has been created with the maximum entropy (ME) approach. Note that all columns (portfolios of borrowers) are proportional to each other, reflecting the relative size of the banks' liabilities; the same is true for all rows (portfolios of lenders): bank E is twice the size of F, $z_{Ej} = 2 \cdot z_{Fj} \forall j = 1, \dots, N$. By construction, this is a fully connected network with 36 (out of a possible 36) active links, resulting in a density of 100 percent, and where each lender is linked to any borrower and vice versa. Any bank, lender and borrower, therefore, has a degree of 6. With fixed constant costs of 1 per link, this adds up to total network costs of 36. If costs are incurred by lenders and their decay factor is $\gamma_A = 0.7$, the marginal costs for additional links go down, and each lender's costs are reduced from 36 to 2.94, causing overall network costs of 17.65 (imprecision due to rounding).

Network Z_{MD} in Subtable 1(b) is one possible solution when the network is being reconstructed under minimum density (MD).⁴ Any bank i with positive assets needs to have at least one link. If there is another bank j with liabilities $L_j = A_i$, then both can do with just this one link, otherwise, either i or j needs at least one additional link, depending on whose position is larger. In this example, (at least) three of these additional links are required. It can be seen that a density

³For general presentation of non-deterministic search methods for economic optimization problems, see, e.g., [Gilli et al. \(2019\)](#).

⁴This solution has been found with the NWCR method presented in Section 3.2 and alphabetical ordering (here identical to decreasing size) of the banks. Note that several solutions exist which all have the same optimal overall degree of 9.

Table 1: Reconstructed networks for a simple example with six lenders (A–F) and six borrowers (K–P), respectively. d_i^A is the (out-) degree of lender i , C_i^A are i 's total costs, depending on decay factor γ_A . Costs are incurred by lenders only ($\alpha = 1$).

(a) Z_{ME} : Reconstructed under maximum entropy.

		liabilities						d_i^A	C_i^A $\gamma_A = 1$	C_i^A $\gamma_A = 0.7$	
		K	L	M	N	O	P				
		51	14	13	10	9	3				
assets	A	50	25.5	7	6.5	5	4.5	1.5	6	6	2.94
	B	15	7.65	2.1	1.95	1.5	1.35	0.45	6	6	2.94
	C	12	6.12	1.68	1.56	1.2	1.08	0.36	6	6	2.94
	D	11	5.61	1.54	1.43	1.1	0.99	0.33	6	6	2.94
	E	8	4.08	1.12	1.04	0.8	0.72	0.24	6	6	2.94
	F	4	2.04	0.56	0.52	0.4	0.36	0.12	6	6	2.94
d_j^L		6	6	6	6	6	6	36			
$\sum_i C_i^A$									36	17.65	

(b) Z_{MD} : Reconstructed under minimum density.

		liabilities						d_i^A	C_i^A $\gamma_A = 1$	C_i^A $\gamma_A = 0.7$	
		K	L	M	N	O	P				
		51	14	13	10	9	3				
assets	A	50	50	1	1	1.0
	B	15	1	14	2	2	1.7
	C	12	.	.	12	.	.	.	1	1	1.0
	D	11	.	.	1	10	.	.	2	2	1.7
	E	8	8	.	1	1	1.0
	F	4	1	3	2	2	1.7
d_j^L		2	1	2	1	2	1	9			
$\sum_i C_i^A$									9	8.1	

(c) Z_{DC} : Reconstructed under Decreasing Marginal Fixed Costs for $\gamma_A = 0.7$.

		liabilities						d_i^A	C_i^A $\gamma_A = 1$	C_i^A $\gamma_A = 0.7$	
		K	L	M	N	O	P				
		51	14	13	10	9	3				
assets	A	50	1	14	13	10	9	3	6	6	2.94
	B	15	15	1	1	1.00
	C	12	12	1	1	1.00
	D	11	11	1	1	1.00
	E	8	8	1	1	1.00
	F	4	4	1	1	1.00
d_j^L		6	1	1	1	1	1	11			
$\sum_i C_i^A$									11	7.94	

of 25 percent (9 active out of 36 possible links) suffices to generate a valid solution. In this particular solution, lenders can do with just one or at most two links, reducing their respective costs to 1 and 2 when there are constant fixed costs, adding up to overall costs of 9, which is optimal under constant fixed costs: for this market, there exists no solution with fewer than nine links.

When additional fixed costs decrease with $\gamma_A = 0.7$, the lender's costs are 1 (when $d_i^A = 1$) and $1+0.7 = 1.7$ (when $d_i^A = 2$), respectively. In this case, the overall costs for the MD network are now just 8.1—which is no longer optimal: under decreasing marginal fixed costs, there exist solutions that have more links, yet lower overall costs, thanks to exploiting reductions in the additional fixed costs. Simply speaking, if there is a need for one extra link, it is cheaper to have it not as a second link, but to ride the learning curve and have this extra link with a lender who already has several links. In fact, one second link (additional costs: 0.7) can be more expensive than a fourth and fifth link combined ($0.7^3 + 0.7^4 = 0.58$). Hence, a slightly higher density is not necessarily more expensive if links are well chosen.

The optimal network under decreasing marginal fixed costs (DC) \mathbb{Z}_{DC} in Subtable 1(c) exhibits exactly that. It has a total of 11 links, but they are more unevenly distributed among the lenders: while most have just one link, the first (and largest) lender has six. With $\gamma_A = 0.7$, this lender's first contract comes with unit cost, the second one costs 0.7, the third just 0.49 and so on, down to less than 0.17 for the sixth. These five extra links cost a total of 1.94. In \mathbb{Z}_{MD} , on the other hand, there were three lenders (B, C, and F) who all had one extra link; each of these cost them 0.7, totaling 2.1 and exceeding A's five additional links. In other words, the lenders now have out-degrees either at the bare minimum with just one link, or they are the provider(s) of as many of the required extra links as possible. Overall, parsimonious networks are still highly desirable; however, concentration can make additional links acceptable.

The consequences of decreasing marginal fixed costs manifest themselves in the distribution of the lenders' degrees. Revisiting the ME network, the Herfindahl index, H , and, for easier comparison, its normalized version, H^* , for the out-degrees are

$$H(\mathbb{Z}_{ME}) = \frac{\sum_i d_i^2}{(\sum_i d_i)^2} = 0.167 = \frac{1}{6} = \frac{1}{N} \quad H^*(\mathbb{Z}_{ME}) = \frac{H - \frac{1}{N}}{1 - \frac{1}{N}} = 0.0.$$

which shows perfect evenness. For the MD network, $H = 0.185$ ($H^* = 0.02$), indicating that the out-degrees are still quite even. For the DC network, however, $H = 0.339$ ($H^* = 0.21$), highlighting the strong unevenness in the lenders' number of links.

Another way of looking at \mathbb{Z}_{DC} is that there are many banks (both lenders and borrowers) with few links (and, in particular, no links between them), and

one bank on either side with many links (with links between these two and all the other ones). This is typical for core-periphery networks where a small group of banks (core) has many links to any other banks, while the large group of all other banks (periphery) is typically linked to some of the cores, but with hardly any links to other periphery banks.

Note that $\gamma_A < 1$ will lower the average costs per contract: a lender with just one link faces costs of 1.0; the lender with 6 links, on the other hand, faces average costs of just $2.94/6 = 0.49$. If lenders pass their costs on to the borrowers, then a core lender can offer more attractive conditions than one from the periphery. However, in this model, we only consider where the costs originate and not how they are redistributed within the system, and we are refraining from additional interpretations of this aspect.

In short, \mathcal{Z}_{DC} found under the decreasing marginal fixed costs is quite sparse, but not at its absolute minimum, and it resembles a core-periphery structure. To test whether this is a typical outcome, we performed a large-scale computational experiment with artificial data, described in the following sections.

4.2 Experimental Setup for Artificial Markets and Preliminary Tests

For the computational experiments, numerous artificial markets were generated. The main experiments reported in this paper are based on 100 markets, each consisting of $N = 100$ banks where assets and liabilities follow a log-normal distribution. Also, it is assumed that the largest lender is the largest borrower, implying that self-links can be identified when $i = j$ and the constraint $z_{ii} = 0$ is enforced. The parameter for shifting the costs between assets and liabilities was chosen from $\alpha \in \{1, 0.5, 0\}$; the decay factors were selected from $\gamma_A, \gamma_L \in \{0.5, 0.55, \dots, 0.85, 0.90, 0.925, 0.95, 0.975, 1.0\}$, including the combinations when both sides incur costs (i.e., $\alpha = 0.5$).

In addition, markets with $N \in \{25, 50, 100, 200\}$ banks have been simulated, where assets and liabilities were drawn from parametric distributions such as the log-normal, Pareto, chi-squared, and uniform, and also by bootstrapping from anonymized empirical data. We found that the main findings can be observed in all of these markets, yet to various degrees: reactions to changes in the decay parameter(s) were more abrupt in smaller, and smoother in larger, markets, but with the latter requiring substantial CPU time in the optimization process and, perhaps, leaving more room for convergence. Similar variations could be found when the distributions of assets and liabilities were more or less skewed.

Stochastic optimization methods are not guaranteed to find the global optimum, but they have an increased chance of finding the optimum or a solution

close to it when granting more iterations and when allowing for restarts and reporting only the best of repeated attempts. This is why all problems for the main experiments have undergone numerous (typically hundreds of) restarts with random initial solutions, but also with initial solutions that were found to be optimal in previous runs (and with the renewed threshold sequence allowing for more than just a refined local search) as well as optimal solutions for similar problems (identical market, but different settings of γ_A , γ_L , and α).

Also, it should be noted that the suggested procedure of creating a network by searching a permutation of banks and mapping it via the NWCR into a network does not guarantee that the reported result is the global optimum. Numerous preliminary experiments, however, suggest that it is more efficient than other methods in the sense that it finds solutions that are at least as good, but in substantially less time, in particular for larger problems. It can therefore be assumed that the reported results typically are close enough to the actual optima to reflect important properties. Nonetheless, results will be interpreted cautiously, also because in these problems, global optima are not necessarily unique, and even while they share the same minimum costs, their other characteristics might differ.

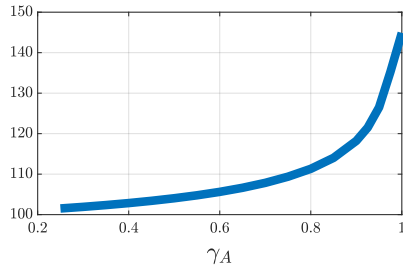
All implementations were done in Matlab version 2018b.

4.3 Findings for Artificial Markets

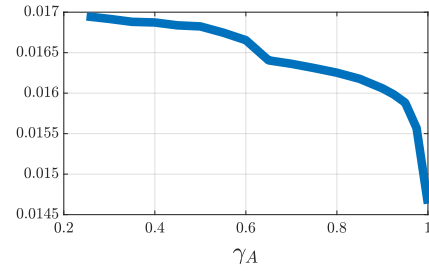
The toy example in Table 1 illustrated that overall costs are lower when fixed costs for links are subject to decay: taking the minimum density solution already lowers costs whenever there are banks that require more than one link. They benefit from the reduced costs, and it might be beneficial to have more links with a highly connected bank than two links with a bank that has only a few connections. This means that, on a macro level, the overall number of links and the density of the network will go up. On a micro level, poorly connected banks will have even fewer links (perhaps just one) and become (even more) peripheral, while the highly connected ones (typically also those with large balance sheets) will have even more links.

These effects are confirmed by the experiments with the 100 artificial markets with $N = 100$ banks each. Figure 1 looks at the overall effects when costs are incurred by the lenders ($\alpha = 1$). As can be seen from Subfigure 1(a), the stronger the decay in marginal fixed costs (i.e., the lower γ_A), the lower the overall cost of the network; this is particularly noticeable when compared to the minimum density network (i.e., when $\gamma_A = 1$). Note that, for lower γ_A , the costs converge to 100: the market consists of $N = 100$ banks, and every bank needs to have at least one link. The first link has unit costs of $c = 1$, while the additional ones will be (substantially) cheaper; their contribution to the overall network costs

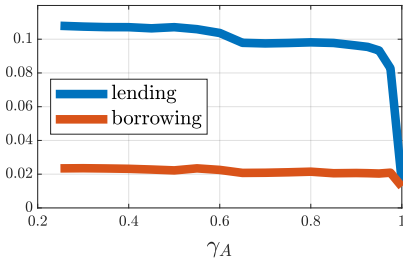
Figure 1: Aggregate results for 100 markets consisting of $N = 100$ banks each with log-normally distributed assets and liabilities when all costs are incurred by lenders ($\alpha = 1$).



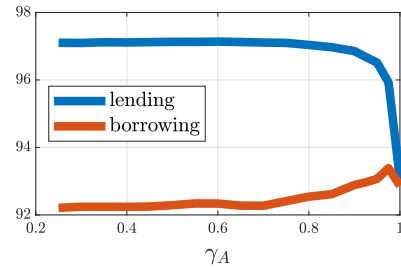
(a) Average overall costs.



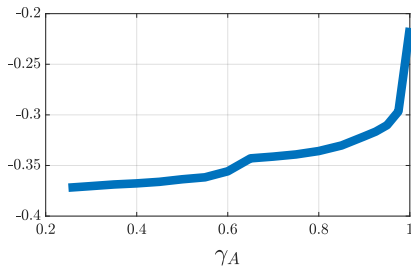
(b) Average network density.



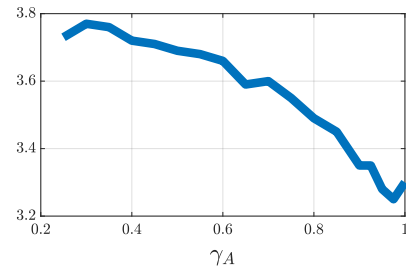
(c) Normalized Herfindahl index of banks' degrees.



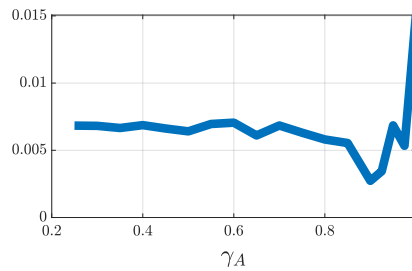
(d) Lending and borrowing depth.



(e) Pearson symmetry.



(f) Core size.



(g) Cluster MIT.

become less noticeable. By construction, every bank has a positive budget and therefore requires at least one link. At the same time, optimal networks tend to have overall more links, which is equivalent to a higher density; cf. panel 1(b).

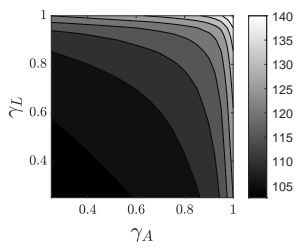
Larger markets allow a more subtle analysis of the results. Panel 1(c) shows that lenders differ more in how many links they have (i.e., the lenders' out-degree), but for the same reason as in the small example: larger banks with many links already in place are the preferred partners for borrowers' additional (or even first) links, as this lowers the additional costs. Borrowers' numbers of links are also slightly more uneven, but the effect is much less pronounced than on the lenders' side (or in the toy example where alternatives were very limited). With more overall links now in the networks, borrowers now get their liabilities from either a single or two (or, occasionally, more) lenders. In particular, these additional lenders tend to be large ones with many outlinks. Note that very strong decay might soften that effect somewhat: when γ_A is (very) low and the number of links is already high, the marginal fixed costs will be virtually negligible, as will be the marginal decrease in the absolute (dollar) values. Hence, the second most connected lender is effectively as favorable as the lender with the highest degree, and lenders with slightly lower assets can also gain core status. This is underlined by two additional statistics: comparing the active links and the rank (with respect to size) of the lender and borrower involved, it is generally true that larger lenders cooperate with smaller borrowers and vice versa. The larger the decay in marginal fixed costs, the more obvious this pattern becomes, and the negative rank correlations become substantially more pronounced; see panel 1(e). At the same time, more banks can be considered core (panel 1(f)).

When costs are entirely incurred by borrowers, $\alpha = 0$, then sides are swapped, and they encounter all the effects just seen for lenders, and vice versa.

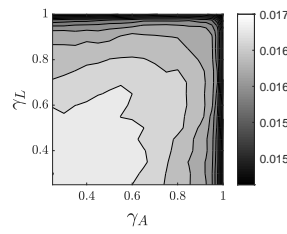
When both sides contribute to the network costs, with $\alpha = 0.5$, and fixed costs for a lender's and a borrower's first link on either side are $\alpha c = (1 - \alpha)c = \frac{1}{2}c$, respectively, effects are mixed, largely influenced by the respective decay factors. Figure 2 summarizes the main results.

Figure 3 provides the adjacency matrices and network structures for one of the sample markets. Lenders are along the vertical axis (sorted by size; largest on top), borrowers along the horizontal axis (also sorted by size, largest on the left). A dot indicates an existing link ($z_{ij} > 0$, $1_{ij} = 1$). The top left network is for the minimum density case; top right and bottom left are for the case where costs decay with a factor of $\gamma_A = \gamma_L = 0.7$ and are incurred by lenders ($\alpha = 1$) and borrowers ($\alpha = 0$) only, respectively; bottom right is the case where both incur costs ($\alpha = 0.5$). A core-periphery structure emerges in particular in the last of these cases: large banks (low indices) tend to be highly linked (albeit not among themselves, which here is a side-effect of the NWCR assignment rule), while small banks have few links, which are mostly to large banks.

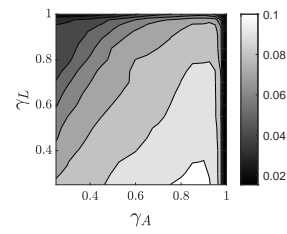
Figure 2: Aggregate results for 100 markets consisting of $N = 100$ banks each with log-normally distributed assets and liabilities when both lenders and borrowers contribute to the network costs ($\alpha = 0.5$).



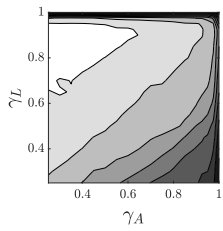
(a) Average overall costs.



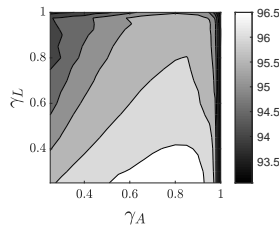
(b) Average network density.



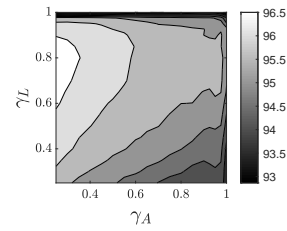
(c) Normalized Herfindahl index of lenders' degrees.



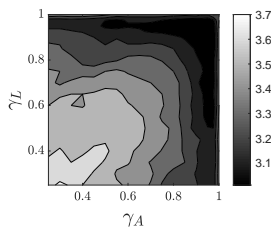
(d) Normalized Herfindahl index of borrowers' degrees.



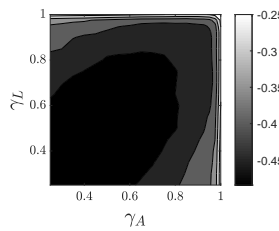
(e) Depth lenders.



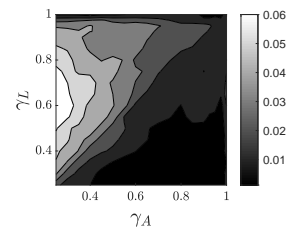
(f) Depth borrowers.



(g) Core size.

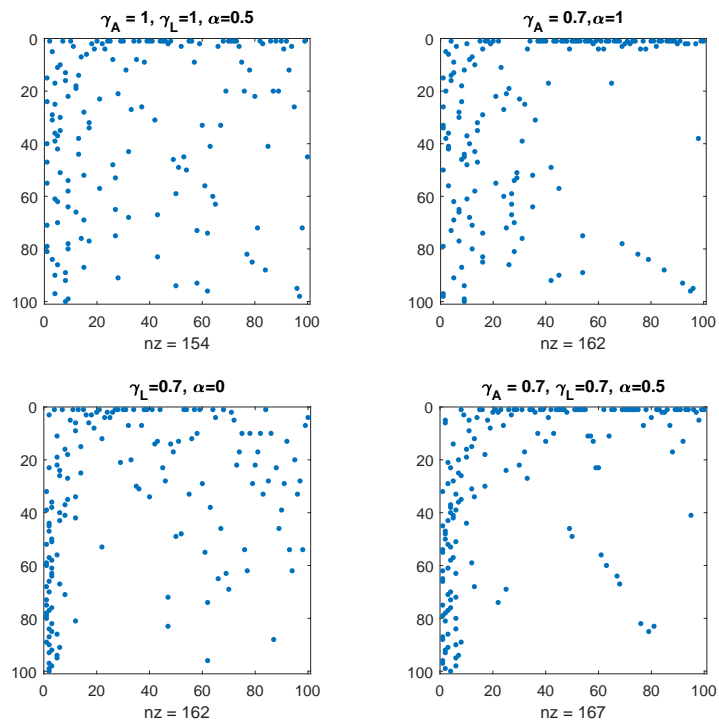


(h) Symmetry.



(i) Cluster MIT.

Figure 3: Adjacency matrices and network structures for one of the artificial markets under different cost regimes.



5 Conclusion

This paper introduces a model with decreasing marginal fixed costs in order to re-create banking networks with realistic characteristics. We argue that establishing a link between a borrower and a lender is costly, yet with decreasing marginal costs: banks face a learning curve, and the process for setting up an additional link is cheaper than establishing the first link. This makes large banks attractive partners. A large lender has the assets to be the sole partner for many small borrowers, both as borrower and as lender. The larger partner therefore is well-linked and benefits from the learning curve. The model is stated as an optimization problem for the social planner, which turns out to be numerically challenging. To approach this problem, we introduce a new algorithm.

Numerical experiments for artificial markets show that the resulting networks exhibit some typical stylized facts observed in real-world banking networks: they are quite, but not too, sparse with densities similar to those observed in real-world networks. A core-periphery structure emerges where a few banks constitute the core (the large ones) and are connected to many other banks, while periphery banks have very few links, which are mostly to core banks. Our algorithm has many potential applications that can be extended to agent-based modeling and empirical structural estimation. The fact that our focus has been on a cost structure facing the system assists in this extension, with some important caveats.

First, the cost structure is aggregated so that it is unclear whether the algorithm can be used to solve problems where the banks or agents are individually optimizing over their cost structure. Work needs to be done on our model where the social planner's problem is related to that of the equilibrium achieved by the optimizing behavior of the separate agents. Second, while the costs of a link are important, they are not the only costs associated with financial networks. Future research will extend our model and algorithm to incorporate aspects such as diversification, explicit limits on contract sizes (e.g., to reflect regulatory requirements), different interest rates, and dynamic aspects such as multiperiod network formation and adaptation.

Acknowledgments

We are grateful to Goetz von Peter and to seminar and conference audiences in London, Bergamo, and Trento for valuable discussions and comments. We would also like to thank Red Laviste and Monica Reusser for carefully proofreading the manuscript.

A Multiple Global Optima

Both under the minimum density model and the decreasing marginal fixed-costs model, solutions are not unique. Typically, there exist several or even many solutions all of which exhibit the same (globally optimal) network costs and distribution of lenders' and borrowers' degree, yet have different adjacency matrices. Building on the small illustrative example in Section 4.1, Table 1 illustrates this for the minimum density model, and Table 2 for the minimum cost model. It is important to note that descriptive statistics for the network structure (and adjacency matrix) can therefore vary.

Table 1: Some of the globally optimal solutions, alternative to the solution reported in Table 1(b), under minimum density for the illustrative example in Section 4.1 with $[A_i] = [50, 15, 12, 11, 8, 4]$ and $[L_j] = [51, 14, 13, 10, 9, 3]$; more exist. All have a degree of 9, i.e., a density of $\frac{9}{36} = 25\%$.

50 1 14 9 . . 3 . . . 10 1 8 . . . 4 . . .	50 14 . . 1 . . . 2 10 11 8 . 1 3	50 13 2 9 3 1 10 8 . . . 4
50 1 14 12 10 1 8 . . . 1 . . 3	50 1 14 9 . . 3 . . . 2 9 8 4 . . .	50 14 1 12 2 9 8 . . 1 3
47 3 . 14 . . 1 . . . 12 1 10 8 . 4	47 3 5 10 9 3 . 11 8 4	47 3 . 2 13 12 10 1 8 . 4
36 14 15 9 3 . . 1 10 8 4	36 14 15 9 3 . . 11 2 6 4	36 14 15 9 3 . . 5 6 8 4

Table 2: Some of the globally optimal solutions under decreasing marginal fixed costs with $\gamma_A = 0.7$ and $\alpha = 1$, alternative to the solution reported in Table 1(c), for the illustrative example in Section 4.1; more exist. In all of these cases, the first lender has 6 links, all other lenders just 1, amounting to overall network costs of 7.94.

28	3	1	10	5	3	13	14	9	10	1	3	9	14	13	2	9	3
15	15	15
.	.	12	.	.	.	12	12
.	11	11	11
8	8	8	.	.
.	.	.	.	4	.	.	.	4	.	.	.	4

B The search and optimization algorithms

Algorithm 1 Network optimization with Simulated Annealing

```

1: function NETWORKOPTIMIZER( $\mathbf{A}, \mathbf{L}, \gamma_A, \gamma_L, \alpha_A$ )
2:   set temperature and nIter
3:    $\mathbf{s}_A \leftarrow \text{random\_permutation}(1:N), \mathbf{s}_L \leftarrow \text{random\_permutation}(1:N)$ 
4:    $f \leftarrow \infty, f^* \leftarrow \infty$ 
5:   for  $t \leftarrow 1$  to nIter do
6:     create mutations of current sequences  $(\mathbf{s}'_A, \mathbf{s}'_L) \leftarrow \mathbf{N}(\mathbf{s}_A, \mathbf{s}_L)$ 
7:      $\mathbf{Z}' \leftarrow \text{NWCR}(\mathbf{A}, \mathbf{L}, \mathbf{s}'_A, \mathbf{s}'_L)$ 
8:      $f' \leftarrow \text{cost}(\mathbf{Z}', \gamma_A, \gamma_L, \alpha_A)$ 
9:     if  $u \leq \exp((f - f')/\text{Temp})$  then
10:      keep mutations,  $(\mathbf{s}_A, \mathbf{s}_L) \leftarrow (\mathbf{s}'_A, \mathbf{s}'_L), f \leftarrow f'$ 
11:      if  $f < f^*$  then
12:        update acting optimum:  $\mathbf{Z}^* \leftarrow \mathbf{Z}', f^* \leftarrow f$ 
13:      lower temperature
14:   return  $\mathbf{Z}^* \leftarrow \text{NWCR}(\mathbf{A}, \mathbf{L}, \mathbf{s}_A^*, \mathbf{s}_L^*)$ 

```

Algorithm 2 Modified North-West-Corner-Rule with provided sequence

```

1: function NWCR( $\mathbf{A}, \mathbf{L}, \mathbf{s}_A, \mathbf{s}_L$ )
2:    $\mathbf{Z} \leftarrow \mathbf{0}_{N \times N}$ 
3:    $r \leftarrow 1, c \leftarrow 1,$ 
4:   while  $(r \leq N)$  and  $(c \leq N)$  do
5:      $i \leftarrow \mathbf{s}_A[r], j \leftarrow \mathbf{s}_L[c]$ 
6:      $z_{ij} \leftarrow \min(A_i, L_j)$ 
7:      $A_i \leftarrow A_i - z_{ij};$ 
8:     if  $A_i = 0$  then  $r \leftarrow r + 1$ 
9:      $L_j \leftarrow L_j - z_{ij};$ 
10:    if  $L_j = 0$  then  $c \leftarrow c + 1$ 
11:   return  $\mathbf{Z}$ 

```

Algorithm 3 Computing a network's costs

```
1: function COST( $Z'$ ,  $\gamma_A$ ,  $\gamma_L$ ,  $\alpha_A$ )
2:    $c \leftarrow 0$ 
3:   if  $\alpha_A > 0$  (lenders contribute to costs) then
4:     for  $i \leftarrow 1$  to  $N$  do
5:       compute degree of asset bank  $i$ ,  $d$ 
6:        $c \leftarrow c + (1 - \gamma_A^d)/(1 - \gamma_A)$ 
7:   if  $(1 - \alpha_A) > 0$  (borrowers contribute to costs) then
8:     for  $j \leftarrow 1$  to  $N$  do
9:       compute degree of asset bank  $i$ ,  $d$ 
10:       $c \leftarrow c + (1 - \gamma_L^d)/(1 - \gamma_L)$ 
11:  if self-lending prohibited then
12:     $p \leftarrow \sum_i z_{ii}$ 
13:     $c \leftarrow c + c \cdot p$  (punishment for violating constraint)
14:  return  $c$ 
```

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