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Job Heterogeneity and Aggregate Labor Market Fluctuations

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This paper disciplines a model with search over match quality using microeconomic evidence on worker mobility patterns and wage dynamics. In addition to capturing these individual data, the model provides an explanation for aggregate labor market patterns. Poor match quality among first jobs implies large fluctuations in unemployment due to a responsive job destruction margin. Endogenous job destruction generates a burst of layoffs at the onset of a recession and, together with on-the-job search, generates a negative comovement between unemployment and vacancies. A significant job ladder, consistent with the empirical wage dispersion, provides ample scope for the propagation of vacancies and unemployment.

JEL codes: E24, E32, J63, J64.

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1 Introduction

The aggregate labor market displays remarkably robust patterns. At the onset of a recession, separations into unemployment rise quickly. The number of job vacancies falls and unemployment rises, a negative comovement referred to as the Beveridge curve. The rate at which workers exit nonemployment and the rate at which they switch employers both fall, and recover sluggishly.[1]

The Mortensen-Pissarides (henceforth MP) search framework has become the dominant approach to understanding these dynamics. The literature starts with Mortensen and Pissarides (1994) and has developed significantly over the last few years.[2] The present paper takes this search and matching framework as the point of departure and incorporates two additional features that are crucial to its success: two-dimensional job heterogeneity and on-the-job search. The resulting structure is disciplined using microeconomic facts about worker flows, wage dynamics, including realistic wage dispersion, and worker mobility patterns.

The model's implications are broadly consistent with observed aggregate labor market outcomes. Owing to endogenous separations, at the onset of a recession, the employment-to-unemployment (E-U) probability rises as low-quality matches become unprofitable. The model delivers significant volatility along this job destruction margin because newly hired individuals are assumed to transition into jobs with poor match quality, a notion that leaves many workers susceptible to layoff during an economic downturn. This volatility persists in the face of significant, empirically relevant wage dispersion.

The current paper features the observed negative relationship between job openings and unemployment due to on-the-job search. Under realistic calibrations, the standard MP model implies a positive comovement between steady-state unemployment and vacancies in response to variations in the exogenous separation rate.[3] In the MP model, a rise in the exogenous destruction rate increases the number of unemployed workers and thus stimulates vacancies. Even in a model with endogenous job destruction, Fujita and Ramey (2012) show that a reasonably calibrated model without on-the-job search also delivers a positive correlation between unemployment and vacancies. In the current paper, on-the-job search

[1]Fujita (2009) documents the response of the separation rate. Elsby, Michaels, and Ratner (2015) provide a survey on the Beveridge curve. Fujita and Ramey (2007) provide evidence for the sluggish response of vacancies and the job-finding rate. Nagypál (2007) shows that the average employer-to-employer transition probability is procyclical.

[2]For a rough progression of research, see Merz (1995), Andolfatto (1996), den Haan, Ramey, and Watson (2000), Shimer (2005), Hall (2005), Yashiv (2006) and Hagedorn and Manovskii (2008). Moretensen (1994), Pissarides (1994), Nagypál (2007), Krause and Lubik (2007) and Tasci (2007) study models with on-the-job search.

[3]See the discussion in Elsby, Michaels, and Ratner (2015),' for example.

implies that, although unemployment rises in a recession and thus makes vacancy posting relatively more attractive, the downward movement in aggregate productivity during the recession dominates this effect and causes vacancies to fall. As a result, the model delivers the observed Beveridge curve.

Since the model features on-the-job search, it also has implications for the average employer-to-employer (E-E) transition probability over the business cycle. As aggregate productivity falls during a recession, the payoff to filling a vacancy falls, and therefore firms post fewer openings and the contact rate for the employed falls. This reduces the probability of E-E transitions and delivers the observed procyclicality of the E-E probability. Similar intuition suggests that the model delivers a procyclical job-finding probability for the unemployed, consistent with the aggregate data.

Hornstein, Krusell, and Violante (2011) document significant wage dispersion. To match this, the calibrated model features a large dispersion in match quality. This "long" job ladder induces a strong internal propagation mechanism and implies that, after a separation shock, it takes workers significant time to attain their pre-separation match. Intuitively, this equilibrium match-quality distribution exhibits slow-moving behavior following a negative aggregate productivity shock as low-quality matches are destroyed, and it takes workers time to find new jobs and slowly climb up the job ladder. In turn, this sluggish movement in the match-quality distribution implies slow-moving incentives for vacancy posting, and therefore the job-finding rate, as the payoff to meeting an employed worker changes with the match-quality distribution. This is in accordance with observed data on vacancies and the job-finding rate. For example, Fujita and Ramey (2007) document significant propagation in the data: The movement of vacancies and the unemployment rate do not occur contemporaneously with productivity movements. The standard MP model cannot deliver these dynamics because market tightness and the job-finding rate are jump variables that correlate perfectly with aggregate productivity.

The success of the model in simultaneously generating observed wage dispersion and significant volatility of the job destruction margin points to a possible resolution of a tension in the search and matching literature. On the one hand, Hornstein, Krusell, and Violante (2011) document significant wage dispersion in the observed data. On the other hand, Bils, Chang, and Kim (2011) point out that the baseline MP model faces a trade-off between matching the observed dispersion in wage growth across workers and realistic cyclical fluctuations in unemployment. In particular, with a large wage dispersion, which requires a large dispersion in match quality, few individuals are at the destruction threshold. In the context of Bils, Chang, and Kim (2011), this means that an aggregate productivity shock affects few relationships and therefore engenders only small movements in unemployment. The model in this paper helps resolve some of these tensions by introducing two-dimensional job heterogeneity in the form of match quality and idiosyncratic productivity. Low-quality matches among new hires give rise to the notion of first jobs as "stepping stones." In the face of aggregate shocks, this serves to increase the volatility of the job destruction margin, and to increase the hazard of separation into nonemployment after the first employment-to-nonemployment event.[4] Moreover, this "stepping stone" nature grants workers access to jobs with a better match-quality component, which can reduce initial wages and thereby raise equilibrium wage dispersion.

Perhaps the most closely related paper is Barlevy (2002), in which employer-to-employer transitions are also incorporated into a search and matching model with aggregate fluctuations. Barlevy shows that recessions may be accompanied by a sullying effect as depressed vacancy postings, and therefore job-finding rates, contribute to a slower reallocation of workers to their ideal jobs, thus reducing average match quality in the economy. The current paper shares many themes with Barlevy (2002), and I engage more with the results from that paper in Section 6.3. For one, the implications for match quality in response to an aggregate productivity shock are almost identical, despite distinct calibration approaches. One main difference is that the current framework draws out the implications of micro phenomena for the aggregate labor market, whereas Barlevy (2002) targets aggregate unemployment fluctuations in the data.[5] In order to generate realistic fluctuations in unemployment, Barlevy (2002) uses implausibly large shocks to aggregate productivity. This highlights the chief contribution of the present paper, which is its ability to generate observed fluctuations in unemployment given realistic shocks to aggregate productivity.

Nagypál (2007) also studies a model with on-the-job search, and suggests that if the payoff from hiring an employed worker exceeds the payoff from hiring an unemployed worker, on-the-job search can increase the amplification of the baseline model. I show that the current framework delivers significant propagation and volatility of the job destruction margin without resorting to a hiring cost, a crucial component of the model in Nagypál (2007). The key difference is that in the current framework, newly hired workers enter jobs with low match quality and hence a large mass of employees are susceptible to job destruction when aggregate productivity moves downward.

The rest of this paper is organized as follows. Section 2 presents the steady-state version

[4] A similar notion of recurring job loss appears in Pries (2004), albeit for different reasons. That paper omits on-the-job search and therefore cannot speak to issues like the cyclicality of the average E-E probability.

[5] This is also what distinguishes the present paper from Fujita and Ramey (2012), where the volatility of the job destruction margin is a targeted outcome.

of the model, which features exogenous contact rates. Section 3 extends the model and endogenizes the contact rates. Section 4 describes the data used in the analysis. Section 5 elaborates on calibration. Section 6 presents the transition dynamics of key endogenous variables in response to an aggregate productivity shock. Section 7 presents a simpler version of the baseline model with only one state variable, but still featuring a job ladder and E-E transitions. The simpler model delivers less amplification and propagation than the baseline model, consistent with work by Bils, Chang, and Kim (2011). Section 8 summarizes and discusses potential avenues for future research.

2 Model with Fixed Contact Rates

2.1 Setup

To set the stage for the rest of the paper, this section summarizes the partial equilibrium model, which features exogenously determined contact rates.[6]

The model is based on the search and matching framework developed by Diamond, Mortensen, and Pissarides. Workers look for jobs and firms post vacancies to attract workers. Unemployed workers receive utility from leisure and encounter vacancies at an exogenous probability p_U. Employed workers receive a flow payment w and produce a flow output. Employed workers participate in on-the-job search and contact vacancies at a different probability p_E. All employer-employee matches are characterized by two state variables: match quality denoted by y, and an idiosyncratic component denoted by x that can be interpreted as either productivity or demand. The product of x and y ($x \cdot y$) provides the flow output of the match. When an unemployed worker contacts a firm, the match draws an initial, nonstochastic, match quality equal to a fixed and deterministic y_0. Match quality remains constant within a job. The notion that newly hired workers transition to jobs with low match quality is necessary for the model to match observed wage dispersion as documented by Hornstein, Krusell, and Violante (2007). On way to motivate this low match quality for workers coming out of nonemployment is in the context of internal labor markets as described by Doeringer and Piore (1971) and more recently by Martins, Solon, and Thomas (2010). These initial jobs can be thought of as "port-of-entry" jobs, jobs into which employers are consistently observed to hire new workers.

All initial idiosyncratic productivities (demands) for nonemployed workers are fixed at a

[6] For brevity I omit timing, the bargaining structure, and most of the recursive equations. For more elaboration, see Krolikowski (2017).

deterministic value, x_0, and then exhibit persistence within a match evolving according to $F_x(x'|x)$. Setting x to x_0 in all new matches out of nonemployment follows Mortensen and Pissarides (1994). In this model, the match quality, y, provides a mean productivity level within a match and x provides some variance around this mean productivity. Over time on-the-job search results in offers to the employed with probability p_E and match quality drawn from $y \sim F_y(\tilde{y})$. This induces a job ladder that agents climb over time. This can be interpreted as finding more suitable jobs while employed and slowly transitioning to one's "ideal" job. For employed workers who switch employers, the starting productivity is fixed to x_{EE}. This allows for temporary changes in productivity when workers switch jobs.

The idiosyncratic component delivers endogenous flows into unemployment; when the realization of the idiosyncratic random variable is low enough, the worker and the firm decide to part ways. The worker prefers to flow into unemployment and search for a new vacancy, and the firm prefers to let the worker go and find a new worker from the pool of searchers. Involuntary endogenous separations on either side of the market do not occur in this model. Whenever there exists positive surplus in a match, the worker and firm can negotiate a wage both parties find agreeable. The model does incorporate exogenous separations with probability $p_s > 0$, however.

Formally, the model can be summarized by the central functional equation, which is the surplus from a match:

$$\begin{aligned} S(x,y) = {} & x \cdot y + \delta \underbrace{(1-p_E)}_{\substack{\text{No outside}\\ \text{offer}}} \underbrace{(1-p_s)}_{\substack{\text{No separation}\\ \text{shock}}} \int \max\{0, \underbrace{S(x',y)}_{\substack{\text{Match}\\ \text{continues}}}\} dF_x(x'|x) \\ & + \delta p_E (1-p_s) \int\int \Bigg[\underbrace{\mathbb{I}\{S(x',y) \geq S(x_{EE},\tilde{y})\}}_{\substack{\text{Worker turns down}\\ \text{poaching firm}}} \underbrace{\max\{0, S(x',y)\}}_{\substack{\text{Match continues or}\\ \text{terminates}}} \\ & + \underbrace{\mathbb{I}\{S(x',y) < S(x_{EE},\tilde{y})\}}_{\substack{\text{Worker leaves}\\ \text{current firm}}} \underbrace{\max\{0, \beta S(x_{EE},\tilde{y})\}}_{\substack{\text{Match at new firm}\\ \text{or unemployment}}} \Bigg] dF_x(x'|x) dF_y(\tilde{y}) \\ & - \underbrace{[b + \delta p_U \beta \max\{0, S(x_0,y_0)\}]}_{\text{Worker's outside option}} \end{aligned} \tag{1}$$

The first part of the right-hand side is the flow payoff from a match, $x \cdot y$. The second piece captures the event of no outside job offer, no exogenous separation shock and the continuation surplus of the match, where δ is the discount rate. In this case, the match either comes to an end or the match continues with the new idiosyncratic productivity (demand).

The third piece captures the event of the worker receiving an outside offer and potentially moving to the poaching firm. When the worker moves to the poaching firm, he uses unemployment as a threat point, and then the current firm has zero continuation value and the worker's continuation value is $\beta S(x_{EE}, \tilde{y})$, where β is the bargaining power of the worker. The final piece is the outside option of an employed worker: He forgoes the value of unemployment, b, and the possibility of finding a job at a new firm with surplus $S(x_0, y_0)$ and receiving β of this surplus.

3 Endogenizing the Contact Rates

This section outlines how to couch the model of Section 2 in a general equilibrium framework, including an aggregate matching function and optimal vacancy posting by firms. These additions allow the firm's decisions to affect the number of matches every period. Suppose that two matching functions, with the same functional forms but different matching efficiencies, determine the number of contacts that occur between unemployed and employed workers and firms in the economy every period. Let v denote the number of vacancies in the economy. I assume Cobb-Douglas matching functions so that the number of contacts equals:

$$m_i(s, v) = m_0^i v^\alpha s^{1-\alpha}, i \in \{U, E\} \tag{2}$$

where s denotes the measure of searchers and m_0^i is the matching efficiency. In the current framework both unemployed and employed agents search and therefore $s = 1$ and the matching function satisfies:[7]

$$m_i(1, v) = m_0^i v^\alpha, i \in \{U, E\} \tag{3}$$

The aggregate meeting rate is:

$$p_i(v) = \frac{m_i(1, v)}{1} = m_i(1, v) = m_0^i v^\alpha, i \in \{U, E\} \tag{4}$$

[7]See Mortensen and Nagypál (2005) for a similar setup. An alternative would be to allow employed workers to optimally choose their search intensity, as in Fujita and Ramey (2012). This usually implies that search intensity is procyclical because when aggregate productivity rises, the returns to search rise. As Elsby, Michaels, and Ratner (2015) point out, this induces vacancy amplification and helps the baseline model deliver the Beveridge curve. Nevertheless, Elsby, Michaels, and Ratner (2015) provide cursory evidence that search intensity may in fact be countercyclical, exacerbating the tensions present in the baseline model. Given this inconclusive evidence, I assume that search intensity does not vary with aggregate conditions.

and the vacancy filling rate is:

$$q_i(v) = \frac{m_i(1, v)}{v} = \frac{f_i(v)}{v} = m_0^i v^{\alpha-1}, i \in \{U, E\} \tag{5}$$

With this addition to the model, I can determine the contact rates given the number of vacancies in the economy.

I still need to determine how many vacancies firms open in equilibrium. To determine the equilibrium vacancy rate I introduce the vacancy creation condition. This condition represents the costs and benefits from opening a vacancy for an individual firm. There exists a flow cost, c, to maintaining an open vacancy. The benefit from posting a vacancy has three parts: the payoff from meeting an unemployed worker, $\mathbb{E}_u$, the payoff from meeting an employed worker, $\mathbb{E}_e$, and the payoff from meeting both workers simultaneously, $\mathbb{E}_{u,e}$. The payoff to meeting an unemployed worker is simply the portion of the surplus, $1 - \beta$, that the firm receives at combination (x_0, y_0). The payoff from meeting an employed worker depends on whether the poaching firm successfully attracts the worker and the poaching firm's payoff from this new employment relationship. The probability of poaching a worker, in turn, depends on the distribution of (x, y) among employed workers, which I denote by $\pi(x, y)$, with an associated cumulative density function $\Pi(x, y)$.[8] When meeting both workers simultaneously, the firm chooses between the potential employees optimally. Hence, the vacancy creation condition can be written as:

$$V = -c + \delta q_U u[1 - q_E(1-u)]\mathbb{E}_u + \delta q_E(1-u)(1 - q_U u)\mathbb{E}_e + \delta q_U u q_E(1-u)\mathbb{E}_{u,e} \tag{6}$$

where u is the unemployment rate, and

$$\begin{aligned}
\mathbb{E}_u &= \max\{0, (1-\beta)S(x_0, y_0)\} \\
\mathbb{E}_e &= \int\int\int\int \mathbb{I}\{S(x_{EE}, \tilde{y}) > S(x', y)\} \max\{0, (1-\beta)S(x_{EE}, \tilde{y})\} dF_x(x'|x) dF_y(\tilde{y}) d\Pi(x, y) \\
\mathbb{E}_{u,e} &= \int\int\int\int \Big[\mathbb{I}\{S(x_{EE}, \tilde{y}) > S(x', y)\}(1-\beta)\max\{S(x_0, y_0), S(x_{EE}, \tilde{y})\} \\
&\quad + \mathbb{I}\{S(x', y) \geq S(x_{EE}, \tilde{y})\} \max\{0, (1-\beta)S(x_0, y_0)\}\Big] dF_x(x'|x) dF_y(\tilde{y}) d\Pi(x, y)
\end{aligned}$$

I assume that in equilibrium $V = 0$ because of free entry into vacancies so that equation

[8] $\pi(x, y)$ is in fact the distribution of (x, y) conditioned on employment so that it sums to one. When finding this distribution I iterate on the unconditional distribution, $\tilde{\pi}(x, y)$ so that together the unemployment rate and $\tilde{\pi}(x, y)$ sum to one. The two functions are related by the simple equation: $\frac{\tilde{\pi}(x,y)}{1-u} = \pi(x, y)$.

(6) implies that the flow cost of opening a vacancy must equal the expected benefit from maintaining that open vacancy. This free-entry condition also simplifies the equation because we can ignore the outcome when the firm meets neither an unemployed nor an employed worker. If we know the expected benefit from posting a vacancy, this equation pins down the equilibrium vacancy rate. Notice that the expected payoff from meeting an employed worker enters the optimal decision of the firm. This presents an important deviation from the MP model that features no on-the-job search. In this context, searching while employed has consequences for aggregate dynamics.

4 Data

Aside from the estimates of wage dispersion from Hornstein, Krusell, and Violante (2007), the individual data come exclusively from the Panel Study of Income Dynamics (PSID) family and individual merged files. The PSID began in 1968 with interviews of approximately 5,000 families, and it follows any new families formed from the original group of families. I use the 1988-1997 waves of the PSID. In the survey years prior to 1988, the PSID did not collect monthly strings on employment status at different employers so it is not possible to calculate monthly E-E probabilities for those years. To avoid the complication of biennial interviews, I only use data up to the 1997 survey. To obtain results that are comparable to alternative data sources, I restrict the sample to working-age males, aged 18 through 65. I omit the self-employed and use individual weights to account for the PSID's poverty over-sample and nonrandom attrition. For the years 1988-1997, respondents were asked to report their employment status in each month of the previous calendar year, as well as monthly employment strings for up to two main employers.[9] From these monthly strings I construct transition probabilities between employment and nonemployment, as well as between employers.[10]

The elasticities of labor market aggregates with respect to output per worker are taken from Fujita and Ramey (2012), who use data from the Bureau of Labor Statistics on unemployment (Current Population Survey), vacancies (Help-Wanted Index constructed by Barnichon, 2010), and the aggregate labor productivity divide (real GDP divided by CPS

[9]To remain consistent with the model, I treat unemployment and out-of-the-labor-force as the same state when calculating layoff rates. This is also why I use the terms nonemployment and unemployment interchangeably in the text.

[10]In other work (Krolikowski, 2017) I have shown that these data imply average transition probabilities that are broadly consistent with values obtained from the Survey of Income and Program Participation (SIPP) and the Current Population Survey (CPS).

employment). The sample period is 1976Q1 to 2005Q4. For the statistics on the average aggregate E-E probability, I use data from Fallick and Fleischman (2004), which have been made publically available and updated to include more recent months (up to 2014Q2).

5 Calibration Strategy

This section discusses the processes of state variables and calibration.

5.1 Exogenous Processes

The model period length is one month. Idiosyncratic productivity starts out at a fixed and deterministic level x_0 (x_{EE}) in all new matches out of nonemployment (employment), and then within the match follows a log AR(1) process:

$$\ln x' = \rho_x \ln x + \epsilon'_x \tag{7}$$

where $\epsilon'_x \sim \mathcal{N}(0, \sigma^2_{\epsilon_x})$. This process captures the intuition that productivity at the match level, or demand for the match's output, exhibits some persistence. Match quality follows the process:

$$\ln y' = \begin{cases} \ln y_0 & \text{for jobs out of unemployment (U} \to \text{E)} \\ \ln y & \text{if no job change} \\ \epsilon'_y & \text{if changes jobs (E} \to \text{E)} \end{cases} \tag{8}$$

where $\epsilon'_y \sim \mathcal{N}(0, \sigma^2_{\epsilon_y})$. Hence, match quality remains constant within a job, and is log-normally distributed when a worker meets a new firm. In the first job coming out of nonemployment, match quality is set to y_0.

I consider the following log AR(1) process for aggregate productivity:

$$\ln z' = \rho_z \ln z + \epsilon'_z \tag{9}$$

where $\epsilon'_z \sim \mathcal{N}(0, \sigma^2_{\epsilon_z})$.

5.2 Calibration

Given the optimal decisions of workers and firms, the model generates simulated data at a monthly frequency. In particular, I simulate 6,000 agents for 480 months (40 years). To remove the effects of initial conditions, I simulate the model for 1280 months and then discard the first 800 months of the sample. This simulation provides a time path of wages and annual earnings, as well as an employment history.

I calibrate the parameters of the model using simulated method of moments, except for the aggregate parameters, namely, the parameters associated with the aggregate productivity process, ρ_z and σ_{ϵ_z}, the flow cost of posting a vacancy, c, and the elasticity of the matching function with respect to vacancies, α. These aggregate parameters are chosen externally. I calibrate $\rho_z = 0.983$ and $\sigma_{\epsilon_z} = 0.005$. This calibration of the monthly productivity process yields Hodrick-Prescott (HP) detrended series of logged labor productivity using a 10^5 smoothing parameter that has a monthly persistence of 0.89 and a standard deviation of 0.015, very close to the empirical counterpart in the US: 0.88 and 0.02, respectively. I choose the elasticity of the matching function with respect to vacancies, α, to be 0.524 which is taken from Mortensen and Nagypál (2005) and captures the empirical elasticity of the job-finding rate with respect to vacancies. Finally, I choose c to normalize the vacancy rate in the steady state to one.

The parameters of the individual's problem are well identified, and here I describe the intuition of how the parameters affect observable moments.[11] To accurately capture the extent to which an initial employment separation induces subsequent nonemployment spells for workers, I target the probability of E-N separations after an initial E-N transition. This model delivers increased hazard rates of separation after an initial E-N transition because of the slippery "stepping stone" nature of the first job out of nonemployment. For a given initial match-quality level, y_0, and idiosyncratic volatility, σ_{ϵ_x}, the starting idiosyncratic component, x_0, will determine the E-N probability in the month right after individuals find jobs. A smaller value of x_0 implies a larger E-N probability after the initial E-N transition. For a given match-quality level and idiosyncratic volatility, the persistence of the idiosyncratic process, ρ_x, will determine how quickly this serial correlation in E-N transitions dissipates. Finally, the volatility of the idiosyncratic process, σ_{ϵ_x}, will shift the entire E-N profile up and down as it affects the probability of separation into nonemployment at all match-quality levels. Hence, the average E-N probability profile after the first E-N transition helps identify the idiosyncratic process.

To accurately capture the empirical mobility patterns, I use the average E-N probability for different levels of tenure to discipline the model. I use the separation rate of high-tenured workers to identify the exogenous separation rate p_s. This is well identified because well-matched (high-tenured) individuals will only experience exogenous separations due to high surplus within their match. The rest of the separation-tenure profile puts additional restric-

[11] This section and the next are elaborated on more fully in Krolikowski (2017).

tions on the idiosyncratic process, much like the E-N profile after the first E-N transition. In particular, for a given initial match-quality level and idiosyncratic volatility, individuals with low levels of tenure will experience higher separation rates if the starting idiosyncratic productivity is low. Raising the volatility of the idiosyncratic process will raise the separation rate at each level of tenure, thus shifting the separation-tenure profile upward. The persistence of the idiosyncratic process will help determine the rate at which the separation rate declines with tenure.

To calibrate the standard deviation of match quality, σ_{ϵ_y}, I use the empirically observed wage dispersion, as documented by Hornstein, Krusell, and Violante (2011) (henceforth HKV). In their working paper, Hornstein, Krusell, and Violante (2007) use data from the Census, Occupational Employment Survey and PSID to document mean-min wage ratios between 1.5 and 2.[12] I choose the standard deviation of match quality to target a mean-min wage ratio of 1.75. The current model has several features that deliver this substantial wage dispersion. First, consistent with HKV, the model features on-the-job search, which helps raise the sustainable mean-min wage ratio. Second, the "stepping stone" nature of the first job grants workers access to jobs with a better match-quality component. This investment motive of initial jobs can reduce initial wages, as workers are willing to pay for this when bargaining with their employer, thereby raising the equilibrium wage dispersion. Moreover, nonemployed workers do not sample the entire wage distribution, with initial match quality fixed at $y = y_0$. This allows for a high job-finding probability for the nonemployed while simultaneously maintaining a large wage dispersion.

The starting idiosyncratic productivity after an E-E transition, x_{EE}, is chosen to match the average wage gains associated with switching employers. When switching jobs and moving up the job quality ladder, a lower starting idiosyncratic productivity will induce smaller wage gains. When calculating the wage gains associated with an E-E transition, I use a method outlined in Topel and Ward (1992) and quantify the quarterly wage growth between jobs. In an independent regression, I estimate the within-job wage growth using annual changes in quarterly earnings in main jobs among job stayers, beginning with the sixth quarter on a job, exactly as in Topel and Ward (1992). Then I assess the wage gains associated with switching employers by looking at wage changes at job transitions (that result in a new job lasting more than one quarter) and subtracting the expected wage growth on the new and old jobs. Based on results from Topel and Ward (1992) and Jung and Kuhn (2018), I target 3 percent quarterly wage gains from E-E transitions to identify the x_{EE}

[12]They use a conservative estimate of the mean-min wage by using the observed ratio of the mean wage to the fifth percentile of the wage distribution. I follow this approach in the simulated data.

parameter.

The value of leisure, b, is chosen to target an average layoff rate into nonemployment of 1.4 percent in the PSID data. The intuition here is straightforward: The higher the value of nonemployment, the more attractive non-employment becomes and hence the higher the average layoff rate. As HKV point out, this parameter is important for the amount of wage dispersion in search models. Reducing the value of nonemployment can raise the amount of wage dispersion because large nonemployment-to-employment (N-E) flows may be the result of very small (even negative) flow payments while out of work, not necessarily little wage dispersion.

The observed E-E transition probability of 1.7 percent is targeted using the contact rate for the employed, p_E, while the contact probability for the nonemployed, p_U, is determined by targeting a 6 percent nonemployment rate, consistent with Bils, Chang, and Kim (2011). The starting match quality is normalized to the expected value of the stochastic process for y, which is close to one. The bargaining power of the worker, β, is set to 0.5, realistic adjustments of which I have found to be immaterial. Finally, δ targets a 5 percent annual interest rate.

5.3 Calibration Outcomes

The model has nine free parameters targeting 58 moments.[13] This section outlines how well the model delivers these targeted moments.

Table 1 summarizes the baseline parameters and the targeted empirical outcomes. Table 2 displays the simulated moments at the calibrated parameter values and shows that the model matches relatively well the calibration targets. The model delivers the empirical mean-min wage ratio. The average E-E probability and the average layoff probability for high-tenured workers are close to their empirical targets, although the E-E probability comes in relatively low.

Figure 1 compares the separation-tenure profile in the model and the data. It takes simulated data based on the calibration in Table 1, and plots the (smoothed) results of averaging the E-N dummies for each month of tenure. The model delivers the observed separation rates by tenure with around a 3 percent E-N probability for new hires, and around a 0.4 percent E-N probability for workers with five years of tenure. High-tenured

[13]The nine parameters are ρ_x, σ_{ϵ_x}, x_0, x_{EE}, σ_{ϵ_y}, p_E, p_U, p_s, and b. The moments include the first two years of the separation-tenure profile (24), the separation probability at 36, 48 and 60 months of tenure (3), the E-N probability in months 2-24 after the first E-N transition (23), the E-N probability 36, 48 and 60 months after the first E-N transition (3), the average E-E probability (1), the average layoff rate (1), average non-employment (1), the mean-min wage ratio (1), and quarterly wage gains from E-E transitions (1).

workers separate from their employers owing to exogenous separations, and therefore, the exogenous separation probability pins down the E-N probability for these workers. The model delivers the rest of the profile via low match qualities in initial matches and the idiosyncratic productivity component.

Figure 2 compares the serial correlation in E-N transitions. As with Figure 1, this plot takes simulated data from the model and plots the (smoothed) results of averaging the E-N dummies for each month after the first E-N transition. Recall that the average layoff rate is 1.4 percent, which means that the average E-N probability roughly triples after an initial E-N event. This is true both in the model and in the data. The model delivers this drastic rise in the layoff rate shortly after an initial E-N transition via low initial match qualities in first jobs and the idiosyncratic productivity component, and I use this aspect of the data as evidence for a low match quality in first jobs out of nonemployment. The model accurately captures that the E-N probability returns to average levels after around five years.

Owing to movements in idiosyncratic productivity at the match level, this model also implies movements in earnings within matches, similar to the model in Bils, Chang, and Kim (2011). Using a method from Topel and Ward (1992), the calibrated model implies a standard deviation of annual growth of quarterly earnings within a match equal to 17 percent, similar to the 19 percent figure obtained by Topel and Ward (1992). This is worth mentioning because Bils, Chang, and Kim (2011) stress that in a model with one-dimensional job heterogeneity, the MP model faces a trade-off between matching this relatively high degree of earnings volatility within matches and matching realistic fluctuations in aggregate unemployment. The intuition for that model is that large match-quality dispersion leaves few individuals at the job destruction threshold, implying small movements in the E-U margin during economic downturns. This highlights the need for two-dimensional job heterogeneity, which allows the present model to deliver realistic movements in earnings within matches while at the time delivering significant volatility in the separation margin.

6 Aggregate Fluctuations

This section presents the business cycle movements of the baseline model. Significant detail can be found in Appendix A, and here I present a cursory treatment.

6.1 Incorporating Aggregate Shocks

Extending the model to incorporate aggregate productivity shocks is not a trivial exercise because, even in the steady state, the equilibrium vacancy rate, determined by equation (6), depends on the distribution of idiosyncratic productivity and match quality among employed workers, $\Pi(x, y)$. As I point out in detail in the appendix, with aggregate shocks, today's vacancy rate depends on tomorrow's distribution, $\Pi'(x, y)$, because tomorrow's surplus depends on tomorrow's vacancy rate, v'. Hence individuals need to forecast the entire distribution, $\Pi(x, y)$, to make optimal decisions. Intuitively, to determine the number of vacancies to open today, firms need to know contact rates tomorrow, and these depend on the endogenous evolution of $\Pi'(x, y)$.

To fix ideas, consider the Bellman equation for the surplus from a match when aggregate productivity, z, follows a stochastic process according to $F_z(z'|z)$:

$$\begin{aligned}
&S(x, y, z, v, \Omega) = zxy \\
&+ \delta(1 - p_E(v))(1 - p_s) \int\int \max\{0, S(x', y, z', v'(\Pi', z'), \Omega)\} dF_x(x'|x) dF_z(z'|z) \\
&+ \delta p_E(v)(1 - p_s) \int\int\int \Big[\mathbb{I}\{S(x', y, z', v'(\Pi', z'), \Omega) \geq S(x_{EE}, \tilde{y}, z', v'(\Pi', z'), \Omega)\} \\
&\max\{0, S(x', y, z', v'(\Pi', z'), \Omega)\} \\
&+ \mathbb{I}\{S(x', y, z', v'(\Pi', z'), \Omega) < S(x_{EE}, \tilde{y}, z', v'(\Pi', z'), \Omega)\} \\
&\max\{0, \beta S(x_{EE}, \tilde{y}, z', v'(\Pi', z'), \Omega)\} dF_x(x'|x) dF_y(\tilde{y}) dF_z(z'|z) \Big] \\
&- [b + \delta p_U(v)\beta \int \max\{0, S(x_0, y_0, z', v'(\Pi', z'), \Omega)\} dF_z(z'|z)]
\end{aligned} \tag{10}$$

where Ω stands for all the constants, the laws of motion for the exogenous processes have been described in Section 5.1 and the endogenous distribution of match quality and idiosyncratic productivity among employed workers evolves according to some unknown equation of motion, Γ:

$$\Pi' = \Gamma(\Pi, z') \tag{11}$$

The surplus equation depends on v through the contact rate. The vacancy creation condition from the steady state could also be amended and written in this recursive, stationary formulation.

The difficulty is that in order to solve for the fixed point of these functional equations, the individuals must forecast the entire $\Pi(x, y)$ distribution. This is shown by the dependence of v' on Π' in the continuation value. This is where the idea of Krusell and Smith (1998) enters.

6.2 Computational Strategy

Forecasting the entire $\Pi(x, y)$ is infeasible so I assume that the agents only use certain moments of the endogenous $\Pi(x, y)$ distribution when making vacancy posting decisions, and I postulate a guess for how these moments determine equilibrium vacancies, v. I assume agents use the average x among the employed (denoted $X = \int x\pi_x(x)dx$ where $\pi_x(x) = \int \pi(x, y)dy$ is the marginal distribution of x), and the average y among the employed (denoted $Y = \int y\pi_y(y)dy$ where $\pi_y(y) = \int \pi(x, y)dx$ is the marginal distribution of y). Furthermore, I conjecture log-linear transition equations for the forecast rules:

$$\ln X' = \chi_0 + \chi_X \ln X + \chi_Y \ln Y + \chi_z \ln z'$$
$$\ln Y' = \nu_0 + \nu_X \ln X + \nu_Y \ln Y + \nu_z \ln z'$$

The firm then uses these forecasts for the aggregate state to determine tomorrow's vacancy rate via the equation:

$$\ln v' = v_0 + v_X \ln X' + v_Y \ln Y' + v_z \ln z' \tag{12}$$

The goal is to find the parameters $\{\chi_0, \chi_X, \chi_Y, \chi_z\}$, $\{\nu_0, \nu_X, \nu_Y, \nu_z\}$ and $\{v_0, v_X, v_Y, v_z\}$ that accurately forecast aggregate variables. Given an arbitrary sequence of aggregate productivity $\{z\}_{t=1}^{T}$, I simulate the economy and estimate the coefficients using ordinary least squares with the simulated data. Once I have coefficients that yield sufficiently accurate forecasts, I use the simulated data to compute the elasticities of the aggregate series with respect to labor productivity. I also use the coefficients to calculate impulse response functions.

6.3 Results

6.3.1 Unemployment and Worker Flows

Table 3 presents the elasticities of aggregate variables with respect to output per worker in the model with match quality. In this table, I also present the baseline results from Shimer (2005) for comparison.[14] The model delivers a significant amount of amplification in the E-U rate and unemployment. Quantitatively, the model delivers around two-thirds of the

[14] These results are based on a numerical replication at monthly frequency performed by the author.

observed volatility of the E-U probability and the unemployment rate. This is largely due to the mass of recently hired unemployed workers who start at the bottom of the job ladder and are therefore susceptible to downward movements in aggregate productivity. As a result of this volatile job destruction margin, the model delivers significant amplification of the unemployment rate.

Owing to the procyclicality of job openings, the model also delivers a procyclical average E-E probability, which is what one observes in the data.[15] Quantitatively, the model performs remarkably well in this dimension, explaining over 90 percent of the volatility of the average E-E probability over the business cycle.

While managing to perform well on the job destruction margin, the model yields a volatility of the job-finding probability that falls far short of the the observed data. This is not surprising. As Fujita and Ramey (2012) point out, a model with endogenous job destruction and on-the-job search does little to resolve the so-called "Shimer puzzle" that realistic aggregate productivity shocks generate small fluctuations in the job-finding rate in a standard MP model. The model outlined here is subject to the same criticism. Addressing this shortcoming is left for future research, and I describe a promising resolution in Section 8.

The model also has implications for the relationships between key macroeconomic variables, as well as the time-series properties of these variables. Table 4 presents the results of the model alongside empirical counterparts, and a model with endogenous job destruction and on-the-job search from Fujita and Ramey (2012). The framework delivers a negative correlation between the E-U probability and labor productivity, one that is quantitatively consistent with observed data. As a result, the model also delivers a strong negative relationship between unemployment and productivity. Along this dimension, the model overstates the strength of this negative relationship because when aggregate productivity changes, the job-finding rate does not respond sufficiently. The model delivers a positive correlation between productivity and vacancies, but quantitatively overstates this link for reasons described in Fujita and Ramey (2007).

As for the serial correlation of unemployment and vacancies, the model performs remarkably well, replicating the observed time-series properties of these variables. It understates the autocorrelation of the average E-U probability, largely because of the responsiveness of the E-U probability to labor productivity declines.

[15] See, for example, Nagypál (2008).

6.3.2 The Beveridge Curve

Aside from the cyclicality of worker flows, Shimer (2005) also notes that with countercyclical separations, the baseline MP model fails to deliver the observed procyclicality of vacancies. This implies that the model fails to deliver the observed negative relationship between vacancies and unemployment, called the Beveridge curve. The model with match quality presented here, because of procyclical vacancies, delivers a negative relationship between vacancies and unemployment. The model overcomes the weakness of the MP model by allowing firms to contact employed workers. As in the standard MP model, as the E-U rate rises, firms want to post more vacancies due to a higher job-filling rate. However, in the current setup, firms also face lower productivity of employed workers when aggregate productivity falls, and since these workers make up the majority of the potential applicant pool, this serves to reduce vacancies.

Using data from Elsby and Michaels (2013) on job openings from the Job Openings and Labor Turnover Survey (JOLTS), and unemployment from the Current Population Sruvey (CPS), from 2001Q1 to 2007Q4, the correlation between unemployment and vacancies (divided by the labor force) is around -0.98. The correlation between unemployment and vacancies in the simulated data is -0.90. Hence, the model delivers the qualitative relationship between vacancies and unemployment, but falls slightly short quantitatively. This is because vacancies exhibit too little volatility in the baseline model compared to the data.

6.3.3 Propagation

Figure 3 depicts the outcomes of key aggregate variables in response to a permanent 1 percent reduction in aggregate productivity. That is, aggregate productivity falls by 1 percent and remains at that level forever, although agents continue to have expectations about aggregate productivity consistent with equation (9). The model displays a sharp rise in E-U separation in the wake of a recession that then falls slightly and continues to rise thereafter to its new steady-state value. The rise in the E-U rate results from a discrete mass of jobs becoming unprofitable and being destroyed immediately. The new steady state of the E-U probability is above the original steady state because of a lower aggregate productivity, which means more jobs are fragile. The response of the E-U rate is very large, reflecting that many employment relationships are near the destruction threshold because of the low match quality of new hires. As a result of the large rise in the E-U rate, unemployment also reacts strongly. The response of the E-U rate in this model stands in sharp contrast to the response in the basic MP model. In that model, separations are exogenous so that the E-U rate does not deviate at all from its initial value when aggregate productivity falls.

The economy delivers significant propagation of aggregate productivity shocks. It takes unemployment around four years to complete most of its adjustment to the one-time change in aggregate productivity. After four years, vacancies, and therefore the job-finding rate, have only completed half of their transition to their new values. This significant propagation is due to the slow-moving nature of the distribution of idiosyncratic productivity and match quality among the employed, and is more in line with empirical observation.[16] This propagation of the job-finding rate is a marked improvement over the standard MP model, which features a job-finding rate that adjusts instantaneously to aggregate productivity shocks. Figure 3 plots the response of vacancies and unemployment in the standard MP model. As mentioned in Section 1, the MP model delivers almost no propagation: Both unemployment and vacancies adjust to their new level within a year. The figure also reinforces that the model with match quality delivers more amplification than the standard MP model, especially in the unemployment rate.

It takes years for the E-E rate to recover. Workers coming out of unemployment initially transition quickly up the ladder: since they encounter many favorable outside offers, they tend to raise the average E-E rate in the economy. Put differently, because of a lower average match quality, more workers are available for poaching, thereby raising the average E-E rate after the initial shock.

6.3.4 The Sullying Effect

The model with match quality also exhibits a slight cleansing effect as low-quality matches are destroyed. Because of lower aggregate productivity, firms post fewer vacancies, which results in lower job-finding rates. This causes the average match quality to fall as agents make their way up the job ladder at a reduced rate. This is the central point of a related paper, Barlevy (2002). As for the impact of aggregate productivity on the match-quality distribution, and its effect on the aggregate job-finding rate, the results presented by Barlevy are very similar to the results presented here. For example, Figure 5 of Barlevy (2002) is very similar to the bottom right graph of Figure 3 in the present paper. On impact, match quality rises because low-quality matches are destroyed. It takes around seven quarters for the average match quality to dip below its original stochastic steady state in Barlevy (2002) and the present model. Quantitatively, the two approaches are difficult to compare because the aggregate productivity decline in Barlevy's work is 72 percent, whereas in the present context it is 1 percent. Nevertheless, average match quality peaks around 0.2 percent above

[16]See, for example, Fujita and Ramey (2007).

the stochastic steady state in Barlevy's model. In the present context, assuming a linear response, average match quality would peak around 2.8 percent. This underscores that the present model delivers significant unemployment volatility when faced with realistic aggregate productivity fluctuations, whereas Barlevy's model requires implausibly large aggregate productivity shocks to generate observed fluctuations in unemployment.

7 A Model Without Match Quality

To gain intuition for the aggregate dynamics of the model with match quality, this section presents a simpler version of that model without match quality. The framework still incorporates search and matching, and is in large part the same as the model with match quality, with one exception: The model presented here features no match quality, denoted by y previously. The output of every match is a linear function of the idiosyncratic productivity, denoted by x, and distributed according to $F(x'|x)$. Aggregate productivity, z, still affects all matches. The model still features E-E transitions and a job ladder.[17] There are substantive differences between this model and the model with match quality. The addition of a second state variable in the model with match quality allows the economy to lose many productive matches in a downturn while still allowing new matches to survive. In the model without match quality, one cannot shed relationships at the starting productivity level because then no new employment relationships would form. Moreover, the model with match quality features two-dimensional heterogeneity, which leaves room for more propagation.

This model can be characterized by a series of Bellman equations. The value of work to the employee, $W(x)$, satisfies:

$$\begin{aligned} W(x) = w + \delta(1-p_E)(1-p_s)\int \max\{U, W(x')\}dF_x(x'|x) + \delta p_s U \\ + \delta p_E(1-p_s)\int\int \max\{U, W(x'), W(\tilde{x})\}dF_x(x'|x)dG(\tilde{x}) \end{aligned} \tag{13}$$

The intuition here is straightforward. The flow payoff to a job equals the wage, w. In the future, given no outside offer, and no exogenous separation shock, idiosyncratic productivity changes according to distribution $F(x'|x)$ and, depending on the level of this future shock, the worker decides whether to remain at the current firm, or flow into unemployment and look for an alternative match. With an outside offer, and no separation shock, the worker decides whether to remain at the current firm or to move to the new firm, with unemployment always

[17] In other words this is a simplification of the model with match quality with $y_0 = 1$ and $\sigma_{\epsilon_y} = 0$.

remaining as an option. The right-hand side also includes the possibility of an exogenous separation shock occurring with probability p_s.

The value of unemployment satisfies:

$$U = b + \delta(1 - p_U)U + \delta p_U W(x_0) \tag{14}$$

The unemployed receive flow payoff b. They either receive an offer and take it or remain unemployed. Notice that all jobs start at the same level of idiosyncratic productivity, x_0. The calibration presented later guarantees that $W(x_0) > U$, so that the worker prefers employment to unemployment at the starting productivity level.

The value of a filled job to the firm, $J(x)$, satisfies:

$$\begin{aligned} J(x) = z \cdot x - w + \delta(1 - p_E)(1 - p_s) \int \max\{0, J(x')\} dF_x(x'|x) \\ + \delta p_E(1 - p_s) \int \int \mathbb{I}\{J(x') \geq J(\tilde{x})\} \max\{0, J(x')\} dF_x(x'|x) dG(\tilde{x}) \end{aligned} \tag{15}$$

where z denotes aggregate productivity. The payoff to the firm includes the output, $z \cdot x$, less the wage paid to the worker, w. In the next period, given no outside offer and no separation shock, the firm decides to continue with the match or to let the worker go and open a vacancy, depending on the level of the idiosyncratic shock. In equilibrium vacancies are assumed to have value zero, which is guaranteed by a free-entry condition into vacancy posting. With an outside offer, the worker does not leave unless the value of the current job exceeds the value of the outside job.

These value functions can be summarized by one equation, the surplus from a match:

$$\begin{aligned} S(x) &= W(x) - U + J(x) \\ &= z \cdot x + \delta(1 - p_E)(1 - p_s) \int \max\{0, S(x')\} dF_x(x'|x) \\ &+ \delta p_E(1 - p_s) \int \int \Big[\mathbb{I}\{S(x') \geq S(\tilde{x})\} \max\{0, S(x')\} \\ &+ \mathbb{I}\{S(\tilde{x}) > S(x')\} \max\{0, \beta S(\tilde{x})\} \Big] dF_x(x'|x) dG(\tilde{x}) \\ &- [b + \delta p_U \beta S(x_0)] \end{aligned} \tag{16}$$

which is the flow payoff from the match and the continuation value of the match, less the worker's outside option that includes a flow payoff from unemployment and the chance of finding a new job with idiosyncratic productivity at x_0.

In the calibration below, it turns out that $S(x)$ is increasing in x, so define the reservation cutoff as the level of productivity that makes the worker and the firm indifferent between maintaining the current match and terminating the current employment relationship:

$$S(x_R) = 0 \tag{17}$$

Wage bargaining follows the standard Nash bargaining protocol so that:

$$w = \arg\max_{w} [W(x) - U]^{\frac{1}{2}} [J(x) - V^s]^{\frac{1}{2}} \tag{18}$$

This implies that the worker and the firm split the surplus evenly.

The number of new meetings between the unemployed and vacancies is determined by an aggregate matching function, exactly as in the model with match quality:

$$m_i(1, v) = m_0^i v^{\alpha}, i \in \{U, E\} \tag{19}$$

where v is the number of vacancies. The aggregate meeting rate is:

$$p_i(v) = \frac{m(1, v)}{1} = m(1, v) = m_0^i v^{\alpha}, i \in \{U, E\} \tag{20}$$

and the vacancy filling rate is:

$$q_i(v) = \frac{m(1, v)}{v} = \frac{f_i(v)}{v} = m_0^i v^{\alpha - 1}, i \in \{U, E\} \tag{21}$$

In order to determine the number of vacancies in equilibrium, we need the value of posting a vacancy:

$$\begin{aligned}
V^s &= -c + \delta q_U u[1 - q_E(1 - u)](1 - \beta)S(x_0) \\
&\quad + \delta q_E(1 - u)[1 - q_U u] \int\int\int \mathbb{I}\{S(\tilde{x}) > S(x')\} \max\{0, (1 - \beta)S(\tilde{x})\} dF_x(x'|x) dG(\tilde{x}) d\Pi(x) \\
&\quad + q_E(1 - u) q_U u \int\int\int \Big[\mathbb{I}\{S(\tilde{x}) > S(x')\}(1 - \beta) \max\{S(x_0), S(\tilde{x})\} \\
&\quad + \mathbb{I}\{S(x') \geq S(\tilde{x})\} \max\{0, (1 - \beta)S(x_0)\}\Big] dF_x(x'|x) dG(\tilde{x}) d\Pi(x) \\
&= -c + \delta q_U u[1 - q_E(1 - u)]\mathbb{E}_u^s + \delta q_E(1 - u)[1 - q_U u]\mathbb{E}_e^s + \delta q_E(1 - u) q_U u \mathbb{E}_{u,e}^s
\end{aligned} \tag{22}$$

where c is the flow cost of maintaining a vacancy, q_i is the job-filling probability, $\mathbb{E}^s_u$, $\mathbb{E}^s_e$ and $\mathbb{E}^s_{u,e}$ denote the expected payoff to meeting an unemployed, employed, and both workers, respectively, and the s superscript denotes the simple model. $\Pi(x)$ is the distribution of idiosyncratic productivity among employed workers and is a slow-moving endogenous object. Notice that this pins down v uniquely, given $V = 0$, u and Π.

7.1 Calibration of the Model Without Match Quality

Given the optimal decisions of workers and firms, the model generates simulated data at a monthly frequency. In particular, I simulate 6,000 agents for 480 months (40 years). To remove the effects of initial conditions, I simulate the model for 980 months and then discard the first 500 months of the sample. This simulation provides a time-path of wages and annual earnings, as well as an employment history.

The model period length is one month. In what follows, idiosyncratic productivity starts out at a fixed and deterministic level x_0 in all matches, and then within the match follows a log AR(1) process:

$$\ln x' = \rho_x \ln x + \epsilon'_x \tag{23}$$

where $\epsilon'_x \sim \mathcal{N}(0, \sigma^2_{\epsilon_x})$. This process captures the intuition that productivity at the match level, or demand for the match's output, exhibits some persistence.

As in the model with match quality I consider the following log AR(1) process for aggregate productivity:

$$\ln z' = \rho_z \ln z + \epsilon'_z \tag{24}$$

where $\epsilon'_z \sim \mathcal{N}(0, \sigma^2_{\epsilon_z})$.

Aggregate parameters are set as in the model with match quality, namely, $\rho_z = 0.983$ and $\sigma_{\epsilon_z} = 0.005$, yielding, after logging and HP filtering, a 0.93 autocorrelation of aggregate labor productivity, with a standard deviation of 0.019. c is set to normalize $v = 1$ in the steady state.

The calibration follows Hornstein, Krusell, and Violante (2011) and Bils, Chang, and Kim (2011) very closely, although I additionally target the average E-E rate. Table 5 shows the baseline parameter values for the model without match quality. I take the annualized interest rate to be 5 percent. The key targeted outcomes are the average rates of nonemployment and separations, and the average E-E rate. Analogously to the model with match quality, I target an average nonemployment rate of 6 percent, and a monthly separation rate of 1.4 percent that is consistent with work using the PSID. The separation rate and the nonemployment rate pin down the steady-state monthly job-finding rate at 22 percent. I target a monthly

E-E rate of 1.7 percent, as in the model with match-quality. Following Bils, Chang, and Kim (2011), I set the starting idiosyncratic productivity, x_0, to the mean value of the unconditional distribution, denoted by $\bar{x}$.[18]

The vacancy posting cost, c, is chosen to normalize the steady-state vacancy level to one. The matching technologies are Cobb-Douglas; $m_U(1,v) = 0.22v^{\alpha}$ and $m_E(1,v) = 0.059v^{\alpha}$, which hit the steady-state finding rates. The matching power parameter α is set to 0.5. In the model without match quality, I fix the persistence of idiosyncratic productivity at 0.97, the value in Bils, Chang, and Kim (2011), to match highly persistent individual wage earnings. I choose the standard deviation of idiosyncratic productivity to match an observed mean-min wage ratio of 1.75, as documented by Hornstein, Krusell, and Violante (2011). I choose the value of leisure, b, to match the 1.4 percent separation probability.

Notice that in order to match the duration of nonemployment and the mean-min wage ratio, the model requires a low value of leisure. Hornstein, Krusell, and Violante (2011) point out the same phenomenon. Their main message is that the wage dispersion delivered by search models is constrained by preference parameters and the observed size of the transition rates of workers. The intuition is that if we observe very large U-E rates, this must mean that the wage offer distribution is not very dispersed; otherwise, workers would be willing to wait longer for a potentially better offer. With the particular setup outlined here, the U-E probability is calibrated to 22 percent and the E-U probability at 1.4 percent. In order to simultaneously target a large mean-min wage ratio (1.75) and the large U-E flows, the model requires the value of leisure to be around 40 percent of average labor productivity among the employed. This low value of leisure ensures that, despite the large wage dispersion, the model can match the observed U-E probability.

7.2 Steady State of the Model Without Match Quality

This section outlines some steady-state features of the model without match quality. The focus of the analysis is on the ability of the model to match observed cross-sectional wage facts. The distribution of idiosyncratic productivity is largely responsible for the model's earnings implications, and this distribution is described in detail.

Table 6 presents some steady-state moments of the simulated data. The calibration can match the targeted moments exactly, hitting the nonemployment rate, the separation

[18] An alternative would be to target the E-N profile after the first E-N transition as in the model with match quality. This would likely imply a lower value of x_0, and potentially higher job destruction volatility; however, this calibration could mean that during recessions the surplus associated with new jobs is negative, thus causing the job-finding rate for the nonemployed to fall, counterfactually, to zero.

rate and the job-finding rate. The calibration delivers significant wage dispersion, obtaining an observed mean-min wage ratio of 1.75, and the standard deviation of (ln) wages within matches is around 55 percent. The latter is substantially higher than the 19 percent documented by Topel and Ward (1992). This suggests that this simpler model without match quality has difficulty simultaneously delivering observed wage dispersion and reasonable within-match earnings volatility because they are controlled by the same parameter (σ_{ϵ_x}). The model with two-dimensional job heterogeneity performs better along this dimension.

One key to understanding wage dispersion in this calibration, and for understanding the aggregate implications for job destruction, is the underlying steady-state distribution of idiosyncratic productivity. The numerical approach for obtaining this steady-state distribution is detailed in Appendix A.2.1. Figure 4 presents the steady-state distribution above the economy's reservation productivity x_R. The distribution peaks at the starting idiosyncratic productivity x_0, as there exists a mass of unemployed workers entering employment at this productivity level. This figure highlights that the model without match quality features a dispersed idiosyncratic productivity distribution, with very little mass toward the reservation productivity. This means that this economy features realistic wage dispersion. However, this calibration exhibits small fluctuations in unemployment and vacancies because downward aggregate productivity movements affect few matches.

In addition to the job destruction margin, the job creation margin plays an important role in aggregate dynamics. As noted before, the value of leisure is small in the model without match quality. This allows the model to simultaneously hit the job-finding rate, and the large wage dispersion observed in the data. A small value of leisure implies that the surplus from employment relationships is large. Hagedorn and Manovskii (2008) show that the volatility of labor market tightness is closely related to the value of leisure and the size of accounting profits. This calibration features large accounting profits, which means that the vacancy posting incentives in this economy will be muted. The job creation margin also features prominently in the aggregate fluctuations, which I point out in the next section.

7.3 Aggregate Fluctuations in the Model Without Match Quality

This section presents the responses of the calibrated economy without match quality to aggregate productivity shocks. As in the model with match quality, the procedure implements the Krusell and Smith (1998) algorithm in the current context, and I refer the reader to Appendix A.2.3 for the computational algorithm and Section 6.2 for an outline of the method used.

Table 3 presents the elasticities of aggregate variables with respect to output per worker in the model without match quality. The model fails to deliver sufficient amplification of aggregate shocks. This is most starkly visible in the elasticities of the E-U rate and unemployment, which fall far short of the observed values. The model explains around 35 percent of the volatility of unemployment and the E-U probability. Similarly, the model delivers a procyclical E-E probability, but is only able to explain 60 percent of its volatility. The model does, however, manage to deliver a countercyclical E-U rate and a negative relationship between vacancies and unemployment as observed in the data. As in the model with match quality, this is because firms can contact employed workers.

Figure 5 portrays the results for the model without match quality in response to a one-time permanent reduction in aggregate productivity. On impact the returns to posting a vacancy fall immediately as aggregate productivity falls, and hence vacancies jump down on impact. As a result of the reduced aggregate productivity, the model displays a sharp rise in E-U separations that then falls, and continues to rise thereafter to its new value. The rise in the E-U rate results from a discrete mass of jobs becoming unprofitable and being destroyed immediately. With the new aggregate productivity, the E-U probability is above the original value because of more fragile jobs. The model also displays a very small cleansing effect as low productivity jobs are destroyed. This is not obvious, since the values plotted are end-of-period values, and although low match-quality jobs have been destroyed, the E-E probability is also immediately lower, and hence, employed workers have fewer outside opportunities to move up to. It seems that the cleansing effect dominates initially. Owing to lower aggregate productivity, firms post fewer vacancies, which results in lower job-finding probabilities. This causes the average match quality to fall as agents make their way up the job ladder at a reduced rate. As in the model with match quality, this relates to Barlevy (2002). The E-E probability falls on impact because of the lowered vacancy rate; however as the average match quality falls in the economy, and more workers are available for poaching, the E-E probability rises slightly toward its new value.

The model without match quality, however, delivers very little propagation compared to the model with match quality. Notice that vacancies effectively jump down to their new value on impact, while the E-U probability and unemployment complete most of their transition in around one year. The E-E probability is slightly more sluggish, completing its transition after about two years, and the same holds true for the average match quality in the economy. Hence, although the simpler model does perform better than the baseline MP model, quantitatively it falls short of matching the data as well as the model with match quality.

8 Summary and Discussion

This paper investigates the aggregate labor market fluctuations associated with a model that features both idiosyncratic productivity and match quality. Closing the model involves introducing aggregate matching functions for the unemployed and the employed, and introducing the optimal vacancy creation condition for the firm. A simpler version of the model, with only one state variable, delivers far less amplification and propagation of aggregate productivity shocks. With idiosyncratic shocks that deliver the correct mean-min wage ratio, there exist very few matches at the destruction threshold, which implies that aggregate productivity shocks have very little bite. In addition to little job destruction, this calibration of the model implies large accounting profits, which means that vacancies do not respond sufficiently to lower aggregate productivity.

This tension is mitigated in the model with match quality. This is not obvious, since the model with match-quality resembles the model without match quality, but features additional volatility because of variation in match quality. Since variation in idiosyncratic productivity was the reason for a lack of amplification of aggregate productivity shocks in the simpler model, it may seem that adding more volatility via match quality would exacerbate the amplification problem in the model with match quality. Despite this additional volatility, the model with match quality delivers significant unemployment amplification because new matches begin with low match quality, which implies that there exist many relationships at the reservation frontier. Relationships with this starting match quality can be destroyed in the model with match quality because relationships are characterized by two state variables: idiosyncratic productivity and match quality. In the model without match quality eliminating, employment relationships with the starting productivity was not a possibility because then no new matches would form. In the model with match quality, aggregate productivity shocks have large effects on unemployment that are quantitatively consistent with the observed facts.

The model also features the observed negative relationship between job openings and unemployment, despite countercyclical separations into nonemployment due to on-the-job search. Allowing employed workers to search means that during a recession, although unemployment rises and thus makes vacancy posting relatively more attractive, the downward movement in aggregate productivity dominates this effect and causes vacancies to fall. As a result, the model delivers the observed Beveridge curve. On a related note, owing to procyclical job openings, the model delivers a procyclical aggregate E-E probability, as observed in the data.

The model with match quality also delivers significant propagation of aggregate productivity shocks. After a 1 percent unexpected reduction in aggregate productivity, unemployment takes around five years to complete 80 percent of its transition. Vacancies, and therefore the job-finding rate, take even longer to respond to the same shock. The propagation stems from the slow-moving distribution of idiosyncratic productivity and match quality among employed workers. A downturn induces separations for workers who are low on the job ladder. As workers climb back up the job ladder after nonemployment, this induces changes in the vacancy-posting incentives of firms.

As a whole, the model with match quality performs remarkably well. In the wake of a recession, there is a spike in the E-U rate, and the job-finding rate for the employed and the unemployed falls. The model is quantitatively consistent with the observed fluctuations in job destruction and unemployment. The model delivers the Beveridge curve: the observed negative relationship between vacancies and unemployment, as well observed wage dispersion. Related to the large wage dispersion, the model features significant propagation of aggregate productivity shocks due to a slow-moving distribution of productivity and match quality among employed workers.

The current paper has not addressed amplification of the job-finding rate. Standard search models deliver far less volatility of job openings than in the real data. In the context of single-worker firms, the literature has chiefly turned to Hagedorn and Manovskii (2008) to deliver significant volatility of the job-finding rate in the face of aggregate productivity shocks, by reducing the average surplus to matches. In the current framework, significantly raising the value of nonemployment is not possible for the reasons highlighted in Hornstein, Krusell, and Violante (2011). In essence, the value of nonemployment must be relatively low in order for standard search models to simultaneously deliver observed wage dispersion and high job-finding rates, even in models with on-the-job search. An alternative, which I propose for future research, is to amend the model in this paper with costly vacancy entry, as in Coles and Moghaddasi Kelishomi (2014). They show that costly vacancy entry can induce amplification and propagation of vacancies in the standard MP model in response to fluctuations in the separation probability. The current framework endogenously delivers realistic movements in job destruction and, therefore, coupled with costly vacancy entry, promises to deliver the features highlighted in this paper along with amplification of the job-finding rate.

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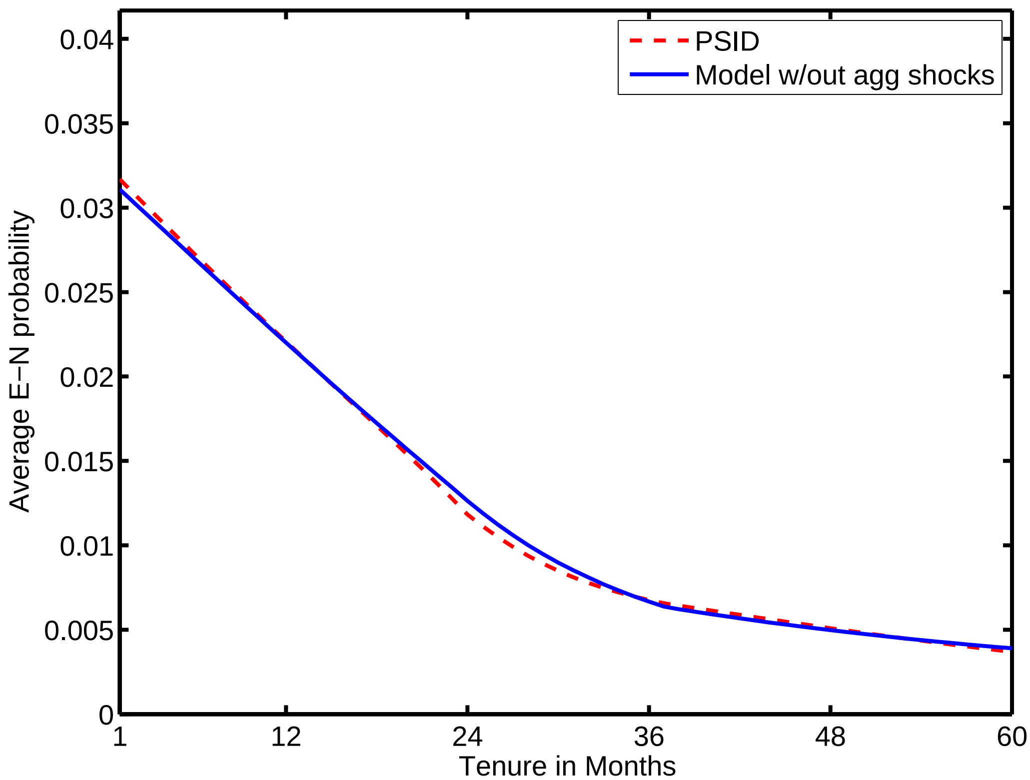

Figure 1: Average E-N Probability by Tenure

Note: The model delivers the observed separation rate into nonemployment by tenure. This is the (smoothed) average E-N probability for each month of tenure in the simulated data and the PSID. Smoothing is performed using a locally weighted (LOWESS) regressions scatter-plot smoothing. This figure is taken from Krolikowski (2017).

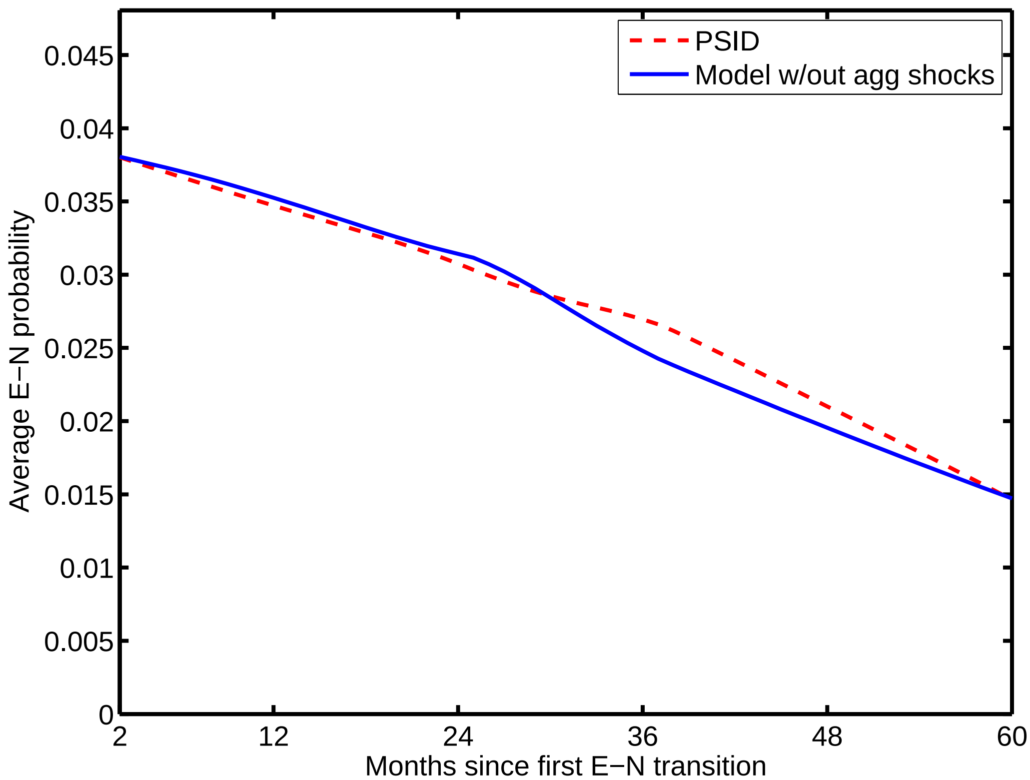

Figure 2: Average E-N Probability after First E-N Transition

Note: The model delivers the observed persistence in E-N probabilities after the first E-N transition. This is the (smoothed) average E-N probability for each month after the first E-N transition in the simulated data and the PSID. Smoothing is performed using a locally weighted (LOWESS) regressions scatter-plot smoothing. This figure is taken from Krolikowski (2017).

MODEL WITH MATCH QUALITY

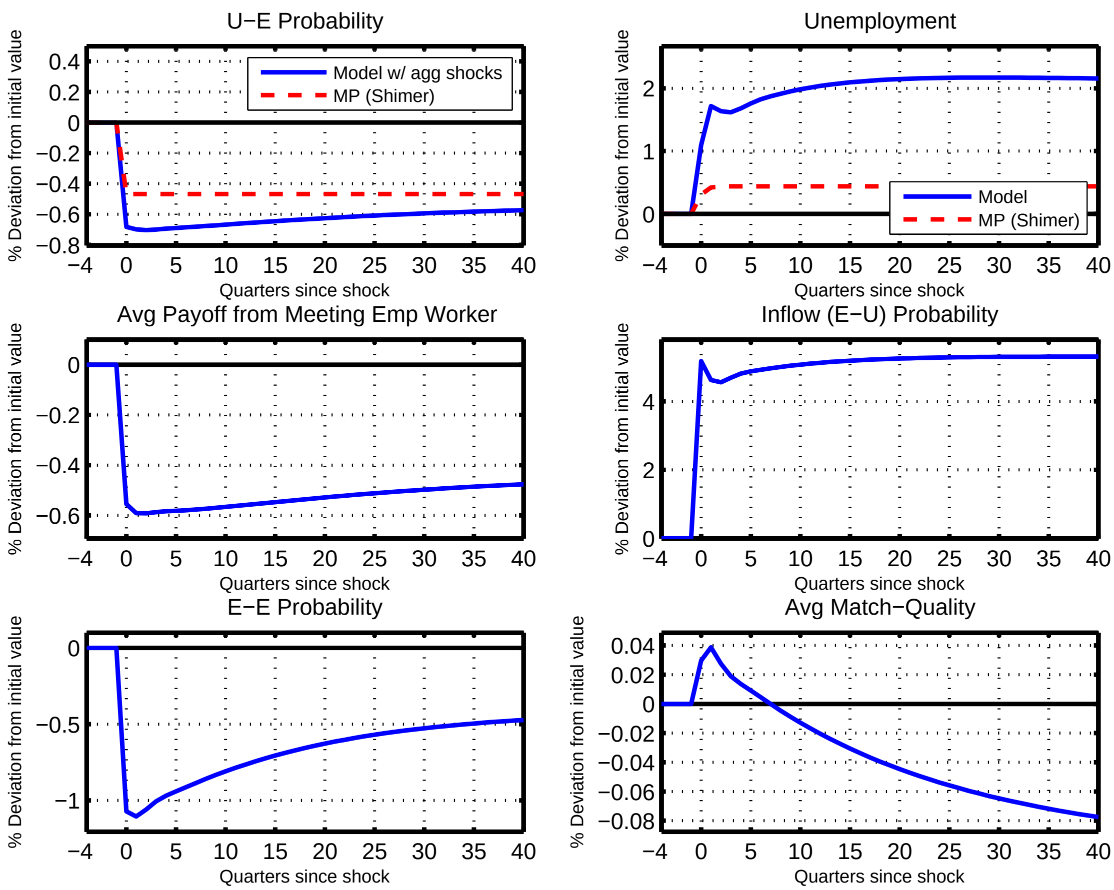

Figure 3: Impulse Response to a 1% Permanent Decrease in Aggregate Productivity: Model with Match Quality

Note: The model with match quality delivers significant propagation of aggregate productivity shocks. The permanent reduction in aggregate productivity occurs at time period 0. Where applicable, I have included the impulse response functions from the basic MP model.

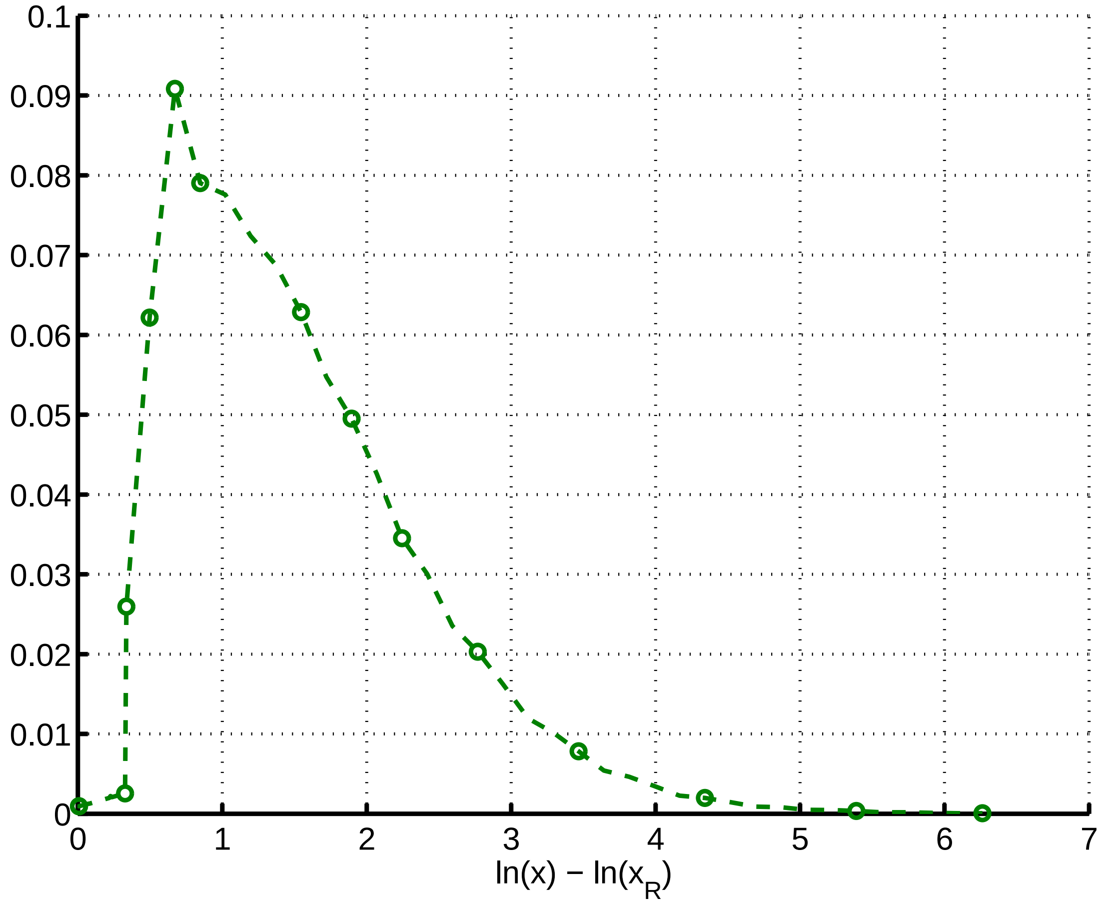

Figure 4: Distribution of Idiosyncratic Productivity (x): Model Without Match Quality

Note: The distribution is very spread out with few matches near the destruction threshold.

MODEL WITHOUT MATCH QUALITY

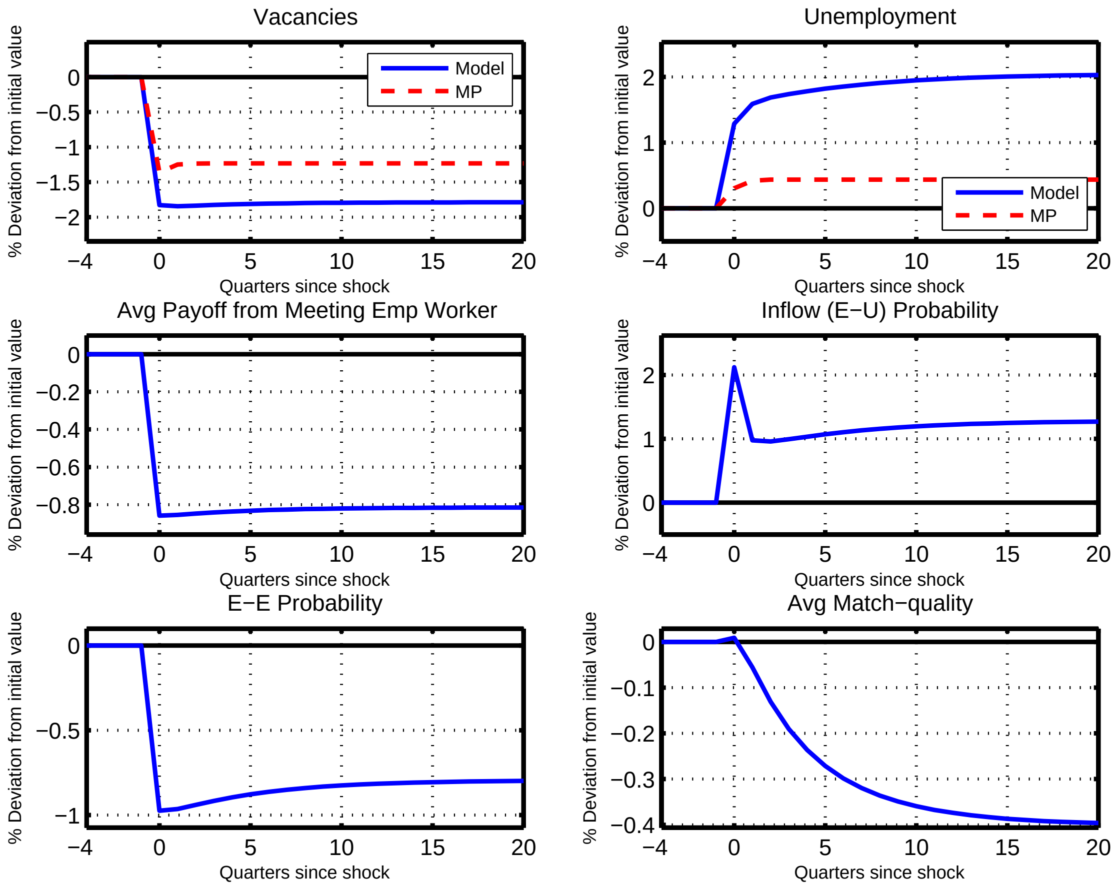

Figure 5: Impulse Response to a 1% Permanent Decrease in Aggregate Productivity: Model Without Match Quality

Note: The model without match quality features some propagation, although most variables jump almost immediately to their new values. The permanent reduction in aggregate productivity occurs at time period 0. Where applicable, I have included the impulse response functions from the basic MP model.

Table 1: Calibrated Model Parameters

Parameter	Meaning	Calibrated Value	Main Source of Identification
ρ_z	Agg prod persistence	0.983	Persistence of Y/L
σ_{ϵ_z}	Std. dev. of agg productivity	0.005	Std. dev of Y/L
α	Matching elasticity	0.524	Mortensen and Nagypál (2005)
c	Flow vacancy cost	0.32	$v = 1$ in steady state
r	Real interest rate	0.0041	Annual interest rate = 0.05
ρ_x	Productivity persistence	0.88	E-N by tenure & E-N profile
σ_{ϵ_x}	Std. dev. of productivity	0.11	E-N by tenure & E-N profile
x_0	Starting productivity (U $\rightarrow$ E)	$0.78 \times \max[x]$	E-N by tenure & E-N profile
σ_{ϵ_y}	Std. dev. of match quality	0.49	Mean-min wage ratio
x_{EE}	Starting productivity (E $\rightarrow$ E)	0.91	Wage growth from E-E transition
p_E	Contact probability (E)	0.20	E-E flow probability
p_U	Contact probability (U)	0.22	Average nonemployment rate
p_s	Exo separation probability	0.0047	E-N prob. for high-tenured workers
b	Value of leisure	1.4	E-N flow probability
y_0	Match quality in first jobs	$\mathbb{E}[y] \approx 1$	Normalization
β	Worker's bargaining power	0.5	Benchmark
δ	Discount factor	0.9959	5% annual interest rate

Note: Calibrated parameters of the model at monthly frequency. The citations and values of these empirical moments appear chiefly in Table 2, along with Figures 1 and 2. Parameters above the dashed line are associated with the model that has endogenous contact rates. Parameters below the dashed line are associated with the model that has exogenous contact rates.

Table 2: Steady-State Features of Model with Match Quality

Moment	Data	Model
Mean-min wage ratio	Hornstein, Krusell & Violante (2007): 1.5-2	1.56
Wage growth from E-E transition	Jung & Kuhn (2014): 3%	2.9%
E-E flow probability	Author (PSID): 1.7%	1.1%
Average nonemployment rate	Bils, Chang & Kim (2011): 6%	4.4%
E-N prob. for high-tenured workers	Author (PSID): 0.4%	0.4%
E-N flow probability	Author (PSID): 1.4%	1.0%
Std. dev. of (ln) wages w/in matches	Topel and Ward (1992): 19%	17%

Note: The model matches the empirical targets relatively well. The middle column of this table presents the value of the moment in the data and the citation. The column on the right presents the value of the equivalent moment in the model at the calibrated parameter values. 'E-E' stands for employer-to-employer, and 'E-N' stands for employment-to-nonemployment. See Figure 1 and Figure 2 for the entire profiles of E-N by tenure and after the first E-N transition.

Table 3: Elasticity With Respect to Output per Worker

Outcome	MP (Shimer)	Model w/out Match Quality	Model w/ Match Quality	Data
E-U probability	0	-0.79	-1.00	-1.55
Unemployment, u	-0.41	-1.31	-1.44	-2.21
E-E probability	0	0.72	1.07	1.18
Job-finding prob, p_U	0.47	0.74	0.72	1.42
Vacancies, v	1.24	1.49	1.37	3.55

Note: The model delivers significant volatility in job destruction and the unemployment rate. Empirical counterparts are taken chiefly from Fujita and Ramey (2012), Table 2, where I multiply the $cor(p_t, X_t)$ by σ_X/σ_p and take $\sigma_p = 0.02$. Column 'SR_t' corresponds to E-U in the current paper, and column 'JFR_t' corresponds to p_U in the current paper. The data on employer-to-employer (E-E) transitions are taken from Fallick and Fleischman (2004). These values represent the elasticities with respect to output per worker. In particular, following Mortensen and Nagypál (2007), these elasticities are computed by regressing the log deviations from trend of the respective series on the log deviation from trend of output per worker. To calculate the trend, I follow Shimer (2005) and use a Hodrick-Prescott filter with smoothing parameter 10^5. After HP detrending, the aggregate productivity series used in the simulation exhibits an autocorrelation of 0.89 and a standard deviation of 0.015. These are very close to US quarterly data: 0.88 and 0.02, respectively. The table also includes elasticities from a replication of Shimer (2005).

Table 4: Correlations

Outcome	Fujita & Ramey (2012)	Model w/ Match Quality	Data
$corr(p_t, u_t)$	-0.87	-0.82	-0.46
$corr(p_t, v_t)$	0.97	0.99	0.56
$corr(p_t, EU_t)$	-0.90	-0.51	-0.54
$corr(u_t, u_{t-1})$	0.85	0.84	0.93
$corr(v_t, v_{t-1})$	0.67	0.90	0.92
$corr(EU_t, EU_{t-1})$	0.70	0.29	0.63

Note: The model delivers correlations with labor productivity and serial correlations that are broadly consistent with the observed data. $corr(p_t, X_t)$: correlation between labor productivity and variable X_t. $corr(X_t, X_{t-1})$: correlation between X_t and X_{t-1}. See Fujita and Ramey (2012), Table 2 (Panel A) for empirical counterparts, and Table 2 (Panel D) for the results of a model with endogenous separations with on-the-job search. All series are logged and HP filtered.

Table 5: Parameter Values for Model Without Match Quality

Parameter	Meaning	Calibrated Value	Main Source of Identification
ρ_z	Agg prod persistence	0.983	Persistence of Y/L
σ_{ϵ_z}	Std. dev. of agg productivity	0.005	Std. dev of Y/L
α	Matching elasticity	0.5	Bils, Chang & Kim (2011)
c	Vacancy posting cost	1.60	$v = 1$ is steady state
r	Real interest rate	0.0041	Annual interest rate = 0.05
ρ_x	Productivity persistence	0.97	Bils, Chang & Kim (2011)
σ_{ϵ_x}	Std. dev. of innovation to $\ln x$	0.23	Mean-min wage ratio
p_U	Contact probability (U)	0.22	Average nonemployment rate
p_s	Exo separation probability	0	Bils, Chang & Kim (2011)
p_E	Contact probability (E)	0.059	E-E flow probability
b	Value of leisure	2.1 ($0.37 \times APL$)	E-N flow probability
x_0	Match quality in first jobs	$\bar{x} \approx 1.6$	Bils, Chang & Kim (2011)

Note: Calibrated parameters of the model without match quality at monthly frequency. Parameters above the dashed line are associated with the model that has endogenous contact rates. Parameters below the dashed line are associated with the model that has exogenous contact rates.

Table 6: Steady-State Features of Model Without Match Quality

Moment	Data	Model
Average nonemployment rate	6%	6%
E-N flow probability	1.4%	1.4%
E-E flow probability	1.7%	1.7%
Std. dev. of (ln) wages w/in matches	19%	55%
Mean-min wage ratio	1.75	1.75

Note: See Table 5 for parameter values for the calibration.

A Numerical Details for the Aggregate Economy

A.1 Numerical Solution to the Model with Match Quality

This appendix details how to solve for the steady state and transitions for the model with match quality.

A.1.1 Steady-State Algorithm: Model with Match Quality

In order to solve the steady state, I start with an economy with aggregate productivity $z = z_1$.[19] Furthermore, as in any standard Mortensen-Pissarides-style model, c can be chosen to normalize steady-state v. I choose c so that the vacancy rate is normalized to one (1) in the steady state. The algorithm for finding the steady state is as follows:

1. Pick the efficiencies of matching, m_0^U and m_0^E to target contact probabilities p_T^U and p_T^E that ensure the correct U-E and E-E transition probabilities, respectively,[20] i.e., $m_0^i = \frac{p_T^i}{v_{SS}^\alpha} = \frac{p_T^i}{1} = p_T^i$, $i \in \{U, E\}$.

2. Solve the match surplus using equation (1). Be sure to use the targeted contact probability p_T^i.

3. Use iteration on the flows into and out of each grid point (x, y) and unemployment to pin down the steady-state distribution of (x, y) among employed workers $\hat{\pi}(x, y)$, and the steady-state unemployment rate $\hat{u}$. See A.1.2 for details.

4. Notice that by choosing m_0^i to target a contact rate, and normalizing v_{SS} to one, the vacancy creation condition implies that the vacancy posting cost satisfies:

$$c = \delta q_U u[1 - q_E(1-u)]\mathbb{E}_u + \delta q_E(1-u)[1 - q_U u]\mathbb{E}_e + \delta q_E(1-u) q_U u \mathbb{E}_{u,e} \tag{25}$$

   Use $\hat{\pi}(x, y)$ and $\hat{u}$ to obtain the right hand side.

The calibration is performed using simulated data and interpolation, so that any value of the state variables is admissible. This is because calibration on a grid results in discontinuous jumps in the simulated moments with continuous movements in the model's parameters. However, the steady-state distribution of x and y is computed on a fixed grid with no interpolation, via flows into and out of these grid points. It is reassuring that when simulation

[19] In practice, $z_1 = 1$.

[20] In practice, this means targeting contact probabilities of around $p_T^U = 0.22$ and $p_T^E = 0.20$, as presented in Table 1.

does occur on a fixed grid, all the moments are identical to the flows approach, which is simulation-free. Moreover, if one increases the number of grid points with the flows approach, one asymptotes to the moments of the simulated data with interpolation. With the number of grid points used, the two approaches render very similar steady-state features.

I use the Rouwenhorst (1995) procedure to discretize the x process on 49 grid points, and to discretize y on 29 grid points for the value function iteration problem. I use a finer grid for x when calculating the steady-state distribution $\Pi(x, y)$. Moreover, I make the grid much finer around the threshold productivity value associated with y_0 because this is where most of the action occurs when aggregate productivity changes.

A.1.2 Flows: Model with Match Quality

Before I proceed to outlining the algorithm used for computing transitions, I want to describe the set of flow balance equations used to solve for steady state $\hat{\pi}(x, y)$ and $\hat{u}$ in the preceding section. I omit the notation of the dependence of the surplus equation on the aggregate productivity, z. The outflows from the unemployment pool include:

- Unemployed agents who find new jobs:

$$up_U$$

The inflows into the unemployment pool include:

- Endogenous separations from movements in x for those with no outside offer:

$$(1 - p_E)(1 - p_s) \sum_x \sum_y \mathbb{I}\{S(x, y) > 0\}\tilde{\pi}(x, y)\mathbb{I}\{S(x', y) \leq 0\}\mathbb{P}[x'|x]$$

- Endogenous separations for those with an outside offer, but the value of neither the outside offer nor the current offer exceeds unemployment:

$$p_E(1 - p_s) \sum_x \sum_y \mathbb{I}\{S(x, y) > 0\}\tilde{\pi}(x, y)\mathbb{I}\{\max\{S(x_{EE}, \tilde{y}), S(x', y)\} \leq 0\}\mathbb{P}_x[x'|x]\mathbb{P}_y(\tilde{y})$$

- Endogenous separations from movements in aggregate productivity:

$$(1 - p_s)\tilde{\pi}(x, y)\mathbb{I}\{S(x, y) < 0\}$$

- Exogenous separations for all the employed:

$$(1-u)p_s$$

The outflows from a particular (x, y) grid point of the distribution $\tilde{\pi}(x, y)$ include:

- Exogenous separations:

$$\tilde{\pi}(x, y)p_s$$

- Those that receive no outside offer, and have their idiosyncratic component change to $x' \neq x$ (including unemployment):

$$\mathbb{I}\{S(x, y) > 0\}(1-p_E)(1-p_s)\tilde{\pi}(x, y)(1 - \mathbb{P}[x' \neq x|x])$$

- Those that receive an outside offer and leave the current firm (including to nonemployment):

$$p_E(1-p_s)\sum_x\sum_y \mathbb{I}\{S(x, y) > 0\}\tilde{\pi}(x, y)\mathbb{I}\{S(x_{EE}, \tilde{y}) > S(x', y)\}\mathbb{P}_x(x'|x)\mathbb{P}_y(\tilde{y})$$

- Those that receive an outside offer and either stay at the current firm with a new x or move to unemployment:

$$p_E(1-p_s)\sum_x\sum_y \mathbb{I}\{S(x, y) > 0\}\tilde{\pi}(x, y)\mathbb{I}\{S(x', y) > S(x_{EE}, \tilde{y})\&x' \neq x\}\mathbb{P}_x(x'|x)\mathbb{P}_y(\tilde{y})$$

- Endgeonous separations from movements in aggregate productivity:

$$\tilde{\pi}(x, y)\mathbb{I}\{S(x, y) < 0\}$$

The inflows into a particular (x, y) grid point of the distribution $\tilde{\pi}(x, y)$ include:

- Inflow from all other cells by a change in x, with no outside offer ensuring that the current (x, y)-cell has positive surplus:

$$(1-p_E)(1-p_s)\sum_{x_i \neq x} \mathbb{I}\{S(x, y) > 0\}\tilde{\pi}(x_i, y)\mathbb{P}_x[x' = x|x_i]$$

- Inflow from all other cells by a change in x, with an outside offer that is rejected ensuring that the current (x, y)-cell has positive surplus:

$$p_E(1-p_s)\sum_{x_i \neq x}\sum_{\tilde{y}} \mathbb{I}\{S(x,y)>0\}\tilde{\pi}(x_i,y)\mathbb{I}\{\max\{0,S(x_{EE},\tilde{y})\} \leq S(x,y)\}\}\mathbb{P}_x[x'=x|x_i]$$

As one special case, the inflow from job changers, i.e., those that get good outside offers (y_i) and switch to current cell. This only happens for idiosyncratic productivity $x = x_{EE}$:

$$\begin{aligned}&p_E(1-p_s)\sum_{x_i}\sum_{y} \mathbb{I}\{S(x_{EE},y_i)>0\}\tilde{\pi}(x_i,y)\times\\ &\mathbb{I}\{S(x_{EE},y_i) \geq S(x,y)\&(x \neq x_0|y_i \neq y)\}\mathbb{P}_x[x'=x|x_i]\mathbb{P}_y[\tilde{y}=y_i|y]\end{aligned}$$

As another special case, the inflow from nonemployment, into cell (x_0, y_0):

$$up_U$$

Iterating on these flows yields the steady-state distribution $\hat{\pi}(x, y)$ and $\hat{u}$ for corresponding value function $\hat{S}(x, y)$.

A.1.3 Transition Algorithm: Model with Match Quality

This appendix details the approach taken to solve the impulse response of the economy to an aggregate productivity shock for the model with match quality with two state variables (x and y). The aggregate productivity process is assumed to follow an AR(1) process.

Once I have the steady state on a grid, I employ the algorithm for aggregate productivity fluctutations. Writing the surplus equation down explicitly in a recursive, *stationary* way yields:

$$\begin{aligned}S(x,y,z,Y,X,v,\Omega) &= zxy+\delta(1-p_E(v))(1-p_s)\int\int \max\{0,S(x',y,z',v'(\Pi',z'),\Omega)\}dF_x(x'|x)dF_z(z'|z)\\ &+\delta p_E(v)(1-p_s)\int\int\int \mathbb{I}\{S(x',y,z',v'(\Pi',z'),\Omega)\geq S(x_{EE},\tilde{y},z',v'(\Pi',z'),\Omega)\}\max\{0,S(x',y,z',v',\Omega)\}\\ &+\mathbb{I}\{S(x',y,z',v'(\Pi',z'),\Omega)<S(x_{EE},\tilde{y},z',v'(\Pi',z'),\Omega)\}\max\{0,\beta S(x_{EE},\tilde{y},z',v'(\Pi',z'),\Omega)\}dF_x(x'|x)dF_y(\tilde{y})dF_z(z'|z)\\ &-[b+\delta p_U(v)\beta\int \max\{0,S(x_{EE},y_0,z',v'(\Pi',z'),\Omega)\}dF_z(z'|z)]\end{aligned} \tag{26}$$

where z is the aggregate productivity, v is the vacancy rate, Ω stands for all the constants, and the laws of motion satisfy:

$$\ln x' = \rho_x \ln x + \epsilon'_x \tag{27}$$

$$\ln y' = \begin{cases} \ln y_0 & \text{for jobs out of unemployment (U} \to \text{E)} \\ \ln y & \text{if no job change} \\ \epsilon'_y & \text{if changes jobs (E} \to \text{E)} \end{cases} \tag{28}$$

$$\ln z' = \rho_z \ln z + \epsilon'_z \tag{29}$$

$$\Pi' = \Gamma(\Pi, z') \tag{30}$$

The surplus equation depends on v through the contact rate. Γ is the equation of motion for the endogenous distribution of x and y among employed workers, $\Pi(x, y)$.

The vacancy creation condition can be written as:

$$\begin{aligned} V(z, v, \Pi, \Omega) = &-c + \delta q_U(v)u[1 - q_E(v)(1-u)](1-\beta) \int \max\{0, S(x_0, y_0, z', v'(\Pi', z'), \Omega)\} dF_z(z'|z) \\ &+ \delta q_E(v)(1-u)(1 - q_U(v)u) \int_{x'} \int_{\tilde{y}} \int_{z'} \int_x \int_y \Big[\mathbb{I}\{S(x_{EE}, \tilde{y}, z', v'(\Pi', z'), \Omega) \geq S(x', y, z', v'(\Pi', z'), \Omega)\} \\ &\max\{0, (1-\beta) S(x_{EE}, \tilde{y}, z', v'(\Pi', z'), \Omega)\} \Big] dF_x(x'|x) dF_y(\tilde{y}) dF_z(z'|z) d\Pi(x, y) \\ &+ \delta q_E(v)(1-u) q_U(v) u \int_{x'} \int_{\tilde{y}} \int_{z'} \int_x \int_y \Big[\mathbb{I}\{S(x_{EE}, \tilde{y}, z', v'(\Pi', z'), \Omega) \geq S(x', y, z', v'(\Pi', z'), \Omega)\} \\ &(1-\beta) \max\{S(x_0, y_0, z', v'(\Pi', z'), \Omega), S(x_{EE}, \tilde{y}, z', v'(\Pi', z'), \Omega)\} \\ &+ \mathbb{I}\{S(x', y, z', v'(\Pi', z'), \Omega) > S(x_{EE}, \tilde{y}, z', v'(\Pi', z'), \Omega)\} \\ &(1-\beta) \max\{0, S(x_0, y_0, z', v'(\Pi', z'), \Omega)\} \Big] dF_x(x'|x) dF_y(\tilde{y}) dF_z(z'|z) d\Pi(x, y) \end{aligned} \tag{31}$$

Notice that since V is a function of (z, v, Π, Ω), the solution to this equation, v, is a function of (Π, z, Ω). The difficult part is that in order to solve the vacancy creation condition for v, the firm must forecast the entire $\Pi(x, y)$ distribution. This is shown by the dependence of v' on Π'. Since Π is infinite-dimensional, the exact equilibrium is not computable. This is where the idea of Krusell and Smith (1998) enters.

Instead of forecasting the entire distribution, I assume that firms only use certain moments of this distribution for their decisions. I assume that firms condition their decisions on the log linear rule:

$$\ln v' = v_0 + v_X \ln X' + v_Y \ln Y' + v_z \ln z' \tag{32}$$

where X is the average idiosyncratic productivity among employed workers, and Y is the average match quality among employed workers. Hence, to forecast vacancies, one must forecast aggregate productivity, average idiosyncratic productivity among employed workers, and average match quality among employed workers. Notice that I have eliminated the need for the firm to forecast the entire Π distribution with this assumption. I have a law of motion for aggregate productivity and I assume that firms use log linear autoregressive rules for the average quantities:

$$\ln X' = \chi_0 + \chi_X \ln X + \chi_Y \ln Y + \chi_z \ln z'$$
$$\ln Y' = \nu_0 + \nu_X \ln X + \nu_Y \ln Y + \nu_z \ln z'$$

Notice that with the conjectured forecasting rule, the problem has different state variables and can be rewritten as:

$$\begin{aligned}
&S(x, y, z, Y, X, v, \Omega) = zxy \\
&+ \delta(1 - p_E(v))(1 - p_s) \int\int \max\{0, S(x', y, z', Y', X', v', \Omega)\} dF_x(x'|x) dF_z(z'|z) \\
&+ \delta p_E(v)(1 - p_s) \int\int\int \Big[\mathbb{I}\{S(x', y, z', Y', X', v', \Omega) \geq S(x_{EE}, \tilde{y}, z', Y', X', v', \Omega)\} \\
&\max\{0, S(x', y, z', Y', X', v', \Omega)\} + \mathbb{I}\{S(x', y, z', Y', X', v', \Omega) < S(x_{EE}, \tilde{y}, z', Y', X', v', \Omega)\} \\
&\max\{0, \beta S(x_{EE}, \tilde{y}, z', Y', X', v', \Omega)\} dF_x(x'|x) dF_y(\tilde{y}) dF_z(z'|z)\Big] \\
&- [b + \delta p_U(v)\beta \int \max\{0, S(x_{EE}, y_0, z', Y', X', v', \Omega)\} dF_z(z'|z)]
\end{aligned} \tag{33}$$

where $Y' = Y'(z', Y, X)$, $X' = X'(z', Y, X)$, $v' = v'(z', Y, X)$, but I have omitted this dependence in the equation for brevity. The problem is now conditioned on the current aggregate state, which is summarized by (z, Y, X, v). Note, however, that for any given (x', y'), the forward value may be predicted from (z', Y, X); the future value of v, the future value of the average x and average y among the employed are summarized entirely by this triple (z', Y, X).

Similarly, the vacancy creation condition can now be rewritten as:

$$\begin{aligned}
V(z, Y, X, v, \Omega) = &-c + \delta q_U(v) u[1 - q_E(v)(1 - u)](1 - \beta) \int \max\{0, S(x_0, y_0, z', Y', X', v', \Omega)\} dF_z(z'|z) \\
&+ \delta q_E(v)(1 - u)(1 - q_U(v)u) \int_{x'}\int_{\tilde{y}}\int_{z'}\int_x\int_y \Big[\mathbb{I}\{S(x_{EE}, \tilde{y}, z', Y', X', v', \Omega) \geq S(x', y, z', Y', X', v', \Omega)\} \\
&\max\{0, (1 - \beta) S(x_{EE}, \tilde{y}, z', Y', X', v', \Omega)\}\Big] dF_x(x'|x) dF_y(\tilde{y}) dF_z(z'|z) d\Pi(x, y) \\
&+ \delta q_E(v)(1 - u) q_U(v) u \int_{x'}\int_{\tilde{y}}\int_{z'}\int_x\int_y \Big[\mathbb{I}\{S(x_{EE}, \tilde{y}, z', Y', X', v', \Omega) \geq S(x', y, z', Y', X', v', \Omega)\} \\
&(1 - \beta) \max\{S(x_0, y_0, z', Y', X', v', \Omega), S(x_{EE}, \tilde{y}, z', Y', X', v', \Omega)\} \\
&+ \mathbb{I}\{S(x', y, z', Y', X', v', \Omega) > S(x_{EE}, \tilde{y}, z', Y', X', v', \Omega)\} \\
&\max\{0, (1 - \beta) S(x_0, y_0, z', Y', X', v', \Omega)\}\Big] dF_x(x'|x) dF_y(\tilde{y}) dF_z(z'|z) d\Pi(x, y)
\end{aligned} \tag{34}$$

So, the algorithm can be neatly summarized as follows:

1. Parameterize your guess for the average x and y among employed workers, and the vacancy rate using log linear equations:

$$\ln X' = \chi_0 + \chi_X \ln X + \chi_Y \ln Y + \chi_z \ln z' \tag{35}$$

$$\ln Y' = \nu_0 + \nu_X \ln X + \nu_Y \ln Y + \nu_z \ln z' \tag{36}$$

$$\ln v' = v_0 + v_X \ln X' + v_Y \ln Y' + v_z \ln z' \tag{37}$$

   where I assume that aggregate productivity takes a log AR(1) form: $\ln z' = \rho_z \ln z + \epsilon'_z$.

2. Set an initial guess for the set of coefficients $\{\chi_i^0, \nu_i^0, v_i^0\}$.

3. Set a guess of the surplus function $S^0(x, y, z, Y, X, v, \Omega)$, which will be used in the vacancy creation condition.

4. Update the surplus equation using the following Bellman operator:[21]

$$\begin{aligned}
&S(x, y, z, Y, X, v, \Omega) = zxy \\
&+ \delta(1 - p_E(v))(1 - p_s) \int\int \max\{0, S(x', y, z', Y', X', v', \Omega)\} dF_x(x'|x) dF_z(z'|z) \\
&+ \delta p_E(v)(1 - p_s) \int\int\int \Big[\mathbb{I}\{S(x', z', Y', X', v', \Omega) \geq S(x_{EE}, \tilde{y}, z', Y', X', v', \Omega)\} \\
&\max\{0, S(x', z', Y', X', v', \Omega)\} \\
&+ \mathbb{I}\{S(x', z', Y', X', v', \Omega) < S(x_{EE}, \tilde{y}, z', Y', X', v', \Omega)\} \\
&\max\{0, \beta S(x_{EE}, \tilde{y}, z', Y', X', v', \Omega)\} \Big] dF_x(x'|x) dF_y(\tilde{y}) dF_z(z'|z) \\
&- [b + \delta p_U(v) \beta \int \max\{0, S(x_0, y_0, z', Y', X', v', \Omega)\} dF_z(z'|z)]
\end{aligned} \tag{38}$$

5. Compare the update with the guess and stop if tolerance is satisfied. Update guess if not at desired tolerance.

6. Simulate the economy. Set the length of the simulation, T. Draw a sequence of aggregate productivities $\{z_t\}_{t=1}^T$ using a random number generator.[22] Set the initial

[21] Do not need to integrate out over X and Y because you have the forecast equation.

[22] I use the same time path of aggregate productivity for all iterations to remove error associated with this aspect of the problem.

distribution of (x, y) among the employed using the steady-state distribution as the initial distribution.

7. Compute the implied realization of vacancies, $\tilde{v}_t$, using the vacancy creation condition (equation (34)). You have current z, current $\Pi(x, y)$, current u as well as v' from the forecast rule for every future z' current X, and current Y. You can evaluate the surplus equation at (z', v') for any given (x', y').

8. Update the distribution to get Π_{t+1} and u_{t+1}.

9. If the simulation has reached the final period, stop and go to the next step. Otherwise, go back to step 7 with the updated distribution and z'.

10. Drop the first T^0 observations of the simulated series of $\{X_t\}_{t=1}^T$, $\{Y_t\}_{t=1}^T$ and $\{v_t\}_{t=1}^T$ to randomize the initial conditions.[23] Run OLS regressions of the form (35), (36) and (37) using the simulated time series. Let $\{\chi_i^1, v_i^1\}$ be the new set of coefficients.

11. Compare $\{\chi_i^0, \nu_i^0, v_i^0\}$ and $\{\chi_i^1, \nu_i^1, v_i^1\}$. If close, stop; otherwise, update coefficients and go back to step 3.

12. If the coefficients do not converge, or the fit of the regression is not high enough, it is necessary to change the functional forms for the forecast equations or increase the set of statistics that we use to simplify the problem.

13. Use the simulated data to compute the elasticities of labor market variables with respect to output per worker.

When computing the surplus equation, I use 49 grid points in the x dimension, 29 grid points in the y dimension, 5 grid points in the X and Y dimensions, 9 grid points in the v dimension, and 11 grid points in the z direction.

For the calibrated parameters, the equilibrium forecast equations are as follows:

$$\begin{aligned}
\ln X' &= -0.0008 + 0.9987 \ln X + 0.0009 \ln Y - 0.0002 \ln z', R^2 = 0.9960 \\
\ln Y' &= 0.0628 - 0.0818 \ln X + 0.9470 \ln Y - 0.0041 \ln z', R^2 = 0.9859 \\
\ln v' &= 2.3986 - 0.2050 \ln X' - 2.4703 \ln Y' + 1.2666 \ln z', R^2 = 0.9969
\end{aligned} \tag{39}$$

I use $\rho_z = 0.983$ and $\sigma_{\epsilon_z} = 0.005$. This calibration of the monthly productivity process yields an HP detrended series using a 10^5 smoothing parameter that has persistence 0.89

[23] In practice, T^0 is 100 years for the full model and 50 years for the simpler model, and T is 500 years for the full model and 450 for the simpler model. Adding more time periods did not affect the results.

and standard deviation 0.015, very close to the empirical counterpart in the US: 0.88 and 0.02, respectively.

The above procedure is going to give me the coefficients consistent with aggregate dynamics, which I can use to simulate impulse response functions. The permanent reduction in aggregate productivity involves simulating the economy with the above forecast rules at the original aggregate productivity for some time, starting in the steady state. This is to ensure that the economy settles down, and because of a stochastic productivity process, this might not be at exactly the steady state. Once the economy settles down, I reduce aggregate productivity by 1 percent and plot out the impulse response functions as percent deviations from this initial stochastic equilibrium. One other complication arises because the aggregate productivity will now fall off its initial grid. In order to deal with this, I use Gauss hermite quadrature nodes to integrate out future aggregate productivities, conditional on today's aggregate productivity. Since I assumed that aggregate productivity followed an AR(1) process with a normal shock, I have the mean and variance of the distribution for tomorrow's aggregate productivity. This is all I need to compute the integral.[24]

A.2 Numerical Solution to the Model Without Match Quality

This appendix details how to solve for the steady state and transition of the model without match quality.

A.2.1 Steady-State Algorithm: Model Without Match Quality

The approach here is the same as that used for the model with match quality; however, it is easier in application because of the reduced state space. The algorithm for finding the original steady state is as follows:

1. Pick the efficiencies of matching, m_0^i to target contact probabilities $p_U(v)$ and $p_E(v)$ that ensure the correct U-E and E-E probabilities,[25] i.e., $m_0^i = \frac{p_T^i}{v_{SS}^\alpha} = \frac{p_T^i}{1} = p_T^i, i \in \{U, E\}$.

2. Solve the match surplus using equation (16). Be sure to use the targeted contact probability p_T^i.

[24] See, for example, Judd (1998), Ch. 7.

[25] In practice, this means targeting a job-finding rate for the unemployed of $p_U(v) = 0.22$ and for the employed of $p_E(v) = 0.059$, as presented in Table 5.

3. Use iteration on the flows into and out of each grid point x and unemployment to pin down the steady-state distribution of x among employed workers $\hat{\pi}(x)$, and the steady-state unemployment rate $\hat{u}$. See Appendix A.2.2 for details.

4. Notice that by choosing m_0^i to target a contact rate, and normalizing v_{SS} to one, the vacancy creation condition implies that the vacancy posting cost satisfies:[26]

$$c = \delta q_U u[1 - q_E(1-u)]\mathbb{E}_u^s + \delta q_E(1-u)[1 - q_U u]\mathbb{E}_e^s + \delta q_E(1-u) q_U u \mathbb{E}_{u,e}^s \tag{40}$$

The calibration is performed using simulated data and interpolation, so that any value of the state variable is admissible. This is because calibration on a grid results in discontinuous jumps in the simulated moments with continuous movements in the model's parameters. However, the steady-state distribution of x is computed on a fixed grid with no interpolation, via flows into and out of these grid points. It is reassuring that when simulation does occur on a fixed grid, all the moments are identical to the flows approach, which is simulation-free. Moreover, if one increases the number of grid points with the flows approach, one asymptotes to the moments of the simulated data with interpolation. With the number of grid points used, the two approaches render very similar steady-state features.

I use the Rouwenhorst (1995) procedure to discretize the x process on 49 grid points for the value function iteration problem. I use a finer grid when calculating the steady-state distribution $\Pi(x)$. Moreover, I make the grid much finer around the threshold productivity value, x_R. Both of these measures serve to capture accurately the movements in the aggregate moments, especially the E-U probability, when aggregate productivity moves around.

A.2.2 Flows: Model Without Match Quality

Before I proceed to outlining the algorithm used for computing transitions, I want to describe the set of flow balance equations used to solve for steady state $\hat{\pi}(x)$ and $\hat{u}$ in the preceding section. Here I omit the notation for the dependence of the surplus equation on aggregate productivity for convenience. The outflows from the unemployment pool include:

[26] Usually, researchers in this literature normalize labor market tightness to 0.72 so that c/q equals 14 percent of quarterly worker compensation, which is in accordance with the results of Silva and Toledo (2007), who use the Saratoga Institute's (2004) estimate of the labor costs of posting vacancies. In this work the labor market tightness normalization is simply rigged up so that c/q turns out to be 14 percent of quarterly worker compensation. In the present setup I could normalize labor market tightness in the steady state so that c/q was also 14 percent of quarterly worker compensation.

- Unemployed agents who find new jobs:

$$up_U$$

The inflows into the unemployment pool include:

- Endogenous separations from movements in x when a worker has no outside offer:[27]

$$(1-p_E)(1-p_s)\sum_x \mathbb{I}\{S(x)>0\}\tilde{\pi}(x)\mathbb{I}\{S(x')\leq 0\}\mathbb{P}[x'|x]$$

- Endogenous separations from movements in x when a worker has an outside offer:

$$p_E(1-p_s)\sum_x \mathbb{I}\{S(x)>0\}\tilde{\pi}(x)\mathbb{I}\{\max\{S(x'),S(\tilde{x})\}\leq 0\}\mathbb{P}[x'|x]\mathbb{P}[\tilde{x}]$$

- Endogenous separations from movements in aggregate productivity:

$$(1-p_s)\tilde{\pi}(x)\mathbb{I}\{S(x)<0\}$$

- Exogenous separations:

$$(1-u)p_s$$

The outflows from a particular (x) grid point of the distribution $\tilde{\pi}(x)$ include:

- Employed agents who experience the exogenous separation shock:

$$\tilde{\pi}(x)p_s$$

- Those who have their idiosyncratic component change to $x'\neq x$ (including unemployment):

$$\mathbb{I}\{S(x)>0\}\tilde{\pi}(x)(1-p_E)(1-p_s)(1-\mathbb{P}[x'\neq x|x])$$

- Those who receive an outside offer that is better than their current job:

$$p_E(1-p_s)\sum_x \mathbb{I}\{S(x)>0\}\tilde{\pi}(x)\mathbb{I}\{S(\tilde{x})>S(x')\}\mathbb{P}[x'|x]\mathbb{P}[\tilde{x}]$$

[27] Since $\tilde{\pi}(x)$ is the unconditional (not conditional on employment) probability of being at x, this quantity does not need to be multiplied by $(1-u)$. Also, all these flows are performed on a fixed grid of x. The summations represent discretized integration.

- Those who receive an unacceptable outside offer but x changes at the current job:

$$p_E(1-p_s)\sum_x \mathbb{I}\{S(x)>0\}\tilde{\pi}(x)\mathbb{I}\{[S(x')\geq S(\tilde{x})]\&[x'\neq x]\}\mathbb{P}[x'|x]\mathbb{P}[\tilde{x}]$$

- Endogenous separations from movements in aggregate productivity:

$$\tilde{\pi}(x)\mathbb{I}\{S(x)<0\}$$

The inflows into a particular (x) grid point of the distribution $\tilde{\pi}(x)$ include:

- Inflow from all other cells by a change in x for those without an outside offer:

$$(1-p_E)(1-p_s)\sum_{x_i\neq x}\mathbb{I}\{S(x)>0\}\tilde{\pi}(x_i)\mathbb{P}_x[x'=x|x_i]$$

- Inflow from all other cells by a change in x for those with an unacceptable outside offer:

$$p_E(1-p_s)\sum_{x_i\neq x}\mathbb{I}\{S(x)>0\}\mathbb{I}\{[S(x)>S(\tilde{x})]\&[S(x_i)>0]\}\tilde{\pi}(x_i)\mathbb{P}_x[x'=x|x_i]$$

- Inflow from job changers:

$$p_E(1-p_s)\sum_{x_i}\mathbb{I}S(x_i)>0\}\mathbb{I}\{S(\tilde{x})>\max\{0,S(x)\}\}\tilde{\pi}(x_i)\mathbb{P}_x[x'=x|x_i]$$

As one special case, the inflow from unemployment, into cell (x_0):

$$up_U\mathbb{I}\{S(x_0)>0\}$$

I also ensure that cells associated with negative surplus have zero mass.

Iterating on these flows yields the steady-state distribution $\hat{\pi}(x)$ and $\hat{u}$ for corresponding value function $\hat{S}(x)$.

A.2.3 Transition Algorithm: Model Without Match Quality

See the description of the transition solution for the model with match-quality as described in Appendix A.1.3. The approach here is virtually identical. A few points are worth mentioning. First, v_t must satisfy the following equation:

$$
\begin{aligned}
&V^s(z,v,X,\Omega) = -c + \delta q_U(v)u[1-q_E(1-u)](1-\beta)\int \max\{0, S(x_0,z',X',v',\Omega)\}dF_z(z'|z) \\
&+ \delta q_E(v)(1-u)[1-q_U u]\int\int\int\int \Big[\mathbb{I}\{S(\tilde{x},z',X',v',\Omega) \geq S(x',z',X',v',\Omega)\} \\
&\max\{0,(1-\beta)S(\tilde{x},z',X',v',\Omega)\}\Big] dF_x(x'|x)dG(\tilde{x})dF_z(z'|z)d\Pi(x) \\
&+ q_E(v)(1-u)q_U(v)u\int\int\int\int \Big[\mathbb{I}\{S(\tilde{x},z',X',v',\Omega) \geq S(x',z',X',v',\Omega)\} \\
&(1-\beta)\max\{S(x_0,z',X',v',\Omega), S(\tilde{x},z',X',v',\Omega)\} \\
&+ \mathbb{I}\{S(x',z',X',v',\Omega) > S(\tilde{x},z',X',v',\Omega)\}(1-\beta)\max\{0,S(x_0,z',X',v',\Omega)\}\Big] dF_x(x'|x)dG(\tilde{x})dF_z(z'|z)d\Pi(x)
\end{aligned}
$$

where $X' = X'(z', X)$ and $v' = v'(z', X)$, as opposed to equation (34), for all t. Second, the surplus equation must satisfy:

$$
\begin{aligned}
S(x,z,X,v,\Omega) = {} & z\cdot x + \delta(1-p_E(v))(1-p_s)\int\int \max\{0,S(x',z',X',v',\Omega)\}dF_x(x'|x)dF_z(z'|z) \\
& + \delta p_E(v)(1-p_s)\int\int\int \Big[\mathbb{I}\{S(x',z',X',v',\Omega) \geq S(\tilde{x},z',X',v',\Omega)\}\max\{0,S(x',z',X',v',\Omega)\} \\
& + \mathbb{I}\{S(\tilde{x},z',X',v',\Omega) > S(x',z',X',v',\Omega)\}\max\{0,\beta S(\tilde{x},z',X',v',\Omega)\}\Big] dF_x(x'|x)dG(\tilde{x})dF_z(z'|z) \\
& - [b + \delta p_U(v)\beta S(x_0,z',X',v',\Omega)]
\end{aligned}
$$

Other than that, the approach here is similar, but simpler, because the steady-state distribution depends on only one state variable.

For the calibrated parameters, the equilibrium forecast equations are as follows:

$$
\begin{aligned}
\ln X' &= 0.0780 + 0.9545\ln X + 0.0189\ln z', R^2 = 0.9993 \\
\ln v' &= 0.2482 - 0.1754\ln X + 1.7876\ln z', R^2 = 0.9998
\end{aligned} \tag{41}
$$