

Supplemental Appendix: Assessing International Commonality in Macroeconomic Uncertainty and Its Effects^{*}

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1 Introduction

This supplemental appendix provides details and results omitted from the paper for space considerations. Section 2 details the MCMC algorithm for the BVAR-GFSV model used to produce the paper’s main results. Section 3 provides priors for this model and spells out the BVAR-SV and BVAR models and priors used in some results in the paper. Section 4 details the historical decomposition used with the BVAR-GFSV estimates. Section 5 summarizes estimates of the correlations of our uncertainty shocks with “known” macroeconomic shocks for the U.S. Section 6 summarizes a robustness check of extending the estimation sample for the GDP-only dataset back in time. The supplement concludes with some additional charts and tables of results mentioned in the paper.

2 MCMC Algorithm for BVAR-GFSV Model

In detailing the algorithm in this appendix, for simplicity we present the more general version with the time-varying idiosyncratic volatility component and then indicate simplifications associated with treating the idiosyncratic component as constant. For simplicity, we describe the computations for a one-factor specification; the second factor is handled with the same basic approach.

For the convenience of self-containment, we begin by repeating the equations of the model:

$$v_t = A^{-1}\Lambda_t^{0.5}\epsilon_t, \quad \epsilon_t \sim iid \ N(0, I), \quad (1)$$

$$\ln \lambda_{i,t} = \beta_{m,i} \ln m_t + \ln h_{i,t}, \quad i = 1, \dots, n, \quad (2)$$

$$\ln m_t = \sum_{i=1}^{p_m} \delta_{m,i} \ln m_{t-i} + \delta'_{m,y} y_{t-1} + u_{m,t}, \quad u_{m,t} \sim iid \ N(0, \phi_m), \quad (3)$$

$$\ln h_{i,t} = \gamma_{i,0} + \gamma_{i,1} \ln h_{i,t-1} + e_{i,t}, \quad i = 1, \dots, n, \quad (4)$$

$$y_t = \sum_{i=1}^p \Pi_i y_{t-i} + \sum_{i=0}^{p_m} \Pi_{m,i} \ln m_{t-i} + v_t. \quad (5)$$

Our exposition of priors, posteriors, and estimation makes use of the following additional notation. Let Π denote the collection of the VAR’s coefficients. The vector a_j , $j = 2, \dots, n$, contains the j^{th} row of the matrix A (for columns 1 through $j - 1$). We define the vector $\gamma = \{\gamma_1, \dots, \gamma_n\}$ as the set of coefficients appearing in the conditional means of the transition equations for the states $h_{1:T}$, and $\delta = \{D(L), \delta'_m\}$ as the set of the coefficients in

the conditional mean of the transition equation for the states $m_{1:T}$. The coefficient matrices Φ_v and Φ_u collect the variances of the shocks to the transition equations for the idiosyncratic states $h_{1:T}$ and the common uncertainty factor $m_{1:T}$; for identification, the value of Φ_u is fixed. In addition, we group the parameters of the model in (1)-(5), except the vector of factor loadings β , into $\Theta = \{\Pi, A, \gamma, \delta, \Phi_v, \Phi_u\}$. Finally, let $s_{1:T}$ denote the time series of the mixture states used (as explained below) to draw $h_{1:T}$.

We use an MCMC algorithm to obtain draws from the joint posterior distribution of model parameters Θ , loadings β , and latent states $h_{1:T}$, $m_{1:T}$, $s_{1:T}$. Specifically, we sample in turn from the following two conditional posteriors (for simplicity, we suppress notation for the dependence of each conditional posterior on the data sample $y_{1:T}$): (1) $h_{1:T}, \beta \mid \Theta, s_{1:T}, m_{1:T}$, and (2) $\Theta, s_{1:T}, m_{1:T} \mid h_{1:T}, \beta$.

The first step relies on a state space system. Defining the rescaled residuals $\tilde{v}_t = Av_t$, taking the log squares of (1), and subtracting out the known (in the conditional posterior) contributions of the common factors yields the observation equations ($\bar{c}$ denotes an offset constant used to avoid potential problems with near-zero values):

$$\ln(\tilde{v}_{j,t}^2 + \bar{c}) - \beta_{m,j} \ln m_t = \ln h_{j,t} + \ln \epsilon_{j,t}^2, \quad j = 1, \dots, n. \quad (6)$$

For the idiosyncratic volatility components, the transition and measurement equations of the state-space system are given by (4) and (6), respectively. The system is linear but not Gaussian, due to the error terms $\ln \epsilon_{j,t}^2$. However, $\epsilon_{j,t}$ is a Gaussian process with unit variance; therefore, we can use the mixture of normals approximation of Kim, Shephard, and Chib (1998) to obtain an approximate Gaussian system, conditional on the mixture of states $s_{1:T}$. To produce a draw from $h_{1:T}, \beta \mid \Theta, s_{1:T}, m_{1:T}$, we then proceed as usual by (a) drawing the time series of the states given the loadings using $h_{1:T} \mid \beta, \Theta, s_{1:T}, m_{1:T}$, following Del Negro and Primiceri's (2015) implementation of the Kim, Shephard, and Chib (1998) algorithm, and by then (b) drawing the loadings given the states using $\beta \mid h_{1:T}, \Theta, s_{1:T}, m_{1:T}$, using the conditional posterior detailed below in (16).

In specifications in which the idiosyncratic components $h_{1:T}$ are restricted to be constant over time, the algorithm simplifies as follows. In this case, the measurement equation (6) simplifies to

$$\ln(\tilde{v}_{j,t}^2 + \bar{c}) - \beta_{m,j} \ln m_t = \ln h_j + \ln \epsilon_{j,t}^2, \quad j = 1, \dots, n, \quad (7)$$

and we no longer have a transition equation for the idiosyncratic components. Rather, given normally distributed priors on the idiosyncratic constants of each variable and the mixture

states $s_{1:T}$ and their associated means and variances, we draw the idiosyncratic constants from a conditionally normal posterior using a GLS regression based on (7).

The second step conditions on the idiosyncratic volatilities and factor loadings to produce draws of the model coefficients Θ , common uncertainty factor $m_{1:T}$, and the mixture states $s_{1:T}$. Draws from the posterior $\Theta, s_{1:T} \mid h_{1:T}, \beta$ are obtained in three substeps from, respectively: (a) $\Theta \mid m_{1:T}, h_{1:T}, \beta$; (b) $m_{1:T} \mid \Theta, h_{1:T}, \beta$; and (c) $s_{1:T} \mid \Theta, m_{1:T}, h_{1:T}, \beta$. More specifically, for $\Theta \mid m_{1:T}, h_{1:T}, \beta$ we use the posteriors detailed below, in equations (14), (15), (17), (18), and (19). For $m_{1:T} \mid \Theta, h_{1:T}, \beta$, we use the particle Gibbs step proposed by Andrieu, Doucet, and Holenstein (2010). For $s_{1:T} \mid \Theta, m_{1:T}, h_{1:T}, \beta$, we use the 10-state mixture approximation of Omori, et al. (2007).

2.1 Coefficient Priors and Posteriors

We specify the following (independent) priors for the parameter blocks of the model:

$$\text{vec}(\Pi) \sim N(\text{vec}(\underline{\mu}_\Pi), \underline{\Omega}_\Pi), \quad (8)$$

$$a_j \sim N(\underline{\mu}_{a,j}, \underline{\Omega}_{a,j}), \quad j = 2, \dots, n, \quad (9)$$

$$\beta_{m,j} \sim N(\underline{\mu}_\beta, \underline{\Omega}_\beta), \quad j = 1, \dots, n, \quad (10)$$

$$\gamma_j \sim N(\underline{\mu}_\gamma, \underline{\Omega}_\gamma), \quad j = 1, \dots, n, \quad (11)$$

$$\delta \sim N(\underline{\mu}_\delta, \underline{\Omega}_\delta), \quad (12)$$

$$\phi_j \sim IG(d_\phi \cdot \underline{\phi}, d_\phi), \quad j = 1, \dots, n. \quad (13)$$

Under these priors, the parameters Π , A , β , γ , δ , and Φ_v have the following closed form conditional posterior distributions:

$$\text{vec}(\Pi) \mid A, \beta, m_{1:T}, h_{1:T}, y_{1:T} \sim N(\text{vec}(\bar{\mu}_\Pi), \bar{\Omega}_\Pi), \quad (14)$$

$$a_j \mid \Pi, \beta, m_{1:T}, h_{1:T}, y_{1:T} \sim N(\bar{\mu}_{a,j}, \bar{\Omega}_{a,j}), \quad j = 2, \dots, n, \quad (15)$$

$$\beta_{m,j} \mid \Pi, A, \gamma, \Phi, m_{1:T}, h_{1:T}, s_{1:T}, y_{1:T} \sim N(\bar{\mu}_\beta, \bar{\Omega}_\beta), \quad j = 1, \dots, n, \quad (16)$$

$$\gamma_j \mid \Pi, A, \beta, \Phi, m_{1:T}, h_{1:T}, y_{1:T} \sim N(\bar{\mu}_\gamma, \bar{\Omega}_\gamma), \quad j = 1, \dots, n, \quad (17)$$

$$\delta \mid \Pi, A, \gamma, \beta, \Phi, m_{1:T}, h_{1:T}, y_{1:T} \sim N(\bar{\mu}_\delta, \bar{\Omega}_\delta), \quad (18)$$

$$\phi_j \mid \Pi, A, \beta, \gamma, m_{1:T}, h_{1:T}, y_{1:T} \sim IG(d_\phi \cdot \underline{\phi} + \sum_{t=1}^T \nu_{jt}^2, d_\phi + T), \quad j = 1, \dots, n. \quad (19)$$

Expressions for $\bar{\mu}_{a,j}$, $\bar{\mu}_\delta$, $\bar{\mu}_\gamma$, $\bar{\Omega}_{a,j}$, $\bar{\Omega}_\delta$, and $\bar{\Omega}_\gamma$ are straightforward to obtain using standard results from the linear regression model. To save space, we omit details for these posteriors;

general solutions are readily available in other sources (e.g., Cogley and Sargent 2005 for $\bar{\mu}_{a,j}$).

In the posterior for the factor loadings β , the mean and variance take a GLS-based form, with dependence on the mixture states used to draw volatility. For the VAR coefficients Π , with smaller models it is common to rely on a GLS solution for the posterior mean (e.g., Carriero, Clark, and Marcellino 2016a). However, with large models it is far faster to exploit the triangularization — obtaining the same posterior provided by standard system solutions — developed in Carriero, Clark, and Marcellino (2016b) and estimate the VAR coefficients on an equation-by-equation basis.

Specifically, using the factorization given below allows us to draw the coefficients of the matrix Π in separate blocks. Let $\pi^{(j)}$ denote the j -th column of the matrix Π , and let $\pi^{(1:j-1)}$ denote all the previous columns. Then draws of $\pi^{(j)}$ can be obtained from:

$$\pi^{(j)} \mid \pi^{(1:j-1)}, A, \beta, m_{1:T}, h_{1:T}, y_{1:T} \sim N(\bar{\mu}_{\pi^{(j)}}, \bar{\Omega}_{\pi^{(j)}}), \quad (20)$$

$$\bar{\mu}_{\pi^{(j)}} = \bar{\Omega}_{\pi^{(j)}} \left\{ \sum_{t=1}^T X_t \lambda_{j,t}^{-1} y_{j,t}^{*'} + \underline{\Omega}_{\pi^{(j)}}^{-1} (\underline{\mu}_{\pi^{(j)}}) \right\}, \quad (21)$$

$$\bar{\Omega}_{\pi^{(j)}}^{-1} = \underline{\Omega}_{\pi^{(j)}}^{-1} + \sum_{t=1}^T X_t \lambda_{j,t}^{-1} X_t', \quad (22)$$

where $y_{j,t}^* = y_{j,t} - (a_{j,1}^* \lambda_{1,t}^{0.5} \epsilon_{1,t} + \dots + a_{j,j-1}^* \lambda_{j-1,t}^{0.5} \epsilon_{j-1,t})$, with $a_{j,i}^*$ denoting the generic element of the matrix A^{-1} and $\underline{\Omega}_{\pi^{(j)}}^{-1}$ and $\underline{\mu}_{\pi^{(j)}}$ denoting the prior moments on the j -th equation, given by the j -th column of $\underline{\mu}_{\Pi}$ and the j -th block on the diagonal of $\underline{\Omega}_{\Pi}^{-1}$.

2.2 Unobservable States

For the unobserved common volatility states m_t , given the law of motion in (3) and priors on the period 0 values, draws from the posteriors can be obtained using the particle Gibbs sampler of Andrieu, Doucet, and Holenstein (2010). In the particle Gibbs sampler of the uncertainty factors, we use 50 particles, which appears sufficient for efficiency and mixing.

For the unobserved idiosyncratic volatility states $h_{j,t}$, $j = 1, \dots, n$, given the law of motion for the unobservable states in (4) and priors on the period 0 values, draws from the posteriors can be obtained using the algorithm of Kim, Shephard, and Chib (1998). As noted above, in specifications in which the idiosyncratic components $h_{1:T}$ are restricted to be constant over time, the algorithm simplifies. In this case, given normally distributed priors on the idiosyncratic constants of each variable and the mixture states $s_{1:T}$ and their associated means and variances, we draw the idiosyncratic constants from a conditionally

normal posterior using a GLS regression based on (7).

2.3 Drawing the Loadings

Finally, we note that in drawing the loadings, we make use of the information in the observable $\ln(\tilde{v}_{j,t}^2)$, with the following transformation of the observation equations:

$$\ln(\tilde{v}_{j,t}^2 + \bar{c}) - \ln h_{j,t} = \beta_{m,j} \ln m_t + \ln \epsilon_{j,t}^2, \quad j = 1, \dots, n.$$

With the conditioning on $h_{1:T}$ and $s_{1:T}$ in the posterior for β , we use this equation, along with the mixture mean and variance associated with the draw of $s_{1:T}$, for sampling the factor loadings with a conditionally normal posterior with mean and variance represented in a GLS form. The same applies in the specifications in which the idiosyncratic volatilities $h_{j,t}$ are restricted to be constant over time.

2.4 Triangularization for Estimation

In this subsection we briefly summarize the VAR triangularization that is needed to handle a large system with asymmetric priors and time-varying volatilities, such as the model used here.¹ More details can be found in Carriero, Clark, and Marcellino (2016b). With the triangularization, the estimation algorithm will block the conditional posterior distribution of the system of VAR coefficients in n different blocks. In the step of the typical Gibbs sampler that involves drawing the set of VAR coefficients Π , all of the remaining model coefficients are given. Consider again the reduced-form residuals:

$$\begin{bmatrix} v_{1,t} \\ v_{2,t} \\ \dots \\ v_{n,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ a_{2,1}^* & 1 & & \dots \\ \dots & & 1 & 0 \\ a_{n,1}^* & \dots & a_{n,n-1}^* & 1 \end{bmatrix} \begin{bmatrix} \lambda_{1,t}^{0.5} & 0 & \dots & 0 \\ 0 & \lambda_{2,t}^{0.5} & & \dots \\ \dots & & \dots & 0 \\ 0 & \dots & 0 & \lambda_{n,t}^{0.5} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \dots \\ \epsilon_{n,t} \end{bmatrix}, \quad (23)$$

¹Since the triangularization obtains computational gains of order n^2 , the cross-sectional dimension of the system can be extremely large, and indeed Carriero, Clark, and Marcellino (2016b) present results for a VAR with 125 variables.

where $a_{j,i}^*$ denotes the generic element of the matrix A^{-1} , which is available under knowledge of A . The VAR can be written as:

$$\begin{aligned}
y_{1,t} &= \sum_{i=1}^n \sum_{l=1}^p \pi_{1,l}^{(i)} y_{i,t-l} + \sum_{l=0}^{p_m} \pi_{l,1}^{(m)} \ln m_{t-l} + \sum_{l=0}^{p_f} \pi_{l,1}^{(f)} \ln f_{t-l} + \lambda_{1,t}^{0.5} \epsilon_{1,t} \\
y_{2,t} &= \sum_{i=1}^n \sum_{l=1}^p \pi_{2,l}^{(i)} y_{i,t-l} + \sum_{l=0}^{p_m} \pi_{l,2}^{(m)} \ln m_{t-l} + \sum_{l=0}^{p_f} \pi_{l,2}^{(f)} \ln f_{t-l} + a_{2,1}^* \lambda_{1,t}^{0.5} \epsilon_{1,t} + \lambda_{2,t}^{0.5} \epsilon_{2,t} \\
&\dots \\
y_{n,t} &= \sum_{i=1}^n \sum_{l=1}^p \pi_{n,l}^{(i)} y_{i,t-l} + \sum_{l=0}^{p_m} \pi_{l,n}^{(m)} \ln m_{t-l} + \sum_{l=0}^{p_f} \pi_{l,n}^{(f)} \ln f_{t-l} + a_{n,1}^* \lambda_{1,t}^{0.5} \epsilon_{1,t} + \dots \\
&\dots + a_{n,n-1}^* \lambda_{n-1,t}^{0.5} \epsilon_{n-1,t} + \lambda_{n,t}^{0.5} \epsilon_{n,t},
\end{aligned}$$

with the generic equation for variable j :

$$\begin{aligned}
&y_{j,t} - (a_{j,1}^* \lambda_{1,t}^{0.5} \epsilon_{1,t} + \dots + a_{j,j-1}^* \lambda_{j-1,t}^{0.5} \epsilon_{j-1,t}) \\
&= \sum_{i=1}^n \sum_{l=1}^p \pi_{j,l}^{(i)} y_{i,t-l} + \sum_{l=0}^{p_m} \pi_{l,j}^{(m)} \ln m_{t-l} + \sum_{l=0}^{p_f} \pi_{l,j}^{(f)} \ln f_{t-l} + \lambda_{j,t} \epsilon_{j,t}.
\end{aligned} \tag{24}$$

Consider estimating these equations in order from $j = 1$ to $j = n$. When estimating the generic equation j , the term of the left-hand side in (24) is known, since it is given by the difference between the dependent variable of that equation and the estimated residuals of all the previous $j - 1$ equations. Therefore we can define:

$$y_{j,t}^* = y_{j,t} - (a_{j,1}^* \lambda_{1,t}^{0.5} \epsilon_{1,t} + \dots + a_{j,j-1}^* \lambda_{j-1,t}^{0.5} \epsilon_{j-1,t}), \tag{25}$$

and equation (24) becomes a standard generalized linear regression model for the variable in equation (25) with Gaussian disturbances with mean 0 and variance $\lambda_{j,t}$.

Accordingly, drawing on results detailed in Carriero, Clark, and Marcellino (2016b), the posterior distribution of the VAR coefficients can be factorized as:

$$\begin{aligned}
p(\Pi|A, \beta, m_{1:T}, h_{1:T}, y_{1:T}) &= p(\pi^{(n)}|\pi^{(n-1)}, \pi^{(n-2)}, \dots, \pi^{(1)}, A, \beta, m_{1:T}, h_{1:T}, y_{1:T}) \\
&\times p(\pi^{(n-1)}|\pi^{(n-2)}, \dots, \pi^{(1)}, A, \beta, m_{1:T}, h_{1:T}, y_{1:T}) \\
&\times \dots \times p(\pi^{(1)}|A, \beta, m_{1:T}, h_{1:T}, y_{1:T}),
\end{aligned} \tag{26}$$

where the vector β collects the loadings of the uncertainty factors and $m_{1:T}, h_{1:T} = (h_{1,T}, \dots, h_{n,T})$, and $y_{1:T}$ denote the history of the states and data up to time T .² As a result, we are able to

²Note we have implicitly used the fact that the matrix Ω_{Π}^{-1} is block diagonal, which is the case in our application, as our prior on the conditional mean coefficients is independent across equations, with a Minnesota-style form.

estimate the coefficients of the VAR on an equation-by-equation basis. This greatly speeds estimation and permits us to consider much larger systems than we would otherwise be able to consider.

Importantly, although the expression (23) and the following triangular system are based on a Cholesky-type decomposition of the variance Σ_t , the decomposition is simply used as an estimation device, not as a way to identify structural shocks. The ordering of the variables in the system does not change the joint (conditional) posterior of the reduced-form coefficients, so changing the order of the variables is inconsequential to the results.³ Moreover, since a shock to uncertainty is uncorrelated with shocks to the conditional mean of the variables, the ordering of the variables in the system has no influence on the shape of impulse responses in our application.

3 Prior settings

3.1 BVAR-GFSV

For the VAR coefficients contained in Π , we use a Minnesota-type prior. With the variables of interest transformed for stationarity, we set the prior mean of all the VAR coefficients to 0. We make the prior variance-covariance matrix $\underline{\Omega}_\Pi$ diagonal. The variances are specified to make the prior on the $\ln m_t$ terms fairly loose and the prior on the lags of y_t take a Minnesota-type form. Specifically, for the $\ln m_t$ terms of equation i , the prior variance is $\theta_3^2 \sigma_i^2$. For lag l of variable j in equation i , the prior variance is $\frac{\theta_1^2}{l^2}$ for $i = j$ and $\frac{\theta_1^2 \theta_2^2}{l^2} \frac{\sigma_i^2}{\sigma_j^2}$ otherwise. In line with common settings, we set overall shrinkage $\theta_1 = 0.1$ and cross-variable shrinkage $\theta_2 = 0.5$; we set factor coefficient shrinkage $\theta_3 = 10$. Finally, consistent with common settings, the scale parameters σ_i^2 take the values of residual variances from AR(p) models fit over the estimation sample.

Regarding priors attached to the volatility-related components of the model, for the

³This statement refers to drawing from the conditional posterior of the conditional mean parameters, when Σ_t belongs to the conditioning set. One needs also to keep in mind that the joint distribution of the system might be affected by the ordering of the variables in the system due to an entirely different reason: the diagonalization typically used for the error variance Σ_t in stochastic volatility models. Since priors are elicited separately for A and Λ_t , the implied prior of Σ_t will change if one changes the equation ordering, and therefore different orderings would result in different prior specifications and then potentially different joint posteriors. This problem is not a feature of our triangular algorithm, but rather it is inherent to all models using the diagonalization of Σ_t . As noted by Sims and Zha (2006) and Primiceri (2005), this problem will be mitigated in the case (as the one considered in this paper) in which the covariances A do not vary with time, because the likelihood information will soon dominate the prior.

rows a_j of the matrix A , we follow Cogley and Sargent (2005) and make the prior fairly uninformative, with prior means of 0 and variances of 10 for all coefficients.

For the loading $\beta_{i,m}$, $i = 1, \dots, n$, on the uncertainty factor $\ln m_t$, we use a prior mean of 1 and a standard deviation of 0.5. The prior is meant to be consistent with average volatility approximating aggregate uncertainty. In the two-factor model, for the loading $\beta_{i,f}$, $i = 1, \dots, n$, on the uncertainty factor $\ln f_t$, we assign a lower prior mean and larger standard deviation, of 0.5 and 1.0, respectively. For the coefficients of the processes of the factors, we use priors consistent with some persistence in volatility. For the coefficients on lags 1 and 2 of $\ln m_t$ and $\ln f_t$, we use means of 0.9 and 0.0, respectively, with standard deviations of 0.2. For the coefficients on y_{t-1} , we use means of 0 and standard deviations of 0.4. For the period 0 values of $\ln m_t$ and $\ln f_t$, we set the means at 0 and in each draw use the variances implied by the AR representations of the factors and the draws of the coefficients and error variance matrix.

For the idiosyncratic volatility component, in the model for the 3-economy macroeconomic dataset in which it is constant at h_i , the prior mean is $\ln \sigma_i^2$, where σ_i^2 is the residual variance of an $\text{AR}(p)$ model over the estimation sample, and the prior standard deviation is 2. In the model for the 19-country GDP dataset in which the idiosyncratic component is time-varying as in (4), the prior mean is $(\ln \sigma_i^2, 0.0)$, where σ_i^2 is the residual variance of an $\text{AR}(p)$ model over the estimation sample. In this specification, for the variance of innovations to the log idiosyncratic volatilities, we use a mean of 0.03 and 15 degrees of freedom.

3.2 BVAR-SV

The conventional BVAR with stochastic volatility, referred to as a **BVAR-SV** specification, takes the following form, for the $n \times 1$ data vector y_t :

$$\begin{aligned} y_t &= \sum_{i=1}^p \Pi_i y_{t-i} + v_t, \\ v_t &= A^{-1} \Lambda_t^{0.5} \epsilon_t, \quad \epsilon_t \sim N(0, I_n), \quad \Lambda_t \equiv \text{diag}(\lambda_{1,t}, \dots, \lambda_{n,t}), \\ \ln(\lambda_{i,t}) &= \gamma_{0,i} + \gamma_{1,i} \ln(\lambda_{i,t-1}) + \nu_{i,t}, \quad i = 1, \dots, n, \\ \nu_t &\equiv (\nu_{1,t}, \nu_{2,t}, \dots, \nu_{n,t})' \sim N(0, \Phi), \end{aligned} \tag{27}$$

where A is a lower triangular matrix with ones on the diagonal and non-zero coefficients below the diagonal, and the diagonal matrix Λ_t contains the time-varying variances of conditionally Gaussian shocks. This model implies that the reduced-form variance-covariance matrix of

innovations to the VAR is $\text{var}(v_t) \equiv \Sigma_t = A^{-1}\Lambda_t A^{-1'}$. Note that, as in Primiceri's (2005) implementation, innovations to log volatility are allowed to be correlated across variables; Φ is not restricted to be diagonal. Estimates derived from the BVAR-SV model are based on samples of 5,000 retained draws, obtained by sampling a total of 30,000 draws, discarding the first 5,000, and retaining every 5th draw of the post-burn sample.⁴

We set the priors for the BVAR-SV model to generally align with those of the baseline model with factor volatility detailed above. For the VAR coefficients contained in Π , we use a Minnesota-type prior. With the variables of interest transformed for stationarity, we set the prior mean of all the VAR coefficients to 0. We make the prior variance-covariance matrix Ω_Π diagonal. For lag l of variable j in equation i , the prior variance is $\frac{\theta_1^2}{l^2}$ for $i = j$ and $\frac{\theta_1^2 \theta_2^2}{l^2} \frac{\sigma_i^2}{\sigma_j^2}$ otherwise. In line with common settings for large models, we set overall shrinkage $\theta_1 = 0.1$ and cross-variable shrinkage $\theta_2 = 0.5$. Consistent with common settings, the scale parameters σ_i^2 take the values of residual variances from $\text{AR}(p)$ models fit over the estimation sample.

For each row a_j of the matrix A , we follow Cogley and Sargent (2005) and make the prior fairly uninformative, with prior means of 0 and variances of 10 for all coefficients. The variance of 10 is large enough for this prior to be considered uninformative. For the coefficients $(\gamma_{i,0}, \gamma_{i,1})$ (intercept, slope) of the log volatility process of equation i , $i = 1, \dots, n$, the prior mean is $(0.05 \times \ln \sigma_i^2, 0.95)$, where σ_i^2 is the residual variance of an $\text{AR}(p)$ model over the estimation sample; this prior implies the mean level of volatility is $\ln \sigma_i^2$. The prior standard deviations (assuming 0 covariance) are $(2^{0.5}, 0.3)$. For the variance matrix Φ of innovations to log volatility, we use an inverse Wishart prior with mean of $0.03 \times I_n$ and $n + 2$ degrees of freedom. For the period 0 values of $\ln \lambda_t$, we set the prior mean and variance at $\ln \sigma_i^2$ and 2.0, respectively.

3.3 BVAR

The homoskedastic BVAR used in the two-step approach to impulse response assessment takes the following form:

$$y_t = \sum_{i=1}^p \Pi_i y_{t-i} + v_t, \quad v_t \sim i.i.d. \ N(0, \Sigma). \quad (28)$$

⁴To speed computation, we estimate the model with the triangularization approach developed in Carriero, Clark, and Marcellino (2016b).

Regarding the priors on the homoskedastic BVAR's coefficients, we set them to be the same as with the BVAR-GFSV and BVAR-SV models, with the same Minnesota-type prior. For the innovation variance matrix Σ , we use $n + 2$ degrees of freedom and a prior mean of a diagonal matrix with elements equal to 0.8 times the values of the residual variances from AR(p) models fit over the estimation sample.

4 Historical Decomposition with BVAR-GFSV Model

This section details the computation of the paper's estimated historical decomposition. As a starting point, consider a simple one-factor model with lag orders of 1:

$$\begin{cases} y_t = \Pi y_{t-1} + \Gamma_1 \ln m_t + \Gamma_2 \ln m_{t-1} + v_t \\ \ln m_t = \delta y_{t-1} + \gamma \ln m_{t-1} + u_t \end{cases}, \quad (29)$$

where v_t and u_t are independent, with variances Σ_t and Φ_u , respectively. So we can replace v_t above with $\Sigma_t^{0.5} \epsilon_t$, where $\Sigma_t^{0.5}$ is a short-cut notation for the Cholesky decomposition of Σ_t and ϵ_t is $N(0, I_n)$. The one-step-ahead forecast errors are $y_{t+1} - E_t y_{t+1} = \Sigma_{t+1}^{0.5} \epsilon_{t+1} + \Gamma_1 u_{t+1}$. Now let $\hat{\Sigma}_{t+s|t}$ denote the future error variance matrix that would prevail in the absence of future shocks to uncertainty. This would be constructed from forecasts of future uncertainty accounting for movements in y driven by ϵ shocks and the path of idiosyncratic volatility terms (incorporating shocks to these terms). The following decomposition can be obtained by adding and subtracting $\hat{\Sigma}_{t+1|t}$ terms in the forecast error:

$$y_{t+1} - E_t y_{t+1} = \Gamma_1 u_{t+1} + \hat{\Sigma}_{t+1|t}^{0.5} \epsilon_{t+1} + (\Sigma_{t+1}^{0.5} - \hat{\Sigma}_{t+1|t}^{0.5}) \epsilon_{t+1}. \quad (30)$$

In this decomposition, the first term gives the direct contribution of the uncertainty shock, the second term gives the direct contribution of the structural shocks to the VAR, and the third term gives the interaction component. The third term can be simply measured as a residual contribution, as the data less the direct contributions from the uncertainty shock and the structural shocks to the VAR. We apply this basic decomposition to our more general model to obtain historical decompositions.

One potential complication with this approach is that, in the interaction components, there is not a good way to separate the roles of aggregate uncertainty and idiosyncratic volatility, because Σ_t is the product of terms containing innovations to aggregate uncertainty and innovations to idiosyncratic components. Since the terms are multiplicative and not additive, there isn't a clear way to isolate the role of aggregate uncertainty from the role

of idiosyncratic components. Moreover, any attempt to do so would be dependent on the ordering of the variables within the VAR because the effect of uncertainty on the conditional variance of y_t is influenced by the matrix A^{-1} , and hence the ordering of the variables within the VAR matters. Because of these complications, and because the interaction effects are empirically much less pronounced than the direct effects, we chose to leave the interaction component as is, without attempting to separate the roles of aggregate uncertainty and idiosyncratic volatility in the interaction component.

5 Correlations of Uncertainty Estimates with Known Macro Shocks

This section reports correlations of our estimated global macroeconomic uncertainty shocks with some well-known and available macro shocks for the U.S. (estimates for other countries do not seem to be widely available). Specifically, we consider productivity shocks (Fernald’s updates of Basu, Fernald, and Kimball 2006), oil supply shocks (Hamilton 2003 and Kilian 2008), monetary policy shocks (Gurkaynak, et al. 2005 and Coibion, et al. 2017), fiscal policy shocks (Ramey 2011 and Mertens and Ravn 2012), shocks to credit conditions (the excess bond premium of Gilchrist and Zakrajsek 2012), and economic news shocks (Barsky and Sims 2011).⁵

As indicated by the results in Appendix Table 3, our international uncertainty shocks are not very correlated with “known” macroeconomic shocks in the U.S. At least in this sense, to the extent shocks in the U.S. bear on the international business cycle, our estimated uncertainty shocks seem to truly represent a second-order “variance” phenomenon, rather than a first-order “level” shock. While it would be interesting to also assess the correlation of our uncertainty shocks with macroeconomic shocks for other countries or the global economy, we are not aware of standard sources of shocks like those that exist for U.S. data.

⁵The productivity shocks correspond to growth rates of utilization-adjusted TFP. The oil price shock measure of Hamilton (2003) is the net-oil price increase series. The monetary policy shocks of Coibion, et al. (2017) update the estimates of Romer and Romer (2004). We constructed the excess bond premium shock as in the VAR analysis in section IV.B of Gilchrist and Zakrajsek (2012).

6 Results for GDP Growth in 19 Countries over Longer Sample

Although one might be concerned with the stability of a VAR in data on GDP growth across countries extending back to 1960, as an additional robustness check we have examined the international factor structure of uncertainty and its effects on GDP for a sample of 1960:Q4 through 2016:Q3. According to the basic measures of a factor structure, results are very similar for the alternative 1960-2016 and the baseline 1985-2016 samples. In the longer sample, as in the baseline, the measures of factor structure suggest one strong factor in the international volatility of the business cycle as captured by GDP, with the first factor accounting for an average of about 74 percent of the variation in volatility and the second accounting for 13 percent, and the Ahn-Horenstein ratio peaking at one factor.

In BVAR-GFSV estimates over the 1960-2016 sample, the influence of the Great Moderation appears to pose some challenges in estimating macroeconomic uncertainty as it relates to the business cycle. With a one-factor specification, the estimated factor contains a sizable Great Moderation component, declining steadily from the early 1960s through the mid-1980s. A shock to that factor has mixed effects across countries, with GDP declining as expected in some countries but rising in others. We obtain estimates more in line with conventional wisdom on uncertainty's effects with a two-factor BVAR-GFSV specification.⁶ In this case, the estimated first factor continues to have a sizable Great Moderation component in it, and a shock to that factor has essentially no effects on the levels of macroeconomic variables. The second factor looks more like a measure of business cycle-relevant uncertainty; in fact, it is very similar to the estimate from the baseline one-factor model for the 1985-2016 sample. A shock to the second factor reduces GDP across countries, with impulse responses qualitatively similar to those from the baseline one-factor model for the 1985-2016 sample.

⁶These two-factor estimates display no evident MCMC convergence problems. In addition, we considered two-factor estimates in which a tight prior is used to effectively eliminate a second factor from the VAR's conditional mean. In this case, the estimated first factor becomes the uncertainty measure with significant macroeconomic effects, and the second factor picks up the Great Moderation's influence on volatility.

7 References

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Appendix Table 1: Variables in the 3-economy macroeconomic dataset

<i>U.S. variables</i>	<i>E.A. variables</i>	<i>U.K. variables</i>
real GDP ($\Delta \ln$)	real GDP ($\Delta \ln$)	real GDP ($\Delta \ln$)
real consumption ($\Delta \ln$)	real consumption ($\Delta \ln$)	real consumption ($\Delta \ln$)
real government consumption ($\Delta \ln$)	real government consumption ($\Delta \ln$)	real government consumption ($\Delta \ln$)
real investment ($\Delta \ln$)	real investment ($\Delta \ln$)	real investment ($\Delta \ln$)
real exports ($\Delta \ln$)	real exports ($\Delta \ln$)	real exports ($\Delta \ln$)
real imports ($\Delta \ln$)	real imports ($\Delta \ln$)	real imports ($\Delta \ln$)
real inventories	real inventories	unit labor costs ($\Delta \ln$)
unit labor costs ($\Delta \ln$)	unit labor costs ($\Delta \ln$)	industrial confidence
employment ($\Delta \ln$)	employment ($\Delta \ln$)	consumer confidence
hours worked ($\Delta \ln$)	unemployment rate	employment ($\Delta \ln$)
unemployment rate	Eonia rate	unemployment rate
Federal funds rate	2-year bond yield	producer prices ($\Delta \ln$)
2-year bond yield	10-year bond yield	retail price index ($\Delta \ln$)
10-year bond yield	M3 ($\Delta \ln$)	official bank rate
M2 ($\Delta \ln$)	GDP deflator ($\Delta \ln$)	10-year bond yield
oil price ($\Delta \ln$)	consumer prices ($\Delta \ln$)	stock price index ($\Delta \ln$)
commodity prices ($\Delta \ln$)	core consumer prices ($\Delta \ln$)	
consumer prices ($\Delta \ln$)	producer prices ($\Delta \ln$)	
core consumer prices ($\Delta \ln$)	real housing investment ($\Delta \ln$)	
producer prices ($\Delta \ln$)	stock price index ($\Delta \ln$)	
real housing investment ($\Delta \ln$)	capacity utilization	
stock price index ($\Delta \ln$)	consumer confidence	
capacity utilization	industrial confidence	
consumer confidence	purchasing managers' index	
industrial confidence	labor shortages	
purchasing managers' index		

Note: For those variables transformed for use in the model, the table indicates the transformation in parentheses following the variable description.

Appendix Table 2: Summary statistics on commonality in volatility

<i>Prin. comp.</i>	19-country GDP dataset		3-economy macroeconomic dataset	
	R^2	<i>A-H ratio</i>	R^2	<i>A-H ratio</i>
1	0.786	7.431	0.417	1.621
2	0.106	1.746	0.258	2.452
3	0.061	3.041	0.105	1.789
4	0.020	1.905	0.059	1.193
5	0.010	1.296	0.049	1.275

Appendix Table 3: Correlations of uncertainty shocks with other shocks

known shock	19-country GDP dataset uncert. shock	3-economy macro dataset uncert. shock
Productivity: Fernald TFP (1985:Q1-2016:Q3, 1985:Q4-2013:Q3)	-0.097 (0.279)	-0.049 (0.496)
Oil supply: Hamilton (2003) (1985:Q1-2016:Q3, 1985:Q4-2013:Q3)	0.056 (0.561)	-0.017 (0.812)
Oil supply: Kilian (2008) (1985:Q1-2004:Q3, 1985:Q4-2004:Q3)	-0.038 (0.776)	0.022 (0.834)
Monetary policy: Guykaynak, et al. (2005) (1990:Q1-2004:Q4, 1990:Q1-2004:Q4)	-0.070 (0.359)	-0.112 (0.284)
Monetary policy: Coibion, et al. (2016) (1985:Q1-2008:Q4, 1985:Q4-2008:Q4)	-0.181 (0.036)	-0.046 (0.589)
Fiscal policy: Ramey (2011) (1985:Q1-2008:Q4, 1985:Q4-2008:Q4)	-0.175 (0.239)	0.050 (0.649)
Fiscal policy: Mertens and Ravn (2012) (1985:Q1-2006:Q4, 1985:Q4-2006:Q4)	0.198 (0.002)	0.013 (0.845)
Excess bond premium: Gilchrist and Zakrajsek (2012) (1985:Q1-2016:Q3, 1985:Q4-2013:Q3)	0.059 (0.609)	-0.106 (0.451)
News: Barsky and Sims (2011) (1985:Q1-2007:Q3, 1985:Q4-2007:Q3)	0.023 (0.793)	0.069 (0.551)

Notes: The table provides the correlations of the shocks to uncertainty (measured as the posterior medians of $u_{m,t}$) with selected macroeconomic shocks for the U.S. Entries in parentheses provide (in column 1) the sample periods of the correlation estimates, first for the 19-country GDP dataset and then for the 3-economy macroeconomic dataset and (in columns 2 and 3) the p -values of t -statistics of the coefficient obtained by regressing the uncertainty shock on the macroeconomic shock (and a constant). The variances underlying the t -statistics are computed with the prewhitened quadratic spectral estimator of Andrews and Monahan (1992).

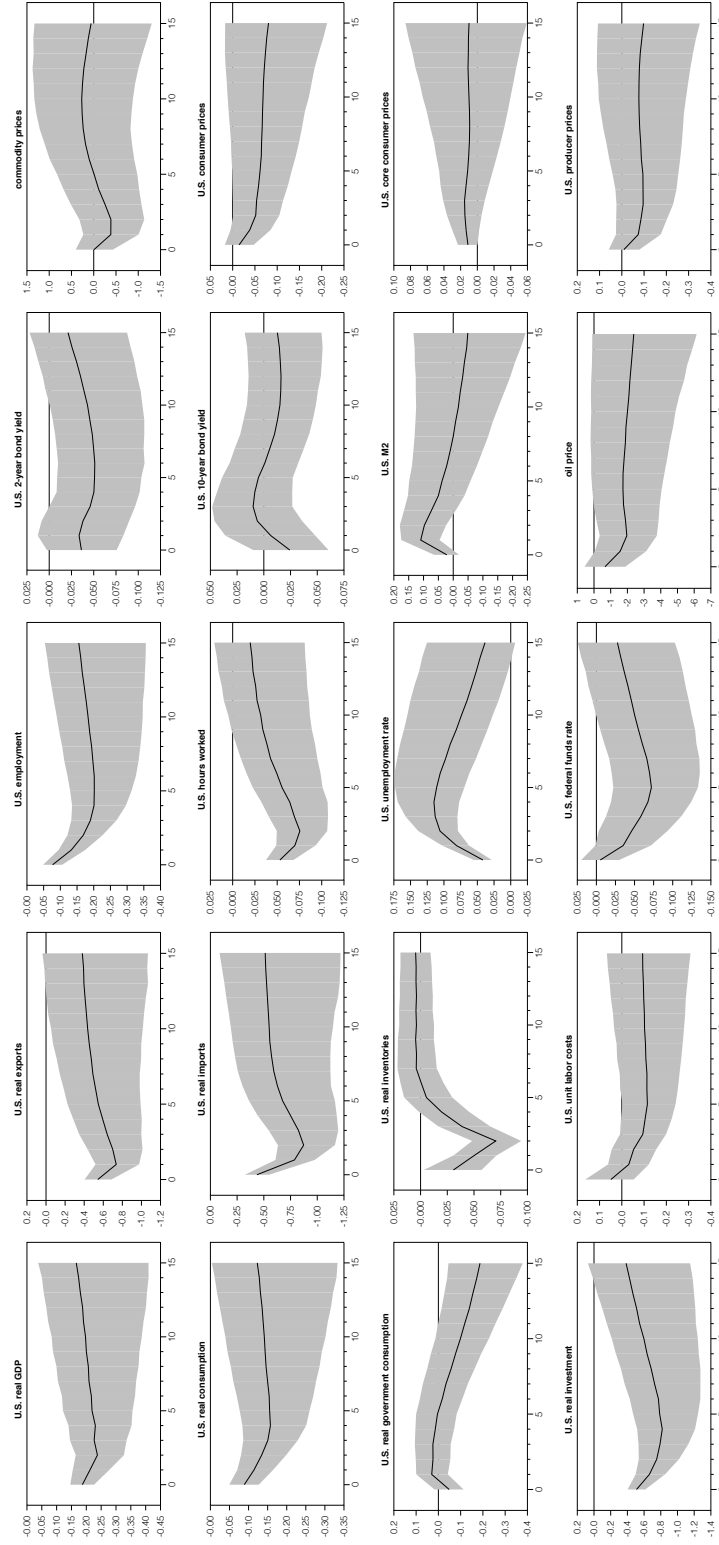


Figure 1: Impulse responses for international uncertainty shock: BVAR-GFSV estimates for 3-economy macroeconomic dataset, posterior median (black line) and 15%/85% quantiles

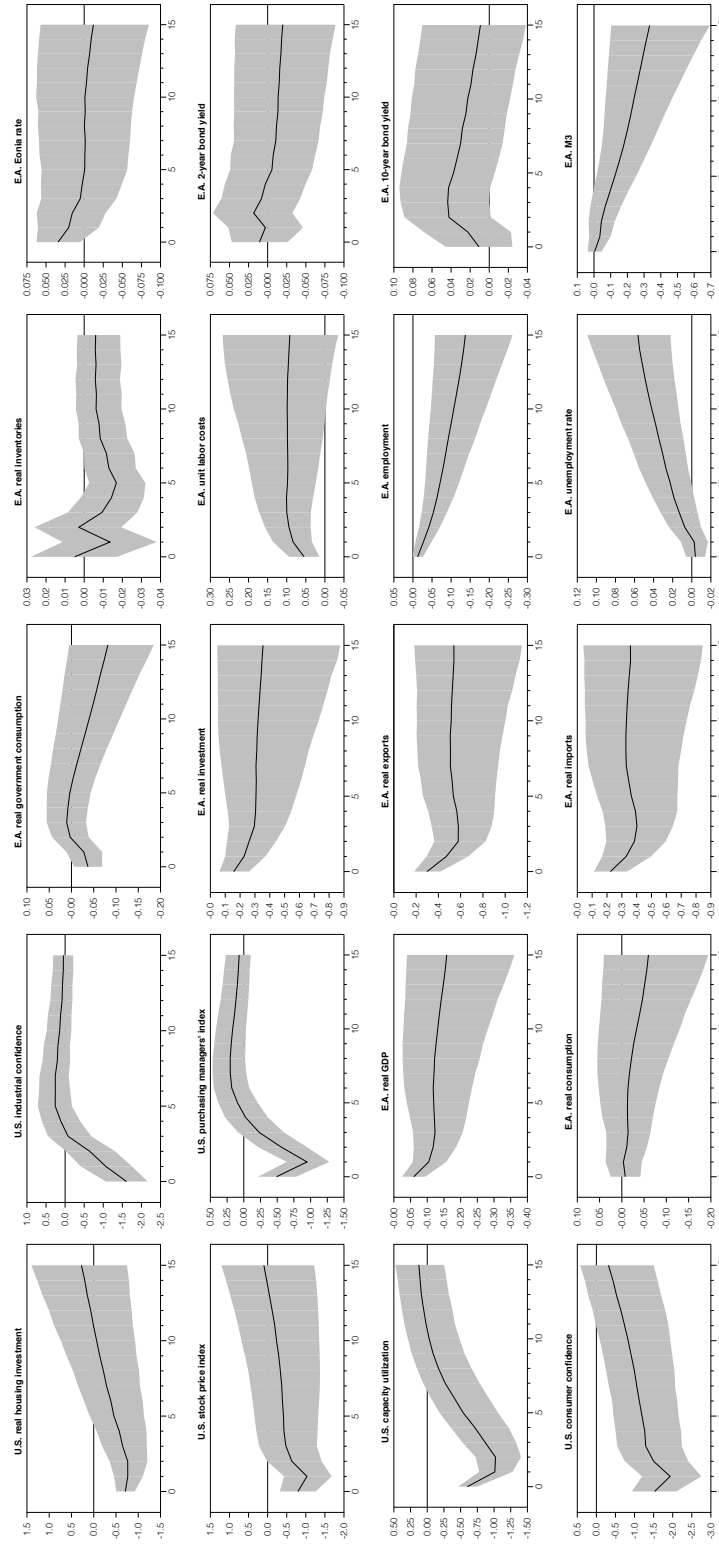


Figure 1: Continued, impulse responses for international uncertainty shock: BVAR-GFSV estimates for 3-economy macroeconomic dataset, posterior median (black line) and 15%/85% quantiles

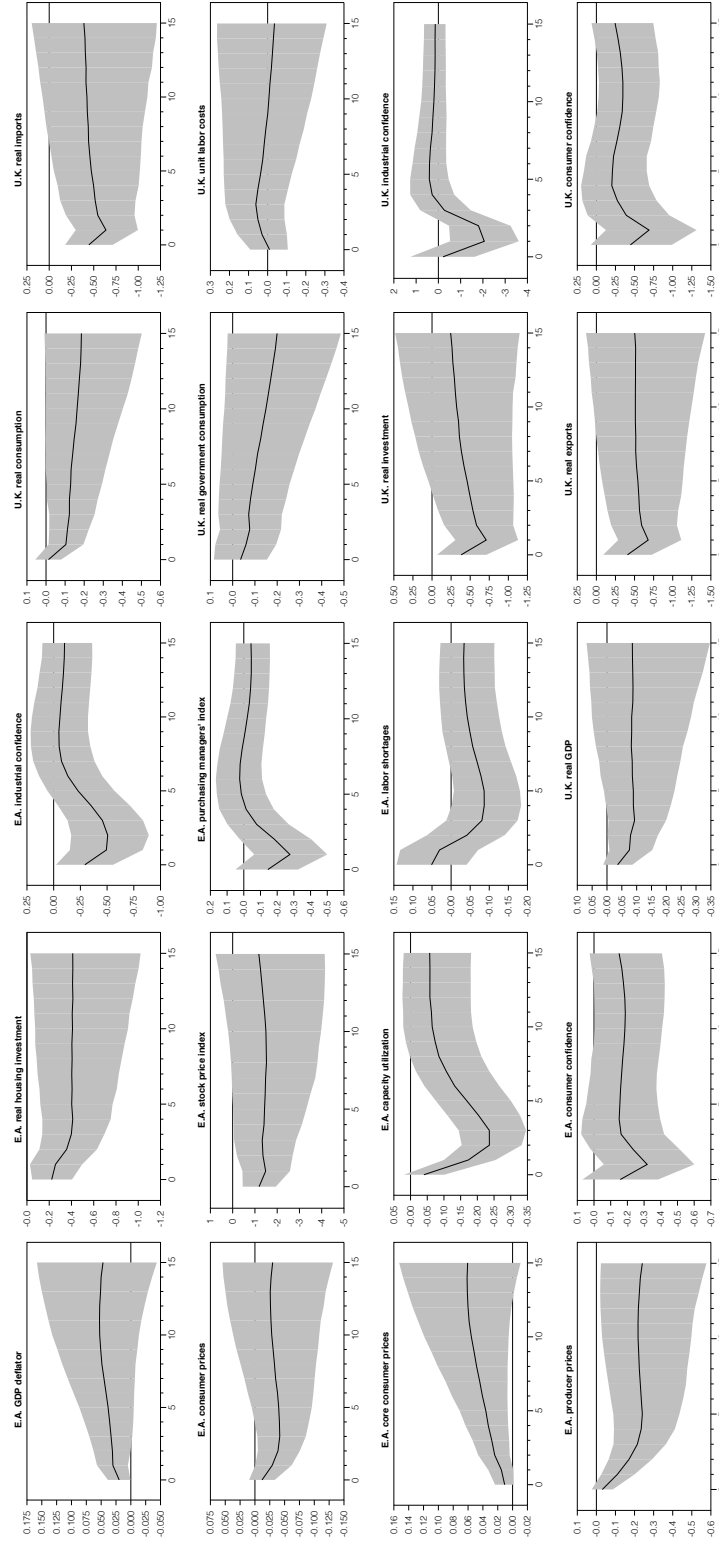


Figure 1: Continued, impulse responses for international uncertainty shock: BVAR-GFSV estimates for 3-economy macroeconomic dataset, posterior median (black line) and 15%/85% quantiles

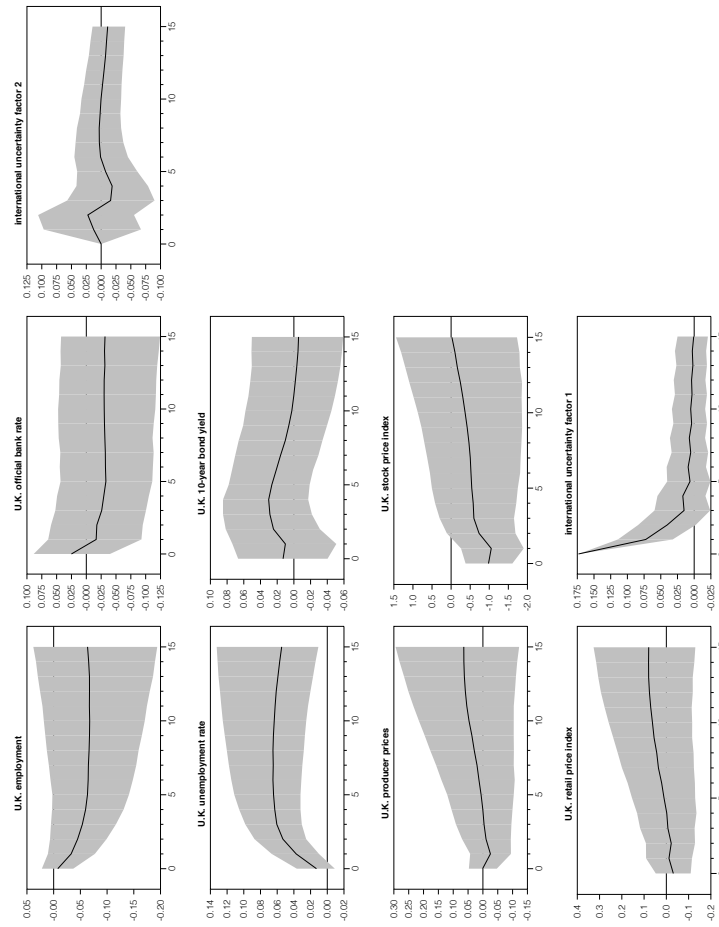
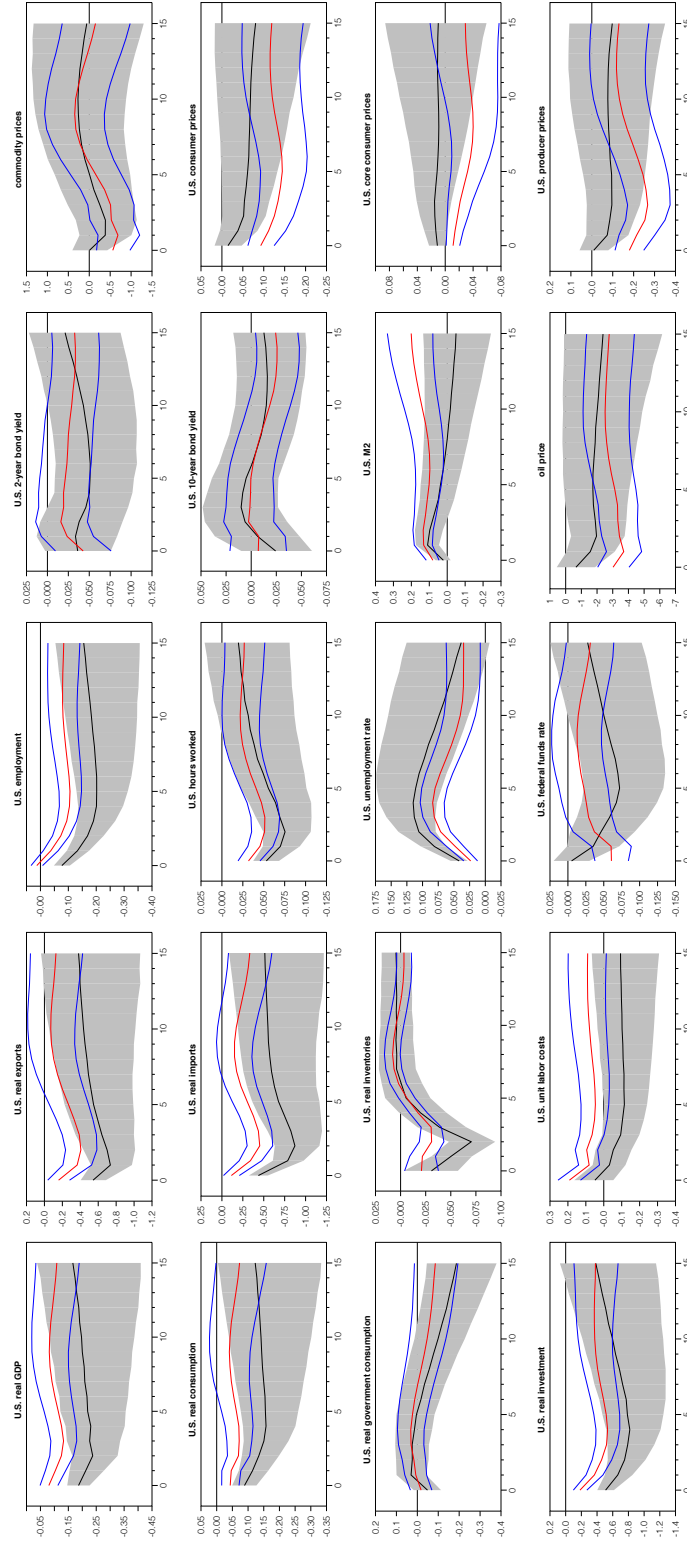
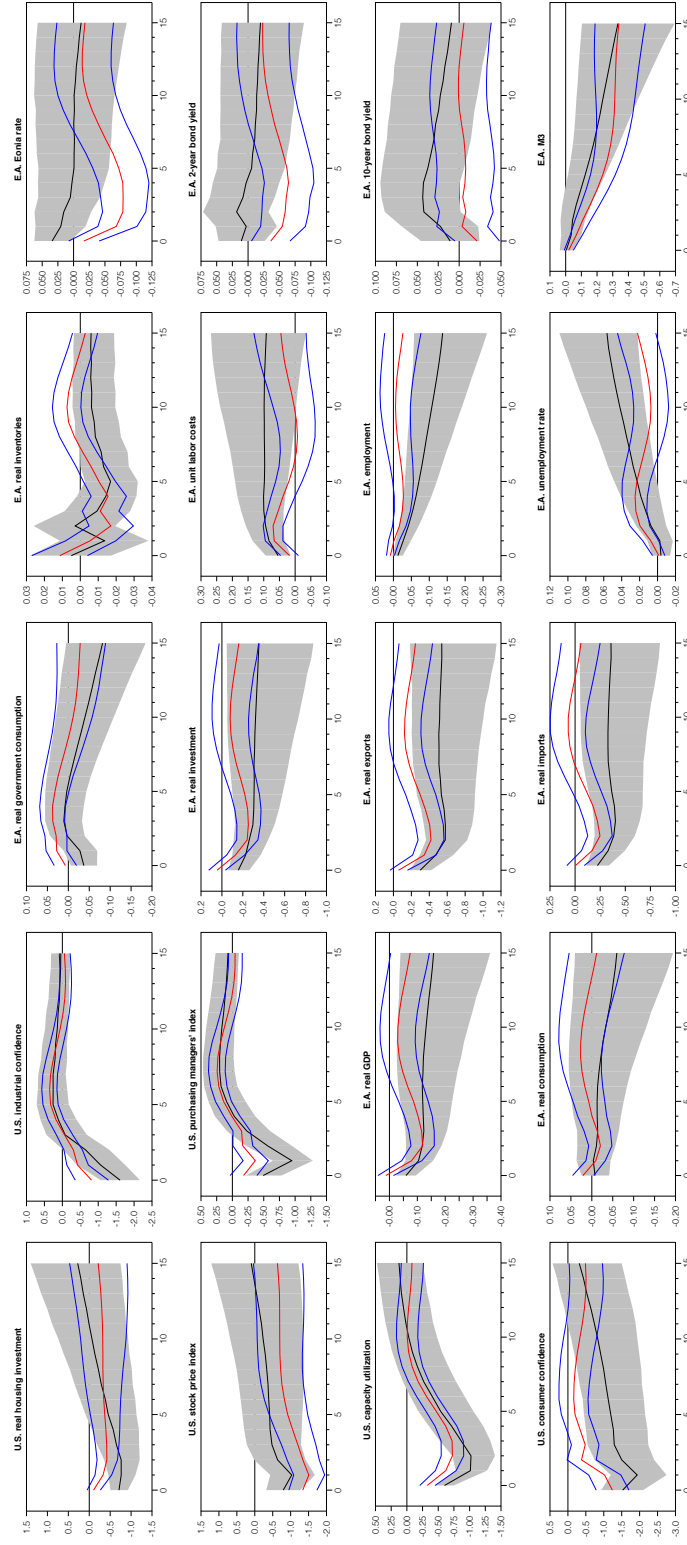


Figure 1: Continued, impulse responses for international uncertainty shock: BVAR-GFSV estimates for 3-economy macroeconomic dataset, posterior median (black line) and 15%/85% quantiles



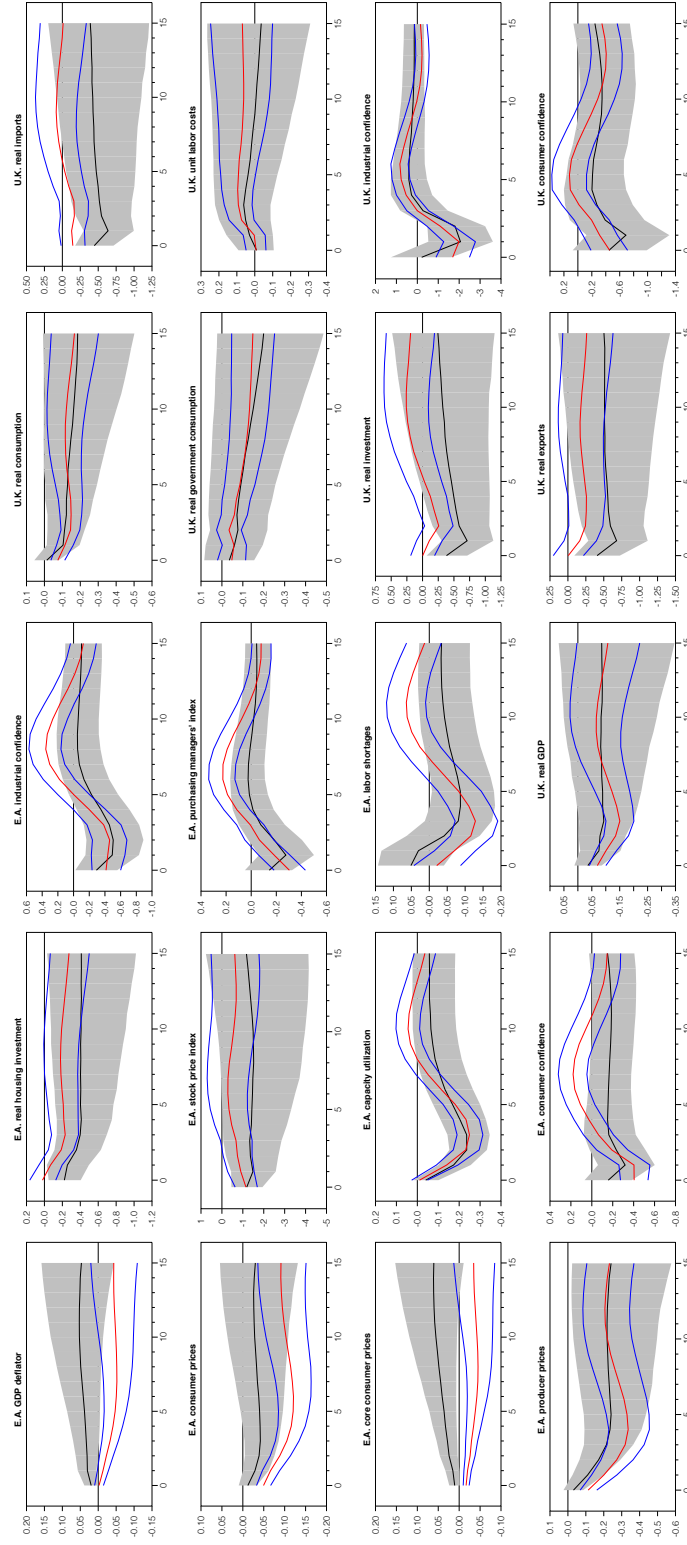
Black line: GFSV estimate. Red: 2-step estimate, using average BVAR-SV volatility

Figure 2: Impulse responses for international uncertainty shock in 3-economy macroeconomic dataset: Comparison of two-step estimates with BVAR-GFSV estimates



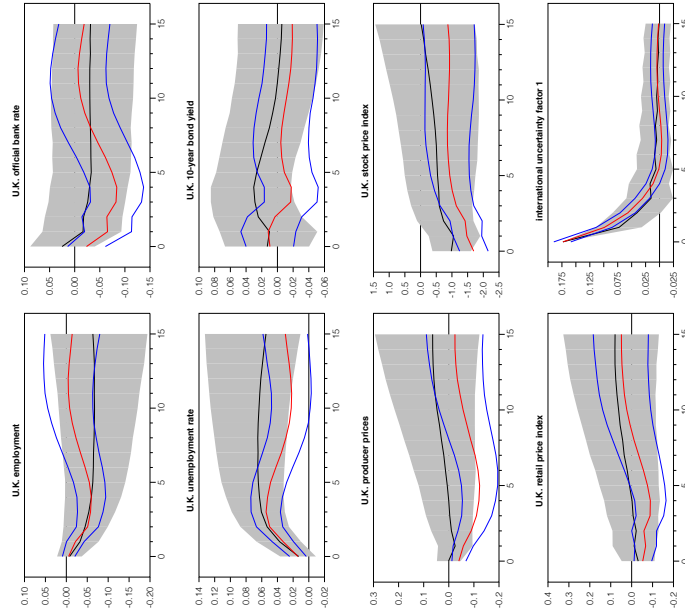
Black line: GFSV estimate. Red: 2-step estimate, using average BVAR-SV volatility

Figure 2: Continued, impulse responses for international uncertainty shock in 3-economy macroeconomic dataset: Comparison of two-step estimates with BVAR-GFSV estimates



Black line: GFSV estimate. Red: 2-step estimate, using average BVAR-SV volatility

Figure 2: Continued, impulse responses for international uncertainty shock in 3-economy macroeconomic dataset: Comparison of two-step estimates with BVAR-GFSV estimates



Black line: GFSV estimate. Red: 2-step estimate, using average BVAR-SV volatility

Figure 2: Continued, impulse responses for international uncertainty shock in 3-economy macroeconomic dataset: Comparison of two-step estimates with BVAR-GFSV estimates

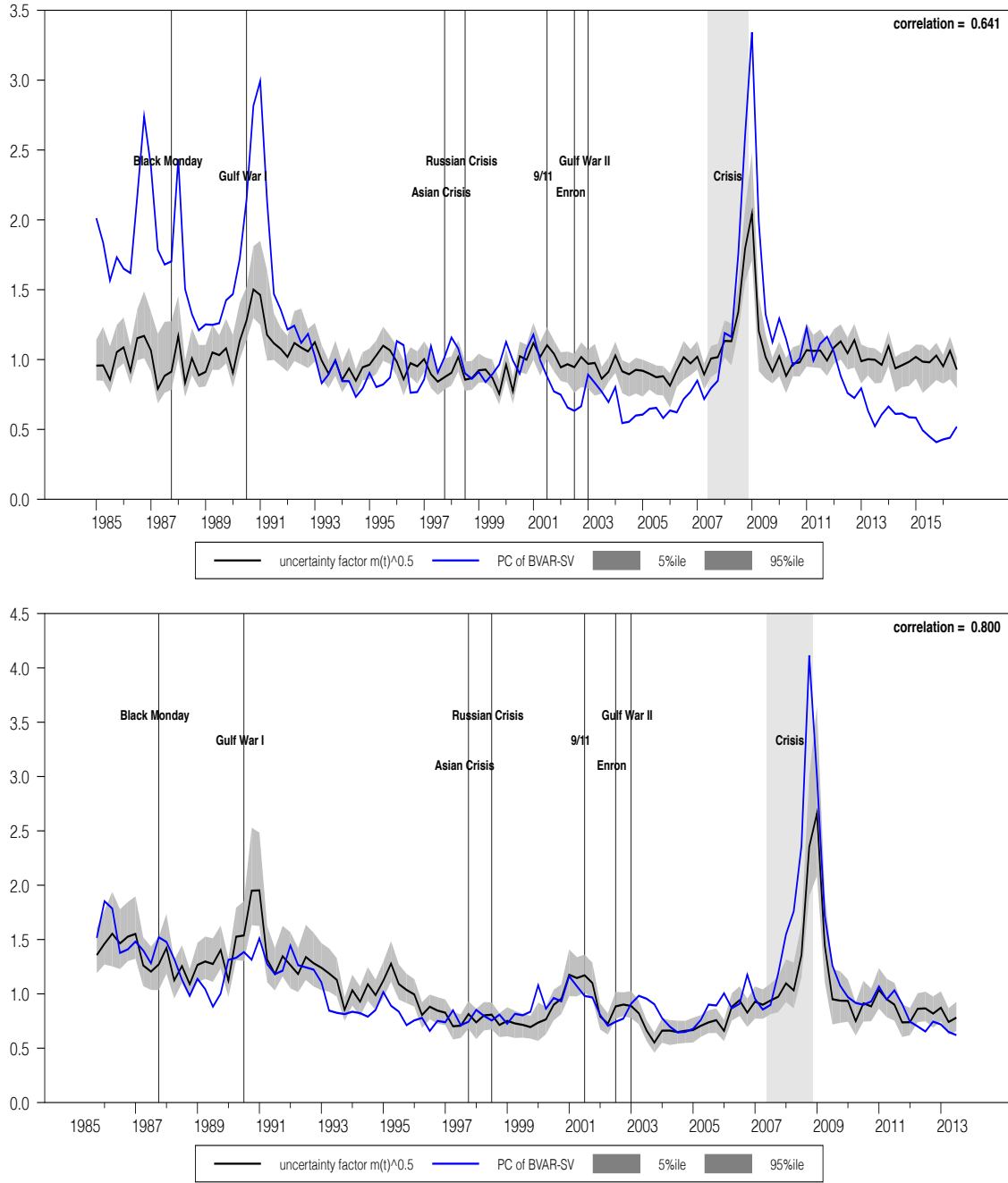


Figure 3: Uncertainty estimates for 19-country GDP dataset in the top panel and for 3-economy macroeconomic dataset in the bottom panel. In each panel, the blue line provides an estimate obtained from the first principal component of the BVAR-SV estimates of log volatility. The solid black line and gray-shaded regions provide the posterior median and 5%/95% quantiles of the BVAR-GFSV estimate of macroeconomic uncertainty ($m_t^{0.5}$). The periods indicated by black vertical lines or regions correspond to the uncertainty events highlighted in Bloom (2009). Labels for these events are indicated in text horizontally centered on the event's start date.