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Goods-Market Frictions and International Trade

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We add goods-market frictions to a general equilibrium dynamic model with heterogeneous exporting producers and identical importing retailers. Our tractable framework leads to endogenously unmatched producers, which attenuate welfare responses to foreign shocks but increase the trade elasticity relative to a model without search costs. Search frictions are quantitatively important in our calibration, attenuating welfare responses to tariffs by 40 percent and increasing the trade elasticity by 50 percent. Eliminating search costs raises welfare by 1 percent and increasing them by only a few dollars has the same effects on welfare and trade flows as a 10 percent tariff.

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1 Introduction

Locating and building connections with overseas buyers is a prevalent firm-level barrier to exporting.[1] Firms pursue costly activities to overcome these barriers.[2] Despite the prevalence and magnitude of these costs at the firm level, how these barriers affect aggregate welfare and trade flows in general equilibrium is not well understood.

In this paper, we formalize this barrier to exporting as a goods-market friction between importing retailers and exporting producers in a Melitz-style model. The key insight is that an endogenous fraction of producers are actively looking for retailers but have yet to match with a partner. This unmatched rate alters the *levels* of aggregate variables and the *changes* in aggregate variables in response to shocks, because when producers are unmatched, their associated varieties cannot be traded. We derive analytic expressions for the welfare response to foreign shocks, the elasticity of consumption and imports with respect to variable trade (iceberg) costs, and the gravity equation, showing that search frictions have first-order effects. Finally, we quantify and decompose the general equilibrium effects of search frictions with a calibrated version of the model.

Our search theoretic framework draws on a literature that documents the important role these frictions play in input markets, especially as presented in Pissarides (2000, Ch. 1). We embed these frictions into a general equilibrium model with heterogeneous producers and identical retailers in the style of Hopenhayn (1992), Melitz (2003), and Chaney (2008). Our model includes many destination-origin markets, and we assume that all retailers and producers, including those in domestic markets, face search frictions. Each search market in our model is summarized by an endogenous sufficient statistic called "market tightness," defined as the ratio of searching retailers to searching producers, and it is determined by retailer entry. Market tightness determines the rate at which producers and retailers contact one another, which in turn determines the unmatched rate of producers and the associated mass of unmatched product varieties. Unmatched varieties cannot be consumed and are therefore absent from the indirect utility (welfare) function, price index, and other aggregates. This feature sets our work apart from standard trade models, in which every firm that chooses to export finds a buyer, but our framework nests those models when we remove the search friction. Our theory remains analytically tractable and has rich implications for firm-level trading relationships and economic aggregates.

[1] Kneller and Pisu (2011) find that "identifying the first contact" and "establishing initial dialogue" are more common obstacles to exporting than "dealing with legal, financial and tax regulations overseas" in a survey of U.K. firms.

[2] Eaton et al. (2014) report that the four most expensive costs for Colombian exporters (in order) are maintaining foreign sales offices, supporting sales representatives abroad, researching potential foreign buyers, and sustaining a web presence.

We find that endogenous unmatched rates attenuate the ex-post welfare response to foreign shocks relative to a model without search frictions. For example, increasing foreign tariffs raises the general equilibrium price index in the domestic market. Protecting the domestic market with tariffs hurts consumers by raising the price index but helps them by increasing the number of domestic varieties that can be consumed. A higher price index allows domestic retailers to sell domestic consumers more and raises the incentive for retailers to enter the domestic market. Having more searching domestic retailers raises domestic market tightness, which raises domestic producers' finding rate, and lowers the domestic producer unmatched rate. More generally, our analytic expression extends the welfare results in Arkolakis, Costinot, and Rodríguez-Clare (2012) to an environment that includes endogenous goods-market frictions between importers and exporters.

We also find that search frictions magnify the response of consumption and imports to iceberg trade costs. In many standard trade models, the consumption and import elasticities are the same. This need not be the case in general, and in our model, they differ because consumption is evaluated at the final sales price of each variety, while imports are evaluated at negotiated prices, which are the outcome of a generalized Nash bargaining game between matched retailers and producers. Both consumption and import elasticities are affected by the unmatched rate and are always at least as negative as the analogous elasticity in a model without goods-market frictions. For example, as tariffs to a destination increase on products from a specific country of origin, retailers in the destination country have less incentive to enter the search market. Having fewer retailers implies a looser search market and a higher unmatched rate for producers, which reduces consumption and imports even more than in a model without search. The import elasticity, but not the consumption elasticity, is also affected by the endogenous markup between import and final prices. This markup magnifies the effects of tariffs on trade flows because raising tariffs on a foreign country, for example, reduces the markup for imports in the foreign market but raises the markup that can be earned in the domestic market. Because the effects of both markups on the import elasticity are weakly negative, the import elasticity in our model is more negative than our consumption elasticity.

Unsurprisingly, goods-market frictions reduce aggregate import flows relative to a model without them. They do so in three ways. First, the unmatched rate reduces aggregate imports because a fraction of foreign varieties are not matched to importing retailers in equilibrium. While aggregate imports are lower, the quantity of any variety traded (intensive margin) between *matched* retailers and producers in our model is the same as in a model without search because Nash bargaining implies that the two parties still seek to maximize the profits earned from consumers. Second, the negotiated import price is always lower than the final sales price paid by consumers and ensures that the importing retailer can at least pay its search costs. Many standard models evaluate aggregates at consumer prices, whereas our model has a gap between import and consumer prices that reduces the value of imports. Finally, by raising the up-front costs associated with entering foreign markets, search costs deter low-productivity producers from searching for a trading partner.

Using an approach advanced by Su and Judd (2012) and Dubé, Fox, and Su (2012) – mathematical programming with equilibrium constraints – we simultaneously recover parameters of the model and solve for the accompanying equilibrium endogenous variables to match U.S. and Chinese data. These data include economic aggregates, business start-up costs, and trading partner separation rates, among other measures. To calibrate importing and domestic retailers' search costs and the matching technology, we use the fraction of U.S. (Chinese) firms exporting to China (the United States) and manufacturing capacity utilization rates in each country, respectively. Average retailer search costs are about 4 percent of average revenue in domestic markets and double that amount in international markets. Our internally calibrated parameters suggest that the technology matching exporters and importers is similar to the technology matching workers and firms (Petrongolo and Pissarides, 2001). As a whole, the model delivers a realistic economic environment for the United States and China.

Search frictions play an important quantitative role for welfare, trade flows, and the consumption elasticity in four results from the calibrated version of our model. First, reducing international retailers' search costs to their domestic levels would increase U.S. welfare by 1.4 percent and Chinese welfare by 2.6 percent. Second, Chinese welfare changes in response to unilateral tariff increases are about 85 percent smaller in our model than in a model without search frictions, despite larger import responses. This is because, relative to the standard model, the domestic producer matched rates attenuate the response of welfare but the international producer matched rates magnify the response of imports. Furthermore, the domestic consumption share response is smaller in the model with search because the matched rate in the foreign market, which is weakly less than one, serves to mute the response of the domestic price index and welfare to tariff changes. Third, small increases in the average search costs retailers face can have impacts on welfare that are commensurate with large increases in bilateral tariffs. For example, raising the average equilibrium costs for importing retailers to contact foreign producers by no more than 6 percent has the same effect on trade flows and welfare as a 10 percent increase in bilateral tariffs. Increasing search costs has these large effects on trade flows because it reduces retailer entry, lowers producers' finding rates, and thereby raises the unmatched rate. Fourth, search frictions, through their effect on the unmatched rate, more than double the consumption elasticity with respect to iceberg costs.

There is a recent expanding literature on search between international trading partners.

Our paper builds on the results of Antràs and Costinot (2011), who model intermediation between traders (retailers) and farmers (producers) in a Ricardian model by accommodating many countries and differentiated goods, and presenting quantitative exercises. Our framework is similar to Drozd and Nosal (2012), who use search frictions between producers and retailers to explain several pricing puzzles in international macroeconomics. Our model also shares many features with Benguria (2015), who shows that goods-market frictions provide a micro foundation for the costs of entering foreign markets. Allen (2014) rationalizes search frictions as costly information acquisition about agricultural market conditions across regions in the Philippines. Eaton et al. (2014) and Eaton et al. (2016) structurally estimate complex search models with endogenous contact rates, many-to-many matches, and learning about foreign markets but their models are partial equilibrium. Lim (2018) endogenizes production networks between heterogeneous firms, but focuses on domestic relationships using Compustat data. Brancaccio, Kalouptsidi, and Papageorgiou (2018) focus on a partial equilibrium search model of the transportation sector, while Startz (2018) includes both contracting and search frictions. Lenoir, Mejean, and Martin (2018) focus on explaining empirical patterns in French export data using a partial equilibrium Eaton and Kortum (2002) model with search frictions. Bernard and Moxnes (2018) survey research on search and matching in trade.

Our model provides search-theoretic micro foundations resembling Rauch (1999) for several empirical estimation approaches used by previous authors. In particular, our simple closed-form gravity equation is consistent with papers that estimate gravity equations and include proxies that might capture search frictions, as in Rauch and Trindade (2002) and Portes and Rey (2005). Our framework is also consistent with the empirical relevance of international intermediaries that move goods from producers to final consumers, as documented by Bernard et al. (2010a) and Ahn, Khandelwal, and Wei (2011).

The remainder of the paper is organized as follows. Section 2 describes the model and section 3 characterizes optimal search and matching behavior by producers and retailers. Section 4 discusses aggregation and the model's equilibrium. Section 5 derives the analytic implications of our framework for changes in welfare to foreign shocks, the consumption and import elasticities, and the gravity equation. Section 6 discusses the calibration strategy and model fit. Section 7 presents several general equilibrium exercises that quantify the role of goods-market frictions. Lastly, section 8 presents a discussion of further research.

2 Model

2.1 Setup

Our model features many countries and is similar to Melitz (2003) and Chaney (2008). In particular, we are motivated by the facts summarized in Bartelsman and Doms (2000) and Syverson (2011): Even within similar industries, firms exhibit persistent differences in measured productivity. We index producers of goods by their productivity, φ. This permanent productivity is exogenously given and known to producers.

As is standard, each country has a representative consumer that has utility over products, including a homogeneous good and differentiated varieties from all countries. Our model, however, assumes that these consumers can access differentiated goods only via ex-ante homogeneous intermediaries called retailers.[3] Moreover, as in the work by Diamond (1982), Pissarides (1985), and Mortensen (1986), a costly process of search governs how producers and retailers find one another. Aside from this goods-market friction, our model nests Melitz (2003) and Chaney (2008). We develop a continuous-time framework and focus on steady-state implications. As such, all events take place simultaneously. Our framework allows for search frictions in domestic and international goods markets.

We index each differentiated-goods market using do to denote destination-origin country pairs. This market includes exporting producers in country o and importing retailers in country d. We will sometimes omit this notation to conserve space.

2.2 Consumers

We assume the representative consumer in destination market d has Cobb-Douglas utility, U_d, over a homogeneous good and a second good that is a constant elasticity of substitution (CES) aggregate of differentiated varieties, indexed by ω, from all origins, indexed by $k \in \{1, \ldots, O\}$. The two goods are combined with exponents $1 - \alpha$ and α, respectively. The differentiated goods are substitutable with constant elasticity, $\sigma > 1$, across varieties and destinations and we denote the value of total consumption as C_d in destination country d.

[3] Although in principle producers could circumvent retailers and contact final consumers directly, we avoid this possibility by assuming that the net value of matching with a retailer is always greater than the net value of forming a relationship directly with a final consumer. This approach is similar to earlier work by Wong and Wright (2014), who assume that a middleman is necessary rather than deriving the conditions under which this is the case.

Formally the consumer's problem is

$$\begin{aligned} \max_{q_d(1), q_{dk}(\omega)} \quad & q_d(1)^{1-\alpha} \left[\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} q_{dk}(\omega)^{\left(\frac{\sigma-1}{\sigma}\right)} d\omega \right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)} \\ s.t.\ C_d \quad = \quad & p_d(1)\, q_d(1) + \sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} p_{dk}(\omega)\, q_{dk}(\omega)\, d\omega, \end{aligned} \tag{1}$$

which results in the following demand for the homogeneous good and each differentiated variety, respectively

$$q_d(1) = \frac{(1-\alpha)\, C_d}{p_d(1)}, \qquad q_{do}(\omega) = \alpha C_d \frac{p_{do}(\omega)^{-\sigma}}{P_d^{1-\sigma}}. \tag{2}$$

Cobb-Douglas preferences across sectors imply that the consumer allocates share $1-\alpha$ of total consumption expenditure to the homogeneous good and share α to the differentiated goods. We could easily extend our framework to any number of Cobb-Douglas sectors, as in Chaney (2008).

The homogeneous good has price $p_d(1)$. Define P_d as the price index for the bundle of differentiated varieties, which is given by

$$P_d = \left[\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} p_{dk}(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}. \tag{3}$$

The ideal price index that minimizes expenditure to obtain utility level $U_d = 1$ is $\Xi_d = \left(\frac{p_d(1)}{1-\alpha}\right)^{1-\alpha} \left(\frac{P_d}{\alpha}\right)^{\alpha}$. To derive these equations, we solve the consumer's utility maximization and expenditure minimization problems explicitly in appendix A.1.

2.3 The matching function, producers, retailers, and bargaining

In section 2.3.1 we describe the technology that determines how matches between producers and retailers form. In sections 2.3.2 and 2.3.3 we depart from the standard environment by introducing a goods-market friction between these foreign producers and domestic retailers, which affects their optimal decisions. In section 2.3.4 we describe how price and quantity are negotiated between a matched retailer and producer.

To maintain tractability, we assume that searching or matching in one market does not affect the costs of searching across other markets. In particular, there are no economies of scale in one market for individual producers and retailers from currently being in a match or from searching in other markets. These assumptions ensure that we can study each "segmented" market independently because, although individual behavior will affect (and be affected by) aggregate variables, they are taken as given by atomistic producers and retailers.

Segmented markets could be relaxed to allow for increasing returns to search for either producers or retailers, but doing so would not change the qualitative results of our paper. As long as the search costs for retailers are positive, producers' finding rate will be finite (proposition 4).[4]

Our approach also assumes that every matched producer must have one, and only one, retailer as its counterpart. We are aware that international retailers and producers can simultaneously engage with several business partners. These many-to-many relationships have been highlighted in Eaton et al. (2014) and Eaton, Kortum, and Kramarz (2017). However, Sugita, Teshima, and Seira (2017) find that, while U.S. importers and Mexican exporters in textiles transact with multiple firms, the main seller and buyer account for the bulk of each firm's total trade. These authors conclude that "a one-to-one matching model is a fair approximation of product-level matching in Mexico-U.S. textile/apparel trade." Similarly, Eaton et al. (2014) find that roughly 80 percent of matches are one-to-one in Colombia-U.S. manufacturing trade. In light of this evidence, we think that a model with one-to-one matches is a reasonable starting point for analysis.

2.3.1 The matching function

The matching function, denoted by $m\left(u_{do}N_o^x, v_{do}N_d^m\right)$, gives the flow number of relationships formed at any moment in time as a function of the stock number of unmatched producers, $u_{do}N_o^x$, and unmatched retailers, $v_{do}N_d^m$, in the *do* market. N_o^x and N_d^m represent the total mass of producing firms in country o and retailing firms in country d, respectively, that exist regardless of their match status. The fraction of producers in country o looking for retailers in country d is u_{do}. The fraction of retailers searching for producing firms in this market is v_{do}.

As in many studies of the labor market (Pissarides, 1985; Shimer, 2005), we assume that the matching function takes a Cobb-Douglas form:

$$m\left(u_{do}N_o^x, v_{do}N_d^m\right) = \xi\left(u_{do}N_o^x\right)^{\eta}\left(v_{do}N_d^m\right)^{1-\eta}, \tag{4}$$

in which ξ is the matching efficiency and η is the elasticity of the matching function with respect to the number of searching producers. Stevens (2007) presents microfoundations for a Cobb-Douglas matching function in a setting of heterogeneous matches when marginal

[4]Furthermore, recent empirical evidence is not definitive as to whether there exists increasing returns in the number of trading partners. The lack of increasing returns to the number of matches is consistent with the results of Arkolakis (2010) and McCallum (2017). Using other approaches or focusing on particular industries, there is some evidence for increasing returns (Moxnes, 2010; Hanson and Xiang, 2011; Chaney, 2014; Morales, Sheu, and Zahler, 2019).

search costs are approximately constant. We think that the process of search between international trading partners is consistent with this framework.

The matching function in equation (4) is homogeneous of degree one. Therefore, market tightness, $\kappa_{do} = v_{do}N_d^m / u_{do}N_o^x$, which is the ratio of the mass of searching retailers to the mass of producers in a given market, is sufficient to determine contact rates on both sides of that market.[5] In particular, the rate at which retailers in country d contact producers in country o, $\chi(\kappa_{do})$, is the number of matches formed each instant over the number of searching retailers:

$$\chi(\kappa_{do}) = \frac{m(u_{do}N_o^x, v_{do}N_d^m)}{v_{do}N_d^m} = \frac{\xi(u_{do}N_o^x)^\eta (v_{do}N_d^m)^{1-\eta}}{v_{do}N_d^m} = \xi\kappa_{do}^{-\eta}.$$

Notice that retailers' contact rate falls with market tightness ($\chi'(\kappa_{do}) < 0$) because as there are more retailers relative to producers, the search market becomes congested with retailers.

The rate at which producers in country o contact retailers in country d is the number of matches formed each instant over the number of searching producers:

$$\frac{m(u_{do}N_o^x, v_{do}N_d^m)}{u_{do}N_o^x} = \frac{\xi(u_{do}N_o^x)^\eta (v_{do}N_d^m)^{1-\eta}}{u_{do}N_o^x} = \xi\kappa_{do}^{1-\eta} = \kappa_{do}\chi(\kappa_{do}),$$

in which producers' contact rate rises with tightness ($d\kappa_{do}\chi(\kappa_{do})/d\kappa_{do} > 0$), also called a market thickness effect. Market tightness is defined from the perspective of producers so that the market is tighter when there are relatively more retailers than producers.

2.3.2 Producers

We assume the homogeneous good is produced with one unit of labor under constant returns to scale in each country. We also assume that there is free entry into the production of that good, there are no search frictions in that sector, and this good is freely traded. Since it is costless to trade, a no-arbitrage condition implies that the price of the homogeneous good must be the same in all countries ($p_d(1) = p(1)\,\forall d$), and because it is made with one unit of labor in each country, it must also be the case that $w_d = p(1)\,\forall d$. As in Chaney (2008), and to simplify our analysis, we only consider equilibria in which every country produces some of the numeraire. Therefore, the homogeneous good will serve as the global numeraire with $p_d(1) = 1\forall d$. We could solve the model without the homogeneous good sector and endogenize wages using market clearing conditions for labor, but the analysis would become analytically intractable and this complication would not alter our main finding that the

[5] We use continuous time Poisson processes to model the random matching of retailers and producers. Thus, the contact rate defines the average number of counterparty meetings during one unit of time. Appendix A.2 contains more details.

endogenous unmatched fraction of producers is important for the levels of and changes in aggregate variables.

For producers of the differentiated good, we use the familiar variable cost function indexed by productivity φ:

$$t\left(q_{do}, w_o, \tau_{do}, \varphi\right) = q_{do} w_o \tau_{do} \varphi^{-1}. \tag{5}$$

Here w_o is the competitive wage in the exporting (origin) country, $\tau_{do} \geq 1$ is a parameter capturing one plus the iceberg transport cost between destination d and origin o, and q_{do} is the amount produced and traded between destination d and origin o. This variable cost function implies a constant-returns-to-scale production function in which labor is the only input. The firm that produces quantity $q_{do}\left(\omega\right)$ of variety ω has productivity φ and marginal cost equal to $w_o \tau_{do} \varphi^{-1}$. Following Melitz (2003), we interpret higher productivity firms as producing a symmetric variety at a lower marginal cost. Total production cost is $t\left(q_{do}, w_o, \tau_{do}, \varphi\right) + f_{do}$ in which f_{do} is the fixed cost of production. We could include nontradeable goods in our framework by increasing the number of sectors and setting the iceberg trade costs in some of these sectors to infinity.

We assume that productivity is exogenous and Pareto distributed with the same cumulative density function in all countries:

$$G\left[\tilde{\varphi} < \varphi\right] = 1 - \varphi^{-\theta}, \tag{6}$$

in which $\varphi \in [1, +\infty)$. The probability density function is $g\left(\varphi\right) = \theta \varphi^{-\theta-1}$. We assume that $\theta > \sigma - 1$ so that aggregate variables determined by the integral $\int_{\bar{\varphi}}^{\infty} z^{\sigma-1} dG\left(z\right)$ are bounded. The Pareto distribution has been widely used in trade models and describes firms' size well (Axtell, 2001; Gabaix, 2009).

The value of a producer with productivity φ being matched to a retailer, $X_{do}\left(\varphi\right)$, can be summarized by a value function in continuous time:

$$rX_{do}\left(\varphi\right) = n_{do} q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} + \lambda\left(U_{do}\left(\varphi\right) - X_{do}\left(\varphi\right)\right). \tag{7}$$

This asset equation states that the flow return at the risk-free rate, r, from the value of producing must equal the flow payoff plus the expected capital gain from operating as an exporting producer. Each producer is indexed by exogenous productivity, φ. The flow payoff consists of $n_{do} q_{do}$, the revenue obtained from selling q_{do} units of the good at negotiated price n_{do} to retailers, less the variable, $t\left(q_{do}, w_o, \tau_{do}, \varphi\right)$, and fixed cost of production, f_{do}. The negotiated price, n_{do}, and the quantity traded, q_{do}, are determined through a bargaining process that we describe in sections 2.3.4, 3.1, and 3.2. The last term in equation (7) captures the event of a dissolution of the match, which occurs at exogenous rate λ and leads to a capital loss of $U_{do}(\varphi) - X_{do}(\varphi)$ as the producer loses value $X_{do}(\varphi)$ but gains the value of being an unmatched producer, $U_{do}(\varphi)$.

The value that an unmatched producing firm receives from looking for a retail partner without being in a business relationship, $U_{do}(\varphi)$, satisfies

$$rU_{do}(\varphi) = -l_{do} + \kappa_{do}\chi(\kappa_{do})(X_{do}(\varphi) - U_{do}(\varphi) - s_{do}). \tag{8}$$

The flow search cost, l_{do}, is what the producer pays when looking for a retailer; it captures the costs we highlighted in the introduction — namely, maintaining foreign sales offices, sending sales representatives abroad, researching potential foreign buyers, and establishing a web presence. The second term captures the expected capital gain, in which $\kappa_{do}\chi(\kappa_{do})$ is the endogenous rate at which producing firms contact retailers, and s_{do} is the sunk cost of starting up the relationship.

The producing firm also has the option of remaining idle and not expending resources to look for a retailer. For producers, the value of not searching, $I_{do}(\varphi)$, satisfies

$$rI_{do}(\varphi) = h_{do}. \tag{9}$$

Producers can always choose this outside option and not search for retailers. Idle firms in this context are analogous to workers who are out of the labor force. Choosing to remain idle provides the flow payoff, h_{do}. The value to a producer of remaining idle can be interpreted, for example, as the value of the stream of payments after liquidation or the flow payoff from home production if these firms are viewed as entrepreneurs.

2.3.3 Retailers

All retailers are ex-ante identical but have values that vary ex-post only because producers are heterogeneous. The value of a retailing firm in a business relationship with a producer of productivity φ, is defined by the asset equation,

$$rM_{do}(\varphi) = p_{do}q_{do} - n_{do}q_{do} + \lambda(V_{do} - M_{do}(\varphi)). \tag{10}$$

The flow payoff from being in a relationship is the revenue generated by selling q_{do} units of the product to a representative consumer at a final sales price p_{do} (determined by their inverse demand curve from equation 2) less the cost of acquiring these goods from producers at negotiated price n_{do}. Retailers do not use the product as an input in another stage of production but only facilitate the match between producers and consumers. We show in section 3 that including an additional intermediate input does not substantively affect our main conclusions. In the event that the relationship undergoes an exogenous separation, the retailing firm loses the capital value of being matched, $V_{do} - M_{do}(\varphi)$. This match destruction occurs at rate λ.

The value of being an unmatched retailer, V_{do}, satisfies

$$rV_{do} = -c_{do} + \chi(\kappa_{do}) \int [\max\{V_{do}, M_{do}(\varphi)\} - V_{do}]\, dG(\varphi). \tag{11}$$

Retailers need to pay a flow cost, c_{do}, to search for a producing affiliate. At Poisson rate $\chi(\kappa_{do})$, retailing firms meet a producer of unknown productivity.

Producers' productivities are ex-ante unknown to retailers so retailers take the expectation over all productivities they might encounter when computing the expected continuation value of searching. As a result, the value, V_{do}, is not a function of a producer's productivity, φ, but rather a function of the expected payoff. We assume that upon meeting, but before consummating a match, retailers learn the productivity of the producer. Upon meeting, and depending on the producer's productivity, φ, retailers choose between matching with that producer, which generates value $M_{do}(\varphi)$, and continuing the search, which generates V_{do}. Hence, the capital gain to retailers from meeting a producer with productivity φ can be expressed as $\max\{V_{do}, M_{do}(\varphi)\} - V_{do}$. In an equilibrium with free entry into retailing, this approach is equivalent to retailers observing producers' productivity after matches are formed.

We conjecture that our model allows for the endogenous existence of retailers and wholesalers, in which the latter intermediate trade between producers and retailers. Retailing and wholesaling would be associated with different payoffs, and with free entry into both sectors, these payoffs would be equal in any equilibrium. This extension would introduce search frictions into a framework related to Ahn, Khandelwal, and Wei (2011). Our model also has skewed sales for both retailers and producers, as shown in appendix B.4.1, and is consistent with facts documented in Bernard et al. (2010b).

2.3.4 Bargaining

Upon meeting, the retailer and producer bargain over the negotiated price and quantity simultaneously. We assume that these objects are determined by the generalized Nash bargaining solution, which, as shown by Nash (1950) and Osborne and Rubinstein (1990), is equivalent to maximizing the following Nash product:

$$\max_{q_{do}, n_{do}} [X_{do}(\varphi) - U_{do}(\varphi)]^{\beta} [M_{do}(\varphi) - V_{do}]^{1-\beta}, 0 \leq \beta < 1, \tag{12}$$

in which β is producers' bargaining power. The total surplus created by a match, which is the value of the relationship to the retailer and the producer less their outside options, is $S_{do}(\varphi) = M_{do}(\varphi) - V_{do} + X_{do}(\varphi) - U_{do}(\varphi)$. In appendix A.3, we derive an expression for the match surplus and for the value of a relationship, $R_{do}(\varphi)$, in terms of model primitives, which also provides theoretical underpinnings for results in Monarch and Schmidt-Eisenlohr (2018). In the next two sections we derive how our bargaining protocol pins down the quantity traded within a business relationship, q_{do}, and the negotiated price, n_{do}. Although different approaches to sharing match surplus (Burdett and Mortensen, 1998; Cahuc, Postel-Vinay, and Robin, 2006) will lead to changes in details and specific expressions, our main results about the effects of endogenous market tightness (section 5) will remain.

3 Optimal search and matching in equilibrium

The retailing and producing firms use backward induction to maximize their value. The second stage is the solution that results from bargaining over price and quantity after a retailer and producer meet and decide to match, which we describe in section 2.3.4. We solve this bargaining problem in sections 3.1 and 3.2.

In the first stage, retailers and producers, taking the solution to this second-stage bargaining problem as given, choose whether to search for a business partner, or to remain idle. Because producers are heterogeneous, their decision to search or not depends on their productivity. Section 3.3 shows that there is a minimum productivity threshold, akin to the entry condition defined in Melitz (2003), that makes searching worthwhile. In section 3.4 we present the condition that characterizes retailers' decisions to search and defines equilibrium market tightness. Finally, there exists a steady-state fraction of unmatched producers that are actively looking for a retail partner and unmatched retailers that are actively looking for a producer. We define these concepts in section 3.5.

3.1 Bargaining over price

Producers and retailers bargaining over the negotiated price, n_{do}, results in a price that divides the total surplus created by a match between the parties according to

$$\begin{aligned} X_{do}(\varphi) - U_{do}(\varphi) &= \beta S_{do}(\varphi), \\ M_{do}(\varphi) - V_{do} &= (1-\beta) S_{do}(\varphi). \end{aligned} \tag{13}$$

Here, producers receive β of the total surplus, while retailers receive the remainder. Therefore, we refer to this expression as the "surplus sharing rule." We have relegated the details regarding the derivation of equation (13) to appendix A.4.1.[6]

The negotiated price, n_{do}, which generates the surplus sharing rule in equation (13), is given by the following proposition.

Proposition 1. *The negotiated price, n_{do}, at which producers sell their good to retailers satisfies*

$$n_{do} = [1 - \gamma_{do}]\, p_{do} + \gamma_{do} \frac{t\left(q_{do}, w_o, \tau_{do}, \varphi\right) + f_{do} - l_{do} - \kappa_{do}\chi\left(\kappa_{do}\right) s_{do}}{q_{do}}, \tag{14}$$

in which $\gamma_{do} \equiv \dfrac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\kappa_{do}\chi\left(\kappa_{do}\right)} \in [0,1]$.

Proof. Use $V_{do} = 0$, equations (7), (8), (10), and the surplus sharing rule defined by equation (13). See appendix A.4.2 for detailed derivations. See appendix A.4.3 for a proof that $\gamma_{do} \in [0,1]$. □

We remind the reader that equation (14) is a function of a producer's productivity, φ, but we have not written it as such to conserve on notation.

The equilibrium negotiated price, n_{do}, is a convex combination of the final sales price and the average total production cost less producers' search costs. A price outside of this range would be unsustainable. The highest negotiated price, n_{do}, that retailers are willing to pay is the final sales price, p_{do}, and the lowest negotiated price that producers are willing to accept is the average total production cost, $\left(t\left(q_{do}, w_o, \tau_{do}, \varphi\right) + f_{do}\right)/q_{do}$, net of the cost of looking for a retailer, l_{do}, and the expected sunk cost, $\kappa_{do}\chi(\kappa_{do})s_{do}$. The search costs of producers, l_{do} and s_{do}, enter negatively in equation (14) because they erode producers' bargaining position and thereby allow retailers to negotiate a lower transaction price.

The negotiated price also depends on the bargaining power and the finding rate of producers. As producers gain all the bargaining power ($\beta \to 1$), then $\gamma_{do} \to 0$ and $n_{do} \to p_{do}$, so producers take all the profits from the business relationship. Similarly, if producers find retailers immediately (no search frictions) so that the finding rate $\kappa_{do}\chi(\kappa_{do}) \to \infty$, and the sunk cost, s_{do}, is set to zero, then the negotiated price also converges to the final sales price, $n_{do} \to p_{do}$. We provide details in appendix A.4.4. Importantly, the case in which $n_{do} \to p_{do}$ recovers the standard trade model (Melitz, 2003; Chaney, 2008), as there is, in effect, no intermediate retailer; producers can be seen as selling their goods directly to the final consumer at price p_{do}.

[6] We also point out that the reasoning behind the restriction that $\beta < 1$ in equation (12) is evident in equation (13). Retailing firms have no incentive to search if $\beta = 1$, as they get none of the resulting match surplus and therefore cannot recoup search costs, $c_{do} > 0$. Any solution to the model with $c_{do} > 0$ and positive trade between retailers and producers also requires $\beta < 1$.

The fact that import prices are lower than final sales prices, as given by equation (14), is consistent with the empirical findings of Berger et al. (2012). This pricing approach, among other model features, is also similar to that of Drozd and Nosal (2012), who use a trade model with search frictions to account for several pricing puzzles of international macroeconomics.

3.2 Bargaining over quantity

We show that bargaining over quantity, q_{do}, together with equation (13), yields the following proposition.

Proposition 2. *The quantity traded, q_{do}, satisfies*

$$p_{do} + \frac{\partial p_{do}}{\partial q_{do}} q_{do} = \frac{\partial t\left(q_{do}, w_o, \tau_{do}, \varphi\right)}{\partial q_{do}}. \tag{15}$$

Proof. See appendix A.5.1. □

The quantity exchanged *within* matches, q_{do}, equates marginal revenue obtained by retailers with the marginal production cost. Equation (15), together with our differentiated demand curve from equation (2), and our cost function from equation (5) imply that the final sales price charged for the imported good in the domestic market takes the standard form of a markup over marginal cost:

$$p_{do}\left(\varphi\right) = \mu w_o \tau_{do} \varphi^{-1}, \tag{16}$$

in which $\mu = \sigma / \left(\sigma - 1\right) > 1$. We present the details of this derivation in appendix A.5.2.

The quantity traded *within* matches in our model is the same as one would obtain in a model without search frictions. The quantity depends on consumers' demand curve p_{do}, the pricing power of retailers, and the production cost function, $t\left(q_{do}, w_o, \tau_{do}, \varphi\right)$. We show in appendix A.5.3 that including an additional input in retailers' production function does not change this result. Nevertheless, although the quantity exchanged does not depend on search frictions, these frictions do affect the mass of matches formed. We turn to this topic in the next section.

3.3 Producers' search productivity thresholds

Given the outcome of bargaining in the second stage, we derive whether retailers and producers will search for a business partner at all in the first stage. Because producers differ by productivity, this first stage leads to a productivity threshold that makes the producer indifferent between searching and remaining idle.[7] This productivity threshold is defined by $U_{do}(\bar{\varphi}_{do}) - I_{do}(\bar{\varphi}_{do}) = 0$ and can be expressed as in the following proposition.

Proposition 3. *In general, the threshold productivity, $\bar{\varphi}_{do}$, is determined by the implicit function*

$$\pi(\bar{\varphi}_{do}) = F(\kappa_{do}), \tag{17}$$

in which variable profits are $\pi(\bar{\varphi}_{do}) \equiv p_{do}(\bar{\varphi}_{do})\, q_{do}(\bar{\varphi}_{do}) - t(q_{do}, w_o, \tau_{do}, \bar{\varphi}_{do})$ and the effective entry cost, $F(\kappa_{do})$, is

$$F(\kappa_{do}) \equiv f_{do} + \left(\frac{r+\lambda}{\beta\kappa_{do}\chi(\kappa_{do})}\right) l_{do} + \left(1 + \frac{r+\lambda}{\beta\kappa_{do}\chi(\kappa_{do})}\right) h_{do} + \left(\frac{r+\lambda}{\beta}\right) s_{do}.$$

Proof. See appendix A.6.2. □

Equation (17) is akin to the entry condition defined in Melitz (2003), even though retailers earn $p_{do}q_{do}$ revenue from the consumer and producers pay the production cost. Our condition defines a threshold productivity that ensures that total flow profits cover what we call the "effective entry cost," $F(\kappa_{do})$, which is the fixed cost of production, f_{do}, and the (appropriately discounted) flow cost of searching for a partner, l_{do}, the opportunity cost of remaining idle, h_{do}, and the sunk cost of starting up a business relationship, s_{do}.

Proposition 3 implies that the effective entry cost, and therefore the threshold productivity, depends endogenously on producers' finding rate $\kappa_{do}\chi(\kappa_{do})$. We define $\kappa_{do} = v_{do}N_d^m / u_{do}N_o^x$ so that as the number of searching retailers increases (or the number of searching producers decreases), it becomes easier for a producer to meet a retailer. Intuitively, higher κ_{do} reduces the time spent searching by producers and, along with it, the effective entry cost. Related to this, if producers' finding rate is exogenous, proposition 3 provides a novel micro-level interpretation of the effective entry cost, but this cost remains a combination of exogenous parameters. Benguria (2015) makes a closely related point.

Another innovation of our model is that the opportunity cost of remaining idle, h_{do}, is an important determinant of the productivity threshold and the fraction of active producers. As pointed out by Armenter and Koren (2014), the fraction of exporting firms is an important moment for parameter identification and one that has been exploited by Eaton et al. (2014) and Eaton et al. (2016), among others. Allowing for the possibility that producers optimally choose not to search could change the estimates in these important papers. Our model also

[7] There exists an alternative threshold, $\underline{\varphi}_{do}$, which makes the producer and retailer indifferent between consummating a relationship upon contact and continuing to search, $X_{do}(\underline{\varphi}_{do}) - U_{do}(\underline{\varphi}_{do}) = 0$. We show in appendix A.6.1 that the binding threshold is defined by $\bar{\varphi}_{do}$, because $\bar{\varphi}_{do} > \underline{\varphi}_{do}$ if $l_{do} + h_{do} + \kappa_{do}\chi(\kappa_{do})\, s_{do} > 0$.

implies that the bargaining power, β, and the match destruction rate, λ, are determinants of the effective entry cost.

Proposition 3 also nests the conditions defining the threshold productivity in many trade models. In particular, with $l_{do} = 0$, $h_{do} = -l_{do}$, and $s_{do} = 0$, we recover the equation defining the productivity threshold in Melitz (2003). We present a more complete discussion of this result and relate proposition 3 to expressions in other standard trade frameworks in appendix A.6.3.[8]

In the context of our functional form assumptions, because the equilibrium price for each variety is a constant markup over marginal cost and the cost function is linear in quantities, variable profits $\pi(\bar{\varphi}_{do})$ are a constant share of revenues. As such, profits become $\pi_{do}(\varphi) = (\alpha/\sigma) C_d P_d^{\sigma-1} (\mu w_o \tau_{do})^{1-\sigma} \varphi^{\sigma-1}$. Using this expression and the implicit function that defines the productivity threshold in equation (17) implies that the threshold productivity is

$$\bar{\varphi}_{do} = \mu \left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}} \left(\frac{w_o \tau_{do}}{P_d}\right) \left(\frac{F(\kappa_{do})}{C_d}\right)^{\frac{1}{\sigma-1}}. \tag{18}$$

We present details in appendix A.6.4. Notice that equation (18) is analogous to equation (7) in Chaney (2008) except that our effective entry cost, $F(\kappa_{do})$, is now endogenous and therefore differs from the fixed entry cost given there.

3.4 Retailer entry

Here we specify the conditions under which unmatched retailers search in order to match with producers. As is standard in the labor literature (Pissarides, 1985; Shimer, 2005), we assume free entry into retailing so that in equilibrium, the value of being an unmatched retailer, V_{do}, is driven to zero. The ability to expand retail shelf space or post a product online until it is no longer valuable to do so provides an intuitive basis for this assumption.

Using equation (11) together with our assumption of free entry into the market of unmatched retailers, $V_{do} = 0$, implies that

$$\frac{c_{do}}{\chi(\kappa_{do})} = \int_{\bar{\varphi}_{do}} M_{do}(\varphi)\, dG(\varphi). \tag{19}$$

This equation defines the equilibrium market tightness, κ_{do}, that equates the expected cost of being an unmatched retailer, on the left, with the expected benefit from matching, on the right. In defining equation (19), we removed the maximum over V_{do} and $M_{do}(\varphi)$ from

[8] It may seem surprising that a trade model without search frictions requires that the flow value of remaining idle be the negative of producers' search cost, $h_{do} = -l_{do}$, instead of having zero idle value, $h_{do} = 0$. Intuitively, without search frictions, idling and searching must have the same flow cost in order for search frictions not to have any effect.

equation (11) and simply integrated from the threshold productivity level defined by equation (18). This simplification is possible as long as $M_{do}(\varphi)$ is strictly increasing in φ so that the ex-post value of being matched is strictly increasing in producers' productivity. In appendix A.7 we prove this result. We emphasize that equation (19) does not inform the binding productivity threshold $\bar{\varphi}_{do}$, which is solely determined by proposition 3.

To get intuition from equation (19), notice that as the expected benefit (the right-hand side) from retailing rises, free entry implies that retailers enter the search market, which raises market tightness, $\kappa_{do} = v_{do}N_d^m / u_{do}N_o^x$, and, through congestion effects, reduces the rate at which searching retailers contact searching producers, $\chi\left(\kappa_{do}\right)$. This, in turn, increases retailers' expected cost of search (the left-hand side). Hence, free entry ensures that V_{do} is zero at all times and that κ_{do} always satisfies equation (19).

If searching for producers was free ($c_{do} = 0$) but matching was associated with positive expected payoff, then free entry would lead to an infinite number of retailers in the economy driving producers' finding rate to infinity. Conversely, if there were an infinite number of retailers in the search market, then the flow cost of search must be zero. These two thought experiments lead to the following proposition.

Proposition 4. *With free entry into retailer search, market tightness, κ_{do}, is finite if and only if retailers' search cost, c_{do}, is positive.*

Proof. See appendix A.8. □

Equation (19), together with proposition 4, highlights that retailers' cost of searching for producers, c_{do}, along with our assumption of free entry into retailing is at the heart of our model. As the retailer cost $c_{do} \to 0$, producers find retailers instantly, relieving the search friction.

One way we motivate goods-market frictions is from survey reports of the high cost of "identifying the first contact" and "establishing initial dialogue" reported by producers (Kneller and Pisu, 2011). Proposition 4 focuses on the role of retailer search costs as the origin of the search friction. However, any reported producer costs, which in our model are captured by the effective entry cost in proposition 3, are influenced by equilibrium variables, and in particular market tightness, κ_{do}. Therefore, retailers' flow search costs, c_{do}, will affect producers' equilibrium costs as well.

Free entry also interacts with assumptions about how firms of both types come into existence. We describe those assumptions in detail in appendix A.9, showing in appendix A.9.1 that, for retailers, free entry into search implies free entry into existence. In appendix A.9.2 we consider the alternative assumption of free entry into search for producers and show that it yields additional restrictions on equilibrium market tightness. We find our baseline approach of setting $V_{do} = 0$ to be a natural starting point, but other approaches lead to similar effects of search frictions, and the major implications of our paper remain the same.

3.5 Matching in equilibrium

In the steady state, there exists a set of unmatched producers that are actively looking for a retail partner and unmatched retailers that are actively looking for a producer. These steady-state fractions of unmatched retailers and producers correspond to frictional unemployment and unfilled vacancies in the labor literature, and will be positive as long as the finding rates are finite and the separation rate is nonzero. The mass of producers that are matched to retailers and selling their products is $(1 - u_{do} - i_{do})\, N_o^x$, in which a fraction u_{do} are unmatched and actively searching for retailers and a fraction i_{do} choose not to search and therefore remain idle.

To determine the steady-state fraction of unmatched producers, it is useful to think about the flow into and out of the unmatched-producer state. In particular, in any given instant, $(1 - u_{do} - i_{do})\, N_o^x$ matched producers separate exogenously at rate λ. Consequently, the inflow into the unmatched state is $\lambda\,(1 - u_{do} - i_{do})\, N_o^x$. Flows out of this state are $\kappa_{do}\chi\,(\kappa_{do})\, u_{do} N_o^x$ because $u_{do} N_o^x$ producers find matches at rate $\kappa_{do}\chi\,(\kappa_{do})$. In the steady state, the inflows must equal the outflows. After re-arranging, we get

$$\frac{u_{do}}{1 - i_{do}} \quad = \quad \frac{\lambda}{\lambda + \kappa_{do}\chi\,(\kappa_{do})}. \tag{20}$$

The fraction of idle producers, i_{do}, that choose not to search is defined by the steady-state productivity threshold, $\bar{\varphi}_{do}$, and the exogenous distribution of productivity:

$$i_{do} = \int_1^{\bar{\varphi}_{do}} dG\,(\varphi) = G\,(\bar{\varphi}_{do})\,. \tag{21}$$

The fraction of producers that are active, $1 - i_{do}$, corresponds to the labor force participation rate in the labor literature. While u_{do} is the fraction of producers that are unmatched, $u_{do}/\,(1 - i_{do})$ is the fraction of active producers that are unmatched and is equivalent to the labor unemployment rate, which is characterized as the fraction of the labor force that is actively searching for a job. Equation (20) implies different predictions about the extensive margin relative to standard trade models because in our model some highly productive varieties are endogenously and randomly unmatched. In this way, we provide a search theoretic explanation for what Armenter and Koren (2014) refer to as “balls-and-bins” facts about the extensive margin of trade.

As mentioned in section 2.3.1, we assume that every matched producer must have one, and only one, retailer as its counterpart. Doing so implies that the mass of matched producers and retailers must be equal in the steady state:

$$\left(1-u_{do}-i_{do}\right)N_o^x=\left(1-v_{do}\right)N_d^m. \tag{22}$$

4 Model aggregation and general equilibrium

4.1 Aggregate resource constraint

The aggregate resource constraint in this economy can be expressed using either the income or expenditure approach to aggregate accounting. Typically, models of international trade highlight the income perspective. We find it more natural to focus on the expenditure approach:

$$\begin{aligned}Y_d=&\underbrace{p_d\left(1\right)q_d\left(1\right)+\sum_{k=1}^{O}\left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right)N_k^x\int_{\bar{\varphi}_{dk}}p_{dk}\left(\varphi\right)q_{dk}\left(\varphi\right)dG\left(\varphi\right)}_{\text{Aggregate consumption }(C_d)}\\&+\underbrace{N_d^xe_d^x+\sum_{k=1}^{O}\kappa_{dk}u_{dk}N_k^xc_{dk}+u_{kd}N_d^x\left(l_{kd}+s_{kd}\kappa_{kd}\chi\left(\kappa_{kd}\right)\right)+\left(1-u_{kd}-i_{kd}\right)N_d^xf_{kd}}_{\text{Aggregate investment }(I_d)}.\end{aligned} \tag{23}$$

Consumption expenditure, C_d, is the total resources devoted to consumption, to both the homogeneous good and the differentiated varieties, evaluated at final consumer prices. Investment expenditure, I_d, is the resources devoted to creating producing firms, to creating retailer-producer relationships, and to paying for the per-period fixed costs of goods production. Here, e_d^x is the sunk, one-time “exploration” cost paid by producers to become an entrepreneur, similar to di Giovanni and Levchenko (2012). As in Chaney (2008), we assume that the number of differentiated-goods producers, N_d^x, is proportional to aggregate consumption expenditure, C_d, so that there is no entry decision by producers, but equation (23) accounts for these expended resources. We present more details in appendix A.10.1. In section 3.4, we mention that free entry into retailing implies that e_o^m must be zero (appendix A.9).[9]

To account for all resources in the economy, we assume all costs incurred by firms for investment and production, including iceberg transport costs, are paid to labor. Therefore,

[9]The mass of producers that are matched to retailers and selling their products is $\left(1-u_{do}-i_{do}\right)N_o^x$. Producers that are idle or searching for retailers but are currently not in a business relationship do not contribute to aggregate output, consumption, or prices. The integral term times $\left(1-i_{do}\right)^{-1}$ captures the conditional average sales of producers that have productivity above the cutoff necessary to match. Another way to see that all aggregate variables must be scaled in this way is to compute the mass of matched producers $\left[\left(1-u_{do}-i_{do}\right)/\left(1-i_{do}\right)\right]N_o^x\int_{\bar{\varphi}_{do}}^{\infty}dG\left(\varphi\right)=\left(1-u_{do}-i_{do}\right)N_o^x$.

we do not have iceberg costs that "melt away" in transit or that are levied and then wasted by the government. Our structure ensures that changes in iceberg costs do not change total resources but instead only introduce distortions. An alternative and identical setup would be to assume that iceberg costs are not paid by firms to workers but are instead levied by the government and then rebated to consumers as lump-sum transfers, which is related to the approach in Irarrazabal, Moxnes, and Opromolla (2015). In that setting, both the expenditure and income approaches would include a government term and aggregate profits would be reduced by the amount of the government's revenue, but total payments to labor would remain the same.

We also treat total payments to idle producers, $\sum_{k=1}^{O}(1-i_{kd})\,N_d^x h_{kd}$, as balanced lump-sum transfers. They enter negatively in the expenditure approach as a lump-sum tax on consumers or firms and enter positively as an additional lump-sum expenditure by the government. As such, these cancel out on the expenditure side of the accounting identity. Finally, we impose balanced trade, so that net exports, NX_d, do not appear in the accounting identity (23).

Total resources are defined by $Y_d = w_d L_d (1+\pi)$, in which L_d is the exogenous size of the economy, w_d is the equilibrium wage, and

$$\pi = \Pi / \sum_{k=1}^{O} w_k L_k, \tag{24}$$

is the value of a share of global profits, Π, which we define in appendix A.10.2. Consumers in destination d get a share of global profits in proportion to the value of labor in the economy, $w_d L_d / \sum_{k=1}^{O} w_k L_k$, as in Chaney (2008).

Additional details about the income and expenditure approaches to accounting, resources available for consumption and investment, and the global mutual fund are included in appendix A.10.2.

4.2 The ideal price index

We can move from indexing over the unordered set of varieties in equation (3) to the distribution of productivities using the steps in appendix A.11.1. We can then use the optimal final sales price that results from Nash bargaining over quantity given in equation (16) along with the productivity threshold from (18) to derive the price index for differentiated goods in country d:

$$P_d = \lambda_2 \times C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \times \rho_d, \tag{25}$$

in which

$$\rho_d \equiv \left(\sum_{k=1}^{O} \frac{C_k}{C} \left(1 - \frac{u_{dk}}{1 - i_{dk}} \right) (w_k \tau_{dk})^{-\theta} F_{dk}^{-\left[\frac{\theta}{\sigma-1}-1\right]} \right)^{-\frac{1}{\theta}},$$

$$\lambda_2 \equiv \left(\frac{\theta}{\theta - (\sigma - 1)} \right)^{-\frac{1}{\theta}} \left(\frac{\sigma}{\alpha} \right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}} \mu \left(\frac{C}{1+\pi} \right)^{-\frac{1}{\theta}},$$

and $C = \sum_{k=1}^{O} C_k$ is global consumption. More details appear in appendix A.11.2 and to conserve on notation, we will sometimes refer to $F(\kappa_{do})$ as F_{do}. Equation (25) closely resembles the price index in Chaney (2008, equation 8) and our model also includes a "multilateral resistance" term, ρ_d. Importantly, the introduction of search frictions makes the price index a function of the consumption weighted average of the equilibrium matched rates of all producers throughout the world in addition to the usual iceberg and entry costs. Introducing search frictions increases the price level by increasing the multilateral resistance term because the fraction of producers that are matched is always less than one $1 - u_{do}/(1 - i_{do}) \leq 1$. Search frictions mute the effect of tariff changes on the price index because these changes only affect matched firms. We quantify the effect of search frictions on the response of the consumption share and the price index to tariff changes in section 7.1.1.

4.3 Defining the general equilibrium

A steady-state general equilibrium consists of threshold productivities, $\bar{\varphi}_{do}$ $\forall do$, market tightnesses, κ_{do} $\forall do$, aggregate consumptions, C_d $\forall d$, and the per-capita dividend distribution, π, which jointly solve the zero-profit conditions (equation 18), the free-entry conditions (equation 19), the aggregate resource constraints (equation 23), and the global mutual fund dividend (equation 24). The exogenous parameters are β, λ, r, η, ξ, θ, σ, α, e_d^x, L_d, c_{do}, f_{do}, h_{do}, l_{do}, s_{do}, and τ_{do}, in which d and o vary with country. We elaborate on this definition in appendix A.12.

5 Analytic general equilibrium results

5.1 Welfare changes in response to foreign shocks

In this section, we discuss how adding search frictions changes the response of welfare to foreign shocks. We relate this to Arkolakis, Costinot, and Rodríguez-Clare (2012), who show that, in a large class of trade models, welfare (indirect utility) changes can be summarized by two sufficient statistics: the change in the domestic consumption share in response to a shock and the elasticity of trade with respect to variable trade costs.

Definition 1. *Define a foreign shock in country d as a change from* $(\mathbf{L}, \mathbf{e^x}, \mathbf{f}, \mathbf{c}, \mathbf{h}, \mathbf{l}, \mathbf{s}, \tau)$ *to* $(\mathbf{L}', \mathbf{e^x}', \mathbf{f}', \mathbf{c}', \mathbf{h}', \mathbf{l}', \mathbf{s}', \tau')$ *such that* $(L_d, e^x_d, f_d, c_d, h_d, l_d, s_d, \tau_d) = (L'_d, e^{x\prime}_d, f'_d, c'_d, h'_d, l'_d, s'_d, \tau'_d)$.

Proposition 5. *The change in welfare associated with any foreign shock in country d in our model can be computed as*

$$\hat{W}_d = \hat{\lambda}_{dd}^{-\frac{\alpha}{\theta}} \left(1 - \widehat{\frac{u_{dd}}{1 - i_{dd}}}\right)^{\frac{\alpha}{\theta}} \hat{C}_d^{1+\frac{\alpha}{\theta}\left(1-\frac{\theta}{\sigma-1}\right)}, \tag{26}$$

in which $\hat{x} \equiv x'/x$ denotes the change in any variable x between the initial and the new equilibrium, $\lambda_{dd} \equiv C_{dd}/C_d$ is the share of country d's total expenditure on differentiated goods produced domestically, and we assume that: 1) $l_{dd} = -h_{dd}$ so that $F(\kappa_{dd})$ is a parameter, 2) the number of producers in d do not change so that $d\ln(N^x_d) = 0$, and 3) productivity, φ, has a Pareto distribution given by equation (6).

Proof. Appendix B.1 derives the proof with the general result in B.1.6 and proposition 5 in B.1.7. □

Equation (26) states that the change in welfare in country d, $\hat{W}_d$, is a function of the changes in the share of domestic expenditure at final prices, $\hat{\lambda}_{dd}$, changes in the rate at which domestic producers are matched in the domestic market, $1 - \widehat{u_{dd}/(1 - i_{dd})}$, and the change in consumption itself, $\hat{C}_d$.

There are a few differences between the Arkolakis, Costinot, and Rodríguez-Clare (2012) welfare expression and equation (26). First, knowing only changes in the consumption ratio, λ_{dd}, and the parameters α, θ, and σ is insufficient for ex-post welfare analysis. One would also need to know the changes in the matched rate and changes in the level of consumption. The change in consumption enters into equation (26) but not the welfare equation in Arkolakis, Costinot, and Rodríguez-Clare (2012) because search and sunk costs imply that profits are not proportional to output. Second, if the rates at which partners find one another are exogenous parameters and profits are proportional to output, equation (26) collapses to the expression in Arkolakis, Costinot, and Rodríguez-Clare (2012) and any welfare effects are the same as in the standard model. Sending the search cost c to zero would also result in the standard expression as long as profits are proportional to output. Third, the matched rate in equation (26) could serve to attenuate the welfare change in response to a change in variable trade costs in comparison with the standard model. Consider, for example, the effect of destination d raising tariffs on products from origin o in a model with search. Higher tariffs result in a higher price index, which makes being a retailer in the domestic market more valuable and induces more retailers to enter the domestic market. With more retailers in the market, the rate at which domestic producers find domestic partners increases, and the matched rate, $1 - u_{dd}/(1 - i_{dd})$, increases. A higher domestic matched rate attenuates the welfare losses from higher tariffs. Fourth, the effects of tariffs could also be attenuated through attenuated changes in the domestic consumption share, $\hat{\lambda}_{dd}$, because tariff changes only affect matched varieties instead of all varieties above the exporting threshold. In section 7.1.1 we quantify the effects of the matched rate in response to specific foreign shocks in a calibrated version of our model showing that welfare attenuation can be quantitatively large.

5.2 Consumption and trade elasticities

The elasticity of trade with respect to variable trade costs is an important quantity itself and is one of the two inputs needed to evaluate the gains from trade in a large class of models (Arkolakis, Costinot, and Rodríguez-Clare, 2012). The general form of this elasticity is provided by Arkolakis, Costinot, and Rodríguez-Clare (2012, equation 21) for Melitz (2003) models under slight restrictions on the number of producers. Starting with their general elasticity and assuming productivity, φ, has a distribution given by equation (6) implies that the trade elasticity is the negative of the Pareto shape parameter,

$$\frac{\partial \ln \left(IM_{do}/IM_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} = \frac{\partial \ln \left(C_{do}/C_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} = \varepsilon_o^{ACRdd'} = \begin{cases} -\theta & if\ d' = d \\ \\ 0 & if\ d' \neq d \end{cases}. \tag{27}$$

It is generally not true that the consumption elasticity needed to evaluate welfare and the trade elasticities are equal (Melitz and Redding, 2015). In our model, they differ because consumption is evaluated at final sales prices, while imports are evaluated at negotiated prices. We present consumption and trade elasticities in our model under simplifying assumptions in order to ease the comparison to models without search frictions and to highlight the quantitative importance of search in our calibration exercises. None of those simplifications are necessary to derive these elasticities. Proposition 6 presents the consumption elasticity, while proposition 7 presents the trade elasticity.

Proposition 6. *The elasticity of consumption shares to iceberg trade costs in our model with goods-market frictions is given by*

$$\frac{\partial \ln \left(C_{do}/C_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} = \begin{cases} -\theta + \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{do}\right)}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln \left(\tau_{do}\right)}\right) & if\ d' = d \\ \\ \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right) & if\ d' \neq d \end{cases}, \tag{28}$$

in which we assume that: 1) $l_{do} = -h_{do}$ so that $F\left(\kappa_{dd}\right)$ and $F\left(\kappa_{do}\right)$ are parameters, 2) the number of producers in d and o do not change with tariff changes so that

$\partial \ln (N_d^x) / \partial \ln (\tau_{d'o}) = \partial \ln (N_o^x) / \partial \ln (\tau_{d'o}) = 0$, *and 3) productivity,* φ*, has a Pareto distribution given by equation* (6).

Proof. Appendix B.2 derives the proof with the general result in B.2.14, proposition 6 is in appendix B.2.15, and a comparison with equation (27) is provided in appendix B.2.16. □

Our consumption elasticity depends not only on the usual trade elasticity, but also on the fraction of unmatched producers and the elasticity of producers' finding rate in the *do* and *dd* product markets. Using equation (19), we know that raising tariffs, τ_{do}, reduces the value of importing, $M_{do}(\varphi)$, and therefore reduces market tightness, κ_{do}, and producers' finding rate, $\kappa_{do}\chi(\kappa_{do})$. This comparative static implies that $\partial \ln \kappa_{do}\chi(\kappa_{do}) / \partial \ln (\tau_{do}) \leq 0$. Conversely, raising tariffs in the *do* market raises the price index, P_d, making the domestic market more attractive for retailers, thereby encouraging domestic retailer entry, and thus raising domestic market tightness, κ_{dd}, and the domestic producers' finding rate, which implies $\partial \ln \kappa_{dd}\chi(\kappa_{dd}) / \partial \ln (\tau_{do}) \geq 0$. Because both *do* and *dd* unmatched rates of producers are weakly positive, the consumption elasticity in our model is at least as negative as the analogous trade elasticity in the class of models from Arkolakis, Costinot, and Rodríguez-Clare (2012) that satisfy equation (27).

Proposition 7. *The elasticity of trade shares to iceberg trade costs in our model with goods-market frictions is given by*

$$\frac{\partial \ln (IM_{do}/IM_{dd})}{\partial \ln (\tau_{d'o})} = \begin{cases} -\theta + \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln (\tau_{do})}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln (\tau_{do})}\right) & \\ + \frac{\partial \ln (1-b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}))}{\partial \ln (\tau_{d'o})} - \frac{\partial \ln (1-b(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd}))}{\partial \ln (\tau_{d'o})} & \text{if } d'=d \\ \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln (\tau_{d'o})}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln (\tau_{d'o})}\right) & \\ + \frac{\partial \ln (1-b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}))}{\partial \ln (\tau_{d'o})} - \frac{\partial \ln (1-b(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd}))}{\partial \ln (\tau_{d'o})} & \text{if } d' \neq d \end{cases}, \tag{29}$$

in which we make the same simplifying assumptions as in proposition 6.

Proof. Appendix B.3 derives the proof, defines the import markup terms, $1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do})$, and shows that these terms are bounded between $\mu(1 - 2/\sigma)$ and one, in which $\mu = \sigma/(\sigma - 1)$ is the final sales price markup. □

Our trade elasticity depends on the usual Pareto parameter θ, as well as on the endogenous change in the fraction of unmatched producers and the elasticity of producers' finding rate in the relevant product market as discussed in proposition 6. Because negotiated import prices are lower than final sales prices, the import share elasticity also depends on the endogenous markup terms in the foreign and domestic markets, $1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do})$ and $1 - b(\sigma, \theta, \gamma_{dd}, \delta_{dd}, F_{dd})$, respectively.

Using an argument similar to our previous logic, we know that raising tariffs, τ_{do}, reduces producers' finding rate, $\kappa_{do}\chi\left(\kappa_{do}\right)$, and that higher marginal costs reduce the markup in the *do* market in our calibration, $\partial \ln\left(1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\right)/\partial \ln\left(\tau_{do}\right) \leq 0$. Higher tariffs, τ_{do}, also increase the markup in the *dd* market so that $\partial \ln\left(1-b\left(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd}\right)\right)/\partial \ln\left(\tau_{do}\right) \geq 0$. Intuitively, this is because raising tariffs in the *do* market makes being a retailer in the *dd* market more valuable, inducing entry and increasing the finding rate for producers in the *dd* market, which allows producers to negotiate higher prices. Because the effects of both *do* and *dd* price markups on the import elasticity are weakly negative, the import elasticity in our model is more negative than our consumption elasticity and the analogous trade elasticity in many standard trade models.

We consider the relative magnitude of the matched and markup effects on consumption and trade elasticities in section 7.2. Under our calibration, we find that while both these effects have the signs we discussed, only the matched margin is quantitatively important. Finally, we point out that a search model with exogenous matching rates or markups will give the same consumption and trade elasticities as the standard model.

5.3 The gravity equation

The gravity structure in our model, albeit more complicated, is similar to the gravity structure common to many trade models. To show this, we begin with the definition of total imports (free on board) by destination d from origin o in the differentiated goods sector:

$$IM_{do} = \left(1-\frac{u_{do}}{1-i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} n_{do}\left(\varphi\right) q_{do}\left(\varphi\right) dG\left(\varphi\right). \tag{30}$$

Performing the required integration in equation (30) gives the following proposition.

Proposition 8. *The gravity equation in our model is:*

$$IM_{do} = \left(1-\frac{u_{do}}{1-i_{do}}\right)\left(1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\right)\alpha\left(\frac{C_oC_d}{C}\right)\left(\frac{w_o\tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}, \tag{31}$$

in which the fraction of matched exporters $1-u_{do}/\left(1-i_{do}\right)\in[0,1]$ *and the import markup* $1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\in\left[\mu\left(1-2/\sigma\right),1\right]$.

Proof. See appendix B.4.1 for the derivation of the gravity equation. □

The main message is clear: Search frictions have a first-order effect on the level of total imports. Search frictions reduce trade flows in three ways. First, search frictions give rise to a fraction of unmatched exporters, $1-u_{do}/\left(1-i_{do}\right)$. Second, trade flows are diminished because imports are computed using negotiated import prices, n_{do}, as opposed to final sales prices, p_{do}. Negotiated import prices are lower than final sales prices. These lower import prices lead to the endogenous import price markup term, which is bounded between one and the final sales price markup as, $1 + b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}) \in [1, \mu]$. Third, search frictions reduce imports because of the negative exponent on the effective entry cost, F_{do}, which is increasing in search frictions, as shown in equation (3). We present further details in appendix B.4.2.

Even if imports are measured at final sales prices, as assumed in the typical gravity equation, search frictions have a significant effect on final consumption. By evaluating equation (30) at p_{do} instead of n_{do}, and using our functional form assumptions, we compute imports at final sales prices in appendix B.4.3 as

$$C_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) \alpha \left(\frac{C_o C_d}{C}\right) \left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma - 1} - 1\right)}. \tag{32}$$

By definition, this provides consumption expenditure in destination d on differentiated goods produced in origin o. Even without the difference between final and import prices caused by $b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do})$, search frictions lead to a mass of unmatched and searching producers $u_{do}/(1 - i_{do})$, which lowers consumption. Search frictions also affect imports through the effective entry cost, $F_{do}(\kappa_{do})$, but with the same exponent as existing models.

Consumption expenditure must equal imports plus the period profits of matched importers, $C_{do} = IM_{do} + \Pi_{do}^m$. Combining equations (31) and (32) gives total period profits accruing to importers in matched relationships:

$$\Pi_{do}^m = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}) \alpha \left(\frac{C_o C_d}{C}\right) \left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma - 1} - 1\right)}. \tag{33}$$

We could also obtain this quantity if we integrate profits to each variety over all imported varieties. Equation (33) determines the expected benefit of retailing in equation (19) because $\int_{\bar{\varphi}_{do}} M_{do}(\varphi) \, dG(\varphi) = \Pi_{do}^m / (r + \lambda) \left[1 - u_{do}/(1 - i_{do})\right] N_o^x$. Hence, the expected value of becoming a matched retailer is equal to the discounted average flow profits to retailers. Despite the value of posting a vacancy being driven to zero by free entry, $V_{do} = 0$, flow profits, Π_{do}^m, are always positive as long as retailers' search costs, c_{do}, are positive, so importers can recoup the costs expended while searching.

6 U.S. and China calibration and model fit

This section presents our calibration methodology, intuition for parameter identification, and model fit. This calibration provides a realistic economic environment that we use in section 7 to undertake quantitative general equilibrium experiments and to decompose the analytic results about welfare changes and consumption elasticities from sections 5.1 and 5.2, respectively. We use data for China and the United States in 2016 to demonstrate the effects of trade policy in a model with and without search frictions. For simplicity, we study a world economy with only two countries but our procedure can be generalized to include more trading partners or a different time period.

6.1 Calibration methodology

The calibration proceeds in two steps. First, we externally calibrate parameters that can be normalized or that are standard in the literature. Table 1 summarizes these parameters. As pointed out by Shimer (2005), we could find alternative values for retailers' flow cost of search such that the equilibrium is unchanged for different values of the matching efficiency, ξ, and so we normalize this efficiency to one. Following Drozd and Nosal (2012), Eaton et al. (2014), and Eaton et al. (2016), we benchmark the producers' bargaining power, β, at 0.5. We set the elasticity of substitution between differentiated varieties, σ, to six, consistent with evidence in Anderson and van Wincoop (2004) and Broda and Weinstein (2006). This elasticity of substitution implies a final sales price markup of 20 percent over the marginal production cost of each variety. We assume a Pareto productivity distribution with the firm-size distribution governed by $\theta/(\sigma-1)$ equal to 1.06, which is consistent with estimates from Axtell (2001) and implies θ equals 5.3. We set the fraction of consumption expenditure spent on differentiated goods, α, to 0.5.

We parameterize the iceberg cost as a function of tariffs and distance:

$$\tau_{do} = a_1^o \times \text{tariff}_{do} \times \text{distance}_{do}^{a_2}. \tag{34}$$

In 2016, effective ad valorem tariff rates on Chinese imports from the United States were 6.3 percent and they were 2.9 percent on U.S. imports from China, according to the World Integrated Trade Solution database (WITS) published by the World Bank (WB, 2019b). There are no domestic tariffs so that $\text{tariff}_{uu} = \text{tariff}_{cc} = 1$. The symmetric distance between the U.S. and China is the population-weighted distance normalized by the U.S. internal distance from Head, Mayer, and Ries (2010). The parameters a_1^o and a_2 will be internally calibrated. Parameters of the model are at annual frequency and we set the annual interest rate to 5 percent.

The second step in our approach internally calibrates the remaining parameters by solving a mathematical program with equilibrium constraints (MPEC) following Dubé, Fox, and Su (2012) and Su and Judd (2012). MPEC simultaneously recovers parameters of a model and solves for the accompanying equilibrium endogenous variables. Our parameters will minimize the distance between moments in the data and the model subject to constraints that define the model's equilibrium:

$$\begin{aligned} \left(\hat{\Omega}, \hat{\Phi}\right) = & \underset{\Omega,\Phi}{\arg\min} \left(M\left(\Omega,\Phi\right) - M\right)' W \left(M\left(\Omega,\Phi\right) - M\right) \\ \text{subject to } & \Gamma\left(\Phi;\Omega\right) = 0 \\ & \Psi\left(\Phi,\Omega\right) \leq 0, \end{aligned} \tag{35}$$

in which $\hat{\Omega}$ are internally calibrated parameters, $\hat{\Phi}$ are endogenous variables solving the model at parameter values $\hat{\Omega}$, $M\left(\Omega,\Phi\right)$ are the moments implied by the model at parameters Ω and endogenous variables Φ, M are observed moments, and W is a weighting matrix. $\Gamma\left(\Phi;\Omega\right)$ captures the equilibrium conditions defined in section 4.3, which depend on the endogenous variables, Φ, for any given set of parameters Ω, and these conditions must hold with equality. $\Psi\left(\Phi,\Omega\right)$ defines nonlinear equilibrium and parameter inequality constraints, examples of which are that the idle rate i_{do} cannot be negative in equilibrium and that the elasticity of the matching function with respect to the number of searching producers is $\eta \in [0,1]$. Appendix C.1 describes these inequality constraints and contains numerical details related to solving equation (35).

6.2 Intuition for parameter identification

The parameters that minimize the MPEC in equation (35) are jointly determined by all the moments, but it is informative to relate certain moments to particular parameters as we discuss intuition for parameter identification. This section provides that intuition, and Table 2 summarizes this discussion.

The search frictions in our model are governed by retailers' flow search cost, c_{do}. If the fraction of matched exporters is low, it implies that there are few searching retailers, market tightness is low, and that international search costs are high. Consequently, we use the fact that 21 percent of Chinese firms export (WB, 2018) and that 6 percent of U.S. firms export to China (CB, 2016a,b) to identify c_{uc} and c_{cu}, respectively. The labor-search literature also uses labor underutilization (the unemployment rate) to identify search costs (Pissarides, 2009). The analogous measure in our model is manufacturing capacity underutilization. We target 75 and 74 percent manufacturing capacity utilization in the United States and China in 2016, respectively, to inform c_{uu} and c_{cc} (FRB, 2016; NBSC, 2016a). We also assume that international search costs are simply the domestic search cost plus a symmetric international premium so that $c_{uc} = c' + c_{uu}$, $c_{cu} = c' + c_{cc}$, and $c' \geq 0$. This symmetry assumption implies, for example, that the cost a Chinese retailer faces to search for a U.S. producer is the same as the cost that that retailer would face to find a Chinese producer plus c'. We present further details about identification of the retailers' flow search costs in appendix C.2.

The elasticity of the matching function with respect to the number of searching producers, η, affects the finding rates for producers and retailers in every *do* market and is identified by the joint level of U.S. and Chinese capacity utilization along with the percent of U.S. (Chinese) firms exporting to China (the United States).

The average duration of a Chinese and U.S. trading relationship is about five quarters (Monarch and Schmidt-Eisenlohr, 2018, Figure 9), which identifies our separation parameter, λ, because average match duration in the model is $1/\lambda$. This observed expected duration is also broadly consistent with survival probabilities among Colombian-U.S. trading relationships (Eaton et al., 2014).

The Doing Business Indicators (DBI) database (WB, 2019a) informs the cost of business startups and the fixed costs of foreign trade, f_{do}, as in di Giovanni and Levchenko (2012) and di Giovanni and Levchenko (2013). We discuss the details of this approach, along with the calibration of l_{do} and s_{do}, in appendix C.3.

The relative magnitude of imports IM_{cu} and IM_{uc} informs a_1^o in equation (34), which serves as a proportional constant that captures any relative U.S. nontariff input cost premium. For example, a high IM_{cu}/IM_{uc} ratio implies that input costs in the United States are high relative to China. Trade in both directions between China and the U.S. together with the level of absorption of domestic production (IM_{uu} and IM_{cc}), as well as tariffs and distance between the two countries, identifies the elasticity on distance, a_2. We define IM_{uu} and IM_{cc} as manufacturing value added minus merchandise exports plus merchandise imports similar to Dekle, Eaton, and Kortum (2008).

Labor endowments, L_c and L_u, and the exploration costs, e_u^x and e_c^x, are informed by the levels of gross domestic products (GDPs), aggregate consumptions, and the ratio of consumption to GDP in China and the U.S., as reported in the national accounts of each country (BEA, 2016a; WB, 2016; BEA, 2016b; NBSC, 2016b).

In section 2.3.2, we assume that the minimum draw from the productivity distribution is one. The minimum draw informs the flow value associated with being idle, h_{do}, because we assume that if search costs, tariffs, and the U.S. input cost premium are zero and countries are in autarky, then the exporting threshold in equation (18) is at its minimum. These steps are similar to the fixed production cost normalization in di Giovanni and Levchenko (2013). Appendix C.4 has details about this normalization.

6.3 Parameter values

The parameters that minimize equation (35) are presented in Table 2. In this section we discuss the most relevant parameters, including the elasticity of the matching function and retailers' search costs. Our internally calibrated parameters point to important search frictions between U.S. and Chinese firms.

We internally calibrate the elasticity of the matching function with respect to the number of searching producers, η, to 0.45. This value is similar to the elasticity of the matching function with respect to the number of unemployed workers estimated in the macro-labor literature, which lies between 0.5 and 0.7 (Petrongolo and Pissarides, 2001). This result suggests that the matching technology might be similar in the two contexts.

The levels of the retailer search costs, c_{do}, reported in Table 2 best match the observed moments for our normalized value of the matching efficiency, $\xi = 1$. Another normalization, however, would yield identical values for all parameters except c_{do} and the same equilibrium except for κ_{do}, which is unit-less and inherits its level from the normalization (Shimer, 2005). In contrast, average retailer search costs, $c_{do}/\chi\left(\kappa_{do}\right)$, do not depend on the normalization of ξ and are about \$30 million in the domestic markets (uu and cc), about \$50 million in the uc market, and about \$15 million in the cu market. As a fraction of average firm revenue, the costs in the domestic markets (uu and cc) are about 4 percent and are similar in magnitude to the labor costs of posting vacancies (Silva and Toledo, 2009). Average search costs in the international markets (cu and uc) are about 9 percent of average firm revenue. It is intuitive that average international search costs as a fraction of average firm revenue are higher than average domestic search costs. Total retailer search costs paid by all retailers are about 0.1 percent of GDP in each country. Lastly, our calibrated search costs are difficult to compare with estimates from the structural literature that uses micro-level data (Eaton et al., 2014) because the models and parameter identification strategies differ substantially.

Without considering the effect of search frictions, the elasticity of trade flows with respect to distance in our calibration is $-0.265 = -\theta \times a_2$ because the effect of iceberg trade costs on trade flows is $-\theta = -5.3$ (equation 29) and the effect of distance on iceberg trade costs is $a_2 = 0.05$ (equation 34). This value is less negative than the mean estimate of -0.9 from a meta-analysis of gravity equation regressions in Disdier and Head (2008). Our lower distance elasticity is consistent with our results in appendix C.5, which shows that any reduced-form gravity equation that does not include adequate proxies for the effects of search frictions on trade prices and matched rates would suffer from negative omitted variable bias (proposition 9). Finally, our calibrated iceberg trade costs, τ_{do}, are 1.1, 1.3, 1.1, and 1.0 in the uu, cu, uc, and cc markets, respectively.

As in Ghironi and Melitz (2005) and di Giovanni and Levchenko (2012), our exploration costs, e_d^x, are well above the effective entry costs in the domestic market, F_{dd}. For example, in the United States, exploration costs are about six times higher than the effective entry cost in the uu market. Our calibrated values of the business start-up costs and fixed foreign trade costs, f_{do}, are small relative to retailers' search costs, c_{do}. As we mention in appendix C.3, the quantitative results depend on the effective entry cost, F_{do}, and less so on its individual components. Changing the fixed cost, f_{do}, or relaxing our restrictions on producer search costs, l_{do}, or sunk costs, s_{do}, results in similar quantitative results.

6.4 Model fit

There are 20 parameters in Table 2 but the model has 15 parameters that are internally calibrated because we have one restriction on the c_{do}, and the h_{do} does not target empirical moments. These 15 parameters are internally calibrated to match 23 empirical moments, subject to 18 equilibrium conditions defining 18 endogenous variables.

Table 3 presents the moments from the model using the calibrated parameter values from Table 2 and shows that the model matches the calibration targets well. The calibrated model matches fractions of exporting firms, manufacturing capacity utilization rates, economic aggregates, such as GDP, consumption, and international trade flows, as well as business start-up costs, fixed foreign trade costs, and separation rates among trading partners. To some extent, the model implies domestic absorption of production in China (the United States) that is lower (higher) than in the data because we use observed data, which include many countries, to calibrate a model with only two countries. Nevertheless, the model provides a realistic economic environment for general equilibrium exercises, a topic we pursue in the next section.

7 Quantitative general equilibrium results

7.1 Welfare analysis

In this section we present several exercises that emphasize the important role of search frictions for aggregate welfare. In section 7.1.1 we decompose the response of ex-ante welfare to a unilateral tariff using our analytic results in proposition 5. Table 4 presents this decomposition for China following a tariff increase on goods from the United States. In the subsequent two sections we examine how welfare changes when we eliminate search frictions and what changes in search frictions mimic a 10 percent increase in bilateral tariffs. Tables 5 and 6, respectively, present a summary of these additional results.

7.1.1 Decomposing the welfare response to unilateral tariffs

Search frictions attenuate welfare changes in response to a 10 percent unilateral tariff by about 85 percent relative to an environment without search frictions. This occurs because, relative to the standard model, both the domestic consumption share and the domestic producer matched rate attenuate the response of welfare. This quantifies our analytic results from section 5.1.

To compute the equilibrium in our model without search frictions, we set all parameters to the baseline values listed in Table 2, but reduce retailers' search costs to zero in domestic and foreign markets, $c_{do} = 0,\ do = \{uu, uc, cu, cc\}$, reproducing the model of Chaney (2008). Because search is free, retailers enter the search market, raising market tightness and sending the contact rates for producers to infinity, $\kappa_{do}\chi(\kappa_{do}) \to \infty$, $\forall do$. As a result, producers find retailers instantly and the fraction of unmatched producers in each market falls to zero.

Column (1) of Table 4 shows that without search frictions, Chinese welfare falls by 1.8 percent in response to a 10 percent tariff increase on imported U.S. goods. This reduction in welfare is governed by the 22 percent increase in the domestic consumption share, along with the parameters α, θ, and σ, and is consistent with the results in Arkolakis, Costinot, and Rodríguez-Clare (2012). This model features no search frictions so the domestic matched rate is always one.

Column (2) of Table 4 shows that in the model with search frictions, Chinese welfare falls by 0.3 percent when China raises unilateral tariffs on U.S. goods by 10 percent. Decomposing this welfare reduction using our analytic results in proposition 5 suggests that welfare changes for three reasons. First, the domestic consumption share rises by about 3.7 percent because foreign goods are more expensive after the tariff increase and this reduces welfare to 99.7 percent of the pre-tariff level. Second, the tariff raises the Chinese price index, allowing Chinese retailers to earn higher revenue from Chinese consumers. The increased value of being a matched retailer in the Chinese market leads to more retailer entry and a higher matched rate for Chinese producers. This higher matched rate serves to attenuate some of the reduction in welfare caused by the lower domestic consumption share. Quantitatively, the tariff raises the domestic market matched rate by 0.6 percent, which boosts welfare by 0.06 percent. Third, the change in Chinese aggregate consumption is quantitatively trivial in both the model with and the model without search.

Comparing results with and without search frictions shows that the economy with search frictions exhibits a smaller decline in welfare in response to the same increase in tariffs. Overall, search frictions attenuate the welfare reduction by about 85 percent and this is largely driven by the change in the domestic consumption share in our calibration.

The domestic consumption share response is smaller in the model with search because the matched rate in the foreign market, which is always less than one, serves to mute the response of the domestic price index to tariff changes (equation 25). For example, extremely high search costs would result in only a few matched firms being affected by tariffs, and would dramatically reduce tariffs' effects on the price index. Moreover, tariff changes endogenously reduce the matched rate in the foreign market, which further mutes the price index relative to a model without search frictions. All endogenous quantities change in response to tariffs and some of those changes work in the opposite direction of the price index's effect on the consumption share. In our calibration, those effects are much smaller than the direct effect of tariffs on the price index.

7.1.2 Eliminating retailers' search costs

Entirely eliminating domestic and international search frictions raises U.S. welfare by about 5.5 percent and Chinese welfare by about 8.7 percent. More realistically, we show that reducing international search frictions to domestic levels raises U.S. welfare by 1.4 percent and Chinese welfare by about 2.6 percent. These are sizable effects and are similar in magnitude to the changes in welfare that would be associated with moving to autarky in a simple Armington model with one or multiple sectors (Costinot and Rodríguez-Clare, 2014).

At first we set all parameters to the baseline values listed in Table 2, but reduce retailers' search costs to zero in domestic and foreign markets, $c_{do} = 0 \; \forall do$, as in the frictionless example of the previous section. Column (1) of Table 5 reports that the value of imports into the United States from China increases by 230 percent, and the value of imports into China from the United States rises by about 800 percent. Welfare in the United States is roughly 5.5 percent higher, and welfare in China rises by about 8.7 percent. Chinese welfare rises significantly more than U.S. welfare because the United States is a relatively large trading partner.

Eliminating all search frictions is an extreme case, so in the next exercise we set all parameters to the baseline values listed in Table 2, but reduce retailers' international search costs to their domestic levels, $c_{do} = c_{dd}$. Column (2) of Table 5 reports that the value of imports into the United States from China increases by 210 percent, and the value of imports into China from the United States rises by about 500 percent. Welfare in the United States is 1.4 percent higher and welfare in China rises by about 2.6 percent.

7.1.3 Replicating tariffs' effects with higher search costs

Increasing retailers' average search costs by no more than 6 percent mimics reductions in trade flows and aggregate welfare of a 10 percent increase in bilateral tariffs. Thus, the recent focus on firm-to-firm relationships in the trade literature (Eaton et al., 2014; Monarch and Schmidt-Eisenlohr, 2018; Heise, 2016) is warranted because these trading relationships, and the costs paid to form them, have economically significant implications for aggregate quantities.

We first quantify the welfare losses associated with a 10 percent increase in bilateral tariffs, so that $\tau'_{do} = 1.1 \times \tau_{do}$ for $d \neq o$. These results appear in column (1) of Table 6. Solving the model with these higher bilateral tariffs, but keeping all other parameters at the baseline values from Table 2, implies that welfare in both countries falls. The United States experiences a 0.4 percent reduction in welfare, whereas China's welfare falls by 0.3 percent. The value of imports into the United States from China falls by about 60 percent, whereas the value of imports into China from the United States falls by about 70 percent.

To understand the importance of importing retailers' search costs, we set all parameters to the baseline values in Table 2, but select flow costs of search for importing retailers, c_{cu} and c_{uc}, so that trade values decline by the same amount as in the experiment with a 10 percent increase in bilateral tariffs. Matching the reduction in trade flows in the cu and uc markets requires raising c_{cu} and c_{uc} by about 140 and 155 percent, respectively. As discussed throughout section 6, more important than the level of these parameters are the associated equilbrium values of the average search costs, which rise by no more than six percent in both markets. This increase in U.S. retailers' search costs lowers retailer entry and reduces producer finding rates, thereby raising the fraction of unmatched producers. These higher unmatched rates have first-order effects on welfare, which fall by the same amount in the two countries as in the example with higher tariffs. We present other equilibrium quantities in column (2) of Table 6.

7.2 Analysis of consumption and trade elasticities

Search frictions more than double the consumption elasticity to -11.9 from -5.3 in a model without them. This is because the change in the domestic and international producer matched rates magnifies the effects of a tariff increase on consumption shares. This quantifies our analytic results from section 5.2 in which we show that the consumption elasticity in our model is at least as negative as the analogous elasticity in the standard trade model (proposition 6).

First, we set all parameters to the baseline values in Table 2 but remove all search costs ($c_{do} = 0\ \forall do$). In this frictionless calibration, a 10 percent (9.5 log percent) unilateral tariff on imports into China from the United States ($\tau'_{cu} = 1.1 \times \tau_{cu}$) reduces the consumption share, C_{cu}/C_{cc}, which is equivalent to the import share, by about 50 log percent. This reduction implies that the consumption elasticity is -5.3 (column 1 of Table 7), which is exactly equal to the negative of the Pareto shape parameter ($-\theta$) that we derive analytically in B.2.17, and matches the predictions in Chaney (2008, p. 1716).

In our model with search frictions, a 10 percent (9.5 log percent) unilateral tariff on imports into China from the United States reduces the consumption share, C_{cu}/C_{cc}, by about 113 log percent, implying that the elasticity is -11.87 (column 2 of Table 7). The elasticity for trade flows is slightly more negative than the consumption elasticity at -11.89 because it also includes the effect on the changes in the markup term, $b(\cdot)$.

We can decompose the consumption elasticity with search frictions into the (negative of the) Pareto shape parameter ($-\theta$), the elasticity of the matched rate in the cu and cc

markets with respect to τ_{cu}, the effect of the effective entry cost in the cu market, and the elasticity of the number of producers in China and the United States with respect to τ_{cu}. This decomposition relies on proposition 6 and the general expression in appendix B.2.14.

The decomposition highlights a large decline, about 58 log percent, in the fraction of U.S. producers that are matched with Chinese retailers, implying an elasticity in the cu market with respect to τ_{cu} of about -6 (line 4, column 2 of Table 7). This decline in the matched rate results from higher τ_{cu} tariffs reducing the benefit to Chinese retailers of being matched with U.S. producers, leading to less Chinese retailer entry, lower market tightness, and a lower finding rate in the cu market. This lower finding rate reduces the matched rate in the cu market.

The decomposition shown in proposition 6 also has an indirect protectionism effect that operates through the matched rate in the domestic, cc, market. As tariffs on U.S. imports rise, the Chinese price index increases, making it more valuable to be a matched domestic Chinese retailer, which leads to entry into the domestic retailing market. Greater entry raises the Chinese domestic finding rate for Chinese producers and subtracts from the standard elasticity. In the calibration of our model, this protectionism effect raises the matched rate in the cc market by about 0.6 log percent.

The decline in the cu matched rate and the increase in the cc matched rate imply that the effective entry cost rises in the cu market and falls in the cc market. These effects are small in our calibration but also weigh negatively on the consumption and trade elasticities. Finally, the effects of tariff changes on the numbers of producers in the U.S. and Chinese markets are trivial in our calibration.

Altogether, the consumption share elasticity in our model is -11.87, which is 6.57 more negative and about 124 percent larger in magnitude than in the model without search frictions. Furthermore, while the elasticity in our framework with search frictions is more negative than the Pareto shape parameter would imply, it remains within the range of values estimated in prior work (Eaton and Kortum, 2002; Imbs and Mejean, 2015).

Lastly, while explaining the rapid increase in worldwide trade during the past few decades is not the goal of our paper, search models with endogenous market tightness have the ability to magnify the effect of tariffs on trade flows in a way that is similar to the role of vertical specialization in Yi (2003).

8 Conclusion

The international trade literature has recently made substantial progress in modeling and estimating the costs of forming relationships at the micro-level (Eaton et al., 2014; Monarch and Schmidt-Eisenlohr, 2018; Heise, 2016). However, far less is known about how these costs affect aggregate quantities in a general equilibrium framework. To improve understanding on this score, we have combined canonical models from search and trade in a rich and tractable framework, which nests workhorse models of trade (Melitz, 2003; Chaney, 2008) when we remove search frictions. This framework shares the central tenet of any search model: In equilibrium, there exists a mass of unmatched agents who are actively looking for partners. This simple observation leads to profound implications because the product varieties associated with these unmatched producers cannot be consumed and are therefore absent from indirect utility (welfare), the price index, trade flows, and the levels of other aggregates. Additionally, if the mass of unmatched producers is endogenous, the absence of these varieties affects not only the levels of aggregates, but also their changes.

Specifically, we show that changes in the mass of unmatched varieties have first-order implications for welfare responses to any foreign shock. As a result, we generalize the welfare findings of Arkolakis, Costinot, and Rodríguez-Clare (2012) to include search frictions. We also provide a tractable baseline model for assessing the welfare implications of searching for new trading partners. This baseline can be compared to richer frameworks like Chaney (2014) and Eaton et al. (2016) that leave aggregate welfare implications to future work.

In particular, we find that search frictions attenuate the response of welfare to changes in tariffs. This result follows because higher tariffs, for example, result in a higher price index, which makes being a retailer in the domestic market more valuable and induces more retailers to enter the domestic market. With more retailers in the market, the rate at which domestic producers find domestic retailers increases, increasing domestic consumption and attenuating the welfare losses from higher tariffs. The effects of tariffs could also be attenuated through smaller changes in the domestic consumption share because tariff changes only affect matched varieties instead of all varieties above the exporting threshold.

In addition to these welfare results, we show that the consumption and import elasticities in our model are at least as negative as the analogous elasticities in a model without search frictions. Search frictions magnify these elasticities because as a destination raises tariffs on products from a specific country of origin, for example, retailers in the destination country have less incentive to enter the search market. Having fewer retailers implies a looser search market and a higher unmatched rate for producers, which reduces consumption and imports even more than in a model without search.

We also find that adding search frictions reduces aggregate trade flows in three ways. First, they reduce the mass of varieties traded, because some producers are unmatched. Second, search frictions introduce a markup between import and final sales prices, which reduces the value of imports. Third, they raise the effective entry cost. All of these effects are evident in the closed-form gravity equation derived from our model.

Beyond aggregate predictions, our framework provides a micro-foundation for the import price of each imported variety, which nests standard pricing results. The model also provides a micro-foundation for the up-front costs firms face when entering foreign markets. These up-front costs can include the fixed, search, opportunity, and sunk costs of serving a foreign market and depend on the rate at which producers find retailers.

When calibrated to U.S and Chinese data, our model implies that search frictions play an important quantitative role for welfare, trade flows, and the consumption elasticity. We find that reducing international search costs to their domestic level would increase U.S. and Chinese welfare by 1.4 percent and 2.6 percent, respectively. Eliminating search frictions entirely would increase U.S. welfare by about 5.5 percent and Chinese welfare by about 8.7 percent. Search frictions also attenuate Chinese welfare responses to tariff increases on U.S. goods by about 85 percent. Finally, through their effect on the unmatched rate, search frictions raise the consumption elasticity by about 6 log percentage points and account for about half of the overall consumption elasticity.

Following the development of search models in labor and monetary economics, we propose three specific directions for future research. First, we have focused on the steady state of the model, but the framework is dynamic and could be extended to include the transition path after a relevant exogenous shock. This direction would dovetail nicely with Chaney (2014) and would go a long way toward providing a dynamic, continuous-time model that admits easy aggregation and retains the basic features of Melitz (2003). Second, the model can be extended to incorporate endogenous separations, in the spirit of Mortensen and Pissarides (1994), so that larger, more productive firms are in more stable trading relationships. Third, other matching and bargaining protocols, as in Burdett and Judd (1983) and Moen (1997), may present alternative implications for the mass of unmatched varieties relative to our model.

Locating and building connections with overseas buyers is a prevalent and costly barrier to exporting. We formalize costly search for international partners as a goods-market friction between producers and retailers. Our tractable setting provides a baseline for analyzing the aggregate implications of search frictions in models of trade.

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Table 1: Externally calibrated parameters

Parameter	**Value**	**Reason**
Efficiency of matching function (ξ)	1	Normalization
Producers' bargaining power (β)	0.5	Benchmark
Elasticity of substitution (σ)	6	Demand curve estimation
Pareto shape parameter (θ)	5.3	US firm size distribution
Cobb-Douglas power (α)	0.5	Benchmark
Tariffs rate faced by US in US (τ_{uu})	1	Normalization
Tariffs rate faced by US in CH (tariff_{cu})	1.063	WITS database
Tariffs rate faced by CH in US (tariff_{uc})	1.029	WITS database
Tariffs rate faced by CH in CH (τ_{cc})	1	Normalization
Distance between US and CH (distance_{cu})	6.03	Relative to US internal distance
Risk-free rate (r)	0.05	5% annual interest rate

Note: Externally calibrated parameters of the model are at annual frequency. See section 6.1 for further details. 'WITS' stands for World Integrated Trade Solution (WB, 2019b), "US" stands for the United States, and "CH" stands for China.

Table 2: Internally calibrated parameters

Parameter	**Value**	**Main source of identification**
US domestic search cost (c_{uu})	\$16.9mil	US mfg. capacity utilization rate
CH importers' search cost (c_{cu})	\$166.6mil	Percent of US firms exporting to CH
US importers' search cost (c_{uc})	\$168.2mil	Percent of CH firms exporting to US
CH domestic search cost (c_{cc})	\$15.3mil	CH mfg. capacity utilization rate
Elasticity of matching function (η)	0.45	Percent of firms exporting; capacity utilization
Separation rate (λ)	0.8	Average relationship duration
US domestic fixed cost (f_{uu})	\$550	Cost of business start up in US
US export fixed cost (f_{cu})	\$683	Fixed foreign trade costs (CH-US)
CH export fixed cost (f_{uc})	\$664	Fixed foreign trade costs (US-CH)
CH domestic fixed cost (f_{cc})	\$28	Cost of business start up in CH
US input cost premium (a_1^u)	1.1	Relative imports (IM_{cu}/IM_{uc})
Effect of distance on trade costs (a_2)	0.05	Imports; absorption of domestic production (IM_{do})
Labor endowment in US (L_u)	\$18.2tril	Consumption and GDP in US
Labor endowment in CH (L_c)	\$10.9tril	Consumption and GDP in CH
US exploration cost (e_u^x)	\$471mil	US consumption to GDP share
CH exploration cost (e_c^x)	\$1.6bil	CH consumption to GDP share
US producers' idle flow payoff (h_{uu})	\$41mil	Absent other barriers, no US-US firms are idle
US exporters' idle flow payoff (h_{cu})	\$26mil	Absent other barriers, no CH-US firms are idle
CH exporters' idle flow payoff (h_{uc})	\$26mil	Absent other barriers, no US-CH firms are idle
CH producers' idle flow payoff (h_{cc})	\$41mil	Absent other barriers, no CH-CH firms are idle

Note: Internally calibrated parameters of the model are at annual frequency. The middle column of this table presents the value of the calibrated parameter. The column on the right provides the main source of identification. The levels of the retailer search costs, c_{do}, in this table depend on the normalization of the matching efficiency, ξ, as in Shimer (2005). We report average retailer search costs, $c_{do}/\chi\left(\kappa_{do}\right)$, which have intrinsic meaning, in section 6.3, along with other parameter values. As a fraction of average firm revenue, these average search costs are about 4 percent in the domestic markets and about 9 percent in the international markets. We discuss the calibration methodology in section 6.1 and intuition for parameter identification in section 6.2. "CH" stands for China, "US" stands for the United States, and "GDP" stands for Gross Domestic Product.

Table 3: Model fit

Moment in the data	**Data**	**Model**
US mfg. capacity utilization rate	75%	72%
Percent of US firms exporting to CH	6%	6%
Percent of CH firms exporting to US	21%	21%
CH mfg. capacity utilization rate	74%	73%
Cost of business start up in US	\$550	\$550
Fixed foreign trade costs (CH-US)	\$683	\$683
Fixed foreign trade costs (US-CH)	\$664	\$664
Cost of business start up in CH	\$28	\$28
US absorption of domestic prod. (IM_{uu})	\$2.8tril	\$5.8tril
CH imports from US (IM_{cu})	\$116 bil	\$111bil
US imports from CH (IM_{uc})	\$463 bil	\$479bil
CH absorption of domestic prod. (IM_{cc})	\$2.7tril	\$2.1tril
US dom. absorp. consump. ratio (IM_{uu}/C_u)	22.2%	44.3%
CH-US imports consump. ratio (IM_{cu}/C_u)	0.9%	0.8%
US-CH imports consump. ratio (IM_{uc}/C_c)	10.5%	10.5%
CH dom. absorp. consump. ratio (IM_{cc}/C_c)	61.5%	45.6%
Average relationship duration	1.25	1.24
GDP in US	\$18.7tril	\$19.1tril
GDP in CH	\$11.2tril	\$11.4tril
Consumption in US	\$12.8tril	\$13.2tril
Consumption in CH	\$4.4tril	\$4.6tril
US consumption to GDP share	68%	69%
CH consumption to GDP share	39%	40%

Note: The model matches the empirical targets relatively well. The middle column of this table presents the value of the moment in the data. The column on the right presents the value of the equivalent moment in the model at the calibrated parameter values in Table 2. We discuss model fit in section 6.4. "CH" stands for China, "US" stands for the United States, and "GDP" stands for Gross Domestic Product.

Table 4: Decomposing the Chinese welfare response to a unilateral tariff increase

Determinants of welfare change	(1) No search frictions and 10% unilateral tariff	(2) Baseline search frictions and 10% unilateral tariff
Pre-tariff dom. consump. share (λ_{cc})	0.277	0.474
Post-tariff dom. consump. share (λ'_{cc})	0.337	0.491
Ratio of consump. shares $\left(\hat{\lambda}_{cc} = \lambda'_{cc}/\lambda_{cc}\right)$	1.215	1.037
Dom. consump. shares' effect $\left(\hat{\lambda}_{cc}^{-\frac{\alpha}{\theta}}\right)$	0.982	0.997
Pre-tariff dom. matched rate $\left(1 - \frac{u_{cc}}{1-i_{cc}}\right)$	1	0.734
Post-tariff dom. matched rate $\left(1 - \frac{u_{cc}}{1-i_{cc}}\right)'$	1	0.738
Ratio of dom. matched rates $\widehat{\left(1 - \frac{u_{cc}}{1-i_{cc}}\right)}$	1	1.006
Dom. matched rates' effect $\widehat{\left(1 - \frac{u_{cc}}{1-i_{cc}}\right)}^{\frac{\alpha}{\theta}}$	1	1.0006
Pre-tariff dom. consump. level (C_c)	\$4.6tril	\$4.6tril
Post-tariff dom. consump. level (C'_c)	\$4.6tril	\$4.6tril
Ratio of dom. consump. levels $\left(\hat{C}_c = C'_c/C_c\right)$	1	1
Dom. consump. levels' effect $\left(\hat{C}_c^{1+\frac{\alpha}{\theta}\left(1-\frac{\theta}{\sigma-1}\right)}\right)$	1	1
Welfare as fraction of pre-tariff welfare $\left(\hat{W}_c\right)$	0.982	0.997
Welfare percent change $\left(100 \times \left[\hat{W}_c - 1\right]\right)$	-1.82	-0.29

Note: Search frictions attenuate the Chinese welfare response to a 10 percent tariff by about 85 percent, lowering the welfare loss from 1.8 percent to 0.3 percent. The table presents equilibrium variables in response to a 10 percent increase in unilateral tariffs on imports to China from the United States. Using proposition 5, we know that the complete welfare response in our baseline calibration is:

$$\hat{W}_c = \underbrace{\hat{\lambda}_{cc}^{-\frac{\alpha}{\theta}}}_{\substack{\text{Dom. consump.} \\ \text{share effect}}} \underbrace{\widehat{\left(1 - \frac{u_{cc}}{1 - i_{cc}}\right)}^{\frac{\alpha}{\theta}}}_{\substack{\text{Dom. matched} \\ \text{rate effect}}} \underbrace{\hat{C}_c^{1+\frac{\alpha}{\theta}\left(1-\frac{\theta}{\sigma-1}\right)}}_{\substack{\text{Dom. consump.} \\ \text{level effect}}}$$

Column (1) presents the response without search frictions, which is the same as Arkolakis, Costinot, and Rodríguez-Clare (2012) and is completely determined by the ratio of the domestic consumption shares and model parameters α, θ, and σ. Some rows in column (1) are exactly 1 because those factors do not change in a model without search frictions. Column (2) presents the decomposition of the effect in our model with search frictions. Domestic consumption rises by about 4 percent after the tariff increase and this reduces welfare to 99.7 percent of the pre-tariff level. Protection of the domestic market raises the domestic matched rate by 0.6 percent and serves to boost welfare by 0.06 percent, offsetting some of the tariff's negative effects. See section 7.1.1 for further details.

Table 5: Changes in welfare, imports, and the unmatched rate when search frictions are reduced

	Experiment (1) No search frictions	Experiment (2) Reducing int'l search frictions to domestic search frictions
US welfare (%Δ)	5.5	1.4
Chinese welfare (%Δ)	8.7	2.6
US imports from China (%Δ)	230	210
Chinese imports from US (%Δ)	816	521
Unmatched rate in US-US mkt (pp. Δ)	-28	2
Unmatched rate in US-CH mkt (pp. Δ)	-79	-47
Unmatched rate in CH-US mkt (pp. Δ)	-94	-37
Unmatched rate in CH-CH mkt (pp. Δ)	-27	4

Note: Search frictions play an important role in the level of welfare. The table presents deviations from the baseline calibration in section 6. Column (1) eliminates search frictions altogether and shows that the associated welfare gains are large. Column (2) reduces retailers' search costs in international markets to their domestic levels. For example, US retailers' search cost for a partner in China are reduced to search costs for a partner in the US. See section 7.1.2 for further details. "%Δ" stands for percent change. "pp. Δ" stands for percentage point change. "CH" stands for China and "US" stands for the United States.

Table 6: Changes in welfare, imports, and the unmatched rate in response to tariff and search cost changes

	Experiment (3) Baseline search frictions and 10% bilateral tariff	Experiment (4) Search costs equiv. to 10% bilateral tariff
US welfare (%Δ)	-0.4	-0.4
Chinese welfare (%Δ)	-0.3	-0.3
US imports from China (%Δ)	-61	-61
Chinese imports from US (%Δ)	-67	-67
Unmatched rate in US-US mkt (pp. Δ)	-0.6	-0.6
Unmatched rate in US-CH mkt (pp. Δ)	7.8	12.9
Unmatched rate in CH-US mkt (pp. Δ)	2.7	4.0
Unmatched rate in CH-CH mkt (pp. Δ)	-0.4	-0.4

Note: Search frictions play an important role in the response of welfare to tariff changes. The table presents deviations from the baseline calibration in section 6. Columns (1) and (2) present the two exercises in section 7.1.3. Experiment (3) increases bilateral tariffs by 10 percent. Experiment (4) shows that, by affecting the unmatched rate, increases in the average cost for retailers to contact foreign producers attain the same welfare changes as in column (1). "%Δ" stands for percent change. "pp. Δ" stands for percentage point change, "CH" stands for China, and "US" stands for the United States.

Table 7: Decomposing the Chinese consumption and trade elasticities

	(1) No search frictions and 10% unilateral tariff	(2) Baseline search frictions and 10% unilateral tariff
Pareto shape parameter ($-\theta$)	-5.3	-5.3
Elasticity of CH producers	0.00	0.00
Elasticity of US producers	0.00	0.00
Elasticity of the CH-US matched rate	0	-6.12
Elasticity of the CH-CH matched rate	0	0.06
Effect of the CH-US effective entry cost	0	-0.38
Consumption elasticity	-5.3	-11.87
Elasticity of CH-US markup	0	-0.01
Elasticity of CH-CH markup	0	0
Trade elasticity	-5.3	-11.89

Note: Search frictions more than double the consumption and trade elasticities and around 50 percent of the overall consumption or trade elasticity is explained by the elasticity of the matched rate in the *cu* market. The table presents equilibrium variables in response to a 10 percent increase in unilateral tariffs on imports to the United States from China. The decomposition is based on proposition 6, along with (76) and (80) in appendix B.2. Column (1) presents the response of the consumption and trade shares to a foreign tariff shock with no search frictions, which is $-\theta$ (equation 80). Column (2) presents the decomposition of these elasticities into their components in our model with search frictions; the elasticity of the *cu* and *cc* matched rates play an important role in the decomposition even though the effective entry cost and markup terms respond to the tariff increase. The elasticity of the CH-US and CH-CH matched rates and markups in column (1) are exactly zero because these results have no search frictions. The other zeros in the table are rounded to the second decimal point. See section 7.2 for further details.

A Model appendix

A.1 Utility maximization and the ideal price index

A.1.1 Utility maximization

Here we present the solution to the utility maximization problem in section 2.2. The representative consumer's maximization problem can be stated as:

$$\begin{aligned}
\max_{q_d(1), q_{dk}(\omega)} \quad & q_d(1)^{1-\alpha} \left[\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} q_{dk}(\omega)^{\left(\frac{\sigma-1}{\sigma}\right)} d\omega \right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)} \\
s.t. \quad & \\
C_d \quad = \quad & p_d(1)\, q_d(1) + \sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} p_{dk}(\omega)\, q_{dk}(\omega)\, d\omega.
\end{aligned}$$

We can solve this problem by maximizing the following Lagrangian

$$\mathcal{L} = q_d(1)^{1-\alpha} \left[\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} q_{dk}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\alpha 2 \sigma}{\sigma-1}} - \lambda \left[p_d(1)\, q_d(1) + \sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} p_{dk}(\omega)\, q_{dk}(\omega)\, d\omega - C \right].$$

The first-order conditions (FOCs) for the homogeneous good and two arbitrary varieties from the same origin, ω and ω', are:

$$\mathcal{L}_{q_d(1)} = \alpha q_1^{-\alpha} \left[\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} q_{dk}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\alpha \frac{\sigma}{\sigma-1}} - \lambda p_d(1) = 0$$

$$\begin{aligned}
\mathcal{L}_{q_{dk}(\omega)} &= q_d(1)^{1-\alpha} \alpha \left(\frac{\sigma}{\sigma-1} \right) \left[\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} q_{dk}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)-1} \left(\frac{\sigma-1}{\sigma} \right) q_{dk}(\omega)^{\frac{\sigma-1}{\sigma}-1} - \lambda p_{dk}(\omega) = 0 \\
\mathcal{L}_{q_{dk}(\omega')} &= q_d(1)^{1-\alpha} \alpha \left(\frac{\sigma}{\sigma-1} \right) \left[\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} q_{dk}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)-1} \left(\frac{\sigma-1}{\sigma} \right) q_{dk}(\omega')^{\frac{\sigma-1}{\sigma}-1} - \lambda p_{dk}(\omega') = 0.
\end{aligned}$$

Dividing the last two FOCs and performing some algebra yields $q_{dk}(\omega)$ in terms of $q_{dk}(\omega')$:

$$q_{dk}(\omega) = q_{dk}(\omega') \left[\frac{p_{dk}(\omega')}{p_{dk}(\omega)} \right]^{\sigma}.$$

Using the ratio of the first and third FOCs delivers a relationship between $q_d(1)$ and $q_{dk}(\omega')$:

$$q_d(1) = \frac{p_{dk}(\omega')}{p_d(1)} \left(\frac{1-\alpha}{\alpha} \right) \left[\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} q_{dk}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right] q_{dk}(\omega')^{\frac{1}{\sigma}}.$$

Using our solution for $q_{dk}(\omega)$ for the term in brackets yields:

$$\int_{\omega\in\Omega_{dk}} q_{dk}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega = \int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma} d\omega p_{dk}(\omega')^{\sigma-1} q_{dk}(\omega')^{\frac{\sigma-1}{\sigma}}$$

and plugging this in gives

$$q_d(1) = \left(\frac{1}{p_d(1)}\right)\left(\frac{1-\alpha}{\alpha}\right)\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma} d\omega p_{dk}(\omega')^{\sigma} q_{dk}(\omega').$$

Now we can write the budget constraint in terms of $q_{dk}(\omega')$ and after some algebra this gives

$$C_d = \left\{\left(\frac{1}{\alpha}\right)\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma} d\omega p_{dk}(\omega')^{\sigma}\right\} q_{dk}(\omega').$$

So demand for $q_{dk}(\omega')$ is given by

$$q_{dk}(\omega') = \alpha C_d \frac{p_{dk}(\omega')^{-\sigma}}{\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma} d\omega}.$$

There are a couple of things to notice here. The first is that the demand for the CES good $q_{dk}(\omega')$ is not a function of the price of the good $q_d(1)$. Also notice that we can interpret αC_d as the consumer using the fraction of total expenditure from the Cobb-Douglas level of the utility function to define the fraction of total consumption resources that are devoted to this particular variety of the differentiated good.

As we show in appendix A.1.2 the price index for the differentiated goods from origin k to destination d is

$$P_{dk} = \left\{\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma} d\omega\right\}^{\frac{1}{1-\sigma}}$$

and the overall price index for the differentiated goods in country d is

$$P_d = \left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}.$$

This means the demand for each CES variety is the function

$$q_{dk}(\omega) = \alpha C_d \frac{p_{dk}(\omega)^{-\sigma}}{P_d^{1-\sigma}}$$

The demand for the homogeneous good $q_d(1)$:

$$\begin{aligned} q_d(1) &= \left(\frac{1}{p_1}\right)\left(\frac{1-\alpha}{\alpha}\right)\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma}\, d\omega p_{dk}(\omega')^{\sigma} q_{dk}(\omega') \\ &= (1-\alpha)\frac{C_d}{p_d(1)} \end{aligned}$$

which is just the amount $(1-\alpha)C_d$ (Cobb-Douglas) spent on the good that has price $p_d(1)$. These are the demand functions in equation (2) of the main text.

A.1.2 Expenditure minimization and the price index

Here we derive, in full, the price index associated with our utility function. First we deal with the price index for the differentiated goods. Then we obtain the overall price index for the homogeneous and the differentiated goods.

The expenditure minimization problem for the differentiated goods looks as follows:

$$\begin{aligned} \min_{q_{dk}(\omega)} \quad & \sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)\, q_{dk}(\omega)\, d\omega \\ s.t. & \\ U_d^{\rho} &= \sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} q_{dk}(\omega)^{\rho}\, d\omega \end{aligned}$$

in which, for ease of notation, we have temporarily defined $\rho \equiv \dfrac{\sigma-1}{\sigma}$. The following steps resemble the steps taken in Varian (1992) pg. 55.

The Lagrangian is:

$$\mathcal{L} = \sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)\, q_{dk}(\omega)\, d\omega + \lambda\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} q_{dk}(\omega)^{\rho}\, d\omega - U_d^{\rho}\right].$$

The first-order conditions (FOCs) are therefore:

$$\mathcal{L}_{q_{dk}(\omega)} = p_{dk}(\omega) - \lambda\rho q_{dk}(\omega)^{\rho-1} = 0$$

$$\mathcal{L}_{\lambda} : \int_{\omega\in\Omega_{dk}} q_{dk}(\omega)^{\rho}\, d\omega = U_d^{\rho}.$$

Rearrange the first FOC to get:

$$q_{dk}\left(\omega\right)^{\rho} = p_{dk}\left(\omega\right)^{\frac{\rho}{\rho-1}}\left(\lambda\rho\right)^{-\frac{\rho}{\rho-1}}.$$

Put this back into the utility function to get:

$$\left(\lambda\rho\right)^{-\frac{\rho}{\rho-1}} = \frac{U_d^{\rho}}{\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}\left(\omega\right)^{\frac{\rho}{\rho-1}} d\omega}.$$

Substitute this back into the equation above to get:

$$\begin{aligned} q_{dk}\left(\omega\right)^{\rho} &= p_{dk}\left(\omega\right)^{\frac{\rho}{\rho-1}}\left(\lambda\rho\right)^{-\frac{\rho}{\rho-1}} \\ q_{dk}\left(\omega\right)^{\rho} &= p_{dk}\left(\omega\right)^{\frac{1}{\rho-1}}\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}\left(\omega\right)^{\frac{\rho}{\rho-1}} d\omega\right]^{-\frac{1}{\rho}} U_d. \end{aligned}$$

Now we have the demand functions in terms of prices and utility. Substitute this back into the objective function and collect terms to obtain the expenditure function:

$$\begin{aligned} e\left(p_{dk}\left(\omega\right),U_d\right) &= \sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}}^{\infty} p_{dk}\left(\omega\right) q_{dk}\left(\omega\right) d\omega \\ &= \sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}\left(\omega\right) p_{dk}\left(\omega\right)^{\frac{1}{\rho-1}}\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}\left(\omega\right)^{\frac{\rho}{\rho-1}} d\omega\right]^{-\frac{1}{\rho}} U_d d\omega \\ &= U_d\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}\left(\omega\right)^{\frac{\rho}{\rho-1}} d\omega\right]^{\frac{\rho-1}{\rho}}. \end{aligned}$$

Substitute $\rho \equiv \dfrac{\sigma-1}{\sigma}$ back into this expression to get that

$$e\left(p_{dk}\left(\omega\right),U_d\right) = U_d\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}\left(\omega\right)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}.$$

And the ideal price index for the differentiated good from country k to country d is

$$P_{dk} = e\left(p_{dk}\left(\omega\right),1\right) = \left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}\left(\omega\right)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}.$$

Note that this is consistent with

$$P_d = \left[\sum_{k=1}^{O} P_{dk}^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

in which $P_{dk} = \left[\int_{\omega\in\Omega_{dk}}^{\infty} p_{dk}(\omega)^{1-\sigma}\, d\omega\right]^{\frac{1}{1-\sigma}}$, which is equation (3) in the main text.

Now we move on to deriving the overall price index, including the homogeneous good. From our previous work in appendix A.1.1, we know that the optimal quantity demanded of the differentiated good is

$$q_{dk}(\omega) = \alpha C_d \frac{p_{dk}(\omega)^{-\sigma}}{P_d^{1-\sigma}}.$$

We also know that the optimal quantity demanded of the homogeneous good is:

$$q_d(1) = (1-\alpha)\frac{C_d}{p_d(1)}.$$

Using these, we can derive the indirect utility function with some algebra:

$$\begin{aligned} W_d\left(p_d(1), p_{dk}(\omega), C_d\right) &= q_d(1)^{1-\alpha}\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} q_{dk}(\omega)^{\frac{\sigma-1}{\sigma}}\, d\omega\right]^{\alpha\frac{\sigma}{\sigma-1}} \\ &= \left((1-\alpha)\frac{C_d}{p_d(1)}\right)^{1-\alpha}\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}}\left(\alpha C_d\frac{p_{dk}(\omega)^{-\sigma}}{P_d^{1-\sigma}}\right)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\alpha\frac{\sigma}{\sigma-1}} \\ &= \left(\frac{1-\alpha}{p_d(1)}\right)^{1-\alpha}\left(\frac{\alpha}{P_d}\right)^{\alpha} C_d. \end{aligned}$$

Now, we know that our utility function is HOD 1 so our welfare expression can also be written as

$$W_d\left(\Xi_d, C_d\right) = \frac{C_d}{\Xi_d},$$

in which Ξ_d is the overall price index. Setting these two welfare expressions equal to each other gives us:

$$\begin{aligned} \frac{C_d}{\Xi_d} &= \left(\frac{1-\alpha}{p_d(1)}\right)^{1-\alpha}\left(\frac{\alpha}{P_d}\right)^{\alpha} C_d \\ \Xi_d &= \left(\frac{p_d(1)}{1-\alpha}\right)^{1-\alpha}\left(\frac{P_d}{\alpha}\right)^{\alpha}. \end{aligned}$$

A.2 Poisson process

Consider a continuous time Poisson process in which the number of events, n, in any time interval of length t is Poisson distributed according to

$$P\{N(t+s)-N(s)=n\}=e^{-\lambda t}\frac{(\lambda t)^n}{n!}\quad n=0,1,\ldots$$

in which $s,t\geq 0$, $N(0)=0$, and the process has independent increments. The mean number of events that occur by time t is

$$E[N(t)]=\lambda t.$$

Notice that λ is defined in units of time as λ events per t. For example, if producers in our model contact nine retailers every six months, on average, then we could recast our model measured in years with $t=1$ and $\lambda=4.5$ because $\lambda t=9\times 1/2=4.5$.

Define t_1 as the time at which the first event occurs. Using the Poisson process, the probability that the first event occurs after time t equals the probability no event has happened before. Thus, the arrival time of the first event is an exponential random variable with parameter λ given by

$$P\{t_1>t\}=P[N(t)=0]=e^{-\lambda t}.$$

Conversely, the probability the first event occurs between time 0 and time t is $P\{t_1\leq t\}=1-e^{-\lambda t}$. Let t_n denote the time between the $(n-1)$st and nth events, which is also consistent with the definition of t_1 as the time of the first event. Because the Poisson process has independent increments, the distribution of time between any two events, t_n, for $n=1,2,\ldots$ will also be an exponential random variable with parameter λ. The sequence of times between all events, $\{t_n, n\geq 1\}$, also known as the sequence of inter-arrival times, will be a sequence of *i.i.d.* exponential random variables with parameter λ. Given this distribution, the mean time between events is

$$E[t_n]=\frac{1}{\lambda}$$

For example, if producers in our model contact nine retailers every six months, on average, so that $\lambda t=9/2$, then the average time between contacts is $1/\lambda=2/9$ years (or about $365.25\times 2/9=81.17$ days). The arrival time of the nth event, S_n, also called the waiting time, is the sum of the time between preceding events

$$S_n=\sum_{i=1}^{n}t_i.$$

Because S_n is the sum of n *i.i.d.* exponential random variables in which each has parameter λ and the number of events n is an integer, S_n has an Erlang distribution with cumulative density function

$$P\{S_n \leq t\} = P[N(t) \geq n] = \sum_{i=n}^{\infty} e^{-\lambda t} \frac{(\lambda t)^i}{i!},$$

and probability density function

$$f(t) = \lambda e^{-\lambda t} \frac{(\lambda t)^{n-1}}{(n-1)!}.$$

which has mean $E[S_n] = \frac{n}{\lambda}$. The Erlang distribution is a special case of the gamma distribution in which the gamma allows the number of events n to be any positive real number, while the Erlang distribution restricts n to be an integer. The above discussion relies heavily on (Ross, 1995, Chapter 2).

A.3 The surplus, value, and expected duration of a relationship

Denote the joint surplus accruing to both sides of a match as $S_{do}(\varphi)$. The bargain will divide this surplus such that the value of being a retailer equals $M_{do}(\varphi) - V_{do} = (1-\beta) S_{do}(\varphi)$ and the value of being a producer is $X_{do}(\varphi) - U_{do}(\varphi) = \beta S_{do}(\varphi)$, in which β is the producer's bargaining power. Using the value functions presented in the main text (7), (8), (10), and (11), we can write the surplus equation as

$$S_{do}(\varphi) = \frac{p_{do}q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + s_{do}\kappa_{do}\chi(\kappa_{do})}{r + \lambda + \beta\kappa_{do}\chi(\kappa_{do})}. \tag{36}$$

The surplus created by a match is the appropriately discounted flow profit, with the search cost l_{do} and the sunk cost s_{do} also entering the surplus equation because being matched avoids paying these costs. There are three things to notice here. First, the surplus from a match is a function of productivity. We show in appendix A.7 that matches that include a more productive exporting firm lead to greater surplus, that is, $S'_{do}(\varphi) > 0$. Second, the value of the relationship will fluctuate over the business cycle as shocks hit the economy and change the finding rate $\kappa_{do}\chi(\kappa_{do})$. Finally, surplus is greater than or equal to zero when

$$p_{do}q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + s_{do}\kappa_{do}\chi(\kappa_{do}) \geq 0.$$

Specifically, at the binding productivity cutoff we can use equation (46) and the surplus sharing rule to write

$$\beta S_{do}(\bar{\varphi}_{do}) = \frac{l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + s_{do},$$

which, in order for surplus to be positive, puts a restriction on the parameter choices and the equilibrium value of market tightness, κ_{do}.

With the definition of surplus in hand, the value of a matched relationship, $R_{do}(\varphi) = X_{do}(\varphi) + M_{do}(\varphi)$, can be expressed as $R_{do}(\varphi) = S_{do}(\varphi)\left(\frac{r+\kappa_{do}\chi(\kappa_{do})\beta}{r}\right) - \frac{l_{do}}{r}$. The value of the relationship to the producer is, of course, $X_{do}(\varphi)$ and to the retailer $M_{do}(\varphi)$. The value of a relationship in product markets has been of recent interest in Monarch and Schmidt-Eisenlohr (2015) and Heise (2016).

Relationships are destroyed at Poisson rate λ in the model, which implies the average duration of each match is $1/\lambda$. Because the destruction rate is exogenous and does not vary in our model, the average duration of each match is constant.

A.4 Bargaining over the negotiated price

A.4.1 Surplus sharing rule

Take equation (12), log and differentiate with respect to the price n_{do} and rearrange to get

$$\beta\frac{q_{do}}{X_{do}(\varphi) - U_{do}(\varphi)} + (1-\beta)\frac{-q_{do}}{M_{do}(\varphi) - V_{do}} = 0, \tag{37}$$

which implies the simple surplus sharing rule, equation (13): The retailer receives β of the total surplus from the trading relationship, $S_{do}(\varphi) = M_{do}(\varphi) - V_{do} + X_{do}(\varphi) - U_{do}(\varphi)$. The producer receives the rest of the surplus, $(1-\beta)\,S_{do}(\varphi)$.

In section 3.1 of the main text, we point out the restriction that $\beta < 1$ in equation (12) is evident in equation (13), which results from equation (37). Retailing firms have no incentive to search if $\beta = 1$ because they get none of the resulting match surplus and therefore cannot recoup search costs $c_{do} > 0$. Any solution to the model with $c_{do} > 0$ and positive trade between retailers and producers also requires $\beta < 1$. This result can be shown explicitly by using equations (10), (11), and (14) together with $\beta = 1$ to show that for productivity, φ, levels above the reservation productivity, $\bar{\varphi}_{do}$, (defined in section 3.3), the retailing firm has no incentive to search.

Finally, we do not need to calculate the partial derivative with respect to $U_{do}(\varphi)$ or $V_{do}(\varphi)$ because the individual firms are too small to influence aggregate values. Hence, when they meet, the firms bargain over the negotiated price-taking behavior in the rest of the economy as given. In particular, the outside option of the firms does not vary with the individual's bargaining problem.

A.4.2 Proof of proposition 1: Solving for the equilibrium negotiated price

Equations (7), (8), (10), and the equilibrium free-entry condition $V_{do} = 0$ imply that

$$M_{do}(\varphi) - V_{do} = \frac{p_{do}(q_{do})\, q_{do} - n_{do} q_{do}}{r + \lambda} \tag{38}$$

and

$$X_{do}(\varphi) - U_{do}(\varphi) = \frac{n_{do} q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})\, s_{do}}{r + \lambda + \kappa_{do}\chi(\kappa_{do})}. \tag{39}$$

Bargaining over price results in equation (37) and delivers the surplus sharing rule given by equation (13), which we can rewrite as $\beta(M_{do}(\varphi) - V_{do}) = (1-\beta)(X_{do}(\varphi) - U_{do}(\varphi))$. Using this transformation of equation (13) and the definitions given by equations (38) and (39) we can write

$$\begin{aligned}
\beta \frac{p_{do}(q_{do})\, q_{do} - n_{do} q_{do}}{r + \lambda} &= (1-\beta) \frac{n_{do} q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})\, s_{do}}{r + \lambda + \kappa_{do}\chi(\kappa_{do})} \\
\Rightarrow n_{do} q_{do} &= p_{do}(q_{do})\, q_{do}(1 - \gamma_{do}) + \gamma_{do}\left[t(q_{do}, w_o, \tau_{do}, \varphi) + f_{do} - l_{do} - \kappa_{do}\chi(\kappa_{do})\, s_{do}\right] \\
\Rightarrow n_{do} &= [1 - \gamma_{do}]\, p_{do} + \gamma_{do} \frac{t(q_{do}, w_o, \tau_{do}, \varphi) + f_{do} - l_{do} - \kappa_{do}\chi(\kappa_{do})\, s_{do}}{q_{do}}
\end{aligned}$$

in which $\gamma_{do} \equiv \dfrac{(r+\lambda)(1-\beta)}{r + \lambda + \beta\kappa_{do}\chi(\kappa_{do})}$.

A.4.3 Bounding the search friction

Recall the definition

$$\gamma_{do} \equiv \frac{(r+\lambda)(1-\beta)}{r + \lambda + \beta\kappa_{do}\chi(\kappa_{do})}.$$

Here we show that $\gamma_{do} \in [0, 1]$. First, because all parameters are positive, $\gamma_{do} \geq 0$. The lower bound, $\gamma_{do} = 0$, is reached only when $\beta = 1$ and $c_{do} = 0$ simultaneously. Second, prove that $\gamma_{do} \leq 1$ by contradiction. Assuming $\gamma_{do} > 1$ implies that $0 > \beta\kappa_{do}\chi(\kappa_{do})$, which is a contradiction, as $\beta \geq 0$ and $\kappa_{do}\chi(\kappa_{do}) \geq 0$.

A.4.4 Negotiated price when producers' finding rate goes to infinity

The limit of γ_{do} when the finding rate $\kappa_{do}\chi(\kappa_{do}) \to \infty$ is simply $\gamma_{do} \equiv \dfrac{(r+\lambda)(1-\beta)}{r + \lambda + \beta\kappa_{do}\chi(\kappa_{do})} \to 0$. More complicated is the limit of $\gamma_{do}\kappa_{do}\chi(\kappa_{do})$ as $\kappa_{do}\chi(\kappa_{do}) \to \infty$. First rewrite the expression as

$$\gamma_{do}\kappa_{do}\chi(\kappa_{do}) = \frac{(r+\lambda)(1-\beta)}{r + \lambda + \beta\kappa_{do}\chi(\kappa_{do})}\kappa_{do}\chi(\kappa_{do}).$$

Dividing the top and bottom of this expression by $\kappa_{do}\chi(\kappa_{do})$ yields

$$\gamma_{do}\kappa_{do}\chi(\kappa_{do}) = \frac{(r+\lambda)(1-\beta)}{\dfrac{r+\lambda}{\kappa_{do}\chi(\kappa_{do})} + \beta}.$$

Now use this to derive the limit

$$\begin{aligned}\lim_{\kappa_{do}\chi(\kappa_{do})\to\infty} \gamma_{do}\kappa_{do}\chi(\kappa_{do}) &= \lim_{\kappa_{do}\chi(\kappa_{do})\to\infty} \frac{(r+\lambda)(1-\beta)}{\dfrac{r+\lambda}{\kappa_{do}\chi(\kappa_{do})} + \beta} \\ &= \frac{(r+\lambda)(1-\beta)}{\beta}.\end{aligned}$$

This can be used to derive the limit of the negotiated price, n_{do}, as $\kappa_{do}\chi(\kappa_{do}) \to \infty$:

$$\begin{aligned}\lim_{\kappa_{do}\chi(\kappa_{do})\to\infty} n_{do} &= \lim_{\kappa_{do}\chi(\kappa_{do})\to\infty} \left[[1-\gamma_{do}] p_{do} + \gamma_{do} \frac{t(q_{do}, w_o, \tau_{do}, \varphi) + f_{do} - l_{do} - \kappa_{do}\chi(\kappa_{do}) s_{do}}{q_{do}} \right] \\ &= \lim_{\kappa_{do}\chi(\kappa_{do})\to\infty} \left[p_{do} - \gamma_{do} p_{do} + \gamma_{do} \left[\frac{t(q_{do}, w_o, \tau_{do}, \varphi) + f_{do} - l_{do}}{q_{do}} \right] - \frac{\gamma_{do}\kappa_{do}\chi(\kappa_{do}) s_{do}}{q_{do}} \right] \\ &= p_{do} - p_{do} \lim_{\kappa_{do}\chi(\kappa_{do})\to\infty} \gamma_{do} \\ &+ \left[\frac{t(q_{do}, w_o, \tau_{do}, \varphi) + f_{do} - l_{do}}{q_{do}} \right] \lim_{\kappa_{do}\chi(\kappa_{do})\to\infty} \gamma_{do} - \frac{s_{do}}{q_{do}} \lim_{\kappa_{do}\chi(\kappa_{do})\to\infty} \gamma_{do}\kappa_{do}\chi(\kappa_{do}) \\ &= p_{do} - p_{do} \cdot 0 + \left[\frac{t(q_{do}, w_o, \tau_{do}, \varphi) + f_{do} - l_{do}}{q_{do}} \right] \cdot 0 - \frac{s_{do}(r+\lambda)(1-\beta)}{q_{do}\beta} \\ &= p_{do} - \frac{s_{do}(r+\lambda)(1-\beta)}{q_{do}\beta}.\end{aligned}$$

The negotiated price is the final sales price, less the amount required to compensate the producer for the sunk cost to start up the business relationship. Notice that if $s_{do} = 0$, then the negotiated price would be the final sales price as in standard trade models.

A.5 Bargaining over the quantity

A.5.1 Maximizing surplus

Take equation (12), log and differentiate with respect to the quantity q_{do} to get

$$\beta \frac{1}{X_{do}(\varphi) - U_{do}(\varphi)} \left(n_{do} - t'(q_{do}, w_o, \tau_{do}, \varphi) \right) + (1-\beta) \frac{1}{M_{do}(\varphi) - V_{do}} \left(p_{do}(q_{do}) + p'_{do}(q_{do}) q_{do} - n_{do} \right) = 0, \tag{40}$$

in which we compute the partials of $X_{do}(\varphi)$ and $M_{do}(\varphi)$ using equations (39) and (38). Now, notice that equation (13) implies that $X_{do}(\varphi) - U_{do}(\varphi) = \frac{\beta}{1-\beta}(M_{do}(\varphi) - V_{do})$, and plugging this into equation (40) and rearranging slightly gives

$$p_{do}\left(q_{do}\right)+p_{do}^{\prime}\left(q_{do}\right)q_{do}=t^{\prime}\left(q_{do},w_{o},\tau_{do},\varphi\right). \tag{41}$$

This expression says that the quantity produced and traded is pinned down by equating marginal revenue in the domestic market with marginal production cost in the foreign country. This restriction is the same as what we get from a model without search and therefore implies that adding search does not change the quantity traded within each match. The profit maximization implied by this equation is crucial: Despite being separate entities, the retailer and the producer decide to set marginal revenue equal to marginal cost. The result follows because of the simple sharing rule, the maximization of joint surplus, and the trivial role of the retailer. To maximize surplus, the parties choose to equate marginal revenue and marginal cost.

A.5.2 Profit maximization

Conditional on the consumer's inverse demand (equation 2), the quantity traded between producer and retailer, $q_{do}\left(\omega\right)$, equates marginal revenue obtained by the retailer with the marginal production cost, as in equation (15). In other words, the retailer and producer solve a profit maximization problem. In particular, they seek to maximize profits for a given variety, ω, given that the producer has productivity φ, i.e., the cost function for producing q_{do} units of variety ω for the producer is given by $w_{o}\tau_{do}\dfrac{q_{do}\left(\omega\right)}{\varphi}+w_{o}f_{do}$, and the retailer faces a downward sloping demand curve. Variable profits can be written as:

$$\pi_{do}\left(\omega\right) = r_{do}\left(\omega\right)-w_{o}\tau_{do}\frac{q_{do}\left(\omega\right)}{\varphi}.$$

From the utility maximization solution we know that

$$r_{do}\left(\omega\right) = \alpha C_{d}\left(\frac{p_{do}\left(\omega\right)}{P_{d}}\right)^{1-\sigma}.$$

Since the CES aggregator is HOD 1, we know that welfare from the differentiated goods must be $\tilde{W}_{d}=\dfrac{\alpha C_{d}}{P_{d}}$ or $\alpha C_{d}=P_{d}\tilde{W}_{d}$ (appendix B.5). Further we know, again from our utility maximization solution, that

$$q_{do}\left(\omega\right) = \tilde{W}_{d}\left(\frac{p_{do}\left(\omega\right)}{P_{d}}\right)^{-\sigma}.$$

Plugging these into our profit expression from the top yields:

$$\begin{aligned}\pi_{do}(\omega) &= \alpha C_d \left[\frac{p_{do}(\omega)}{P_d}\right]^{1-\sigma} - \frac{w_o\tau_{do}}{\varphi}\tilde{W}_d\left[\frac{p_{do}(\omega)}{P_d}\right]^{-\sigma} \\ &= P_d^\sigma \tilde{W}_d p_{do}(\omega)^{1-\sigma} - \frac{w_o\tau_{do}}{\varphi}P_d^\sigma \tilde{W}_d p_{do}(\omega)^{-\sigma}.\end{aligned}$$

Differentiating this expression with respect to the price for this particular variety, $p_{do}(\omega)$, and setting this derivative equal to zero we get:

$$\frac{\partial \pi_{do}(\omega)}{\partial p_{do}(\omega)} = 0 = (1-\sigma) P_d^\sigma \tilde{W}_d p_{do}(\omega)^{1-\sigma-1} + \sigma\frac{w_o\tau_{do}}{\varphi}P_d^\sigma \tilde{W}_d p_{do}(\omega)^{-\sigma-1}.$$

Solving this for $p_{do}(\omega)$ yields:

$$p_{do}(\omega) = \mu\frac{w_o\tau_{do}}{\varphi},$$

in which $\mu = \dfrac{\sigma}{\sigma-1}$. Notice that since the right-hand side is not a function of ω (the index), but is a function of the productivity φ, we write

$$p_{do}(\varphi) = \mu\frac{w_o\tau_{do}}{\varphi}$$

throughout the text.

We should also note that since we assume that matches between retailers and producers are one to one, and each producer has a differentiated good, matched retailers have a monopoly in the variety that they import.

A.5.3 Retailer production function

In this section, we show that including another input for the retailer does not affect the conclusions of this paper under some weak additional assumptions. With an additional input, the value of being in a relationship for a retailer changes to

$$rM_{do}(\varphi) = p_{do}(f(q_{do},k_{do}))f(q_{do},k_{do}) - n_{do}q_{do} - \frac{\partial n_{do}}{\partial k_{do}}k_{do} - \lambda(M_{do}(\varphi) - V_{do}), \qquad (42)$$

in which the retailer combines the input, denoted by k_{do}, with the input purchased from the producer, q_{do}, according to production function $f(q_{do},k_{do})$ for the final good sold to consumers. The price of the additional input, $\frac{\partial n_{do}}{\partial k_{do}}$, is determined outside of the search model and is taken as given by the retailer.

With this new Bellman equation, logging and differentiating the Nash product in equation (12) with respect to p_{do} gives the same surplus sharing (13) rule as before. The first-order condition of equation (12) with respect to q_{do}, however, becomes

$$\beta \frac{n_{do} - t'\left(q_{do}, w_o, \tau_{do}, \varphi\right)}{X_{do}(\varphi) - U_{do}(\varphi)} + (1-\beta) \frac{p'_{do}(f(q_{do}, k_{do})) f_{q_{do}}(q_{do}, k_{do}) f(q_{do}, k_{do}) + p_{do}(f(q_{do}, k_{do})) f_{q_{do}}(q_{do}, k_{do}) - n_{do}}{M_{do}(\varphi) - V_{do}} = 0. \quad (43)$$

Combining this with the surplus sharing rule (13) yields an expression similar to equation (15):

$$p'_{do}(f(q_{do}, k_{do})) f_{q_{do}}(q_{do}, k_{do}) f(q_{do}, k_{do}) + p_{do}(f(q_{do}, k_{do})) f_{q_{do}}(q_{do}, k_{do}) = t'\left(q_{do}, w_o, \tau_{do}, \varphi\right). \quad (44)$$

This equation states that retailers and producers will negotiate to trade a quantity of q that ensures that the marginal revenue equals marginal cost. As the price of input k_{do} is taken as given, the firm chooses the optimal level of the input, k^*_{do}, so that $f_{k_{do}}\left(q_{do}, k^*_{do}\right) = \frac{\partial n_{do}}{\partial k_{do}}$. Strict concavity of the function $f(q_{do}, k_{do})$ is sufficient to ensure that $f_{k_{do}}\left(q_{do}, k_{do}\right)$ is invertible. Making this assumption gives $f^{-1}_{k_{do}}\left(q_{do}, n_{do}\right) = k^*_{do}$ which can be substituted into equation (44) to get one equation in one unknown, q_{do}. The quantity traded depends on $\frac{\partial n_{do}}{\partial k_{do}}$, the price of the other input, but search frictions still do not enter equation (44). The result in the main text - that optimal q_{do} is determined by the condition that ensures that marginal revenue from q_{do} equals the marginal cost of producing q_{do} - remains intact.

A.6 Solving for the productivity thresholds

A.6.1 Solving for the lowest productivity threshold

First, let's solve for an expression for $X_{do}(\varphi) - U_{do}(\varphi)$ by plugging in equations (7) and (8):

$$\begin{aligned}
rX_{do}(\varphi) - rU_{do}(\varphi) &= n_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} - \lambda\left(X_{do}(\varphi) - U_{do}(\varphi)\right) + l_{do} - \kappa_{do}\chi(\kappa_{do}(X_{do}(\varphi) - U_{do}(\varphi) - s_{do}) \\
&= n_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do} - (\lambda + \kappa_{do}\chi(\kappa_{do}))(X_{do}(\varphi) - U_{do}(\varphi)) \\
&\Rightarrow (r + \lambda + \kappa_{do}\chi(\kappa_{do}))(X_{do}(\varphi) - U_{do}(\varphi)) = n_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do} \\
&\Rightarrow X_{do}(\varphi) - U_{do}(\varphi) = \frac{n_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do}}{r + \lambda + \kappa_{do}\chi(\kappa_{do})}.
\end{aligned} \quad (45)$$

Now plug this expression into the definition of $\underline{\varphi}_{do}$ from the main text to get

$$\frac{n_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do}}{r + \lambda + \kappa_{do}\chi(\kappa_{do})} = 0$$
$$\Rightarrow n_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do} = 0.$$

By using the fact that $X'_{do}\left(\varphi\right) - U'_{do}\left(\varphi\right) > 0$ from above we can state that this threshold is unique.

We can be sure that for any positive cost of forming a relationship, $\dfrac{l_{do} + h_{do}}{\kappa_{do}\chi\left(\kappa_{do}\right)} + s_{do}$, if and only if $l_{do} + h_{do} + \kappa_{do}\chi(\kappa_{do}) s_{do} > 0$, the expression $X_{do}(\bar{\varphi}_{do}) - U_{do}(\bar{\varphi}_{do})$ exceeds $X_{do}(\underline{\varphi}_{do}) - U_{do}(\underline{\varphi}_{do})$. This result implies that as long as $X_{do}(\varphi) - U_{do}(\varphi)$ is increasing in φ, then $\bar{\varphi}_{do} > \underline{\varphi}_{do}$. In appendix A.7, we show the very general conditions under which $X_{do}(\varphi) - U_{do}(\varphi)$ is increasing in φ. The binding productivity threshold defining the mass of producers that have retail partners is the greater of these two and hence $\bar{\varphi}_{do}$. In other words, the productivity necessary to induce a producer to search for a retail partner is greater than the productivity necessary to consummate a match after meeting a retailer due to the costs that are incurred while searching. Similarly, the productivity necessary to form a match is greater than the productivity to maintain one already in place.

A.6.2 Proof of proposition 3: Solving for the binding productivity threshold

Our threshold productivity, $\bar{\varphi}_{do}$, is given by $U_{do}(\bar{\varphi}) - I_{do}(\bar{\varphi}_{do}) = 0$. Plugging equations (8) and (9) into this definition yields

$$X_{do}(\bar{\varphi}_{do}) - U_{do}(\bar{\varphi}_{do}) = \frac{l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + s_{do} \tag{46}$$

Using equation (45) in equation (46) yields

$$\begin{aligned}
&\frac{n_{do}q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do}}{r + \lambda + \kappa_{do}\chi(\kappa_{do})} = \frac{l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + s_{do} \\
&\Rightarrow n_{do}q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do} = (r + \lambda + \kappa_{do}\chi(\kappa_{do}))\frac{s_{do}\kappa_{do}\chi(\kappa_{do}) + l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} \\
&\Rightarrow n_{do}q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do} = (r + \lambda)\frac{s_{do}\kappa_{do}\chi(\kappa_{do}) + l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + s_{do}\kappa_{do}\chi(\kappa_{do}) + l_{do} + h_{do} \\
&\Rightarrow n_{do}q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} = (r + \lambda)\frac{s_{do}\kappa_{do}\chi(\kappa_{do}) + l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + h_{do} \\
&\Rightarrow n_{do}q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} - (r + \lambda)\frac{l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} - h_{do} = (r + \lambda)s_{do} \\
&\Rightarrow n_{do}q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} = (r + \lambda)s_{do} + (r + \lambda)\frac{l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + h_{do}.
\end{aligned}$$

Now, plug in for the equilibrium import price, n_{do}, from equation (14), to get

$$\begin{aligned}
&(1 - \gamma_{do})\, p_{do}(q_{do})q_{do} + \gamma_{do}\left(t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} - l_{do} - \kappa_{do}\chi(\kappa_{do})s_{do}\right) - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} \\
&\quad = (r + \lambda)\, s_{do} + \frac{(r + \lambda)}{\kappa_{do}\chi(\kappa_{do})} l_{do} + \left(1 + \frac{(r + \lambda)}{\kappa_{do}\chi(\kappa_{do})}\right) h_{do}.
\end{aligned}$$

which can be rearranged to obtain

$$\begin{aligned}
&p_{do}(q_{do})\, q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} \\
&= (1 - \gamma_{do})^{-1}\left[(r + \lambda + \gamma_{do}\kappa_{do}\chi(\kappa_{do}))\, s_{do} + \left(\gamma_{do} + \frac{(r + \lambda)}{\kappa_{do}\chi(\kappa_{do})}\right) l_{do} + \left(1 + \frac{(r + \lambda)}{\kappa_{do}\chi(\kappa_{do})}\right) h_{do}\right].
\end{aligned}$$

Further simplification of the terms with γ_{do} implies that

$$p_{do}\left(q_{do}\right)q_{do}-t\left(q_{do},w_{o},\tau_{do},\varphi\right)=f_{do}+\left(\frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right)l+\left(1+\frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right)h_{do}+\frac{(r+\lambda)}{\beta}s_{do}.$$

which is the expression in the main text.

A.6.3 Comparing our productivity threshold to previous models

Defining $F\left(\kappa_{do}\right)\equiv f_{do}+\left(\frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right)l_{do}+\left(1+\frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right)h_{do}+\frac{(r+\lambda)}{\beta}s_{do}$ in this framework would allow us to replace the fixed cost in the standard models with $F\left(\kappa_{do}\right)$ from here. The key thing to remember when working with the other quantities of our model is we want to work with them in terms of the cutoff and not in terms of these fundamental frictions just yet.

Another interesting comparison is to Eaton et al. (2014). That framework includes a flow search cost, l_{do}, but does not have a sunk cost s_{do} or any idle state. If we set $h_{do}=0$, we are implicitly including an idle state because the producer will have a zero value for being in the idle state but have a negative flow cost, $-l_{do}$, for being in the searching state because that state requires a payment each period of $l_{do}>0$. In other words, because the producer cannot opt out of searching we must set the flow of the idle state to $h_{do}=-l_{do}$ instead of what one might think is the intuitive value of that state $h_{do}=0$. Making this assumption together with $s_{do}=0$ provides:

$$\begin{aligned}p_{do}\left(q_{do}\right)q_{do}-t\left(q_{do},w_{o},\tau_{do},\varphi\right) &= f_{do}+\left(\frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right)l_{do}+\left(1+\frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right)h_{do}+\frac{(r+\lambda)}{\beta}s_{do}\\ &= f_{do}-l_{do}.\end{aligned}$$

This result is the very reason why Eaton et al. (2014) must have that $f_{do}>l_{do}$. Notice that we recover the standard model when we make these assumptions together with $l_{do}=0$:

$$\begin{aligned}p_{do}\left(q_{do}\right)q_{do}-t\left(q_{do},w_{o},\tau_{do},\varphi\right) &= f_{do}+\left(\frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right)l_{do}+\left(1+\frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right)h_{do}+\frac{(r+\lambda)}{\beta}s_{do}\\ &= f_{do}.\end{aligned}$$

Another interesting way to remove just the search friction, l_{do}, from the model is to set the finding rate $\kappa_{do}\chi\left(\kappa_{do}\right)\to\infty$ so that $\frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\to 0$

$$\begin{aligned}p_{do}\left(q_{do}\right)q_{do}-t\left(q_{do},w_{o},\tau_{do},\varphi\right) &= f_{do}+\left(\frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right)l_{do}+\left(1+\frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right)h_{do}+\frac{(r+\lambda)}{\beta}s_{do}\\ &= f_{do}+h_{do}+\frac{(r+\lambda)}{\beta}s_{do}.\end{aligned}$$

The interpretation of the fixed cost includes the sunk cost, the bargaining power of the producer, and the flow from the outside option, even if one finds a partner immediately. If bargaining power differs by market, for example, the bundle of entry costs will as well.

A.6.4 Productivity cutoff and flow profits

Because the equilibrium price for each variety is a constant markup over marginal cost we can write the firms' variable cost function as a proportional function of revenue

$$t_{do}(\varphi) = p_{do}(\varphi)\, q_{do}(\varphi)\, \mu^{-1}.$$

Combine the definition of flow variable profits

$$\pi_{do}(\varphi) = p_{do}(\varphi)\, q_{do}(\varphi) - t_{do}(\varphi)$$

with the relationship between variable costs and revenue to get that

$$\pi_{do}(\varphi) = p_{do}(\varphi)\, q_{do}(\varphi) - p_{do}(\varphi)\, q_{do}(\varphi)\, \mu^{-1},$$

which simplifies to

$$\pi_{do}(\varphi) = p_{do}(\varphi)\, q_{do}(\varphi)\, \sigma^{-1}$$

because $1 - \mu^{-1} = \sigma^{-1}$. Using demand from equation (2) and the pricing rule provides revenue in this model

$$p_{do}(\varphi)\, q_{do}(\varphi) = \alpha C_d P_d^{\sigma-1} (\mu w_o \tau_{do})^{1-\sigma} \varphi^{\sigma-1}.$$

We can use revenue and the profit expression combined with (17) to derive threshold productivity in our search model. We start with the expression

$$\pi_{do}(\bar{\varphi}_{do}) = F_{do}(\kappa_{do}).$$

Then use the functional forms and the relationship between revenues and profits to write

$$\frac{\alpha}{\sigma} C_d P_d^{\sigma-1} (\mu w_o \tau_{do})^{1-\sigma} \bar{\varphi}_{do}^{\sigma-1} = F_{do}(\kappa_{do})$$

before arriving at

$$\bar{\varphi}_{do} = \mu \left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}} \left(\frac{w_o \tau_{do}}{P_d}\right) C_d^{\frac{1}{1-\sigma}} F_{do}(\kappa_{do})^{\frac{1}{\sigma-1}},$$

which is presented in equation (18) in the main text.

A.7 The value of importing is strictly increasing in productivity

Here we show that the value of importing, $M_{do}(\varphi)$, is strictly increasing with the producer's productivity level, φ. This fact allows us to replace the integral of the max over V_{do} and $M_{do}(\varphi)$ (equation 11) with the integral of $M_{do}(\varphi)$ from the productivity threshold, $\bar{\varphi}_{do}$ (equation 19).

Starting with equation (10) and $V_{do} = 0$ we obtain

$$\begin{aligned}
(r+\lambda) M_{do}(\varphi) &= p_{do}q_{do} - n_{do}q_{do} \\
&= p_{do}q_{do} - [1-\gamma_{do}]\, p_{do}q_{do} - \gamma_{do}\left(t\left(q_{do}, w_o, \tau_{do}, \varphi\right) + f_{do} - l_{do} - \kappa_{do}\chi\left(\kappa_{do}\right) s_{do}\right) \\
&= \gamma_{do}p_{do}q_{do} - \gamma_{do}\left(t\left(q_{do}, w_o, \tau_{do}, \varphi\right) + f_{do}\right) + \gamma_{do}\left(l_{do} + \kappa_{do}\chi\left(\kappa_{do}\right) s_{do}\right) \\
&= \gamma_{do}\left(p_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do}\right) + \gamma_{do}\left(l_{do} + \kappa_{do}\chi\left(\kappa_{do}\right) s_{do}\right).
\end{aligned}$$

Remember that $\gamma_{do} \equiv \dfrac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\kappa_{do}\chi\left(\kappa_{do}\right)}$. It is clear from the integral in the import relationship creation equation (19) that neither the finding rate for retailers, $\chi\left(\kappa_{do}\right)$, nor the tightness, κ_{do}, is a function of the productivity, φ. Given this, $M'_{do}(\varphi)$ and $\dfrac{\partial[p_{do}\left(q_{do}\right) q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do}]}{\partial\varphi}$ will have the same sign. As long as flow profits without search frictions are strictly increasing in productivity, $M'_{do}(\varphi) > 0$. Using the specific functional forms for $t\left(q_{do}, w_o, \tau_{do}, \varphi\right) + f_{do}$ used above, as well as the equilibrium values for n_{do}, p_{do}, and q_{do}, we can derive this result explicitly. In this case,

$$M_{do}(\varphi) = \alpha\gamma_{do}\left(\frac{1}{r+\lambda}\right)\left(\frac{\mu^{-\sigma}}{\sigma-1}\right)\left(w_o\tau_{do}\right)^{1-\sigma}\alpha C_d P_d^{\sigma-1}\varphi^{\sigma-1} - \gamma_{do}f_{do} + \gamma_{do}\left(l_{do} + \kappa_{do}\chi\left(\kappa_{do}\right) s_{do}\right).$$

Therefore the derivative is

$$\frac{\partial M_{do}(\varphi)}{\partial\varphi} = \alpha\gamma_{do}\left(\frac{1}{r+\lambda}\right)\mu^{-\sigma}\left(w_o\tau_{do}\right)^{1-\sigma}\alpha C_d P_d^{\sigma-1}\varphi^{\sigma-2}.$$

which is always positive.

As long as $M'_{do}(\varphi) > 0$, we can demonstrate the way in which many other important quantities depend on the producer's productivity level, φ. From the surplus sharing rule (37) can be rewritten as

$$\beta M_{do}(\varphi) = (1-\beta)\left(X_{do}(\varphi) - U_{do}(\varphi)\right), \tag{47}$$

We know that in equilibrium, because $M'_{do}(\varphi) > 0$, it must be that $X'_{do}(\varphi) - U'_{do}(\varphi) > 0$. Differentiating both sides of equation (8) gives $rU'_{do}(\varphi) = \kappa_{do}\chi\left(\kappa_{do}\right)\left(X'_{do}(\varphi) - U'_{do}(\varphi)\right) > 0$. We can combine these facts to show $X'_{do}(\varphi) > U'_{do}(\varphi) > 0$. Using the definition of the joint surplus of a match $S_{do}(\varphi) = X_{do}(\varphi) + M_{do}(\varphi) - U_{do}(\varphi) - V_{do}$ we get $S'_{do}(\varphi) > 0$. Likewise, the value of a relationship, $R_{do}(\varphi) = X_{do}(\varphi) + M_{do}(\varphi)$, has $R'_{do}(\varphi) > 0$.

A.8 Proof of proposition 4: Market tightness and the cost of search

Let's first prove that $\kappa_{do} < \infty$ if $c_{do} > 0$. To do this, let's prove the contrapositive: assume that $c_{do} = 0$ and show that $\kappa_{do} = \infty$. Rearrange equation (19) slightly to get

$$0 = c_{do} = \chi\left(\kappa_{do}\right) \int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right) dG\left(\varphi\right).$$

We have shown that $M_{do}\left(\bar{\varphi}_{do}\right) \geq 0$ for any consummated match in equilibrium (Nash bargaining together with appendix A.6) and $M'_{do}\left(\varphi\right) > 0$ (appendix A.7). Therefore we know that $\int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right) dG\left(\varphi\right) > 0$. Thus, $\chi\left(\kappa_{do}\right)$ must be zero. Because $\chi'\left(\kappa_{do}\right) < 0$ this is true if and only if $\kappa_{do} = \infty$.

To prove that if $c_{do} > 0$ then $\kappa_{do} < \infty$, let's use equation (19) again. In particular, because $c_{do} > 0$ it must mean that $\chi\left(\kappa_{do}\right) \int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right) dG\left(\varphi\right) > 0$. As before, we know that $\int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right) dG\left(\varphi\right) > 0$ so it must be that $\chi\left(\kappa_{do}\right) > 0$ as well, which is true if and only if $\kappa_{do} < \infty$.

A.9 Producer and retailer existence

A.9.1 Retailing firms

Free entry implies that the ex-ante expected value from entering for a potential retailer equals the expected cost of entering. Assume for a moment that the potential retailers consider the value of becoming a retailer as defined by E^m_{do}. This value is characterized by the following Bellman equation

$$rE^m_{do} = -e^m_o + \left(V_{do} - E^m_{do}\right). \tag{48}$$

The potential retailer could sell the value E^m_{do} and invest the proceeds at the interest rate r getting flow payoff rE^m_{do} forever after. Alternatively, they could pay a cost e^m_o to become a retailer, at which point they will begin in the state of having a vacancy with value V_{do} (with certainty) and give up the value of being a potential retailer E^m_{do}. Free entry into becoming a retailer implies that $E^m_{do} = 0$ in equilibrium so that

$$\begin{aligned} 0 &= -e^m_o + V_{do} \\ e^m_o &= V_{do}. \end{aligned}$$

Hence, free entry into vacancies $V_{do} = 0$ implies $e^m_o = 0$ and we cannot have a sunk cost for entry into retailing. In other words, free entry into the search market along with assuming that one must post a vacancy before matching implies free entry into retailing.

Free entry into the search market subsumes free entry into retailing and so we only have one condition defined by free entry on the retailing side given by equation (19) and restated here

$$\frac{c_{do}}{\chi\left(\kappa_{do}\right)}=\int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right)dG\left(\varphi\right).$$

Remember this states that product vacancies continue being created until the expected cost of being an unmatched retailer, $c_{do}/\chi\left(\kappa_{do}\right)$, equals the expected benefit $\int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right)dG\left(\varphi\right)$. Because each potential retailer must post a product vacancy before forming a match, the expected cost of becoming a retailer (entering as a retailer) is the same as the expected cost of being an unmatched retailer. Likewise, the expected benefit of posting a vacancy and the expected benefit of becoming a retailer are also the same because we assume retailers must post a vacancy before matching.

A.9.2 Producing firms

Similar to the entry decision of retailers, the value of entry for producers, E^x_{do}, is defined by

$$\begin{aligned} rE^x_{do} &= -e^x_d+\int \max\left\{I_{do}\left(\varphi\right),U_{do}\left(\varphi\right)\right\}dG\left(\varphi\right)-E^x_{do} \\ &= -e^x_d+\int_1^{\bar{\varphi}_{do}} I_{do}\left(\varphi\right)dG\left(\varphi\right)+\int_{\bar{\varphi}_{do}}^{\infty} U_{do}\left(\varphi\right)dG\left(\varphi\right)-E^x_{do}.\end{aligned}$$

We assume that the potential producer must transit through the unmatched state before forming a match. After paying e^x_d and taking a productivity draw φ, the potential producer loses the value E^x_{do} with certainty and, depending on the drawn productivity, chooses between searching for a retailer and getting value $U_{do}\left(\varphi\right)$ or remaining idle and getting value $I_{do}\left(\varphi\right)$. If we assumed free entry into production, we would get $E^x_{do}=0$ and that

$$e^x_d=\int_1^{\bar{\varphi}_{do}} I_{do}\left(\varphi\right)dG\left(\varphi\right)+\int_{\bar{\varphi}_{do}}^{\infty} U_{do}\left(\varphi\right)dG\left(\varphi\right). \tag{49}$$

which ensures that the expected value of taking a productivity draw equals the expected cost.

Free entry into production, therefore, imposes another restriction on the equilibrium. We can use the facts that $X_{do}\left(\varphi\right)-U_{do}\left(\varphi\right)=\left(1-\beta\right)S_{do}\left(\varphi\right)$ and that $M_{do}\left(\varphi\right)=\beta S_{do}\left(\varphi\right)$ to write $X_{do}\left(\varphi\right)-U_{do}\left(\varphi\right)=\left(\frac{1-\beta}{\beta}\right)M_{do}\left(\varphi\right)$. Applying this to equation (8) gives

$$rU_{do}\left(\varphi\right)=-l_{do}+\kappa_{do}\chi\left(\kappa_{do}\right)\left(\left(\frac{1-\beta}{\beta}\right)M_{do}\left(\varphi\right)-s_{do}\right).$$

Computing the relevant integrals in equation (49) gives

$$\int_{\bar{\varphi}_{do}}^{\infty} U_{do}\left(\varphi\right) dG\left(\varphi\right) = -\left(\frac{l_{do} + s_{do}\kappa_{do}\chi\left(\kappa_{do}\right)}{r}\right)\left(1 - G\left(\bar{\varphi}_{do}\right)\right) + \frac{\kappa_{do}\chi\left(\kappa_{do}\right)}{r}\left(\frac{1-\beta}{\beta}\right)\int_{\bar{\varphi}_{do}}^{\infty} M_{do}\left(\varphi\right) dG\left(\varphi\right).$$

Likewise, from (9) we have

$$\int_{1}^{\bar{\varphi}_{do}} I_{do}\left(\varphi\right) dG\left(\varphi\right) = \frac{h_{do}}{r} G\left(\bar{\varphi}_{do}\right).$$

Combining these with equation (49) gives

$$e_d^x = \frac{h_{do}}{r} G\left(\bar{\varphi}_{do}\right) - \left(\frac{l_{do} + s_{do}\kappa_{do}\chi\left(\kappa\right)}{r}\right)\left(1 - G\left(\bar{\varphi}_{do}\right)\right) + \frac{\kappa_{do}\chi\left(\kappa_{do}\right)}{r}\left(\frac{1-\beta}{\beta}\right)\int_{\bar{\varphi}_{do}}^{\infty} M_{do}\left(\varphi\right) dG\left(\bar{\varphi}_{do}\right). \quad (50)$$

which is the restriction that free entry into production for producers would place on equilibrium market tightness κ_{do}.

From equation (49), we can see that free entry into search for producers would require $\int_{\bar{\varphi}_{do}}^{\infty} U_{do}\left(\varphi\right) dG\left(\varphi\right) = 0$, in which case we would be left with

$$e_d^x = \frac{h_{do}}{r} G\left(\bar{\varphi}_{do}\right). \quad (51)$$

which still places a restriction on market tightness because $\bar{\varphi}_{do}$ from proposition (17) includes κ_{do}.

Finally, we note that simultaneous combinations of free entry on both sides of the market are possible. Combining free entry into both existence and search for retailers from equation (19) with free entry into existence for producers from equation (50) gives

$$e_d^x = \frac{h_{do}}{r} G\left(\bar{\varphi}_{do}\right) - \left(\frac{l_{do} + s_{do}\kappa_{do}\chi\left(\kappa\right)}{r}\right)\left(1 - G\left(\bar{\varphi}_{do}\right)\right) + \frac{c_{do}\kappa_{do}}{r}\left(\frac{1-\beta}{\beta}\right).$$

Likewise, allowing for free entry into both existence and search for retailers and producers would give the following system that defines κ_{do}

$$\begin{aligned} \frac{c_{do}}{\chi\left(\kappa_{do}\right)} &= \int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right) dG\left(\varphi\right) \\ e_d^x &= \frac{h_{do}}{r} G\left(\bar{\varphi}_{do}\right). \end{aligned}$$

A.10 Aggregate resources

A.10.1 Number of producers

Similar to Chaney (2008), we assume that the number of producers in the origin market that take a draw from the productivity distribution is proportional to consumption expenditure in the economy, C_o. The basic intuition behind this is that larger economies have a larger stock of potential entrepreneurs. To make this explicit, we denote the total mass of potential entrants as $N_o^x = \xi_o C_o$, in which the proportionality constant $\xi_o \in [0, \infty)$ captures exogenous structural factors that affect the number of potential entrants in country k. Among others, these could include such factors as literacy levels and attitudes toward entrepreneurship. As discussed in section A.10.2, because the number of producers is fixed, the economy has profits. We assume that a global mutual fund collects worldwide profits and redistributes them as π dividends per share to each worker who owns w_o shares. We assume that $\xi_o = \dfrac{1}{1+\pi}$ so that

$$N_o^x = \frac{C}{(1+\pi)} \frac{C_o}{C} \tag{52}$$

in which we have multiplied and divided by global consumption, C.

A.10.2 Aggregate accounting and the global mutual fund

Our economy has profits because we restrict producer entry and the model features monopolistic competition. We define a global mutual fund that collects all profits in the economy and rebates them back to consumers. In order to calculate total profits, we first define variable profits earned in each market pair as

$$\Pi_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}} \pi_{do}(\varphi)\, dG(\varphi)$$

in which $\pi_{do}(\varphi) = p_{do}(\varphi)\, q_{do}(\varphi) - t_{do}(\varphi)$. Our functional form assumptions and the pricing rule in (16) ensure that profits are proportional to sales:
$p_{do}(\varphi)\, q_{do}(\varphi) - t_{do} = p_{do}(\varphi)\, q_{do}(\varphi)\, \sigma^{-1}$. Aggregating profits from each variety provides

$$\Pi_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}} p_{do}(\varphi)\, q_{do}(\varphi)\, \sigma^{-1} dG(\varphi) = \frac{C_{do}}{\sigma} \tag{53}$$

in which we define the value of total consumption in destination d of the differentiated good from origin o as

$$C_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}} p_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi).$$

This definition for the value of consumption is consistent with equation (1) in the main text.

The income that consumers in o earn and can spend on consumption, C_o, comes from three sources. The first two sources are labor income in the production and investment sectors of the economy, $w_d L_d$. The third source is dividends from the global mutual fund, which we assume owns all firms in all countries. Each country gets a share, π, of total global profits proportional to labor income in the economy. Explicitly GDP can be written as

$$Y_o = w_o L_o (1 + \pi),$$

in which

$$\pi = \Pi / \sum w_k L_k. \tag{54}$$

Notice that the dividend per unit value of labor π is proportional to the value of the global labor endowment and so also matches Chaney (2008) equation (6) in our model. Wage income is derived providing the fixed cost of production, the formation of relationships, creating new retailers and producers, and the variable cost of production:

$$w_o L_o = \sum_{k=1}^{D} \Phi^i_{ko} + \sum_{k=1}^{D} \Phi^e_{ko} + \sum_{k=1}^{D} \Phi^p_{ko} + w_o q_o (1)$$

in which

$$\begin{aligned}
\Phi^i_{do} &= \kappa_{od} u_{od} N^x_d c_{od} + u_{do} N^x_o (l_{do} + s_{do} \kappa_{do} \chi(\kappa_{do})) + (1 - u_{do} - i_{do}) N^x_o f_{do} \\
\Phi^e_{do} &= N^x_o e^x_o \\
\Phi^p_{do} &= \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N^x_o \int_{\bar{\varphi}_{do}} t_{do}(\varphi) \, dG(\varphi)
\end{aligned}$$

The production structure for the homogeneous good is undetermined because it is freely traded and has constant returns to scale production. Like Chaney (2008), we only consider equilibrium in which every country produces some of that good. In order to simplify and make accounting for resources in every country symmetric, we also assume that each country produces what it would like to consume itself, namely $q_o(1)$. While the good can be freely traded, in equilibrium there is no international trade of the homogeneous good. Despite no trade in this good, its price is the same in all countries because of a no-arbitrage condition. Each unit of the homogeneous good requires one unit of labor to produce so the cost of producing $q_o(1)$ units of the homogeneous good is given by $w_o q_o(1)$. Importantly, our definition of the income earned from labor used in producing the differentiated good, Φ^p_{do}, includes the iceberg transport costs so that labor is compensated for transporting goods.

Summing payments to labor across all countries of the world gives

$$\Phi^h = \sum_{k=1}^{O} w_k q_k (1), \quad \Phi^i = \sum_{k=1}^{O} \sum_{j=1}^{D} \Phi^i_{jk}, \quad \Phi^e = \sum_{k=1}^{O} \sum_{j=1}^{D} \Phi^e_{jk}, \quad \Phi^p = \sum_{k=1}^{O} \sum_{j=1}^{D} \Phi^p_{jk}. \tag{55}$$

Similarly, we can define global variable profits from operation in each market either as equation (53) or as

$$\Pi_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}} p_{do}(\varphi)\, q_{do}(\varphi) - t_{do}(\varphi)\, dG(\varphi) = C_{do} - \Phi_{do}^p \tag{56}$$

Summing variable profits throughout the world provides global profits

$$\Pi = \sum_{k=1}^{O} \sum_{j=1}^{D} \Pi_{jk} = \sum_{k=1}^{O} \sum_{j=1}^{D} C_{jk} - \Phi_{jk}^p = \sum_{k=1}^{O} \sum_{j=1}^{D} \frac{C_{jk}}{\sigma} = \frac{\alpha}{\sigma} C \tag{57}$$

The last two equalities come from our functional form assumptions. We can check that we have treated the global mutual fund correctly by ensuring that global income equals global expenditure. Start by defining investment in each market

$$I_{do} = \kappa_{od} u_{od} N_d^x c_{od} + u_{do} N_o^x \left(l_{do} + s_{do} \kappa_{do} \chi(\kappa_{do})\right) + \left(1 - u_{do} - i_{do}\right) N_o^x f_{do} + N_o^x e_o^x$$

in which it is also clear that $I_{do} = \Phi_{do}^i + \Phi_{do}^e$ and global investment is

$$I = \sum_{k=1}^{O} \sum_{j=1}^{D} I_{jk}.$$

Global consumption of both homogeneous and differentiated goods is

$$C = \sum_{k=1}^{O} C_k = \sum_{k=1}^{O} p_k(1)\, q_k(1) + \sum_{k=1}^{O} \sum_{j=1}^{D} C_{jk}$$

To check that we have everything correct, start with total resources available in the economy $Y_o = w_o L_o (1+\pi)$ and sum across economies

$$
\begin{aligned}
\sum_{k=1}^{O} Y_k &= \sum_{k=1}^{O} w_k L_k (1+\pi) \\
Y &= (1+\pi) \sum_{k=1}^{O} w_k L_k \\
Y &= \left(1 + \frac{\Pi}{\sum_{k=1}^{O} w_k L_k}\right) \sum_{k=1}^{O} w_k L_k \\
Y &= \sum_{k=1}^{O} w_k L_k + \Pi \\
Y &= \Pi + \sum_{k=1}^{O} \left(\sum_{k=1}^{D} \Phi_{ko}^{i} + \sum_{k=1}^{D} \Phi_{ko}^{e} + \sum_{k=1}^{D} \Phi_{ko}^{p} + w_o q_o (1) \right) \\
Y &= \Pi + \Phi^i + \Phi^e + \Phi^p + \Phi^h
\end{aligned}
$$

We can finish the proof by starting with the last line, which is the income approach to accounting, and showing that this expression also gives the expenditure approach

$$
\begin{aligned}
Y &= \Pi + \Phi^h + \Phi^i + \Phi^e + \Phi^p \\
Y &= \Pi + \Phi^h + I + \Phi^p \\
Y &= \sum_{k=1}^{O} \sum_{j=1}^{D} \left(C_{jk} - \Phi_{jk}^{p} \right) + \Phi^h + I + \Phi^p \\
Y &= \sum_{k=1}^{O} \sum_{j=1}^{D} C_{jk} + \Phi^h + I \\
Y &= C + I
\end{aligned}
$$

so that

$$Y = C + I, \tag{58}$$

and

$$C = Y - I. \tag{59}$$

Notice that we used

$$\Phi^h = \sum_{k=1}^{O} w_k q_k (1) = \sum_{k=1}^{O} p_k (1) q_k (1)$$

in the last line. Costless trading of the homogeneous good delivers a "no arbitrage condition," implying that its price must be the same in all countries, $p_k (1) = p (1)$. Because the homogeneous good is made with one unit of labor in each country, it must also be that $w_k = p_k(1) = p(1)$ pinning down the equilibrium wage in every country.

Finally, we point out that total resources in each economy are given by $Y_o = w_o L_o (1+\pi)$. Total resources are larger than the labor endowment because the definitions of payments to labor do not account for an existing mass of firms. With an existing mass of firms, the global economy is endowed not only with labor but also with that mass. This nonlabor endowment is reflected in profits made by those firms. If there is a pre-existing mass of firms that does not make profits, the additional resources are paid to labor in the form of production costs. Without a pre-existing mass of firms, the cost of creating new firms is captured in the payments to labor, Φ^e, when creating those firms.

Notice that for one country, equation (58) can be written as

$$\begin{aligned}
Y_d &= C_d + I_d \\
&= p_d(1)\, q_d(1) + \sum_{k=1}^{O} \left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}} p_{dk}(\varphi)\, q_{dk}(\varphi)\, dG(\varphi) \\
&+ N_d^x e_d^x + \sum_{k=1}^{O} \kappa_{dk} u_{dk} N_k^x c_{dk} + u_{kd} N_d^x \left(l_{kd} + s_{kd}\kappa_{kd}\chi(\kappa_{kd})\right) + (1-u_{kd}-i_{kd})\, N_d^x f_{kd},
\end{aligned}$$

which is equation (23) in the main text.

A.11 The ideal price index with our productivity distribution

A.11.1 Moving from an index to a distribution of goods

Melitz uses the following steps to move from index ω_{do} over a continuum of goods available to consume, Ω, which we assume has measure $M_{do} = |\Omega_{do}|$, to the cumulative distribution of productivity $G(\varphi)$ and the measure of goods available for consumption $(1-i_{do})\, M_{do}$.

The following steps keep the notation in Melitz's original work. Begin with the definition for the change of variables, also known as integration by substitution, which states

$$\int_a^b f(h(\varphi))h'(\varphi)d\varphi = \int_{i(a)}^{i(b)} f(\omega)d\omega$$

Choose to index the goods ω with the indexing number $G(\varphi)\, M_{do}$ which is differentiable in φ such that $\omega = h(\varphi) = G(\varphi)M_{do}$. Then we can apply the rule from left to right to get

$$\int_0^\infty f(G(\varphi)M_{do})\frac{\partial G(\varphi)M_{do}}{\partial \varphi}d\varphi = \int_{G(0)M_{do}}^{G(\infty)M_{do}} f(\omega)d\omega = \int_0^{M_{do}} f(\omega)d\omega = \int_{\omega\in\Omega_{do}} f(\omega)d\omega.$$

We choose $G(0) = 0$ and $G(\infty) = 1$ in our context because $G(\varphi)$ is a cumulative distribution function and it allows us to start the continuous indexing such that the upper bound of the integral is the measure of Ω_{do}. More generally, change of variables allows for any $G(\varphi)$ as long as $G(\varphi)$ is differentiable in φ.

Remember that in our context $f(\omega)$ is a function that simply indexes the continuum of goods ω so that $f(\omega)$ does not vary with ω even though $f(\varphi)$ will vary with φ. Therefore, we can reassign the indexing number $G(\varphi)M_{do}$ to φ to get

$$\int_0^\infty f(G(\varphi)M_{do})\frac{\partial G(\varphi)M_{do}}{\partial\varphi}d\varphi = \int_0^\infty f(\varphi)\frac{\partial G(\varphi)M_{do}}{\partial\varphi}d\varphi,$$

We often integrate over $[\bar{\varphi}_{do}, \infty)$ and not $[0, \infty)$ because some goods are not available in equilibrium. As long as $f(\varphi) = 0$ when $\varphi < \bar{\varphi}_{do}$, we can ignore those goods and

$$\int_0^\infty f(\varphi)\frac{\partial G(\varphi)M_{do}}{\partial\varphi}d\varphi = \int_{\bar{\varphi}_{do}}^\infty f(\varphi)\frac{\partial G(\varphi)M_{do}}{\partial\varphi}d\varphi$$

In order to relate this expression to economically meaningful concepts, it is helpful to rewrite this as

$$\int_{\bar{\varphi}_{do}}^\infty f(\varphi)\frac{\partial G(\varphi)M_{do}}{\partial\varphi}d\varphi = (1 - i_{do})\, M_{do}\int_{\bar{\varphi}_{do}}^\infty f(\varphi)\frac{g(\varphi)}{(1 - i_{do})}d\varphi$$

in which $i_{do} = G(\bar{\varphi}_{do})$, $g(\varphi) = \partial G(\varphi)/\partial\varphi$, and $g(\varphi)$ is a proper density because $1 = \int_{\bar{\varphi}_{do}}^\infty g(\varphi)\,(1 - i_{do})^{-1}\,d\varphi$. This implies that the measure of goods available to consume is $(1 - i_{do})\, M$ and the density of goods available to consume is given by $g(\varphi)\,(1 - i_{do})^{-1}$. The analogous measure of goods available to consume in our model is $(1 - u_{do} - i_{do})\, N_o^x$ and we have the same density of goods as Melitz because the unmatched fraction of products, u_{do}, is still available to consumers.

A.11.2 Differentiated goods price index

We are able to map from the price index defined using varieties, ω, in equation (3) to a price index in terms of firm productivities, φ, using the approach in appendix A.11.1 to obtain:

$$P_d = \left[\sum_{k=1}^{O}\left(\frac{1 - u_{dk} - i_{dk}}{1 - i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}}^\infty p_{dk}(\varphi)^{1-\sigma}\, dG(\varphi)\right]^{\frac{1}{1-\sigma}},$$

in which $G(\cdot)$ is a cumulative density function that is defined as Pareto distributed in section 2.1. With our assumptions about demand and the production structure in sections 2.2 and 2.1 we get equation (16), which is $p_{do}(\varphi) = \mu w_o \tau_{do}\varphi^{-1}$. Plugging this into the price index gives

$$P_d = \left[\sum_{k=1}^{O}\left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right)N_k^x\int_{\bar{\varphi}_{dk}}^{\infty}\left(\mu w_k\tau_{dk}\right)^{1-\sigma}dG\left(\varphi\right)\right]^{\frac{1}{1-\sigma}}$$

then we can use the moment $\int_{\bar{\varphi}_{dk}}^{\infty}z^{\sigma-1}dG\left(z\right)=\dfrac{\theta\bar{\varphi}_{dk}^{\sigma-\theta-1}}{\theta-\sigma+1}$ to get

$$P_d = \left[\sum_{k=1}^{O}\left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right)N_k^x\left(\mu w_k\tau_{dk}\right)^{1-\sigma}\frac{\theta\bar{\varphi}_{dk}^{\sigma-\theta-1}}{\theta-\sigma+1}\right]^{\frac{1}{1-\sigma}}$$

The threshold productivity is given in equation (18) in the main text, which is

$$\bar{\varphi}_{do} = \mu\left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}}\left(\frac{w_o\tau_{do}}{P_d}\right)\left(\frac{F_{do}\left(\kappa_{do}\right)}{C_d}\right)^{\frac{1}{\sigma-1}}$$

By substituting the threshold into the price index we get

$$P_d = \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{-\frac{1}{\theta}}\left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}}\mu C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}}\left(\sum_{k=1}^{O}\left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right)N_k^x\left(w_k\tau_{dk}\right)^{-\theta}F_{dk}\left(\kappa_{dk}\right)^{-\left[\frac{\theta}{\sigma-1}-1\right]}\right)^{-\frac{1}{\theta}}$$

Then we can employ our definition for the number of producers from section A.10.1 to derive

$$P_d = \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{-\frac{1}{\theta}}\left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}}\mu C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}}\left(\sum_{k=1}^{O}\left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right)\left(\frac{C}{1+\pi}\right)\frac{C_k}{C}\left(w_k\tau_{dk}\right)^{-\theta}F_{dk}\left(\kappa_{dk}\right)^{-\left[\frac{\theta}{\sigma-1}-1\right]}\right)^{-\frac{1}{\theta}}$$

Slightly rearranging terms and using the fact that $\left(1-\dfrac{u_{dk}}{1-i_{dk}}\right)=\dfrac{\kappa_{dk}\chi\left(\kappa_{dk}\right)}{\lambda+\kappa_{dk}\chi\left(\kappa_{dk}\right)}$ the price index becomes

$$\begin{aligned}
P_d &= \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{-\frac{1}{\theta}}\left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}}\mu\left(\frac{C}{1+\pi}\right)^{-\frac{1}{\theta}}\\
&\times\ C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}}\\
&\times\ \left(\sum_{k=1}^{O}\frac{C_k}{C}\left(\frac{\kappa_{dk}\chi\left(\kappa_{dk}\right)}{\lambda+\kappa_{dk}\chi\left(\kappa_{do}\right)}\right)\left(w_k\tau_{dk}\right)^{-\theta}F_{dk}\left(\kappa_{dk}\right)^{-\left[\frac{\theta}{\sigma-1}-1\right]}\right)^{-\frac{1}{\theta}}
\end{aligned}$$

The final expression of the differentiated goods price index is a simple function of three terms

$$P_d = \lambda_2\times C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}}\times\rho_d \tag{60}$$

in which

$$\rho_d \equiv \left(\sum_{k=1}^{O} \frac{C_k}{C} \left(\frac{\kappa_{dk}\chi\left(\kappa_{dk}\right)}{\lambda + \kappa_{dk}\chi\left(\kappa_{dk}\right)}\right) \left(w_k \tau_{dk}\right)^{-\theta} F_{dk}\left(\kappa_{dk}\right)^{-\left[\frac{\theta}{\sigma-1}-1\right]}\right)^{-\frac{1}{\theta}},$$

and

$$\lambda_2 \equiv \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{-\frac{1}{\theta}} \left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}} \mu \left(\frac{C}{1+\pi}\right)^{-\frac{1}{\theta}}.$$

The expression in (60) resembles the price index in Chaney (2008), equation (8) closely. We note that ρ_d, the "multilateral resistance term," in that model is an equilibrium object in wages and GDP, whereas now it's an equilibrium object in wages, total consumption expenditure, and market tightness.

A.12 Defining the equilibrium

The equilibrium reduces to these equations in the equilibrium variables:

1. The free-entry condition for retailers, which pins down κ_{do}:

$$\frac{c_{do}}{\chi\left(\kappa_{do}\right)} = \int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right) dG\left(\varphi\right)$$

   Notice that now there are d times o markets and each market has an associated tightness. With our functional form assumptions, this equation can be simplified. Remember that with $V_{do} = 0$

$$M_{do}\left(\varphi\right) = \frac{p_{do}q_{do} - n_{do}q_{do}}{r+\lambda}$$

   so that

$$\begin{aligned} \int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right) dG\left(\varphi\right) &= \left(\frac{1}{r+\lambda}\right) \int_{\bar{\varphi}_{do}} p_{do}q_{do} - n_{do}q_{do} dG\left(\varphi\right) \\ \Rightarrow &= \left(\frac{1}{r+\lambda}\right) \left(1 - \frac{u_{do}}{1-i_{do}}\right)^{-1} \left(\frac{1}{N_o^x}\right) \Pi_{do}^m \end{aligned}$$

   in which Π_{do}^m is defined in equation (33) in the main text and we know that

$$\Pi_{do}^m = b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right) C_{do}$$

Using the equilibrium retailer entry condition gives

$$\begin{aligned}
\frac{c_{do}}{\chi(\kappa_{do})} &= \left(\frac{1}{r+\lambda}\right)\left(1-\frac{u_{do}}{1-i_{do}}\right)^{-1}\left(\frac{1}{N_o^x}\right)\Pi_{do}^m \\
\Rightarrow \kappa_{do} &= \left(\frac{1}{r+\lambda}\right)(\lambda+\kappa_{do}\chi(\kappa_{do}))\frac{b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do})(1+\pi)}{c_{do}C_o}C_{do},
\end{aligned}$$

in which we used $N_o^x = \frac{1}{1+\pi}C_o$ and $C_{do} = \alpha C_d$. In sum, this equilibrium condition can be written neatly as

$$\kappa_{do} = \left(\frac{1}{r+\lambda}\right)(\lambda+\kappa_{do}\chi(\kappa_{do}))\frac{b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do})(1+\pi)}{c_{do}C_o}C_{do}.$$

2. The expression that equates variable profits with the effective entry cost, which pins down $\bar{\varphi}_{do}$:

$$\bar{\varphi}_{do} = \mu\left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}}\left(\frac{w_o\tau_{do}}{P_d}\right)C_d^{\frac{1}{1-\sigma}}F_{do}(\kappa_{do})^{\frac{1}{\sigma-1}}.$$

in which

$$P_d = \lambda_2 \times C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \times \rho_d$$

and

$$\rho_d \equiv \left(\sum_{k=1}^{O}\frac{C_k}{C}\left(\frac{\kappa_{dk}\chi(\kappa_{dk})}{\lambda+\kappa_{dk}\chi(\kappa_{dk})}\right)(w_k\tau_{dk})^{-\theta}F_{dk}(\kappa_{dk})^{-\left[\frac{\theta}{\sigma-1}-1\right]}\right)^{-\frac{1}{\theta}}$$

and

$$\lambda_2 \equiv \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{-\frac{1}{\theta}}\left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}}\mu\left(\frac{C}{1+\pi}\right)^{-\frac{1}{\theta}}.$$

In this simplification we have used the assumption that

$$N_o^x = \frac{1}{1+\pi}C_o$$

3. National accounting/consumer's budget constraint pins down consumption C_d:

$$C_d = Y_d - I_d$$

in which

$$I_d = \sum_{k=1}^{O} I_{dk} = N_d^x e_d^x + \sum_{k=1}^{O}\kappa_{dk}u_{dk}N_k^x c_{dk} + u_{kd}N_d^x(l_{kd}+s_{kd}\kappa_{kd}\chi(\kappa_{kd})) + (1-u_{kd}-i_{kd})N_d^x f_{kd}$$

and

$$N_d^x = \frac{1}{1+\pi} C_d$$

and

$$Y_d = w_d L_d \left(1+\pi\right).$$

4. The global mutual fund pins down π:

$$\pi = \frac{\Pi}{\sum_k w_k L_k}$$

in which Π are the profits from the differentiated goods sold in all countries

$$\begin{aligned}\Pi &= \sum_k \sum_j \Pi_{jk} = \sum_k \sum_j \left(1 - \frac{u_{jk}}{1-i_{jk}}\right) N_k^x \int_{\bar{\varphi}_{jk}} p_{jk}\left(\varphi\right) q_{jk}\left(\varphi\right) - t_{jk}\left(\varphi\right) dG\left(\varphi\right) \\ &= \alpha \frac{C}{\sigma}\end{aligned}$$

B Welfare, consumption elasticity, and import elasticity

B.1 Proof of proposition 5: Changes in welfare

We prove proposition 5 assuming monopolistic competition and following steps similar to those used to prove proposition 1 in Arkolakis, Costinot, and Rodríguez-Clare (2012). With the exception of the search frictions, our functional form assumptions allow us to relate the differentiated goods price index in our model from section 4 to the price index equation (A22) in Arkolakis, Costinot, and Rodríguez-Clare (2012, p. 123)

$$P_{do}^{1-\sigma} = \left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right)\left(P_{do}^{ACR}\right)^{1-\sigma}, \tag{61}$$

in which $\left(P_{do}^{ACR}\right)^{1-\sigma} = N_o^x\left(\mu w_o \tau_{do}\right)^{1-\sigma}\Psi_{do}$, and it will be useful to define the important one-sided moment $\Psi_{do} \equiv \int_{\bar{\varphi}_{do}}^{\infty} z^{\sigma-1} dG(z)$. Also define the elasticity of this integral with respect to the cutoff as $\psi_{do} \equiv \dfrac{\partial \ln(\Psi_{do})}{\partial \ln(\bar{\varphi}_{do})}$. A sufficient condition for $\psi_{do} \leq 0$ is $\sigma > 1$. The overall price index with both homogeneous and differentiated goods is

$$\Xi_d = \left(\frac{p_d(1)}{1-\alpha}\right)^{1-\alpha}\left(\frac{P_d}{\alpha}\right)^{\alpha},$$

in which $p_d(1)$ is the price of the freely traded homogeneous good and α is the share of consumption devoted to the differentiated goods bundle. In section A.11.2, we show that the price of the freely traded good in equilibrium is the same in all countries $p_d(1) = p(1)$. Lastly, it will be useful to denote the total derivative of the log of a variable x, as $d\ln x = \ln(x'/x) = \ln(\hat{x})$ and so $\exp(d\ln x) = \hat{x}$.

B.1.1 Step 1: Small changes in welfare satisfy

$$d\ln(W_d) \quad = \quad d\ln(C_d) - \alpha d\ln(P_d). \tag{62}$$

Because the utility function is homogeneous of degree one, welfare is defined by real consumption expenditure $W_d = \dfrac{C_d}{\Xi_d}$. To derive equation (62) we use the definition of the price index to write

$$W_d \quad = \quad C_d\left(\frac{p(1)}{1-\alpha}\right)^{\alpha-1}\left(\frac{P_d}{\alpha}\right)^{-\alpha}$$

Taking logs gives

$$\ln(W_d) = \ln(C_d) + (\alpha - 1)[\ln(p(1)) - \ln(1-\alpha)] - \alpha[\ln(P_d) - \ln(\alpha)]$$

We define the price of the freely traded good as the numeraire, $p(1) = 1$, and then totally differentiate

$$d\ln(W_d) = d\ln(C_d) - \alpha d\ln(P_d)$$

Arkolakis, Costinot, and Rodríguez-Clare (2012) rely on two additional simplifications that remove consumption from this expression, which we cannot employ. First, they have that $C_d \propto Y_d$ with a proportionality constant that is only a function of exogenous parameters. We lack this simplification because investment in our setting is not exogenously proportional to output. Second, while Arkolakis, Costinot, and Rodríguez-Clare (2012) do not explicitly invoke restriction R2 here, they do rely on it to get that $\Pi_d \propto Y_d$ with a proportionality constant that is only a function of exogenous parameters. Because L_d is an exogenous endowment and w_d can be normalized, using R2 ensures that $w_d L_d + \Pi_d = Y_d \propto w_d L_d$ which ensures that $d\ln(w_d L_d) = 0$ implies $d\ln(C_d) = 0$ and welfare is determined soley by the price index.

B.1.2 Step 2: Small changes in the consumer price index satisfy

$$\begin{aligned} d\ln P_d &= \sum_{k=1}^{O} \frac{\lambda_{dk}}{\alpha\left(1-\sigma+\alpha^{-1}\psi_d\right)}\left[d\ln\left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right)\right. \\ &+ (1-\sigma+\psi_{dk})(d\ln w_k + d\ln\tau_{dk}) + d\ln N_k^x + \psi_{dk}\left(\frac{1}{\sigma-1}\right)d\ln(F(\kappa_{dk})) \\ &+ \left.\psi_{dk}\left(\frac{1}{1-\sigma}\right)d\ln(C_d)\right], \end{aligned} \tag{63}$$

in which ψ_{do} is defined above and $\psi_d \equiv \sum_{k=1}^{O} \lambda_{dk}\psi_{dk}$.

Equation (63) is analogous to equation (A33) in Arkolakis, Costinot, and Rodríguez-Clare (2012, p. 125) (except there is a typo in their first multiplicative term because γ_{ij} should be γ_j). When the utility function has only differentiated goods ($\alpha = 1$) and there are no search frictions ($u_{do} = 0$), equations (A33) and (63) are the same. The signs on ψ_d and ψ_{do} differ between the models because our model is defined in terms of productivity, while theirs is defined in terms of marginal cost.

We derive equation (63) by starting with total consumption in destination country d for the differentiated goods bundle from origin country o by integrating over all varieties at final prices. Because CES preferences define the differentiated goods aggregate given in equation

(1), this integral is the value of CES demand for the bundle of country o products

$$C_{do} = \left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} p_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi) = \alpha \frac{P_{do}^{1-\sigma} C_d}{P_d^{1-\sigma}}.$$

This can be easily derived from equation (2). Define the consumption share λ_{do} as

$$\lambda_{do} \equiv \frac{C_{do}}{C_d} = \alpha \frac{P_{do}^{1-\sigma} C_d}{P_d^{1-\sigma}} \left(\frac{1}{C_d}\right) = \alpha \frac{P_{do}^{1-\sigma}}{P_d^{1-\sigma}}.$$

Our definition of the consumption share differs from Arkolakis, Costinot, and Rodríguez-Clare (2012) in two important ways. First, we use consumer expenditure instead of output because our model does not guarantee that income and consumption are proportional. Second, consumption, which is what matters for welfare, is measured at final sales prices, while the import share is measured at negotiated import prices. We will work with $P_d^{1-\sigma}$ using the definition of the price index for the differentiated good in the destination market d, given by

$$P_d = \left[\sum_{k=1}^{O} P_{dk}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

Take the log of this expression to get $(1-\sigma)\ln P_d = \ln \sum_{k=1}^{O} P_{dk}^{1-\sigma}$ and then totally differentiate to get

$$(1-\sigma)\, d\ln P_d = \sum_{k=1}^{O} \frac{P_{dk}^{1-\sigma}}{P_d^{1-\sigma}} \frac{dP_{dk}^{1-\sigma}}{P_{dk}^{1-\sigma}}.$$

Rearrange λ_{do} to get $\dfrac{\lambda_{do}}{\alpha P_{do}^{1-\sigma}} = \dfrac{1}{P_d^{1-\sigma}}$ and then use this to simplify

$$(1-\sigma)\, d\ln P_d = \sum_{k=1}^{O} \frac{\lambda_{dk}}{\alpha} d\ln P_{dk}^{1-\sigma}.$$

Taking logs of equation (61) and totally differentiating gives

$$d\ln P_{do}^{1-\sigma} = d\ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) + d\ln\left(P_{do}^{ACR}\right)^{1-\sigma}.$$

Employing our functional form assumptions, which gives $\left(P_{do}^{ACR}\right)^{1-\sigma} = N_o^x\left(\mu w_o \tau_{do}\right)^{1-\sigma}\Psi_{do}$, we can derive

$$d\ln\left(P_{do}^{ACR}\right)^{1-\sigma} = d\ln N_o^x + (1-\sigma)\left(d\ln w_o + d\ln \tau_{do}\right) + \psi_{do} d\ln\left(\bar{\varphi}_{do}\right),$$

in which we use the chain rule to get

$$d\ln\Psi_{do}=\psi_{do}d\ln\left(\bar{\varphi}_{do}\right).$$

Putting these parts together gives

$$(1-\sigma)\,d\ln P_d=\sum_{k=1}^{O}\frac{\lambda_{dk}}{\alpha}\left[d\ln\left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right)+(1-\sigma)\left(d\ln w_k+d\ln\tau_{dk}\right)+d\ln N_k^x+\psi_{dk}d\ln\left(\bar{\varphi}_{dk}\right)\right]. \tag{64}$$

If we set $\alpha=1$ and $c_{do}=0$ so that $u_{do}=0$ we match equation (A34) in Arkolakis, Costinot, and Rodríguez-Clare (2012, p. 125).

Next, take the log and total derivative of the cutoff expression from equation (18)

$$d\ln\left(\bar{\varphi}_{do}\right)=-d\ln\left(P_d\right)+d\ln\left(\tau_{do}\right)+\left(\frac{1}{\sigma-1}\right)d\ln\left(F\left(\kappa_{do}\right)\right)-\left(\frac{1}{\sigma-1}\right)d\ln\left(C_d\right)+d\ln\left(w_o\right), \tag{65}$$

which is the analog to equation (A36) in Arkolakis, Costinot, and Rodríguez-Clare (2012, p. 126). There are a few differences between equation (65) here and their equation (A36). First, the signs are reversed because we define everything in terms of productivity, while they use costs. Second, their term ξ_{ij} captures the fixed cost of entry like our term $F\left(\kappa_{do}\right)$ (see equation (A27) on page 124). And while their term ρ_{ij} allows for some foreign labor to be used to enter a foreign country, we do not. Making the same restriction in their model would require setting $h_{ij}=1$ and hence $\rho_{ij}=1$. Finally, our threshold expression includes total consumption.

Combining equations (64) and (65) gives equation (63).

B.1.3 Step 3: Small changes in the consumer price index satisfy

$$\begin{aligned}
d\ln P_d &= \sum_{k=1}^{O}\lambda_{dk}\left(\frac{d\ln\left(\lambda_{dk}\right)-d\ln\left(\lambda_{dd}\right)}{\alpha\left(1-\sigma+\alpha^{-1}\psi_d\right)}\right)+\frac{\left(\alpha\psi_{dd}-\psi_d\right)d\ln\left(\bar{\varphi}_{dd}\right)}{\alpha\left(1-\sigma+\alpha^{-1}\psi_d\right)}\\
&+\frac{d\ln N_d^x}{\left(1-\sigma+\alpha^{-1}\psi_d\right)}\\
&+\frac{\psi_d d\ln\left(F\left(\kappa_{dd}\right)\right)}{\left(\sigma-1\right)\alpha\left(1-\sigma+\alpha^{-1}\psi_d\right)}\\
&+\frac{d\ln\left(\dfrac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right)}{\left(1-\sigma+\alpha^{-1}\psi_d\right)}\\
&+\frac{\psi_d d\ln\left(C_d\right)}{\left(1-\sigma\right)\alpha\left(1-\sigma+\alpha^{-1}\psi_d\right)}
\end{aligned} \tag{66}$$

If we set $\alpha=1$ and $c_{do}=0$ so that $u_{do}=0$ and $l_{dd}=-h_{dd}$ so that $F\left(\kappa_{dd}\right)$ is a constant, then (66) becomes equation (A37) of Arkolakis, Costinot, and Rodríguez-Clare (2012, p.

126).

Start again with the consumption share $\lambda_{do} = \alpha \frac{P_{do}^{1-\sigma}}{P_d^{1-\sigma}}$ and form $\frac{\lambda_{do}}{\lambda_{dd}} = \frac{P_{do}^{1-\sigma}}{P_{dd}^{1-\sigma}}$. Substitute into the ratio $\frac{\lambda_{do}}{\lambda_{dd}}$ our functional form assumptions for the price index, take logs, and then totally differentiate to get

$$\begin{aligned} d\ln\left(\lambda_{do}\right) - d\ln\left(\lambda_{dd}\right) &= (1-\sigma)\left(d\ln w_o + d\ln \tau_{do}\right) + \psi_{do} d\ln\left(\bar{\varphi}_{do}\right) - \psi_{dd} d\ln\left(\bar{\varphi}_{dd}\right) \\ &+ d\ln N_o^x - d\ln N_d^x \\ &+ d\ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) - d\ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right). \end{aligned} \tag{67}$$

In obtaining this expression we have simplified terms by recalling that we are considering a foreign shock so that $d\ln\tau_{dd} = 0$ and that our normalization of the price of the freely traded good ensures that $d\ln w_o = d\ln w_d = 0$. We keep $d\ln w_o$ in the expression and ordered the terms as presented in Arkolakis, Costinot, and Rodríguez-Clare (2012, p. 126) to make comparing the expressions easier.

We can derive two cutoff expressions

$$d\ln\left(\bar{\varphi}_{do}\right) = -d\ln\left(P_d\right) + d\ln\left(\tau_{do}\right) + \frac{d\ln\left(F\left(\kappa_{do}\right)\right)}{\sigma-1} + d\ln\left(w_o\right) - \left(\frac{1}{\sigma-1}\right) d\ln\left(C_d\right),$$

and also

$$d\ln\left(\bar{\varphi}_{dd}\right) = -d\ln\left(P_d\right) + \frac{d\ln\left(F\left(\kappa_{dd}\right)\right)}{\sigma-1} - \left(\frac{1}{\sigma-1}\right) d\ln\left(C_d\right),$$

in which we again impose that $\tau_{dd} = 1$ and $d\ln w_d = 0$. Combining these two cutoff expressions gives

$$d\ln\left(\bar{\varphi}_{do}\right) = d\ln\left(\bar{\varphi}_{dd}\right) + d\ln\left(w_o\right) + d\ln\left(\tau_{do}\right) + \frac{d\ln\left(F\left(\kappa_{do}\right)\right)}{\sigma-1} - \frac{d\ln\left(F\left(\kappa_{dd}\right)\right)}{\sigma-1}, \tag{68}$$

which is akin to the last equation of Arkolakis, Costinot, and Rodríguez-Clare (2012, p. 126) with the exception that they have a typo because the equal sign should be a minus sign. In our model, it is not necessarily the case that $d\ln\left(F\left(\kappa_{dd}\right)\right) = 0$ in response to a foreign shock because the effective entry cost, $F\left(\kappa_{dd}\right)$, is an endogenous variable and not a parameter.

Combine expression (68) with (67) to get

$$\begin{aligned} d\ln(\lambda_{do}) - d\ln(\lambda_{dd}) &= (1-\sigma+\psi_{do})(d\ln w_o + d\ln\tau_{do}) \\ &+ \psi_{do}\left(\frac{d\ln(F(\kappa_{do}))}{\sigma-1} - \frac{d\ln(F(\kappa_{dd}))}{\sigma-1}\right) \\ &+ (\psi_{do}-\psi_{dd})\,d\ln(\bar{\varphi}_{dd}) + d\ln N_o^x - d\ln N_d^x \\ &+ d\ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) - d\ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right). \end{aligned} \tag{69}$$

Equation (69) is analogous to equation (A38) in Arkolakis, Costinot, and Rodríguez-Clare (2012, p. 127), which has a typo because α^*_{ij} should be α^*_{jj}. Substituting equation (69) into equation (63) and performing algebra gives equation (66).

B.1.4 Step 4: Small changes in the consumer price index satisfy

$$\begin{aligned} d\ln P_d &= \frac{d\ln(\lambda_{dd})}{\theta} \\ &- \frac{d\ln N_d^x}{\theta} \\ &- \frac{(\sigma-1-\theta)\,d\ln(F(\kappa_{dd}))}{(\sigma-1)\,\theta} \\ &- \frac{d\ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right)}{\theta} \\ &- \frac{(\sigma-1-\theta)\,d\ln(C_d)}{(1-\sigma)\,\theta} \end{aligned} \tag{70}$$

We depart somewhat from the approach taken in step 4 of Arkolakis, Costinot, and Rodríguez-Clare (2012, p. 127) in simplifying equation (66) to derive equation (70). They invoke macro-level restriction number 3, “R3: The import demand system is such that for any importer j and any pair of exporters $i \neq j$ and $i' \neq j$, $\varepsilon_j^{ii'} = \varepsilon < 0$ if $i = i'$, and zero otherwise.” As they describe on page 103, this restriction imposes symmetry on the elasticity of the consumption ratio to changes in variable trade costs. That elasticity in our model in general is given by equation (76) and need not be symmetric across countries. A sufficient condition to derive equation (70), however, is that productivity distributions and consumer preferences are symmetric. For now, we impose those restrictions in the following steps but could likely relax them in future work.

The term we need to consider from equation (66) is $\psi_d \equiv \sum_{k=1}^{O} \lambda_{dk}\psi_{dk}$, which is the consumption share weighted average of the elasticity of the moment of the productivity distribution, in which $\psi_{do} = \dfrac{d\ln(\Psi_{do})}{d\ln(\bar{\varphi}_{do})}$. We assume that productivity $\varphi \in [1, +\infty)$ is Pareto distributed with CDF $G[\tilde{\varphi} < \varphi] = 1 - \varphi^{-\theta}$ and PDF $g(\varphi) = \theta\varphi^{-\theta-1}$ in which, as usual, $\theta > \sigma - 1$ in order to close the model. With this distribution, the moment $\Psi_{do} = \dfrac{\theta\bar{\varphi}_{do}^{\sigma-\theta-1}}{\theta-\sigma+1}$ and the elasticity $\psi_{do} = -\dfrac{\bar{\varphi}_{do}^{\sigma}\theta\bar{\varphi}_{do}^{-\theta-1}}{\Psi_{do}} = -(\theta-\sigma+1)$. Notice that the restriction that $\theta > \sigma - 1$ ensures $\psi_{do} < 0$ and $\psi_d < 0$. Also notice that $\psi_{do} = \psi_{dd}$ and the term we are actually interested in becomes

$$\psi_d \equiv \sum_{k=1}^{O} \lambda_{dk}\psi_{do} = \alpha(\sigma - 1 - \theta), \tag{71}$$

because by definition consumption shares $\alpha = \sum_{k=1}^{O} \lambda_{dk}$. Substituting equation (71) into (66) and also using the fact that Euler's homogeneous function theorem gives $\sum_{k=1}^{O} \lambda_{dk} d\ln(\lambda_{do}) = 0$ provides (70).

B.1.5 Step 5: Small changes in the number of producers

We cannot make the simplification $d\ln N_d^x = 0$ as done in step 5 of Arkolakis, Costinot, and Rodríguez-Clare (2012, p. 127) because we have assumed that $N_d^x = \dfrac{C}{(1+\pi)}\left(\dfrac{C_d}{C}\right)$ and both C_d and $\pi = \Pi/\sum_k w_k L_k$ are endogenous objects. Allowing free entry into the market for producers would be an alternative assumption but would then require an additional equation for determining equilbrium market tightness. We discuss that extension more in appendix A.9.

B.1.6 Combining step 1 to step 4 into the general welfare expression

Combining equation (62) with equation (70) provides the change in welfare in response to a foreign shock in our model

$$\begin{aligned} d\ln(W_d) &= -\left(\frac{\alpha}{\theta}\right) d\ln(\lambda_{dd}) \\ &+ \left(1 + \left(\frac{\alpha}{\theta}\right)\left(1 - \frac{\theta}{\sigma-1}\right)\right) d\ln(C_d) \\ &+ \left(\frac{\alpha}{\theta}\right) d\ln N_d^x \\ &+ \left(\frac{\alpha}{\theta}\right)\left(1 - \frac{\theta}{\sigma-1}\right) d\ln(F(\kappa_{dd})) \\ &+ \left(\frac{\alpha}{\theta}\right) d\ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right) \end{aligned} \tag{72}$$

We derive this by substituting the change in the price index from (70) into the welfare expression from (62) and simplifying the algebra using, in particular, $\frac{\alpha\left(\sigma-1-\theta\right)}{\left(\sigma-1\right)\theta}=\left(\frac{\alpha}{\theta}\right)\left(1-\frac{\theta}{\sigma-1}\right)$.

B.1.7 The change in welfare in proposition 5

We made an assumption that productivity, $\varphi\in[1,+\infty)$, follows a Pareto distribution with CDF $G\left[\tilde{\varphi}<\varphi\right]=1-\varphi^{-\theta}$ in appendix B.1.4 in order to derive the general equation (72). Two additional assumptions are needed to derive proposition 5 in the main text from the general welfare change in equation (72). The first of these assumptions is that the cost of remaining idle, $-h_{dd}$, is the same as the flow search costs that producers pay to find retailers, l_{dd}, so that $l_{dd}=-h_{dd}$. With this assumption, the effective entry costs become a function of exogenous parameters and $d\ln\left(F\left(\kappa_{dd}\right)\right)=0$. The second assumption is that the number of domestic producers does not respond to a foreign shock, $d\ln N_d^x=0$. One could rationalize this assumption by assuming free entry into production or that N_d^x is exogenous.

Applying these two additional assumptions to the general welfare changes in equation (72) gives

$$d\ln\left(W_d\right)=-\left(\frac{\alpha}{\theta}\right)d\ln\left(\lambda_{dd}\right)+\left(1+\left(\frac{\alpha}{\theta}\right)\left(1-\frac{\theta}{\sigma-1}\right)\right)d\ln\left(C_d\right)+\left(\frac{\alpha}{\theta}\right)d\ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right), \tag{73}$$

which we can integrate to get the welfare response to any foreign shock in proposition 5 of the main text

$$\hat{W}_d=\hat{\lambda}_{dd}^{-\frac{\alpha}{\theta}}\left(1-\widehat{\frac{u_{dd}}{1-i_{dd}}}\right)^{\frac{\alpha}{\theta}}\hat{C}_d^{1+\frac{\alpha}{\theta}\left(1-\frac{\theta}{\sigma-1}\right)} \tag{74}$$

B.2 Proof of proposition 6: Consumption elasticity

B.2.1 Relating price indexes

To derive an analogous elasticity in our model, start with the functional form assumptions detailed in section 2. Because, with the exception of the search frictions, these functional form assumptions are the same as in Arkolakis, Costinot, and Rodríguez-Clare (2012), we can relate the price index in our model given in section 4 to the price index equation (A22) in Arkolakis, Costinot, and Rodríguez-Clare (2012, p. 123)

$$\left(P_{do}^{ACR}\right)^{1-\sigma}\quad=\quad N_o^x\left(\mu w_o\tau_{do}\right)^{1-\sigma}\Psi_{do},$$

in which it will be useful to define $\Psi_{do}=\int_{\bar{\varphi}_{do}}^{\infty}\varphi^{\sigma-1}dG\left(\varphi\right)$ and the elasticity of this integral with respect to the cutoff $\psi_{do}=\frac{\partial\ln\left(\Psi_{do}\right)}{\partial\ln\left(\bar{\varphi}_{do}\right)}\leq 0$ a sufficient condition for which is $\sigma>1$.

B.2.2 Demand for a country's bundle of goods

We can derive total consumption in destination country d for the goods bundle from origin country o by integrating over all varieties at final prices. Because we have CES preferences, this integral is the value of CES demand for the bundle of country o products

$$C_{do} = \left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} p_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi) = \alpha \frac{P_{do}^{1-\sigma} C_d}{P_d^{1-\sigma}}.$$

Define the consumption share (which we note in our model is different from the observed trade share) as λ_{do}

$$\lambda_{do} = \frac{C_{do}}{C_d} = \alpha \frac{P_{do}^{1-\sigma} C_d}{P_d^{1-\sigma} C_d} = \alpha \frac{P_{do}^{1-\sigma}}{P_d^{1-\sigma}}.$$

We can also form relative consumption ratios, which is equivalent to Arkolakis, Costinot, and Rodríguez-Clare (2012) equation (21), page 110, and is just the ratio of the price indexes raised to a power

$$\frac{\lambda_{do}}{\lambda_{dd}} = \frac{C_{do}}{C_{dd}} = \frac{P_{do}^{1-\sigma}}{P_{dd}^{1-\sigma}}.$$

Using the country-specific price indexes given above we have

$$\frac{P_{do}^{1-\sigma}}{P_{dd}^{1-\sigma}} = \frac{\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right)\left(P_{do}^{ACR}\right)^{1-\sigma}}{\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right)\left(P_{dd}^{ACR}\right)^{1-\sigma}},$$

in which we used the definition of P_{do}^{ACR}. Taking the log of relative consumption ratios therefore gives

$$\ln\left(\frac{C_{do}}{C_{dd}}\right) = \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) + \ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) - \ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right). \tag{75}$$

B.2.3 Derivative of consumption ratio with respect to tariffs

The goal is to derive two derivatives. The first is the direct effect of a change in the tariffs τ_{do} on the consumption ratio

$$\frac{\partial \ln\left(C_{do}/C_{dd}\right)}{\partial \ln\left(\tau_{do}\right)} = \varepsilon_o^{dd}.$$

The second is the indirect effect, which documents how changing tariffs between a third country d' and the origin o changes relative consumption in country d

$$\frac{\partial \ln \left(C_{do}/C_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} = \varepsilon_o^{dd'}.$$

B.2.4 Direct effect of tariff changes ($d' = d$ case)

We begin by deriving $\frac{\partial \ln \left(C_{do}/C_{dd}\right)}{\partial \ln \left(\tau_{do}\right)} = \varepsilon_o^{dd}$ in the most general form and then apply a few restrictions to compare it to the elasticity in Arkolakis, Costinot, and Rodríguez-Clare (2012). Normalizing the price of the homogeneous good ensures that $\frac{\partial \ln \left(w_d\right)}{\partial \ln \left(\tau_{do}\right)} = 0$ and $\frac{\partial \ln \left(w_o\right)}{\partial \ln \left(\tau_{do}\right)} = 0$.

B.2.5 First and second terms of equation (75) ($d' = d$ case)

Differentiating and simplifying the first term of equation (75) gives

$$\frac{\partial}{\partial \ln \left(\tau_{do}\right)} \ln \left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) = \frac{\partial \ln \left(N_o^x\right)}{\partial \ln \left(\tau_{do}\right)} + (1-\sigma) + \psi_{do} \frac{\partial \ln \left(\bar{\varphi}_{do}\right)}{\partial \ln \left(\tau_{do}\right)},$$

and similarly

$$\frac{\partial}{\partial \ln \left(\tau_{do}\right)} \ln \left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) = \frac{\partial \ln \left(N_d^x\right)}{\partial \ln \left(\tau_{do}\right)} + \psi_{dd} \frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{do}\right)}.$$

Combining these gives

$$\begin{aligned} \frac{\partial}{\partial \ln \left(\tau_{do}\right)} \ln \left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \frac{\partial}{\partial \ln \left(\tau_{do}\right)} \ln \left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) &= (1-\sigma) + \psi_{do} \frac{\partial \ln \left(\bar{\varphi}_{do}\right)}{\partial \ln \left(\tau_{do}\right)} - \psi_{dd} \frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{do}\right)} \\ &+ \frac{\partial \ln \left(N_o^x\right)}{\partial \ln \left(\tau_{do}\right)} - \frac{\partial \ln \left(N_d^x\right)}{\partial \ln \left(\tau_{do}\right)}. \end{aligned}$$

The elasticities of the cutoffs $\bar{\varphi}_{do}$ and $\bar{\varphi}_{dd}$ are related because changing tariff τ_{do} changes the price index P_d which changes the cutoff $\bar{\varphi}_{dd}$. We can derive this relationship by differentiating the explicit expression for the cutoff given in equation (18)

$$\begin{aligned} \frac{\partial \ln \left(\bar{\varphi}_{do}\right)}{\partial \ln \left(\tau_{do}\right)} &= 1 + \frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{do}\right)} \\ &+ \left(\frac{1}{\sigma-1}\right) \left[\frac{\partial \ln \left(F_{do}\right)}{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)} \frac{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}{\partial \ln \left(\tau_{do}\right)} - \frac{\partial \ln \left(F_{dd}\right)}{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)} \frac{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}{\partial \ln \left(\tau_{do}\right)}\right]. \end{aligned}$$

Substituting this into the elasticity of the general expression for the ratio of relative price indexes and simplifying gives

$$\frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) = (1-\sigma) + \psi_{do} + (\psi_{do} - \psi_{dd}) \frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{do})}$$
$$+ \left(\frac{\psi_{do}}{\sigma - 1}\right)\left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}\frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{do})} - \frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}\frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{do})}\right]$$
$$+ \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{do})} - \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{do})}.$$

The first line on the right is the same as equation (21) in Arkolakis, Costinot, and Rodríguez-Clare (2012) except that $\psi_{do} \leq 0$ in our case, while $\gamma_{ij} \geq 0$ in Arkolakis et al.'s expressions because we define our model in terms of productivity, φ, while they define theirs in terms of marginal cost.

B.2.6 Elasticity of destination–origin market unmatched rate

Next, we calculate the elasticity of the destination–origin market unmatched producers' rate. Because we are studying a steady state, we use the definition $\dfrac{1-u_{do}-i_{do}}{1-i_{do}} = \dfrac{\kappa_{do}\chi(\kappa_{do})}{\lambda + \kappa_{do}\chi(\kappa_{do})}$ to derive

$$\frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) = \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})}\right),$$

in which we used the chain rule to write

$$\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})} = \frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \kappa_{do}\chi(\kappa_{do})}\frac{\partial \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})} = \frac{1}{\kappa_{do}\chi(\kappa_{do})}\left(\frac{\partial \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})}\right).$$

B.2.7 Elasticity of destination–destination market unmatched rate

Calculating the elasticity of the destination–destination market unmatched producers' rate with respect to τ_{do} also relies on the definition of $\dfrac{1-u_{dd}-i_{dd}}{1-i_{dd}} = \dfrac{\kappa_{dd}\chi(\kappa_{dd})}{\lambda + \kappa_{dd}\chi(\kappa_{dd})}$. The steps to derive this will be identical to the ones we took in calculating the destination–origin market unmatched rate with only the sub-indexes changing. The final derivative is

$$\frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right) = \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{do})}\right),$$

in which we used the chain rule again to calculate

$$\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{do})} = \frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \kappa_{dd}\chi(\kappa_{dd})}\frac{\partial \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{do})} = \frac{1}{\kappa_{dd}\chi(\kappa_{dd})}\left(\frac{\partial \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{do})}\right).$$

B.2.8 General expression for $d' = d$ case

Here we try to write the most general possible expression only assuming that $\frac{\partial \ln(w_d)}{\partial \ln(\tau_{do})} = 0$ and $\frac{\partial \ln(w_o)}{\partial \ln(\tau_{do})} = 0$. Combining the general term expression in Arkolakis et al. with the elasticity of the finding rate with respect to tariffs gives

$$\begin{aligned}
\frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\frac{C_{do}}{C_{dd}}\right) &= (1-\sigma) + \psi_{do} + (\psi_{do} - \psi_{dd}) \frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{do})} + \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{do})} - \frac{\partial \ln\left(N_d^x\right)}{\partial \ln(\tau_{do})} \\
&+ \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{do})}\right) \\
&+ \left(\frac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}\frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{do})} - \frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}\frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{do})}\right].
\end{aligned}$$

B.2.9 Indirect effect of tariff changes ($d' \neq d$ case)

The second derivative is the indirect effect, which documents how changing tariffs between a third country d' and the origin o changes relative consumption in country d

$$\frac{\partial \ln(C_{do}/C_{dd})}{\partial \ln(\tau_{d'o})} = \varepsilon_o^{dd'}.$$

B.2.10 First and second terms of equation (75) ($d' \neq d$ case)

Following the general pattern used previously, we first derive the change in the price indexes in Arkolakis et al. as

$$\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) = \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{d'o})} + \psi_{do}\frac{\partial \ln(\bar{\varphi}_{do})}{\partial \ln(\tau_{d'o})},$$

and similarly

$$\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) = \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{d'o})} + \psi_{dd}\frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})}.$$

Combining these gives

$$\begin{aligned}
\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) &= \psi_{do}\frac{\partial \ln(\bar{\varphi}_{do})}{\partial \ln(\tau_{d'o})} - \psi_{dd}\frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})} \\
&+ \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{d'o})}.
\end{aligned}$$

The elasticities of the cutoffs $\bar{\varphi}_{do}$ and $\bar{\varphi}_{dd}$ with respect to $\tau_{d'o}$ are also related because changing tariff $\tau_{d'o}$ changes the price index P, which changes the cutoff $\bar{\varphi}_{dd}$. We can derive this relationship by differentiating the explicit expression for the cutoff given in equation (18)

$$\frac{\partial \ln \left(\bar{\varphi}_{do}\right)}{\partial \ln \left(\tau_{d'o}\right)} = -\frac{\partial \ln \left(P\right)}{\partial \ln \left(\tau_{d'o}\right)} + \left(\frac{1}{\sigma - 1}\right) \frac{\partial \ln \left(F_{do}\right)}{\partial \ln \left(\tau_{d'o}\right)}$$

and symmetrically

$$\frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} = -\frac{\partial \ln \left(P_d\right)}{\partial \ln \left(\tau_{d'o}\right)} + \left(\frac{1}{\sigma - 1}\right) \frac{\partial \ln \left(F_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)}.$$

So the relationship between the two cutoff elasticities is

$$\begin{aligned}\frac{\partial \ln \left(\bar{\varphi}_{do}\right)}{\partial \ln \left(\tau_{d'o}\right)} &= \frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} \\ &+ \left(\frac{1}{\sigma - 1}\right)\left[\frac{\partial \ln \left(F_{do}\right)}{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}\frac{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}{\partial \ln \left(\tau_{d'o}\right)} - \frac{\partial \ln \left(F_{dd}\right)}{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}\frac{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}{\partial \ln \left(\tau_{d'o}\right)}\right],\end{aligned}$$

and we use the chain rule to expand the derivatives with respect to the finding rate. Substituting the relationship between the cutoffs into the general expression for the ratio of relative prices and simplifying gives

$$\begin{aligned}\frac{\partial}{\partial \ln \left(\tau_{d'o}\right)} \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \frac{\partial}{\partial \ln \left(\tau_{d'o}\right)} \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) &= \left(\psi_{do} - \psi_{dd}\right)\frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} + \frac{\partial \ln \left(N_o^x\right)}{\partial \ln \left(\tau_{d'o}\right)} - \frac{\partial \ln \left(N_d^x\right)}{\partial \ln \left(\tau_{d'o}\right)} \\ &+ \left(\frac{\psi_{do}}{\sigma - 1}\right)\left[\frac{\partial \ln \left(F_{do}\right)}{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}\frac{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}{\partial \ln \left(\tau_{d'o}\right)} - \frac{\partial \ln \left(F_{dd}\right)}{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}\frac{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}{\partial \ln \left(\tau_{d'o}\right)}\right].\end{aligned}$$

B.2.11 Elasticity of destination–origin market matched rate

We continue to follow the pattern used previously and calculate the elasticity of the destination–origin market matched producers' rate. Because we are studying a steady state, we use the definition $\frac{1 - u_{do} - i_{do}}{1 - i_{do}} = \frac{\kappa_{do}\chi\left(\kappa_{do}\right)}{\lambda + \kappa_{do}\chi\left(\kappa_{do}\right)}$ to derive

$$\frac{\partial}{\partial \ln \left(\tau_{d'o}\right)} \ln\left(\frac{1 - u_{do} - i_{do}}{1 - i_{do}}\right) = \left(\frac{u_{do}}{1 - i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right),$$

in which we used the chain rule to write

$$\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{d'o}\right)} = \frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \kappa_{do}\chi\left(\kappa_{do}\right)}\frac{\partial \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{d'o}\right)} = \frac{1}{\kappa_{do}\chi\left(\kappa_{do}\right)}\frac{\partial \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{d'o}\right)}.$$

For the record, the elasticity of the third term boils down to

$$\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) = \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{d'o})}\right).$$

The things that matter are the unmatched rate and the elasticity of the finding rate with respect to tariffs.

B.2.12 Elasticity of destination–destination market matched rate

The fourth term requires that we calculate

$$\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right).$$

The steps are identical to the ones we took in calculating the destination–origin market matched rate derivative with only the sub-indexes changing. The end result is

$$\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right) = \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{d'o})}\right),$$

in which we again used the chain rule to write

$$\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{d'o})} = \frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \kappa_{dd}\chi(\kappa_{dd})}\frac{\partial \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{d'o})} = \frac{1}{\kappa_{dd}\chi(\kappa_{dd})}\left(\frac{\partial \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{d'o})}\right).$$

B.2.13 General expression for $d' \neq d$ case

Here we try to write the most general possible expression only assuming that $\frac{\partial \ln(w_d)}{\partial \ln(\tau_{d'o})} = 0$ and $\frac{\partial \ln(w_o)}{\partial \ln(\tau_{d'o})} = 0$. The general term expression in Arkolakis et al. was

$$\begin{aligned}\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) &= (\psi_{do}-\psi_{dd})\frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})} + \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{d'o})} \\ &+ \left(\frac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}\frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}\frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{d'o})}\right].\end{aligned}$$

Combining these with the elasticity of unmatched rates gives

$$\begin{aligned}\frac{\partial \ln(C_{do}/C_{dd})}{\partial \ln(\tau_{d'o})} &= (\psi_{do}-\psi_{dd})\frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})} + \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{d'o})} \\ &+ \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{d'o})}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{d'o})}\right) \\ &+ \left(\frac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}\frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}\frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{d'o})}\right].\end{aligned}$$

B.2.14 Final general elasticity

The final expression is

$$\frac{\partial \ln \left(C_{do}/C_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} = \varepsilon_o^{dd'} = \begin{cases} \begin{aligned} & (1-\sigma) + \psi_{do} + \left(\psi_{do} - \psi_{dd}\right) \frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{do}\right)} + \frac{\partial \ln \left(N_o^x\right)}{\partial \ln \left(\tau_{do}\right)} - \frac{\partial \ln \left(N_d^x\right)}{\partial \ln \left(\tau_{do}\right)} \\ + & \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{do}\right)}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln \left(\tau_{do}\right)}\right) \\ + & \left(\frac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln \left(F_{do}\right)}{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)} \frac{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}{\partial \ln \left(\tau_{do}\right)} - \frac{\partial \ln \left(F_{dd}\right)}{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)} \frac{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}{\partial \ln \left(\tau_{do}\right)}\right] \end{aligned} & if\ d' = d \\ \\ \begin{aligned} & \left(\psi_{do} - \psi_{dd}\right) \frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} + \frac{\partial \ln \left(N_o^x\right)}{\partial \ln \left(\tau_{d'o}\right)} - \frac{\partial \ln \left(N_d^x\right)}{\partial \ln \left(\tau_{d'o}\right)} \\ + & \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right) \\ + & \left(\frac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln \left(F_{do}\right)}{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)} \frac{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}{\partial \ln \left(\tau_{d'o}\right)} - \frac{\partial \ln \left(F_{dd}\right)}{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)} \frac{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}{\partial \ln \left(\tau_{d'o}\right)}\right] \end{aligned} & if\ d' \neq d \end{cases}. \tag{76}$$

B.2.15 The elasticity in proposition 6

Three additional assumptions are needed to derive proposition 6 in the main text from the general elasticity equation (76). The first of these is that the cost of remaining idle, $-h_{do}$, is the same as the flow search costs that producers pay to find retailers, l_{do}, so that $l_{do} = -h_{do}\ \forall do$. With this assumption, the effective entry costs become a function of exogenous parameters and $\partial \ln \left(F_{do}\right)/\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right) = 0\ \forall do$. The second assumption is that $\frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{do})} = \frac{\partial \ln\left(N_d^x\right)}{\partial \ln(\tau_{do})}$ and $\frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{d'o})} = \frac{\partial \ln\left(N_d^x\right)}{\partial \ln(\tau_{d'o})}$. One could rationalize equality between these elasticities by studying symmetric equilibria or ensure that the elasticities are zero by either assuming free entry into production or that N_d^x and N_o^x are exogenous. The third assumption is that productivity, $\varphi \in [1, +\infty)$, follows a Pareto distribution with CDF $G\left[\tilde{\varphi} < \varphi\right] = 1 - \varphi^{-\theta}$. Appendix B.2.17 shows that this assumption implies that the terms in the elasticity that depend on moments of the productivity distribution simplify to

$$(1-\sigma) + \psi_{do} + \left(\psi_{do} - \psi_{dd}\right) \frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{do}\right)} = -\theta \qquad \text{and} \qquad \left(\psi_{do} - \psi_{dd}\right) \frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} = 0$$

for the $d' = d$ and $d' \neq d$ cases of the consumption elasticity, respectively.

Applying these three assumptions to the general elasticity equation (76) gives

$$\frac{\partial \ln \left(C_{do}/C_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} = \begin{cases} -\theta + \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{do}\right)}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln \left(\tau_{do}\right)}\right) & if\ d' = d \\ \\ \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right) & if\ d' \neq d \end{cases}, \tag{77}$$

which is the consumption share elasticity equation in proposition 6 of the main text.

B.2.16 Consumption elasticity as retailer search costs approach zero

As the search costs that retailers pay to find producers approaches zero in all destination-origin markets, $c_{do} \to 0 \; \forall do$, the following three things happen: 1) the fraction of unmatched searching producers goes to zero, $u_{do} \to 0 \; \forall do$, 2) the effective entry costs become a function of exogenous parameters, $\partial \ln (F_{do}) / \partial \ln (\kappa_{do} \chi (\kappa_{do})) \to 0 \; \forall do$, and 3) the value of imports converges to the value of consumption, $IM_{do} \to C_{do} \; \forall do$. These three facts together imply that the consumption elasticity converges to

$$\frac{\partial \ln (C_{do}/C_{dd})}{\partial \ln (\tau_{d'o})} = \varepsilon_o^{ACRdd'} = \begin{cases} (1-\sigma) + \psi_{do} + (\psi_{do} - \psi_{dd}) \dfrac{\partial \ln (\bar{\varphi}_{dd})}{\partial \ln (\tau_{do})} + \dfrac{\partial \ln (N_o^x)}{\partial \ln (\tau_{do})} - \dfrac{\partial \ln \left(N_d^x\right)}{\partial \ln (\tau_{do})} \; if \; d' = d \\ \\ (\psi_{do} - \psi_{dd}) \dfrac{\partial \ln (\bar{\varphi}_{dd})}{\partial \ln (\tau_{d'o})} + \dfrac{\partial \ln (N_o^x)}{\partial \ln (\tau_{d'o})} - \dfrac{\partial \ln \left(N_d^x\right)}{\partial \ln (\tau_{d'o})} \; if \; d' \neq d \end{cases}. \tag{78}$$

First, we highlight that this is the elasticity of imports with respect to variable trade costs that would result in a model that has the same structure but no search frictions. Second, if we are willing to assume that $\frac{\partial \ln (N_o^x)}{\partial \ln (\tau_{do})} = \frac{\partial \ln \left(N_d^x\right)}{\partial \ln (\tau_{do})}$ and $\frac{\partial \ln (N_o^x)}{\partial \ln (\tau_{d'o})} = \frac{\partial \ln \left(N_d^x\right)}{\partial \ln (\tau_{d'o})}$, then equation (78) becomes

$$\frac{\partial \ln (C_{do}/C_{dd})}{\partial \ln (\tau_{d'o})} = \frac{\partial \ln (IM_{do}/IM_{dd})}{\partial \ln (\tau_{d'o})} = \varepsilon_o^{ACRdd'} = \begin{cases} (1-\sigma) + \psi_{do} + (\psi_{do} - \psi_{dd}) \dfrac{\partial \ln (\bar{\varphi}_{dd})}{\partial \ln (\tau_{do})} & if \; d' = d \\ \\ (\psi_{do} - \psi_{dd}) \dfrac{\partial \ln (\bar{\varphi}_{dd})}{\partial \ln (\tau_{d'o})} & if \; d' \neq d \end{cases}, \tag{79}$$

in which $\psi_{do} = \dfrac{\partial \ln (\Psi_{do})}{\partial \ln (\bar{\varphi}_{do})} \geq 0$ and $\Psi_{do} = \int_{\bar{\varphi}_{do}}^{\infty} \varphi^{\sigma-1} dG(\varphi)$. Equation (79) is exactly the trade elasticity in the Melitz (2003) model as derived in Arkolakis, Costinot, and Rodríguez-Clare (2012), equation (21) except that $\psi_{do} \leq 0$ while $\gamma_{ij} \geq 0$. This sign difference occurs because we define our model in terms of productivity, while they define theirs in terms of marginal cost.

Our baseline calibration assumes that productivity, φ, follows a Pareto distribution with CDF $G[\tilde{\varphi} < \varphi] = 1 - \varphi^{-\theta}$. This assumption simplifies terms in equation (79) that depend on moments of the productivity distribution as shown in B.2.17 and leads to

$$\frac{\partial \ln (C_{do}/C_{dd})}{\partial \ln (\tau_{d'o})} = \frac{\partial \ln (IM_{do}/IM_{dd})}{\partial \ln (\tau_{d'o})} = \varepsilon_o^{ACRdd'} = \begin{cases} -\theta & if \; d' = d \\ \\ 0 & if \; d' \neq d \end{cases}. \tag{80}$$

Consumption and trade elasticities are equivalent in these models because trade and consumption are both evaluated at final sales prices. Equation (80), equation (27) in the main text, is the consumption and trade elasticity if $c_{do} \to 0 \; \forall do$ and productivity is Pareto distributed. This elasticity is identical to the Melitz (2003) model with the same productivity distribution. We compare the effects of search frictions on the consumption and trade elasticities from propositions 6 and 7 to standard trade models without search frictions in section 5.2 using our baseline calibration and equation (80).

B.2.17 Consumption elasticity with Pareto distributed productivity

The elasticity of the moment of the productivity distribution defined by $\psi_{do} = \frac{d\ln(\Psi_{do})}{d\ln(\bar{\varphi}_{do})}$ takes a particularly simple form if productivity $\varphi \in [1, +\infty)$ is Pareto distributed with CDF $G[\tilde{\varphi} < \varphi] = 1 - \varphi^{-\theta}$ and PDF $g(\varphi) = \theta\varphi^{-\theta-1}$. As usual, assume that $\theta > \sigma - 1$ in order to close the model, which also ensures that $\psi_{do} < 0$. With this distribution, the moment $\Psi_{do} \equiv \int_{\bar{\varphi}_{do}}^{\infty} z^{\sigma-1} dG(z) = \frac{\theta\bar{\varphi}_{do}^{\sigma-\theta-1}}{\theta-\sigma+1}$ and the elasticity $\psi_{do} = -\frac{\bar{\varphi}_{do}^{\sigma}\theta\bar{\varphi}_{do}^{-\theta-1}}{\Psi_{do}} = \sigma - 1 - \theta$. Importantly, this implies that $\psi_{do} = \psi_{dd}$. The $d' = d$ case of the consumption elasticity therefore simplifies to

$$(1-\sigma) + \psi_{do} + (\psi_{do} - \psi_{dd})\frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{do})} = (1-\sigma) + (\sigma - 1 - \theta) = -\theta, \tag{81}$$

and the $d' \neq d$ case of the consumption elasticity simplifies to

$$(\psi_{do} - \psi_{dd})\frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})} = 0. \tag{82}$$

B.3 Proof of proposition 7: Trade elasticity

B.3.1 Relating the consumption and trade elasticities

We derive the trade elasticity by relating it to the consumption elasticity. Imports evaluated at negotiated prices and total sales evaluated at final prices are related through the gravity equation (31) and equation (32) as

$$IM_{do} = (1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}))\, C_{do},$$

in which

$$(1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do})) = \left(1 - \frac{\gamma_{do}}{\sigma\theta}\left(\theta - \frac{\delta_{do}}{F_{do}}(\theta - (\sigma - 1))\right)\right).$$

This equation is not a general relationship and depends on the assumptions we have made about preferences, bargaining, and the productivity distribution. Forming the import ratio in markets do and dd in our model therefore gives

$$\frac{IM_{do}}{IM_{dd}} = \frac{\left(1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)\right) C_{do}}{\left(1 - b\left(\sigma, \theta, \gamma_{dd}, \delta_{dd}, F_{dd}\right)\right) C_{dd}}.$$

It is straightforward to see that the trade elasticity is related to the consumption elasticity according to

$$\frac{\partial \ln\left(IM_{do}/IM_{dd}\right)}{\partial \ln\left(\tau_{d'o}\right)} = \frac{\partial \ln\left(C_{do}/C_{dd}\right)}{\partial \ln\left(\tau_{d'o}\right)} + \frac{\partial \ln\left(1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)\right)}{\partial \ln\left(\tau_{d'o}\right)} - \frac{\partial \ln\left(1 - b\left(\sigma, \theta, \gamma_{dd}, \delta_{dd}, F_{dd}\right)\right)}{\partial \ln\left(\tau_{d'o}\right)}.$$

In particular, the baseline trade elasticity presented in proposition 6 of the main text is simply

$$\frac{\partial \ln\left(IM_{do}/IM_{dd}\right)}{\partial \ln\left(\tau_{d'o}\right)} = \begin{cases} -\theta + \left(\frac{u_{do}}{1 - i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln\left(\tau_{do}\right)}\right) - \left(\frac{u_{dd}}{1 - i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln\left(\tau_{do}\right)}\right) & \\ + \frac{\partial \ln\left(1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)\right)}{\partial \ln\left(\tau_{d'o}\right)} - \frac{\partial \ln\left(1 - b\left(\sigma, \theta, \gamma_{dd}, \delta_{dd}, F_{dd}\right)\right)}{\partial \ln\left(\tau_{d'o}\right)} & if\ d' = d \\ \left(\frac{u_{do}}{1 - i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln\left(\tau_{d'o}\right)}\right) - \left(\frac{u_{dd}}{1 - i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln\left(\tau_{d'o}\right)}\right) & \\ + \frac{\partial \ln\left(1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)\right)}{\partial \ln\left(\tau_{d'o}\right)} - \frac{\partial \ln\left(1 - b\left(\sigma, \theta, \gamma_{dd}, \delta_{dd}, F_{dd}\right)\right)}{\partial \ln\left(\tau_{d'o}\right)} & if\ d' \neq d \end{cases}, \tag{83}$$

The trade elasticity differs from the standard trade elasticity because it is affected by the endogenous markup change between the negotiated and final sales prices in addition to the change in the mass of unmatched varieties that also affect the final consumption elasticity as discussed in proposition 6.

We can also show that

$$1 + b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right) \leq \mu = \frac{\sigma}{\sigma - 1}$$

Using proof by contradiction, begin by assuming that $1 + b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right) > \frac{\sigma}{\sigma - 1}$. Applying the definition of $b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)$, this assumption implies that

$$\gamma_{do}\left(\theta - \frac{\delta_{do}}{F_{do}}\left(\theta - (\sigma - 1)\right)\right) > \frac{\sigma\theta}{\sigma - 1}$$

To close the model, we assume that $\theta - (\sigma - 1) > 0$ so we also know that $\theta > \sigma - 1$ and that $\sigma > 1$. Together these imply that

$$\theta\left(\frac{\sigma}{\sigma - 1}\right) > (\sigma - 1)\left(\frac{\sigma}{\sigma - 1}\right) = \sigma > 1$$

Hence, our initial assumption implies that

$$\gamma_{do}\left(\theta-\frac{\delta_{do}}{F_{do}}\left(\theta-(\sigma-1)\right)\right)>1$$

but we show in section B.4.2 that $\left(\theta-\frac{\delta_{do}}{F_{do}}\left(\theta-(\sigma-1)\right)\right)\in[0,1]$ and in section A.4.3 that $\gamma_{do}\in[0,1]$ so we have derived a contradiction and proved our desired result. Note that these steps also imply that $b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\in[0,\mu-1]$ and $1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\in\left[(\sigma-2)/(\sigma-1),1\right]$ so that the difference between negotiated and final sales prices could reduce imports by, at most, a factor of $1-(\mu-1)=(\sigma-2)/(\sigma-1)$.

B.3.2 Markup response to tariff changes in our baseline

The sign of the elasticity of the markup between consumption and imports with respect to iceberg costs,

$$\frac{\partial\ln\left(1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\right)}{\partial\ln\left(\tau_{do}\right)}$$

and

$$\frac{\partial\ln\left(1-b\left(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd}\right)\right)}{\partial\ln\left(\tau_{do}\right)},$$

respectively, only depend on market tightness κ_{do} because tariffs do not directly affect the $b(\cdot)$ term. The relevant partial derivative in the first case is

$$\frac{\partial\ln\left(1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\right)}{\partial\ln\left(\tau_{do}\right)} = -\frac{\partial b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)}{\partial\kappa_{do}}\frac{\kappa_{do}}{\left(1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\right)}\frac{\partial\ln\left(\kappa_{do}\right)}{\partial\ln\left(\tau_{do}\right)}$$

We showed that $\frac{\partial\ln\kappa_{do}\chi\left(\kappa_{do}\right)}{\partial\ln\left(\tau_{do}\right)}\leq 0$, which implies that $\frac{\partial\ln\kappa_{do}}{\partial\ln\left(\tau_{do}\right)}\leq 0$ as well. The term $\frac{\kappa_{do}}{\left(1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\right)}\geq 0$ and so it remains to consider the sign of $\frac{\partial b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)}{\partial\kappa_{do}}$. Our baseline calibration has $l_{do}=-h_{do}$ so that $F_{do}\left(\kappa_{do}\right)$ is not a function of κ_{do},

$$F_{do} = f_{do}+h_{do}+\frac{(r+\lambda)}{\beta}s_{do}.$$

This assumption simplifies the desired derivative to

$$\frac{\partial b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)}{\partial\kappa_{do}} = \frac{\gamma_{do}}{\sigma\theta}\left(\frac{\theta-(\sigma-1)}{F_{do}}\right)\left[\frac{\partial}{\partial\kappa_{do}}\kappa_{do}\chi\left(\kappa_{do}\right)s_{do}\right]+\frac{b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)}{\gamma_{do}}\frac{\partial}{\partial\kappa_{do}}\gamma_{do}$$

in which $\frac{\partial}{\partial\kappa_{do}}\kappa_{do}\chi\left(\kappa_{do}\right)\geq 0$ as mentioned above and $\frac{\partial\gamma_{do}}{\partial\kappa_{do}}\leq 0$ because $\gamma_{do}=\frac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\kappa_{do}\chi\left(\kappa_{do}\right)}$ so that $\frac{\partial\gamma_{do}}{\partial\kappa_{do}}=-\frac{\gamma_{do}}{r+\lambda+\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\leq 0$ Even with our restriction $l_{do} = -h_{do}$, the sign is ambiguous. In particular, $\dfrac{\partial b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)}{\partial \kappa_{do}}$ will be negative if $s_{do} = 0$ or if the first term is smaller than the second. Our baseline parameterization has that $\dfrac{\partial b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)}{\partial \kappa_{do}} \leq 0$. This fact implies that

$$\frac{\partial \ln\left(1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\right)}{\partial \ln\left(\tau_{do}\right)} \quad \leq \quad 0$$

As tariffs increase, the aggregate markup term, $1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)$, the difference between final sales prices and negotiated prices, declines. Similar logic applies for $\dfrac{\partial \ln\left(1-b\left(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd}\right)\right)}{\partial \ln\left(\tau_{do}\right)}$ because each term will have the same sign as before except that $\dfrac{\partial \ln\left(\kappa_{dd}\right)}{\partial \ln\left(\tau_{do}\right)} \geq 0$ so that

$$\begin{aligned}\frac{\partial \ln\left(1-b\left(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd}\right)\right)}{\partial \ln\left(\tau_{do}\right)} &= -\frac{\partial b\left(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd}\right)}{\partial \kappa_{do}}\frac{\kappa_{dd}}{\left(1-b\left(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd}\right)\right)}\frac{\partial \ln\left(\kappa_{dd}\right)}{\partial \ln\left(\tau_{do}\right)} \\ &\geq 0\end{aligned}$$

B.4 The gravity equation with search frictions

B.4.1 Proof of proposition 8: Deriving the gravity equation

The value of total imports will be

$$IM_{do} = \left(1-\frac{u_{do}}{1-i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} n_{do}\left(\varphi\right) q_{do}\left(\varphi\right) dG\left(\varphi\right).$$

We need to integrate over the varieties to get the total value of imports going into the domestic market. Demand for a variety, φ, in the differentiated goods sector is given in equation (2): $q_{do}\left(\varphi\right) = p_{do}\left(\varphi\right)^{-\sigma}\dfrac{\alpha C_d}{P_d^{1-\sigma}}$. Given this demand, monopolistic competition, and constant returns-to-scale production imply that producers set optimal prices according to equation (16): $p_{do}\left(\varphi\right) = \mu w_o \tau_{do}\varphi^{-1}$. For notational simplicity, define $B_{do} \equiv \alpha\left(\mu w_o \tau_{do}\right)^{-\sigma} C_d P_d^{\sigma-1}$ and combine the optimal price with the demand curve to get $q_{do}\left(\varphi\right) = B_{do}\varphi^{\sigma}$. Evaluated at final prices, the value of sales of each variety is $p_{do}\left(\varphi\right) q_{do}\left(\varphi\right) = \mu w_o \tau_{do} B_{do}\varphi^{\sigma-1}$ and the cost to produce $q_{do}\left(\varphi\right)$ units of this variety is $t_{do}\left(\varphi\right) + f_{do} = w_o\tau_{do}B_{do}\varphi^{\sigma-1} + f_{do}$. These expressions imply that total profits generated by each variety are $p_{do}\left(\varphi\right) q_{do}\left(\varphi\right) - t_{do}\left(\varphi\right) = A_{do}\varphi^{\sigma-1}$, in which it is also useful to define $A_{do} = w_o\tau_{do}B_{do}\left[\mu-1\right]$. Using this profits expression, the productivity cutoff is $\bar{\varphi}_{do} = \left(\dfrac{F_{do}}{A_{do}}\right)^{\frac{1}{\sigma-1}}$, in which F_{do} is given in equation (18). The value of total imports from the negotiated price curve in equation (14) is

$$n(\varphi)\, q_{do}(\varphi) = [1-\gamma_{do}]\, p_{do}(\varphi)\, q_{do}(\varphi) + \gamma_{do}\left[t_{do}(\varphi) + f_{do} - l_{do} - \kappa_{do}\chi(\kappa_{do})\, s_{do}\right].$$

Using the functional forms assumptions from above this becomes

$$n(\varphi)\, q_{do}(\varphi) = (\sigma-\gamma_{do})\, A_{do}\varphi^{\sigma-1} - \gamma_{do}\left[-f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})\, s_{do}\right].$$

Substituting the value of imports for a particular variety into the integral defining the value of total imports gives

$$IM_{do} = \left(1-\frac{u_{do}}{1-i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} (\sigma-\gamma_{do})\, A_{do}\varphi^{\sigma-1} - \gamma_{do}\left[-f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})\, s_{do}\right] dG(\varphi).$$

We assume productivity, φ, has a Pareto distribution over $[1,+\infty)$ with cumulative density function $G[\tilde{\varphi}<\varphi] = 1-\varphi^{-\theta}$ and probability density function $g(\varphi) = \theta\varphi^{-\theta-1}$. The Pareto parameter and the elasticity of substitution are such that $\theta > \sigma - 1$, which ensures that the integral $\int_{\bar{\varphi}}^{\infty} z^{\sigma-1} dG(z)$ is bounded. Using these assumptions we can compute the integral to get

$$IM_{do} = \left(1-\frac{u_{do}}{1-i_{do}}\right) N_o^x \left[(\sigma-\gamma_{do})\, A_{do}\frac{\theta\bar{\varphi}_{do}^{\sigma-\theta-1}}{\theta-\sigma+1} - \gamma_{do}\left[-f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})\, s_{do}\right]\bar{\varphi}_{do}^{-\theta}\right],$$

in which we use the relevant moment of the productivity distribution $\int_{\bar{\varphi}_{do}}^{\infty} z^{\sigma-1} dG(z) = \dfrac{\theta\bar{\varphi}_{do}^{\sigma-\theta-1}}{\theta-\sigma+1}$. Define $\delta_{do} \equiv f_{do} - l_{do} - \kappa_{do}\chi(\kappa_{do})\, s_{do}$ to conserve on notation, substitute the export productivity threshold into this expression, and simplify to get

$$IM_{do} = \left(1-\frac{u_{do}}{1-i_{do}}\right) N_o^x \left[(\sigma-\gamma_{do})\frac{\theta}{\theta-\sigma+1} + \gamma_{do}\frac{\delta_{do}}{F_{do}}\right] F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)} A_{do}^{\frac{\theta}{\sigma-1}}. \tag{84}$$

Next, utilize the assumption that the number of producers in the origin market is proportional to output in that market $N_o^x = \left(\dfrac{C}{1+\pi}\right)\dfrac{C_o}{C}$ and the definition for $A_{do} = \mu^{-\sigma}\alpha(w_o\tau_{do})^{1-\sigma} C_d P_d^{\sigma-1}[\mu-1]$ to write

$$\begin{aligned} IM_{do} &= \left(1-\frac{u_{do}}{1-i_{do}}\right)\left(\frac{C}{1+\pi}\right)\frac{C_o}{C}\left[(\sigma-\gamma_{do})\frac{\theta}{\theta-\sigma+1} + \gamma_{do}\frac{\delta_{do}}{F_{do}}\right] \\ &\times\ F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}\left(\mu^{-\sigma}\alpha(w_o\tau_{do})^{1-\sigma} C_d P_d^{\sigma-1}[\mu-1]\right)^{\frac{\theta}{\sigma-1}}. \end{aligned}$$

We presented the price index earlier as

$$P_d = \lambda_2 \times C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \times \rho_d.$$

Substituting that into our value of imports and simplifying gives

$$\begin{aligned} IM_{do} &= \left(1-\frac{u_{do}}{1-i_{do}}\right)\left[(\sigma-\gamma_{do})\frac{\theta}{\theta-\sigma+1}+\gamma_{do}\frac{\delta_{do}}{F_{do}}\right] \\ &\times \left(\mu^{-\sigma}\alpha\left[\mu-1\right]\right)^{\frac{\theta}{\sigma-1}}\left(\frac{C}{1+\pi}\right)\lambda_2^{\theta}\left(\frac{C_oC_d}{C}\right)\left(\frac{w_o\tau_{do}}{\rho_d}\right)^{-\theta}F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}. \end{aligned}$$

In the price index section earlier we also define λ_2, which we can now substitute in here and then simplify to get

$$IM_{do}=\left(1-\frac{u_{do}}{1-i_{do}}\right)\left(1-\frac{\gamma_{do}}{\sigma\theta}\left(\theta-\frac{\delta_{do}}{F_{do}}\left(\theta-(\sigma-1)\right)\right)\right)\alpha\left(\frac{C_oC_d}{C}\right)\left(\frac{w_o\tau_{do}}{\rho_d}\right)^{-\theta}F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}. \quad (85)$$

Define the bundle of search parameters

$$b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)=\frac{\gamma_{do}}{\sigma\theta}\left(\theta-\frac{\delta_{do}}{F_{do}}\left(\theta-(\sigma-1)\right)\right)$$

and substitute it into (85) in order to write the gravity equation more compactly as

$$IM_{do}=\left(1-\frac{u_{do}}{1-i_{do}}\right)\left(1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\right)\alpha\left(\frac{C_oC_d}{C}\right)\left(\frac{w_o\tau_{do}}{\rho_d}\right)^{-\theta}F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}.$$

which is equation (31) in proposition 8.

B.4.2 Search frictions reduce imports

Search costs reduce imports through the unmatched rate and difference between final and negotiated prices. In order to show this result, we show that the matched rate and the aggregate markup terms are weakly in the unit interval.

First, it is easy to see that

$$\left(1-\frac{u_{do}}{1-i_{do}}\right)=\left(\frac{\kappa_{do}\chi\left(\kappa_{do}\right)}{\lambda+\kappa_{do}\chi\left(\kappa_{do}\right)}\right)\in[0,1],$$

because the finding and destruction rates must be finite in any model with positive search costs, $c_{do}>0\forall do$.

Second, proving the bundle of search parameters

$$1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\in[0,1].$$

takes a few steps. Begin by proving that $\left(1-\frac{\gamma_{do}}{\sigma\theta}\left(\theta-\frac{\delta_{do}}{F_{do}}\left(\theta-(\sigma-1)\right)\right)\right)\leq 1$. We can prove this by noting that $\delta_{do}\equiv f_{do}-l_{do}-\kappa_{do}\chi\left(\kappa_{do}\right)s_{do}$ so it must be that $\delta_{do}\leq F_{do}$ and therefore $\delta_{do}F_{do}^{-1} \leq 1$. Also, the restriction that $\sigma > 1$ ensures that $\theta - (\sigma - 1) < \theta$. Together, these ensure $\theta - \frac{\delta_{do}}{F_{do}}(\theta - (\sigma - 1)) \geq 0$. Combining this with the fact that $\gamma_{do} \in [0,1]$ ensures that $1 - \frac{\gamma_{do}}{\sigma\theta}\left(\theta - \frac{\delta_{do}}{F_{do}}(\theta - (\sigma - 1))\right) \leq 1$.

Next, show that $\left(1 - \frac{\gamma_{do}}{\sigma\theta}\left(\theta - \frac{\delta_{do}}{F_{do}}(\theta - (\sigma - 1))\right)\right) \geq 0$ by showing that $1 \geq \frac{\gamma_{do}}{\sigma\theta}\left(\theta - \frac{\delta_{do}}{F_{do}}(\theta - (\sigma - 1))\right)$. Because $\gamma_{do} \in [0,1]$ and $\sigma > 1$ we know that $\frac{\gamma_{do}}{\sigma} < 1$. Likewise, $\sigma > 1$ ensures $\theta - (\sigma - 1) < \theta$ so that $\frac{(\theta - (\sigma - 1))}{\theta} < 1$. We assume above that $\theta - (\sigma - 1) > 0$ in order to close the model. Together these imply that $\frac{(\theta - (\sigma - 1))}{\theta} \in [0,1]$. Finally, because $\delta_{do}F_{do}^{-1} \leq 1$ we have that $\frac{\delta_{do}}{F_{do}}\frac{(\theta - (\sigma - 1))}{\theta} \leq 1$ and we have proved the result.

B.4.3 Consumption is imports evaluated at final sales prices

We could have also evaluated the quantity of goods imported at final sales prices $p_{do}(\varphi)$ instead of negotiated prices $n_{do}(\varphi)$. From equation (14), we can see $p_{do}(\varphi) = n_{do}(\varphi)$ if $\gamma_{do} = 0$. Setting $\gamma_{do} = 0$ in equation (85) then gives

$$C_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right)\alpha\left(\frac{C_o C_d}{C}\right)\left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}.$$

We can obtain the same result by integrating

$$C_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} p_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi).$$

B.5 Deriving aggregate welfare

Here we outline the steps to show that the indirect utility function (welfare) is C_d/P_d, in which C_d is total consumption expenditure, n is the vector of prices for each good, and P_d is the ideal price index. Assume that preferences are homothetic, which is defined in Mas-Colell, Whinston, and Green (1995), section 3.B.6, page 45. This means that they can be represented by a utility function that is homogeneous of degree one in quantities and that the corresponding indirect utility function is linear in total consumption expenditure. We can begin with the indirect utility function and then manipulate it as follows

$$\begin{aligned} W_d(p, C_d) &= W_d(p, 1)\, C_d \\ W_d(p, e(p, u)) &= W_d(p, 1)\, e(p, u) \\ u &= W_d(p, 1)\, e(p, u) \\ 1 &= W_d(p, 1)\, e(p, 1) \\ \frac{1}{e(p, 1)} &= W_d(p, 1), \end{aligned}$$

in which the first line comes from homothetic preferences; the second line follows by plugging in for consumption expenditure $C_d = e(p, u)$; the third line comes from equation (3.E.1) in MWG that says $W_d(p, e(p, u)) = u$ (also known as duality); and in the fourth line we plug in for utility level $u = 1$. The function $e(p, u)$ is the consumption expenditure function that solves the expenditure minimization problem. Using this result and the fact that the price index is defined as $e(p, 1) \equiv P_d$ we can show that

$$W_d(p, C_d) = W_d(p, 1)\, C_d = \frac{1}{e(p, 1)} C_d = \frac{C_d}{P_d}.$$

Hence, as long as preferences are homothetic, we will always get welfare equal to consumption expenditure divided by the price index, $W_d(p, Y) = C_d/P_d$. The expenditure approach to accounting can be particularly useful for computing aggregate welfare in this setting because $W_d(p, C_d) = \dfrac{C_d}{P_d} = \dfrac{Y - I_d}{P_d}$.

C Calibration appendix

C.1 Numerical details

C.1.1 Solution algorithm and weighting matrix

We use MATLAB's *fmincon* function to solve the constrained optimization problem in equation (35) starting at 1,500 randomly selected parameter vectors. The 1,500 solutions associated with these starting points are local minima. Our baseline internally calibrated parameter values reported in Table 2 are the parameters associated with the minimum objective function value among all the local 1,500 local minima solutions.

In our weighting matrix in equation (35), W, we choose relatively high weights for the manufacturing capacity utilization and fraction of exporting firms moments because they are influential in determining the retailers' search cost, c_{do}. In particular, we choose a weight of five for these four moments and a weight of one for all other moments.

C.1.2 Nonlinear constraints

Equation (35) includes additional linear and nonlinear equilibrium and parameter inequality constraints, $\Psi(\Phi, \Omega)$, which we list here:

1. The fraction of matched producers cannot be negative and cannot exceed one:

$$0 \leq 1 - u_{do} - i_{do} \leq 1 \; \forall do$$

2. The effective entry cost must be nonnegative:

$$F_{do} \geq 0 \; \forall do$$

3. The effective entry cost must be weakly less than total imports:

$$F_{do} \leq IM_{do} \; \forall do$$

4. Retailers' profit margin must be weakly smaller than the overall profit margin:

$$1 + b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}) \leq \mu \; \forall do$$

5. Retailers' profit margin must be weakly larger than one:

$$1 + b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}) \geq 1 \; \forall do$$

6. Iceberg costs must be weakly greater than one:

$$\tau_{do} \geq 1 \; \forall do$$

7. Investment cannot exceed output:

$$I_d \leq Y_d \; \forall d$$

8. Output must be nonnegative:

$$Y_d \geq 0 \; \forall d$$

9. The labor endowment must be weakly smaller than output:

$$L_d \leq Y_d \; \forall d$$

10. The idle rate must be nonnegative:

$$i_{do} \geq 0 \; \forall do$$

11. The threshold productivity in the domestic market must be weakly less than the threshold productivity in the foreign market:

$$\bar{\varphi}_{oo} \leq \bar{\varphi}_{do} \forall o$$

12. Persistence in export status cannot exceed one and must be weakly greater than zero:

$$0 \leq \beta_{du} \leq 1 \; \forall d \in \{u, c\}$$

See appendix C.1.3 for a definition of export persistence, β_{do}, in the context of the model.

C.1.3 Export persistence

Suppose we have a linear regression that relates export status of a firm this period, y_{it}, with export status last period, $y_{i,t-1}$:

$$y_{it} = \alpha + \beta y_{i,t-1} + \epsilon_{it}.$$

in which we drop *do* notation. Notice that

$$\mathbb{E}\left[y_{it}|y_{i,t-1}\right] = \alpha + \beta y_{i,t-1}$$

and recall that

$$\begin{aligned}\mathbb{E}\left[y_{it}|y_{i,t-1}\right] &= \mathbb{E}\left[y_{it}|y_{it}=1, y_{i,t-1}\right]\mathbb{P}\left[y_{it}=1|y_{i,t-1}\right] + \mathbb{E}\left[y_{it}|y_{it}=0, y_{i,t-1}\right]\mathbb{P}\left[y_{it}=0|y_{i,t-1}\right] \\ &= 1\times\mathbb{P}\left[y_{it}=1|y_{i,t-1}\right] + 0\times\mathbb{P}\left[y_{it}=0|y_{i,t-1}\right] \\ &= \mathbb{P}\left[y_{it}=1|y_{i,t-1}\right].\end{aligned}$$

This implies that

$$\mathbb{P}\left[y_{it}=1|y_{i,t-1}\right] = \alpha + \beta y_{i,t-1}.$$

For reference, note that

$$\mathbb{P}\left[y_{it}=1|y_{i,t-1}=1\right] = \alpha+\beta$$

$$\mathbb{P}\left[y_{it}=0|y_{i,t-1}=1\right] = 1-\mathbb{P}\left[y_{it}=1|y_{i,t-1}=1\right] = 1-(\alpha+\beta)$$

$$\mathbb{P}\left[y_{it}=1|y_{i,t-1}=0\right] = \alpha$$

$$\mathbb{P}\left[y_{it}=0|y_{i,t-1}=0\right] = 1-\mathbb{P}\left[y_{it}=1|y_{i,t-1}=0\right] = 1-\alpha.$$

In our model, we know that separation shocks occur at Poisson rate λ, which means that the probability that separation occurs during one unit of time (one unit is one year in our calibration) is $1-e^{-\lambda}$:

$$\mathbb{P}\left[y_{it}=0|y_{i,t-1}=1\right] = \left(1-e^{-\lambda}\right).$$

Therefore

$$\begin{aligned}\mathbb{P}\left[y_{it}=1|y_{i,t-1}=1\right] &= 1-\mathbb{P}\left[y_{it}=0|y_{i,t-1}=1\right] \\ &= 1-\left(1-e^{-\lambda}\right) \\ &= e^{-\lambda}.\end{aligned}$$

The probability of becoming an exporter means that you have to make contact with a retailer, which occurs at rate $\kappa\chi(\kappa)$, that the producer has productivity above the threshold, and the producer was searching:

$$\begin{aligned}\mathbb{P}\left[y_{it}=1|y_{i,t-1}=0\right] &= \left(1-e^{-\kappa\chi(\kappa)}\right)\mathbb{P}\left[\varphi>\bar{\varphi}\right]\frac{u}{1-i} \\ &= \left(1-e^{-\kappa\chi(\kappa)}\right)(1-i)\frac{u}{1-i} \\ &= \left(1-e^{-\kappa\chi(\kappa)}\right)u.\end{aligned}$$

Finally, notice that

$$\begin{aligned}\beta &= (\alpha+\beta)-\alpha \\ &= \mathbb{P}\left[y_{it}=1|y_{i,t-1}=1\right]-\mathbb{P}\left[y_{it}=1|y_{i,t-1}=0\right] \\ &= e^{-\lambda}-\left(1-e^{-\kappa\chi(\kappa)}\right)u \\ &= e^{-\lambda}-\left(1-e^{-\kappa\chi(\kappa)}\right)u.\end{aligned}$$

C.2 Identifying retailers' search costs

C.2.1 Identifying importing retailers' search costs

In this section we describe how data on the fraction of exporters in uc and cu markets can be used to help inform the importing retailers' search cost parameters, c_{uc} and c_{cu}. As an example, consider the probability that a U.S. firm exports to China:

$$\mathbb{P}\left[export_{cu}\right]=\mathbb{P}\left[export_{cu}|\varphi>\bar{\varphi}_{cu}\right]\mathbb{P}\left[\varphi>\bar{\varphi}_{cu}\right]+\mathbb{P}\left[export_{cu}|\varphi\leq\bar{\varphi}_{cu}\right]\mathbb{P}\left[\varphi\leq\bar{\varphi}_{cu}\right]$$
$$\mathbb{P}\left[export_{cu}\right]=1-u_{cu}-i_{cu}$$

We divide this term by $1-i_{cu}$ to obtain the fraction of producers that are matched in the cu market. We know that this matched rate is monotonically increasing in κ_{cu} because the producers' finding rate, $\kappa_{cu}\chi\left(\kappa_{cu}\right)$, is increasing in market tightness. From the free-entry condition (equation 19) we know that c_{cu} is important in determining equilibrium κ_{cu}. In particular, as the retailers' search cost rises, there is less entry into retailing and κ_{cu} falls. Therefore, we can use observed data on the fraction of U.S. firms that export to China to pin down the c_{cu} parameter. Similarly, we can use the fraction of Chinese firms that export to the United States to pin down the c_{uc} parameter.

C.2.2 Identifying domestic retailers' search costs

In this section, we describe how data on manufacturing capacity utilization can help identify domestic retailers' search cost parameters, c_{uu} and c_{cc}. The capacity utilization rate is mainly determined from two measures collected from manufacturing plants. The first measure is the market value of actual production during a time period. The second measure is the full production capability for that time period assuming normal downtime, fully available inputs, and with currently available machinery and equipment. The manufacturing capacity utilization rate is the sum of all plants' market value of actual production divided by the sum of all plants' full production capability.

The quantity in our model that is analogous to the capacity utilization rate in the data for producers in country o is the value of all sales divided by the value of sales if there were no search frictions:

$$\frac{\sum_k IM_{ko}}{\sum_k IM_{ko}/\left(1-\frac{u_{ko}}{1-i_{ko}}\right)}.$$

In the main text, we restrict our attention to capacity utilization in the domestic market only to make the exercise transparent. For the United States, domestic capacity utilization is defined as:

$$\frac{IM_{uu}}{IM_{uu}/\left(1-\frac{u_{uu}}{1-i_{uu}}\right)}=1-\frac{u_{uu}}{1-i_{uu}}=\frac{\kappa_{uu}\chi\left(\kappa_{uu}\right)}{\lambda+\kappa_{uu}\chi\left(\kappa_{uu}\right)}.$$

As mentioned before, this quantity is monotonically increasing in κ_{uu}, which is negatively related to c_{uu}, and so monotonically decreasing in c_{uu}. We use observed data on U.S. and Chinese manufacturing capacity utilization to identify the domestic retailers' search costs in each country.

C.2.3 Search cost symmetry

Intuitively, the cost retailers pay to search for producers should be similar whether they are searching for a domestic or foreign producer. As such, we assume that international search costs are simply the domestic search cost plus a symmetric international premium so that $c_{uc}=c'+c_{uu}$, $c_{cu}=c'+c_{cc}$, and $c'\geq 0$. This symmetry assumption implies, for example, that the cost a Chinese retailer pays to search for a U.S. producer is the same as the cost that Chinese retailer would pay to search for a Chinese producer plus c'. We find this structure for these costs intuitively appealing because, as Kneller and Pisu (2011) report, ''identifying the first contact'' and ''establishing initial dialogue'' are examples of search costs and these are likely to be mainly symmetric. We are comfortable imposing that international retailers' search costs at least exceed domestic retailers' search costs. Finally, we note that this restriction provides additional identification for the elasticity of the matching function with respect to the number of searching producers, η.

C.3 Calibrating producers' fixed, sunk, and flow search costs

We follow di Giovanni and Levchenko (2012) and di Giovanni and Levchenko (2013) and calibrate the fixed costs of production, f_{do}, by using data from the Doing Business Indicators (DBI) database (WB, 2019a). For each country in the database, these measures document the time and costs associated with starting a new business and with exporting and importing a 20-foot dry-cargo container. We use the cost to start a business in the United States and China to discipline f_{uu} and f_{cc}, respectively. This cost is about \$600 in the United States and about \$30 in China. To identify the fixed costs associated with international production, f_{uc} and f_{cu}, we use the sum of the cost of exporting and importing from the Trading Across

Borders module of the DBI. For example, to discipline f_{uc}, we use the cost of exporting from China plus the cost of importing into the United States. These trading costs are about $675 in both the United States and China.

The threshold productivity in proposition 3 is defined by the effective entry cost, F_{do}, which is a linear function of the producers' fixed cost, f_{do}, search cost, l_{do}, their idle flow payoff, h_{do}, and their sunk cost, s_{do}. The fact that these costs enter linearly makes it difficult to separately identify them. Because we are ultimately interested in the effective entry cost, F_{do}, as a whole and are less concerned about its individual components, we set l_{do} and s_{do} to zero. Setting $l_{uu} = l_{cc} = 0$ is also consistent with small domestic search costs found in Eaton et al. (2014) and $s_{do} = 0\ \forall do$ also matches the treatment of sunk costs in most steady-state trade models such as Melitz (2003), Chaney (2008), and Allen (2014).

C.4 Identifying producers' flow idle benefit

The minimum productivity draw identifies the flow payoff from being idle, h_{do}. In particular, we assume that when search costs, tariffs, and the U.S. input cost premium are all zero and each country has a price index equal to the autarky price index, the threshold productivity in each country is equal to one. These assumptions are contradictory — no tariffs but autarky price indexes — but are exactly the restrictions we want to impose when solving for h_{do} because they ensure we choose an h_{do} such that all equilibria in our counterfactual exercises will have $\bar{\varphi}_{do} \geq 1$.

We implement this procedure by computing another set of equilibrium variables ($\bar{\varphi}_{do}$ $\forall do$, $C_d\ \forall d$, and π) under the additional restrictions, using equations (18), (23), and (24). Since search costs are zero, we know that market tightness and producers' findings rates in this equilibrium will be infinite using equation (19).

With these restrictions, the cutoff in the do market is determined only by global variables, including consumption, and the ratio of h_{dd} and h_{do}. Further imposing the constraint that the cutoff is equal to one implies that the domestic producers' flow idle benefit, h_{dd}, does not vary by country, and that the international producers' flow idle benefit is symmetric, $h_{do} = h_{od}$.

C.5 Implications for gravity equation estimation

The fact that introducing search frictions into a model of trade results in a scalar times the typical gravity equation has a few interesting implications for estimation.

First, if the fraction of matched exporters and the bundle of search friction parameters do not vary by destination–origin pairs, then their effect on trade would be lost in the constant term of a gravity regression. In this case, while estimates of the other coefficients in the model would be unbiased, search frictions could be a pervasive feature of international trade but would not be identifiable using the gravity equation.

Second, if the fraction of unmatched producers and the bundle of search frictions vary by importer–exporter pair, they may provide an additional rationale for why language, currency, common legal origin, historical colonial ties or other variables often included in gravity equations have an effect on aggregate trade flows. In particular, Rauch and Trindade (2002) argue that populations of ethnic Chinese within a country facilitate the flow of information, provide matching and referral services, and otherwise reduce informal barriers to trade. Their empirical specification matches the gravity equation with search that we have derived here if the destination–origin search frictions are a function of the ethnically Chinese population.

Third, any gravity regression that does not include adequate proxies for search frictions would suffer from omitted variable bias. In particular, suppose that a researcher omits search frictions, as measured by the matched rate, and estimates the following equation:

$$\ln IM_{do} = A_d + B_o + \beta_0 \ln \tau_{do} + \nu_{do}, \tag{86}$$

in which I_{do} are the imports from origin o to destination d, A_d is an importer-specific term, B_o is an exporter-specific term, β_0 is the partial elasticity of bilateral imports with respect to variable trade costs, and ν_{do} is an error term. Econometric theory suggests that the omitted variable, $Z_{do} = 1 - \dfrac{u_{do}}{1 - i_{do}}$, will introduce bias into the ordinary least squares estimate of $\hat{\beta}_0$, according to the well-known formula:

$$\mathbb{E}[\hat{\beta}_0 | \mathbf{X}] = \beta_0 + \rho(\tau_{do}, Z_{do})\rho(I_{do}, Z_{do}), \tag{87}$$

in which $\mathbf{X}$ is a vector of all right-hand-side variables and $\rho\,(X, Y)$ is the correlation between X and Y. We know that $\rho(\tau_{do}, Z_{do}) < 0$ (higher variable trade costs, τ_{do}, raise the threshold productivity, $\bar{\varphi}_{do}$, increasing the fraction of idle firms, i_{do}, and lowering the matched rate) and $\rho(I_{do}, Z_{do}) > 0$ (increasing the matched rate increases trade flows) so the sign of the bias is negative.

Proposition 9. *Omitting the matched rate from a standard gravity equation implies that the estimate of trade elasticity with respect to variable trade costs is more negative than if one included the matched rate in the estimating equation.*

Proof. This result follows from the discussion in the text. □