Online Appendix

Combining Survey Long-Run Forecasts and Nowcasts with BVAR Forecasts Using Relative Entropy*

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1 A.1. Results Pre-Crisis (1994Q1-2006Q4) Sample

To gauge the sensitivity of our results to the financial crisis/Great Recession, we report results corresponding to the real-time out-of-sample forecast evaluation sample spanning 1994Q1 to 2006Q4. Tables A1 and A2 confirm that our results are robust to a sample that excludes the Great Recession and the subsequent recovery.

Table A1: Real-Time Out-of-Sample **Point** Forecasting Performance: **Small BVAR**

Pre-Crisis Sample (Recursive evaluation: 1994.Q1-2006.Q4)							
Panel A: BVAR (Now Only) vs. Raw BVAR							
Series	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q	h=20Q
Relative MSE: **BVAR (Now Only)** Forecast / BVAR Raw Forecast							
Real GDP	0.74*	1.02	1.10**	1.01	0.97***	0.98**	0.99
CPI Inflation	0.44***	0.76***	0.92	0.85**	0.93	0.96	1.04
Unemployment rate	0.25***	0.63	0.73	0.84	0.91	0.92***	0.98
Federal Funds rate	0.01***	0.81	0.90	0.93	0.96	0.96	1.01
Credit Spread	0.91	0.87	0.93**	0.97***	1.00	1.00	1.01
Panel B: BVAR (Now and LR) vs. Raw BVAR							
Series	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q	h=20Q
Relative MSE: **BVAR (Now and LR)** Forecast / BVAR Raw Forecast							
Real GDP	0.74*	0.98	1.01	1.03	1.02	1.05	1.01
CPI Inflation	0.44***	0.65***	0.63***	0.53***	0.43***	0.47***	0.47***
Unemployment rate	0.25***	0.64*	0.69	0.73	0.82	0.87	0.91
Federal Funds rate	0.01***	0.91	0.96	0.85	0.76	0.70	0.63**
Credit Spread	0.92	0.96	0.86**	0.80**	0.78*	0.79*	0.91
Panel C: BVAR (Now and LR) vs. BVAR (Now Only)							
Series	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q	h=20Q
Relative MSE: **BVAR (Now and LR)** Forecast / **BVAR (Now Only)** Forecast							
Real GDP	1.00	0.96	0.91	1.02	1.05	1.07	1.02
CPI Inflation	1.00	0.86**	0.68***	0.63***	0.46***	0.49***	0.46***
Unemployment rate	1.00	1.01	0.94	0.87	0.90	0.95	0.93
Federal Funds rate	1.00	1.11*	1.07	0.92	0.79	0.73	0.62***
Credit Spread	1.02	1.10	0.93*	0.82*	0.78*	0.79*	0.90

Notes for Table: Panel A reports the relative mean squared error defined as mean squared error (MSE) of BVAR forecast conditional on survey nowcasts only divided by MSE of unconditional BVAR forecast; a ratio of less than 1 suggests that tilting the BVAR forecasts to survey nowcasts only is on average more accurate compared to unconditional BVAR forecasts. Panel B reports the relative MSE defined as MSE of BVAR forecast conditional on both survey nowcasts and long-horizon forecasts divided by MSE of unconditional BVAR forecast; a ratio of less than 1 suggests that tilting the BVAR forecasts to survey nowcasts and long-horizon forecasts is on average more accurate compared to unconditional BVAR forecasts. Panel C reports the relative MSE defined as MSE of BVAR forecast conditional on survey nowcasts and long-horizon forecasts divided by MSE of BVAR forecast conditional on survey nowcasts only; a ratio of less than 1 suggests tilting the BVAR forecasts to survey nowcasts and long-horizon forecasts is on average more accurate compared to tilting on just the survey nowcasts. The table reports statistical significance based on the Diebold-Mariano and West test with the lag $h-1$ truncation parameter of the HAC variance estimator and adjusts the test statistic for the finite sample correction proposed by Harvey, Leybourne, and Newbold (1997); *10 percent, **5 percent, and ***1 percent significance levels, respectively. The test statistics use two-sided standard normal critical values.

Table A2: Real-Time Out-of-Sample **Density** Forecasting Performance: **Small BVAR**

Pre-Crisis Sample (Recursive evaluation: 1994.Q1-2006.Q4)							
Panel A: BVAR (Now Only) vs. Raw BVAR							
Series	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q	h=20Q
Mean (Relative CRPS Score: **BVAR Now Only** Forecast - BVAR Raw Forecast)							
Real GDP	-0.13*	0.00	0.04**	0.00	0.00	0.00	0.00
CPI Inflation	-0.23***	-0.08**	-0.03	-0.06**	-0.02	-0.02	0.02
Unemployment rate	-0.05***	-0.05	-0.05	-0.04	-0.03	-0.03**	0.00
Federal Funds rate	-0.07***	-0.05	-0.03*	-0.03	-0.02	-0.01	0.00
Credit Spread	-0.01	-0.02	-0.01**	-0.01***	0.00	0.00	0.01

Panel B: BVAR (Now and LR) vs. Raw BVAR							
Series	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q	h=20Q
Mean (Relative CRPS Score: **BVAR Now and LR** Forecast - BVAR Raw Forecast)							
Real GDP	-0.12*	0.01	0.00	0.01	0.01	0.01	0.00
CPI Inflation	-0.22***	-0.12***	-0.11***	-0.15***	-0.25***	-0.23***	-0.18***
Unemployment rate	-0.05***	-0.05	-0.06	-0.06	-0.05	-0.05	-0.03
Federal Funds rate	-0.07***	-0.01	0.00	-0.04	-0.09	-0.16	-0.28*
Credit Spread	-0.01	-0.01	-0.03**	-0.06**	-0.07*	-0.08*	-0.04

Panel C: BVAR (Now and LR) vs. BVAR (Now Only)							
Series	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q	h=20Q
Mean (Relative CRPS Score: **BVAR Now and LR** Forecast - **BVAR Now Only** Forecast)							
Real GDP	0.00	0.01	-0.04	0.00	0.01	0.01	0.00
CPI Inflation	0.00	-0.04*	-0.08**	-0.09***	-0.23***	-0.21***	0.20***
Unemployment rate	0.00	0.01	0.00	-0.02	-0.02	-0.01	-0.03
Federal Funds rate	0.00	0.03*	0.03**	-0.01	-0.07	-0.14	-0.27***
Credit Spread	0.00	0.01	-0.02*	-0.05*	-0.07*	-0.09*	-0.05

Notes for Table: Panel A reports the mean relative CRPS between CRPS of BVAR forecast conditional on survey nowcasts and CRPS of unconditional BVAR forecast; a negative value suggests that tilting the BVAR forecasts to survey nowcasts only is on average more accurate compared to unconditional BVAR forecasts. Panel B reports the mean relative CRPS between CRPS of BVAR forecast conditional on both survey nowcasts and long-horizon forecasts and CRPS of unconditional BVAR forecast; a negative value suggests that tilting the BVAR forecasts to survey nowcasts and long-horizon forecasts is on average more accurate compared to unconditional BVAR forecasts. Panel C reports the mean relative CRPS between CRPS of BVAR forecast conditional on survey nowcasts and long-horizon forecasts and CRPS of BVAR forecast conditional on survey nowcasts only; a negative value suggests tilting the BVAR forecasts to survey nowcasts and long-horizon forecasts is on average more accurate compared to tilting on just the survey nowcasts. The table reports statistical significance based on the Diebold-Mariano and West test with the lag $h-1$ truncation parameter of the HAC variance estimator and adjusts the test statistic for the finite sample correction proposed by Harvey, Leybourne, and Newbold (1997); *10 percent, **5 percent, and ***1 percent significance levels, respectively. The test statistics use two-sided standard normal critical values.

2 A.2. Results Based on BVAR Model Estimation with Post-1985 Sample

In this section we assess the forecast accuracy of our modeling approach using the estimation sample beginning in 1985. The use of a 1985 estimation start date is motivated by the fact that many empirical studies have documented a structural break in the relationships among various macroeconomic variables. Therefore, a priori we would expect our hybrid approach to be relatively less effective in improving forecast accuracy compared to those reported in the main section of the paper. Indeed, results below confirm our prior expectations in that the forecast gains from the hybrid approach relative to the baseline (i.e., tilting on the nowcasts only) are smaller in magnitude compared to those reported in the main part of the paper. That said, the hybrid forecast continues to produce the most accurate forecasts with the exception of the unemployment rate for which the hybrid forecasts are inferior to baseline forecasts in absolute sense but the gains are not statistically significant.

Tables A3 and A4 report the accuracy results from the model estimated from 1985 onward for the full-sample evaluation (i.e., 1994-2016) for point forecast and density forecast accuracy, respectively. The hybrid approach on average generates more accurate forecasts of CPI inflation compared to the baseline (Panel C Tables A3 and A4) and the accuracy gains are statistically significant. The hybrid approach also generates more accurate forecasts for the federal funds rate, but the accuracy gains are statistically significant through the four quarters ahead only. To provide some explanation of why the hybrid approach generates more accurate forecasts, also plotted in Figures 1 and 2 are the evolution of the implied long-run forecasts (i.e. sample mean) of real GDP, the unemployment rate, CPI inflation, and the federal funds rate from the BVAR estimated with post-1985 data. A few things to note: (1) The implied long-run forecasts from the BVAR with post-1985 sample are generally closer to the survey expectations compared to the implied long-run forecasts from the BVAR with post-1959 data. (2) The movements in the implied long-run forecasts from the BVAR with post-1985 data track quite closely the long-run estimates from the TVP-VAR SV. (3) Even though the implied long-run forecasts for CPI inflation from the BVAR post-1985 is notably lower compared to the BVAR post-1959 they are still materially higher than the survey expectations. The latter fact (slower adjustment in the trend inflation from the BVAR post-1985 compared to survey expectations) partly explains the more accurate forecasts of CPI inflation from the hybrid approach.

Table A3: Real-Time Out-of-Sample **Point** Forecasting Performance: **Small BVAR**

Full Sample (Recursive evaluation: 1994.Q1-2016.Q4)

Panel A: BVAR (Now Only) vs. Raw BVAR

Series	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q	h=20Q
Relative MSE: **BVAR (Now Only)** Forecast / BVAR Raw Forecast							
Real GDP	0.70**	0.95**	0.98	0.97	1.01	1.05**	1.02**
CPI Inflation	0.29***	1.00	1.01	1.00	0.97*	0.98*	1.08***
Unemployment rate	0.28***	0.79**	0.83*	0.84	0.86	0.91	1.05
Federal Funds rate	0.01***	0.57***	0.70***	0.79**	0.87**	0.95	1.20
Credit Spread	0.74**	0.92*	0.91*	0.95	0.98	1.03*	1.06***

Panel B: BVAR (Now and LR) vs. Raw BVAR

Series	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q	h=20Q
Relative MSE: **BVAR (Now and LR)** Forecast / BVAR Raw Forecast							
Real GDP	0.70**	0.99	0.93	0.84*	0.88	0.94	1.07**
CPI Inflation	0.29***	0.97	0.93*	0.90***	0.84***	0.83**	1.02
Unemployment rate	0.28***	0.91	1.01	1.01	0.99	0.98	1.01
Federal Funds rate	0.01***	0.51***	0.60***	0.65*	0.69	0.69	0.90
Credit Spread	0.72**	0.92*	0.84	0.83	0.81	0.83	1.05

Panel C: BVAR (Now and LR) vs. BVAR (Now Only)

Series	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q	h=20Q
Relative MSE: **BVAR (Now and LR)** Forecast / **BVAR (Now Only)** Forecast							
Real GDP	1.00	1.04	0.95	0.86*	0.87	0.90	1.04
CPI Inflation	1.00	0.98	0.92*	0.91***	0.87**	0.85**	0.95
Unemployment rate	1.00	1.16	1.22	1.21	1.15	1.08	0.97**
Federal Funds rate	1.00	0.88***	0.85	0.83	0.79	0.73	0.75
Credit Spread	0.98	1.00	0.93	0.87	0.83*	0.80*	0.98

Notes for Table: Panel A reports the relative mean squared error defined as mean squared error (MSE) of BVAR forecast conditional on survey nowcasts only divided by MSE of unconditional BVAR forecast; a ratio less than 1 suggests that tilting the BVAR forecasts to survey nowcasts only is on average more accurate compared to unconditional BVAR forecasts. Panel B reports the relative MSE defined as MSE of BVAR forecast conditional on both survey nowcasts and long-horizon forecasts divided by MSE of unconditional BVAR forecast; a ratio of less than 1 suggests that tilting the BVAR forecasts to survey nowcasts and long-horizon forecasts is on average more accurate compared to unconditional BVAR forecasts. Panel C reports the relative MSE defined as MSE of BVAR forecast conditional on survey nowcasts and long-horizon forecasts divided by MSE of BVAR forecast conditional on survey nowcasts only; a ratio less than 1 suggests tilting the BVAR forecasts to survey nowcasts and long-horizon forecasts is on average more accurate compared to tilting on just the survey nowcasts. The table reports statistical significance based on the Diebold-Mariano and West test with the lag $h-1$ truncation parameter of the HAC variance estimator and adjusts the test statistic for the finite sample correction proposed by Harvey, Leybourne, and Newbold (1997); *10 percent, **5 percent, and ***1 percent significance levels, respectively. The test statistics use two-sided standard normal critical values.

Table A4: Real-Time Out-of-Sample **Density** Forecasting Performance: **Small BVAR**

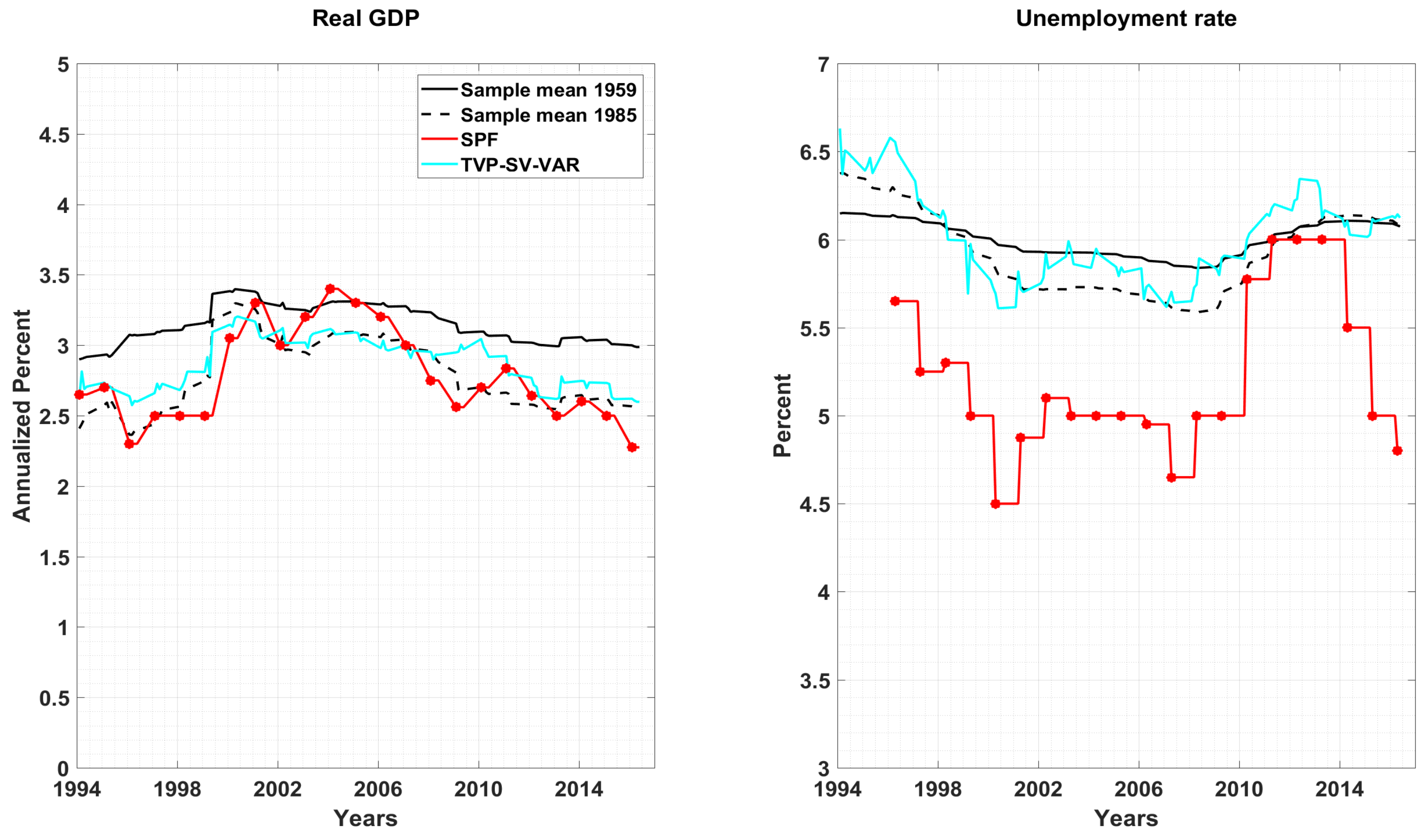
Full Sample (Recursive evaluation: 1994.Q1-2016.Q4)							
Panel A: BVAR (Now Only) vs. Raw BVAR							
Series	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q	h=20Q
Mean (Relative CRPS Score: **BVAR Now Only** Forecast - BVAR Raw Forecast)							
Real GDP	-0.15**	-0.04**	-0.01	-0.02	0.02**	0.04***	0.01*
CPI Inflation	-0.42***	0.00	0.00	-0.01	-0.01	-0.01	0.06***
Unemployment rate	-0.06***	-0.05**	-0.06*	-0.07	-0.07	-0.06	0.03
Federal Funds rate	-0.10***	-0.18***	-0.19***	-0.15**	-0.10*	-0.05	0.12
Credit Spread	-0.02***	-0.03*	-0.02*	-0.01	0.00	0.02**	0.02***

Panel B: BVAR (Now and LR) vs. Raw BVAR							
Series	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q	h=20Q
Mean (Relative CRPS Score: **BVAR Now and LR** Forecast - BVAR Raw Forecast)							
Real GDP	-0.16***	-0.03	-0.07	-0.13*	0.10	-0.05	0.04***
CPI Inflation	-0.42***	-0.02	-0.05*	-0.07**	-0.11***	-0.12**	0.02
Unemployment rate	-0.06***	-0.03*	-0.02	-0.02	-0.03	-0.04	0.00
Federal Funds rate	-0.10***	-0.21***	-0.25***	-0.26**	-0.27*	-0.29	-0.08
Credit Spread	-0.02***	-0.02*	-0.05*	-0.05	-0.06*	-0.06*	0.01

Panel C: BVAR (Now and LR) vs. BVAR (Now Only)							
Series	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q	h=20Q
Mean (Relative CRPS Score: **BVAR Now and LR** Forecast - **BVAR Now Only** Forecast)							
Real GDP	0.00	0.01	-0.05	-0.12**	-0.11*	-0.09**	0.03
CPI Inflation	0.00	-0.02	-0.06*	-0.06***	-0.10**	-0.12**	-0.05
Unemployment rate	0.00	0.02	0.04	0.05	0.04	0.03	-0.02
Federal Funds rate	0.00	-0.03**	-0.06	-0.11	-0.17	-0.24*	-0.20
Credit Spread	0.00	0.00	-0.02	-0.04*	-0.06*	-0.08*	-0.01

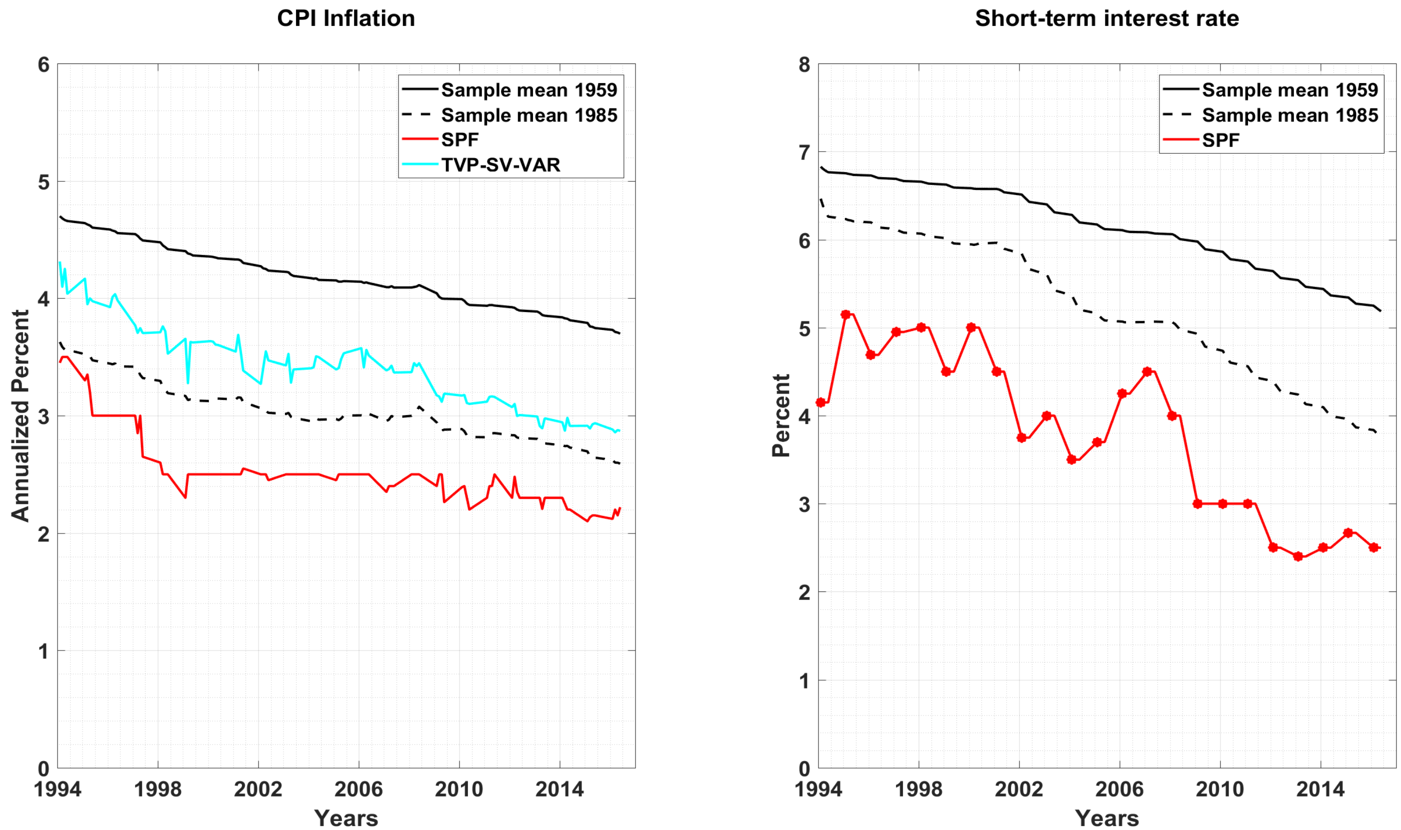
Notes for Table: Panel A reports the mean relative CRPS between CRPS of BVAR forecast conditional on survey nowcasts and CRPS of unconditional BVAR forecast; a negative value suggests that tilting the BVAR forecasts to survey nowcasts only is on average more accurate compared to unconditional BVAR forecasts. Panel B reports the mean Relative CRPS between CRPS of BVAR forecast conditional on both survey nowcasts and long-horizon forecasts and CRPS of unconditional BVAR forecast; a negative value suggests that tilting the BVAR forecasts to survey nowcasts and long-horizon forecasts is on average more accurate compared to unconditional BVAR forecasts. Panel C reports the Mean Relative CRPS between CRPS of BVAR forecast conditional on survey nowcasts and long-horizon forecasts and CRPS of BVAR forecast conditional on survey nowcasts only; a negative value suggests tilting the BVAR forecasts to survey nowcasts and long-horizon forecasts is on average more accurate compared to tilting on just the survey nowcasts. The table reports statistical significance based on Diebold-Mariano and West test with the lag $h-1$ truncation parameter of the HAC variance estimator and adjusts the test statistic for the finite sample correction proposed by Harvey, Leybourne, and Newbold (1997); *10 percent, **5 percent, and ***1 percent significance levels, respectively. The test statistics use two-sided standard normal critical values.

Figure 1: Real-Time Long-Run Forecasts: GDP and Unemployment Rate



Notes for the figure: The figure plots the real-time long-run forecasts from the Survey of Professional Forecasters (SPF), 10-year annual average forecast for real GDP growth and estimate of the natural rate of unemployment. To facilitate comparison also plotted are the real-time implied estimates of the long-run forecasts from a Small BVAR recursively estimated with data beginning in 1959 (denoted as Sample Mean 1959) and recursively estimated with data beginning in 1985 (denoted as Sample Mean 1985) respectively.

Figure 2: Real-Time Long-Run forecasts: CPI Inflation and the Short-Term Interest Rate



Notes for the figure: The figure plots the real-time long-run forecasts from the Survey of Professional Forecasters (SPF), 10-year annual average forecast for CPI inflation and 3-month T-bill rate. To facilitate comparison also plotted are the real-time implied estimates of the long-run forecasts from a Small BVAR recursively estimated with data beginning in 1959 (denoted as Sample Mean 1959) and recursively estimated with data beginning in 1985 (denoted as Sample Mean 1985) respectively.

3 A.3. Horse Race Between Steady-State BVAR and Small BVAR (Hybrid Approach)

Wright (2013) proposed using the steady-state BVAR technology developed by Villani (2009) to combine VAR forecasts with long-term survey expectations. Accordingly we compare the performance of Wright's approach to that of ours. A priori we would expect the two approaches to be competitive with each other. Indeed the results reported below (Table A5) confirm our expectations.

The steady-state BVAR model includes the same set of variables as used in our Small BVAR. The steady states are informed by the long-term forecasts of the Survey of Professional Forecasters (i.e., same values to which the Small BVAR is tilted). To run a fair horserace, we tilt both the steady-state BVAR and Small BVAR in the near term on the same nowcasts to ensure that both models start with the same jumping-off point.

The estimation procedure and the prior settings are the same as in Clark (2011) with the exception that prior variances around the steady-state priors are set very tight (as proposed in Wright, 2013). This will ensure that variables converge to the modeler's specified steady-states which in this case are the long-term forecasts from the SPF. (The prior variances are set at a value of 0.001).

Table A5 reports the forecast accuracy comparison between the steady-state BVAR and the hybrid approach from the Small BVAR. Panel A reports the point forecast accuracy comparison using relative MSE: MSE Small BVAR / MSE steady-state BVAR. A ratio of less than one suggests that the hybrid forecast from the Small BVAR is on average more accurate compared to the forecast from the steady-state BVAR. Panel B reports the density forecast accuracy comparison in the form of the mean relative CRPS, where negative numbers indicate that density forecasts from the hybrid approach (of Small BVAR) are on average more accurate compared to the steady-state BVAR.

Table A5: Real-Time Out-of-Sample Forecasting Performance: **Small BVAR (Hybrid) vs. Steady-State BVAR**

Full Sample (Recursive evaluation: 1994.Q1-2016.Q4)

Panel A: **Point** Forecast Accuracy of BVAR (Now and LR) vs. Steady-State BVAR (Now Only)

Series	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q	h=20Q	h=32Q
Relative MSE: **BVAR (Now and LR)** Forecast / **Steady-State BVAR (Now Only)** Forecast								
Real GDP	1.00	0.90	0.84*	0.87*	0.88	0.95*	0.98	1.00
CPI Inflation	1.00	1.02	1.00	1.04	0.94	1.03	0.90*	1.00
Unemployment	1.00	0.98	0.96	0.95	0.96	1.00	1.07	0.93
Federal Funds	1.00	1.15*	1.06	0.98	0.92	0.88	0.87	1.02
Credit Spread	1.04	1.07	1.05	1.03	1.03	1.07	1.06***	1.14**

Panel B: **Density** Forecast Accuracy of BVAR (Now and LR) vs. Steady-State BVAR (Now Only)

	h=1Q	h=4Q	h=5Q	h=8Q	h=10Q	h=12Q	h=20Q	h=32Q
Mean (Relative CRPS Score: **BVAR Now and LR** Forecast - **Steady-State (Now Only)** Forecast)								
Real GDP	-0.02	-0.07	-0.11	-0.08*	-0.07	-0.04**	-0.02*	-0.01
CPI Inflation	-0.01	0.00	0.02	0.02	0.00	0.03**	-0.02	0.04
Unemployment	0.00	-0.01	-0.02	-0.04	-0.04	-0.02	0.04	-0.04
Federal Funds	-0.00	0.04*	0.03	0.02	-0.01	-0.05	-0.03	0.25**
Credit Spread	0.00	0.02	0.03	0.03	0.03	0.03	0.04**	0.06*

Notes for Table: The numbers reported in the top panel of the table are relative mean squared errors: mean squared error conditional on nowcasts and long-horizon survey forecasts from Small BVAR (Hybrid) / mean squared error from the steady-state BVAR conditional on nowcasts only. So a ratio of less than 1 indicates that point forecasts from the hybrid approach corresponding to fixed-coefficient Small VAR are on average more accurate compared to the forecasts from the steady-state BVAR. The numbers reported in the bottom panel of the table are mean relative CRPS: CRPS from the Hybrid - CRPS from steady-state BVAR conditional on nowcasts only. So a negative number indicates that density forecasts from the hybrid approach corresponding to fixed-coefficient Small VAR are on average more accurate compared to the forecasts from the steady-state BVAR. The table reports statistical significance based on the Diebold-Mariano and West test with the lag $h-1$ truncation parameter of the HAC variance estimator and adjusts the test statistic for the finite sample correction proposed by Harvey, Leybourne, and Newbold (1997); *10 percent, **5 percent, and ***1 percent significance levels, respectively. The test statistics use two-sided standard normal critical values.

4 A.4. Horse Race Between BVAR Modeled in Gaps and Small BVAR (Hybrid Approach)

Another popular approach to anchor model forecasts to survey expectations is to model variables by first transforming them into a gap form (i.e., deviation from the respective long-run survey expectations) and then estimating them using a VAR (or a univariate regression for the single variable of interest). The forecasts of the gap coming out of the VAR are then transformed back to the units of interest by adding the latest estimate of the survey expectations available as of the forecast origin to construct the corresponding implied forecasts (see Faust and Wright 2013 in the context of the univariate inflation case; Clark and McCracken 2010 in the case of the VAR). The trend estimate (proxied by the survey expectations measure) is assumed to follow a random walk over the forecast horizon. By construction, the implied long-run forecasts from this approach would be close to the latest available estimate of the survey expectations plugged in as of the time forecast is generated. The advantage of this approach is its simplicity, and therefore, it has gained traction over the past few years. A key drawback is that it requires a time series of survey expectations as long as the estimation sample (necessary for constructing the transformed gap variable). This issue may be more likely to bind for regions outside the United States and Europe for which publicly available survey forecasts have a shorter history.

We construct a BVAR model in gaps that includes the same set of variables as used in our Small BVAR. The variables (with the exception of the credit spread) are transformed to the gap by taking a deviation from their respective time series of the survey expectations. To run a fair horserace, we condition both the BVAR in gaps and Small BVAR in the near-term on the same nowcasts. This will ensure that both models start with the same jumping-off points.
We construct the expectation series for real GDP, CPI inflation, the unemployment rate, and the federal funds rate going back to 1959 as follows:
From **1959 to 1993**:
1. Real GDP growth trend=constant 3%
2. Unemployment rate trend is computed using an exponential smoother with a smoothing parameter of 0.02 (as in Clark, 2011):

$$u_t^* = u_{t-1}^* + 0.02(u_t - u_{t-1}^*)$$

3. CPI inflation trend is the PTR (long-term inflation expectations series used in the Federal Reserve Board's FRB/US econometric model)
4. Nominal federal funds rate trend is assumed to be the same as the CPI inflation trend
From **1994 to 2016**:
The respective trend estimates are the long-run forecasts from the SPF.

The prior settings are the same as those of the Small BVAR. Table A6 reports the forecast accuracy comparison between the Small BVAR in gaps and the hybrid approach from the Small BVAR. Panel A reports the point forecast accuracy comparison using relative MSE: MSE Small BVAR / MSE Small BVAR in Gaps. A ratio of less than one suggests that the hybrid forecast from the Small BVAR is on average more accurate compared to the forecast from the BVAR in Gaps. Panel B reports the density forecast accuracy comparison in the form of mean relative CRPS where negative numbers indicate density forecasts from the hybrid approach (of Small BVAR) are on average more accurate compared to the BVAR in Gaps. A priori we would expect the two econometric approaches to perform comparably. Based on the results reported in

Table A6, for real GDP growth, CPI Inflation, and the federal funds rate, the hybrid approach generates more accurate forecasts and the gains are statistically significant. To facilitate visual assessment of the forecast accuracy comparison between the two approaches, Figures 3 to 6 plot the forecast trajectories corresponding to real GDP growth, the unemployment rate, CPI inflation and the federal funds rate.

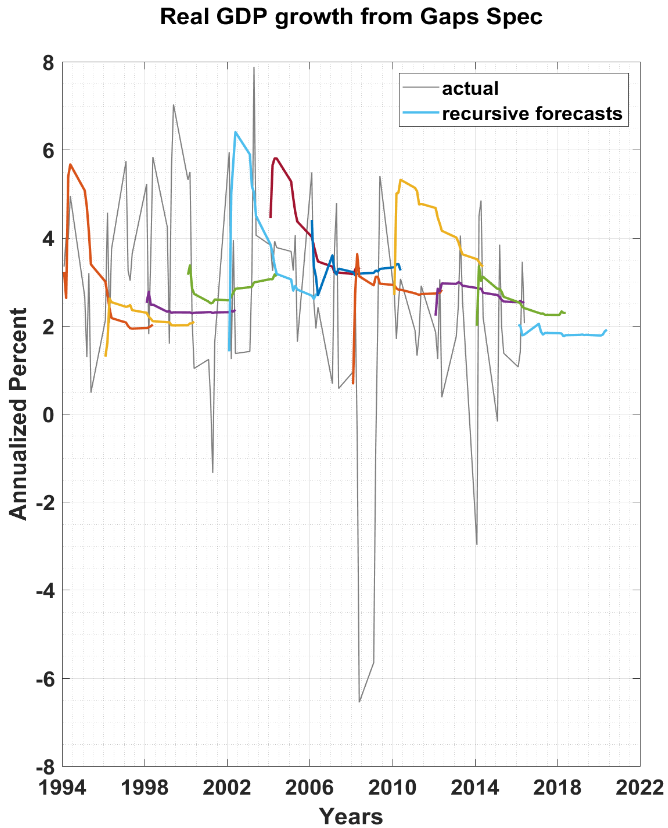
It is worth noting that even though an attempt is made to anchor the forecasts closer to the survey expectations (through modeling the variables in gap transformation), there is no guarantee that the medium- to long-term forecast would converge to the survey expectations. It may even settle far from the assumed underlying trend. This is due to the presence of an intercept term in the gap equation (e.g., inflation gap) that captures the long-run historical deviation of the gap from zero within the estimation sample. The estimate of the intercept term will be positive if the variable (e.g., inflation) has exceeded its trend (informed by the survey) on average during the sample, while it will be negative if the variable has been below trend on average. So an inflation forecast three years out may settle at a level that is lower than the trend estimate informed by the survey expectations (and the modeler's desired level).

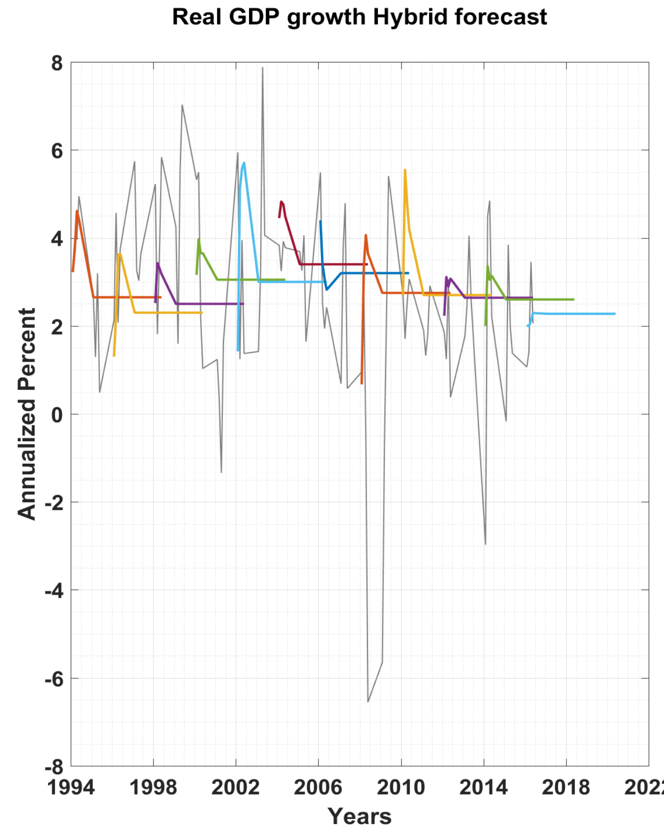
Table A6: Real-Time Out-of-Sample Forecasting Performance: **Small BVAR (Hybrid) vs. Small BVAR in Gaps**

Full Sample (Recursive evaluation: 1994.Q1-2016.Q4)							
Panel A: **Point** Forecast Accuracy of BVAR (Now and LR) vs. BVAR in Gaps (Now Only)							
Series	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q	h=20Q
Relative MSE: **BVAR (Now and LR)** Forecast / **BVAR in Gaps (Now Only)** Forecast							
Real GDP	1.00	0.91	0.89*	0.87***	0.92**	0.94***	0.97*
CPI Inflation	1.00	0.94***	0.87***	0.82***	0.78***	0.83**	0.80***
Unemployment rate	1.00	0.95	0.99	1.00	1.01	1.03	1.13
Federal Funds rate	1.00	0.95	0.89	0.80*	0.73**	0.69***	0.67***
Credit Spread	0.91	0.97	0.96	0.95	0.94	0.98	1.08
Panel B: **Density** Forecast Accuracy of BVAR (Now and LR) vs. BVAR in Gaps (Now Only)							
	h=1Q	h=4Q	h=5Q	h=8Q	h=10Q	h=12Q	h=20Q
Mean (Relative CRPS Score: **BVAR Now and LR** Forecast - **BVAR in Gaps (Now Only)** Forecast)							
Real GDP	0.01	-0.07	-0.07*	-0.08***	-0.05**	-0.03	-0.01
CPI Inflation	0.00	-0.02	-0.03	-0.06*	-0.06*	0.00	0.04
Unemployment rate	0.00	-0.01	-0.01	-0.01	0.00	0.00	0.04
Federal Funds rate	0.00	0.00	-0.03	-0.09	-0.17	-0.26*	-0.40***
Credit Spread	-0.01	0.00	-0.01	-0.01	-0.01	0.00	0.04

Notes for Table: The numbers reported in the top panel of the table are relative mean squared errors: mean squared error conditional on nowcasts and long-horizon survey forecasts from Small BVAR (Hybrid) / mean squared error from the BVAR in Gaps conditional on nowcasts only. So a ratio of less than 1 indicates that point forecasts from the hybrid approach corresponding to fixed-coefficient Small VAR are on average more accurate compared to the forecasts from the BVAR in gaps. The numbers reported in the bottom panel of the table are mean relative CRPS: CRPS from the Hybrid - CRPS from BVAR in Gaps conditional on nowcasts only. So a negative number indicates that density forecasts from the hybrid approach corresponding to fixed-coefficient Small VAR are on average more accurate compared to the forecasts from the BVAR in gaps. The table reports statistical significance based on the Diebold-Mariano and West test with the lag $h-1$ truncation parameter of the HAC variance estimator and adjusts the test statistic for the finite sample correction proposed by Harvey, Leybourne, and Newbold (1997); *10 percent, **5 percent, and ***1 percent significance levels, respectively. The test statistics use two-sided standard normal critical values.

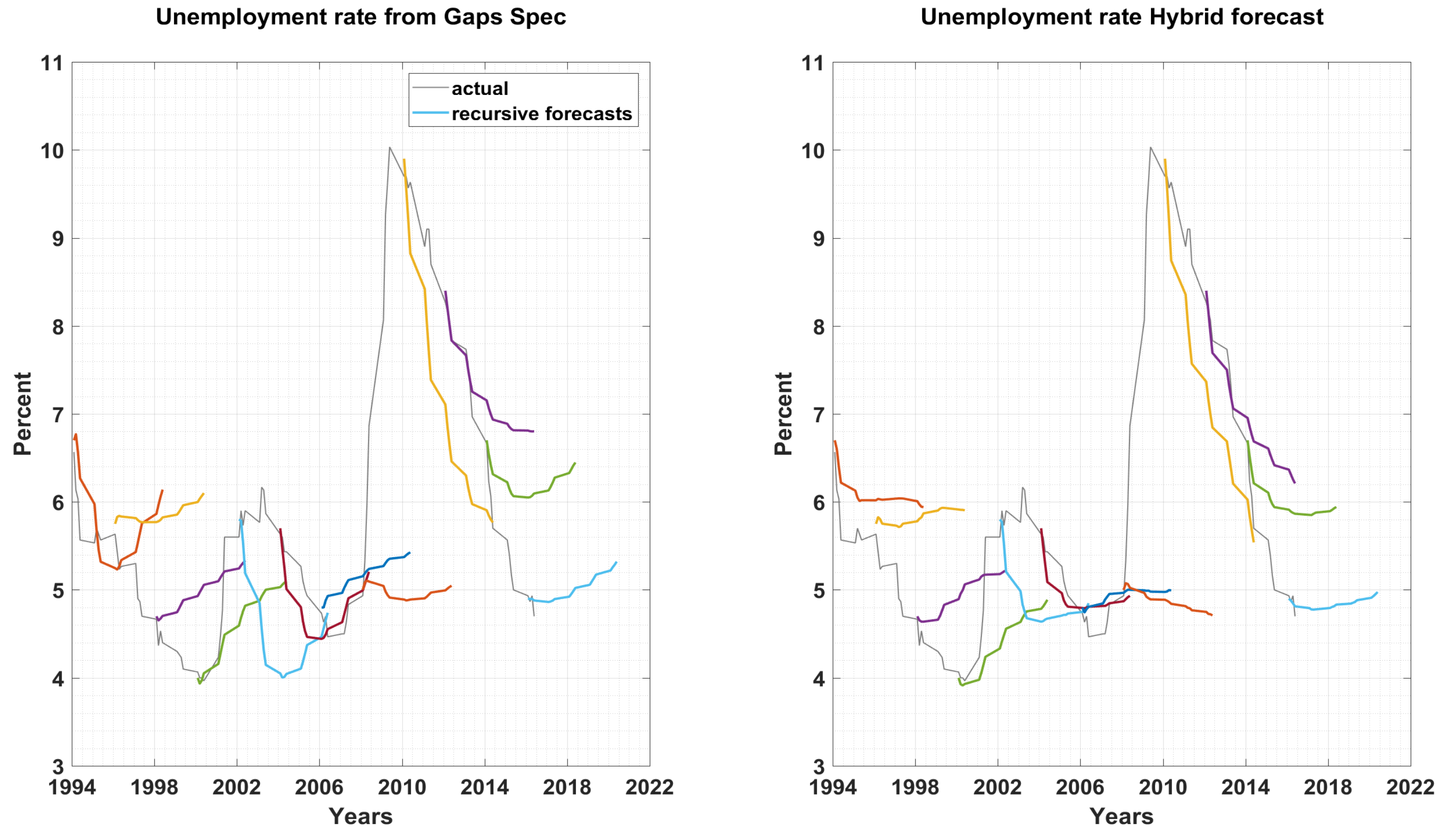
Figure 3: Recursive Point Forecasts for Real GDP Growth





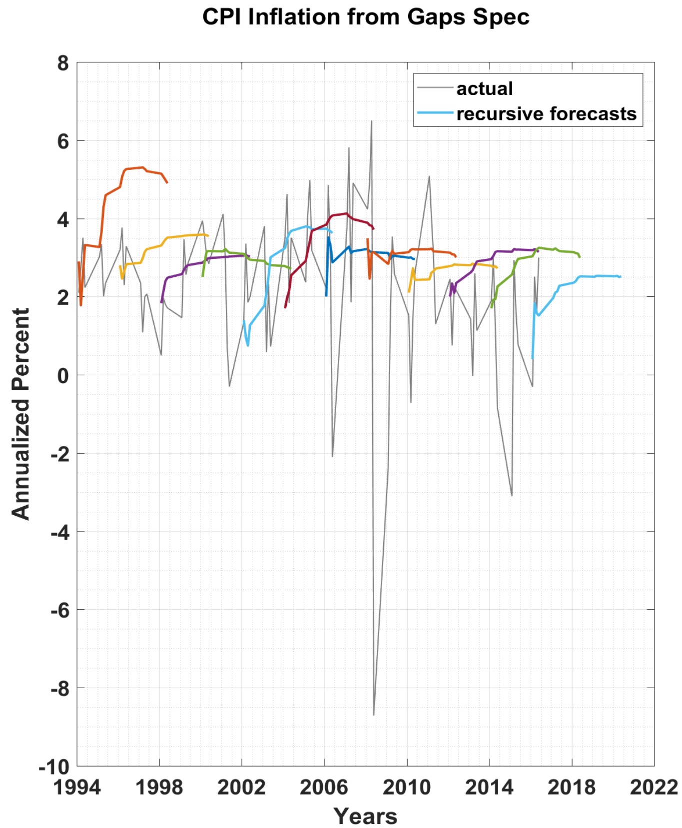
Notes: The figure plots the recursive point forecasts of real GDP growth annualized from the Small BVAR and the Small BVAR in Gaps, respectively at different points in time. Also plotted are the actual data (solid gray line). The forecasts are recursively generated beginning in 1994.Q1 through 2016.Q4, giving us a total of 92 recursive forecasts. Each forecast is up to 20 quarters ahead. For presentation purposes, every 7th forecast is displayed, i.e., 12 out of 92. The left panel corresponds to the forecast from the Small BVAR in Gaps that conditions only on the survey nowcast. The right panel corresponds to the forecast from the Small BVAR that conditions on both the survey nowcast and survey long-horizon forecast.

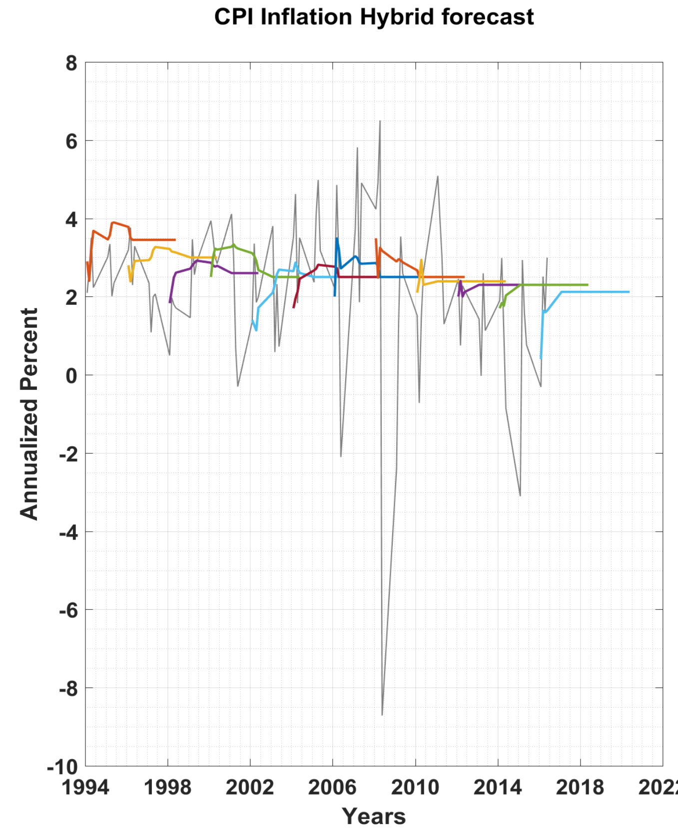
Figure 4: Recursive Point Forecasts for Unemployment rate



Notes: The figure plots the recursive point forecasts of the unemployment rate from the Small BVAR and the Small BVAR in Gaps, respectively at different points in time. Also plotted are the actual data (solid gray line). The forecasts are recursively generated beginning in 1994.Q1 through 2016.Q4, giving us a total of 92 recursive forecasts. Each forecast is up to 20 quarters ahead. For presentation purposes, every 7th forecast is displayed, i.e., 12 out of 92. The left panel corresponds to the forecast that conditions only on the survey nowcast. The right panel corresponds to the forecast that conditions on both the survey nowcast and the survey long-horizon forecast.

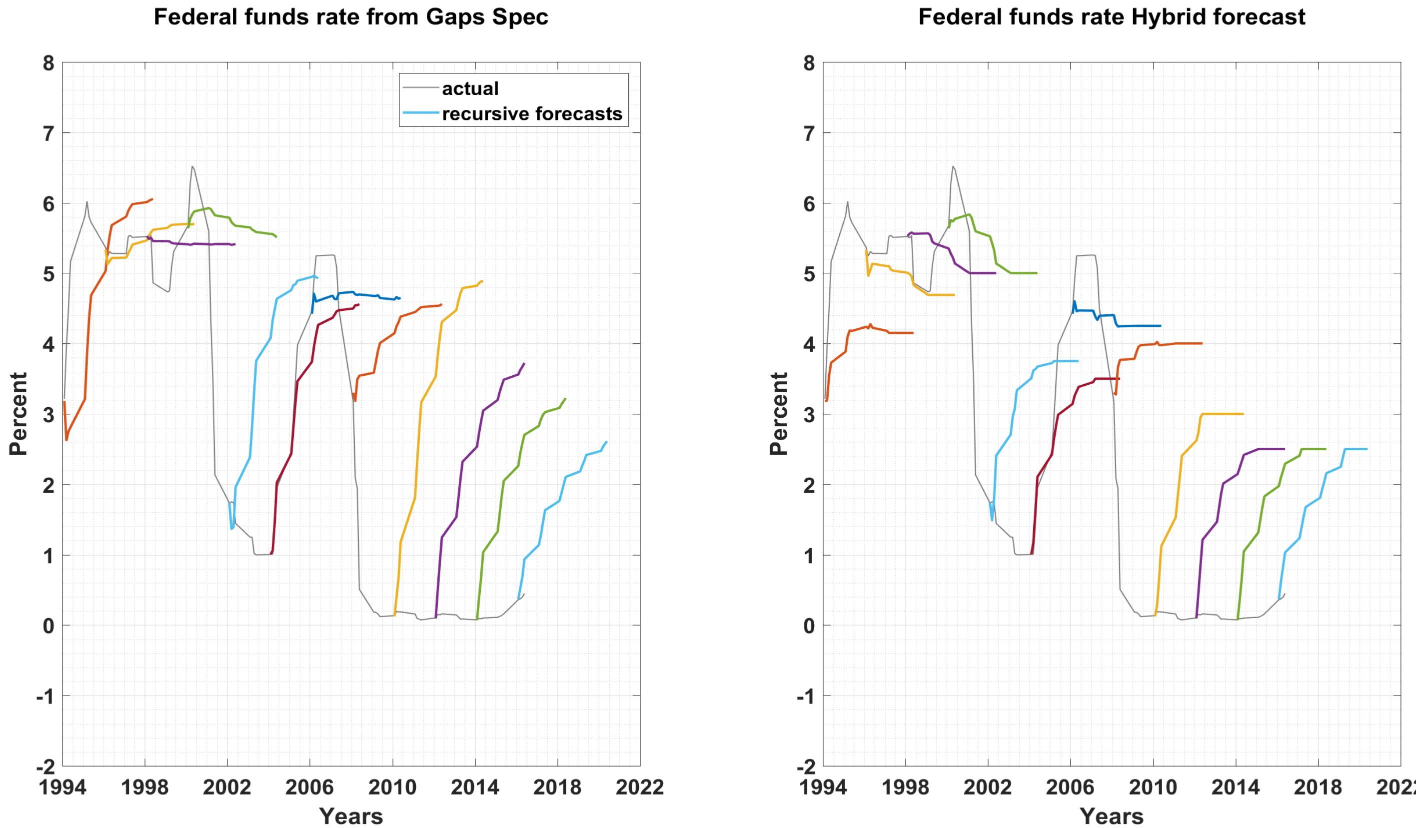
Figure 5: Recursive Point Forecasts for CPI Inflation





Notes: The figure plots the recursive point forecasts of headline CPI inflation at an annualized rate from the Small BVAR and the Small BVAR in Gaps, respectively at different points in time. Also plotted are the actual data (solid gray line). The forecasts are recursively generated beginning in 1994.Q1 through 2016.Q4, giving us a total of 92 recursive forecasts. Each forecast is up to 20 quarters ahead. For presentation purposes, every 7th forecast is displayed, i.e., 12 out of 92. The left panel corresponds to the forecast from the Small BVAR in Gaps that conditions only on the survey nowcast. The right panel corresponds to the forecast from the Small BVAR that conditions on both the survey nowcast and survey long-horizon forecast.

Figure 6: Recursive Point Forecasts for the Federal Funds Rate



Notes: The figure plots the recursive point forecasts of the federal funds rate from the Small BVAR and the Small BVAR in Gaps, respectively at different points in time. Also plotted are the actual data (solid gray line). The forecasts are recursively generated beginning in 1994.Q1 through 2016.Q4, giving us a total of 92 recursive forecasts. Each forecast is up to 20 quarters ahead. For presentation purposes, every 7th forecast is displayed, i.e., 12 out of 92. The left panel corresponds to the forecast from Small BVAR in Gaps that conditions only on the survey nowcast. The right panel corresponds to the forecast from the Small BVAR that conditions on both the survey nowcast and survey long-horizon forecast.

5 A.5. The role of Nowcast Uncertainty

To gauge the usefulness of nowcast variance conditions on the multi-horizon forecast accuracy, we compare the forecast accuracy between two sets of forecasts: a BVAR forecast that tilts on the nowcast mean only and BVAR forecast that tilts on both the nowcast mean and the variance around that mean. The variance conditions corresponding to the nowcast mean are constructed as detailed below.

Uncertainty around the nowcast mean

To construct the variance conditions, we follow Clements (2014) and Kruger et al. (2017) by computing the variance of the SPF forecast errors over a rolling period preceding the forecast origin.[1] A variance of forecast errors constructed through this approach is defined as an ex-post forecast uncertainty measure.

Specifically, if we denote $\hat{Y}_{t+h}^{SPF}$ as the median SPF forecast for indicator $Y_{t,h}$, then the variance condition is formed as follows,

$$\sum_{q=0}^{19}(Y_{t-Delay-q} - \hat{Y}_{t-Delay-q,h}^{SPF})^2 \tag{1}$$

where q reflects the number of past forecasts used to compute the variance of errors, and Delay indicates the number of quarters before the relevant actual data is released. In our exercises, we set Delay=2 quarters for all macroeconomic variables and Delay=1 quarter for financial variables.

Table A7 reports the forecast accuracy comparison between the baseline forecast (i.e., the BVAR tilted to the nowcast mean only) from Small BVAR and the Small BVAR forecast tilted to match both the nowcast mean and the nowcast variance (denoted BVAR NowOnly with Uncertainty). Panel A reports the point forecast accuracy comparison using relative MSE: MSE Small BVAR NowOnly with Uncertainty / MSE Small BVAR NowOnly. A ratio of less than one suggests that tilting BVAR forecasts toward nowcast variance in addition to nowcast mean leads to more accurate point forecasts on average compared to tilting just on the nowcast mean. Panel B reports the density forecast accuracy comparison in the form of the mean relative CRPS where negative numbers indicate that density forecasts from tilting toward both the nowcast mean and the variance are more accurate compared to just tilting on the nowcast mean. As would be expected, tilting toward the nowcast variance in addition to the nowcast mean leads to improved density forecast accuracy and the improvements persist far into the future for the unemployment rate, CPI inflation, and the federal funds rate. For real GDP growth the improvements in density forecast accuracy die out by five quarters out, which would be expected because it is well established that real GDP growth displays very little persistence. Given the similarity of our exercise to that of Kruger et al. (2017) we are able to relate our results to their findings. The main difference between our exercise and that of Kruger et al. (2017) is the BVAR model specification: in our exercise we use a constant-parameter BVAR, whereas they use a time-varying parameter BVAR. It is worth noting that tilting toward the

[1] If we were to use mean conditions informed from a different model then the variance conditions can be informed from that model's corresponding predictive density provided the model is able to generate one.

nowcast variance does not affect the point forecast accuracy.

Table A7: Real-Time Out-of-Sample Forecasting Performance: **Small BVAR (NowOnly with Uncertainty) vs. Small BVAR (NowOnly)**

Full Sample (Recursive evaluation: 1994.Q1-2016.Q4)

Panel A: **Point** Forecast Accuracy of BVAR (NowOnly with Uncertainty) vs. BVAR (NowOnly)

Series	h=1Q	h=4Q	h=6Q	h=8Q	h=10Q	h=12Q	h=20Q	h=32Q
Relative MSE: **BVAR (NowOnly with Uncertainty)** Forecast / **BVAR (NowOnly)** Forecast								
Real GDP	1.00	1.00	1.00	1.00	1.00	0.99	1.09	0.99
CPI Inflation	1.00	1.00	1.03	1.01	0.98*	0.99	1.01	1.02
Unemployment	1.00	1.00	0.99	0.99	0.99	0.99	1.00	1.01
Federal Funds	1.00	1.00	1.00	0.99	0.99	0.99	1.00	1.01
Credit Spread	1.00	1.00	0.99	0.99	1.00	0.99*	1.00	1.01

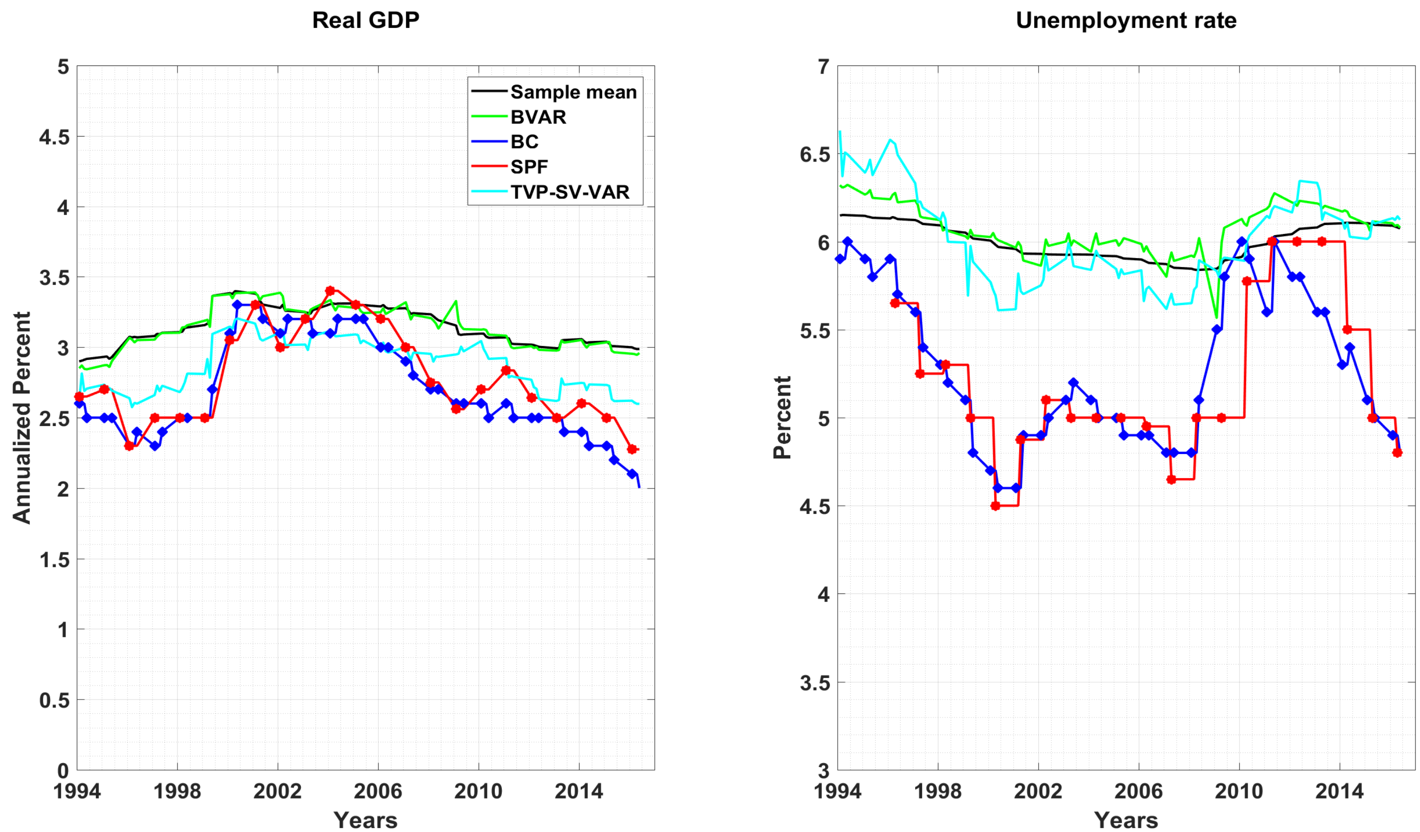
Panel B: **Density** Forecast Accuracy of BVAR (NowOnly with Uncertainty) vs. BVAR (NowOnly)

	h=1Q	h=4Q	h=5Q	h=8Q	h=10Q	h=12Q	h=20Q	h=32Q
Mean (Relative CRPS Score: **BVAR (NowOnly with Uncertainty) - BVAR (NowOnly)**)								
Real GDP	-0.10***	-0.01*	0.00	0.01	-0.01	0.00	0.04	0.00
CPI Inflation	-0.03**	-0.01**	0.01	-0.01	-0.02**	-0.02**	0.01	0.00
Unemployment	-0.01***	-0.01**	-0.01***	-0.01*	-0.01*	0.00	0.00	0.00
Federal Funds	-0.01***	-0.01**	-0.02***	-0.02	-0.01	-0.01**	0.02*	0.01
Credit Spread	0.00	0.00	0.00	0.00	0.00	-0.01*	0.00	0.01

Notes for Table: The numbers reported in the top panel of the table are relative mean square errors: mean squared error from the Small VAR conditional on the nowcast mean and the variance / mean squared error from the Small VAR conditional on the nowcast mean only. So a ratio of less than 1 indicates that conditioning on the nowcast variance in addition to the nowcast mean helps improve forecast accuracy on average. The numbers reported in the bottom panel of the table are mean relative CRPS: CRPS from the Small VAR conditioning on both the nowcast mean and the variance - CRPS from Small VAR conditioning on the nowcast mean only. So a negative number indicates that conditioning on the nowcast variance in addition to the nowcast mean helps improve density forecast accuracy on average. The table reports statistical significance based on the Diebold-Mariano and West test with the lag $h-1$ truncation parameter of the HAC variance estimator and adjusts the test statistic for the finite sample correction proposed by Harvey, Leybourne, and Newbold (1997); *10 percent, **5 percent, and ***1 percent significance levels, respectively. The test statistics use two-sided standard normal critical values.

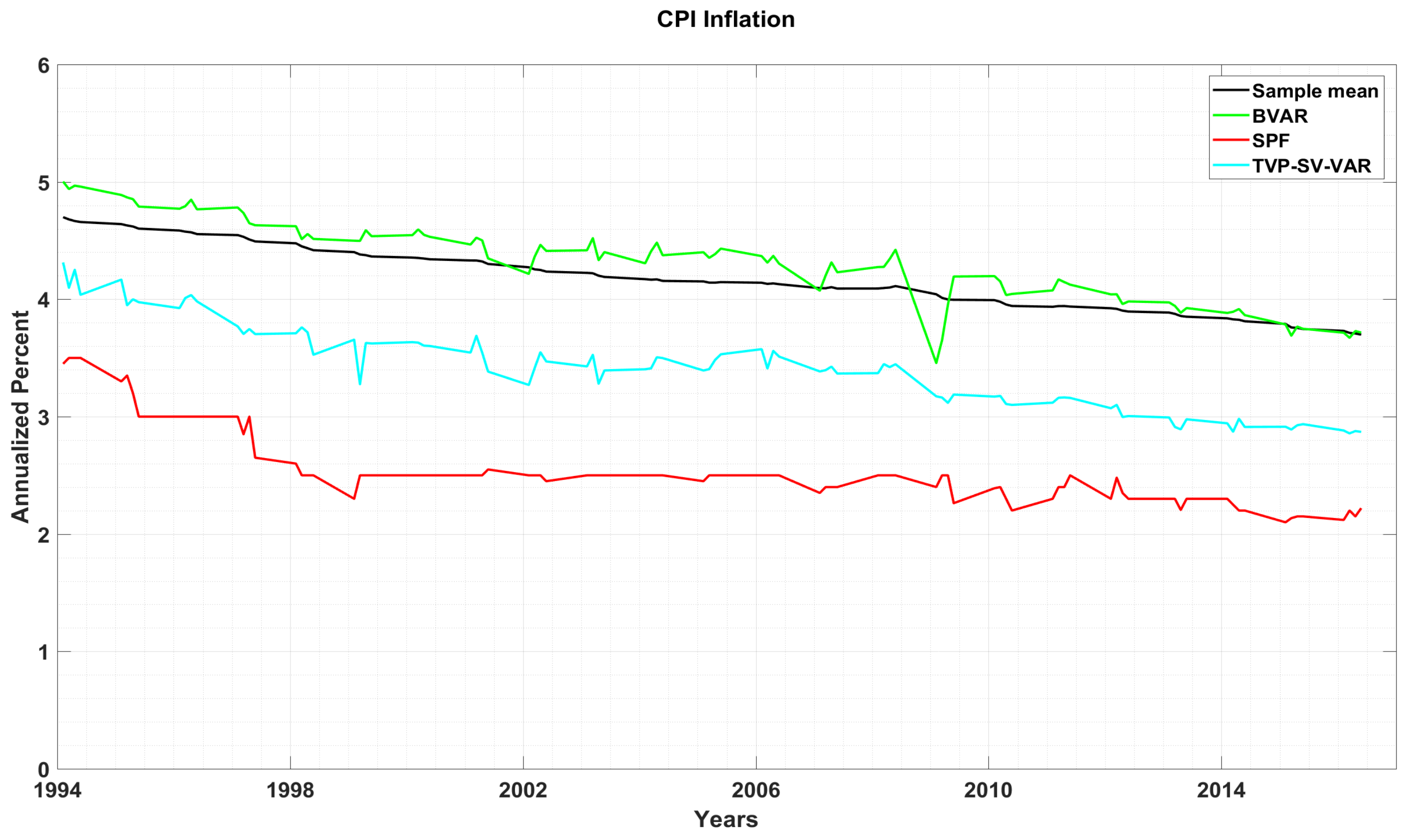
6 A.6. Comparing the Evolution of Survey Expectations to Trend Estimates from TVP-VAR SV

Figure 7: Real-Time Long-Run Forecasts: GDP and Unemployment Rate



Notes for the figure: The figure plots the real-time long-run forecasts from the Survey of Professional Forecasters (SPF), 10-year average forecast for real GDP growth and the estimate of the natural rate of unemployment; from the Blue Chip survey, the figure plots the 7 to 11 years ahead average. To facilitate comparison also plotted are the real-time estimates of the long-run forecasts (40 quarters out) from a Small BVAR and the time-varying VAR with SV recursively estimated with data beginning in 1959 and a simple arithmetic sample mean computed recursively.

Figure 8: Real-Time Long-Run Forecasts: CPI Inflation



Notes for the figure: The figure plots the real-time long-run forecasts from the Survey of Professional Forecasters (SPF), 10-year average forecast; from Blue Chip survey, the figure plots the 7 to 11 years ahead average forecast. To facilitate comparison also plotted are the real-time estimates of the long-run forecasts (40 quarters out) from a Small BVAR and the time-varying VAR with SV recursively estimated with data beginning in 1959 and a simple arithmetic sample mean computed recursively.

7 A.7. Illustrating the Spillover Effects of Tilting: Gaussian Example

Restricting the elements of the forecast matrix by imposing conditions on some future horizon will influence the forecast starting from the jumping-off point all the way to the tilted forecast horizon. For example, if we tilt real GDP growth at forecast horizon h=6, then tilting it will potentially impact the forecast trajectory from forecast horizons h=1 to h=5 and from h=7 and beyond for all the variables. The extent and degree of the spillover effects will be determined importantly by the BVAR's implied estimates of the covariances and autocorrelations among the variables and across forecast horizons.

To provide an intuition of the mechanics behind the spillover effects below, we illustrate using an example of a multivariate normal density (as would be obtained from a constant coefficient VAR model). Our example below generalizes the examples provided in Robertson, Tallman, and Whiteman (2005) and Kruger, Clark and Ravazzolo (2017). For convenience, we keep the same notation where possible.

We begin with a multivariate normal density $f(Y) = N\ (\theta, \Sigma)$ corresponding to the H-variate vector Y of forecast, $Y = [y_1, y_2, ..., y_H]$; Σ is positive definite and $\theta = [\theta_1, ...\theta_H]'$.

We obtain a KLIC-closest density $f(Y)^* = N\ (\mu, \Omega)$ such that it satisfies the restriction that the mean and the variance of the first element of vector Y, y^1 equals μ_1 and $\Omega_{1,1}$, respectively (e.g., a nowcast informed by the survey expectations).

The parameters of the tilted density f^* are defined as follows,

$$\mu_{2:H} = \theta_{2:H} + \Sigma_{1,1}^{-1}\Sigma_{1,2:H}(\mu_1 - \theta_1) \tag{2}$$

$$\Omega_{2:H,2:H} = \Sigma_{2:H,2:H} - \Sigma_{2:H,1}\Sigma_{1,1}^{-1}\Sigma_{1,2:H} \times (\frac{\Sigma_{1,1} - \Omega_{1,1}}{\Sigma_{1,1}}) \tag{3}$$

$$\Omega_{2:H,1} = \Sigma_{2:H,1}\Sigma_{1,1}^{-1}\Omega_{1,1} \tag{4}$$

The matrices indexed by $i : j, a : b$ represents a matrix containing rows from i to j, and columns a to b. Accordingly, matrices indexed by $i : j, a$ correspond to column vector and those indexed by $i, a : b$ correspond to row vector. The elements of column vector $\Sigma_{2:H,1}$ reflect the correlation between the nowcast horizon and the forecast horizons beyond the nowcast.

From the above definitions of the parameters, it can be easily seen that imposing the moment restriction $\Omega_{1,1} = 0$ is equivalent to the standard conditional forecasting. Also, by imposing the mean moment condition **only** as we have done in all the exercises reported in the paper (and not the variance condition), it can be seen that the tilted variance is the same as variance of the untilted density (i.e. the original variance). The Online Appendix A5 is where we also assessed the impact of imposing the variance condition in addition to the mean condition.

8 A.8. Sampling from the Tilted Predictive Density: Multinomial Resampling Algorithm

To sample from the modified (i.e., tilted) predictive density g(.), we follow the approach suggested in Cogley et al. (2005). Specifically, they suggest using the multinomial resampling algorithm of Gordon et al. (1993) to redraw from the original predictive density p(.) using the modified weights, ω^*, to obtain a sample corresponding to the tilted density g(.)

Algorithm

Given a sample $Y_i^{T+1,T+H}/ensapce, i = 1, ...D$ from the predictive density p(.) along with the weights, ω_i^* corresponding to the tilted density g(.) the steps listed below are used to obtain a sample from g(.)

Step 1: Define NC as a $D \times 1$ vector representing the number of offspring corresponding to each draw obtained from the original density p(.)

Step 2: Define a value for D^* such that $D^* > D$. D^* represents the number of draws for the tilted predictive density g(.), our object of interest.
The above two steps ensure that $\sum_{i=1}^{D} NC_i = D^*$.

Step 3: Draw D number of draws for NC from a multinomial distribution $NC\ MN(D_*; w_1^*, w_2^*, ..., w_D^*)$. (Matlab function *mnrnd* is used to draw from the MN distribution.)

Step 4: Given a sample for NC obtained in the previous step, construct a density g(.) by replicating $Y_i^{T+1,T+H}$ NC_i times for $i = 1, ...D$

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