

w o r k i n g
p a p e r

17 21R

**Costly Information Intermediation as a
Natural Monopoly**

Daniel Monte and Roberto Pinheiro



FEDERAL RESERVE BANK OF CLEVELAND

ISSN: 2573-7953

Working papers of the Federal Reserve Bank of Cleveland are preliminary materials circulated to stimulate discussion and critical comment on research in progress. They may not have been subject to the formal editorial review accorded official Federal Reserve Bank of Cleveland publications. The views expressed herein are solely those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Cleveland or the Federal Reserve System.

Working papers are available on the Cleveland Fed's website:

<https://clevelandfed.org/wp>

Costly Information Intermediation as a Natural Monopoly

Daniel Monte and Roberto Pinheiro

Many markets rely on information intermediation to sustain cooperation between large communities. We identify a key trade-off in costly information intermediation: intermediaries can create trust by incentivizing information exchange, but with too much information acquisition, intermediation becomes expensive, with a resulting high equilibrium default rate and a low fraction of agents buying this information. The particular pricing scheme and the competitive environment affect the direct and indirect costs of information transmission, represented by fees paid by consumers and the expected loss due to imperfect information, respectively. Moreover, we show that information trade has characteristics similar to a natural monopoly, where competition may be detrimental to efficiency either because of the duplication of direct costs or the slowing down of information spillovers. Finally, a social-welfare-maximizing policymaker optimally chooses a low information sampling frequency in order to maximize the number of partially informed agents. In other words, maximizing information spillovers, even at the cost of slow information accumulation, enhances welfare.

Keywords: Costly Information Trade, Market Structure, Natural Monopoly.

JEL Codes: D47, D83, D85.

Suggested citation: Monte, Daniel, and Roberto Pinheiro, 2018. "Costly Information Intermediation as a Natural Monopoly," Federal Reserve Bank of Cleveland, Working Paper no. 17-21R. <https://doi.org/10.26509/frbc-wp-201721r>.

Daniel Monte is at the São Paulo School of Economics, FGV (daniel.monte@fgv.br), and Roberto Pinheiro is at the Federal Reserve Bank of Cleveland (roberto.pinheiro@clev.frb.org). The authors thanks Joyee Deb, Bruce Fallick, Renato Gomes, Pawel Krolkowski, Lucas Maestri, Joao Ramos, and Jan-Peter Siedlerek for many helpful comments as well as seminar participants at Bristol University, 2018 Canadian Economic Association Meetings, IIOC 2018, 2018 Midwest Economic Theory, 2018 RES meetings, and the 2018 Summer Workshop on Money, Banking, Payments, and Finance, among others.

*First version November 2017.

1 Introduction

Many important market interactions are infrequent in that each consumer only requires services sporadically and the parties involved do not engage in long-term relationships. In these environments, the expected reputational cost of providing bad service or defaulting on small loans is small, in particular, when the market size is large (see Kandori [1992]). For example, the cost of suing a plumber for bad service, a store for poorly provided services, or a debtor that has not paid a small loan may be prohibitive, in particular, once we introduce the cost of going through the legal process (see Schmitz [2016]). In these markets, investing in reputation is important only insofar as it might help spread the word to other customers. While word-of-mouth is important to sustain cooperation in many markets, in others, for example, in large and mobile populations, it might serve less well. In many of these situations, markets develop information sharing and reporting systems that allow the harmed party to disclose information. This information can be used by other agents in the future in order to avoid trading with a dishonest or incompetent counterpart. This reporting system provides not only an incentive to agents in developing a reputation as good providers or debtors, but also a mechanism that allows consumers and creditors to reduce the likelihood that they will be harmed by faulty service.

In this paper, we show that the trade of costly information in a market with atomistic agents has characteristics similar to a natural monopoly. First, the direct cost of information acquisition can be seen as a high fixed cost that may be duplicated because of competition among information intermediaries (hereinafter referred to as bureaus). Second, even in cases where the bureau designs an information pricing mechanism that attempts to reduce direct the costs of information acquisition, competition may have a negative effect on information accumulation. The reason is that alternative pricing schemes reduce the direct cost of information acquisition by promptly obtaining only a fraction of the available information while learning the remaining information over time. Hence, these pricing schemes introduce an indirect cost from partial information. Competition among bureaus increases these indirect costs by slowing down the process of information dissemination. When bureaus with partial information compete, the average number of agents that transact with each bureau goes down. Consequently, the number of agents that learn about a previously faulty service at each round is smaller, slowing down information diffusion.

Furthermore, we are able to disentangle the impact of quality and the degree of spillover in the information transmission. In particular, we consider the optimal information sampling design by a social-welfare-maximizing bureau. We show that the bureau optimally chooses to reduce the sampling frequency in order to minimize the cost of information acquisition and maximize the measure of (partially) informed agents. In other words, maximizing information dissemination or spillover improves social welfare, even at the cost of a slow learning rate by the bureau. Consequently, the social welfare loss due to competition with pricing schemes that introduce less than full information is mostly due to the reduction in the bureau's equilibrium size, reducing the information spillover across agents in the

economy.

We model the introduction of an information sharing bureau in a random matching set-up in which the population is large enough that a folk theorem such as the one presented in Kandori [1992] and Ellison [1994] is not possible. Specifically, we consider two large populations of consumers and providers in an infinite horizon economy. At the start of each period, each consumer is randomly matched to a single service provider and they play a sequential game in which the provider has a short-run incentive to shirk. Agents are assumed to be risk-neutral and forward-looking.¹ All providers are rational and only earnings enter the utility function. Hence, providers consider the loss of future earnings as their only motivation for good behavior. We assume that the revelation of public information by consumers is costly, so in an equilibrium without a bureau, no messages are sent, even when costs are arbitrarily small. As a consequence, without a bureau, the market for services collapses and services are never hired. Our focus is to study the implications of the introduction of a bureau in this environment.

We consider the introduction of a bureau through different market structures: 1. non-profit bureau that aims to maximize social welfare while subject to a balanced-budget restriction and where participation is voluntary, and 2. for-profit bureaus in a competitive market. We also consider different pricing methods: a. membership fees that require that members provide information whenever they receive the service from a provider, and b. buy and sell information, where the bureau pays a fee for reports of services provided, while charging another fee whenever a consumer or creditor would like to access the bureau's database. In the case of buy and sell information, agents that sell information are not required to buy information or vice-versa. We consider the case in which all fees are paid by the side of the market that faces potential harm (consumers or creditors). Our goal is normative: we see the problem through the lens of a policymaker choosing the market structure in order to maximize a social welfare function that equally weighs consumers' and providers' well-being. Consequently, we are concerned with how the different market structures affect the number of informed consumers or creditors in the economy, the number of providers or debtors that default in equilibrium, and the cost of implementing the different policies.

The results from our benchmark model show that a non-profit bureau that offers memberships would provide the highest welfare, as well as the highest number of informed consumers or creditors and the lowest number of defaulting providers or debtors in an equilibrium with risk-neutral agents. A non-profit bureau with a buy and sell pricing framework would be a second-best among the considered mechanisms.²

We then consider competition among for-profit bureaus. We start with the case in which bureaus implement a buy and sell pricing mechanism. We show that, once we take into account the pricing mechanism, competition does not affect the equilibrium number of informed consumers and defaulting

¹We focus on the case of an economy with risk-neutral agents in order to avoid issues of insurance against negative shocks that may complicate the analysis.

²A monopolistic bureau would deliver the lowest social welfare, regardless of the pricing mechanism.

providers. However, social welfare is lower because information acquisition costs are duplicated. In contrast, in the case of membership pricing with competition, we show that the equilibrium number of providers that default is the same as in the monopoly case. Moreover, the case of competing bureaus with membership is, in equilibrium, equivalent to a monopoly with a for-profit bureau, delivering the lowest social welfare among the studied mechanisms.

It is important to highlight that the different pricing and competition mechanisms not only induce equilibria with different numbers of informed and uninformed consumers, but also differ in terms of the quality of the information (or coarseness of the information set) that informed consumers obtain. For example, let's consider the case of a non-profit bureau. While more consumers are informed in an equilibrium with membership than in buy and sell, the quality of the information held by members is worse. In particular, members have no information about poor service delivered to non-members, so the knowledge about bad providers builds slowly over time in a membership equilibrium, as members interact with providers. In a buy and sell equilibrium, since all consumers sell information, the bureau has all information about previous deviations at once, so the quality of information is high from the beginning.

The counterpart of the differences in information quality is the cost of information acquisition. There are two components to the cost of information. First, there is a direct cost, which is captured by the membership fee or the information buying fee. This direct cost is strictly increasing in information quality, since the quality depends on the number of consumers that must be compensated for reporting a transaction with a provider. Hence, the buy and sell arrangement – by buying information from all consumers in the economy – naturally has a higher direct cost than a membership arrangement. We should also emphasize that the direct cost does not fluctuate much over time, although it shrinks a bit over time in the membership case, as members learn about defaulting providers and stop buying their services. On the other side, the indirect cost is given by the expected loss due to faulty service that informed consumers may face. Notice that this cost is strictly decreasing in the information's quality. In the case of the buy and sell arrangement, the indirect cost is always zero. In contrast, in a membership arrangement the indirect cost is strictly positive and approaches zero as time passes and the bureau learns over time. However, the speed at which the bureau's quality improves varies with the bureau's size. Large bureaus learn faster, so indirect costs converge to zero at a much faster rate. Consequently, competition among membership bureaus, by decreasing the individual bureau's size, reduces the speed at which both indirect and direct costs shrink over time.

In summary, the policymaker's choice between a non-profit bureau and for-profit competitive bureaus, as well as different pricing mechanisms – buy and sell vs. membership – turns out to generate a trade-off between information quality and information costs. Moreover, different mechanisms also imply a trade-off between direct and indirect costs and the speed at which indirect costs diminish over time. Our results for the case of risk-neutral agents, i.e., a case that disregards the need for insuring consumers against consumption volatilities, show that trading information may present features similar to a natural

monopoly, as competition implies either a duplication of information acquisition costs or no improvement over a pure monopolistic case. Moreover, the fact that a non-profit altruistic bureau prefers membership to buy and sell indicates that if a bureau is large enough and there are no concerns about insuring consumers, the reduction in direct costs more than compensates for the expected losses through indirect costs.

In order to further understand the effects of information quality and the degree of dissemination, we allow a non-profit social-welfare-maximizing bureau with membership pricing to optimally choose its information sampling scheme. In particular, we assume that every period, the bureau optimally chooses the probability that it's going to buy information from its members. We show that the optimal sampling scheme involves a low buying probability that induces a small membership fee that maximizes the bureau's size, equalling the entire consumer population. As a result, while the bureau learns about providers' deviations very slowly, whenever deviations are detected, knowledge of them is spread across all consumers in the economy. Consequently, the social welfare loss due to competition with membership is likely because of a reduction in the information spillover once a default is detected instead of a decline in the bureau's learning rate.

We further extend the model in several dimensions. First, we consider the case where the bureau is allowed to buy only negative feedback, instead of having to purchase information from all consumers regardless of their experience. Since in our model all providers are *ex ante* identical, only negative information (defaults) is important to incentivize providers to exert effort. Consequently, buying only negative information may be social welfare improving by reducing information acquisition costs. In this case, we show that we are able to reach the first best in the case in which consumers face no cost of accessing information. However, once we introduce a small but positive cost of accessing information, all the results we present in the benchmark cases are recovered. Second, we consider the case in which membership is observable by providers. This extension is relevant since it is at the core of the services provided by some bureaus – such as Angie's list – as well as it is relevant for some historical examples, such as the Maghribi trader's coalitions (see Greif [1993]). In this case, we again reach the first best if consumers costlessly access information and the bureau is able to buy only negative information. In the case of costly information access, we consider two cases: the bureau can either enforce information access or not. The case in which information access is enforceable is identical to the case of costless access, apart from the decline in social welfare due to the cost of access. In contrast, in the case of unenforceable costly access of information, we show that known membership becomes counterproductive as agents become patient. The reason is that, in this environment, even effectively “uninformed” consumers are compelled to become members. Consequently, these effectively “uninformed” members report deviations over time. As a result, the overall cost for consumers increases, reducing social welfare. In contrast, making membership unobservable allows uninformed consumers to free-ride on the incentives to providers delivered by informed consumers, without paying a membership fee. Similarly, unobservable membership allows bureaus to save on the direct costs of acquiring information from uninformed consumers, relying

on the learning process over time in order to induce effort by providers.

Our benchmark model is related to the literature on community enforcement starting with Okuno-Fujiwara and Postlewaite [1995], who showed that the folk theorem holds in a random matching game. Recent papers on community enforcement by Takahashi [2010], Awaya [2014], Bhaskar and Thomas [2018], Ali and Miller [2016] and Deb [2017] are also related to our benchmark model.

Our paper is also related to the literature on information sharing in credit markets, as presented by Pagano and Jappelli [1993], Jappelli and Pagano [2002], and Brown et al. [2009]. A few papers have studied how markets that depend on permanent reputations are affected by different information sharing mechanisms. Some important papers in this area that link to ours are Vercammen [1995], Ekmekci [2011], Liu and Skrzypacz [2014], Elul and Gottardi [2015], and Kovbasyuk and Spagnolo [2017]. Finally, our paper is related to mechanisms that were developed by society throughout history in order to overcome the lack of community enforcement in large societies, as pointed out by Milgrom et al. [1990], Araujo [2004], and Araujo and Minetti [2011], among others.

Finally, we contribute to the literature on information reporting systems - a topic that, in our opinion, has not received enough attention from the theoretical literature. Empirically, we observe that the implementation of information reporting systems has taken different forms. Different market reporting systems have evolved across markets and countries. For example, in many countries, credit reporting systems for both consumer credit and business credit are done through public credit registries (see Miller [2000]) with mandatory participation. In this case, not only is all information available – once counterparts are obliged to report – but also the information quality is high because of potential legal penalties. In the US, credit reporting systems for both consumer credit and business credit are done through the private sector. However, the sectors’ structures are quite different. While the consumer credit reporting sector has three major players – Experian, Equifax, and TransUnion – the business credit reporting system is concentrated in only one firm – Dun & Bradstreet. Similarly, distinct market structures are seen on the consumer protection side, where we have the Better Business Bureau, a non-profit that provides information to consumers about the quality of services received by previous customers, and Angie’s List, a for-profit organization that earns its income through memberships and advertising, among others. Moreover, the way these bureaus charge customers for the services provided also varies considerably. While most privately owned credit bureaus provide services through either membership fees or a fee per credit report requested, services like Angie’s list have been funded mostly through membership fees and advertising. Finally, the Better Business Bureau charges membership fees from the businesses that are accredited by the bureau. Consequently, evaluating the social welfare impact of different market designs becomes a relevant question.

The paper is divided into 6 sections. Section 2 introduces the environment without an information bureau. Section 3 considers the case in which a non-profit altruistic bureau is introduced. Section 4 introduces competition among profit-seeking bureaus. Section 5 discusses model extensions, focusing on the bureau’s optimal information sampling. Finally, Section 6 concludes by summarizing the paper’s

results. All proofs are presented in the appendix.

2 Basic Model

Consider two populations indexed by i , where i lies in $I_i = [0, 1]$, $\forall i \in \{1, 2\}$. We assume that $x \in I_1$ is a consumer and $y \in I_2$ is a provider. Agents are distributed according to a Lebesgue measure. In each period $t = 1, 2, \dots$, each consumer is randomly matched to a provider to play a stage game Γ . We assume that the probability distribution over opponents in each period is uniform regardless of the matching history. Therefore, the probability that a currently matched pair of players will match again is zero.

The stage game Γ is represented by the game tree in Figure 1. Consumers initially decide whether or not they should hire the provider. Not hiring the services generates a payoff of zero to both parts. Hiring the provider's services implies that the consumer must pay w to the provider, irrespective of the quality of the service. If the provider is hired, then the provider must decide whether or not he will put in effort into the service. If the provider puts in effort, the service is high quality, inducing a payoff of $P > w$ to the consumer. If no effort is exerted, the service turns out to be low quality, generating no benefit to the consumer. Putting effort into a task generates a disutility e to the provider. We assume that $0 < e < w$, but $P - e > 0$, so hiring the service and exerting effort are socially optimal. Effort is verifiable but the expected cost of a lawsuit is too high to be used as a credible threat. After the service is provided, the consumer must decide whether or not she will send a message to others about the quality of the service received. These messages consist of saying whether or not the provider made effort. We must be aware that this is not related to any intrinsic quality of the provider, but only to his immediate previous action (all providers are *ex ante* identical, acting rationally to maximize their payoffs). We assume that these messages are public to the consumers (but not to the providers). The consumers incur a fixed cost $c > 0$ for sending each message.

This is an infinite horizon problem in which each agent discounts future periods by the same rate $\delta \in (0, 1)$. We assume a minimum patience level throughout the paper. Precisely, we assume that $\delta > e/w$. Most of our results, unless explicitly mentioned, do not depend on a high discount factor, except for this minimum threshold.³ Histories are private and a history observed by an agent i at time $\tau > 1$, denoted by $h_{i,\tau}$, is a sequence of private interactions that agent i observed from periods $1, 2, \dots, \tau - 1$. The set of all (private histories) at time t is denoted by \mathcal{H}_t and the set of all histories by $\mathcal{H} = \cup_{t=1}^{\infty} \mathcal{H}_t$. Let $\sigma = \{\sigma_t\}_{t=1}^{\infty}$ denote the consumer's behavior strategy. In each stage game, behavior strategy σ encompasses the actions the consumer must take, conditional on all the possible histories. In particular, it includes her initial decision of whether or not to hire the provider, conditional on the information set at that period t ; her pure strategy at this node is characterized by $s_{h,i} : \mathcal{H} \rightarrow \{\text{hire}, \text{don't hire}\}$. It also includes her next decision node in the stage game, where

³When this threshold is not met, the provider would prefer to default even in a world in which all consumers would immediately become informed about the default. When we look at the competitive bureau case, it will sometimes be convenient to assume that $\delta > \sqrt{e/w}$.

she must decide whether or not to send a costly message to inform others about the received service: $s_{m,i} : \mathcal{H} \times \{\text{hire}, \text{don't hire}\} \times \{\text{effort}, \text{no-effort}\} \rightarrow \{\text{good}, \text{bad}, \emptyset\}$. A pure strategy for the consumer can be denoted simply as $s_i = (s_{h,i}, s_{m,i})$. Similarly, a behavior strategy for the provider is denoted by $\sigma_i : \mathcal{H} \times \{\text{hire}, \text{don't hire}\} \rightarrow \{\text{effort}, \text{no-effort}\}$. A strategy profile σ^* is a *perfect Bayesian equilibrium* if, for every $t \geq 1$, every $h \in \mathcal{H}$, every pair (i, j) , every play of the stage game and every σ it holds that: $U_t(\sigma^*|h_t) \geq U_t(\sigma_i, \sigma_j^*|h_t)$, where $U_t(\cdot)$ represents the expected continuation payoff of the repeated game.

Given that there is a continuum of agents, there is a zero probability of rematching with a former partner. Thus, there is no incentive to either insure oneself against former deviations or to obtain gains punishing former deviators by sending messages. Therefore, since messages are costly, no consumer would send a message. Hence, there is no way to punish a former deviator. Even if we try to apply Kandori [1992]'s contagious equilibrium, it would fail because of the continuum of agents hypothesis. Formally, suppose, by contradiction, that there exists a history in which a provider is hired with positive probability. This means that the consumer expects that the provider will exert effort with positive probability. At this history, if this provider shirks, he has a higher current payoff than if he exerts effort (by avoiding the cost of effort). Moreover, his expected continuation payoff is given by the payoff that he expects to get by matching to each consumer in each future period. Given that actions are private to the interaction, the maximum number of players that have been exposed to the defection after t periods is at most 2^{t-1} . In particular, there is a countable number of such agents, and the measure of the union of these agents is zero. Thus, if f is the probability density function (henceforth p.d.f.) over all possible consumers and f_t^* is the p.d.f. over consumers who have not been exposed by the original defector at time t , then f and f_t^* are equal almost everywhere (a.e.) for any t . Simply put, the chance that one such consumer will meet the original defector again is zero for any time t , so that there is no incentive to cooperate. Therefore, the only equilibrium would be the infinite repetition of the stage game Nash equilibrium. We state the result in the following proposition.

Proposition 1 (No Trade) *The only equilibrium in this game is one in which there is no trade: Providers are never hired on the equilibrium path and providers shirk and consumers do not send messages off the equilibrium path.*

In summary, the market collapses in the absence of an information sharing bureau, regardless of how small the cost of providing information is. In this sense, the introduction of an information sharing bureau is likely to improve social welfare. The question becomes: what bureau design is socially optimal and could a profit-seeking bureau improve welfare? We focus on two forms that the bureau might take. First, we consider a bureau that posts a price to buy information from consumers that have experienced the product and sells verification information to whoever wants to buy it. Second, we consider a bureau that operates with a membership system: members must pay a membership fee, and then they can access information at no additional costs. Moreover, members are compensated for sending information

to their bureaus. To carry on our analysis, we assume that the bureau can keep track of individual providers and their past behaviors. Formally, this assumption means that a bureau can distinguish between two distributions even if they differ by measure zero.⁴

3 Information Sharing Bureau: Non-profit Altruistic Bureau

We now extend our basic model by introducing an information sharing bureau. Throughout the paper we focus on equilibria that are stationary in the sense that a given agent i always takes the same action when she is indifferent.⁵ Further, we focus on stationary equilibria in which uninformed players choose to hire on the equilibrium path. Before we define a stationary equilibrium, it is convenient to define the set $A(h)$ to be the set of actions available to player i after history h .

Definition 1 (Stationary Equilibrium) *A behavior strategy σ_i is a stationary strategy if for any two histories h^t and h^τ (of equal or different lengths) with $A(h^t) = A(h^\tau)$, player i chooses the same action, that is, $\sigma_i(h^t) = \sigma_i(h^\tau)$. The strategy profile σ is a stationary equilibrium if σ is a stationary strategy profile and a perfect Bayesian equilibrium.*

In these equilibria, stationary strategies are best responses when all other agents are playing stationary strategies. In this class of equilibria, there is always a trivial equilibrium in which every provider defaults if hired, but none are hired on the equilibrium path. We focus on the non-trivial equilibria in stationary strategies.

We approach the problem of establishing an information sharing bureau from the view of a policymaker that is trying to maximize the economy's social welfare. We focus on a social welfare function that equally weights consumers and providers. In order to do that, the policymaker can design the features of the market for information sharing bureaus, granting permits to potential competitors, fostering competition, creating a government-run non-profit bureau, etc. However, the policymaker cannot force consumers to participate in the information exchange, i.e., bureau participation must be voluntary. Moreover, in the case of a government-run non-profit bureau, we also impose the condition that the bureau must be self-sustaining apart from an initial endowment. This restriction means that the bureau must raise enough money to generate at least zero profits every period. We consider two pricing mechanisms: buy and sell information and membership. We call the government-run bureau an altruistic bureau, since its goal is to maximize social welfare.

⁴Technically, we generally assume that two random variables are equal if they are equal almost everywhere (a.e.), using some notion of measure zero. Here we assume that two random variables are equal only if they are equal everywhere. A similar assumption is present in Kocherlakota [1998].

⁵Note that this restriction rules out certain equilibria that are not robust to small shocks, such as belief-free equilibria.

3.1 Buying and Selling Information

We assume that the altruistic agency pays $f_1 \geq c$ for reported information and asks a fee of f_2 to disclose information about any given provider. If the agency doesn't have the asked for information, the consumer pays nothing. Without loss of generality, we impose the condition that if a consumer is indifferent between selling information or not, she sells it. Fees charged by the bureau are known by all agents in the economy and the bureau can credibly commit to posted fees. In this sense, a functioning bureau must set $f_1 \geq c$, implying that all consumers that hire the providers' services sell information to the bureau in equilibrium. As presented in Figure 2, the extended game tree has an additional decision node at the beginning of the tree, in which the consumer decides whether or not she will purchase information from the bureau. After this node, all of the remaining tree is identical to the one presented in Figure 1, apart from the payoffs in the terminal nodes, where we must include the paid and received fees. Therefore, if the consumer decides to purchase information from the bureau, we subtract f_2 from her final payoff. Similarly, if the consumer decides to sell information to the bureau, we must add the received fee f_1 to her payoff. No changes are needed for the provider's payoffs or decisions.⁶

Providers' Problem

Assume that a fraction $X_{A, buy}$ of consumers buy the information from the bureau once it is established. Informed consumers only buy services from providers with no history of default. Let's also assume that uninformed consumers hire any provider with whom they match, free-riding on the discipline imposed by informed consumers. We must show that in equilibrium it is optimal for uninformed consumers to hire the service.

Let's consider the decision problem of a provider that has never defaulted before. His only possible stage game action is $\eta = \{effort, no-effort\}$. As previously mentioned, consumers that buy information never hire the service of providers that have previously defaulted and all customers sell information. Consequently, providers know that after putting in no effort once, no informed consumer will hire their services henceforth. As a result, we can focus on the once for all decision of effort or no effort. A provider prefers putting in no effort if:

$$(1 - \delta)w + \delta[(1 - X_{A, buy}) \times w + X_{A, buy} \times 0] > w - e \quad (1)$$

where the left-hand side (henceforth, LHS) of equation (1) is the payoff to always putting in no effort, while the expression on the right-hand side (henceforth, RHS) is the payoff to always delivering

⁶The game presented in Figure 2 has some abuse of notation, considering that we assumed that the bureau would not charge the consumer if there was no information about the match provided. As we will see, the bureau's information is complete in the case of buying and selling, so the abuse of notation is without loss of generality.

high-quality service. Simplifying the expression in equation (1), we have:

$$\delta < \frac{e}{wX_{A,buy}} \quad (2)$$

If the fraction of informed consumers $X_{A,buy}$ is high enough, a provider always puts in effort. However, this would kill the incentive to buy information in the first place. Therefore, given that providers are *ex ante* identical, there is no equilibrium in which all providers follow the same pure strategy and a positive fraction of consumers buy information. Consequently, if in equilibrium a fraction of providers put in no effort, while the remainder deliver high-quality services, we must have that all providers are indifferent between putting in effort or not. Therefore, from equation (2), the measure of informed consumers in equilibrium is:

$$X_{A,buy} = \frac{e}{\delta w} \quad (3)$$

Notice from equation (3) that the more costly the effort, the higher the measure of informed consumers must be in order to keep providers indifferent between delivering high-quality service or not. In contrast, the more costly it is to lose business – the higher the w – and the more patient providers are – the higher the δ – the smaller is the needed fraction of informed consumers.

Consumers' Problem

We now consider the consumer's decision. Keep in mind that in the stage game, the consumer has three decision nodes. First, she decides whether or not she will buy information from the bureau, paying a fee f_2 . Then, based on the information in hand, the consumer must decide whether or not to hire the provider. Finally, if the consumer hires the provider, she must decide whether or not to sell the information to the bureau.

We focus on equilibria in which a bureau is sustained in equilibrium. Consequently, in the equilibria we look at, consumers sell information and uninformed consumers hire providers on the equilibrium path. Let's start with the decision to sell information. As we mentioned before, we assume that all consumers sell information if they are indifferent between selling information or not. Consequently, a consumer sells information as long as $f_1 \geq c$. Let's then consider the decision of an uninformed consumer in purchasing the provider's services. Let $Y_{A,buy}$ denote the fraction of providers that make no effort in equilibrium once the bureau is installed. A consumer that buys no information still prefers hiring a provider if:

$$Y_{A,buy} \leq \frac{P - w + f_1 - c}{P} \quad (4)$$

Consequently, as long as the fraction of providers that put in no effort is below the threshold presented in equation (4), uninformed consumers hire the matched providers. If this restriction is not satisfied, the market unravels. First, because even after the announcement that a bureau will be installed, no agent would buy services in the period prior to the bureau's establishment. Consequently, no information is

aggregated by the bureau. Second, if in equilibrium uninformed consumers decide not to hire the service and informed ones only purchase services from providers that always put in effort, there is no incentive for providers to put in no effort. Unfortunately, this pattern also eliminates the consumers' incentive to buy information in the first place.

Then, assuming that equation (4) and $f_1 \geq c$ are satisfied, we move toward the decision of whether or not to buy information. Since there is no punishment for not buying information in a given period, we just need to compare the consumer's payoff to buying and selling information with the payoff to just selling it. Since the consumer's problem presents a recursive environment, we don't need to evaluate strategies in which the agent presents a mixture of both. So, the payoff to buying and selling information is given by:

$$[-f_2 Y_{A,buy} + (P - w + f_1 - f_2 - c)(1 - Y_{A,buy})] \quad (5)$$

while the payoff to just selling information is given by:

$$(-w + f_1 - c) Y_{A,buy} + (P - w + f_1 - c)(1 - Y_{A,buy}) \quad (6)$$

Therefore, the consumer is indifferent between buying information or not if $Y_{A,buy} = \frac{f_2}{w - f_1 + c}$.

Altruistic Bureau's Problem

The bureau's objective is to maximize the social welfare of consumers and providers. In particular, we consider an egalitarian social welfare function that weights equally consumers and providers. The two populations are equally weighted and normalized to 1 (i.e., $I_1 = I_2 \equiv 1$). Consequently, the social welfare in period t is given by:

$$SW_t = \frac{1}{2} \{U_{consumer}(t) + U_{providers}(t)\} \quad (7)$$

Once we are focusing on the set of equilibria that has a functioning bureau, we consider the equilibria in which providers are indifferent between putting in effort or not and consumers are indifferent between buying information or not. Consequently, we have that:

$$U_{consumer}(t) = (1 - Y_{A,buy})P - w + f_1 - c \quad \text{and} \quad U_{provider}(t) = w - e, \quad \forall t > 0 \quad (8)$$

where we set $U_{consumer}$ equal to the uninformed consumer's utility, while $U_{provider}$ is set to equal the utility earned by a provider that puts in effort in equilibrium. Substituting back into equation (7) and rearranging:

$$SW_t = \frac{1}{2} \{(1 - Y_{A,buy})P + f_1 - c - e\} \quad (9)$$

Consequently, the bureau's problem is given by:

$$\mathbf{SW}_{A,\text{buy}} \equiv \max_{f_1, f_2} \frac{1}{2} \sum_{t=0}^{\infty} \delta^t SW_t \quad (10)$$

subject to:

$$\frac{f_2 \delta X_{A,\text{buy}} - f_1 [1 - \delta X_{A,\text{buy}} Y_{A,\text{buy}}]}{1 - \delta} \geq 0 \quad (\text{C.1})$$

$$Y_{A,\text{buy}} \leq \frac{P - w + f_1 - c}{P} \quad (\text{C.2})$$

$$X_{A,\text{buy}} \in [0, 1] \quad (\text{C.3})$$

$$Y_{A,\text{buy}} \in [0, 1] \quad (\text{C.4})$$

$$X_{A,\text{buy}} = \frac{e}{\delta w} \quad (\text{C.5})$$

$$Y_{A,\text{buy}} = \frac{f_2}{w - f_1 + c} \quad (\text{C.6})$$

$$f_1 \geq c \quad (\text{C.7})$$

where (C.1) is the break-even condition, implying that the bureau must be self-funded once established. Since all informed consumers matched to providers that put in no effort do not hire the services, the bureau buys information from a fraction $1 - X_{A,\text{buy}} Y_{A,\text{buy}}$ of informed consumers each period. Restriction (C.2) says that uninformed consumers must still buy information in equilibrium, as presented in equation (4). Then, simplifying the bureau's problem we have:

$$\max_{f_1, f_2} \frac{1}{2(1 - \delta)} \left\{ \left[1 - \frac{f_2}{w + c - f_1} \right] P - f_1 + c - e \right\}$$

subject to:

$$\frac{\frac{e}{w} f_2 - f_1 \left[1 - \frac{e f_2}{w(w + c - f_1)} \right]}{1 - \delta} \geq 0 \quad (\text{C.1}')$$

$$f_1 \geq c \quad (\text{C.2}')$$

$$0 \leq \frac{f_2}{w + c - f_1} \leq \min \left\{ \frac{P + f_1 - (w + c)}{P}, 1 \right\} \quad (\text{C.3}')$$

We can now show the following result:

Lemma 1 *In an economy in which it is optimal to establish information trade, the bureau's optimal paying fee is c , i.e., $f_1 = c$*

Based on the proof of Lemma 1, we observe that in the case in which we have an operating market, at the optimum we must have (C.2') and (C.1') satisfied with equality. Therefore, we have that:

$$f_1 = c \quad \text{and} \quad f_2 = \frac{w^2 c}{e(w + c)} \quad (11)$$

while the social welfare is given by:

$$\mathbf{SW}_{A,\text{buy}} = \frac{1}{2(1 - \delta)} \left\{ \left[1 - \frac{w c}{e(w + c)} \right] P - e \right\} \quad (12)$$

Notice that we must still satisfy restriction (C.3'). Consequently, we must have:

$$\frac{wc}{e(w+c)} \leq \frac{P-w}{P} \quad (13)$$

If this restriction is not satisfied, we have that (C.3') is binding and $f_2 = \frac{P-w}{P}w$. However, based on the proof of Lemma 1, we can show the following lemma:

Lemma 2 *In the cases where an operating market is possible, the inequality (13) is trivially satisfied. In other words, whenever there is an active bureau and a functioning market, (C.3') is non-binding.*

In order to simplify the presentation, Corollary 1 collects the parameter restrictions obtained in the proof of Lemma 1 that allow us to obtain a functioning provider's market with an active information bureau.

Corollary 1 *All restrictions (C.1) – (C.7) of the bureau's problem presented in equation (10) can be jointly satisfied if:*

$$c \leq \frac{ew(P-w)}{P(w-e)+ew} \quad (14)$$

otherwise, a buy-and-sell bureau cannot be installed and the market collapses.

Consequently, from Lemma 2 and Corollary 1 we conclude that, for the relevant set of parameters – i.e., the ones in which it is optimal for the policymaker to establish an information bureau – uninformed consumers prefer hiring the provider. Moreover, unless (14) is satisfied with equality, consumers strictly prefer hiring the services and are better off if the market for providers exists.

3.2 Membership

Suppose that the bureau requires that consumers pay a membership fee before they can either access or sell information. The bureau commits to a sequence of fees, which we denote by $\{f_{ee}^t\}_{t=1}^{\infty}$, where by f_{ee}^{τ} we mean the fee that is required to join the bureau at time τ . We assume that the membership fee is characterized as a per period fee in order to avoid additional present value costs for up-front fees. However, the results presented in the paper are robust to an up-front membership fee unless otherwise mentioned. Bureau members always receive compensation for giving information, just enough to cover their costs of sending the information. *Only members can receive or report information.* Assume that there are two technological constraints in the environment: (1) it is impossible for the bureau to credibly reveal to the service provider who is a member and who is not.⁷ Indeed, think of this as a rating system where the service provider cannot really tell where the customer got her information from; and (2) it is not possible to make information public market-wide.

⁷In Section 5 we discuss an extension that considers the case of observable membership in detail.

Let $Y_{A,member}$ be the fraction of providers who default in equilibrium and $X_{A,member}$ be the fraction of consumers who buy a membership. If a consumer is a member, her period payoff depends on the fraction of providers who put in effort ($1 - Y_{A,member}$), the fraction of providers who default every period and have served a bureau's member at least once, and the fraction of providers who default, but have never previously served a bureau's member. The latter measure is the source of an indirect cost for informed consumers in the membership case. In contrast to the buy and sell case, bureau members have just partial information, facing the possibility of default even after acquiring information. As a counterpart, this reduction in information quality induces a lower membership fee. Moreover, the likelihood of members facing default decreases over time. Since providers that default eventually meet a bureau's member, the information becomes available to all other members, reducing the likelihood of a member facing default in the future.

Consumers We initially focus on a stationary equilibrium in which all consumers that join the membership do so in period 1; we show that this is without loss of generality later in this section. Thus, in the first period, there is a $1 - Y_{A,member}$ chance that the consumer faces a provider who does not default and a $Y_{A,member}$ chance that the matched provider defaults. Given that this is the first period, no consumer knows which provider she's facing. In the second period, assuming that a fraction $X_{A,member}$ has bought the membership, there is a $1 - Y_{A,member}$ chance of facing a provider that does not default; a $X_{A,member}Y_{A,member}$ chance of facing a provider who is known to default – therefore, the bureau's member will not hire his services; and a $(1 - X_{A,member})Y_{A,member}$ chance of facing a provider who defaults, but was not caught in the previous period. Summing up for all periods, this means that the payoff of the consumer joining the bureau in period 1 is:

$$\begin{aligned} & (1 - Y_{A,member})(P - w) - f_{ee}^1 + (1 - \delta) \sum_{t=0}^{\infty} \delta^t (1 - X_{A,member})^t Y_{A,member} (-w) \\ & = (1 - Y_{A,member})(P - w) - f_{ee}^1 - (1 - \delta) \frac{Y_{A,member}w}{1 - \delta(1 - X_{A,member})} \end{aligned} \quad (15)$$

If a consumer does not buy a membership, her payoff when hiring is:

$$(1 - Y_{A,member})(P - w) + Y_{A,member}(-w) = (1 - Y_{A,member})P - w. \quad (16)$$

Therefore, if the consumer is indifferent between joining the bureau or not in period 1, equations (15) and (16) give us the following condition:

$$Y_{A,member}w \frac{\delta X_{A,member}}{1 - \delta(1 - X_{A,member})} = f_{ee}^1. \quad (17)$$

Note that the membership becomes more attractive over time (there is more information about providers), so the flat per period fee charged to members that joined the bureau at different times must increase over time. Otherwise, consumers would wait and purchase the membership at a later date. We show in

the lemma below that there is a sequence of fees such that the discounted change in fees is just enough to absorb all of the delay's potential benefits or losses. Since the consumer is indifferent about when to enroll in the bureau, we focus on enrollments in period 1.

Lemma 3 *There exists a sequence of strictly increasing fees $\{f_{ee}^t\}_{t=1}^\infty$, such that if the bureau announces (and credibly commits to) such a sequence, consumers are indifferent between enrolling in the bureau in period τ instead of $\tau + 1$.*

We now restrict the parameters to the case in which non-members as well as members purchase the service in the period before the membership kicks in. Notice that the expected payoff to hiring a service with no record on the provider is $(1 - Y_{A,member})P - w$. Thus, in the equilibrium that we are considering, $(1 - Y_{A,member})P - w \geq 0$, i.e.:

$$Y_{A,member} \leq 1 - \frac{w}{P}. \quad (18)$$

Providers Each provider chooses whether or not to default. If he chooses not to default, he gets a payoff of $w - e$ for every period (recall that in the equilibrium we focus on, non-members also hire every period). If he decides to default in any period t , he may default against either a member or a non-member. If he defaults against a member, he will never be hired by members again. Consequently, the payoff to defaulting can be obtained using the following recursive equation:

$$U_{Default} = w + \delta \left\{ X_{A,member} \frac{(1 - X_{A,member})w}{1 - \delta} + (1 - X_{A,member})U_{Default} \right\} \quad (19)$$

therefore, the provider is indifferent between defaulting and not defaulting if:

$$\frac{(1 - \delta)w}{1 - \delta(1 - X_{A,member})} + \delta \frac{X_{A,member}(1 - X_{A,member})w}{(1 - \delta(1 - X_{A,member}))} = w - e. \quad (20)$$

Simplifying it, we get:

$$\frac{\delta X_{A,member}^2 w}{1 - \delta(1 - X_{A,member})} = e \quad (21)$$

Solving the equation for $X_{A,member}$ and keeping in mind that $X_{A,member} \in [0, 1]$, we have

$$X_{A,member} = \frac{e\delta + \sqrt{e^2\delta^2 + 4\delta(1 - \delta)we}}{2\delta w}. \quad (22)$$

Note that given the parameters e, δ, w , there is only one value of $X_{A,member}$ that is consistent with a stationary equilibrium. Moreover, in a stationary equilibrium, conditions (17) and (18) must also hold. Combining both conditions gives us:

$$f_{ee}^1 \leq \left(1 - \frac{w}{P}\right) \frac{w\delta X_{A,member}}{1 - \delta(1 - X_{A,member})}, \quad (23)$$

with equality when $Y_{A,member} = (1 - \frac{w}{P})$.

Altruistic Bureau's Problem

As in Section 3.1, the altruistic bureau's problem is to maximize an egalitarian social welfare function that equally weights consumers' and providers' utilities, conditional on some restrictions that include a break-even condition, i.e., that the bureau must be self-funded. Consequently, let's start by looking at the bureau's profit function:

$$\Pi_{A,member} = X_{A,member} \left\{ \frac{f_{ee}^1}{1-\delta} - c - \frac{\delta(1-Y_{A,member})c}{1-\delta} - Y_{A,member}c \sum_{t=1}^{\infty} \delta^t (1 - X_{A,member})^t \right\} \quad (24)$$

from equation (17), we have that, after a few simplifications:

$$\Pi_{A,member} = \frac{X_{A,member}}{1-\delta} \left\{ f_{ee}^1 \frac{c+w}{w} - c \right\} \quad (25)$$

where $X_{A,member}$ in equilibrium does not depend on f_{ee}^1 , so in order to keep the expression simple, we are not going to substitute it here.

Then, moving to the social welfare function, we have that the per period social welfare function is given by

$$SW_{A,member}(t) = \left\{ \frac{1}{2} \left[\begin{array}{l} (1 - Y_{A,member})(P - w) \\ + Y_{A,member}(-w) \\ + \frac{1}{2}(w - e) \end{array} \right] \right\} = \frac{1}{2} [(1 - Y_{A,member})P - e] \quad (26)$$

Consequently, the altruistic bureau's problem in the case of membership is given by:

$$\mathbf{SW}_{A,member} \equiv \max_{f_{ee}^1} \frac{1}{2(1-\delta)} [(1 - Y_{A,member})P - e] \quad (27)$$

subject to:

$$\frac{X_{A,member}}{1-\delta} \left\{ f_{ee}^1 \frac{c+w}{w} - c \right\} \geq 0 \quad (C.1)$$

$$0 \leq Y_{A,member} \leq \frac{P-w}{P} \quad (C.2)$$

$$Y_{A,member} = \frac{[1-\delta(1-X_{A,member})]}{\delta X_{A,member} w} f_{ee}^1 \quad (C.3)$$

where, recall, (C.1) is the break-even constraint and the second constraint is obtained by a combination of $Y_{A,member} \in [0, 1]$ and equation (18). Restriction (C.3) is given by equation (17). Substituting (C.3) into (C.2) and the objective function, we can see that the objective function is linearly decreasing in f_{ee}^1 . Therefore, at the optimum (C.1) must be binding:

$$f_{ee}^1 = \frac{cw}{w+c} \quad (28)$$

We are now able to show the following proposition:

Proposition 2 (Membership Is Better Than Buy and Sell) *Assume that the restriction presented in Corollary 1 is satisfied with inequality. The policymaker would prefer a pricing mechanism based on membership instead of buy and sell. Moreover, the fraction of providers offering high-quality services and the fraction of informed consumers are both higher with membership.*

Moreover, from the proof of Proposition 2, jointly with Lemma 2 and Corollary 1, we can easily show the following corollary:

Corollary 2 *In equilibrium, the constraint that guarantees that uninformed consumers buy the providers' services is non-binding in a membership set-up.*

Consequently, the policymaker strictly prefers membership. Notice, however, that there is a clear trade-off between membership and buy-and-sell schemes. By creating a bureau based on membership, the policymaker avoids spending too much money by purchasing the information of non-members. While the bureau's information set is not as fine as in the case of buy and sell information, the fact that it is cheaper implies that in equilibrium more consumers will become informed and the fraction of times that providers that default will not be hired actually goes up. However, since, with membership, not all information is aggregated by the bureau, even members face default in equilibrium. However, given that we have only one large bureau, the information propagates relatively fast, as we can observe in Figure 3.

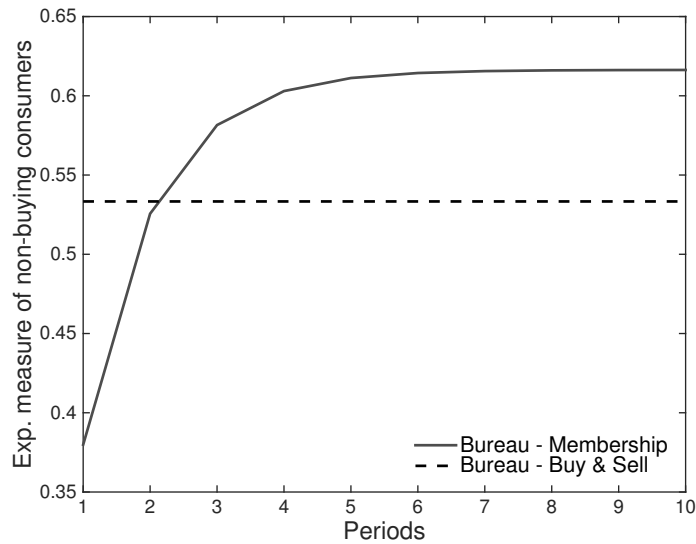


Figure 3: Speed of learning with membership

4 Profit-Oriented Bureau: Monopoly Case

The consumers' and providers' problems only depend on the pricing mechanism and actual fees charged by the bureau (and how these features affect the number of informed consumers and providers that

put in effort in equilibrium). Therefore, nothing that was presented in Sections 3.1 and 3.2 about the consumers' and providers' problems need to be changed. Consequently, we just need to focus on the bureau's profit maximization and social welfare issues.

4.1 Buying and Selling Information

4.1.1 Bureau's Profit Maximization

The bureau's profit maximization problem is given by:

$$\Pi_{M, buy} = -f_1 + \sum_{t=1}^{\infty} \delta^t \{X_{M, buy} [f_2 - (1 - Y_{M, buy})f_1] + (1 - X_{M, buy})(-f_1)\} \quad (29)$$

subject to:

$$0 \leq \frac{f_2}{w+c-f_1} \leq \min \left\{ \frac{P+f_1-(w+c)}{P}, 1 \right\} \quad (C.1)$$

$$f_1 \geq c \quad (C.2)$$

$$X_{M, buy} = \frac{e}{\delta w} \quad (C.3)$$

The bureau's profit maximization takes into account that it must buy information from all consumers in period 0. From period 1 on, it must buy information only from consumers that hire the service, corresponding to all uninformed consumers as well as all informed consumers matched with a provider that has not yet defaulted, i.e. $X_{M, buy} \times (1 - Y_{M, buy})$. The restrictions (C.1)–(C.3) are just a simplified version of the restrictions (C.2)–(C.7) presented at the altruistic bureau's problem, i.e., the same restrictions apart from the break-even condition. Rearranging the profit function, we have:

$$\Pi_{M, buy} = \frac{\delta X_{M, buy} f_2 - f_1 [1 - \delta X_{M, buy} Y_{M, buy}]}{1 - \delta} \quad (30)$$

We can then show the following lemma:

Lemma 4 *Assume that the parameter constraint defined in Corollary 1 holds. A monopolist buy and sell bureau would optimally choose $f_1 = c$.*

Therefore, once we substitute f_1 , $X_{M, buy}$, and $Y_{M, buy}$, the bureau's problem becomes:

$$\max_{f_2} \frac{\frac{e}{w} [f_2 + (1 - \frac{w}{P}) c] - c}{1 - \delta} \quad (29')$$

subject to:

$$f_2 \leq \frac{(P-w)w}{P} \quad (C.1')$$

Given the linearity of the profit function, the constraint should be binding, i.e. $f_2 = \frac{(P-w)w}{P}$. Conse-

quently, in equilibrium, we have:

$$Y_{M,buy} = 1 - \frac{w}{P}, \quad X_{M,buy} = \frac{e}{\delta w}, \quad f_1 = c, \quad f_2 = \frac{(P-w)w}{P}. \quad (31)$$

While the bureau's profit is:

$$\Pi_{M,buy} = \frac{1}{1-\delta} \left\{ \left[\frac{(w+c)(P-w)}{Pw} \right] e - c \right\} \quad (32)$$

Social Welfare

Let's now consider the social welfare function. Apart from the measure of providers that default in equilibrium, $Y_{M,buy}$, the social welfare function in the monopoly case with buy and sell is the same as the one presented for the altruistic case in equations (7) to (9). Consequently, we have that

$$\mathbf{SW}_{M,buy} = \frac{1}{2(1-\delta)} \{ (1 - Y_{M,buy})P + f_1 - c - e \} \quad (33)$$

Substituting f_1 and $Y_{M,buy}$, using equation (31), we have:

$$\mathbf{SW}_{M,buy} = \frac{1}{2(1-\delta)} (w - e) \quad (34)$$

4.2 Membership

Bureau's Profit Maximization

The bureau's profit maximization problem in the case of membership is given by:⁸

$$\Pi_{M,member} \equiv \max_{f_{ee}^M} X_{M,member} \left[\frac{f_{ee}^M}{1-\delta} - c - \sum_{t=1}^{\infty} \delta^t \{ (1 - Y_{M,member}) + Y_{M,member} (1 - X_{M,member})^t \} c \right] \quad (35)$$

subject to:

$$f_{ee}^M \leq \left(1 - \frac{w}{P}\right) \frac{w\delta X_{M,member}}{1-\delta(1-X_{M,member})} \quad (C.1)$$

The profit function takes into account the fact that, while all members must pay the membership fee, only the ones that hire the provider's service must be compensated for sending information to the bureau. Consequently, the only members that send information to the bureau at time t are the ones matched to providers with no registered history of default. There are two types of providers with a clean history at time t : Providers that always offer good services ($1 - Y_{M,member}$) and providers that provide bad service but have not matched with bureau members before, i.e., $Y_{M,member}(1 - X_{M,member})^t$. Moreover, the constraint is just a combined version of constraints (C.2) and (C.3) for altruistic bureau's problem

⁸Recall that the sequence of fees is such that all membership decisions occur at period 1 here.

presented in equation (27). Simplifying and substituting equation (17), the bureau's problem becomes:

$$\Pi_{M,member} \equiv \max_{f_{ee}^M} X_{M,member} \left\{ \left(\frac{w+c}{w} \right) f_{ee}^M - \frac{c}{1-\delta} \right\} \quad (36')$$

subject to:

$$f_{ee}^M \leq \left(1 - \frac{w}{P}\right) \frac{w\delta X_{M,member}}{1-\delta(1-X_{M,member})} \quad (C.1)$$

Since $X_{M,member}$ does not depend on f_{ee}^M , we can see that $\Pi_{M,member}$ is linearly increasing in f_{ee}^M . Consequently, the restriction is binding and we have that:

$$f_{ee}^M = \left(1 - \frac{w}{P}\right) \frac{w\delta X_{M,member}}{1-\delta(1-X_{M,member})} \quad (36)$$

Substituting equation (21) and manipulating it, we obtain:

$$f_{ee}^M = \left(1 - \frac{w}{P}\right) \frac{e}{X_{M,member}} \quad (37)$$

and the profit of the monopolistic bureau that provides membership is then given by:

$$\Pi_{M,member} = \frac{1}{1-\delta} \left\{ \left[\frac{(w+c)(P-w)}{Pw} \right] e - cX_{M,member} \right\} \quad (38)$$

where $X_{M,member} = \frac{e\delta + \sqrt{e^2\delta^2 + 4\delta(1-\delta)we}}{2\delta w}$.

Remark: We assume that $X_{M,member} \in (0, 1)$ which is guaranteed by the assumption that $\delta > \frac{2e}{e+2w}$.

Comparing this expression with the profit of a monopolistic bureau buying and selling information, we can easily show the following result:

Proposition 3 *A monopolistic bureau prefers offering membership to buying and selling information.*

Social Welfare

Let's now consider the social welfare function. Apart from the measure of providers that default in equilibrium, the social welfare function in the monopoly case with membership is the same as the one presented in equation (26) for the altruistic case. Consequently, we have that:

$$\mathbf{SW}_{M,member} = \frac{1}{2(1-\delta)} [(1 - Y_{M,member})P - e] = \frac{1}{2(1-\delta)} (w - e) \quad (39)$$

In this case, since $Y_{M,buy} = Y_{M,member}$, we have that $\mathbf{SW}_{M,buy} = \mathbf{SW}_{M,member}$. Moreover, both components, i.e. consumers' and providers' surpluses are identical in both cases, even though the number of bureau members in the membership equilibrium is different from the number of informed consumers in the buy-sell information equilibrium. The reason for that it is that members have partial information

while information buyers have full information. Consequently, in equilibrium, members may still face default, while information buyers will never face default.

5 Competitive Profit-Maximizing Bureaus

5.1 Buy and Sell

Let's consider that there are two competing bureaus. Define f_1^i as the fee paid by bureau i in order to acquire consumers' information. Similarly, f_2^i is the fee charged by bureau i in order to sell information. Both fees are endogenously pinned down in equilibrium. We assume that bureaus cannot deny buying information from any given consumer or that consumers can punish bureaus by only buying information from bureaus that also acquire information.⁹

We assume that information is non-rival, i.e., consumers can sell the information to multiple bureaus at the same time. Moreover, as in the benchmark case, bureaus can verify that services were purchased and consequently only customers that bought the service are allowed to sell information.

We focus our analysis on the case in which all consumers sell information to all bureaus but only buy from one bureau. We assume that consumers buying information equally randomize across all bureaus. Hence, bureaus equally share the demand for information. Since each bureau buys information from all consumers, we have that the information is the same, irrespective of the bureau a consumer buys information from. Therefore, the restriction that consumers only buy information from at most one bureau is without loss of generality. Moreover, notice that the consumers' and providers' problems are still exactly the same, apart from the fact that consumers buy information from the cheapest bureau. Therefore, we have:

$$X_{C,buy} = \frac{e}{\delta w} \quad \text{and} \quad Y_{C,buy} = \frac{\min\{f_2^i, f_2^{-i}\}}{w + c - \max\{f_1^i, f_1^{-i}\}} \quad (40)$$

Consequently, the main change is in the bureau's profit maximization. Then, we have that bureau i 's demand for service can be given by Bertrand competition, i.e.:

$$D_i(f_2^i, f_2^{-i}; f_1^i, f_1^{-i}) = \begin{cases} X_{C,buy} & \text{if } f_2^i < f_2^{-i} \text{ and } f_1^i \geq c, f_1^{-i} \geq c \\ X_{C,buy} & \text{if } f_1^{-i} < c \text{ and } f_1^i \geq c \\ \frac{X_{C,buy}}{2} & \text{if } f_2^i = f_2^{-i} \text{ and } f_1^i \geq c, f_1^{-i} \geq c \\ 0 & \text{if } f_2^i > f_2^{-i} \text{ and } f_1^i \geq c, f_1^{-i} \geq c \\ 0 & \text{if } f_1^i < c \end{cases} \quad (41)$$

Therefore, D_i measures the fraction of consumers that decide to buy information in equilibrium that buy the information from bureau i . Moreover, notice that D_i gives us additional information. It

⁹We consider that consumers sell information before buying it, so they can observe deviations and punish appropriately.

shows that consumers would not buy information from a bureau that does not pay enough to acquire information, i.e., $D_i(f_2^i, f_2^{-i}; f_1^i, f_1^{-i}) = 0$ if $f_1^i < c$. We consider the simplest sharing rule: if both firms ask the same fee f_2 , they evenly split the market. Hoernig [2007] presents alternative sharing rules. Given the fact that the consumers' problem is still the same, we must have that $X_{C,buy} = \frac{e}{\delta w}$. Then, bureau i 's profit maximization problem is:

$$\Pi_{C,buy} = \max_{f_1^i, f_2^i} \frac{\delta D_i(f_2^i, f_2^{-i}; f_1^i, f_1^{-i}) f_2^i - f_1^i (1 - \delta X_{C,buy} Y_{C,buy})}{1 - \delta} \quad (42)$$

subject to:

$$0 \leq \frac{f_2^i}{w+c-f_1^i} \leq \min \left\{ \frac{P+f_1^i-(w+c)}{P}, 1 \right\} \quad (C.1)$$

$$f_1^i \geq c \quad (C.2)$$

where constraints (C.1) and (C.2) are just a simplified version of the restrictions (C.2)–(C.7) presented in the altruistic bureau's problem, i.e., the problem presents the same restrictions apart from the break-even condition. Notice that if (C.2) is not satisfied for bureau i , we trivially have $\Pi_{buy}^i = 0$, regardless of the other fees. Therefore, the easiest way for a bureau to shut down is to establish a buying fee below cost c . In this case, its competitor becomes a monopoly. As a result, we can focus on the cases in which $f_1^{-i} \geq c$ and $f_1^i \geq c$. However, notice that, as long as $f_1^i \geq c$ is satisfied, f_1^i only affects the cost of acquiring information. Therefore, following a similar argument presented in the proof of Lemma 1, we can show that it is optimal for the bureaus to set $f_1 = c$. As a result, bureau i 's reaction correspondence is given by:

$$R^i(f_1^{-i}, f_2^{-i}) = \begin{cases} f_2^i = \frac{(P-w)w}{P} \text{ and } f_1^i = c & \text{if } f_1^{-i} < c; \\ f_2^i = \frac{(P-w)w}{P} \text{ and } f_1^i = c & \text{if } f_1^{-i} \geq c \text{ and } f_2^{-i} > \frac{(P-w)w}{P}; \\ f_2^i = f_2^{-i} - \varepsilon \text{ and } f_1^i = c & \text{if } f_1^{-i} \geq c \text{ and } \frac{2cw^2}{e(w+2c)} < f_2^{-i} \leq \frac{(P-w)w}{P} \\ f_2^i = \frac{2cw^2}{e(w+2c)} \text{ and } f_1^i = c & \text{if } f_1^{-i} \geq c \text{ and } f_2^{-i} = \frac{2cw^2}{e(w+2c)} \\ f_2^i \in \mathbb{R}_+ \text{ and } f_1^i < c & \text{if } f_1^{-i} \geq c \text{ and } f_2^{-i} < \frac{2cw^2}{e(w+2c)} \end{cases} \quad (43)$$

where ε is arbitrarily small.

Consequently, in an equilibrium in which we have two active bureaus, we have:

$$f_2^1 = f_2^2 = \frac{2cw^2}{e(w+2c)} \text{ and } f_1^1 = f_1^2 = c \quad (44)$$

with both bureaus earning zero profits. The equilibrium share of informed consumers is $X_{C,buy} = \frac{e}{\delta w}$, while the share of defaulting providers is given by $Y_{C,buy} = \frac{2wc}{e(w+2c)}$. Finally, in order for (C.1) in equation (42) not to bind, we must have $\frac{2cw}{e(w+2c)} < \frac{P-w}{P}$.

Social Welfare

Let's now consider the social welfare function. Apart from the measure of providers that default in equilibrium $Y_{C,buy}$, the social welfare function in the competitive case with buy and sell is the same as the one presented for the altruistic case in equations (7) to (9). Consequently, given $f_1^1 = f_1^2 = c$ and substituting $Y_{C,buy}$, we have:

$$\mathbf{SW}_{C,buy} = \frac{1}{2(1-\delta)} \left\{ \left[1 - \frac{2wc}{e(w+2c)} \right] P - e \right\} \quad (45)$$

Then, we are ready to show the following proposition:

Proposition 4 *Assume that the parameter restriction presented in Corollary 1 is satisfied with inequality. Then, we have:*

$$\mathbf{SW}_{A,member} > \mathbf{SW}_{A,buy} > \mathbf{SW}_{C,buy} > \mathbf{SW}_{M,buy} = \mathbf{SW}_{M,member}$$

Therefore, we can rank the different bureau designs and pricing mechanisms, with an altruistic bureau with membership at the top.

5.2 Membership

In this section we develop a model of competition between bureaus with a membership pricing mechanism. Specifically, we consider two bureaus A and B , where bureau i charges f_i for the membership. In the stationary equilibria we consider, each bureau has a consumer base X_i ($X_A + X_B \leq 1$) and there is a fraction $Y_{C,member}$ of providers that choose to default every period. The timing of this game is the following: first, the bureaus post their membership fees simultaneously, then the consumers and providers play an infinitely repeated game with private histories given the fees that were posted.¹⁰ We look for the subgame perfect equilibria of this repeated game.

Providers Let us now have a closer look at the providers' incentives. We introduce the following notation: U^0 is the expected continuation utility of a provider that decides to default and: 1. either has never defaulted before or 2. has defaulted but has never been reported to an information bureau; U^i is a provider that has been reported by at least one member of bureau i , but has not interacted with members of bureau $j \neq i$, and U^{AB} is the expected continuation payoff of a provider that has been reported by at least one member of A and by at least one member of B . These expected continuation

¹⁰For simplicity, we assume here that all membership affiliations are decided at this initial time.

payoffs can be written in recursive form as follows:

$$\begin{aligned}
U^0 &= w + \delta (X_A U^A + X_B U^B + (1 - X_A - X_B) U^0) \\
U^A &= X_A \delta U^A + X_B (w + \delta U^{AB}) + (1 - X_A - X_B) (w + \delta U^A) \\
U^B &= X_A (w + \delta U^{AB}) + X_B \delta U^B + (1 - X_A - X_B) (w + \delta U^B) \\
U^{AB} &= X_A \delta U^{AB} + X_B \delta U^{AB} + (1 - X_A - X_B) (w + \delta U^{AB})
\end{aligned}$$

We are able to recursively solve this system of linear equations in order to obtain U^A , U^B , and U^{AB} . Taking into account that providers are indifferent between defaulting and exerting effort if $U^0 = w - e$, our first stationary equilibrium condition is:

$$\frac{w}{1 - \delta (1 - X_A - X_B)} \left\{ 1 + \delta \left(\frac{X_A(1-X_A)}{1-\delta(1-X_B)} + X_A X_B \delta \frac{(1-X_A-X_B)}{(1-\delta)(1-\delta(1-X_B))} + \frac{X_B(1-X_B)}{1-\delta(1-X_A)} + X_B X_A \delta \frac{(1-X_A-X_B)}{(1-\delta)(1-\delta(1-X_A))} \right) \right\} = \frac{w - e}{1 - \delta} \quad (46)$$

The following result relates the consumer basis of the two bureaus in the stationary equilibria.

Lemma 5 *Equation (46) defines a strictly decreasing relationship between X_A and X_B .*

Consumers In this section, for convenience, we focus on the case where $\delta > \sqrt{e/w}$.¹¹ Whenever there are two bureaus operating in equilibrium, that is, with positive consumer bases, the utility of the consumers who buy from bureau A must be the same as the utility of those who buy from bureau B and the same as not buying at all. This leads us to the following indifference conditions:

$$(1 - Y_{C,member}) (P - w) - f_i + (1 - \delta) \sum_{t=0}^{\infty} \delta^t (1 - X_i)^t Y_{C,member} (-w) = (1 - Y_{C,member}) P - w,$$

where the LHS is the consumer's payoff from joining bureau i and the RHS is the payoff of not joining any bureau. Given that the indifference must hold for both bureaus, we have the following two equations:

$$f_i = Y_{C,member} \frac{w \delta X_i}{1 - \delta (1 - X_i)}, \forall i = A, B \quad (47)$$

The ratio of fees is given by:

$$\frac{f_A}{f_B} = \frac{X_A (1 - \delta) + \delta X_A X_B}{X_B (1 - \delta) + \delta X_A X_B} \quad (48)$$

Proposition 5 *For any pair of fees (f_A, f_B) , there is, at most, one stationary equilibrium with two operating bureaus ($X_A > 0$ and $X_B > 0$) in the continuation game.*

¹¹If $\sqrt{e/w} > \delta > e/w$, there might be an equilibrium in which all consumers join bureaus and some providers default twice: the first time he meets a member of each bureau.

Implicit in the statement of the proposition above is the fact that there might be equilibria in the continuation game in which either only one bureau operates (that is, $X_i > 0$ and $X_j = 0$) or in which there is no bureau operating, so essentially there is no market ($X_A = X_B = 0$).

In any stationary equilibrium, we can partition the set of consumers into two subsets: consumers who join at least one bureau and consumers who do not join any bureau. The last condition that we need to construct a stationary equilibrium with two operating bureaus is the condition that the consumers who do not join any bureau also find it profitable to hire providers, despite the fact that they have no information on the provider's past behavior. This gives us our last equilibrium condition:

$$Y_{C,member} \leq \frac{P - w}{P} \quad (49)$$

It is convenient to define a feasible set for the providers' fees. Let us note that there is an upper bound for a fee of a bureau that operates in equilibrium. We can compute this upper bound using (47) and (49) and the fact that X_i has an upper bound, which is given by $X_i \leq \frac{e\delta + \sqrt{e^2\delta^2 + 4\delta(1-\delta)we}}{2\delta w}$. A fee f_i is *feasible* if it is below this upper bound, that is, if there exists a stationary equilibrium in which consumers buy from firm i at this fee f_i :

$$f_i \leq \frac{P - w}{P} w \frac{\delta \frac{e\delta + \sqrt{e^2\delta^2 + 4\delta(1-\delta)we}}{2\delta w}}{1 - \delta \left(1 - \frac{e\delta + \sqrt{e^2\delta^2 + 4\delta(1-\delta)we}}{2\delta w} \right)}. \quad (50)$$

A very important issue in this section is the role played by beliefs off the equilibrium path. The timing of the duopoly game is such that first the bureaus simultaneously choose fees and then consumers decide whether or not to join. From Proposition 5, we know that for any given pair of fees, there might be a multiplicity of continuation equilibria. Nevertheless, at most one continuation equilibrium has two bureaus operating in equilibrium. We assume an equilibrium refinement in which in every continuation equilibrium two bureaus operate whenever possible. Precisely, this means that starting from a given pair (f_i, f_j) , a deviation by one of the two bureaus, say, bureau i , leads to a new pair (f'_i, f_j) in which it is possible to construct a stationary equilibrium of the continuation game in which 1. both bureaus still operate; 2. only bureau i operates; 3. only bureau j operates or 4. neither one of the two bureaus operates. Our refinement is to consider equilibria in which the equilibrium after the deviation is the one in which both bureaus operate, if such an equilibrium exists. If, after a deviation, there is no equilibrium with two operating bureaus, then we consider only the equilibrium in which only the cheapest bureau operates.¹²

¹²Suppose, instead, that we consider a refinement in which whenever there is no equilibrium with two bureaus, the consumers buy only from the most expensive bureau. Then, there are two equilibria only, one in which bureau i is the monopolist and one in which j is the monopolist. For the i monopolist, the fee is given by $f_i = \frac{P-w}{P} w \frac{\delta X_i}{1-\delta(1-X_i)}$, and the fractions of members and defaulters are, respectively, $X_i = \frac{e\delta + \sqrt{e^2\delta^2 + 4\delta(1-\delta)we}}{2\delta w}$ and $Y_{C,member} = \frac{P-w}{P}$, with $f_j \geq f_i$ and $X_j = 0$.

Given that each bureau's profit is a direct function of $fee \times X$, any deviation that increases f might seem like a profitable deviation. But can the bureau increase its fee without bounds? Note that if $f_A > Y_{C,member} \frac{w\delta X_A}{1-\delta(1-X_A)}$, then nobody buys from bureau A . Therefore, certainly (50) imposes an upper bound on a fee of an operating bureau. This imposes a non-tight bound.

Consider a case in which both bureaus set the same fee and suppose that

$$f_A = f_B = \frac{P-w}{P} \frac{w\delta X}{1-\delta(1-X)},$$

where X solves

$$\frac{(1-\delta)w}{1-\delta(1-2X)} + \delta 2 \frac{wX}{(1-\delta(1-2X))(1-\delta(1-X))} ((1-\delta)(1-X) + X\delta(1-2X)) = w - e \quad (51)$$

Denote the X that solves the above equation by X^{sym} . Thus, consider a situation in which both firms charge

$$\bar{f} = \frac{P-w}{P} \frac{w\delta X^{sym}}{1-\delta(1-X^{sym})} \quad (52)$$

Before we prove the main result in this section, the next lemma will be useful.

Lemma 6 *For any given pair (f_i, f_j) with $f_i \leq f_j \leq \frac{P-w}{P} \frac{w\delta X^{sym}}{1-\delta(1-X^{sym})}$ there exists an equilibrium with two operating bureaus (unique in this class) in the continuation game.*

In an economy in which the two bureaus operate, consumers join at most one of them. The intuition for this result comes from the fact that the marginal benefit of a second bureau is smaller than it is for the first bureau. Since consumers are indifferent between joining and not joining the bureau, the cost is equal to the benefit of joining, but joining a second bureau has negative expected cost. We state this result formally below.

Proposition 6 *Suppose that $\delta > \sqrt{\frac{e}{w}}$. Then, in a stationary equilibrium with two bureaus, each consumer joins at most one bureau.*

We are now ready to prove the main result of this section.

Proposition 7 (Unique Stationary Competitive Equilibrium) *There is a unique stationary equilibrium in which both bureaus operate. In this equilibrium, $f_i = f_j = \frac{P-w}{P} \frac{w\delta X^{sym}}{1-\delta(1-X^{sym})}$, with symmetric consumer bases $X_i = X_j = X^{sym}$ and $Y_{C,member} = \frac{P-w}{P}$.*

Therefore, there is a unique stationary equilibrium in the duopoly competition game where the firms set fees simultaneously and the consumers and providers play an infinitely repeated game following the chosen fees. In this equilibrium, the fraction of providers who default is $Y_{C,member} = \frac{P-w}{P}$, which is the lowest possible fraction that sustains a stationary equilibrium.

5.2.1 Social Welfare

We are focusing on equilibria in which consumers are indifferent between buying a membership from bureau i, j , or not buying at all, but hiring providers nonetheless. Thus, we have that:

$$U_{consumer} = (1 - Y_{C,member})P - w \quad \text{and} \quad U_{provider} = w - e \quad (53)$$

where $Y_{C,member} = \frac{P-w}{P}$, so that $U_{consumer} = (1 - \frac{P-w}{P})P - w = 0$. The social welfare becomes $\mathbf{SW}_{C,member} = \frac{1}{2(1-\delta)} \{w - e\}$. Given that social welfare in the case of a monopoly is $\mathbf{SW}_{M,member} = \mathbf{SW}_{M,buy} = \frac{1}{2(1-\delta)}(w - e)$, we have: $\mathbf{SW}_{C,member} = \mathbf{SW}_{M,buy}$. Thus, assuming that the parameter restriction in Corollary 1 is satisfied with inequality, we have:

$$\mathbf{SW}_{A,member} > \mathbf{SW}_{A,buy} > \mathbf{SW}_{C,buy} > \mathbf{SW}_{M,buy} = \mathbf{SW}_{M,member} = \mathbf{SW}_{C,member}$$

Consequently, competition, while possibly welfare improving in comparison with a monopoly, it is significantly worse than either case of the altruistic bureau. There are several reasons for this. First, as mentioned in Section 5.1, the presence of competition in a buy and sell set-up generates a duplication of direct costs, significantly decreasing the benefits of introducing the bureau. Moreover, in the case of membership, the indirect costs become significantly higher, not only because bureaus are smaller, but also because the measure of defaulting providers must be higher in equilibrium in order to sustain multiple bureaus. In summary, not only is learning slowed down but also the indirect costs that arise from facing default while informed are consistently higher throughout. We can see the negative effects of slow learning and higher expected indirect costs in Figures 4a and 4b, respectively. Overall, information trade in many ways presents the same characteristics as natural monopolies, where, in order to avoid the duplication of costs and harvest the benefits of economies of scale, the optimal number of producers (or, in this case, information brokers) is one, provided it is regulated in order to avoid a concentration of market power.

6 Extensions

The model can be further extended in several dimensions. First, we consider the case where the bureau that operates through membership is allowed to choose a sample of consumers from whom to buy information, instead of constantly buying information from its entire membership base. The advantage of doing so is that information acquisition becomes less costly; thus, a stationary equilibrium with a lower default rate might be possible. Second, we consider the case where the bureau is allowed to buy only negative feedback, instead of having to purchase information from all consumers regardless of their experience. Since in our model all providers are ex ante identical, only negative information (defaults) is important to incentivize providers to exert effort. Consequently, buying only negative information may

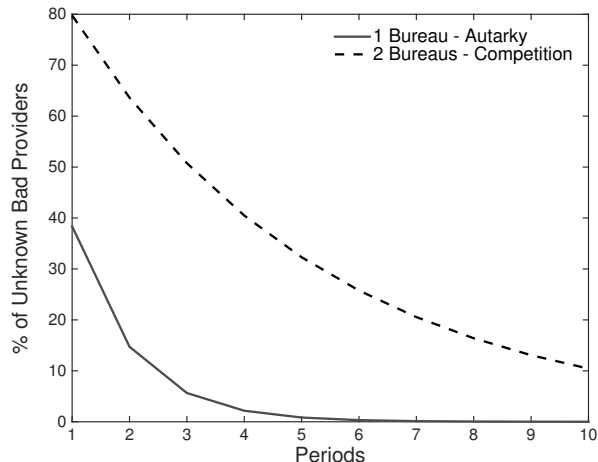


Figure 4a: Speed of learning with membership

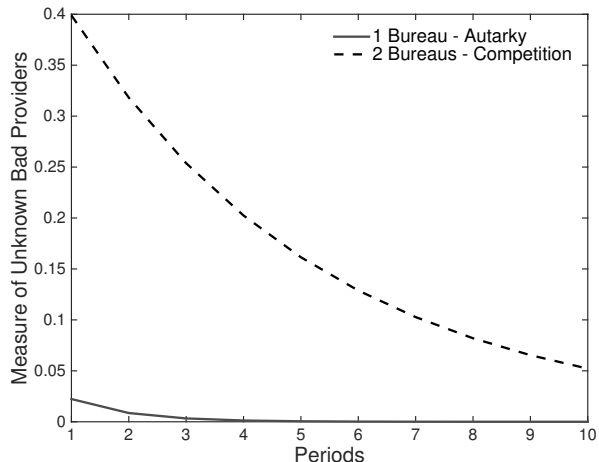


Figure 4b: Normalized indirect costs

be social welfare improving by reducing information acquisition costs. Third, we consider the case in which consumers face a positive cost to access information. This extension is important not only to eliminate an asymmetry in the information transmission mechanism – information is costly to report but costless to access – but it also allows us to purify the equilibrium set from some corner results. Finally, we consider the case in which membership is observable by providers. In particular, we consider the case of observable membership in a set-up where the bureau is allowed to buy only negative information and information access can be either costless or costly. In the case of costly information access, we consider two cases: the bureau can either enforce information access or not.

Equilibrium patterns presented by the model extensions are able to further our understanding of the mechanisms behind information transmission and its relation with market structures and pricing mechanisms. First, our results for the optimal sampling frequency show that social welfare is maximized with a low frequency that minimizes the costs of information acquisition, while maximizing the bureau’s size. Consequently, in terms of welfare, it is better to maximize information spillovers, even though that may mean that the bureau accumulates information about providers more slowly. Second, we show that a bureau only buying negative information in an environment with costless access of information can deliver the first best scenario. However, this result is not robust. Once we introduce a small but positive cost of accessing information, all the results presented in previous sections are recovered. Finally, in the case in which providers can verify consumers’ membership status, our results are significantly more nuanced than the ones presented by previous literature (see Greif [1993]). First, in the case of costless information access, we again reach the first best if the bureau is able to only buy negative information. Second, in the case of costly information access, the bureau’s ability to enforce information access by its members is key. The case in which information access is enforceable is identical to the case of costless access, apart from the decline in social welfare due to the cost of access. In contrast, in the case of unenforceable costly access of information, we show that observable membership status becomes counterproductive as agents become patient. The reason is that, in this environment, even effectively

“uninformed” consumers are compelled to become members. Consequently, these effectively “uninformed” members report deviations over time. As a result, the overall cost for consumers increases, reducing social welfare. In contrast, making membership unobservable allows uninformed consumers to free-ride on the incentives to providers delivered by informed consumers, without paying a membership fee. Similarly, unobservable membership allows bureaus to save on the direct costs of acquiring information from uninformed consumers, relying on the learning process over time in order to induce effort by providers.

6.1 General Case: Optimal Information Sampling

In our benchmark model, we consider the case in which the bureau always buys the information from all members. While this is in principle a natural starting point, it is not necessarily an optimal strategy. In order to simplify the presentation as well as to keep the focus on the social welfare effects, we limit the presentation to the altruistic case. Notice that, since information acquisition is costly, buying too much information can be suboptimal. In particular, buying too much information may have a deleterious effect by inducing a high membership fee and a smaller bureau size in equilibrium. As a result, the number of providers choosing to exercise no effort in equilibrium may be larger, even though informed consumers have more knowledge of past deviations. Therefore, a policymaker establishing a bureau may decide to pin down the optimal sampling scheme in order to maximize social welfare. In particular, let’s assume that, in every period, the bureau buys any given member’s information with probability $(1 - q) \in (0, 1)$.

Provider’s Problem

In this general case, we have that a provider will be indifferent between making effort or not if:

$$(1 - \delta)w + (1 - X_{A,mq})\delta w + X_{A,mq}(1 - \delta) \sum_{t=1}^{\infty} \delta^t w \sum_{t_1=0}^t \binom{t}{t_1} (X_{A,mq}q)^{t-t_1} (1 - X_{A,mq})^{t_1} = w - e \quad (54)$$

Solving this equation for $X_{A,mq}$, we obtain:

$$X_{A,mq} = \frac{\delta e + \sqrt{\delta^2 e^2 + \frac{4\delta(1-\delta)we}{(1-q)}}}{2\delta w} \quad (55)$$

First, note that if $q = 0$, we are in the previous case. Moreover, notice that $\frac{\partial X_{A,mq}}{\partial q} > 0$, i.e., the bureau’s size increases as the likelihood of acquiring information – and consequently the direct cost of membership – declines.

Consumer’s Problem

Now, let's consider the consumer's decision. First, the consumer's payoff to becoming a member in period 1 is:

$$(1 - Y_{A,mq})(P - w) - f_{ee} - Y_{A,mq}(1 - \delta) \sum_{t=1}^{\infty} \delta^t w \sum_{t_1=0}^t \binom{t}{t_1} (X_{A,mq}q)^{t-t_1} (1 - X_{A,mq})^{t_1} \quad (56)$$

In equilibrium, we must have that consumers are indifferent between applying for membership or not. Consequently, in equilibrium we must have:

$$f_{ee} = \frac{\delta Y_{A,mq} w (1 - q) X_{A,mq}}{1 - \delta [1 - (1 - q) X_{A,mq}]} \quad (57)$$

Altruistic Bureau's Problem

As in Section 3.1, the altruistic bureau's problem is to maximize an egalitarian social welfare function that equally weights consumers' and providers' utilities, conditional on some restrictions that include a break-even condition, i.e., that the bureau must be self-funded. Consequently, the altruistic bureau's problem in the case of membership and information sampling is given by:

$$\mathbf{SW}_{\mathbf{A},mq} \equiv \max_{f_{ee}, q} \frac{1}{2} \{ (1 - Y_{A,mq}) P - e \} \quad (58)$$

subject to:

$$f_{ee} \geq c(1 - q) \left\{ 1 - \frac{\delta Y_{A,mq} X_{A,mq} (1 - q)}{1 - \delta [1 - X_{A,mq} (1 - q)]} \right\} \quad (C.1)$$

$$0 \leq Y_{A,mq} \leq \frac{P - w}{P} \quad (C.2)$$

$$Y_{A,mq} = \frac{f_{ee} \{ 1 - \delta [1 - (1 - q) X_{A,mq}] \}}{\delta w (1 - q) X_{A,mq}} \quad (C.3)$$

$$X_{A,mq} = \frac{\delta e + \sqrt{\delta^2 e^2 + \frac{4\delta(1-\delta)we}{(1-q)}}}{2\delta w} \in [0, 1] \quad (C.4)$$

where (C.1) is the break-even condition. Notice that now the bureau can choose not only the fee but also the sampling frequency. We are now able to show the following auxiliary results.

Lemma 7 *At the optimum, (C.1) must be binding.*

Lemma 8 *$Y_{A,mq}$ is decreasing in q .*

In summary, Lemma 7 shows that (C.1) is binding at the optimum, pinning down f_{ee} . Therefore, we just need to pin down q . Then, Lemma 8 shows that $Y_{A,mq}$ is strictly decreasing in q . Finally, $\mathbf{SW}_{\mathbf{A},mq}$ is linearly decreasing in $Y_{A,mq}$. Consequently, in order to maximize $\mathbf{SW}_{\mathbf{A},mq}$, the bureau must choose the highest value of q . Therefore, at the optimum we must have:

$$q^* = \frac{\delta w - e}{\delta(w - e)} \quad (59)$$

However, this implies that $X_{A,mq} = 1$. Hence, all consumers become bureau members. The next theorem collects all of these results:

Theorem 1 (Optimal Sampling Theorem: Full Membership) *If the credit bureau can choose the sampling frequency in order to maximize social welfare in a membership pricing mechanism, we have that all consumers become members. Moreover, the membership fee and the fraction of defaulting providers are given by:*

$$f_{ee} = \frac{c(1-\delta)ew}{\delta w(w-e) + c(1-\delta)e} \quad \text{and} \quad Y_{A,mq} = \frac{(1-\delta)cw}{w\delta(w-e) + c(1-\delta)e}$$

Finally, we can show the following corollary:

Corollary 3 *The speed of learning – determined by the decline in the fraction of unknown bad providers – declines with q .*

Consequently, the policymaker optimally chooses to minimize the costs of information acquisition by reducing the sampling frequency in order to maximize the equilibrium bureau's size. In other words, in terms of welfare effects, it is best to have a large bureau, maximizing information spillovers (dissemination) even though information accumulation may occur more slowly. As a result, we may infer that the social welfare loss in the case of competitive bureaus with membership occurs mainly through the reduction in the bureau's size, instead of through the slower accumulation of information.

6.2 Bureau Buys Only Negative Information

BUYING AND SELLING INFORMATION

In this section, we consider the case in which the bureau is able to only buy negative information, i.e., information from consumers that faced default. For simplicity, let us assume that $f_1 = c$. Apart from these new assumptions, we keep all the other features of our framework. As a result, consumers' and providers' problems are the same as in the benchmark case. Then, the balanced budget condition becomes:

$$\frac{\delta f_2 X_{A,buy}}{1-\delta} - c Y_{A,buy} - \sum_{t=1}^{\infty} \delta^t (1 - X_{A,buy}) Y_{A,buy} c \geq 0$$

Substituting the values for $X_{A,buy}$ and $Y_{A,buy} = \frac{f_2}{w}$ obtained in section 3.1, we have:

$$\frac{\frac{f_2}{w} [e - (1 - \frac{e}{w})c]}{1-\delta} \geq 0 \tag{60}$$

We obtain an equilibrium in which, by setting $f_2 = 0$, the bureau not only establishes a costless bureau, but also maximizes the overall social welfare. The following proposition summarizes these results:

Proposition 8 *Assume the bureau purchases only negative information and there are no payoff-types among providers. An altruistic bureau is able to obtain the first-best by committing to buying information from consumers that faced default and sell it at no cost to other consumers.*

This result is not robust to small changes in the environment. The main issue is that, off the equilibrium path, the bureau would not have money to purchase information in the case of a deviation. There are a few ways in which we can avoid this result. For example, we implicitly imposed an asymmetry between the costs of reporting and accessing information. We assumed that reporting the information is costly, but accessing the information is costless apart from the fee. We may ease this constraint and assume that accessing the information has an intrinsic cost of c_1 apart from the fee. As we show in the next section, this minor extension rules out this equilibrium, once consumers would skip accessing information if they expect no provider to default.

6.3 Costly Information Access

BUYING AND SELLING INFORMATION

Consumers' Problem

In this section, we assume that consumers pay as cost $c_1 > 0$ to access the information gathered by the bureau, which is in principle distinct of the bureau's fee f_2 . Then, consumers are indifferent between buying information or not in the case of buy or sell information if:

$$-(f_2 + c_1)Y_{A,buy} + (P - w - f_2 - c_1)(1 - Y_{A,buy}) = -wY_{A,buy} + (P - w)(1 - Y_{A,buy}) \quad (61)$$

However, notice that, by adjusting f_2 , the bureau is able to try to compensate the consumer for the cost of accessing information as well. So no new fee is necessary. Then, in a mixed equilibrium, we must have:

$$Y_{A,buy} = \frac{f_2 + c_1}{w} \quad (62)$$

Altruistic Bureau's Problem

Notice that provider's and altruistic bureau's problems are still the same as the ones presented in Section 3.1. Then, substituting it back into the budget constraint and manipulating it, we

have:

$$f_2 \geq \frac{\left[1 - \frac{e}{w}\right] c}{e - \left[1 - \frac{e}{w}\right] c} c_1 \quad (63)$$

Again, since the objective function is strictly decreasing in f_2 , at the optimum, equation (63) must be satisfied with equality. Consequently, we have:

$$f_2 = \frac{(w - e)c}{ew - (w - e)c} c_1 \quad \text{and} \quad Y_{A, \text{buy}} = \frac{ec_1}{ew - (w - e)c} \quad (64)$$

Again, (C.3') still needs to be satisfied, implying that:

$$0 \leq \frac{ec_1}{ew - (w - e)c} \leq \frac{P - w}{P} \quad (65)$$

Finally, the social welfare function gives us:

$$\text{SW}_{A, \text{buy}} = \frac{1}{2(1 - \delta)} \left\{ \left[1 - \frac{ec_1}{ew - (w - e)c} \right] P - e \right\} \quad (66)$$

Membership

First of all, let's keep in mind that the utility for consumers is given by:

$$U = (1 - \delta) \sum_{t=0}^{\infty} \delta^t c_t$$

where c_t is the consumption of the numeraire with price normalized to 1. In this sense, we have risk neutral agents, but we normalize the utility function such that consuming one unit of the numeraire every period gives a utility of 1 (see Mailath and Samuelson [2006]).

Consumers' Problem

Consumers pay a cost c_1 to access the information gathered by the bureau beyond their membership fees. As before, we assume that the bureau compensates its members for the cost of reporting defaults. Consequently, a consumer is indifferent between becoming a member or not if:

$$(1 - Y_{A, \text{member}})(P - w) - f_{ee}^1 - \delta c_1 - \frac{(1 - \delta)Y_{A, \text{member}}}{1 - \delta(1 - X_{A, \text{member}})} w = (1 - Y_{A, \text{member}})P - w \quad (67)$$

which leads to the following condition:

$$f_{ee}^1 = \delta \left[\frac{X_{A, \text{member}} Y_{A, \text{member}} w}{1 - \delta(1 - X_{A, \text{member}})} - c_1 \right] \quad (68)$$

Apart from that, the problem is identical to the one presented in Section 3.2. Similarly, the provider's problem is the same as the one presented in Section 3.2.

Altruistic Bureau's Problem

The altruistic bureau's problem is similar to the one presented in Section 3.2, apart from a few details. First, the bureau's profit function must take into account that only negative information is purchased, i.e., the profit function is now given by:

$$\Pi_{A,member} = X_{A,member} \left\{ \frac{f_{ee}^1}{1-\delta} - Y_{A,member} c \sum_{t=0}^{\infty} \delta^t (1 - X_{A,member})^t \right\} \quad (69)$$

which impacts the break even condition. Second, the consumer's problem implies that we must take into account the new fee value presented in (68). Consequently, the altruistic bureau's problem is given by:

$$\mathbf{SW}_{A,member} \equiv \max_{f_{ee}^1} \frac{1}{2(1-\delta)} [(1 - Y_{A,member})P - e] \quad (70)$$

subject to:

$$\frac{f_{ee}^1 X_{A,member}}{1-\delta} - \left(\frac{f_{ee}^1 + \delta c_1}{\delta w} \right) c \geq 0 \quad (C.1)$$

$$0 \leq Y_{A,member} \leq \frac{P-w}{P} \quad (C.2)$$

$$Y_{A,member} = \frac{(f_{ee}^1 + \delta c_1)[1 - \delta(1 - X_{A,member})]}{\delta X_{A,member} w} \quad (C.3)$$

$$X_{A,member} = \frac{e\delta + \sqrt{e^2\delta^2 + 4\delta(1-\delta)we}}{2\delta w} \quad (C.4)$$

Substituting (C.3) into (C.2) and the objective function, we can see that the objective function is linearly decreasing in f_{ee}^1 . Therefore, at the optimum, (C.1) must be binding:

$$f_{ee}^1 = \frac{(1-\delta)\delta c_1 c}{\delta X_{A,member} w - (1-\delta)c} \quad (71)$$

Substituting it back into (C.3), we have:

$$Y_{A,member} = \delta c_1 \frac{[1 - \delta(1 - X_{A,member})]}{\delta X_{A,member} w - (1-\delta)c} \quad (72)$$

The following proposition shows that the results presented in Proposition 2 are still valid in this new environment in which the bureau only acquires negative information but there is costly information access by consumers.

Proposition 9 *The policymaker would prefer a pricing mechanism based on membership instead of buy and sell. Moreover, the fraction of providers offering high-quality services and the fraction of informed consumers are both higher with membership.*

Monopolist Case

The monopolist case is a straightforward extension of the case presented in Section 4, with the bureau extracting all the surplus from consumers. Consequently, we present all details of the extension in the proof of the following proposition, which shows that results are the same as the ones obtained in Proposition 3.

Proposition 10 *A monopolist bureau prefers offering membership to buying and selling information. Social welfare is the same regardless the pricing mechanism.*

Competitive Bureaus

Buy and Sell

The framework of competitive bureaus with a buy-and-sell pricing scheme follows the one presented in Section 5.1, apart from the deviations in the consumer problem as presented in the beginning of Section 6.3. Consequently, from consumer's and provider's problems, we have:

$$X_{C,buy} = \frac{e}{\delta w} \quad \text{and} \quad Y_{C,buy} = \frac{\min\{f_2^i, f_2^{-i}\} + c_1}{w + c - \max\{f_1^i, f_1^{-i}\}} \quad (73)$$

Similarly, the Bertrand competition between bureaus follows the same dynamics as presented in Section 5.1. Consequently, profit maximization follows equation (42). Then, solving bureau i 's problem, we have, given the optimal $f_1^{-i} = f_1^i = c$, we have:

$$\Pi_{C,buy}^i = \begin{cases} - \left[\left(1 - \frac{e}{w}\right) \frac{f_2^{-i} + c_1}{w} \right] \frac{c}{1-\delta} & \text{if } f_2^i > f_2^{-i} \\ \frac{1}{1-\delta} \left\{ \frac{e}{2w} f_2^i - \left[\left(1 - \frac{e}{w}\right) \frac{f_2^i + c_1}{w} \right] c \right\} & \text{if } f_2^i = f_2^{-i} \\ \frac{1}{1-\delta} \left\{ \frac{e}{w} f_2^i - \left[\left(1 - \frac{e}{w}\right) \frac{f_2^i + c_1}{w} \right] c \right\} & \text{if } f_2^i < f_2^{-i} \end{cases} \quad (74)$$

Then bureau i's reaction correspondence is given by:

$$R^i(f_1^{-i}, f_2^{-i}) = \begin{cases} f_2^i = \frac{P(w-c_1)-w^2}{P} \text{ and } f_1^i = c & \text{if } f_1^{-i} < c; \\ f_2^i = \frac{P(w-c_1)-w^2}{P} \text{ and } f_1^i = c & \text{if } f_1^{-i} \geq c \text{ and } f_2^{-i} > \frac{P(w-c_1)-w^2}{P}; \\ f_2^i = f_2^{-i} - \varepsilon \text{ and } f_1^i = c & \text{if } f_1^{-i} \geq c \text{ and } \frac{2(w-e)cc_1}{ew-2c(w-e)} < f_2^{-i} \leq \frac{P(w-c_1)-w^2}{P} \\ f_2^i = \frac{2(w-e)cc_1}{ew-2c(w-e)} \text{ and } f_1^i = c & \text{if } f_1^{-i} \geq c \text{ and } f_2^{-i} = \frac{2(w-e)cc_1}{ew-2c(w-e)} \\ f_2^i \in \mathbb{R}_+ \text{ and } f_1^i < c & \text{if } f_1^{-i} \geq c \text{ and } f_2^{-i} < \frac{2(w-e)cc_1}{ew-2c(w-e)} \end{cases} \quad (75)$$

First of all, notice that a duopoly is only possible if the following restriction is satisfied:

$$c < \frac{ew}{2(w-e)} \quad (76)$$

Otherwise profits are negative for any $f_2 \geq -c_1$ (minimum f_2 required for $Y_{C,buy} \geq 0$).¹³

Membership

In Section 5.2, we showed that there was a unique stationary equilibrium in which two profit-maximizing bureaus offering membership operate in the market. Here, we extend this analysis to the case where bureaus can buy only negative information and there is a cost c_1 to access the bureau's information, even if the consumer is a member. To obtain the uniqueness result, we showed to auxiliary results: 1. the providers' indifference condition generated a decreasing relation between the competing bureaus' sizes, and 2. for any given pair of fees, the consumers' indifference condition generated an increasing relation between the bureaus' sizes. These two results together implied that there was at most one stationary equilibrium with two bureaus.

Here, even though we have the costly access to information and the possibility of the bureaus restricting themselves to buying only negative information, it is still the case that these auxiliary results hold: the providers' indifference condition is unchanged and in any equilibrium with two operating bureaus, we can use (68) and with some algebra obtain that the ratio of fees is now given by:

$$\frac{f_A + \delta c_1}{f_B + \delta c_1} = \frac{X_A(1-\delta) + \delta X_A X_B}{X_B(1-\delta) + \delta X_A X_B} \quad (77)$$

Thus, given any costs c_1 and a ratio of fees, it is still the case that we must have the same relation between the relative sizes of the bureaus X_A and X_B . Therefore, we must have a unique stationary equilibrium and it is the same as the one presented in Section 5.2.

¹³This restriction is always satisfied in the case of a monopolist profit-maximizing bureau.

6.4 Membership Status is Observable

In this case, we assume that bureau membership status can be credibly communicated to providers. Following the case presented in Section 6.2, we also assume that the bureau can purchase only negative information. Moreover, we focus on the case of a non-profit altruistic bureau whose goal it is to maximize social welfare.

First of all, we can easily show that, in this case, there is no incentive for providers to exert effort when facing a consumer that is not a bureau member.

Lemma 9 *There is no equilibrium in which the providers puts effort with positive probability when facing a non-member.*

Consequently, providers never put in effort when facing a non-member. As a result, it is optimal for non-member not to buy the providers' services, as we summarize in the following corollary.

Corollary 4 *Consumers that choose not to become bureau members will optimally choose not to buy the providers' service.*

We consider three sub-cases in terms of the cost of accessing information: 1. costless information access; 2. costly and enforceable information access; and 3. costly and unenforceable information access. We present each sub-case in details in the following sub-sections.

1. Costless access of information: We establish this as the initial benchmark case, by assuming that $c_1 = 0$.
2. Costly and enforceable information access: In this case, while $c_1 > 0$, we assume that the bureau can punish members that do not access information by stripping them of their membership status;
3. Costly and unenforceable information access: In this case, not only $c_1 > 0$, but the bureau cannot punish members that do not access information. As a result, the choice of accessing information must be optimal by itself.

6.4.1 Costless Information Access

In this case, $c_1 = 0$ and the consumer can costlessly access the bureaus' information, conditional on becoming a member and paying the membership fee. In this case, members will optimally access the information whenever available. Therefore, consumer's problem is twofold: 1. the

decision of becoming a member or not, and 2. the decision of hiring the provider's service or not, conditional on the available information about the provider's past behavior.

In terms of the latter decision, let's focus on the case in which the member hires only the services of providers that have never defaulted in the past, since it induces the strongest incentive for providers to put in effort. Let's assume that the bureau collects all the information about deviations against members (no optimal sampling). In this case, it's optimal for the provider to put in effort if:

$$w - e + \delta \frac{X_{A,k-m}(w - e)}{1 - \delta} \geq w \Rightarrow X_{A,k-m} \geq \frac{(1 - \delta)e}{\delta(w - e)} \quad (78)$$

Therefore, there is a threshold size of the bureau that would induce providers to put in effort. In particular, if $\delta > \frac{e}{w}$, the RHS of (78) is strictly less than one and the threshold problem is well-defined. Let's assume that (78) is satisfied and that the bureau pays any incurred cost of reporting default. In this case, a consumer decides to become a member and pay the membership fee if:

$$\frac{P - w}{1 - \delta} - \frac{\delta f_{ee}^{k-m}}{1 - \delta} \geq 0 \Rightarrow f_{ee}^{k-m} \leq \frac{P - w}{\delta} \quad (79)$$

As a result, as long as the membership fee is less than the flow benefit of buying high-effort services, the consumer joins the bureau. Therefore if (79) is satisfied with inequality, $X_{A,k-m} = 1$.

Finally, notice that if inequalities (78) and (79) are satisfied and non-binding, all consumers become members and all providers put in effort whenever facing a member. As a result, in order to maximize social welfare, a bureau can set a zero membership fee and still satisfy the break-even condition, as in the case of Proposition 8 in Section 6.2. The following proposition summarizes the results:

Proposition 11 *Assume that membership status is observable, the bureau is able to buy only negative information, and there is no cost of accessing information available to the bureau. An altruistic bureau is able to obtain the first-best by setting a zero-cost membership fee and committing to repaying members any incurred cost of reporting default.*

6.4.2 Costly and Enforceable Information Access

In this case, while there is a cost of accessing information $c_1 > 0$, if members choose not to access information, the bureau can punish them by expelling them from the bureau. Notice that, while the provider's problem is the same as the one defined by (78), the consumer problem has changed. In particular, assuming that $X_{A,k-m}$ satisfies the inequality (78) and providers exert effort when facing a member, the consumer now decides to join the bureau if:

$$\frac{P - w}{1 - \delta} - \frac{\delta(c_1 + f_{ee}^{k-m})}{1 - \delta} \geq 0 \Rightarrow f_{ee}^{k-m} \leq \frac{P - w}{\delta} - c_1 \quad (80)$$

As in the case with costless information access, notice that if inequalities (78) and (80) are satisfied and non-binding, all consumers become members and all providers put in effort whenever facing a member. As a result, in order to maximize social welfare a bureau can again set a zero fee and still satisfy a break-even condition, in a similar manner as the one presented in Proposition 8 in Section 6.2.

Therefore, apart from a reduction in social welfare due to the incurred cost of information access, the equilibrium outcome is the same as the one obtained in the case of costless information access. In particular, the social welfare becomes:

$$\mathbf{SW}_{\mathbf{A},k-m} = \frac{1}{2(1-\delta)} \{P - e - c_1\} \quad (81)$$

6.4.3 Costly and Unenforceable Information Access

In this case, some consumers that became members may prefer to skip information access in order to save information access costs. It is clear that there is no equilibrium in which either all consumers skip accessing information or all consumers access information. Therefore, we should expect that at least a fraction of members access the information, incurring the cost c_1 . However, in order for this strategy to be optimal, we must have some providers defaulting against members in equilibrium. As a result, we reestablish a mixed strategy equilibrium. Therefore, in equilibrium, while all providers default against non-members, some would also default against members. Define ε the fraction of members that decide to not to access information. Then, the provider's problem becomes:

$$w - e + \delta \frac{X_{A,k-m}(w - e)}{1 - \delta} = w + \delta \frac{X_{A,k-m}\varepsilon w}{1 - \delta} + \sum_{t=1}^{\infty} \delta^t (1 - X_{A,k-m})^t X_{A,k-m} (1 - \varepsilon) w \quad (82)$$

Keep in mind that we are already assuming here that non-members do not purchase the provider's services. Rearranging it, we have:

$$(w - e) \left[1 + \frac{\delta X_{A,k-m}}{1 - \delta} \right] = w \left[1 + \frac{\delta \varepsilon X_{A,k-m}}{1 - \delta} + \frac{\delta X_{A,k-m} (1 - X_{A,k-m})}{1 - \delta (1 - X_{A,k-m})} (1 - \varepsilon) \right] \quad (83)$$

Let's consider the consumer's problem now. First, the consumer must decide whether or not she becomes a bureau member. Furthermore, if a consumer becomes a member, she must to decide whether or not to access information. However, even members that did not access information will report poor service, since they are reimbursed for the cost of reporting. The

consumer utility of becoming a member and regularly accessing the information is:

$$(1 - Y_{A,k-m}) \frac{(P-w)}{1-\delta} - \frac{f_{ee}^{k-m}}{1-\delta} - \delta \frac{c_1}{1-\delta} + \sum_{t=0}^{\infty} \delta^t (1 - X_{A,k-m})^t Y_{A,k-m} (-w) = \frac{1}{1-\delta} \left\{ (1 - Y_{A,k-m})(P - w) - f_{ee}^{k-m} - \delta c_1 - \frac{(1-\delta)Y_{A,k-m}w}{1-\delta(1-X_{A,k-m})} \right\} \quad (84)$$

While the utility of becoming a member and not accessing the information is:

$$(1 - Y_{A,k-m}) \frac{(P - w)}{1 - \delta} - \frac{f_{ee}^{k-m}}{1 - \delta} - \frac{Y_{A,k-m}w}{1 - \delta} \quad (85)$$

Then, the consumer is indifferent between accessing the information or not if:

$$c_1 = \frac{X_{A,k-m}Yw}{1 - \delta(1 - X_{A,k-m})} \Rightarrow Y_{A,k-m} = \frac{[1 - \delta(1 - X_{A,k-m})]c_1}{wX_{A,k-m}} \quad (86)$$

Let's assume that (86) is satisfied. Moreover, notice that the RHS(86) is strictly decreasing in $X_{A,k-m}$. Then, the consumer prefers to become a member of the bureau if:

$$(1 - Y_{A,k-m}) \frac{(P - w)}{1 - \delta} - \frac{f_{ee}^{k-m}}{1 - \delta} - \frac{Y_{A,k-m}w}{1 - \delta} \geq 0 \Rightarrow f_{ee}^{k-m} \leq (1 - Y)P - w \quad (87)$$

Let's assume that (87) is satisfied with inequality. In this case, all consumers prefer becoming members and we have $X_{A,k-m} = 1$. Then, from (86), we have $Y_{A,k-m} = \frac{c_1}{w}$. Similarly, from (83), we have:

$$(w - e) = w [1 - \delta(1 - \varepsilon)] \Rightarrow \varepsilon = 1 - \frac{e}{\delta w} \quad (88)$$

since we have the maintained assumption that $\delta > \frac{e}{\delta w}$, (88) implies that $0 < \varepsilon < 1$.

Finally, looking at the bureau's problem. First, let's look into the break even condition. As previously, let's assume the case in which the bureau only buys negative information. Then, the bureau's profit function is given by:

$$\Pi_{A,k-m} = X_{A,k-m} \left\{ \frac{f_{ee}^{k-m} - Y_{A,k-m}c\varepsilon}{1 - \delta} - \frac{Y_{A,k-m}c(1 - \varepsilon)}{1 - \delta(1 - X_{A,k-m})} \right\} \quad (89)$$

So the break-even condition implies:

$$f_{ee}^{k-m} \geq Y_{A,k-m}c\varepsilon + \frac{Y_{A,k-m}c(1 - \delta)(1 - \varepsilon)}{1 - \delta(1 - X_{A,k-m})} \quad (90)$$

From equation (90), we can see that increasing $X_{A,k-m}$ eases the break-even constraint, allowing the bureau to reduce its membership fee.

Finally, let's consider the bureau's problem. First of all, it's easy to see that there is no

equilibrium in which all members access information. Similarly, there is no subgame perfect equilibrium in which non-members purchase the services. In both cases, there is either optimal one-shot deviations by consumers and providers, respectively. Consequently, in equilibrium we expect non-members choose not to buy the service and a fraction ε of members will skip accessing the information. As a result, we will focus on the bureau's choices in this case. Then, the bureau's problem becomes:

$$\mathbf{SW}_{\mathbf{A},\mathbf{k}-\mathbf{m}} \equiv \max_{f_{ee}^{k-m}} \frac{1}{2(1-\delta)} \left\{ (w-e)[1-\delta(1-X_{A,k-m})] + (1-Y_{A,k-m})P - f_{ee}^{k-m} - w \right\} \quad (91)$$

subject to:

$$f_{ee}^{k-m} \geq Y_{A,k-m}c\varepsilon + \frac{Y_{A,k-m}c(1-\delta)(1-\varepsilon)}{1-\delta(1-X_{A,k-m})} \quad (C.1)$$

$$Y_{A,k-m} = \frac{[1-\delta(1-X_{A,k-m})]c_1}{wX_{A,k-m}} \quad (C.2)$$

$$(w-e) \left[1 + \frac{\delta X_{A,k-m}}{1-\delta} \right] = w \left[1 + \frac{\delta\varepsilon X_{A,k-m}}{1-\delta} + \frac{\delta X_{A,k-m}(1-X_{A,k-m})(1-\varepsilon)}{1-\delta(1-X_{A,k-m})} \right] \quad (C.3)$$

But then, notice that:

$$\frac{\partial \mathbf{SW}_{\mathbf{A},\mathbf{k}-\mathbf{m}}}{\partial X_{A,k-m}} = \delta(w-e) - \frac{\partial Y_{A,k-m}}{\partial X_{A,k-m}} P$$

Since $\frac{\partial Y_{A,k-m}}{\partial X_{A,k-m}} = -\frac{(1-\delta)c_1}{wX_{A,k-m}^2} < 0$, we have that $\frac{\partial \mathbf{SW}_{\mathbf{A},\mathbf{k}-\mathbf{m}}}{\partial X_{A,k-m}} > 0$. Therefore, a bureau that is trying to maximize social welfare would like to set $X_{A,k-m} = 1$. In order to obtain that, we must satisfy (87) with inequality. Therefore, the bureau's problem becomes:

$$\mathbf{SW}_{\mathbf{A},\mathbf{k}-\mathbf{m}} \equiv \max_{f_{ee}^{k-m}} \frac{1}{2(1-\delta)} \left\{ (w-e)[1-\delta(1-X_{A,k-m})] + (1-Y)P - f_{ee}^{k-m} - w \right\} \quad (92)$$

subject to:

$$f_{ee}^{k-m} \geq Y_{A,k-m}c\varepsilon + \frac{Y_{A,k-m}c(1-\delta)(1-\varepsilon)}{1-\delta(1-X_{A,k-m})} \quad (C.1)$$

$$Y_{A,k-m} = \frac{[1-\delta(1-X_{A,k-m})]c_1}{wX_{A,k-m}} \quad (C.2)$$

$$(w-e) \left[1 + \frac{\delta X_{A,k-m}}{1-\delta} \right] = w \left[1 + \frac{\delta\varepsilon X_{A,k-m}}{1-\delta} + \frac{\delta X_{A,k-m}(1-X_{A,k-m})(1-\varepsilon)}{1-\delta(1-X_{A,k-m})} \right] \quad (C.3)$$

$$f_{ee}^{k-m} < (1-Y)P - w \quad (C.4)$$

$$X_{A,k-m} = 1 \quad (C.5)$$

Simplifying the problem by substituting (C.5) and (C.3), we have:

$$\mathbf{SW}_{\mathbf{A},\mathbf{k}-\mathbf{m}} \equiv \max_{f_{ee}^{k-m}} \frac{1}{2(1-\delta)} \left\{ \left(1 - \frac{c_1}{w}\right)P - f_{ee}^{k-m} - e \right\} \quad (93)$$

subject to:

$$\frac{c_1}{w}c[1 - \delta + \delta\varepsilon] \leq f_{ee}^{k-m} < (1 - \frac{c_1}{w})P - w \quad (C.1')$$

$$\varepsilon = 1 - \frac{e}{\delta w} \quad (C.3')$$

Finally, since $\mathbf{SW}_{\mathbf{A},k-m}$ is strictly decreasing in f_{ee}^{k-m} , (C.1') lower bound restriction must be satisfied with equality at the optimum. Therefore, we have that:

$$f_{ee}^{k-m} = \frac{c_1}{w}c[1 - \delta + \delta\varepsilon] = \frac{c_1}{w}c \left[1 - \frac{e}{w}\right] \quad (94)$$

where the second equality in (94) is obtained by substituting (C.3'). Finally, from previous calculations, we obtained $X_{A,k-m} = 1$ and $Y_{A,k-m} = \frac{c_1}{w}$.

Let's now present a few auxiliary results in terms of welfare:

Lemma 10 *The fraction of providers that choose to exert no effort is lower in the case of known membership.*

Corollary 5 *As $\delta \rightarrow 1$ we have that $Y_{A,u-m} \rightarrow \frac{c_1}{w}$.*

Therefore, notice that $Y_{A,u-m} \rightarrow Y_{A,k-m}$ as $\delta \rightarrow 1$. Consequently, we obtain the following result.

Proposition 12 *In the case in which information access is costly and unenforceable, we have that $\mathbf{SW}_{\mathbf{A},k-m} - \mathbf{SW}_{\mathbf{A},u-m} \rightarrow -\infty$ as $\delta \rightarrow 1$.*

Consequently, in the case in which information access is costly but unenforceable, known membership becomes counterproductive as agents become patient. The fact that even effectively “uninformed” consumers feel compelled to become members and consequently report deviations overtime induces an increase in the overall cost for consumers that reduces social welfare. Making membership unknown allows uninformed consumers to free-ride on the incentives to providers delivered by informed consumers without paying a membership fee. Similarly, it allows bureaus to save on the direct costs of acquiring information from uninformed consumers, relying on the learning process overtime in order to induce effort by providers. Finally, notice that as agents – both consumers and providers – become more patient, the gap between the incentives for provider's efforts induced by known and unknown membership narrows, being equal at the limit. In contrast, notice that $X_{A,u-m} \rightarrow \frac{e}{w}$ and in fact, $\frac{\partial X_{A,u-m}}{\partial \delta} < 0$ for $\delta > \frac{1}{2}$. Therefore, as agents become increasingly patient, fewer and fewer informed consumers are necessary in order to induce effort.

7 Conclusion

In this paper, we show that not only does the availability of information matter for a well-functioning market, but also how information is negotiated. The pricing and selling mechanisms, as well as the number of information brokers in the market, are important to determine not only how many agents choose to become informed, but also the quality of information available to them. At the end, these features pin down how much discipline the information trade imposes on both sides of the market, affecting providers' incentives and ultimately the social welfare in the economy. These results are true even in an environment in which we disregard insurance issues. In particular, we consider different information pricing mechanisms – membership vs. buy and sell information – and a competitive environment – non-profit, monopoly, and competitive – in an economy with random matching between a large population of consumers and providers. We show that both dimensions affect direct and indirect costs, represented by fees and expected loss due to default while informed, respectively. In particular, we show that information trade has characteristics similar to a natural monopoly, where competition may hurt because of the duplication of costs as well as by slowing down the information aggregation by each individual information broker. Moreover, we show that there is a trade-off between information quality and cost. In particular, in a world with only one non-profit information bureau, a membership set-up, while having lower information quality, induces low enough direct costs through fees that more than compensate for the initially high indirect costs. However, this is true only because the bureau is large enough to quickly disseminate information and reduce indirect costs.

Finally, we would like to emphasize that risk aversion may significantly change our results, since the bureau may be able to provide insurance against losses through default by paying more for the reported information. However, risk aversion introduces an additional trade-off between insurance and the incentive to buy information, potentially influencing providers' incentives to exercise effort. More research is needed in order to disentangle these additional complications.

References

- S. Nageeb Ali and David A. Miller. Ostracism and forgiveness. *American Economic Review*, 106 (8):2329–2348, August 2016. doi:10.1257/aer.20130768.
- Luis Araujo. Social norms and money. *Journal of Monetary Economics*, 51(2):241–256, 2004. ISSN 0304-3932. doi:10.1016/j.jmoneco.2003.01.005.
- Luis Araujo and Raoul Minetti. On the essentiality of banks. *International Economic Review*, 52 (3):679–691, 2011. doi:10.1111/j.1468-2354.2011.00645.x.

- Yu Awaya. Community enforcement with observation costs. *Journal of Economic Theory*, 154: 173–186, 2014. doi:10.1016/j.jet.2014.09.007.
- V. Bhaskar and Caroline Thomas. Community enforcement of trust with bounded memory. *The Review of Economic Studies*, October 2018. doi:10.1093/restud/rdy048.
- Martin Brown, Tullio Jappelli, and Marco Pagano. Information sharing and credit: Firm-level evidence from transition countries. *Journal of Financial Intermediation*, 18(2):151–172, 2009. doi:10.1016/j.jfi.2008.04.002.
- Joyee Deb. Cooperation and community responsibility: A folk theorem for repeated matching games with names. Working Paper, 2017.
- Mehmet Ekmekci. Sustainable reputations with rating systems. *Journal of Economic Theory*, 146(2):479–503, 2011. doi:10.1016/j.jet.2010.02.015.
- Glenn Ellison. Cooperation in the prisoner’s dilemma with anonymous random matching. *The Review of Economic Studies*, 61(3):567–588, 1994. doi:10.2307/2297904.
- Ronel Elul and Piero Gottardi. Bankruptcy: Is it enough to forgive or must we also forget? *American Economic Journal: Microeconomics*, 7(4):294–338, 2015. doi:10.1257/mic.20130139.
- Avner Greif. Contract enforceability and economic institutions in early trade: The Maghribi traders’ coalition. *The American Economic Review*, 83(3):525–548, 1993. URL <https://www.jstor.org/stable/2117532>.
- Steffen H. Hoernig. Bertrand games and sharing rules. *Economic theory*, 31(3):573–585, 2007. doi:10.1007/s00199-006-0112-8.
- Tullio Jappelli and Marco Pagano. Information sharing, lending and defaults: Cross-country evidence. *Journal of Banking & Finance*, 26(10):2017–2045, 2002. doi:10.1016/S0378-4266(01)00185-6.
- Michihiro Kandori. Social norms and community enforcement. *The Review of Economic Studies*, 59(1):63–80, 1992. doi:10.2307/2297925.
- Narayana R. Kocherlakota. Money is memory. *Journal of Economic Theory*, 81(2):232–251, 1998. doi:10.1006/jeth.1997.2357.
- Sergei Kovbasyuk and Giancarlo Spagnolo. Memory and markets. EIEF Working Papers Series 1606, Einaudi Institute for Economics and Finance (EIEF), 2017.

- Qingmin Liu and Andrzej Skrzypacz. Limited records and reputation bubbles. *Journal of Economic Theory*, 151:2–29, 2014. doi:10.1016/j.jet.2013.12.014.
- George J Mailath and Larry Samuelson. *Repeated games and reputations: long-run relationships*. Oxford university press, 2006.
- Paul R. Milgrom, Douglass C. North, and Barry R. Weingast. The role of institutions in the revival of trade: The law merchant, private judges, and the champagne fairs. *Economics & Politics*, 2(1):1–23, March 1990. doi:10.1111/j.1468-0343.1990.tb00020.x.
- Margaret Miller. Credit reporting systems around the globe: the state of the art in public and private credit registries. In *World Bank. Presented at the Second Consumer Credit Reporting World Conference, held in San Francisco, California, October, 2000*. URL http://siteresources.worldbank.org/INTRES/Resources/469232-1107449512766/Credit_Reporting_Systems_Around_The_Globe.pdf.
- Masahiro Okuno-Fujiwara and Andrew Postlewaite. Social norms and random matching games. *Games and Economic behavior*, 9(1):79–109, 1995. doi:10.1006/game.1995.1006.
- Marco Pagano and Tullio Jappelli. Information sharing in credit markets. *The Journal of Finance*, 48(5):1693–1718, 1993. doi:10.1111/j.1540-6261.1993.tb05125.x.
- Amy J. Schmitz. Remedy realities in business-to-consumer contracting. *Arizona Law Review*, 58: 213–261, 2016. URL <http://arizonalawreview.org/pdf/58-1/58arizlrev213.pdf>.
- Satoru Takahashi. Community enforcement when players observe partners’ past play. *Journal of Economic Theory*, 145(1):42–62, 2010. doi:10.1016/j.jet.2009.06.003.
- James A. Vercammen. Credit bureau policy and sustainable reputation effects in credit markets. *Economica*, pages 461–478, 1995. doi:10.2307/2554671.

Appendix - Proofs

Proof of Lemma 1. Initially, let's consider the case in which (C.3') is not binding. In this case, we focus on (C.1') and (C.2'). Manipulating (C.1'), we obtain:

$$f_2 \geq \frac{w}{e} f_1 \left[\frac{w+c-f_1}{w+c} \right] \quad (\text{C.1}'')$$

Then, notice that:

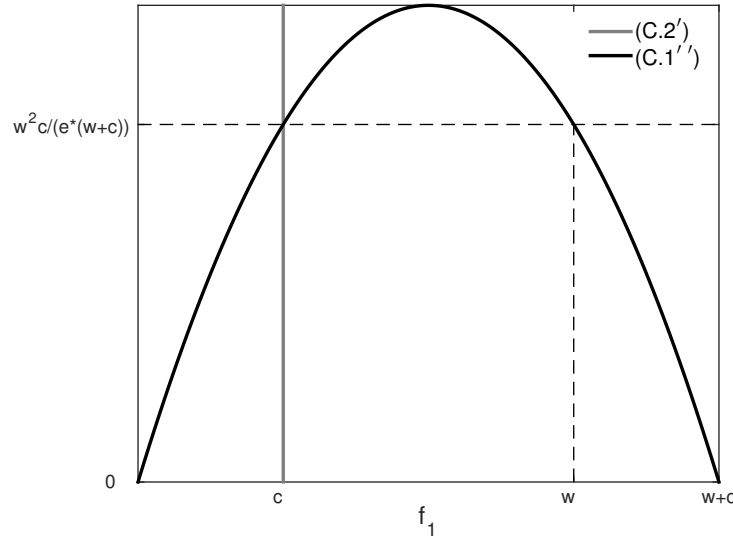
$$\frac{\partial \mathbf{SW}_{\mathbf{A},\text{buy}}}{\partial f_2} = -\frac{P}{w+c-f_1} < 0$$

and

$$\frac{\partial \mathbf{SW}_{\mathbf{A},\text{buy}}}{\partial f_1} = -\frac{f_2}{[w+c-f_1]^2} P - 1 < 0$$

So $\mathbf{SW}_{\mathbf{A},\text{buy}}$ is strictly decreasing in both f_2 and f_1 . Moreover, from (C.1''), we have that RHS(C.1'') is strictly concave and it has roots at $f_1 = 0$ and $f_1 = w+c$. Moreover, since $\mathbf{SW}_{\mathbf{A},\text{buy}}$ is strictly decreasing in f_2 , in order to maximize social welfare, (C.1'') must be satisfied with equality. Then, let's consider two cases:

Case 1: $w \geq c$: In this case, constraints (C.1'') and (C.2') can be graphically represented as:



Notice that, while $f_1 < c$ does not satisfy (C.2'), we also have that for $f_1 \in (c, w)$, in order to satisfy (C.1'') we must have that both $f_1 > c$ and $f_2 > \frac{w^2c}{e(w+c)}$, implying a lower $\mathbf{SW}_{\mathbf{A},\text{buy}}$. Similarly, notice that

$$\mathbf{SW}_{\mathbf{A},\text{buy}}(f_1 = c) - \mathbf{SW}_{\mathbf{A},\text{buy}}(f_1 = w) = \frac{w-c}{2(1-\delta)} \left\{ \frac{w}{e(w+c)} P + 1 \right\} > 0.$$

Then, we just need to evaluate the cases in which $f_1 = w + \alpha c$, with $\alpha \in (0, 1)$. In this case, we have:

$$\mathbf{SW}_{\mathbf{A},\text{buy}}(f_1 = c) - \mathbf{SW}_{\mathbf{A},\text{buy}}(f_1 = w + \alpha c) = \frac{w - (1 - \alpha)c}{2(1 - \delta)} \left\{ \frac{w}{e(w + c)} P + 1 \right\} > 0.$$

Consequently, we are unable to increase $\mathbf{SW}_{\mathbf{A},\text{buy}}$ by setting $f_1 > c$.

Case 2: $w < c$: In this case, we just need to consider setting $f_1 = c + \alpha w$ with $\alpha \in (0, 1)$. In this case, we have:

$$\mathbf{SW}_{\mathbf{A},\text{buy}}(f_1 = c) - \mathbf{SW}_{\mathbf{A},\text{buy}}(f_1 = c + \alpha w) = \frac{\alpha w}{2(1 - \delta)} \left\{ \frac{w}{e(c + w)} P + 1 \right\} > 0.$$

Consequently, again we are unable to increase $\mathbf{SW}_{\mathbf{A},\text{buy}}$ by setting $f_1 > c$.

Now, let's consider the possibility that (C.3') is binding. First, from (C.1'') and (C.3') notice that, in order to satisfy both constraints, we must have that:

$$\frac{w}{e} f_1 \left[\frac{w + c - f_1}{w + c} \right] \leq f_2 \leq \left[\frac{P - (w + c - f_1)}{P} \right] (w + c - f_1)$$

In order for this restriction to be satisfied, we must have:

$$\text{Gap} = \left[\frac{Pw - e(w + c)}{Pe(w + c)} \right] f_1^2 + \left[\frac{2e(w + c) - P(w + c)}{Pe} \right] f_1 + (w + c) \left[\frac{P - (w + c)}{P} \right] \geq 0$$

that has roots: $\underline{f}_1 = \left(\frac{P - (w + c)}{P - \frac{e}{w}(w + c)} \right) e \left(\frac{w + c}{w} \right)$ and $\bar{f}_1 = w + c$. Then, notice that the range in which the solution is satisfied depends on the concavity of Gap. Let's consider the different cases:

case a: $\left[\frac{Pw - e(w + c)}{Pe(w + c)} \right] > 0$. In this case, Gap is strictly convex. In order to satisfy both constraints, we must have $f_1 < \underline{f}_1$. But then, as long as $\underline{f}_1 \geq c$, we are back to the previous cases and the optimal $f_1 = c$. Otherwise, if $\underline{f}_1 < c$ there is no solution that satisfies all constraints and the market collapses.

case b: $\left[\frac{Pw - e(w + c)}{Pe(w + c)} \right] < 0$. In this case, Gap is strictly concave. We can in principle eliminate this case through the following reasoning. If $\left[\frac{Pw - e(w + c)}{Pe(w + c)} \right] < 0 \Rightarrow P < (w + c)$, since $e < w$. Then, given that we must have $f_1 < w + c$ – otherwise constraints (C.1), (C.4), and (C.6) cannot be jointly satisfied – we have that restrictions (C.2) and (C.4) cannot be jointly satisfied. Therefore, in this case the market collapses.

For the same reasons presented in case b, if $\left[\frac{Pw - e(w + c)}{Pe(w + c)} \right] = 0$ the market collapses.

Consequently, $f_1 = c$ is optimal for the bureau. ■

Proof of Lemma 2.

From the proof of Lemma 1, we showed that, in order for both restrictions (C.1'') and (C.3') to be satisfied, we must have:

$$\frac{P - (w + c)}{P - \frac{e}{w}(w + c)} \geq \frac{wc}{e(w + c)} \Rightarrow \frac{wc}{e(w + c)} \leq \frac{P - w}{P} \quad (95)$$

Consequently, for the cases in which we have a functioning market, (13) is satisfied and (C.3') is non-binding. ■

Proof of Corollary 1. From the proof of Lemma 1, we showed that, in order for both restrictions (C.1'') and (C.3') to be satisfied, we must have, after some algebraic manipulations:

$$c \leq \frac{ew(P - w)}{P(w - e) + ew} \quad (96)$$

Similarly, from the proof of Lemma 1, case b, we showed that in order to satisfy jointly restrictions (C.2) and (C.4), we also needed:

$$Pw - e(w + c) > 0 \Rightarrow c < \frac{Pw - ew}{e} \quad (97)$$

Now, let's compare the RHS((96)) against the RHS((97)). Then notice that:

$$\frac{Pw - ew}{e} - \frac{ew(P - w)}{P(w - e) + ew} = \frac{P^2w(w - e)}{e[P(w - e) + ew]} > 0$$

Therefore, whenever (96) is satisfied, (97) is also satisfied, concluding that our only constraint is (96). ■

Proof of Lemma 3. Suppose that the consumer is deciding on whether or not to join at some random time τ . If she does not join, her payoff is $(1 - Y_{A,member})P - w$. If she does join, her payoff is:

$$\begin{aligned} & (1 - Y_{A,member})(P - w) - f_{ee}^\tau + (1 - \delta)Y_{A,member}(-w) \left[1 + \sum_{t=1}^{\infty} \delta^t (1 - X_{A,member})^{t+\tau-1} \right] \\ & = (1 - Y_{A,member})(P - w) - f_{ee}^\tau - (1 - \delta)Y_{A,member}w \left[1 + \frac{\delta(1 - X_{A,member})^\tau}{1 - \delta(1 - X_{A,member})} \right]. \end{aligned} \quad (98)$$

Indifference between joining at time τ and not joining requires that:

$$\left\{ \begin{array}{l} (1 - Y_{A,member})(P - w) - f_{ee}^\tau \\ -(1 - \delta)Y_{A,member}w \left[1 + \frac{\delta(1 - X_{A,member})^\tau}{1 - \delta(1 - X_{A,member})} \right] \end{array} \right\} = (1 - Y_{A,member})P - w \quad (99)$$

Therefore, if the bureau announces (and credibly commits to) a sequence of fees $\{f_{ee}^t\}_{t=1}^\infty$, in this stationary equilibrium that we are looking at, they must satisfy:

$$f_{ee}^\tau = w\delta Y_{A,member} \left[1 - \frac{(1-\delta)(1-X_{A,member})^\tau}{1-\delta(1-X_{A,member})} \right] \quad (100)$$

Consequently, f_{ee}^τ is strictly increasing with τ and $\lim_{\tau \rightarrow \infty} f_{ee}^\tau = w\delta Y_{A,member}$.

■

Proof of Proposition 2. Substituting equation (21) into restriction (C.3) in the bureau's problem shown in (27), we have:

$$Y_{A,member} = \frac{[1 - \delta(1 - X_{A,member})]}{\delta X_{A,member} w} f_{ee} = \frac{X_{A,member}}{e} f_{ee}$$

Substituting f_{ee} :

$$Y_{A,member} = \frac{X_{A,member}}{e} \frac{cw}{(w+c)} = X_{A,member} \frac{cw}{e(w+c)} = X_{A,member} Y_{A,buy}$$

Consequently, $Y_{A,member} < Y_{A,buy} \Rightarrow SW_{A,member} > SW_{A,buy}$.

Finally, we just need to show that the fraction of informed consumers is higher with membership, i.e., $X_{A,member} > X_{A,buy}$. In order to show that, first notice that the LHS of (1) is strictly decreasing in $X_{A,buy}$. Moreover, notice that the LHS(1) = w if $X_{A,buy} = 0$ and the LHS(1) = $(1-\delta)w$ if $X_{A,buy} = 1$. In contrast, rearranging equation (20), we have:

$$w \left\{ 1 - \frac{\delta X_{A,member}^2}{1 - \delta [1 - X_{A,member}]} \right\} = w - e \quad (25')$$

Again, notice that the LHS(25') = w if $X_{A,member} = 0$ and the LHS(25') = $(1-\delta)w$ if $X_{A,member} = 1$. Moreover, taking the derivative of the LHS(25') with respect to $X_{A,member}$, we have:

$$\frac{dLHS(25')}{dX_{A,member}} = -\delta w \left\{ \frac{\delta X_{A,member}^2 + 2X_{A,member}(1-\delta)}{[1 - \delta(1 - X_{A,member})]^2} \right\} < 0. \quad (101)$$

Moreover:

$$\frac{d^2LHS(25')}{dX_{A,member}^2} = -\frac{2\delta(1-\delta)w}{[1 - \delta(1 - X_{A,member})]^3} < 0 \quad (102)$$

Consequently, the LHS(25') is strictly decreasing and concave in $X_{A,member}$. Then, given that the RHS(1) = RHS(25'), we must have that $X_{A,member} > X_{A,buy}$. Figure A.1 illustrates the result.

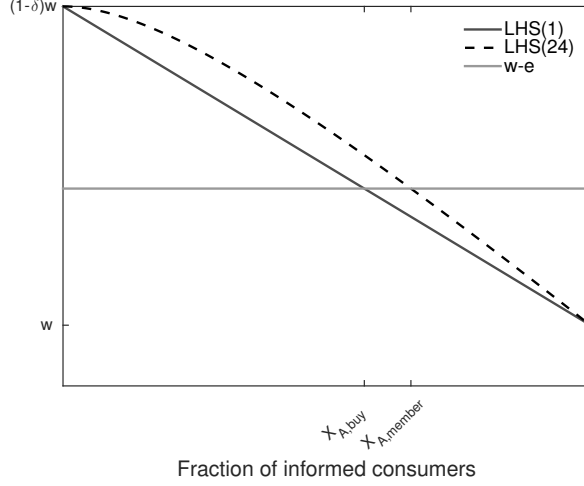


Figure A.1: Comparison $X_{A,buy}$ and $X_{A,member}$

■

Proof of Proposition 4. First, notice that Proposition 2 and Corollary 2 showed the first inequality in the left, while we showed the last equality in section 4. Therefore, we just need to show the two inequalities in the middle. In order to show that $SW_{A,buy} \geq SW_{C,buy}$, notice that:

$$Y_{C,buy} - Y_{A,buy} = \frac{2wc}{e(w+2c)} - \frac{wc}{e(w+c)} = \frac{wc}{e} \left[\frac{2}{w+2c} - \frac{1}{w+c} \right] = \frac{w^2c}{e(w+c)(w+2c)} > 0$$

Since $SW = \frac{1}{2\delta}(1-Y)P - e$, once $Y_{A,buy} < Y_{C,buy}$, we must have $SW_{A,buy} > SW_{C,buy}$. The equality occurs if we have that the constraint on uninformed consumers having to buy information is binding. However, as we mentioned in the last remark, this is only binding for $Y_{A,buy}$ if it is also binding for $Y_{C,buy}$. Finally, let's look at $SW_{C,buy} \geq SW_{M,buy}$. Notice that if $\frac{2cw}{e(w+2c)} < \frac{P-w}{P}$ is satisfied, then $Y_{C,buy} < Y_{M,buy}$ and $SW_{C,buy} > SW_{M,buy}$. If $\frac{2cw}{e(w+2c)} < \frac{P-w}{P}$ is not satisfied, the constraint on uninformed consumers hiring providers is binding and we have that $SW_{C,buy} = SW_{M,buy}$, concluding our proof. ■

Proof of Lemma 5.

Let us define the differentiable function $F : [0, 1] \times [0, 1] \rightarrow \mathbb{R}$ as

$$F(X_A, X_B) = \frac{w}{1 - \delta(1 - X_A - X_B)} \left\{ 1 + \delta \left(\frac{X_A(1-X_A)}{1-\delta(1-X_B)} + X_A X_B \delta \frac{(1-X_A-X_B)}{(1-\delta)(1-\delta(1-X_B))} + \frac{X_B(1-X_B)}{1-\delta(1-X_A)} + X_B X_A \delta \frac{(1-X_A-X_B)}{(1-\delta)(1-\delta(1-X_A))} \right) \right\} - \frac{w-e}{1-\delta}$$

and denote the level set of F corresponding to 0 by the set $\{(X_A, X_B) : X_A + X_B \leq 1 \text{ and } F(X_A, X_B) = 0\}$. In words, these are the possible combinations of each pair of consumer bases that makes the providers indifferent between exerting effort or not.

$$\frac{w}{1-\delta(1-X_A-X_B)} + \delta \frac{w}{1-\delta(1-X_A-X_B)} \left(\frac{X_A(1-X_A)}{1-\delta(1-X_B)} + X_A X_B \delta \frac{(1-X_A-X_B)}{(1-\delta)(1-\delta(1-X_B))} + \frac{X_B(1-X_B)}{1-\delta(1-X_A)} + X_B X_A \delta \frac{(1-X_A-X_B)}{(1-\delta)(1-\delta(1-X_A))} \right) = \frac{w-e}{1-\delta}$$

Note that $F(X_A, 0) = F(0, X_B)$ and x that solves $F(x, 0) = F(0, x) = 0$ is $X_A = \frac{e\delta + \sqrt{e^2\delta^2 + 4\delta(1-\delta)we}}{2\delta w} < 1$.

Rearranging it, we have:

$$\times \left\{ \frac{w}{1-\delta(1-X_A-X_B)} \times \frac{1}{(1-\delta)(1-\delta+\delta X_A)(1-\delta+\delta X_B)} \times \begin{aligned} &(1-\delta)(1-\delta+\delta X_A)(1-\delta+\delta X_B) + \\ &\delta(1-\delta)X_A(1-X_A)(1-\delta+\delta X_A) + \\ &+\delta^2 X_A X_B(1-X_A-X_B)(1-\delta+\delta X_A) + \\ &\delta(1-\delta)X_B(1-X_B)(1-\delta+\delta X_B) + \\ &\delta^2 X_B X_A(1-X_A-X_B)(1-\delta+\delta X_B) \end{aligned} \right\} = \frac{w-e}{1-\delta}$$

Notice then that:

$$\begin{aligned} &\left\{ \begin{aligned} &(1-\delta)(1-\delta+\delta X_A)(1-\delta+\delta X_B) + \\ &\delta(1-\delta)X_A(1-X_A)(1-\delta+\delta X_A) + \\ &+\delta^2 X_A X_B(1-X_A-X_B)(1-\delta+\delta X_A) + \\ &\delta(1-\delta)X_B(1-X_B)(1-\delta+\delta X_B) + \\ &\delta^2 X_B X_A(1-X_A-X_B)(1-\delta+\delta X_B) \end{aligned} \right\} = \\ &= (1-\delta(1-X_A-X_B)) \left\{ \begin{aligned} &-\delta^2 X_A^2 X_B - \delta(1-\delta)X_A^2 - \delta^2 X_A X_B^2 + \\ &+\delta^2 X_A X_B + \delta(1-\delta)X_A - \delta(1-\delta)X_B^2 \\ &+\delta(1-\delta)X_B + (1-\delta)^2 \end{aligned} \right\} \end{aligned}$$

Substituting back and rearranging, we have:

$$\frac{1}{(1-\delta+\delta X_A)(1-\delta+\delta X_B)} \left\{ \begin{aligned} &-\delta^2 X_A^2 X_B - \delta(1-\delta)X_A^2 - \delta^2 X_A X_B^2 + \\ &+\delta^2 X_A X_B + \delta(1-\delta)X_A - \delta(1-\delta)X_B^2 \\ &+\delta(1-\delta)X_B + (1-\delta)^2 \end{aligned} \right\} = \frac{w-e}{w}$$

Then, notice that:

$$\left\{ \begin{aligned} &-\delta^2 X_A^2 X_B - \delta(1-\delta)X_A^2 - \delta^2 X_A X_B^2 + \\ &+\delta^2 X_A X_B + \delta(1-\delta)X_A - \delta(1-\delta)X_B^2 \\ &+\delta(1-\delta)X_B + (1-\delta)^2 \end{aligned} \right\} = \left\{ \begin{aligned} &-\delta X_A^2(1-\delta+\delta X_B) \\ &-\delta X_B^2(1-\delta+\delta X_A) \\ &+(1-\delta+\delta X_A)(1-\delta+\delta X_B) \end{aligned} \right\}$$

Substituting it back and rearranging, we have:

$$\frac{e}{w} - \frac{\delta X_A^2}{1 - \delta + \delta X_A} - \frac{\delta X_B^2}{1 - \delta + \delta X_B} = 0 \quad (\star)$$

Then (\star) defines a functional F . Notice that:

$$F_A = -\frac{\delta X_A(2(1 - \delta) + \delta X_A)}{(1 - \delta + \delta X_A)^2} < 0$$

and

$$F_B = -\frac{\delta X_B(2(1 - \delta) + \delta X_B)}{(1 - \delta + \delta X_B)^2} < 0$$

Since (\star) implicitly defines X_B as a function of X_A , from the implicit function theorem, we have:

$$\frac{dX_B}{dX_A} = -\frac{F_A}{F_B} = -\left(\frac{-\frac{\delta X_A(2(1-\delta)+\delta X_A)}{(1-\delta+\delta X_A)^2}}{-\frac{\delta X_B(2(1-\delta)+\delta X_B)}{(1-\delta+\delta X_B)^2}}\right)$$

Simplifying it, we have:

$$\frac{dX_B}{dX_A} = -\frac{(1 - \delta + \delta X_B)^2 X_A(2(1 - \delta) + \delta X_A)}{(1 - \delta + \delta X_A)^2 X_B(2(1 - \delta) + \delta X_B)} < 0$$

■

Proof of Proposition 5. We will show that for each given pair of fees f_A and f_B , there is a unique pair (X_A, X_B) that simultaneously solves equations (46) and (48). First, let us show that for each given pair of fees f_A and f_B , equation (48) above defines a strictly increasing function $X_B(X_A)$. For convenience, let us define $\frac{f_A}{f_B} = \frac{1}{l}$.

$$\frac{X_A(1 - \delta) + \delta X_A X_B}{X_B(1 - \delta) + \delta X_A X_B} = \frac{1}{l}$$

Therefore, for each given pair of fees, the indifference condition of the consumers defines a relation between the consumer bases of the bureaus, and we can write X_B as an explicit function of X_A :

$$X_B = \frac{lX_A(1 - \delta)}{(1 - \delta) + \delta X_A(1 - l)} \quad (103)$$

Thus, the function is increasing in X , but it is discontinuous:

$$\frac{\partial X_B(X_A)}{\partial X_A} = \frac{l(1 - \delta)^2}{((1 - \delta) + \delta X_A(1 - l))^2} > 0, \forall l > 0$$

The shape of this function depends on l . Let us look at this function for each of the possible three cases.

- (1) If $f_A > f_B$ ($l < 1$), then $X_A > X_B > 0$ and $X_B(X_A)$ is a continuous and concave function;
- (2) If $l = 1$, then $X_B = X_A$;
- (3) Finally, if $f_A < f_B$ ($l > 1$) then $X_A < X_B$. Moreover, the function is discontinuous at $(1 - \delta) + \delta X_A(1 - l) = 0$, that is, there is a value $\hat{X}_A > 0$, given by

$$\hat{X}_A = \frac{(1 - \delta)}{\delta(l - 1)},$$

where if $X_A < \hat{X}_A$, then X_B that solves (103) is an increasing function from zero and increasing asymptotically to ∞ as X_A approaches \hat{X}_A .

Lemma (5) completes the proof, i.e., the pair (X_A, X_B) that solves (46) is a strictly decreasing function with both X_A and X_B positives, so there is a unique point at which this decreasing function crosses the curve $X_B(X_A)$ defined by (103). ■

Proof of Lemma 6. First, note that (f_i, f_j) with $f_i = f_j = \frac{P-w}{P} \frac{\delta w X^{sym}}{1-\delta(1-X^{sym})}$ with $X_i = X_j = X^{sym}$ and $Y = \frac{P-w}{P}$ is an equilibrium. Second, let us look at the case where $f_i < f_j = \frac{P-w}{P} \frac{\delta w X^{sym}}{1-\delta(1-X^{sym})}$. There is a unique pair (X_i, X_j) that solves both conditions (46) and (48). This pair is such that $X_i < X^{sym} < X_j$. Also, let

$$Y_{C,member} = \bar{f} \frac{1 - \delta(1 - X_j)}{\delta w X_j},$$

where $Y_{C,member} = \frac{P-w}{P} \frac{\delta X^{sym}}{1-\delta(1-X^{sym})} \frac{1-\delta(1-X_j)}{\delta X_j} < \frac{P-w}{P}$, since $\frac{\delta X^{sym}}{1-\delta(1-X^{sym})} < \frac{\delta X_j}{1-\delta(1-X_j)}$, so condition (49) is also satisfied.

Now suppose that $f_i = f_j < \frac{P-w}{P} \frac{\delta w X^{sym}}{1-\delta(1-X^{sym})}$. Then, let $X_i = X_j = X^{sym}$ and Y be given by

$$\begin{aligned} Y_{C,member} &= f_i \frac{1 - \delta(1 - X^{sym})}{\delta w X^{sym}} \\ &< \frac{P-w}{P} \frac{\delta w X^{sym}}{1-\delta(1-X^{sym})} \frac{1 - \delta(1 - X^{sym})}{\delta w X^{sym}} \\ &= \frac{P-w}{P}, \end{aligned}$$

so, again, condition (49) is satisfied. Finally, let $f_i < f_j < \frac{P-w}{P} \frac{\delta w X^{sym}}{1-\delta(1-X^{sym})}$. Again, there is a unique pair (X_i, X_j) that solves both conditions (46) and (48). This pair is such that

$X_i < X^{sym} < X_j$. Also, let

$$\begin{aligned}
Y_{C,member} &= f_j \frac{1 - \delta(1 - X_j)}{\delta w X_j} \\
&< \frac{P - w}{P} \frac{\delta X^{sym}}{1 - \delta(1 - X^{sym})} \frac{1 - \delta(1 - X_j)}{\delta X_j} \\
&< \frac{P - w}{P}.
\end{aligned}$$

■

Proof of Proposition 7. Suppose that this is not an equilibrium. First, assume that there is a profitable deviation for A in which A increases its fee. Thus, $f_A > f_B = \bar{f}$. In this case, to satisfy both conditions (46) and (47), we need a higher X_A and smaller X_B . However, note that for firm B to operate in a market with such fees, we need condition (47) to be satisfied. Given that $f_B = \bar{f}$ and that we require a smaller X_B , we need the new mass of providers buying in equilibrium to be higher, that is, $Y' > Y = \frac{P-w}{P}$, but this cannot be an equilibrium, since it violates (49). There are only two equilibria following such a deviation: one in which A is a monopolist and one in which B is a monopolist. Given our refinement, we will assume that following such a deviation, only bureau B (because it has a lower fee) will operate, a contradiction.

Suppose that the profitable deviation is one in which A decreases its fee. Thus, $f_A < f_B = \bar{f}$. From Lemma (6) we know that there exists a unique equilibrium with two operating bureaus. Now, to satisfy both conditions (46) and (47), we need a lower X_A and a higher X_B . A lower X_A , together with a lower f_A , implies that A has decreased its profit, so this is not a profitable deviation either. This proves that the proposed candidate is indeed a stationary competitive equilibrium. Below, we prove that it is unique.

Suppose that bureau j is a monopolist and is charging a feasible fee f_j . Then, any fee $0 < f_i < f_j$ is a profitable deviation for firm i , since it either accommodates two bureaus in the continuation game or it shifts the monopoly to firm i ; in either case, firm i will make positive profits.

Now suppose that two firms are operating and $f_i < f_j$. If i increases its fee (but such that it is still lower than f_j) the new pair (f'_i, f_j) will increase the consumer basis of firm i , which, together with the higher fee, increases its profit. This proves that a stationary competitive equilibrium must be symmetric. Finally, suppose that $f_i = f_j < \bar{f}$. Then, $X_i = X_j = X^{sym}$, and $Y < \frac{P-w}{P}$. Consider a deviation in which firm i increases f_i such that both firms still operate (otherwise, given our refinement, only j will operate). We know from Lemma (6) that such a deviation in which the equilibrium in the continuation game has two operating bureaus exists. Then, the new consumer basis of firm i must increase $X'_i > X^{sym} > X'_j$. Given that this is a profitable deviation

for firm i , such an equilibrium cannot exist either. Therefore, the symmetric equilibrium with $f_i = f_j = \bar{f}$ is the unique equilibrium in which two bureaus operate. ■

Proof of Lemma 7. Toward a contradiction, assume that (C.1) is non-binding. Then, from (C.3), we have that:

$$\frac{\partial Y_{A,mq}}{\partial f_{ee}} > 0 \quad \text{and} \quad \frac{\partial Y_{A,mq}}{\partial q} = \frac{(1-\delta)^2 e f_{ee}}{\delta^2 w [(1-q)X_{A,mq}]^2 (X_{A,mq}w - e)^2} \frac{\partial X_{A,mq}}{\partial q} > 0$$

Consequently, we can reduce $Y_{A,mq}$ by either reducing f_{ee} or q without violating (C.1). Since $\frac{\partial \text{SW}_{A,mq}}{\partial Y_{A,mq}} < 0$, we conclude that at the optimum we must have (C.1) binding. ■

Proof of Lemma 8. After some manipulations, taking a total derivative of equation(54) with respect to q and manipulating it, we have:

$$\frac{dX_{A,mq}}{dq} = \frac{X_{A,mq}(wX_{A,mq} - e)}{(1-q)\{2wX_{A,mq} - e\}} \quad (104)$$

Again, from the provider's problem equation (54), after rearranging, we obtain:

$$\frac{1 - \delta[1 - (1-q)X_{A,mq}]}{\delta X_{A,mq}} = \frac{(1-\delta)w}{\delta(wX_{A,mq} - e)}$$

Once we assume that (C.1) is binding, from (C.3), we have:

$$Y_{A,mq} = \frac{c}{w + c(1-q)} \left\{ \frac{1 - \delta[1 - (1-q)X_{A,mq}]}{\delta X_{A,mq}} \right\} \quad (105)$$

Substituting (104) into (105), we have:

$$Y_{A,mq} = \frac{(1-\delta)cw}{\delta(wX_{A,mq} - e)} \times \frac{1}{w + c(1-q)} \quad (106)$$

Then, taking the derivative with respect to q , we have:

$$\frac{\partial Y_{A,mq}}{\partial q} = \frac{(1-\delta)cw}{\delta(wX_{A,mq} - e)^2 (w + c(1-q))^2} \times \left\{ w \frac{dX_{A,mq}}{dq} (w + c(1-q)) - c(wX_{A,mq} - e) \right\}$$

Therefore, $\frac{\partial Y_{A,mq}}{\partial q} < 0$ if:

$$w \frac{dX_{A,mq}}{dq} (w + c(1-q)) - c(wX_{A,mq} - e) > 0 \quad (107)$$

Rearranging it and substituting (104), we have:

$$c(1 - q) < \frac{[w + c(1 - q)]wX_{A,mq}}{2wX_{A,mq} - e} \quad (108)$$

Rearranging it:

$$c(1 - q) < \frac{w^2 X_{A,mq}}{wX_{A,mq} - e} \quad (109)$$

Note that $\frac{w^2 X_{A,mq}}{wX_{A,mq} - e} > \frac{w^2}{w - e}$, since $\frac{\partial \left\{ \frac{w^2 X_{A,mq}}{wX_{A,mq} - e} \right\}}{\partial X_{A,mq}} < 0$. Thus, if we show that $c(1 - q) < \frac{w^2}{w - e}$ we are done. But then, notice that:

$$c(1 - q) < c < \frac{we}{w - e} < \frac{w^2}{w - e}$$

where $c < \frac{we}{w - e}$ is trivially satisfied by the parameter restriction established in Lemma 3. ■

Proof of Corollary 3. Keep in mind that the fraction of unknown bad providers in period t is given by $[1 - X(1 - q)]^t$. Then, the rate at which the fraction of unknown bad providers declines over time is given by:

$$\frac{[1 - X(1 - q)]^{t+1} - [1 - X(1 - q)]^t}{[1 - X(1 - q)]^t} = -X(1 - q), \quad \forall t \in \mathbb{N}$$

Consequently, the decline in the fraction of unknown bad providers is constant at $X(1 - q)$ for all t . But then, totally differentiating equation (54), we have:

$$\frac{dX(1 - q)}{dq} = -\frac{(1 - \delta)e}{\delta(Xw - e)^2} \times \frac{dX}{dq} w < 0.$$

Consequently, the speed of learning declines with q . ■

Proof of Proposition 9

Proof. First, from the LHS of equation (20), we have that:

$$w \left\{ 1 - \frac{\delta X_{A,member}^2}{1 - \delta [1 - X_{A,member}]} \right\} = w - e \quad (14')$$

Notice that the LHS(14') = w if $X_{A,member} = 0$ and the LHS(14') = $(1 - \delta)w$ if $X_{A,member} = 1$. Moreover, taking the derivative of LHS(14') with respect to $X_{A,member}$, we have:

$$\frac{dLHS(14')}{dX_{A,member}} = -\delta w \left\{ \frac{\delta X_{A,member}^2 + 2X_{A,member}(1 - \delta)}{[1 - \delta(1 - X_{A,member})]^2} \right\} < 0. \quad (110)$$

Moreover:

$$\frac{d^2 LHS(14')}{dX_{A,member}^2} = -\frac{2\delta(1-\delta)w}{[1-\delta(1-X_{A,member})]^3} < 0 \quad (111)$$

Consequently, the LHS(14') is strictly decreasing and concave in $X_{A,member}$.

While, in the buy and sell case, a provider is indifferent between whether or not to exert effort if:

$$(1-\delta)w + \delta[(1-X_{A,buy}) \times w + X_{A,buy} \times 0] = w - e \quad (112)$$

Again, notice that the LHS(112) = w if $X_{A,buy} = 0$ and the LHS(112) = $(1-\delta)w$ if $X_{A,buy} = 1$. Moreover, notice the derivative of the LHS(112) with respect to $X_{A,buy}$ is $-\delta w$ and the function is linearly decreasing.

Then, given that RHS(112) = RHS(14'), we must have that $X_{A,member} > X_{A,buy}$. Figure A.2 below illustrates the result.

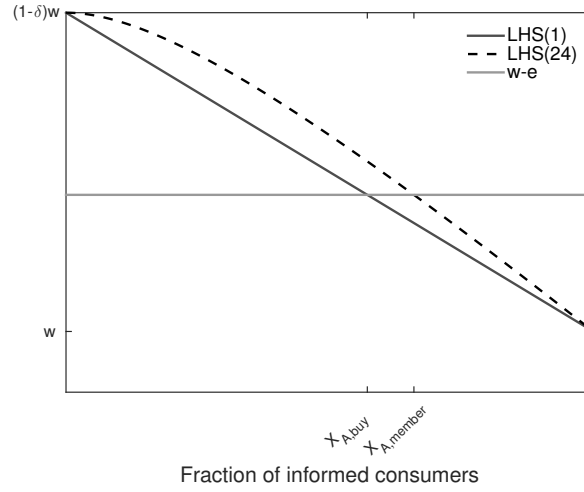


Figure A.2: Comparison $X_{A,buy}$ and $X_{A,member}$

Then, let's compare $Y_{A,member}$ and $Y_{A,buy}$. First, from (72), we have:

$$\frac{\partial Y_{A,member}}{\partial X} = \frac{-\delta(1-\delta)\delta c_1(w+c)}{\{\delta Xw - (1-\delta)c\}^2} < 0 \quad (113)$$

Therefore, the higher X , the lower $Y_{A,member}$. As we showed before, $X_{A,member} > X_{A,buy}$. Then, we have that:

$$Y_{A,member} < \delta c_1 \frac{[1-\delta(1-X_{A,buy})]}{\delta X_{A,buy}w - (1-\delta)c}$$

Substituting $X_{A,buy} = \frac{e}{\delta w}$:

$$Y_{A,member} < \delta c_1 \frac{[1-\delta + \frac{e}{w}]}{e - (1-\delta)c} \quad (114)$$

From (114), notice that:

$$\frac{\partial \left(\frac{Z + \frac{e}{w}}{1 - Zc} \right)}{\partial Z} = \frac{(1 - Zc + c(Z + \frac{e}{w}))}{(1 - Zc)^2} > 0 \quad (115)$$

Given that $X_{A,buy} < 1$ and $X_{A,buy} = \frac{e}{\delta w} \Rightarrow \delta > \frac{e}{w} \Rightarrow 1 - \delta < \frac{w-e}{w}$. Then, from the inequality obtained in (115):

$$Y_{A,member} < \delta c_1 \frac{w}{we - (w - e)c} \quad (116)$$

Since $\delta > \frac{e}{w}$:

$$Y_{A,member} < \frac{e}{w} c_1 \frac{w}{we - (w - e)c} \equiv Y_{A,buy} \quad (117)$$

So $Y_{A,member} < Y_{A,buy}$. Finally, from equations (66) and (70), we have that $\mathbf{SW}_{A,member} > \mathbf{SW}_{A,buy}$. ■

Proof of Proposition 10

Proof. *Buying and selling Information*

Assuming $f_1 = c$, bureau's profits are given by:

$$\Pi_{M,buy} = \max_{f_2 \geq 0} \frac{\delta}{1 - \delta} f_2 X_{M,buy} - \frac{(1 - \delta X_{M,buy}) Y_{M,buy} c}{1 - \delta} \quad (118)$$

subject to:

$$0 \leq \frac{f_2 + c_1}{w} \leq \min \left\{ \frac{P-w}{P}, 1 \right\} \quad (C.1)$$

$$X_{M,buy} = \frac{e}{\delta w} \quad (C.2)$$

$$Y_{M,buy} = \frac{f_2 + c_1}{w} \quad (C.3)$$

Notice that $\Pi_{M,buy}$ is linearly increasing in f_2 if $c < \frac{ew}{w-e}$, which is the same restriction that we obtained in the autarky case with buy and sell.

Then, since $\frac{P-w}{P} < 1$, we have that:

$$f_2 = \frac{P(w - c_1) - w^2}{P} \quad (119)$$

Then, social welfare gives us:

$$SW_{M,buy} = \frac{1}{2(1 - \delta)} \{(1 - Y_{M,buy})P - w + w - e\}$$

Substituting $Y_{M,buy}$:

$$SW_{M,buy} = \frac{1}{2(1 - \delta)} \{w - e\} \quad (120)$$

While Bureau's profit is given by:

$$\Pi_{M,buy} = \frac{1}{1-\delta} \left\{ \left(\frac{P-w}{P} \right) \left[e - \left(1 - \frac{e}{w} \right) c \right] - \frac{e}{w} c_1 \right\} \quad (121)$$

which is positive as long restriction (65) is satisfied.

Membership Case

In this case, the bureau's profit is given by:

$$\Pi_{M,member} = X_{M,member} \left\{ f_{ee}^M - Y_{M,member} c + \sum_{t=1}^{\infty} \delta^t [f_{ee}^M - Y_{M,member} (1 - X_{M,member})^t c] \right\} \quad (122)$$

Rearranging it, the bureau's problem becomes:

$$\Pi_{M,member} = \max_{f_{ee}^M \geq 0} X_{M,member} \left\{ \frac{f_{ee}^M}{1-\delta} - \frac{Y_{M,member}}{1-\delta(1-X_{M,member})} c \right\} \quad (123)$$

subject to:

$$f_{ee}^M \leq \delta \left[\frac{X_{M,member} w}{1-\delta(1-X_{M,member})} \left(1 - \frac{w}{P} \right) - c_1 \right] \quad (C.1)$$

From (21), we have:

$$\frac{Y_{M,member}}{1-\delta(1-X_{M,member})} = \frac{f_{ee}^M + \delta c_1}{\delta X_{M,member} w} \quad (124)$$

Substituting it back into (123), we have:

$$\Pi_{M,member} = \max_{f_{ee}^M \geq 0} X_{M,member} \left\{ \frac{f_{ee}^M}{1-\delta} - \frac{(f_{ee}^M + \delta c_1)}{\delta X_{M,member} w} c \right\} \quad (125)$$

Notice that whenever $\Pi_{M,member} > 0$ we must have $\frac{\partial \Pi_{M,member}}{\partial f_{ee}^M} > 0$. Consequently, for the relevant cases, Π is linearly increasing in f_{ee}^M . So, the optimal membership fee has (C.1) binding, i.e.:

$$f_{ee}^M = \delta \left[\frac{X_{M,member} w}{1-\delta(1-X_{M,member})} \left(1 - \frac{w}{P} \right) - c_1 \right] \quad (126)$$

Substituting (21) into (126), we have:

$$f_{ee}^M = \frac{e}{X_{M,member}} \left(\frac{P-w}{P} \right) - \delta c_1 \quad (127)$$

Then the social welfare gives us:

$$SW_{M,member} = \frac{1}{2(1-\delta)} \{w - e\} \quad (128)$$

So $SW_{M,buy} = SW_{M,member}$.

While profits are given by:

$$\Pi_{M,member} = \frac{1}{1-\delta} \left(\frac{P-w}{P} \right) \left\{ e - \frac{(1-\delta)e}{\delta X_{M,member} w} c \right\} - \frac{\delta X_{M,member}}{1-\delta} c_1 \quad (129)$$

From (21) we have:

$$\delta X_{A,member}^2 w = (1-\delta)e + \delta X_{A,member} e \quad (130)$$

Then, we have that:

$$\frac{(1-\delta)e}{\delta X_{A,member} w} = X_{A,member} - \frac{e}{w} \quad (131)$$

Substituting (131) into (129), we have:

$$\Pi_{M,member} = \frac{1}{1-\delta} \left\{ \left(\frac{P-w}{P} \right) \left[e - \left(X_{A,member} - \frac{e}{w} \right) c \right] - \delta X_{M,member} c_1 \right\} \quad (132)$$

Comparing (132) and (121), keeping in mind that $X_{M,member} < 1$, we can easily see that $\Pi_{M,member} > \Pi_{M,buy}$. ■

Proof of Lemma 9

Proof. Consider the case in which the providers puts in effort when facing a non-member with probability $\theta \in (0, 1)$. Similarly, assume that a non-member hires the service of a provider with probability $\gamma_{non-member}$, while members hire services with probability γ_{member} .

Without loss of generality for the proof at hand, let's consider that the providers puts in effort in the first period (before memberships are established). In this case, the expected payoff of following a strategy in which the provider always puts in effort when facing a member, but only puts in effort when facing a non-member with probability θ is:

$$w - e + \delta \frac{X_{A,k-m} \gamma_{member}}{1-\delta} (w - e) + \delta \frac{(1 - X_{A,k-m}) \gamma_{non-member}}{1-\delta} w - \delta \frac{(1 - X_{A,k-m}) \gamma_{non-member}}{1-\delta} \theta e \quad (133)$$

Notice that (133) is weakly decreasing in θ and strictly decreasing whenever $(1 - X_{A,k-m}) \gamma_{non-member} > 0$. Consequently, setting $\theta = 0$ is an optimal deviation, concluding our proof. ■

Proof of Lemma 10

Proof. From the calculations for the case of unknown membership, we have:

$$Y_{A,u-m} = \delta c_1 \frac{[1 - \delta(1 - X_{A,u-m})]}{\delta X_{A,u-m} w - (1 - \delta)c} \quad (134)$$

Then, from (134) and $Y_{A,k-m} = \frac{c_1}{w}$, we have:

$$Y_{A,k-m} - Y_{A,u-m} = -\frac{(1 - \delta)c_1 [\delta(1 - X_{A,u-m})w + c]}{w [\delta X_{A,u-m} w - (1 - \delta)c]} < 0 \quad (135)$$

■

Proof of Proposition 12

Proof. From previous calculations, we have:

$$SW_{A,k-m} - SW_{A,u-m} = \frac{1}{2(1 - \delta)} [(Y_{A,u-m} - Y_{A,k-m})P - f_{ee}^{k-m}]$$

From Corollary 5, we have that $Y_{A,u-m} \rightarrow Y_{A,k-m}$ as $\delta \rightarrow 1$. So the first term in squared brackets goes to zero as $\delta \rightarrow 1$. In contrast, $f_{ee}^{k-m} = \frac{c_1}{w} c [1 - \frac{e}{w}]$ and therefore constant as δ approaches 1. Finally, $2(1 - \delta) \rightarrow 0$ as $\delta \rightarrow 1$, completing the proof. ■

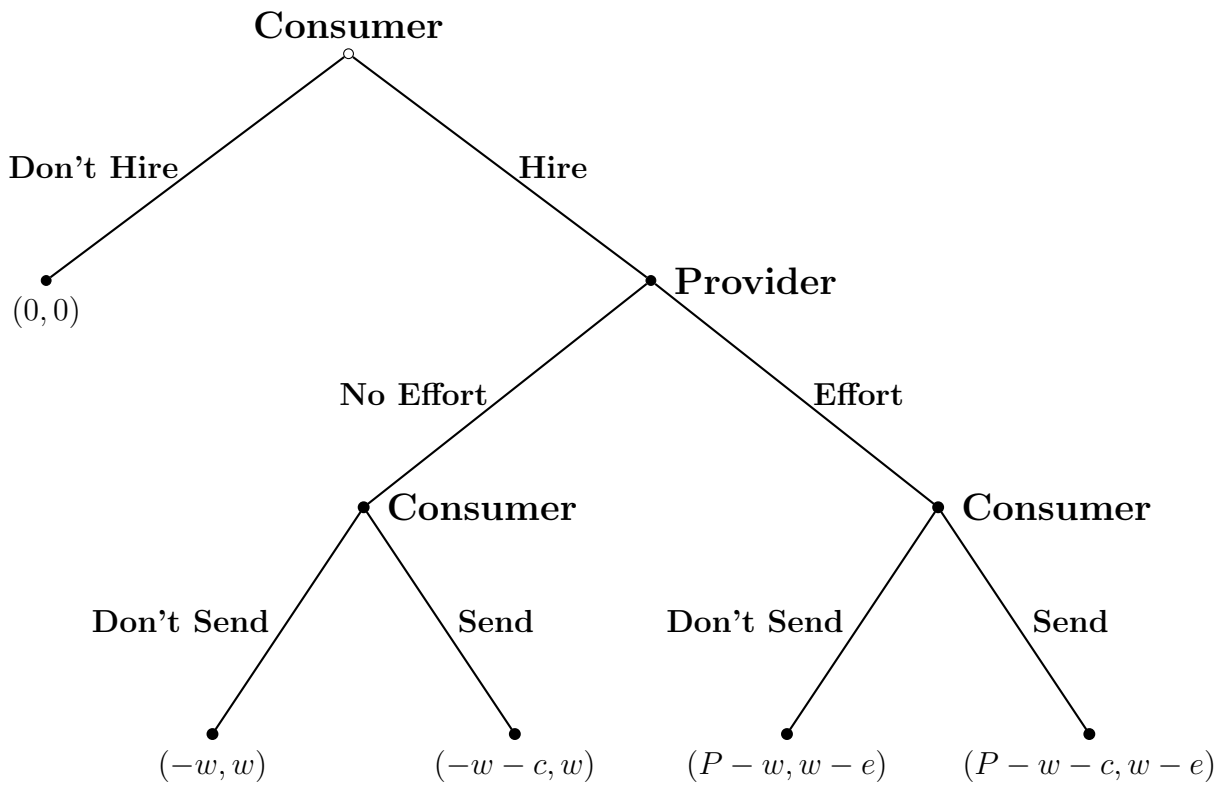


Figure 1: Game Tree – No Bureau Case

Figure 2: Extended Game Tree – Bureau That Buys and Sells Information

