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Goods-Market Frictions and International Trade

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We add goods-market frictions to a general equilibrium dynamic model with heterogeneous exporting producers and identical importing retailers. Our tractable framework leads to endogenously unmatched producers, which attenuate welfare responses to foreign shocks but increase the trade elasticity relative to a model without search costs. Search frictions are quantitatively important in our calibration, attenuating welfare responses to tariffs by 40 percent and increasing the trade elasticity by 50 percent. Eliminating search costs raises welfare by 1 percent and increasing them by only a few dollars has the same effects on welfare and trade flows as a 10 percent tariff.

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1 Introduction

Locating and building connections with overseas buyers is a prevalent firm-level barrier to exporting.[1] Moreover, firms pursue a number of costly activities to overcome these barriers.[2] Despite the prevalence and magnitude of these costs at the firm level, how these barriers affect aggregate welfare and trade flows in general equilibrium is not well understood.

In this paper, we formalize this barrier to exporting as a goods-market friction between importing retailers and exporting producers in a Melitz-style model. The key insight is that there exists an endogenous fraction of producers that are actively looking for retailers but are yet to match with a partner. This unmatched rate alters the *levels* of aggregate variables and the *changes* in aggregate variables in response to shocks because when producers are unmatched their associated varieties cannot be traded. We derive analytic expressions for the welfare response to foreign shocks, the elasticity of consumption and imports with respect to iceberg costs, and the gravity equation, showing that search frictions have first-order effects. Finally, we quantify and decompose the general equilibrium effects of search frictions with a calibrated version of the model.

Our search theoretic framework is motivated by a literature that documents the important role these frictions play in input markets and most closely resembles Pissarides (2000, Ch. 1). We embed these frictions into a general equilibrium model with heterogeneous producers and identical retailers in the style of Hopenhayn (1992) and Melitz (2003). Our model includes many destination-origin markets and we assume that all retailers and producers, including those in each domestic-domestic market, face search frictions. Each search market in our model is summarized by an endogenous sufficient statistic called "market tightness," which is defined as the ratio of searching retailers to searching producers

[1] Kneller and Pisu (2011) find that "identifying the first contact" and "establishing initial dialogue" are more common obstacles to exporting than "dealing with legal, financial and tax regulations overseas" in a survey of U.K. firms.

[2] Eaton et al. (2014) report that the four most expensive costs for Colombian exporters (in order) are maintaining foreign sales offices, supporting sales representatives abroad, researching potential foreign buyers, and sustaining a web presence.

and is determined by retailer entry. Market tightness determines the rate at which producers and retailers contact one another, which in turn determines the unmatched rate of producers and the associated mass of unmatched product varieties. Unmatched varieties cannot be consumed and are therefore absent from the indirect utility (welfare) function, price index, and other aggregates. This feature sets our work apart from standard new trade models, in which every firm that chooses to export finds a buyer, but our framework nests those models when we remove the search friction. Our model also differs in that the equilibrium negotiated import price for each variety is the outcome of a generalized Nash bargaining game between matched retailers and producers. Our theory remains analytically tractable and has rich implications for firm-level trading relationships and economic aggregates.

We find that endogenous unmatched rates attenuate the welfare response to foreign shocks relative to a model without search frictions. For example, increasing foreign tariffs raises the general equilibrium price index in the domestic market. A higher price index allows domestic retailers to sell domestic consumers more and raises the incentive for retailers to enter the domestic market. More searching domestic retailers raises domestic market tightness, which raises domestic producers' finding rate, and lowers the domestic producer unmatched rate. Protecting the domestic market with tariffs hurts consumers by raising the price index but helps them by increasing the number of domestic varieties that can be consumed. More generally, our analytic expression extends the welfare results in Arkolakis, Costinot, and Rodríguez-Clare (2012) to an environment that includes endogenous goods-market frictions between importers and exporters.

We also find that search frictions magnify the response of consumption and imports to iceberg trade costs. In many standard trade models, the consumption and import elasticities are the same. This need not be the case in general, and in our model, they differ because consumption is evaluated at final sales prices, while imports are evaluated at negotiated prices. Both elasticities are affected by the unmatched rate and are always at least as negative as the analogous elasticity in a model without goods-market frictions. As a destination raises tariffs on products from a specific country of origin, retailers in the destination country have less incentive to enter the search market. Having fewer retailers implies a looser search market and a higher unmatched rate for producers, which reduces consumption and imports even more than in a model without search.

Unsurprisingly, goods-market frictions reduce aggregate import flows relative to a model without them. They do so in two ways. First, the negotiated import price is always lower than the final sales price paid by consumers and ensures that the importing retailer can at least pay their search costs. Many standard models evaluate aggregates at consumer prices whereas our model has a gap between import and final sales prices that reduces the value of imports. Second, the unmatched rate reduces aggregate imports because a fraction of exported varieties are not matched to importing retailers in equilibrium. While aggregate imports are lower, the quantity of any variety traded (intensive margin) between *matched* retailers and producers in our model is the same as in a model without search because the two parties still seek to maximize the profits earned from consumers.

We calibrate the model using aggregate U.S. and Colombian data, estimates from partial equilibrium micro-level models using the same countries, and standard trade and labor-search parameter values. To calibrate the unmatched rates we use the fraction of U.S. (Colombian) firms exporting to Colombia (the United States) and manufacturing capacity utilization rates in each country. We assume the technology matching exporters and importers is similar to the technology matching workers and firms and therefore calibrate the matching function using estimates from Petrongolo and Pissarides (2001). Many of our search cost parameters are from Eaton et al. (2014), while structural estimates of sunk costs are from Das, Roberts, and Tybout (2007). We calibrate the model's additional trade parameters using standard estimates from Axtell (2001), Broda and Weinstein (2006), Chaney (2008), and Baily and Bosworth (2014). As a whole, the model delivers a realistic economic environment for the United States and Colombia by matching several important aggregate facts.

Search frictions play an important quantitative role for welfare, trade flows, and the consumption elasticity in four results from the calibrated version of our model. First, eliminating search costs for all retailers would increase U.S. welfare by about 1 percent. Second, welfare changes in response to unilateral tariff increases are about 40 percent smaller in our model than in a model without search frictions despite larger import responses. This is because, relative to the standard model, the domestic producer matched rates attenuate the response of welfare but the international producer matched rates magnify the response of imports. Third, small increases in the search costs retailers face can have impacts on welfare that are commensurate with large increases in bilateral tariffs. For example, raising the average equilibrium costs for importing retailers to contact foreign producers by only a few dollars has the same impact on trade flows and welfare as a 10 percent increase in bilateral tariffs. Increasing search costs has these large effects on trade flows because it reduces retailer entry, lowers producers' finding rates, and thereby raises the unmatched rate. Fourth, search frictions, through their effect on the unmatched rate, raise the consumption elasticity with respect to iceberg costs by over 50 percent.

There is a recent expanding literature on search between international trading partners. Our paper builds upon the results of Antràs and Costinot (2011), who model intermediation between traders (retailers) and farmers (producers) in a Ricardian model by accommodating many countries and differentiated goods, and presenting quantitative exercises. Our model also shares many features with Benguria (2015), who shows that goods-market frictions provide a micro foundation for the costs of entering foreign markets. Allen (2014) rationalizes search frictions as costly information acquisition about agricultural market conditions across regions in the Philippines. Eaton et al. (2014) and Eaton et al. (2016) structurally estimate complex search models with endogenous contact rates, many-to-many matches, and learning about foreign markets but are restricted to partial equilibrium frameworks. Our model differs from these two papers because it presents a more stylized search process that admits straightforward aggregation and shows analytically how exogenous changes affect aggregates in general equilibrium. Brancaccio, Kalouptsidi, and Papageorgiou (2017) also have a setup similar to ours, but their focus is on a partial equilibrium microeconomic model of transportation through shipping, while Startz (2018) includes both contracting and search frictions. Lenoir, Mejean, and Martin (2018) focus on explaining empirical patterns in French export data using a partial equilibrium Eaton and Kortum (2002) model with search frictions.

Our model provides search-theoretic micro foundations resembling Rauch (1999) for several empirical estimation approaches used by previous authors. In particular, our simple closed-form gravity equation is consistent with papers that estimate gravity equations and include proxies that might capture search frictions, as in Rauch and Trindade (2002) and Portes and Rey (2005). Our framework is also consistent with the empirical relevance of international intermediaries that move goods from producers to final consumers, as documented by Bernard et al. (2010) and Ahn, Khandelwal, and Wei (2011).

The remainder of the paper is organized as follows. Section 2 describes the model and section 3 characterizes optimal search and matching behavior by producers and retailers. Section 4 discusses aggregation and the model's equilibrium. Section 5 derives the analytic implications of our framework for changes in welfare to foreign shocks, the consumption and import elasticities, and the gravity equation. Section 6 discusses the calibration strategy and model fit. Section 7 presents several general equilibrium exercises that quantify the role of goods-market frictions for welfare, trade flows, the unmatched rate, and the consumption and import elasticities. Lastly, section 8 presents a discussion of further research.

2 Model

2.1 Setup

Our model features many countries and is similar to Melitz (2003) and Chaney (2008). In particular, we are motivated by the facts summarized in Bartelsman and Doms (2000) and Syverson (2011): Even within similar industries, firms exhibit persistent differences in measured productivity. We index producers of goods by their productivity, φ. This permanent productivity is exogenously given and known to producers.

As is standard, each country has a representative consumer that has utility over products, including a homogeneous good and differentiated varieties from all countries. Our model, however, assumes that these consumers can access differentiated goods only via ex-ante homogenous intermediaries called retailers.[3] Moreover, as in the work by Diamond (1982), Pissarides (1985), and Mortensen (1986), a costly process of search governs how producers and retailers find one another. Aside from this goods-market friction, our model nests Melitz (2003) and Chaney (2008). We develop a continuous-time framework and focus on steady-state implications.

We index each differentiated-goods market using *do* to denote destination-origin country pairs. This market includes exporting producers in country o and importing retailers in country d. We will sometimes omit this notation to conserve space. To maintain tractability, we assume that searching or matching in one market does not affect the costs of searching across other markets. In particular, there are no economies of scale in one market for individual producers and retailers from currently being in a match or from searching in other markets. These assumptions ensure that we can study each "segmented" market independently because, although individual behavior will affect (and be affected by) aggregate variables, they are taken as given by atomistic producers and retailers.

2.2 Consumers

We assume the representative consumer in destination market d has Cobb-Douglas utility, U_d, over a homogeneous good and a second good that is a constant elasticity of substitution (CES) aggregate of differentiated varieties, indexed by ω, from all origins, indexed by $k \in \{1, \dots, O\}$. The two goods are combined with exponents $1 - \alpha$ and α, respectively. The differentiated goods are substitutable with constant elasticity, $\sigma > 1$, across varieties and destinations and we denote the value of total consumption as C_d in destination country d.

[3] Although in principle producers could circumvent retailers and contact final consumers directly, we avoid this possibility by assuming that the net value of matching with a retailer is always greater than the net value of forming a relationship directly with a final consumer. This approach is similar to earlier work by Wong and Wright (2014), who assume that a middleman is necessary rather than deriving the conditions under which this is the case.

Formally the consumer's problem is

$$\begin{aligned} \max_{q_d(1), q_{dk}(\omega)} \quad & q_d(1)^{1-\alpha} \left[\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} q_{dk}(\omega)^{\left(\frac{\sigma-1}{\sigma}\right)} d\omega \right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)} \\ s.t.\ C_d \quad = \quad & p_d(1)\, q_d(1) + \sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} p_{dk}(\omega)\, q_{dk}(\omega)\, d\omega, \end{aligned} \tag{1}$$

which results in the following demand for the homogeneous good and each differentiated variety, respectively

$$q_d(1) = \frac{(1-\alpha)\, C_d}{p_d(1)}, \qquad q_{do}(\omega) = \alpha C_d \frac{p_{do}(\omega)^{-\sigma}}{P_d^{1-\sigma}}. \tag{2}$$

Cobb-Douglas preferences across sectors imply that the consumer allocates share $1-\alpha$ of total consumption expenditure to the homogeneous good and share α to the differentiated goods. The homogeneous good has price $p_d(1)$. Define P_d as the price index for the bundle of differentiated varieties, which is given by

$$P_d = \left[\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} p_{dk}(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}. \tag{3}$$

The ideal price index that minimizes expenditure to obtain utility level $U_d = 1$ is $\Xi_d = \left(\frac{p_d(1)}{1-\alpha}\right)^{1-\alpha} \left(\frac{P_d}{\alpha}\right)^{\alpha}$. To derive these equations, we solve the consumer's utility maximization and expenditure minimization problems explicitly in appendix A.1.

2.3 The matching function, producers, retailers, and bargaining

In section 2.3.1 we describe the technology that determines how matches between producers and retailers form. In sections 2.3.2 and 2.3.3 we depart from the standard environment by introducing a goods-market friction between these foreign producers and domestic retailers, which affects their optimal decisions. In section 2.3.4 we describe how price and quantity are negotiated between a matched retailer and producer.

2.3.1 The matching function

The matching function, denoted by $m\left(u_{do}N_o^x, v_{do}N_d^m\right)$, gives the flow number of relationships formed at any moment in time as a function of the stock number of unmatched producers, $u_{do}N_o^x$, and unmatched retailers, $v_{do}N_d^m$, in the *do* market. N_o^x and N_d^m represent the total mass of producing firms in country o and retailing firms in country d, respectively, that exist regardless of their match status. The fraction of producers in country o looking for retailers in country d is u_{do}. The fraction of retailers that are searching for producing firms in this market is v_{do}.

As in many studies of the labor market (Pissarides, 1985; Shimer, 2005), we assume that the matching function takes a Cobb-Douglas form:

$$m\left(u_{do}N_o^x, v_{do}N_d^m\right) = \xi\left(u_{do}N_o^x\right)^{\eta}\left(v_{do}N_d^m\right)^{1-\eta}. \tag{4}$$

Because this matching function is homogeneous of degree one, market tightness, $\kappa_{do} = v_{do}N_d^m/u_{do}N_o^x$, which is the ratio of the mass of searching retailers to the mass of producers in a given market, is sufficient to determine contact rates on both sides of that market.[4] In particular, the rate at which retailers in country d contact producers in country o, $\chi\left(\kappa_{do}\right)$, is the number of matches formed each instant over the number of searching retailers:

$$\chi\left(\kappa_{do}\right) = \frac{m\left(u_{do}N_o^x, v_{do}N_d^m\right)}{v_{do}N_d^m} = \frac{\xi\left(u_{do}N_o^x\right)^{\eta}\left(v_{do}N_d^m\right)^{1-\eta}}{v_{do}N_d^m} = \xi\kappa_{do}^{-\eta}.$$

Notice that retailers' contact rate falls with market tightness ($\chi'\left(\kappa_{do}\right) < 0$) because as there are more retailers relative to producers, the search market becomes congested with retailers.

The rate at which producers in country o contact retailers in country d is the number of

[4] We use continuous time Poisson processes to model the random matching of retailers and producers. Thus, the contact rate defines the average number of counterparty meetings during one unit of time. Appendix A.2 contains more details.

matches formed each instant over the number of searching producers:

$$\frac{m\left(u_{do}N_o^x, v_{do}N_d^m\right)}{u_{do}N_o^x} = \frac{\xi\left(u_{do}N_o^x\right)^{\eta}\left(v_{do}N_d^m\right)^{1-\eta}}{u_{do}N_o^x} = \xi\kappa_{do}^{1-\eta} = \kappa_{do}\chi\left(\kappa_{do}\right),$$

in which producers' contact rate rises with tightness ($d\kappa_{do}\chi\left(\kappa_{do}\right)/d\kappa_{do} > 0$), also called a market thickness effect. Market tightness is defined from the perspective of producers so that the market is tighter when there are relatively more retailers than producers.

2.3.2 Producers

We assume the homogeneous good is produced with one unit of labor under constant returns to scale in each country. We also assume there is free entry into the production of that good, there are no search frictions in that sector, and that this good is freely traded. Since it is costless to trade, a no-arbitrage condition implies that the price of the homogeneous good must be the same in all countries ($p_d\left(1\right) = p\left(1\right)\forall d$), and because it is made with one unit of labor in each country, it must also be the case that $w_d = p\left(1\right)\forall d$. As in Chaney (2008), and to simplify our analysis, we only consider equilibria in which every country produces some of the numeraire. Therefore, the homogeneous good will serve as the global numeraire with $p_d\left(1\right) = 1\forall d$.

For producers of the differentiated good, we use the familiar variable cost function indexed by productivity φ:

$$t\left(q_{do}, w_o, \tau_{do}, \varphi\right) = q_{do}w_o\tau_{do}\varphi^{-1}. \tag{5}$$

Here w_o is the competitive wage in the exporting (origin) country, $\tau_{do} \geq 1$ is a parameter capturing one plus the iceberg transport cost between destination d and origin o, and q_{do} is the amount produced and traded between destination d and origin o. This variable cost function implies a constant-returns-to-scale production function in which labor is the only input. The firm that produces quantity $q_{do}\left(\omega\right)$ of variety ω has productivity φ and marginal cost equal to $w_o\tau_{do}\varphi^{-1}$. Following Melitz (2003), we interpret higher productivity firms as producing a symmetric variety at a lower marginal cost. Total production cost is $t\left(q_{do}, w_o, \tau_{do}, \varphi\right) + f_{do}$ in which f_{do} is the fixed cost of production.

We assume that productivity is exogenous and Pareto distributed with the same cumulative density function in all countries:

$$G\left[\tilde{\varphi} < \varphi\right] = 1 - \varphi^{-\theta}, \tag{6}$$

in which $\varphi \in [1, +\infty)$. The probability density function is $g\left(\varphi\right) = \theta\varphi^{-\theta-1}$. We assume that $\theta > \sigma - 1$ so that aggregate variables determined by the integral $\int_{\bar{\varphi}}^{\infty} z^{\sigma-1} dG\left(z\right)$ are bounded. The Pareto distribution has been widely used in trade models and describes firms' size well (Axtell, 2001; Gabaix, 2009).

The value of a producer with productivity φ being matched to a retailer, $X_{do}\left(\varphi\right)$, can be summarized by a value function in continuous time:

$$rX_{do}\left(\varphi\right) = n_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} + \lambda\left(U_{do}\left(\varphi\right) - X_{do}\left(\varphi\right)\right). \tag{7}$$

This asset equation states that the flow return at the risk-free rate, r, from the value of producing must equal the flow payoff plus the expected capital gain from operating as an exporting producer. Each producer is indexed by exogenous productivity, φ. The flow payoff consists of $n_{do}q_{do}$, the revenue obtained from selling q_{do} units of the good at negotiated price n_{do} to retailers, less the variable, $t\left(q_{do}, w_o, \tau_{do}, \varphi\right)$, and fixed cost of production, f_{do}. The negotiated price, n_{do}, and the quantity traded, q_{do}, are determined through a bargaining process that we describe in sections 2.3.4 to 3.2. The last term in equation (7) captures the event of a dissolution of the match, which occurs at exogenous rate λ and leads to a capital loss of $U_{do}\left(\varphi\right) - X_{do}\left(\varphi\right)$ as the producer loses value $X_{do}\left(\varphi\right)$ but gains the value of being an unmatched producer, $U_{do}\left(\varphi\right)$.

The value that an unmatched producing firm receives from looking for a retail partner without being in a business relationship, $U_{do}(\varphi)$, satisfies

$$rU_{do}(\varphi) = -l_{do} + \kappa_{do}\chi(\kappa_{do})(X_{do}(\varphi) - U_{do}(\varphi) - s_{do}). \tag{8}$$

The flow search cost, l_{do}, is what the producer pays when looking for a retailer; it captures the costs we highlighted in the introduction — namely, maintaining foreign sales offices, sending sales representatives abroad, researching potential foreign buyers, and establishing a web presence. The second term captures the expected capital gain, in which $\kappa_{do}\chi(\kappa_{do})$ is the endogenous rate at which producing firms contact retailers, and s_{do} is the sunk cost of starting up the relationship.

The producing firm also has the option of remaining idle and not expending resources to look for a retailer. For producers, the value of not searching, $I_{do}(\varphi)$, satisfies

$$rI_{do}(\varphi) = h_{do}. \tag{9}$$

Producers can always choose this outside option and not search for retailers. Idle firms in this context are analogous to workers who are out of the labor force. Choosing to remain idle provides the flow payoff, h_{do}. The value to a producer of remaining idle can be interpreted, for example, as the value of the stream of payments after liquidation or the flow payoff from home production if these firms are viewed as entrepreneurs. We denote the fraction of producers that choose not to search, and therefore remain idle in the *do* market, as i_{do}.

2.3.3 Retailers

All retailers are ex-ante identical but have values that vary ex-post only because producers are heterogeneous. The value of a retailing firm in a business relationship with a producer of productivity φ, is defined by the asset equation,

$$rM_{do}(\varphi) = p_{do}q_{do} - n_{do}q_{do} + \lambda(V_{do} - M_{do}(\varphi)). \tag{10}$$

The flow payoff from being in a relationship is the revenue generated by selling q_{do} units of the product to a representative consumer at a final sales price p_{do} (determined by their inverse demand curve from equation 2) less the cost of acquiring these goods from producers at negotiated price n_{do}. To highlight other aspects of the search process, we assume retailers do not use the product as an input in another stage of production but only facilitate the match between producers and consumers. When we solve the model in section 3 (and in particular section 3.2), we show that including an additional intermediate input does not substantively affect our main conclusions. In the event that the relationship undergoes an exogenous separation, the retailing firm loses the capital value of being matched, $V_{do} - M_{do}(\varphi)$. This match destruction occurs at rate λ.

The value of being an unmatched retailer, V_{do}, satisfies

$$rV_{do} = -c_{do} + \chi(\kappa_{do}) \int [\max\{V_{do}, M_{do}(\varphi)\} - V_{do}]\, dG(\varphi). \tag{11}$$

Retailers need to pay a flow cost, c_{do}, to search for a producing affiliate. At Poisson rate $\chi(\kappa_{do})$, retailing firms meet a producer of unknown productivity. Because retailers do not know the productivity of the producers they will meet, they take the expectation over all productivities they might encounter when computing the expected continuation value of searching. As a result, the value, V_{do}, is not a function of a producer's productivity, φ, but rather a function of the expected payoff. We assume that upon meeting, but before consummating a match, retailers learn the productivity of the producer. In the terminology of Menzio and Shi (2011), matches are inspection goods as opposed to experience goods. Upon meeting, and depending on the producer's productivity, φ, retailers choose between matching with that producer, which generates value $M_{do}(\varphi)$, and continuing the search, which generates V_{do}. Hence, the capital gain to retailers from meeting a producer with productivity φ can be expressed as $\max\{V_{do}, M_{do}(\varphi)\} - V_{do}$.

2.3.4 Bargaining

Upon meeting, the retailer and producer bargain over the negotiated price and quantity simultaneously. We assume that these objects are determined by the generalized Nash bargaining solution, which, as shown by Nash (1950) and Osborne and Rubinstein (1990), is equivalent to maximizing the following Nash product:

$$\max_{q_{do},n_{do}} \left[X_{do}\left(\varphi\right) - U_{do}\left(\varphi\right)\right]^{\beta} \left[M_{do}\left(\varphi\right) - V_{do}\right]^{1-\beta}, 0 \leq \beta < 1, \tag{12}$$

in which β is producers' bargaining power. The total surplus created by a match, which is the value of the relationship to the retailer and the producer less their outside options, is $S_{do}\left(\varphi\right) = M_{do}\left(\varphi\right) - V_{do} + X_{do}\left(\varphi\right) - U_{do}\left(\varphi\right)$. In appendix A.3, we derive an expression for the match surplus and for the value of a relationship, $R_{do}\left(\varphi\right)$, in terms of model primitives, which also provides theoretical underpinnings for results in Monarch and Schmidt-Eisenlohr (2015). In the next two sections we derive how our bargaining protocol pins down the quantity traded within a business relationship, q_{do}, and the negotiated price, n_{do}.

3 Optimal search and matching in equilibrium

The retailing and producing firms use backward induction to maximize their value. The second stage is the solution that results from bargaining over price and quantity after a retailer and producer meet and decide to match, which we describe in section 2.3.4. We solve this bargaining problem in sections 3.1 and 3.2.

In the first stage, retailers and producers, taking the solution to this second-stage bargaining problem as given, choose whether to search for a business partner, or to remain idle. Because producers are heterogeneous, their decision to search or not depends on their productivity. Section 3.3 shows that there is a minimum productivity threshold, akin to the entry condition defined in Melitz (2003), that makes searching worthwhile. In section 3.4 we present the condition that characterizes retailers' decisions to search and defines equilibrium market tightness. Finally, in the steady state, there exists a set of unmatched producers that are actively looking for a retail partner and unmatched retailers that are actively looking for a producer. We define these concepts in section 3.5.

3.1 Bargaining over price

Producers and retailers bargaining over the negotiated price, n_{do}, results in a price that divides the total surplus created by a match between the parties according to

$$\begin{aligned} X_{do}(\varphi) - U_{do}(\varphi) &= \beta S_{do}(\varphi), \\ M_{do}(\varphi) - V_{do} &= (1-\beta) S_{do}(\varphi). \end{aligned} \tag{13}$$

Here, producers receive β of the total surplus, while retailers receive the remainder. Therefore, we refer to this expression as the "surplus sharing rule." We have relegated the details regarding the derivation of equation (13) to appendix A.4.1.[5]

The negotiated price, n_{do}, which generates the surplus sharing rule in equation (13), is given by the following proposition.

Proposition 1. *The negotiated price, n_{do}, at which producers sell their good to retailers satisfies*

$$n_{do} = [1-\gamma_{do}]\, p_{do} + \gamma_{do} \frac{t(q_{do}, w_o, \tau_{do}, \varphi) + f_{do} - l_{do} - \kappa_{do}\chi(\kappa_{do})\, s_{do}}{q_{do}}, \tag{14}$$

in which $\gamma_{do} \equiv \dfrac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\kappa_{do}\chi(\kappa_{do})} \in [0,1]$.

Proof. Use $V_{do} = 0$, equations (7), (8), (10), and the surplus sharing rule defined by equation (13). See appendix A.4.2 for detailed derivations. See appendix A.4.3 for a proof that $\gamma_{do} \in [0,1]$. □

[5] We also point out that the reasoning behind the restriction that $\beta < 1$ in equation (12) is evident in equation (13). Retailing firms have no incentive to search if $\beta = 1$, as they get none of the resulting match surplus and therefore cannot recoup search costs, $c_{do} > 0$. Any solution to the model with $c_{do} > 0$ and positive trade between retailers and producers also requires $\beta < 1$.

We remind the reader that equation (14) is a function of a producer's productivity, φ, but we have not written it as such to conserve on notation.

The equilibrium negotiated price, n_{do}, is a convex combination of the final sales price and the average total production cost less producers' search costs. A price outside of this range would be unsustainable. The highest negotiated price, n_{do}, that retailers are willing to pay is the final sales price, p_{do}, and the lowest negotiated price that producers are willing to accept is the average total production cost, $\left(t\left(q_{do}, w_o, \tau_{do}, \varphi\right) + f_{do}\right)/q_{do}$, net of the cost of looking for a retailer, l_{do}, and the expected sunk cost, $\kappa_{do}\chi(\kappa_{do})s_{do}$. The search costs of producers, l_{do} and s_{do}, enter negatively in equation (14) because they erode producers' bargaining position and thereby allow retailers to negotiate a lower transaction price.

The negotiated price also depends on the bargaining power and the finding rate of producers. As producers gain all the bargaining power ($\beta \to 1$), then $\gamma_{do} \to 0$ and $n_{do} \to p_{do}$, so producers take all the profits from the business relationship. Similarly, if producers find retailers immediately (no search frictions) so that the finding rate $\kappa_{do}\chi(\kappa_{do}) \to \infty$, and the sunk cost, s_{do}, is set to zero, then the negotiated price also converges to the final sales price, $n_{do} \to p_{do}$. We provide details in appendix A.4.4. Importantly, the case in which $n_{do} \to p_{do}$ recovers the standard trade model (Melitz, 2003; Chaney, 2008), as there is, in effect, no intermediate retailer; producers can be seen as selling their goods directly to the final consumer at price p_{do}.

The fact that import prices are lower than final sales prices, as given by equation (14), is consistent with the empirical findings of Berger et al. (2012). This pricing approach, among other model features, is also similar to that of Drozd and Nosal (2012), who use a trade model with search frictions to account for several pricing puzzles of international macroeconomics.

3.2 Bargaining over quantity

We show that bargaining over quantity, q_{do}, together with equation (13), yields the following proposition.

Proposition 2. *The quantity traded, q_{do}, satisfies*

$$p_{do} + \frac{\partial p_{do}}{\partial q_{do}} q_{do} = \frac{\partial t\left(q_{do}, w_o, \tau_{do}, \varphi\right)}{\partial q_{do}}. \tag{15}$$

Proof. See appendix A.5.1. □

The quantity exchanged *within* matches, q_{do}, equates marginal revenue obtained by retailers with the marginal production cost. Equation (15), together with our differentiated demand curve from equation (2), and our cost function from equation (5) imply that the final sales price charged for the imported good in the domestic market takes the standard form of a markup over marginal cost:

$$p_{do}\left(\varphi\right) = \mu w_o \tau_{do} \varphi^{-1}, \tag{16}$$

in which $\mu = \sigma / \left(\sigma - 1\right) > 1$. We present the details of this derivation in appendix A.5.2.

We want to note that the quantity traded *within* matches in our model is the same as one would obtain in a model without search frictions. The quantity depends on consumers' demand curve p_{do}, the pricing power of retailers, and the production cost function, $t\left(q_{do}, w_o, \tau_{do}, \varphi\right)$. We show in appendix A.5.3 that including an additional input in retailers' production function does not change this result. Nevertheless, although the quantity exchanged does not depend on search frictions, these frictions do affect the mass of matches formed. We turn to this topic in the next section.

3.3 Producers' search productivity thresholds

Given the outcome of bargaining in the second stage, we derive whether retailers and producers will search for a business partner at all in the first stage. Because producers differ by productivity, this first stage leads to a productivity threshold that makes the producer indifferent between searching and remaining idle.[6] This productivity threshold is defined by

[6]There exists an alternative threshold, $\underline{\varphi}_{do}$, which makes the producer and retailer indifferent between consummating a relationship upon contact and continuing to search, $X_{do}\left(\underline{\varphi}_{do}\right) - U_{do}\left(\underline{\varphi}_{do}\right) = 0$. We show in

$U_{do}(\bar{\varphi}_{do}) - I_{do}(\bar{\varphi}_{do}) = 0$ and can be expressed as in the following proposition.

Proposition 3. *In general, the threshold productivity, $\bar{\varphi}_{do}$, is determined by the implicit function*

$$\pi(\bar{\varphi}_{do}) = F(\kappa_{do}), \tag{17}$$

in which variable profits are $\pi(\bar{\varphi}_{do}) \equiv p_{do}(\bar{\varphi}_{do})\, q_{do}(\bar{\varphi}_{do}) - t(q_{do}, w_o, \tau_{do}, \bar{\varphi}_{do})$ and the effective entry cost, $F(\kappa_{do})$, is

$$F(\kappa_{do}) \equiv f_{do} + \left(\frac{r+\lambda}{\beta\kappa_{do}\chi(\kappa_{do})}\right) l_{do} + \left(1 + \frac{r+\lambda}{\beta\kappa_{do}\chi(\kappa_{do})}\right) h_{do} + \left(\frac{r+\lambda}{\beta}\right) s_{do}.$$

Proof. See appendix A.6.2. □

Equation (17) is akin to the entry condition defined in Melitz (2003), even though retailers earn $p_{do}q_{do}$ revenue from the consumer and producers pay the production cost. Our condition defines a threshold productivity that ensures that total flow profits cover what we call the "effective entry cost," $F(\kappa_{do})$, which is the fixed cost of production, f_{do}, and the (appropriately discounted) flow cost of searching for a partner, l_{do}, the opportunity cost of remaining idle, h_{do}, and the sunk cost of starting up a business relationship, s_{do}.

Proposition 3 implies that the effective entry cost, and therefore the threshold productivity, depends endogenously on producers' finding rate $\kappa_{do}\chi(\kappa_{do})$. We define $\kappa_{do} = v_{do}N_d^m / u_{do}N_o^x$ so that as the number of searching retailers increases (or the number of searching producers decreases), it becomes easier for a producer to meet a retailer. Intuitively, higher κ_{do} reduces the time spent searching by producers and, along with it, the effective entry cost. Related to this, if producers' finding rate is exogenous, proposition 3 provides a novel micro-level interpretation of the effective entry cost, but this cost remains a combination of exogenous parameters. Benguria (2015) makes a closely related point.

Another innovation of our model is that the opportunity cost of remaining idle, h_{do}, is an important determinant of the productivity threshold and the fraction of active producers. As

appendix A.6.1 that the binding threshold is defined by $\bar{\varphi}_{do}$, because $\bar{\varphi}_{do} > \underline{\varphi}_{do}$ if $l_{do} + h_{do} + \kappa_{do}\chi(\kappa_{do})\, s_{do} > 0$.

pointed out by Armenter and Koren (2014), the fraction of exporting firms is an important moment for parameter identification and one that has been exploited by Eaton et al. (2014) and Eaton et al. (2016), among others. Allowing for the possibility that producers optimally choose not to search could change the estimates in these important papers. Our model also implies that the bargaining power, β, and the match destruction rate, λ, are determinants of the effective entry cost.

Proposition 3 also nests the conditions defining the threshold productivity in many trade models. In particular, with $l_{do} = 0$, $h_{do} = -l_{do}$, and $s_{do} = 0$, we recover the equation defining the productivity threshold in Melitz (2003). We present a more complete discussion of this result and relate proposition 3 to expressions in other standard trade frameworks in appendix A.6.3.[7]

In the context of our functional form assumptions, because the equilibrium price for each variety is a constant markup over marginal cost and the cost function is linear in quantities, variable profits $\pi\left(\bar{\varphi}_{do}\right)$ are a constant share of revenues. As such, profits become $\pi_{do}\left(\varphi\right) = \left(\alpha/\sigma\right) C_d P_d^{\sigma-1}\left(\mu w_o \tau_{do}\right)^{1-\sigma}\varphi^{\sigma-1}$. Using this expression and the implicit function that defines the productivity threshold in equation (17) provides

$$\bar{\varphi}_{do} = \mu\left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}}\left(\frac{w_o\tau_{do}}{P_d}\right)\left(\frac{F\left(\kappa_{do}\right)}{C_d}\right)^{\frac{1}{\sigma-1}}. \tag{18}$$

We present details in appendix A.6.4. Notice that equation (18) is analogous to equation (7) in Chaney (2008) except that our effective entry cost, $F\left(\kappa_{do}\right)$, is now endogenous and therefore differs from the fixed entry cost given there.

[7]It may seem surprising that a trade model without search frictions requires that the flow value of remaining idle be the negative of producers' search cost, $h_{do} = -l_{do}$, instead of having zero idle value, $h_{do} = 0$. Intuitively, without search frictions, idling and searching must have the same flow cost in order for search frictions not to have any effect.

3.4 Retailer entry

Here we specify the conditions under which unmatched retailers search in order to match with producers. As is standard in the labor literature (Pissarides, 1985; Shimer, 2005), we assume free entry into retailing so that in equilibrium, the value of being an unmatched retailer, V_{do}, is driven to zero. The ability to expand retail shelf space or post a product online until it is no longer valuable to do so provides an intuitive basis for this assumption.

Using equation (11) together with our assumption of free entry into the market of unmatched retailers, $V_{do} = 0$, implies that

$$\frac{c_{do}}{\chi\left(\kappa_{do}\right)} = \int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right) dG\left(\varphi\right). \tag{19}$$

This equation defines the equilibrium market tightness, κ_{do}, that equates the expected cost of being an unmatched retailer, on the left, with the expected benefit from matching, on the right. In defining equation (19), we removed the maximum over V_{do} and $M_{do}\left(\varphi\right)$ from equation (11) and simply integrated from the threshold productivity level defined by equation (18). This simplification is possible as long as $M_{do}(\varphi)$ is strictly increasing in φ so that the ex-post value of being matched is strictly increasing in producers' productivity. In appendix A.7 we prove this result. It is worth emphasizing that equation (19) does not inform the binding productivity threshold $\bar{\varphi}_{do}$, which is solely determined by proposition 3.

To get intuition from equation (19), notice that as the expected benefit (the right-hand side) from retailing rises, free entry implies that retailers enter the search market, which raises market tightness, $\kappa_{do} = v_{do}N_d^m/u_{do}N_o^x$, reduces the rate at which searching retailers contact searching producers, $\chi\left(\kappa_{do}\right)$, through congestion effects, and increases retailers expected cost of search (the left-hand side). Hence, free entry ensures that V_{do} is zero at all times and that κ_{do} always satisfies equation (19).

If searching for producers was free ($c_{do} = 0$) but matching was associated with positive expected payoff, then free entry would lead to an infinite number of retailers in the economy driving producers' finding rate to infinity. Conversely, if there were an infinite number of retailers in the search market, then the flow cost of search must be zero. These two thought experiments lead to the following proposition.

Proposition 4. *With free entry into retailer search, market tightness, κ_{do}, is finite if and only if retailers' search cost, c_{do}, is positive.*

Proof. See appendix A.8. □

Equation (19), together with proposition 4, highlights that retailers' cost of searching for producers, c_{do}, along with our assumption of free entry into retailing is at the heart of our model. As the retailer cost $c_{do} \to 0$, producers find retailers instantly relieving the search friction.

One way we motivate goods-market frictions is from survey reports of the high cost of "identifying the first contact" and "establishing initial dialogue" reported by producers (Kneller and Pisu, 2011). Proposition 4 focuses on the role of retailer search costs as the origin of the search friction. However, any reported producer costs, which in our model are captured by the effective entry cost in proposition 3, are influenced by equilibrium variables, and in particular market tightness, κ_{do}. Therefore, retailers' flow search costs, c_{do}, will affect producers' equilibrium costs as well.

Free entry also interacts with assumptions about how firms of both types come into existence. We describe those assumptions in detail in appendix A.9, showing in appendix A.9.1 that, for retailers, free entry into search implies free entry into existence. In appendix A.9.2 we consider the alternative assumption of free entry into search for producers and show that it yields additional restrictions on equilibrium market tightness. We find our baseline approach of setting $V_{do} = 0$ to be a natural starting point, but other approaches lead to similar effects of search frictions, and the major implications of our paper remain the same.

3.5 Matching in equilibrium

In the steady state, there exists a set of unmatched producers that are actively looking for a retail partner and unmatched retailers that are actively looking for a producer. These steady-state fractions of unmatched retailers and producers correspond to frictional unemployment and unfilled vacancies in the labor literature, and will be positive as long as the finding rates are finite and the separation rate is non-zero. The mass of producers that are matched to retailers and selling their products is $(1 - u_{do} - i_{do}) N_o^x$, in which a fraction u_{do} are unmatched and actively searching for retailers and a fraction i_{do} choose not to search and therefore remain idle.

To determine the steady-state fraction of unmatched producers, it is useful to think about the flow into and out of the unmatched-producer state. In particular, in any given instant, $(1 - u_{do} - i_{do}) N_o^x$ matched producers separate exogenously at rate λ. Consequently, the inflow into the unmatched state is $\lambda (1 - u_{do} - i_{do}) N_o^x$. Flows out of this state are $\kappa_{do}\chi(\kappa_{do}) u_{do} N_o^x$ because $u_{do} N_o^x$ producers find matches at rate $\kappa_{do}\chi(\kappa_{do})$. In the steady state, the inflows must equal the outflows. After re-arranging, we get

$$\frac{u_{do}}{1 - i_{do}} = \frac{\lambda}{\lambda + \kappa_{do}\chi(\kappa_{do})}. \tag{20}$$

The fraction of idle producers, i_{do}, that choose not to search is defined by the steady-state threshold, $\bar{\varphi}_{do}$, and the exogenous distribution of productivity:

$$i_{do} = \int_1^{\bar{\varphi}_{do}} dG(\varphi) = G(\bar{\varphi}_{do}). \tag{21}$$

The fraction of producers that are active, $1 - i_{do}$, corresponds to the labor force participation rate in the labor literature. While u_{do} is the fraction of producers that are unmatched, $u_{do}/(1 - i_{do})$ is the fraction of active producers that are unmatched and is equivalent to the labor unemployment rate, which is characterized as the fraction of the labor force that is actively searching for a job. Equation (20) implies different predictions about the extensive margin relative to standard trade models because in our model some highly productive varieties are endogenously and randomly unmatched. In this way, we provide a search theoretic explanation for what Armenter and Koren (2014) refer to as "balls-and-bins" facts about the extensive margin of trade.

We assume that every matched producer must have one, and only one, retailer as its counterpart. Doing so implies that the mass of matched producers and retailers must be equal in the steady state:

$$\left(1 - u_{do} - i_{do}\right) N_o^x = \left(1 - v_{do}\right) N_d^m . \tag{22}$$

We are aware that international retailers and producers can simultaneously engage with several business partners. These many-to-many relationships have been highlighted in Eaton et al. (2014) and Eaton, Kortum, and Kramarz (2017). However, Sugita, Teshima, and Seira (2017) find that, while U.S. importers and Mexican exporters in textiles transact with multiple firms, the main seller and buyer account for the bulk of each firm's total trade. These authors conclude that "a one-to-one matching model is a fair approximation of product-level matching in Mexico-U.S. textile/apparel trade." Similarly, Eaton et al. (2014) find that roughly 80 percent of matches are one-to-one in Colombia-U.S. manufacturing trade.

4 Model aggregation and general equilibrium

4.1 Aggregate resource constraint

The aggregate resource constraint in this economy can be expressed using either the income or expenditure approach to aggregate accounting. Typically, models of international trade highlight the income perspective. We find it more natural to focus on the expenditure approach:

$$\begin{aligned} Y_d = & \underbrace{p_d\left(1\right) q_d\left(1\right) + \sum_{k=1}^{O} \left(\frac{1 - u_{dk} - i_{dk}}{1 - i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}} p_{dk}\left(\varphi\right) q_{dk}\left(\varphi\right) dG\left(\varphi\right)}_{\text{Aggregate consumption } (C_d)} \\ & + \underbrace{N_d^x e_d^x + \sum_{k=1}^{O} \kappa_{dk} u_{dk} N_k^x c_{dk} + u_{kd} N_d^x \left(l_{kd} + s_{kd} \kappa_{kd} \chi\left(\kappa_{kd}\right)\right) + \left(1 - u_{kd} - i_{kd}\right) N_d^x f_{kd}}_{\text{Aggregate investment } (I_d)} . \end{aligned} \tag{23}$$

Consumption expenditure, C_d, is the total resources devoted to consumption, to both the homogeneous good and the differentiated varieties, evaluated at final consumer prices. Investment expenditure, I_d, is the resources devoted to creating producing firms, to creating retailer-producer relationships, and to paying for the per-period fixed costs of goods production. Here, e_d^x is the sunk, one-time cost paid by producers to come into existence. As in Chaney (2008), we assume that the number of differentiated-goods producers, N_d^x, is proportional to aggregate consumption expenditure, C_d, so that there is no entry decision by producers, but equation (23) accounts for these expended resources. We present more details in appendix A.10.1. In section 3.4, we mention that free entry into retailing implies that e_o^m must be zero (appendix A.9).[8]

To account for all resources in the economy, we assume all costs incurred by firms for investment and production, including iceberg transport costs, are paid to labor. Therefore, we do not have iceberg costs that "melt away" in transit or that are levied and then wasted by the government. Our structure ensures that changing iceberg costs do not change total resources but instead only introduce distortions. An alternative and identical setup would be to assume that iceberg costs are not paid by firms to workers but are instead levied by the government and then rebated to consumers as lump-sum transfers, which is related to the approach in Irarrazabal, Moxnes, and Opromolla (2015). In that setting, both the expenditure and income approaches would include a government term and aggregate profits would be reduced by the amount of the government's revenue, but total payments to labor would remain the same.

We also treat total payments to idle producers, $\sum_{k=1}^{O}\left(1-i_{kd}\right)N_d^x h_{kd}$, as balanced lump-sum transfers. They enter negatively in the expenditure approach as a lump-sum tax on consumers or firms and enter positively as an additional lump-sum expenditure by the

[8]The mass of producers that are matched to retailers and selling their products is $\left(1-u_{do}-i_{do}\right)N_o^x$. Producers that are idle or searching for retailers but are currently not in a business relationship do not contribute to aggregate output, consumption, or prices. The integral term times $\left(1-i_{do}\right)^{-1}$ captures the conditional average sales of producers that have productivity above the cutoff necessary to match. Another way to see that all aggregate variables must be scaled in this way is to compute the mass of matched producers $\left[\left(1-u_{do}-i_{do}\right)/\left(1-i_{do}\right)\right]N_o^x \int_{\bar{\varphi}_{do}}^{\infty} dG\left(\varphi\right)=\left(1-u_{do}-i_{do}\right)N_o^x$.

government. As such, these cancel out on the expenditure side of the accounting identity. Finally, we impose balanced trade, so that net exports, NX_d, do not appear in the accounting identity (23).

Total resources are defined by $Y_d = w_d L_d (1 + \pi)$, in which L_d is the exogenous size of the economy, w_d is the equilibrium wage, and

$$\pi = \Pi / \sum_{k=1}^{O} w_k L_k, \tag{24}$$

is the value of a share of global profits, Π, which we define in appendix A.10.2. Consumers in destination d get a share of global profits in proportion to the value of labor in the economy, $w_d L_d / \sum_{k=1}^{O} w_k L_k$, as in Chaney (2008).

Additional details about the income and expenditure approaches to accounting, resources available for consumption and investment, and the global mutual fund are included in appendix A.10.2.

4.2 The ideal price index

We can move from indexing over the unordered set of varieties in equation (3) to the distribution of productivities using the steps in appendix A.11.1. We can then use the optimal final sales price that results from Nash bargaining over quantity given in equation (16) along with the productivity threshold from (18) to derive the price index for differentiated goods in country d:

$$P_d \quad = \quad \lambda_2 \times C_d^{\frac{1}{\theta} - \frac{1}{\sigma-1}} \times \rho_d \tag{25}$$

in which

$$\rho_d \quad \equiv \quad \left(\sum_{k=1}^{O} \frac{C_k}{C} \left(1 - \frac{u_{dk}}{1 - i_{dk}} \right) (w_k \tau_{dk})^{-\theta} F_{dk}^{-\left[\frac{\theta}{\sigma-1} - 1\right]} \right)^{-\frac{1}{\theta}},$$

$$\lambda_2 \equiv \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{-\frac{1}{\theta}} \left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}} \mu \left(\frac{C}{1+\pi}\right)^{-\frac{1}{\theta}},$$

and $C = \sum_{k=1}^{O} C_k$ is global consumption. More details appear in appendix A.11.2 and to conserve on notation, we will sometimes refer to $F(\kappa_{do})$ as F_{do}. Equation (25) closely resembles the price index in Chaney (2008, equation 8) and our model also includes a "multilateral resistance" term, ρ_d. Importantly, the introduction of search frictions makes the price index a function of the consumption weighted average of the equilibrium matched rates of all producers throughout the world in addition to the usual iceberg and entry costs.

4.3 Defining the general equilibrium

A steady-state general equilibrium consists of threshold productivities, $\bar{\varphi}_{do}$ $\forall do$, market tightnesses, κ_{do} $\forall do$, aggregate consumptions, C_d $\forall d$, and the per-capita dividend distribution, π, which jointly solve the zero-profit conditions (equation 18), the free-entry conditions (equation 19), the aggregate resource constraints (equation 23), and the global mutual fund dividend (equation 24). The exogenous parameters are β, λ, r, η, ξ, θ, σ, α, e_d^x, L_d, c_{do}, f_{do}, h_{do}, l_{do}, s_{do}, and τ_{do}, in which d and o vary with country. We elaborate on this definition in appendix A.12.

5 Analytic general equilibrium results

5.1 Welfare changes in response to foreign shocks

In this section, we discuss how adding search frictions changes the response of welfare to foreign shocks. We relate this to Arkolakis, Costinot, and Rodríguez-Clare (2012), who show that, in a large class of trade models, welfare (indirect utility) changes can be summarized by two sufficient statistics: the change in the domestic consumption share in response to a shock and the elasticity of trade with respect to variable trade costs.

Definition 1. *Define a foreign shock in country d as a change from $(\mathbf{L}, \mathbf{e^x}, \mathbf{f}, \mathbf{c}, \mathbf{h}, \mathbf{l}, \mathbf{s}, \tau)$ to $(\mathbf{L}', \mathbf{e^x}', \mathbf{f}', \mathbf{c}', \mathbf{h}', \mathbf{l}', \mathbf{s}', \tau')$ such that $(L_d, e_d^x, f_d, c_d, h_d, l_d, s_d, \tau_d) = (L_d', e_d^{x\prime}, f_d', c_d', h_d', l_d', s_d', \tau_d')$.*

Proposition 5. *The change in welfare associated with any foreign shock in country d in our model can be computed as*

$$\hat{W}_d = \hat{\lambda}_{dd}^{-\frac{\alpha}{\theta}} \left(1 - \widehat{\frac{u_{dd}}{1 - i_{dd}}} \right)^{\frac{\alpha}{\theta}} \hat{C}_d^{1 + \frac{\alpha}{\theta}\left(1 - \frac{\theta}{\sigma - 1}\right)}, \tag{26}$$

in which $\hat{x} \equiv x'/x$ denotes the change in any variable x between the initial and the new equilibrium, $\lambda_{dd} \equiv C_{dd}/C_d$ is the share of country d's total expenditure on differentiated goods produced domestically, and we assume that: 1) $l_{dd} = -h_{dd}$ so that $F(\kappa_{dd})$ is a parameter, 2) the number of producers in d do not change so that $d \ln (N_d^x) = 0$, and 3) productivity, φ, has a Pareto distribution given by equation (6).

Proof. Appendix B.1 derives the proof with the general result in B.1.6 and proposition 5 in B.1.7. □

Equation (26) states that the change in welfare in country d, $\hat{W}_d$, is a function of the changes in the share of domestic expenditure at final prices, $\hat{\lambda}_{dd}$, changes in the rate at which domestic producers are matched in the domestic market, $1 - \widehat{u_{dd}/(1 - i_{dd})}$, and the change in consumption itself, $\hat{C}_d$.

There are a few differences between the Arkolakis, Costinot, and Rodríguez-Clare (2012) welfare expression and equation (26). First, knowing only changes in the consumption ratio, λ_{dd}, and the parameters α, θ, and σ is insufficient for ex-post welfare analysis. One would also need to know the changes in the matched rate and changes in the level of consumption. The change in consumption enters into equation (26) but not the welfare equation in Arkolakis, Costinot, and Rodríguez-Clare (2012) because search and sunk costs imply that profits are not proportional to output. Second, if the rates at which partners find one another are exogenous parameters and profits are proportional to output, equation (26) collapses to the expression in Arkolakis, Costinot, and Rodríguez-Clare (2012) and any welfare effects are the same as in the standard model. Sending the search cost c to zero would also result in the standard expression as long as profits are proportional to output. Third, the matched rate in equation (26) could serve to attenuate the welfare change in response to a change in variable trade costs in comparison with the standard model. Consider, for example, the effect of destination d raising tariffs on products from origin o in a model with search. Higher tariffs result in a higher price index, which makes being a retailer in the domestic market more valuable and induces more retailers to enter the domestic market. With more retailers in the market, the rate at which domestic producers find domestic partners increases, and the matched rate, $1 - u_{dd}/\left(1 - i_{dd}\right)$, increases. A higher domestic matched rate *attenuates* the welfare losses from higher tariffs. In section 7.1.1 we quantify the effects of the matched rate in response to specific foreign shocks in a calibrated version of our model showing that welfare attenuation can be quantitatively large.

5.2 Consumption and trade elasticities

The elasticity of trade with respect to variable trade costs is an important quantity itself and is one of the two inputs needed to evaluate the gains from trade in a large class of models (Arkolakis, Costinot, and Rodríguez-Clare, 2012). The general form of this elasticity is provided by Arkolakis, Costinot, and Rodríguez-Clare (2012, equation 21) for Melitz (2003) models under slight restrictions on the number of producers. Starting with their general elasticity and assuming productivity, φ, has a distribution given by equation (6) implies that the trade elasticity is the negative of the Pareto shape parameter,

$$\frac{\partial \ln\left(IM_{do}/IM_{dd}\right)}{\partial \ln\left(\tau_{d'o}\right)} = \frac{\partial \ln\left(C_{do}/C_{dd}\right)}{\partial \ln\left(\tau_{d'o}\right)} = \varepsilon_o^{ACRdd'} = \begin{cases} -\theta & if\ d' = d \\ \\ 0 & if\ d' \neq d \end{cases}. \tag{27}$$

It is generally not true that the consumption elasticity needed to evaluate welfare and the trade elasticities are equal. In our model, they differ because consumption is evaluated at final sales prices, while imports are evaluated at negotiated prices. We present consumption and trade elasticities in our model under simplifying assumptions in order to ease the comparison to models without search frictions and to highlight the quantitative importance of search in our calibration exercises. None of those simplifications are necessary to derive these elasticities. Proposition 6 presents the consumption elasticity, while proposition 7 presents the trade elasticity.

Proposition 6. *The elasticity of consumption shares to iceberg trade costs in our model with goods-market frictions is given by*

$$\frac{\partial \ln \left(C_{do}/C_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} = \begin{cases} -\theta + \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{do}\right)}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln \left(\tau_{do}\right)}\right) & if\ d' = d \\ \\ \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right) & if\ d' \neq d \end{cases}, \quad (28)$$

in which we assume that: 1) $l_{do} = -h_{do}$ so that $F\left(\kappa_{dd}\right)$ and $F\left(\kappa_{do}\right)$ are parameters, 2) the number of producers in d and o do not change with tariff changes so that $\partial \ln \left(N_d^x\right)/\partial \ln \left(\tau_{d'o}\right) = \partial \ln \left(N_o^x\right)/\partial \ln \left(\tau_{d'o}\right) = 0$, and 3) productivity, φ, has a Pareto distribution given by equation (6).

Proof. Appendix B.2 derives the proof with the general result in B.2.14, proposition 6 is in appendix B.2.15, and a comparison with equation (27) is provided in appendix B.2.16. □

Our consumption elasticity depends not only on the usual trade elasticity, but also on the fraction of unmatched producers and the elasticity of producers' finding rate in the *do* and *dd* product markets. Using equation (19), we know that raising tariffs, τ_{do}, reduces the value of importing, $M_{do}\left(\varphi\right)$, and therefore reduces market tightness, κ_{do}, and producers' finding rate, $\kappa_{do}\chi\left(\kappa_{do}\right)$. This comparative static implies that $\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)/\partial \ln \left(\tau_{do}\right) \leq 0$. Conversely, raising tariffs in the *do* market raises the price index, P_d, making the domestic market more attractive, encouraging domestic retailer entry, and thus raising domestic market tightness, κ_{dd}, and the domestic producers' finding rate, which implies $\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)/\partial \ln \left(\tau_{do}\right) \geq 0$. Because both *do* and *dd* unmatched rates of producers are weakly positive, the consumption elasticity in our model is at least as negative as the analogous trade elasticity in the class of models from Arkolakis, Costinot, and Rodríguez-Clare (2012) that satisfy equation (27).

Proposition 7. *The elasticity of trade shares to iceberg trade costs in our model with goods-market frictions is given by*

$$\frac{\partial \ln \left(IM_{do}/IM_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} = \begin{cases} -\theta + \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{do}\right)}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln \left(\tau_{do}\right)}\right) & \\ +\frac{\partial \ln \left(1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\right)}{\partial \ln \left(\tau_{d'o}\right)} - \frac{\partial \ln \left(1-b\left(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd}\right)\right)}{\partial \ln \left(\tau_{d'o}\right)} & if\ d'=d \\ \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right) & \\ +\frac{\partial \ln \left(1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\right)}{\partial \ln \left(\tau_{d'o}\right)} - \frac{\partial \ln \left(1-b\left(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd}\right)\right)}{\partial \ln \left(\tau_{d'o}\right)} & if\ d' \neq d \end{cases}, \tag{29}$$

in which we make the same simplifying assumptions as proposition 6.

Proof. Appendix B.3 derives the proof and defines the bundle of search frictions $\left(1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\right)$. □

Our trade elasticity depends on the usual parameter θ, as well as on the endogenous change between the markup in addition to the fraction of unmatched producers and the elasticity of producers' finding rate in the relevant product market as discussed in proposition 6. Using an argument similar to our previous logic, we know that raising tariffs, τ_{do}, reduces producers' finding rate, $\kappa_{do}\chi\left(\kappa_{do}\right)$, and that higher marginal costs reduce the markup in the *do* market in our calibration, $\partial \ln \left(1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\right)/\partial \ln \left(\tau_{do}\right) \leq 0$. Higher tariffs, τ_{do}, also increase the markup in the *dd* market so that $\partial \ln \left(1-b\left(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd}\right)\right)/\partial \ln \left(\tau_{do}\right) \geq 0$. Intuitively, this is because raising tariffs in the *do* market makes being a retailer in the *dd* market more valuable, inducing entry and increasing the finding rate for producers in the *dd* market, which allows producers to negotiate higher prices. Because the effects of both *do* and *dd* price markups on the import elasticity are weakly negative, the import elasticity in our model is more negative than our consumption elasticity and the analogous trade elasticity in many standard trade models.

We consider the relative magnitude of the matched and markup effects on consumption and trade elasticities in section 7.2. Under that calibration, we find that while both these effects have the signs we discussed, only the matched margin is quantitatively important.

Finally, we point out that a search model without endogenous matching rates or markups will give the same consumption and trade elasticities as the standard model.

5.3 The gravity equation

The gravity structure in our model, albeit more complicated, is similar to the gravity structure common to many trade models. To show this, we begin with the definition of total imports (free on board) by destination d from origin o in the differentiated goods sector:

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} n_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi)\,. \tag{30}$$

Performing the required integration in equation (30) gives the following proposition.

Proposition 8. *The gravity equation in our model is:*

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) \left(1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)\right) \alpha \left(\frac{C_o C_d}{C}\right) \left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}, \tag{31}$$

in which the fraction of matched exporters $1 - u_{do}/\left(1 - i_{do}\right)$ *and the bundle of search frictions* $1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)$ *are each weakly in the unit interval.*

Proof. See appendix B.4.1 for the derivation of the gravity equation. □

The main message is clear: Search frictions have a first-order effect on the level of total imports. Search frictions reduce trade flows in three ways. First, search frictions give rise to a fraction of unmatched exporters, $1 - u_{do}/\left(1 - i_{do}\right)$. Second, trade flows are diminished because imports are computed using negotiated import prices, n_{do}, as opposed to final sales prices, p_{do}. Negotiated import prices are lower than final sales prices. These lower import prices lead to the bundle of search friction parameters $1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right) \in [0, 1]$. Third, search frictions reduce imports because of the negative exponent on the effective entry cost, F_{do}, which is increasing in search frictions, as shown in equation (3). We present further details in appendix B.4.2.

Even if imports are measured at final sales prices, as assumed in the typical gravity equation, search frictions have a significant effect on trade flows. By evaluating equation

(30) at p_{do} instead of n_{do}, and using our functional form assumptions, we compute imports at final sales prices in appendix B.4.3 as

$$C_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) \alpha \left(\frac{C_o C_d}{C}\right) \left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}. \tag{32}$$

By definition, this provides consumption expenditure in destination d on differentiated goods produced in origin o. Even without the difference between final and import prices caused by $b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do})$, search frictions lead to a mass of unmatched and searching producers $u_{do}/(1 - i_{do})$, which lowers imports. Search frictions also affect imports through the effective entry cost, $F_{do}(\kappa_{do})$, but with the same exponent as existing models.[9]

Consumption expenditure must equal imports plus the period profits of matched importers, $C_{do} = IM_{do} + \Pi^m_{do}$. Combining equations (31) and (32) gives total period profits accruing to importers in matched relationships:

$$\Pi^m_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}) \alpha \left(\frac{C_o C_d}{C}\right) \left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}. \tag{33}$$

We could also obtain this quantity if we integrate profit to each variety $p_{do}(\varphi) q_{do}(\varphi) - n_{do}(\varphi) q_{do}(\varphi)$ over all imported varieties. Despite the value of posting a vacancy being driven to zero by free entry, flow profits are always positive as long as retailers' search cost, c_{do}, is positive, so importers can recoup the costs expended while searching.

6 Calibration and model fit

6.1 Calibration methodology

This section presents a baseline model calibration, which is summarized in table I. The unit of time is one year and we study a world economy with two countries for simplicity. Many of our search cost parameters rely on structural estimates based on U.S. and Colombian data

[9]Equation (31) also suggests that any empirical investigation that does not include adequate proxies for the effects of search frictions on trade prices and matched rates would suffer from omitted variable bias. We discuss some of these implications for estimating gravity equations in appendix B.5.

from Eaton et al. (2014) and Das, Roberts, and Tybout (2007). As a result, we calibrate most of our parameters to the United States (u) and Colombia (c) in 2014. We internally calibrate retailer flow search costs and producers' entry costs, and externally calibrate all remaining parameters of the model.

We take Colombia to represent the median economy trading with the rest of the world because Colombia is about at the 60th percentile in terms of imports and exports per capita. Our calibration is also informative for a broad range of U.S. trading partners because Colombia is about at the 40th percentile in terms of imports from and exports to the United States (Head, Mayer, and Ries, 2010).

The search friction in the model is generated by retailers' flow cost of search that is positive, $c_{do} > 0$, as discussed in section 3.4. Data on the fraction of exporters in the uc and cu markets can be used to pin down importing retailers' search costs, c_{uc} and c_{cu}, respectively, as shown in appendix C.1. Therefore, we choose c_{uc} to target that 30 percent of Colombian firms export to the United States (Eaton et al., 2014). Similarly, we choose c_{cu} so that the fraction of U.S. firms exporting to Colombia equals 2.5 percent. This fraction is based on the fact that about 50 percent of U.S. firms export (Lincoln and McCallum, 2018) and that around 5 percent of these firms export to Colombia (Bureau of the Census, 2014). Finally, labor underutilization (the unemployment rate) is often used to inform search costs in the labor-search literature, as in Pissarides (2009). As an analogy, we use manufacturing capacity underutilization to inform our choice of the domestic retailers' search cost, and we present the details of this approach in appendix C.2. We choose c_{uu} so that U.S. capacity utilization in manufacturing is 75 percent (Board of Governors of the Federal Reserve System (US), 2018b). We also assume that domestic retailers' flow search cost in Colombia is the same as in the United States so that $c_{uu} = c_{cc}$.

As pointed out by Shimer (2005), we could find alternative values for retailers' flow cost of search such that the equilibrium is unchanged for different values of the matching efficiency, ξ, and so we normalize the efficiency to one. We assume the matching technology for finding trading partners is similar to the technology used by workers and firms.

Therefore, we follow Petrongolo and Pissarides (2001) and set the elasticity of the matching function with respect to the number of searching producers, η, to 0.5. Motivated by Hall and Milgrom (2008), we set the bargaining power of producers, β, to 0.5, but we have found that realistic adjustments of this parameter yield similar results.

Turning to the search costs that producers face, we rely on the structural estimates provided by Eaton et al. (2014) and assume that producers' flow cost of search are symmetric, so that $l_{uc} = l_{cu}$ and $l_{uu} = l_{cc}$. We set l_{uc} and l_{cu} to \$20,000, which is broadly consistent with the large costs associated with international search by exporters documented in Eaton et al. (2014).[10] We also set $l_{uu} = l_{cc} = 0$, which is consistent with small domestic search costs found in Eaton et al. (2014). We assume that producers' fixed cost, f_{do}, and the sunk cost of production, s_{do}, do not vary by do pair so that $f_{do} = f,\ do = \{uu, uc, cu, cc\}$, and $s_{do} = s$. From Eaton et al. (2014) we assume that producers' fixed cost is \$2,900, and from Das, Roberts, and Tybout (2007) we take producers' sunk cost to be \$300,000. As we discussed in section 3.3, our model is comparable to standard trade models when we set $h_{do} = -l_{do}\ \forall do$, so we make this assumption. We have found that the equilibrium is not sensitive to alternative values for producers' fixed cost, f_{do}, the sunk cost of production, s_{do}, and producers' flow cost of search, l_{do}. Lastly, consistent with Eaton et al. (2014), we set the annual separation rate, λ, to 0.6, which implies that trading relationships in our calibration last just over 1.5 years on average.

We calibrate the model's trade parameters using standard estimates. We set the elasticity of substitution between differentiated varieties, σ, to six, as in Anderson and van Wincoop (2004) and Broda and Weinstein (2006). This implies a final sales price markup of 20 percent over the marginal production cost of each variety. As in Chaney (2008), we assume that there are no iceberg trade costs domestically, so that $\tau_{uu} = \tau_{cc} = 1$. Since U.S. trade-weighted tariffs are low (U.S. International Trade Commission, 2009) and the United States and Colombia signed a free trade agreement in 2012 (U.S. Trade Representative,

[10] Our value for l_{uc} and l_{cu} is consistent with the costs implied by equation (10) in Eaton et al. (2014) when we set the number of current customers, a, to zero – our model has one-to-one matches – and the finding rate for producers, s, to 0.1, which is roughly our implied values for $\kappa_{uc}\chi(\kappa_{uc})$ and $\kappa_{cu}\chi(\kappa_{cu})$ in equilibrium.

2018), we assume that the policy-relevant iceberg trade costs between the countries are zero, so that $\tau_{uc} = \tau_{cu} = 1$. We assume a Pareto productivity distribution consistent with a Pareto firm-size distribution governed by $\theta/(\sigma - 1)$ equal to 1.06, which is consistent with estimates from Axtell (2001) and implies θ equals 5.3. We set α, the fraction of consumption expenditure spent on differentiated goods, to 18 percent, which is consistent with evidence on the manufacturing output share in Baily and Bosworth (2014). We choose the labor endowments, L_c and L_u, so that the model generates the observed ratio of gross domestic products (GDPs) between Colombia and the United States, which is 2.2 percent. The levels of the labor endowments do not matter for key ratios in the model. We choose the cost of taking a productivity draw needed to become a producing firm in both countries, $e_u^x = e_c^x = e^x$, to target a ratio of consumption to output of two-thirds in the United States (Bureau of Economic Analysis, 2018b) and we set the annual interest rate to 4 percent (Board of Governors of the Federal Reserve System (US), 2018a).

6.2 Calibration outcomes and non-targeted moments

The model has four free parameters calibrated to match four empirical moments. The parameters are c_{uc}, c_{cu}, c_{uu} (which is constrained to equal c_{cc}), and e_u^x (which is constrained to equal e_c^x). The moments are manufacturing capacity utilization in the United States, the fraction of U.S. firms that export to Colombia, the fraction of Colombian firms exporting to the United States, and the U.S. consumption share. Table II presents the moments from the model using the calibrated parameter values from table I and shows that the model matches the calibration targets well. The model features a realistic fraction of exporting firms, and matches U.S. manufacturing capacity utilization and aggregate consumption as a share of GDP.

It is worth discussing the calibrated values of c_{do}. As we mentioned before, due to the normalization of the matching efficiency, ξ, it is difficult to interpret the level of these search costs. Rather, we look at the expected average costs for retailers to generate a contact with a producer, $c_{do}/\chi(\kappa_{do})$. These expected costs turn out to be ($\$14, \$29, \$16, \$8.9$) for the

uu, uc, cu, and cc markets, respectively. We think it is reasonable that the average retailer search cost of finding domestic partners is lower than for finding international partners ($14 and $8.9 vs. $29 and $16). We also think that it is reasonable that it costs Colombian retailers less to contact U.S. producers ($16) than the other way around ($29). In terms of magnitudes, these costs are small. This is largely because the exogenous separation rate, λ, is relatively high at 0.6, which means that matches are expected to last for only around 1.5 years. Therefore, the expected benefits from any one relationship in this model are small, and free entry into retailing implies that the expected costs must also be small (equation 19). Despite their small size, we will see that these search costs have quantitatively large implications for welfare, trade flows, and the consumption elasticity.

As a whole, the model delivers a realistic economic environment for the United States and Colombia by matching several important aggregate facts. Table III presents evidence for this by reporting several moments that we did not use in the calibration strategy. The model delivers the consumption share of GDP in Colombia, which was around 62 percent in 2014 (DANE, 2018). The trade flows in the calibrated model are also broadly consistent with observed trade flows.[11] The value of U.S. exports to Colombia as a fraction of U.S. GDP was small at 0.12 percent in 2014 (Bureau of Economic Analysis, 2018a; Bureau of the Census, 2014). The model overshoots this only slightly at 0.14 percent. Colombian exports to the United States as a fraction of Colombian GDP were larger than this at 4.8 percent, which is somewhat above the calibrated model's 2.9 percent (Bureau of the Census, 2014; World Bank, 2018b). The model implies that manufacturing capacity utilization rates are lower in Colombia than in the United States and this matches the data. Capacity utilization among Colombian manufacturers is around 70 percent, compared to 65 percent in the calibrated model, and below the 75 percent level in the United States (World Bank, 2018a; Board of Governors of the Federal Reserve System (US), 2018b).

The calibrated model also implies a distribution of final sales prices to prices at the dock,

[11] We use all exports to and imports from Colombia instead of focusing on manufacturing goods alone. This is consistent with the aggregate nature of our model and the fact that the traded value of the homogeneous good is indeterminate in the model.

p_{do}/n_{do}. The median of this distribution in the model for goods imported to the United States from Colombia is almost 10 percent. Berger et al. (2012) find that aggregate wedges between U.S. retail prices and prices at the dock, which include "both retail and distributor markups and local distribution and marketing costs," are around 50 to 70 percent. Since our model does not feature local distribution costs, it explains only about 20 percent of this aggregate wedge, but also suggests that about 80 percent of the aggregate wedge can be attributed to local distribution and marketing costs.

7 Quantitative general equilibrium results

7.1 Welfare analysis

In this section we present several exercises that emphasize the important role of search frictions for aggregate welfare. In section 7.1.1 we decompose the response of welfare to a unilateral tariff using our analytic results in proposition 5. Table IV presents this decomposition. In the subsequent two sections we examine the role of search costs for the level of aggregate welfare and what changes in search frictions are commensurate with a 10 percent increase in bilateral trade tariffs. Table V presents a summary of these additional results.

7.1.1 Decomposing the welfare response to unilateral tariffs

Search frictions attenuate welfare changes in response to a 10 percent unilateral tariff by about 40 percent relative to an environment without search frictions. This occurs because, relative to the standard model, both the domestic consumption share and the domestic producer matched rate attenuate the response of welfare. This quantifies our analytical results from section 5.1.

To compute the equilibrium in our model without search frictions, we set all parameters to the baseline values listed in table I, but reduce retailers' search costs to zero in domestic and foreign markets, $c_{do} = 0,\ do = \{uu, uc, cu, cc\}$. Because search is free, retailers enter the search market, raising market tightness and sending the contact rates for producers to infinity, $\kappa_{do}\chi(\kappa_{do}) \to \infty$, $\forall do$. As a result, producers find retailers instantly and the fraction of unmatched producers in each market falls to zero.

Column (1) of table IV shows that without search frictions, Colombian welfare falls by 1.7 percent in response to a 10 percent tariff on U.S. goods. This reduction in welfare is governed by the 64 percent increase in the domestic consumption share, along with the parameters α, θ, and σ, and is consistent with the results in Arkolakis, Costinot, and Rodríguez-Clare (2012). This model features no search frictions so the domestic matched rate is always one.

Column (2) of table IV shows that in the model with search frictions, Colombian welfare falls by 1 percent when Colombia raises unilateral tariffs on U.S. goods by 10 percent. Decomposing this welfare reduction using our analytic results in proposition 5 suggests that welfare changes for two reasons. First, the domestic consumption share rises by about 44 percent after the tariff increase and this reduces welfare to 98.8 percent of the pre-tariff level. Second, the tariff raises the Colombian price index, allowing Colombian retailers to earn higher revenue from consumers. The increased value of being a matched retailer in the Colombian market leads to more retailer entry and a higher matched rate for Colombian producers. A higher matched rate for Colombian producers serves to attenuate some of the reduction in welfare caused by the lower domestic consumption share. Quantitatively, the tariff raises the domestic market matched rate by 6.9 percent, which boosts welfare by 0.23 percent, offsetting some of the tariff's negative effects. The change in Colombian aggregate consumption is quantitatively trivial in both the model with and the model without search.

Comparing results with and without search frictions shows that the economy with search frictions exhibits a smaller decline in welfare in response to the same increase in tariffs. Overall, search frictions attenuate the welfare reduction by about 40 percent. About 30 percentage points of this attenuation come from a smaller increase in the domestic consumption share and about 10 percentage points come from the higher domestic matched rate.

The domestic consumption share response is smaller in the model with search because matched rates in both the domestic and foreign markets serve to mute the tariff's effect on the price index. The effect is muted because in the model without search, tariff changes affect all firms above the exporting cutoff, while in the model with search, only the fraction of matched firms are affected and the matched rate declines as tariffs increase. Additionally, protectionism increases the domestic matched rate, which also serves to attenuate the increase in the domestic price index.

7.1.2 Eliminating retailers' search costs

Eliminating search frictions raises U.S. welfare by around 1 percent and Colombian welfare by around 12 percent. These are sizable effects and are similar in magnitude to the changes in welfare that would be associated with moving to autarky in a simple Armington model (Costinot and Rodríguez-Clare, 2014).

In this exercise, we set all parameters to the baseline values listed in table I, but reduce retailers' search costs to zero in domestic and foreign markets, $c_{do} = 0 \ \forall do$, as in the frictionless example of the previous section. Column (1) of table V reports that the value of imports into the United States from Colombia increases by around 300 percent, and the value of imports into Colombia from the United States rises by around 90 percent. Due to the lower domestic consumption share caused by greater imports, welfare in the United States is roughly 1 percent higher, and welfare in Colombia rises by around 12 percent. Colombian welfare rises significantly more than U.S. welfare because the United States is a relatively large trading partner.

7.1.3 Replicating tariffs' effects with higher search costs

Increasing retailers' flow search costs by only a few dollars can have impacts on trade flows and aggregate welfare that are commensurate with a 10 percent increase in bilateral trade tariffs. Thus, the recent focus of international trade on firm-to-firm relationships (Eaton et al., 2014; Monarch and Schmidt-Eisenlohr, 2015; Heise, 2016) is warranted because these trading relationships, and the costs paid to form them, have economically significant implications for aggregate quantities.

We first quantify the welfare losses associated with a 10 percent increase in bilateral tariffs, so that $\tau_{uc} = \tau_{cu} = 1.1$, in column (2) of table V. This was the level of average U.S. tariffs in the 1970s (U.S. International Trade Commission, 2009). Solving the model with these higher bilateral tariffs, but keeping all other parameters at the baseline values from table I, implies that welfare in both countries falls. The United States experiences a 0.01 percent reduction in welfare, whereas Colombia's welfare falls by 1.0 percent. The value of imports into the United States from Colombia falls by around 60 percent, whereas the value of imports into Colombia from the United States falls by around 35 percent.

To understand the importance of importing retailers' search costs, we set all parameters to the baseline values in table I, but select higher flow costs of search for importing retailers, c_{uc} and c_{cu}, so that trade values decline by the same amount as in the experiment with a 10 percent increase in bilateral tariffs. Matching the reduction in trade flows requires raising the average cost for U.S. retailers to contact Colombian producers from \$29 to only \$31, and the average cost for Colombian retailers to contact U.S. producers from \$16 to only \$22. This increase in retailers' search costs lowers retailer entry and reduces producer finding rates, thereby raising the fraction of unmatched producers in the *uc* and *cu* markets. These higher unmatched rates have first-order effects on welfare, which, like trade flows, falls by the same amount in the two countries as in the example with higher tariffs. We present other equilibrium quantities in column (3) of table V.

7.2 Analysis of consumption and trade elasticities

Search frictions raise the consumption elasticity by over 50 percent to negative 8.1 from negative 5.3 in a model without them. This is because the change in the domestic and international producer matched rates magnify the effects of a tariff increase on consumption shares. This quantifies our analytical results from section 5.2 where we showed that the consumption elasticity in our model is at least as negative as the analogous elasticity in the standard trade model (proposition 6).

In our calibrated model without search frictions, a 10 percent (9.5 log percent) unilateral tariff on imports into Colombia from the United States ($\tau_{cu} = 1.1$) reduces the consumption share, C_{cu}/C_{cc}, by about 50 log percent, implying that the elasticity is negative 5.3 (column 1 of table VI). We derive this using a parameterization that is consistent with standard trade models and so has all parameters set to the baseline values in table I but, as before, does not have search frictions ($c_{do} = 0 \; \forall do$). The consumption share elasticity of negative 5.3 is exactly equal to the negative of the Pareto shape parameter ($-\theta$), which is the trade elasticity we derive analytically in our model without search frictions in appendix B.2.17, and matches Chaney (2008, p. 1716).

Comparing this to our model with search frictions, a 10 percent (9.5 log percent) unilateral tariff on imports into Colombia from the United States ($\tau_{cu} = 1.1$) reduces the consumption share, C_{cu}/C_{cc}, by about 80 log percent, implying that the elasticity is negative 8.1 (column 2 of table VI). For an analogous exercise where we increase tariffs by 10 percent on imports into the United States from Colombia ($\tau_{cu} = 1.1$), we find similar effects on the consumption share. We focus on the consumption elasticity because markup responses in this calibration are trivial and so the trade and consumption elasticities are essentially equal despite the differences between them derived in propositions 6 and 7.

We can decompose the consumption elasticity with search frictions into the (negative of the) Pareto shape parameter ($-\theta$), the elasticity of the matched rate in the cu and cc markets with respect to τ_{cu}, and the elasticity of the number of producers in Colombia and the United States with respect to τ_{cu}. This decomposition relies on proposition 6. The decomposition highlights a large decline, about 21 log percent, in the fraction of U.S. producers that are matched with Colombian retailers, implying an elasticity in the cu market with respect to τ_{cu} of around negative 2 (line 4, column 2 of table VI). This decline in the matched rate results from higher τ_{cu} tariffs reducing the benefit to Colombian retailers of being matched with U.S. producers, leading to less Colombian retailer entry, lower market tightness, and a lower finding rate in the cu market. This lower finding rate reduces the matched rate in the *cu* market. Because the consumption elasticity in proposition 6 is more negative in the model with search, the elasticity of the foreign consumption share, λ_{do}, can also be more negative despite the domestic consumption share, λ_{dd}, being less positive as shown in section 7.1.1.

The decomposition shown in proposition 6 also has an indirect protectionism effect that operates through the matched rate in the domestic, *cc*, market. As tariffs on U.S. imports rise, the Colombian price index increases, making it more valuable to be a matched domestic Colombian retailer, which leads to entry into the domestic retailing market. Greater entry raises the Colombian domestic finding rate for Colombian producers and subtracts from the standard elasticity. In the calibration of our model, this protectionism effect raises the matched rate in the *cc* market by 7 log percent (line 5, column 2, table VI). Finally, the effects of tariff changes on the numbers of producers in the U.S. and Colombian markets are trivial in our calibration.

Altogether, the consumption share elasticity in our model is negative 8.1, which is 2.8 more negative and about 50 percent greater than in the model without search frictions. Furthermore, while the elasticity in our framework with search frictions is more negative than the Pareto shape parameter would imply, it remains within the range of values estimated in prior work (Eaton and Kortum, 2002; Imbs and Mejean, 2015).

Lastly, while explaining the rapid increase in worldwide trade during the past few decades is not the goal of our paper, search models with endogenous market tightness have the ability to magnify the effect of tariffs on trade flows in a way that is similar to the role of vertical specialization in Yi (2003).

8 Conclusion

The international trade literature has recently made substantial progress in modeling and estimating the costs of forming relationships at the micro-level (Eaton et al., 2014; Monarch and Schmidt-Eisenlohr, 2015; Heise, 2016). However, far less is known about how these costs affect aggregate quantities in a general equilibrium framework. To improve understanding on this score, we have combined canonical models from search and trade to present a rich and tractable framework. This framework shares the central tenet of any search model: In equilibrium, there exists a mass of unmatched agents who are actively looking for partners. This simple observation leads to profound implications in our model because the product varieties associated with these unmatched agents (producers) cannot be consumed and are therefore absent from indirect utility (welfare), the price index, trade flows, and the levels of other aggregates. Additionally, if the mass of unmatched producers is endogenous, the absence of these varieties affects not only the levels of aggregates, but also their changes.

Specifically, we show that changes in the mass of unmatched varieties have first-order implications for welfare responses to any foreign shock. As a result, we generalize the findings of Arkolakis, Costinot, and Rodríguez-Clare (2012) to include search frictions and derive the welfare implications of search and trade that more complex models, such as Chaney (2014) and Eaton et al. (2016), left to future work. In particular, we find that search frictions attenuate the response of welfare to changes in tariffs. This result follows because higher tariffs, for example, result in a higher price index, which makes being a retailer in the domestic market more valuable and induces more retailers to enter the domestic market. With more retailers in the market, the rate at which domestic producers find domestic retailers increases, increasing domestic consumption and attenuating the welfare losses from higher tariffs.

In addition to these welfare results, we show that the consumption and import elasticities in our model are at least as negative as the analogous elasticities in a model without search frictions. Search frictions magnify these elasticities because as a destination raises tariffs on products from a specific country of origin, retailers in the destination country have less incentive to enter the search market. Having fewer retailers implies a looser search market and a higher unmatched rate for producers, which reduces consumption and imports even more than in a model without search.

We also find that adding search frictions reduces aggregate trade flows in three ways. First, they change the mass of varieties traded, because some producers are unmatched.

Second, search frictions introduce a markup between import and final sales prices. Third, they raise the effective entry cost. All of these effects are evident in the closed-form gravity equation derived from our model. Beyond aggregate predictions, our framework also provides a micro-foundation for the up-front costs firms face when entering foreign markets. These costs can include the fixed, search, opportunity, and sunk costs of serving a foreign market and depend on the rate at which producers find retailers.

When calibrated to U.S and Colombian data, our model implies that search frictions play an important quantitative role for welfare, trade flows, and the consumption elasticity. We find that eliminating retailers' search costs would increase U.S. welfare by over 1 percent, and search frictions attenuate the welfare responses to foreign tariff increases by around 40 percent. Search frictions, through their effect on the unmatched rate, raise the consumption elasticity by over 50 percent and can account for about 1/3 of the overall consumption elasticity.

Following the development of search models in labor and monetary economics, we propose three specific directions for future research. First, we have focused on the steady state of the model, but the framework is a dynamic one and could be extended to include the transition path after a relevant exogenous shock. This direction would dovetail nicely with Chaney (2014) and would go a long way toward providing a dynamic, continuous-time model that admits easy aggregation and retains the basic features of Melitz (2003). Second, the model can be extended to incorporate endogenous separations, in the spirit of Mortensen and Pissarides (1994), so that larger, more productive firms are in more stable trading relationships. Third, other matching and bargaining protocols, as in Burdett and Judd (1983) and Moen (1997), may present alternative implications for the mass of unmatched varieties relative to our model.

Locating and building connections with overseas buyers is a prevalent and costly barrier to exporting. We formalize costly search for international partners as a goods-market friction between producers and retailers. Our tractable setting provides a baseline for analyzing the aggregate implications of search frictions in models of trade.

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TABLE I: Calibrated Model Parameters

Parameter	Meaning	Calibrated Value	Reason
(c_{uu}, c_{cc})	Domestic retailers' flow search cost	$(0.0079, 0.0079)$	US manuf. capacity utilization
(c_{uc}, c_{cu})	Importing retailers' flow search cost	$(0.197, 1.4)$	Fraction of firms exporting
(e^x_u, e^x_c)	Producers' cost of taking a draw	$(0.52, 0.52)$	US consumption GDP share
ξ	Efficiency of matching function	1	Normalization
η	Elasticity of matching function	0.5	Petrongolo and Pissarides (2001)
β	Producers' bargaining power	0.5	Hall and Milgrom (2008)
$(l_{uu}, l_{uc}, l_{cu}, l_{cc})$	Producers' flow search cost	$(0, 20, 20, 0)$	Eaton et al. (2014)
f_{do}	Producers' fixed production cost	$2.9\ \forall do$	Eaton et al. (2014)
s_{do}	Producers' sunk cost	$300\ \forall do$	Das, Roberts, and Tybout (2007)
h_{do}	Producers' idle flow payoff	$-l_{do}\ \forall do$	See section 3.3
λ	Exogenous separation rate	0.6	Eaton et al. (2014)
σ	Elasticity of substitution	6	Broda and Weinstein (2006)
τ_{do}	Iceberg transit costs	$1\ \forall do$	Chaney (2008); U.S. International Trade Commission (2009)
θ	Pareto shape parameter	5.3	Axtell (2001)
α	Cobb-Douglas power on CES aggregate	0.18	Baily and Bosworth (2014)
L_c/L_u	Relative economic size	0.022	Relative GDPs
r	Risk-free rate	0.04	3-month T-bill

Note: Calibrated parameters of the model at annual frequency. We calibrate to data from the United States and Colombia in 2014. Domestic and importing retailers' search costs are calibrated internally, but we constrain the flow search costs of domestic retailers in the United States to be the same as those in Colombia, $c_{uu} = c_{cc}$. Producers' cost of taking a productivity draw is also calibrated internally, but we constrain this cost to be the same for U.S. and Colombian producers, $e^x_u = e^x_c$. The rest of the parameters are exogenously determined. All payoffs and costs are reported in thousands of year 2000 U.S. dollars. The exogenous separation rate, λ, is an annual Poisson rate. The iceberg transit cost, τ_{do}, is defined such that fraction $1/\tau_{do}$ of a unit arrives. "Manuf." stands for manufacturing.

TABLE II: Calibration Targets

Moment in the data	**Data**	**Model**
Fraction of US firms exporting to CO	0.025	0.025
Fraction of CO firms exporting to US	0.30	0.30
US manufacturing capacity utilization	75%	75%
US consumption GDP share	66%	66%

Note: The model matches the empirical targets well. The middle column of this table presents the value of the moment in the data. The column on the right presents the value of the equivalent moment in the model at the calibrated parameter values (table I). "CO" stands for Colombia and "US" stands for the United States.

TABLE III: Nontargeted Moments

Moment	**Data**	**Model**
CO consumption GDP share	62%	66%
US exports to CO as fraction of US GDP	0.12%	0.14%
CO exports to US as fraction of CO GDP	4.8%	2.9%
CO manufacturing capacity utilization	70%	65%

Note: The model performs relatively well in matching nontargeted moments. The middle column of this table presents the value of the moment in the data. The column on the right presents the value of the equivalent moment in the model at the calibrated parameter values (table I). "CO" stands for Colombia and "US" stands for the United States.

TABLE IV: Decomposing the Colombian Welfare Response to a Unilateral Tariff Increase

Determinants of welfare change	(1) No search frictions and 10% unilateral tariff	(2) Baseline search frictions and 10% unilateral tariff
Pre-tariff dom. consump. share (λ_{cc})	0.0038	0.0783
Post-tariff dom. consump. share (λ'_{cc})	0.0063	0.113
Ratio of dom. consump. shares $\left(\hat{\lambda}_{cc} = \lambda'_{cc}/\lambda_{cc}\right)$	1.636	1.438
Dom. consump. shares' effect on welfare $\left(\hat{\lambda}_{cc}^{-\frac{\alpha}{\theta}}\right)$	0.983	0.988
Pre-tariff dom. matched rate $\left(1 - \frac{u_{cc}}{1-i_{cc}}\right)$	1	0.652
Post-tariff dom. matched rate $\left(1 - \frac{u_{cc}}{1-i_{cc}}\right)'$	1	0.698
Ratio of dom. matched rates $\widehat{\left(1 - \frac{u_{cc}}{1-i_{cc}}\right)}$	1	1.069
Dom. matched rates' effect on welfare $\widehat{\left(1 - \frac{u_{cc}}{1-i_{cc}}\right)}^{\frac{\alpha}{\theta}}$	1	1.0023
Pre-tariff dom. consump. level (C_c)	33.097	33.10
Post-tariff dom. consump. level (C'_c)	33.096	33.097
Ratio of dom. consump. levels $\left(\hat{C}_c = C'_c/C_c\right)$	1.00	0.999
Dom. consump. levels' effect $\left(\hat{C}_c^{1+\frac{\alpha}{\theta}\left(1-\frac{\theta}{\sigma-1}\right)}\right)$	1.00	0.999
Welfare as fraction of pre-tariff welfare $\left(\hat{W}_c\right)$	0.983	0.990
Welfare percent change $\left(100 \times \left[\hat{W}_c - 1\right]\right)$	-1.7	-1.0

Note: Search frictions attenuate the Colombian welfare response to a 10 percent tariff by about 40 percent, lowering the welfare loss from 1.7 percent to 1 percent. The table presents equilibrium variables in response to a 10 percent increase in unilateral tariffs on imports to Colombia from the United States. Using proposition 5, we know that the complete welfare response in our baseline calibration is:

$$\hat{W}_c = \underbrace{\hat{\lambda}_{cc}^{-\frac{\alpha}{\theta}}}_{\substack{\text{Dom. consump.}\\ \text{share effect}}} \underbrace{\widehat{\left(1 - \frac{u_{cc}}{1 - i_{cc}}\right)}^{\frac{\alpha}{\theta}}}_{\substack{\text{Dom. matched}\\ \text{rate effect}}} \underbrace{\hat{C}_c^{1+\frac{\alpha}{\theta}\left(1-\frac{\theta}{\sigma-1}\right)}}_{\substack{\text{Dom. consump.}\\ \text{level effect}}}$$

Column (1) presents the response without search frictions, which is the same as Arkolakis, Costinot, and Rodríguez-Clare (2012) and is completely determined by the ratio of the domestic consumption shares and model parameters α, θ, and σ. Some rows in column (1) are exactly 1 because those factors do not change in a model without search frictions. Column (2) presents the decomposition of the effect in our model with search frictions. Domestic consumption rises by about 44 percent after the tariff increase and this reduces welfare to 98.8 percent of the pre-tariff level. Protection of the domestic market raises the domestic matched rate by 6.9 percent and serves to boost welfare by 0.23 percent, offsetting some of the tariff's negative effects.

TABLE V: Changes in Welfare, Imports, and the Unmatched Rate for Three Experiments

	Experiment (1) No search frictions	Experiment (2) Baseline search frictions and 10% bilateral tariff	Experiment (3) Search costs equiv. to 10% bilateral tariff
US welfare (%Δ)	1.0	-0.01	-0.01
Colombian welfare (%Δ)	12.4	-1.0	-1.0
US imports from Colombia (%Δ)	305	-59	-59
Colombian imports from U.S. (%Δ)	88	-34	-34
Unmatched rate in US-US market (pp. Δ)	-25	0	0
Unmatched rate in US-CO market (pp. Δ)	-80	6.3	11.5
Unmatched rate in CO-US market (pp. Δ)	-98	0.3	0.9
Unmatched rate in CO-CO market (pp. Δ)	-35	-4.5	-4.5

Note: Search frictions play an important role in the level of welfare and in the response of welfare to tariff changes. The table presents deviations from the baseline calibration in section 6. Recall that the baseline calibration has $c_{do}/\chi\left(\kappa_{do}\right)$ equal to $(0.014, 0.029, 0.016, 0.0089)$ in the uu, uc, cu and cc markets, respectively, $\tau_{do} = 1\ \forall do$, and equilibrium unmatched rates equal to $(0.25, 0.80, 0.98, 0.35)$ in the uu, uc, cu and cc markets, respectively. Columns (1) through (3) report the percent changes from baseline for three different experiments. Column (1) eliminates search frictions as in section 7.1.2 and shows that the associated welfare gains are large. Columns (2) and (3) present the two exercises in section 7.1.3. Column (2) increases bilateral tariffs by 10 percent. Column (3) shows that, by affecting the unmatched rate, only small increases in the average cost for retailers to contact foreign producers attain the same welfare changes as in column (2) (in this calibration $c_{do}/\chi\left(\kappa_{do}\right)$ equals $(0.014, 0.031, 0.022, 0.011)$ in the uu, uc, cu and cc markets, respectively). "%Δ" stands for percent change. "pp. Δ" stands for percentage point change. "CO" stands for Colombia and "US" stands for the United States.

TABLE VI: Decomposing the Colombian Consumption and Trade Elasticities

	(1) No search frictions and 10% unilateral tariff	(2) Baseline search frictions and 10% unilateral tariff
Pareto shape parameter $(-\theta)$	-5.30	-5.30
Elasticity of CO producers	0.00	0.00
Elasticity of US producers	0.00	0.00
Elasticity of the CO-US matched rate	0	-2.14
Elasticity of the CO-CO matched rate	0	0.70
Consumption elasticity	-5.31	-8.14
Elasticity of CO-US markup	0	-0.03
Elasticity of CO-CO markup	0	0.00
Trade elasticity	-5.31	-8.18

Note: Search frictions raise the consumption and trade elasticities by over 50 percent and around 1/3 of the overall consumption or trade elasticity is explained by the elasticity of the matched rate in the cu market. The table presents equilibrium variables in response to a 10 percent increase in unilateral tariffs on imports to the United States from Colombia. Using proposition 6, along with equations (77) and (78) in appendix B.2, we know that the complete consumption elasticity expression in this baseline calibration is:

$$\frac{\partial \ln (C_{cu}/C_{cc})}{\partial \ln (\tau_{cu})} = -\theta + \underbrace{\frac{\partial \ln (N_u^x)}{\partial \ln (\tau_{cu})}}_{\substack{\text{Elasticity of} \\ \text{US producers}}} - \underbrace{\frac{\partial \ln (N_c^x)}{\partial \ln (\tau_{cu})}}_{\substack{\text{Elasticity of} \\ \text{CO producers}}} + \underbrace{\underbrace{\left(\frac{u_{cu}}{1-i_{cu}}\right)\left(\frac{\partial \ln \kappa_{cu}\chi(\kappa_{cu})}{\partial \ln (\tau_{cu})}\right)}_{\substack{\text{Elasticity of} \\ cu \text{ matched rate}}} - \underbrace{\left(\frac{u_{cc}}{1-i_{cc}}\right)\left(\frac{\partial \ln \kappa_{cc}\chi(\kappa_{cc})}{\partial \ln (\tau_{cu})}\right)}_{\substack{\text{Elasticity of} \\ cc \text{ matched rate}}}}_{\substack{\text{``Market thickness''} \\ \text{effects}}}.$$

The trade elasticity adds to this the elasticity of import markup terms in proposition 7,

$$\frac{\partial \ln (IM_{cu}/IM_{cc})}{\partial \ln (\tau_{cu})} = \underbrace{\frac{\partial \ln (C_{cu}/C_{cc})}{\partial \ln (\tau_{cu})}}_{\substack{\text{Elasticity of} \\ \text{consumption}}} + \underbrace{\underbrace{\frac{\partial \ln (1-b(\sigma,\theta,\gamma_{cu},\delta_{cu},F_{cu}))}{\partial \ln (\tau_{cu})}}_{\substack{\text{Elasticity of } cu \\ \text{markup}}} - \underbrace{\frac{\partial \ln (1-b(\sigma,\theta,\gamma_{cc},\delta_{cc},F_{cc}))}{\partial \ln (\tau_{cu})}}_{\substack{\text{Elasticity of } cc \\ \text{markup}}}}_{\substack{\text{Markup} \\ \text{effects}}}.$$

Column (1) presents the response of the consumption and trade shares to a foreign tariff shock with no search frictions, which is essentially $-\theta$ (equation (78)). Column (2) presents the decomposition of these elasticities into their components in our model with search frictions; the elasticity of the cu and cc matched rates are the only non-trivial components even though the markup terms respond to the tariff increase. The elasticity of the CO-US and CO-CO matched rates and markups in column (1) are exactly zero because these results have no search frictions. The other zeros in the table are rounded to the second decimal point. Notice that the consumption and trade elasticities in column (1) do not equal -5.30 due to numerical imprecision. In column (2) rounding error means that the consumption elasticity, together with the markups, does not quite equal the trade elasticity.

A Model appendix

A.1 Utility maximization and the ideal price index

A.1.1 Utility maximization

Here we present the solution to the utility maximization problem in section 2.2. The representative consumer's maximization problem can be stated as:

$$\begin{aligned}
\max_{q_d(1),q_{dk}(\omega)} \quad & q_d(1)^{1-\alpha}\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} q_{dk}(\omega)^{\left(\frac{\sigma-1}{\sigma}\right)}d\omega\right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)} \\
s.t. \quad & \\
C_d \quad = \quad & p_d(1)\,q_d(1)+\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)\,q_{dk}(\omega)\,d\omega.
\end{aligned}$$

We can solve this problem by maximizing the following Lagrangian

$$\mathcal{L} = q_d(1)^{1-\alpha}\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} q_{dk}(\omega)^{\frac{\sigma-1}{\sigma}}d\omega\right]^{\frac{\alpha 2\sigma}{\sigma-1}} - \lambda\left[p_d(1)\,q_d(1)+\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)\,q_{dk}(\omega)\,d\omega - C\right].$$

The first-order conditions (FOCs) for the homogenous good and two arbitrary varieties from the same origin, ω and ω', are:

$$\mathcal{L}_{q_d(1)} = \alpha q_1^{-\alpha}\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} q_{dk}(\omega)^{\frac{\sigma-1}{\sigma}}d\omega\right]^{\alpha\frac{\sigma}{\sigma-1}} - \lambda p_d(1) = 0$$

$$\begin{aligned}
\mathcal{L}_{q_{dk}(\omega)} &= q_d(1)^{1-\alpha}\,\alpha\left(\frac{\sigma}{\sigma-1}\right)\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} q_{dk}(\omega)^{\frac{\sigma-1}{\sigma}}d\omega\right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)-1}\left(\frac{\sigma-1}{\sigma}\right)q_{dk}(\omega)^{\frac{\sigma-1}{\sigma}-1} - \lambda p_{dk}(\omega) = 0 \\
\mathcal{L}_{q_{dk}(\omega')} &= q_d(1)^{1-\alpha}\,\alpha\left(\frac{\sigma}{\sigma-1}\right)\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} q_{dk}(\omega)^{\frac{\sigma-1}{\sigma}}d\omega\right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)-1}\left(\frac{\sigma-1}{\sigma}\right)q_{dk}(\omega')^{\frac{\sigma-1}{\sigma}-1} - \lambda p_{dk}(\omega') = 0.
\end{aligned}$$

Dividing the last two FOCs and performing some algebra yields $q_{dk}(\omega)$ in terms of $q_{dk}(\omega')$:

$$q_{dk}(\omega) = q_{dk}(\omega')\left[\frac{p_{dk}(\omega')}{p_{dk}(\omega)}\right]^{\sigma}.$$

Using the ratio of the first and third FOCs delivers a relationship between $q_d(1)$ and $q_{dk}(\omega')$:

$$q_d(1) = \frac{p_{dk}(\omega')}{p_d(1)}\left(\frac{1-\alpha}{\alpha}\right)\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} q_{dk}(\omega)^{\frac{\sigma-1}{\sigma}}d\omega\right]q_{dk}(\omega')^{\frac{1}{\sigma}}.$$

Using our solution for $q_{dk}(\omega)$ for the term in brackets yields:

$$\int_{\omega\in\Omega_{dk}} q_{dk}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega = \int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma} d\omega p_{dk}(\omega')^{\sigma-1} q_{dk}(\omega')^{\frac{\sigma-1}{\sigma}}$$

and plugging this in gives

$$q_d(1) = \left(\frac{1}{p_d(1)}\right)\left(\frac{1-\alpha}{\alpha}\right)\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma} d\omega p_{dk}(\omega')^{\sigma} q_{dk}(\omega').$$

Now we can write the budget constraint in terms of $q_{dk}(\omega')$ and after some algebra this gives

$$C_d = \left\{\left(\frac{1}{\alpha}\right)\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma} d\omega p_{dk}(\omega')^{\sigma}\right\} q_{dk}(\omega').$$

So demand for $q_{dk}(\omega')$ is given by

$$q_{dk}(\omega') = \alpha C_d \frac{p_{dk}(\omega')^{-\sigma}}{\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma} d\omega}.$$

There are a couple of things to notice here. The first is that the demand for the CES good $q_{dk}(\omega')$ is not a function of the price of the good $q_d(1)$. Also notice that we can interpret αC_d as the consumer using the fraction of total expenditure from the Cobb-Douglas level of the utility function to define the fraction of total consumption resources that are devoted to this particular variety of the differentiated good.

As we show in appendix A.1.2 the price index for the differentiated goods from origin k to destination d is

$$P_{dk} = \left\{\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma} d\omega\right\}^{\frac{1}{1-\sigma}}$$

and the overall price index for the differentiated goods in country d is

$$P_d = \left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}.$$

This means the demand for each CES variety is the function

$$q_{dk}(\omega) = \alpha C_d \frac{p_{dk}(\omega)^{-\sigma}}{P_d^{1-\sigma}}$$

The demand for the homogeneous good $q_d(1)$:

$$
\begin{aligned}
q_d(1) &= \left(\frac{1}{p_1}\right)\left(\frac{1-\alpha}{\alpha}\right)\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma}\,d\omega p_{dk}(\omega')^{\sigma}\,q_{dk}(\omega') \\
&= (1-\alpha)\frac{C_d}{p_d(1)}
\end{aligned}
$$

which is just the amount $(1-\alpha)C_d$ (Cobb-Douglas) spent on the good that has price $p_d(1)$. These are the demand functions in equation (2) of the main text.

A.1.2 Expenditure minimization and the price index

Here we derive, in full, the price index associated with our utility function. First we deal with the price index for the differentiated goods. Then we obtain the overall price index for the homogeneous and the differentiated goods.

The expenditure minimization problem for the differentiated goods looks as follows:

$$
\begin{aligned}
\min_{q_{dk}(\omega)} \quad & \sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)\,q_{dk}(\omega)\,d\omega \\
s.t. \quad & \\
U_d^{\rho} &= \sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} q_{dk}(\omega)^{\rho}\,d\omega
\end{aligned}
$$

in which, for ease of notation, we have temporarily defined $\rho \equiv \dfrac{\sigma-1}{\sigma}$. The following steps resemble the steps taken in Varian (1992) pg. 55.

The Lagrangian is:

$$
\mathcal{L} = \sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)\,q_{dk}(\omega)\,d\omega + \lambda\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} q_{dk}(\omega)^{\rho}\,d\omega - U_d^{\rho}\right].
$$

The first-order conditions (FOCs) are therefore:

$$
\mathcal{L}_{q_{dk}(\omega)} = p_{dk}(\omega) - \lambda\rho q_{dk}(\omega)^{\rho-1} = 0
$$

$$
\mathcal{L}_{\lambda} : \int_{\omega\in\Omega_{dk}} q_{dk}(\omega)^{\rho}\,d\omega = U_d^{\rho}.
$$

Rearrange the first FOC to get:

$$q_{dk}(\omega)^{\rho} = p_{dk}(\omega)^{\frac{\rho}{\rho-1}} (\lambda\rho)^{-\frac{\rho}{\rho-1}}.$$

Put this back into the utility function to get:

$$(\lambda\rho)^{-\frac{\rho}{\rho-1}} = \frac{U_d^{\rho}}{\sum_{k=1}^{O} \int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{\frac{\rho}{\rho-1}} d\omega}.$$

Substitute this back into the equation above to get:

$$\begin{aligned} q_{dk}(\omega)^{\rho} &= p_{dk}(\omega)^{\frac{\rho}{\rho-1}} (\lambda\rho)^{-\frac{\rho}{\rho-1}} \\ q_{dk}(\omega)^{\rho} &= p_{dk}(\omega)^{\frac{1}{\rho-1}} \left[\sum_{k=1}^{O} \int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{\frac{\rho}{\rho-1}} d\omega\right]^{-\frac{1}{\rho}} U_d. \end{aligned}$$

Now we have the demand functions in terms of prices and utility. Substitute this back into the objective function and collect terms to obtain the expenditure function:

$$\begin{aligned} e(p_{dk}(\omega), U_d) &= \sum_{k=1}^{O} \int_{\omega\in\Omega_{dk}}^{\infty} p_{dk}(\omega)\, q_{dk}(\omega)\, d\omega \\ &= \sum_{k=1}^{O} \int_{\omega\in\Omega_{dk}} p_{dk}(\omega)\, p_{dk}(\omega)^{\frac{1}{\rho-1}} \left[\sum_{k=1}^{O} \int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{\frac{\rho}{\rho-1}} d\omega\right]^{-\frac{1}{\rho}} U_d d\omega \\ &= U_d \left[\sum_{k=1}^{O} \int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{\frac{\rho}{\rho-1}} d\omega\right]^{\frac{\rho-1}{\rho}}. \end{aligned}$$

Substitute $\rho \equiv \dfrac{\sigma-1}{\sigma}$ back into this expression to get that

$$e(p_{dk}(\omega), U_d) = U_d \left[\sum_{k=1}^{O} \int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}.$$

And the ideal price index for the differentiated good from country k to country d is

$$P_{dk} = e(p_{dk}(\omega), 1) = \left[\sum_{k=1}^{O} \int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}.$$

Note that this is consistent with

$$P_d = \left[\sum_{k=1}^{O} P_{dk}^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

in which $P_{dk} = \left[\int_{\omega\in\Omega_{dk}}^{\infty} p_{dk}(\omega)^{1-\sigma}\, d\omega\right]^{\frac{1}{1-\sigma}}$, which is equation (3) in the main text.

Now we move on to deriving the overall price index, including the homogeneous good. From our previous work in appendix A.1.1, we know that the optimal quantity demanded of the differentiated good is

$$q_{dk}(\omega) = \alpha C_d \frac{p_{dk}(\omega)^{-\sigma}}{P_d^{1-\sigma}}.$$

We also know that the optimal quantity demanded of the homogeneous good is:

$$q_d(1) = (1-\alpha)\frac{C_d}{p_d(1)}.$$

Using these, we can derive the indirect utility function with some algebra:

$$\begin{aligned}
W_d\left(p_d(1), p_{dk}(\omega), C_d\right) &= q_d(1)^{1-\alpha}\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} q_{dk}(\omega)^{\frac{\sigma-1}{\sigma}}\, d\omega\right]^{\alpha\frac{\sigma}{\sigma-1}} \\
&= \left((1-\alpha)\frac{C_d}{p_d(1)}\right)^{1-\alpha}\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}}\left(\alpha C_d\frac{p_{dk}(\omega)^{-\sigma}}{P_d^{1-\sigma}}\right)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\alpha\frac{\sigma}{\sigma-1}} \\
&= \left(\frac{1-\alpha}{p_d(1)}\right)^{1-\alpha}\left(\frac{\alpha}{P_d}\right)^{\alpha} C_d.
\end{aligned}$$

Now, we know that our utility function is HOD 1 so our welfare expression can also be written as

$$W_d\left(\Xi_d, C_d\right) = \frac{C_d}{\Xi_d},$$

in which Ξ_d is the overall price index. Setting these two welfare expressions equal to each other gives us:

$$\begin{aligned}
\frac{C_d}{\Xi_d} &= \left(\frac{1-\alpha}{p_d(1)}\right)^{1-\alpha}\left(\frac{\alpha}{P_d}\right)^{\alpha} C_d \\
\Xi_d &= \left(\frac{p_d(1)}{1-\alpha}\right)^{1-\alpha}\left(\frac{P_d}{\alpha}\right)^{\alpha}.
\end{aligned}$$

A.2 Poisson process

Consider a continuous time Poisson process in which the number of events, n, in any time interval of length t is Poisson distributed according to

$$P\left\{N\left(t+s\right)-N\left(s\right)=n\right\}=e^{-\lambda t}\frac{\left(\lambda t\right)^{n}}{n!}\qquad n=0,1,\ldots$$

in which $s,t\geq 0$, $N\left(0\right)=0$, and the process has independent increments. The mean number of events that occur by time t is

$$E\left[N\left(t\right)\right]=\lambda t.$$

Notice that λ is defined in units of time as λ events per t. For example, if producers in our model contact nine retailers every six months, on average, then we could recast our model measured in years with $t=1$ and $\lambda=4.5$ because $\lambda t=9\times 1/2=4.5$.

Using the Poisson process above, the probability that the first event occurs after time t equals the probability no event has happened before

$$P\left\{t_1>t\right\}=P\left[N\left(t\right)=0\right]=e^{-\lambda t},$$

in which t_n denotes the time between the $(n-1)$st and nth events so t_1 is the time of the first event. The arrival time of the first event is an exponential random variable with parameter λ. Conversely, the probability the first event occurs between time 0 and time t is $P\left\{t_1\leq t\right\}=1-e^{-\lambda t}$. Because the Poisson process has independent increments, the distribution of time between any two events, t_n, for $n=1,2,\ldots$ will also be an exponential random variable with parameter λ. The sequence of times between all events, $\{t_n, n\geq 1\}$, also known as the sequence of inter-arrival times, will be a sequence of *i.i.d.* exponential random variables with parameter λ. Given this distribution, the mean time between events is

$$E\left[t_n\right]=\frac{1}{\lambda}$$

For example, if producers in our model contact nine retailers every six months, on average, so that $\lambda t=9/2$, then the average time between contacts is $1/\lambda=2/9$ years (or about $365.25\times 2/9=81.17$ days).

The arrival time of the nth event, S_n, also called the waiting time, is the sum of the time between preceding events

$$S_n=\sum_{i=1}^{n}t_i.$$

Because S_n is the sum of n *i.i.d.* exponential random variables in which each has parameter λ and the number of events n is an integer, S_n has an Erlang distribution with cumulative density function

$$P\{S_n \leq t\} = P[N(t) \geq n] = \sum_{i=n}^{\infty} e^{-\lambda t} \frac{(\lambda t)^i}{i!},$$

and probability density function

$$f(t) = \lambda e^{-\lambda t} \frac{(\lambda t)^{n-1}}{(n-1)!}.$$

The Erlang distribution is a special case of the gamma distribution in which the gamma allows the number of events n to be any positive real number, while the Erlang distribution restricts n to be an integer. The above discussion relies heavily on (Ross, 1995, Chapter 2).

A.3 The surplus, value, and expected duration of a relationship

Denote the joint surplus accruing to both sides of a match as $S_{do}(\varphi)$. The bargain will divide this surplus such that the value of being a retailer equals $M_{do}(\varphi) - V_{do} = (1-\beta) S_{do}(\varphi)$ and the value of being a producer is $X_{do}(\varphi) - U_{do}(\varphi) = \beta S_{do}(\varphi)$, in which β is the producer's bargaining power. Using the value functions presented in the main text (7), (8), (10), and (11), we can write the surplus equation as

$$S_{do}(\varphi) \quad = \quad \frac{p_{do} q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + s_{do} \kappa_{do} \chi(\kappa_{do})}{r + \lambda + \beta \kappa_{do} \chi(\kappa_{do})}. \tag{34}$$

The surplus created by a match is the appropriately discounted flow profit, with the search cost l_{do} and the sunk cost s_{do} also entering the surplus equation because being matched avoids paying these costs. There are three things to notice here. First, the surplus from a match is a function of productivity. We show in appendix A.7 that matches that include a more productive exporting firm lead to greater surplus, that is, $S'_{do}(\varphi) > 0$. Second, the value of the relationship will fluctuate over the business cycle as shocks hit the economy and change the finding rate $\kappa_{do}\chi(\kappa_{do})$. Finally, surplus is greater than or equal to zero when

$$p_{do} q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + s_{do} \kappa_{do} \chi(\kappa_{do}) \geq 0.$$

Specifically, at the binding productivity cutoff we can use equation (44) and the surplus sharing rule to write

$$\beta S_{do}(\bar{\varphi}_{do}) = \frac{l_{do} + h_{do}}{\kappa_{do} \chi(\kappa_{do})} + s_{do},$$

which, in order for surplus to be positive, puts a restriction on the parameter choices and the equilibrium value of market tightness, κ_{do}.

With the definition of surplus in hand, the value of a matched relationship,

$R_{do}(\varphi) = X_{do}(\varphi) + M_{do}(\varphi)$, can be expressed as $R_{do}(\varphi) = S_{do}(\varphi)\left(\frac{r + \kappa_{do}\chi(\kappa_{do})\beta}{r}\right) - \frac{l_{do}}{r}$. The value of the relationship to the producer is, of course, $X_{do}(\varphi)$ and to the retailer $M_{do}(\varphi)$. The value of a relationship in product markets has been of recent interest in Monarch and Schmidt-Eisenlohr (2015) and Heise (2016).

Relationships are destroyed at Poisson rate λ in the model, which implies the average duration of each match is $1/\lambda$. Because the destruction rate is exogenous and does not vary in our model, the average duration of each match is constant.

A.4 Bargaining over the negotiated price

A.4.1 Surplus sharing rule

Take equation (12), log and differentiate with respect to the price n_{do} and rearrange to get

$$\beta\frac{q_{do}}{X_{do}(\varphi) - U_{do}(\varphi)} + (1-\beta)\frac{-q_{do}}{M_{do}(\varphi) - V_{do}} = 0, \tag{35}$$

which implies the simple surplus sharing rule, equation (13): The retailer receives β of the total surplus from the trading relationship, $S_{do}(\varphi) = M_{do}(\varphi) - V_{do} + X_{do}(\varphi) - U_{do}(\varphi)$. The producer receives the rest of the surplus, $(1-\beta)S_{do}(\varphi)$.

In section 3.1 of the main text, we point out the restriction that $\beta < 1$ in equation (12) is evident in equation (13), which results from equation (35). Retailing firms have no incentive to search if $\beta = 1$ because they get none of the resulting match surplus and therefore cannot recoup search costs $c_{do} > 0$. Any solution to the model with $c_{do} > 0$ and positive trade between retailers and producers also requires $\beta < 1$. This result can be shown explicitly by using equations (10), (11), and (14) together with $\beta = 1$ to show that for productivity, φ, levels above the reservation productivity, $\bar{\varphi}_{do}$, (defined in section 3.3), the retailing firm has no incentive to search.

Finally, we do not need to calculate the partial derivative with respect to $U_{do}(\varphi)$ or $V_{do}(\varphi)$ because the individual firms are too small to influence aggregate values. Hence, when they meet, the firms bargain over the negotiated price-taking behavior in the rest of the economy as given. In particular, the outside option of the firms does not vary with the individual's bargaining problem.

A.4.2 Proof of proposition 1: Solving for the equilibrium negotiated price

Equations (7), (8), (10), and the equilibrium free entry condition $V_{do} = 0$ imply that

$$M_{do}(\varphi) - V_{do} = \frac{p_{do}(q_{do})q_{do} - n_{do}q_{do}}{r + \lambda} \tag{36}$$

and

$$X_{do}(\varphi) - U_{do}(\varphi) = \frac{n_{do}q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do}) s_{do}}{r + \lambda + \kappa_{do}\chi(\kappa_{do})}. \tag{37}$$

Bargaining over price results in equation (35) and delivers the surplus sharing rule given by equation (13), which we can rewrite as $\beta(M_{do}(\varphi) - V_{do}) = (1-\beta)(X_{do}(\varphi) - U_{do}(\varphi))$. Using this transformation of equation (13) and the definitions given by equations (36) and (37) we can write

$$\begin{aligned}
\beta\frac{p_{do}(q_{do})q_{do} - n_{do}q_{do}}{r+\lambda} &= (1-\beta)\frac{n_{do}q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do}) s_{do}}{r + \lambda + \kappa_{do}\chi(\kappa_{do})} \\
\Rightarrow n_{do}q_{do} &= p_{do}(q_{do})q_{do}(1-\gamma_{do}) + \gamma_{do}\left[t(q_{do}, w_o, \tau_{do}, \varphi) + f_{do} - l_{do} - \kappa_{do}\chi(\kappa_{do}) s_{do}\right] \\
\Rightarrow n_{do} &= [1-\gamma_{do}]\, p_{do} + \gamma_{do}\frac{t(q_{do}, w_o, \tau_{do}, \varphi) + f_{do} - l_{do} - \kappa_{do}\chi(\kappa_{do}) s_{do}}{q_{do}}
\end{aligned}$$

in which $\gamma_{do} \equiv \dfrac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\kappa_{do}\chi(\kappa_{do})}$.

A.4.3 Bounding the search friction

Recall the definition

$$\gamma_{do} \equiv \frac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\kappa_{do}\chi(\kappa_{do})}.$$

Here we show that $\gamma_{do} \in [0,1]$. First, because all parameters are positive, $\gamma_{do} \geq 0$. The lower bound, $\gamma_{do} = 0$, is reached only when $\beta = 1$ and $c_{do} = 0$ simultaneously. Second, prove that $\gamma_{do} \leq 1$ by contradiction. Assuming $\gamma_{do} > 1$ implies that $0 > \beta\kappa_{do}\chi(\kappa_{do})$, which is a contradiction, as $\beta \geq 0$ and $\kappa_{do}\chi(\kappa_{do}) \geq 0$.

A.4.4 Negotiated price when producers' finding rate goes to infinity

The limit of γ_{do} when the finding rate $\kappa_{do}\chi(\kappa_{do}) \to \infty$ is simply $\gamma_{do} \equiv \dfrac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\kappa_{do}\chi(\kappa_{do})} \to 0$. More complicated is the limit of $\gamma_{do}\kappa_{do}\chi(\kappa_{do})$ as $\kappa_{do}\chi(\kappa_{do}) \to \infty$. First rewrite the expression as

$$\gamma_{do}\kappa_{do}\chi(\kappa_{do}) = \frac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\kappa_{do}\chi(\kappa_{do})}\kappa_{do}\chi(\kappa_{do}).$$

Dividing the top and bottom of this expression by $\kappa_{do}\chi(\kappa_{do})$ yields

$$\gamma_{do}\kappa_{do}\chi(\kappa_{do}) = \frac{(r+\lambda)(1-\beta)}{\dfrac{r+\lambda}{\kappa_{do}\chi(\kappa_{do})} + \beta}.$$

Now use this to derive the limit

$$\begin{aligned}\lim_{\kappa_{do}\chi(\kappa_{do})\to\infty}\gamma_{do}\kappa_{do}\chi\left(\kappa_{do}\right) &= \lim_{\kappa_{do}\chi(\kappa_{do})\to\infty}\frac{(r+\lambda)(1-\beta)}{\dfrac{r+\lambda}{\kappa_{do}\chi\left(\kappa_{do}\right)}+\beta}\\ &= \frac{(r+\lambda)(1-\beta)}{\beta}.\end{aligned}$$

This can be used to derive the limit of the negotiated price, n_{do}, as $\kappa_{do}\chi\left(\kappa_{do}\right)\to\infty$:

$$\begin{aligned}\lim_{\kappa_{do}\chi(\kappa_{do})\to\infty} n_{do} &= \lim_{\kappa_{do}\chi(\kappa_{do})\to\infty}\left[\left[1-\gamma_{do}\right]p_{do}+\gamma_{do}\frac{t\left(q_{do},w_o,\tau_{do},\varphi\right)+f_{do}-l_{do}-\kappa_{do}\chi\left(\kappa_{do}\right)s_{do}}{q_{do}}\right]\\ &= \lim_{\kappa_{do}\chi(\kappa_{do})\to\infty}\left[p_{do}-\gamma_{do}p_{do}+\gamma_{do}\left[\frac{t\left(q_{do},w_o,\tau_{do},\varphi\right)+f_{do}-l_{do}}{q_{do}}\right]-\frac{\gamma_{do}\kappa_{do}\chi\left(\kappa_{do}\right)s_{do}}{q_{do}}\right]\\ &= p_{do}-p_{do}\lim_{\kappa_{do}\chi(\kappa_{do})\to\infty}\gamma_{do}\\ &+ \left[\frac{t\left(q_{do},w_o,\tau_{do},\varphi\right)+f_{do}-l_{do}}{q_{do}}\right]\lim_{\kappa_{do}\chi(\kappa_{do})\to\infty}\gamma_{do}-\frac{s_{do}}{q_{do}}\lim_{\kappa_{do}\chi(\kappa_{do})\to\infty}\gamma_{do}\kappa_{do}\chi\left(\kappa_{do}\right)\\ &= p_{do}-p_{do}\cdot 0+\left[\frac{t\left(q_{do},w_o,\tau_{do},\varphi\right)+f_{do}-l_{do}}{q_{do}}\right]\cdot 0-\frac{s_{do}(r+\lambda)(1-\beta)}{q_{do}\beta}\\ &= p_{do}-\frac{s_{do}(r+\lambda)(1-\beta)}{q_{do}\beta}.\end{aligned}$$

The negotiated price is the final sales price, less the amount required to compensate the producer for the sunk cost to start up the business relationship. Notice that if $s_{do}=0$, then the negotiated price would be the final sales price as in standard trade models.

A.5 Bargaining over the quantity

A.5.1 Maximizing surplus

Take equation (12), log and differentiate with respect to the quantity q_{do} to get

$$\beta\frac{1}{X_{do}(\varphi)-U_{do}(\varphi)}\left(n_{do}-t'\left(q_{do},w_o,\tau_{do},\varphi\right)\right)+(1-\beta)\frac{1}{M_{do}(\varphi)-V_{do}}\left(p_{do}\left(q_{do}\right)+p'_{do}\left(q_{do}\right)q_{do}-n_{do}\right)=0, \tag{38}$$

in which we compute the partials of $X_{do}(\varphi)$ and $M_{do}(\varphi)$ using equations (37) and (36). Now, notice that equation (13) implies that $X_{do}(\varphi)-U_{do}(\varphi)=\frac{\beta}{1-\beta}\left(M_{do}(\varphi)-V_{do}\right)$, and plugging this into equation (38) and rearranging slightly gives

$$p_{do}\left(q_{do}\right)+p'_{do}\left(q_{do}\right)q_{do}=t'\left(q_{do},w_o,\tau_{do},\varphi\right). \tag{39}$$

This expression says that the quantity produced and traded is pinned down by equating marginal revenue in the domestic market with marginal production cost in the foreign country. This restriction is the same as what we get from a model without search and therefore implies that adding search does not change the quantity traded within each match. The profit maximization implied by this equation is crucial: Despite being separate entities, the retailer and the producer decide to set marginal revenue equal to marginal cost. The result follows because of the simple sharing rule, the maximization of joint surplus, and the trivial role of the retailer. To maximize surplus, the parties choose to equate marginal revenue and marginal cost.

A.5.2 Profit maximization

Conditional on the consumer's inverse demand (equation 2), the quantity traded between producer and retailer, $q_{do}(\omega)$, equates marginal revenue obtained by the retailer with the marginal production cost, as in equation (15). In other words, the retailer and producer solve a profit maximization problem. In particular, they seek to maximize profits for a given variety, ω, given that the producer has productivity φ, i.e., the cost function for producing q_{do} units of variety ω for the producer is given by $w_o\tau_{do}\dfrac{q_{do}(\omega)}{\varphi}+w_o f_{do}$, and the retailer faces a downward sloping demand curve. Variable profits can be written as:

$$\pi_{do}(\omega) \quad = \quad r_{do}(\omega) - w_o\tau_{do}\frac{q_{do}(\omega)}{\varphi}.$$

From the utility maximization solution we know that

$$r_{do}(\omega) \quad = \quad \alpha C_d\left(\frac{p_{do}(\omega)}{P_d}\right)^{1-\sigma}.$$

Since the CES aggregator is HOD 1, we know that welfare from the differentiated goods must be $\tilde{W}_d=\dfrac{\alpha C_d}{P_d}$ or $\alpha C_d = P_d\tilde{W}_d$ (appendix B.6). Further we know, again from our utility maximization solution, that

$$q_{do}(\omega) \quad = \quad \tilde{W}_d\left(\frac{p_{do}(\omega)}{P_d}\right)^{-\sigma}.$$

Plugging these into our profit expression from the top yields:

$$\begin{aligned}\pi_{do}(\omega) &= \alpha C_d\left[\frac{p_{do}(\omega)}{P_d}\right]^{1-\sigma}-\frac{w_o\tau_{do}}{\varphi}\tilde{W}_d\left[\frac{p_{do}(\omega)}{P_d}\right]^{-\sigma}\\ &= P_d^{\sigma}\tilde{W}_d p_{do}(\omega)^{1-\sigma}-\frac{w_o\tau_{do}}{\varphi}P_d^{\sigma}\tilde{W}_d p_{do}(\omega)^{-\sigma}.\end{aligned}$$

Differentiating this expression with respect to the price for this particular variety, $p_{do}(\omega)$, and setting this derivative equal to zero we get:

$$\frac{\partial \pi_{do}(\omega)}{\partial p_{do}(\omega)} = 0 \quad = \quad (1-\sigma) P_d^{\sigma} \tilde{W}_d p_{do}(\omega)^{1-\sigma-1} + \sigma \frac{w_o \tau_{do}}{\varphi} P_d^{\sigma} \tilde{W}_d p_{do}(\omega)^{-\sigma-1}.$$

Solving this for $p_{do}(\omega)$ yields:

$$p_{do}(\omega) \quad = \quad \mu \frac{w_o \tau_{do}}{\varphi},$$

in which $\mu = \dfrac{\sigma}{\sigma - 1}$. Notice that since the right-hand side is not a function of ω (the index), but is a function of the productivity φ, we write

$$p_{do}(\varphi) \quad = \quad \mu \frac{w_o \tau_{do}}{\varphi}$$

throughout the text.

We should also note that since we assume that matches between retailers and producers are one to one, and each producer has a differentiated good, matched retailers have a monopoly in the variety that they import.

A.5.3 Retailer production function

In this section, we show that including another input for the retailer does not affect the conclusions of this paper under some weak additional assumptions. With an additional input, the value of being in a relationship for a retailer changes to

$$rM_{do}(\varphi) = p_{do}(f(q_{do}, k_{do}))f(q_{do}, k_{do}) - n_{do}q_{do} - \frac{\partial n_{do}}{\partial k_{do}} k_{do} - \lambda(M_{do}(\varphi) - V_{do}), \qquad (40)$$

in which the retailer combines the input, denoted by k_{do}, with the input purchased from the producer, q_{do}, according to production function $f(q_{do}, k_{do})$ for the final good sold to consumers. The price of the additional input, $\frac{\partial n_{do}}{\partial k_{do}}$, is determined outside of the search model and is taken as given by the retailer.

With this new Bellman equation, logging and differentiating the Nash product in equation (12) with respect to p_{do} gives the same surplus sharing (13) rule as before. The first-order condition of equation (12) with respect to q_{do}, however, becomes

$$\beta \frac{n_{do} - t'(q_{do}, w_o, \tau_{do}, \varphi)}{X_{do}(\varphi) - U_{do}(\varphi)} + (1-\beta) \frac{p'_{do}(f(q_{do}, k_{do}))f_{q_{do}}(q_{do}, k_{do})f(q_{do}, k_{do}) + p_{do}(f(q_{do}, k_{do}))f_{q_{do}}(q_{do}, k_{do}) - n_{do}}{M_{do}(\varphi) - V_{do}} = 0. \quad (41)$$

Combining this with the surplus sharing rule (13) yields an expression similar to equation

(15):

$$p'_{do}(f(q_{do},k_{do}))f_{q_{do}}(q_{do},k_{do})f(q_{do},k_{do})+p_{do}(f(q_{do},k_{do}))f_{q_{do}}(q_{do},k_{do})=t'\left(q_{do},w_o,\tau_{do},\varphi\right). \tag{42}$$

This equation states that retailers and producers will negotiate to trade a quantity of q that ensures that the marginal revenue equals marginal cost. As the price of input k_{do} is taken as given, the firm chooses the optimal level of the input, k^*_{do}, so that $f_{k_{do}}\left(q_{do},k^*_{do}\right)=\frac{\partial n_{do}}{\partial k_{do}}$. Strict concavity of the function $f(q_{do},k_{do})$ is sufficient to ensure that $f_{k_{do}}\left(q_{do},k_{do}\right)$ is invertible. Making this assumption gives $f^{-1}_{k_{do}}\left(q_{do},n_{do}\right)=k^*_{do}$ which can be substituted into equation (42) to get one equation in one unknown, q_{do}. The quantity traded depends on $\frac{\partial n_{do}}{\partial k_{do}}$, the price of the other input, but search frictions still do not enter equation (42). The result in the main text - that optimal q_{do} is determined by the condition that ensures that marginal revenue from q_{do} equals the marginal cost of producing q_{do} - remains intact.

A.6 Solving for the productivity thresholds

A.6.1 Solving for the lowest productivity threshold

First, let's solve for an expression for $X_{do}(\varphi)-U_{do}(\varphi)$ by plugging in equations (7) and (8):

$$\begin{aligned}
rX_{do}(\varphi)-rU_{do}(\varphi)&=n_{do}q_{do}-t\left(q_{do},w_o,\tau_{do},\varphi\right)-f_{do}-\lambda\left(X_{do}(\varphi)-U_{do}(\varphi)\right)+l_{do}-\kappa_{do}\chi(\kappa_{do}(X_{do}(\varphi)-U_{do}(\varphi)-s_{do})\\
&=n_{do}q_{do}-t\left(q_{do},w_o,\tau_{do},\varphi\right)-f_{do}+l_{do}+\kappa_{do}\chi(\kappa_{do})s_{do}-(\lambda+\kappa_{do}\chi(\kappa_{do}))(X_{do}(\varphi)-U_{do}(\varphi))\\
&\Rightarrow(r+\lambda+\kappa_{do}\chi(\kappa_{do}))(X_{do}(\varphi)-U_{do}(\varphi))=n_{do}q_{do}-t\left(q_{do},w_o,\tau_{do},\varphi\right)-f_{do}+l_{do}+\kappa_{do}\chi(\kappa_{do})s_{do}\\
&\Rightarrow X_{do}(\varphi)-U_{do}(\varphi)=\frac{n_{do}q_{do}-t\left(q_{do},w_o,\tau_{do},\varphi\right)-f_{do}+l_{do}+\kappa_{do}\chi(\kappa_{do})s_{do}}{r+\lambda+\kappa_{do}\chi(\kappa_{do})}.
\end{aligned} \tag{43}$$

Now plug this expression into the definition of $\underline{\varphi}_{do}$ from the main text to get

$$\begin{aligned}
&\frac{n_{do}q_{do}-t\left(q_{do},w_o,\tau_{do},\varphi\right)-f_{do}+l_{do}+\kappa_{do}\chi(\kappa_{do})s_{do}}{r+\lambda+\kappa_{do}\chi(\kappa_{do})}=0\\
&\Rightarrow n_{do}q_{do}-t\left(q_{do},w_o,\tau_{do},\varphi\right)-f_{do}+l_{do}+\kappa_{do}\chi(\kappa_{do})s_{do}=0.
\end{aligned}$$

By using the fact that $X'_{do}\left(\varphi\right)-U'_{do}\left(\varphi\right)>0$ from above we can state that this threshold is unique.

We can be sure that for any positive cost of forming a relationship, $\dfrac{l_{do}+h_{do}}{\kappa_{do}\chi\left(\kappa_{do}\right)}+s_{do}$, if and only if $l_{do}+h_{do}+\kappa_{do}\chi\left(\kappa_{do}\right)s_{do}>0$, the expression $X_{do}\left(\bar{\varphi}_{do}\right)-U_{do}\left(\bar{\varphi}_{do}\right)$ exceeds $X_{do}\left(\underline{\varphi}_{do}\right)-U_{do}\left(\underline{\varphi}_{do}\right)$. This result implies that as long as $X_{do}\left(\varphi\right)-U_{do}\left(\varphi\right)$ is increasing in φ, then $\bar{\varphi}_{do}>\underline{\varphi}_{do}$. In appendix A.7, we show the very general conditions under which $X_{do}\left(\varphi\right)-U_{do}\left(\varphi\right)$ is increasing in φ. The binding productivity threshold defining the mass of producers that have retail partners is the greater of these two and hence $\bar{\varphi}_{do}$. In other words, the productivity necessary to induce a producer to search for a retail partner is greater than the productivity necessary to consummate a match after meeting a retailer due to the costs that are incurred while searching. Similarly, the productivity necessary to form a match is greater than the productivity to maintain one already in place.

A.6.2 Proof of proposition 3: Solving for the binding productivity threshold

Our threshold productivity, $\bar{\varphi}_{do}$, is given by $U_{do}(\bar{\varphi}) - I_{do}(\bar{\varphi}_{do}) = 0$. Plugging equations (8) and (9) into this definition yields

$$X_{do}(\bar{\varphi}_{do}) - U_{do}(\bar{\varphi}_{do}) = \frac{l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + s_{do} \tag{44}$$

Using equation (43) in equation (44) yields

$$\begin{aligned}
\frac{n_{do}q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do}}{r + \lambda + \kappa_{do}\chi(\kappa_{do})} &= \frac{l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + s_{do} \\
\Rightarrow n_{do}q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do} &= (r + \lambda + \kappa_{do}\chi(\kappa_{do}))\frac{s_{do}\kappa_{do}\chi(\kappa_{do}) + l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} \\
\Rightarrow n_{do}q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do} &= (r + \lambda)\frac{s_{do}\kappa_{do}\chi(\kappa_{do}) + l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + s_{do}\kappa_{do}\chi(\kappa_{do}) + l_{do} + h_{do} \\
\Rightarrow n_{do}q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} &= (r + \lambda)\frac{s_{do}\kappa_{do}\chi(\kappa_{do}) + l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + h_{do} \\
\Rightarrow n_{do}q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} - (r + \lambda)\frac{l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} - h_{do} &= (r + \lambda)s_{do} \\
\Rightarrow n_{do}q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} &= (r + \lambda)s_{do} + (r + \lambda)\frac{l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + h_{do}.
\end{aligned}$$

Now, plug in for the equilibrium import price, n_{do}, from equation (14), to get

$$(1 - \gamma_{do})p_{do}(q_{do})q_{do} + \gamma_{do}(t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} - l_{do} - \kappa_{do}\chi(\kappa_{do})s_{do}) - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} = (r + \lambda)s_{do} + \frac{(r + \lambda)}{\kappa_{do}\chi(\kappa_{do})}l_{do} + \left(1 + \frac{(r + \lambda)}{\kappa_{do}\chi(\kappa_{do})}\right)h_{do}.$$

which can be rearranged to obtain

$$p_{do}(q_{do})q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} = (1 - \gamma_{do})^{-1}\left[(r + \lambda + \gamma_{do}\kappa_{do}\chi(\kappa_{do}))s_{do} + \left(\gamma_{do} + \frac{(r + \lambda)}{\kappa_{do}\chi(\kappa_{do})}\right)l_{do} + \left(1 + \frac{(r + \lambda)}{\kappa_{do}\chi(\kappa_{do})}\right)h_{do}\right].$$

Further simplification of the terms with γ_{do} implies that

$$p_{do}(q_{do})q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) = f_{do} + \left(\frac{(r + \lambda)}{\beta\kappa_{do}\chi(\kappa_{do})}\right)l + \left(1 + \frac{(r + \lambda)}{\beta\kappa_{do}\chi(\kappa_{do})}\right)h_{do} + \frac{(r + \lambda)}{\beta}s_{do}.$$

which is the expression in the main text.

A.6.3 Comparing our productivity threshold to previous models

Defining $F\left(\kappa_{do}\right) \equiv f_{do} + \left(\frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right) l_{do} + \left(1 + \frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right) h_{do} + \frac{(r+\lambda)}{\beta} s_{do}$ in this framework would allow us to replace the fixed cost in the standard models with $F\left(\kappa_{do}\right)$ from here. The key thing to remember when working with the other quantities of our model is we want to work with them in terms of the cutoff and not in terms of these fundamental frictions just yet.

Another interesting comparison is to Eaton et al. (2014). That framework includes a flow search cost, l_{do}, but does not have a sunk cost s_{do} or any idle state. If we set $h_{do} = 0$, we are implicitly including an idle state because the producer will have a zero value for being in the idle state but have a negative flow cost, $-l_{do}$, for being in the searching state because that state requires a payment each period of $l_{do} > 0$. In other words, because the producer cannot opt out of searching we must set the flow of the idle state to $h_{do} = -l_{do}$ instead of what one might think is the intuitive value of that state $h_{do} = 0$. Making this assumption together with $s_{do} = 0$ provides:

$$\begin{aligned} p_{do}\left(q_{do}\right) q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) &= f_{do} + \left(\frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right) l_{do} + \left(1 + \frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right) h_{do} + \frac{(r+\lambda)}{\beta} s_{do} \\ &= f_{do} - l_{do}. \end{aligned}$$

This result is the very reason why Eaton et al. (2014) must have that $f_{do} > l_{do}$. Notice that we recover the standard model when we make these assumptions together with $l_{do} = 0$:

$$\begin{aligned} p_{do}\left(q_{do}\right) q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) &= f_{do} + \left(\frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right) l_{do} + \left(1 + \frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right) h_{do} + \frac{(r+\lambda)}{\beta} s_{do} \\ &= f_{do}. \end{aligned}$$

Another interesting way to remove just the search friction, l_{do}, from the model is to set the finding rate $\kappa_{do}\chi\left(\kappa_{do}\right) \to \infty$ so that $\frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)} \to 0$

$$\begin{aligned} p_{do}\left(q_{do}\right) q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) &= f_{do} + \left(\frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right) l_{do} + \left(1 + \frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right) h_{do} + \frac{(r+\lambda)}{\beta} s_{do} \\ &= f_{do} + h_{do} + \frac{(r+\lambda)}{\beta} s_{do}. \end{aligned}$$

The interpretation of the fixed cost includes the sunk cost, bargaining power of the producer, and the flow from the outside option, even if one finds a partner immediately. If bargaining power differs by market, for example, the bundle of entry costs will as well.

A.6.4 Productivity cutoff and flow profits

Because the equilibrium price for each variety is a constant markup over marginal cost we can write the firms' variable cost function as a proportional function of revenue

$$t_{do}(\varphi) = p_{do}(\varphi)\, q_{do}(\varphi)\, \mu^{-1}.$$

Combine the definition of flow variable profits

$$\pi_{do}(\varphi) = p_{do}(\varphi)\, q_{do}(\varphi) - t_{do}(\varphi)$$

with the relationship between variable costs and revenue to get that

$$\pi_{do}(\varphi) = p_{do}(\varphi)\, q_{do}(\varphi) - p_{do}(\varphi)\, q_{do}(\varphi)\, \mu^{-1},$$

which simplifies to

$$\pi_{do}(\varphi) = p_{do}(\varphi)\, q_{do}(\varphi)\, \sigma^{-1}$$

because $1 - \mu^{-1} = \sigma^{-1}$. Using demand from equation (2) and the pricing rule provides revenue in this model

$$p_{do}(\varphi)\, q_{do}(\varphi) = \alpha C_d P_d^{\sigma-1} \left(\mu w_o \tau_{do}\right)^{1-\sigma} \varphi^{\sigma-1}.$$

We can use revenue and the profit expression combined with (17) to derive threshold productivity in our search model. We start with the expression

$$\pi_{do}(\bar{\varphi}_{do}) = F_{do}(\kappa_{do}).$$

Then use the functional forms and the relationship between revenues and profits to write

$$\frac{\alpha}{\sigma} C_d P_d^{\sigma-1} \left(\mu w_o \tau_{do}\right)^{1-\sigma} \bar{\varphi}_{do}^{\sigma-1} = F_{do}(\kappa_{do})$$

before arriving at

$$\bar{\varphi}_{do} = \mu \left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}} \left(\frac{w_o \tau_{do}}{P_d}\right) C_d^{\frac{1}{1-\sigma}} F_{do}(\kappa_{do})^{\frac{1}{\sigma-1}},$$

which is presented in equation (18) in the main text.

A.7 The value of importing is strictly increasing in productivity

Here we show that the value of importing, $M_{do}(\varphi)$, is strictly increasing with the producer's productivity level, φ. This fact allows us to replace the integral of the max over V_{do} and

$M_{do}(\varphi)$ (equation 11) with the integral of $M_{do}(\varphi)$ from the productivity threshold, $\bar{\varphi}_{do}$ (equation 19).

Starting with equation (10) and $V_{do}=0$ we obtain

$$\begin{aligned}
(r+\lambda)\,M_{do}(\varphi) &= p_{do}q_{do}-n_{do}q_{do}\\
&= p_{do}q_{do}-[1-\gamma_{do}]\,p_{do}q_{do}-\gamma_{do}\left(t\left(q_{do},w_o,\tau_{do},\varphi\right)+f_{do}-l_{do}-\kappa_{do}\chi\left(\kappa_{do}\right)s_{do}\right)\\
&= \gamma_{do}p_{do}q_{do}-\gamma_{do}\left(t\left(q_{do},w_o,\tau_{do},\varphi\right)+f_{do}\right)+\gamma_{do}\left(l_{do}+\kappa_{do}\chi\left(\kappa_{do}\right)s_{do}\right)\\
&= \gamma_{do}\left(p_{do}q_{do}-t\left(q_{do},w_o,\tau_{do},\varphi\right)-f_{do}\right)+\gamma_{do}\left(l_{do}+\kappa_{do}\chi\left(\kappa_{do}\right)s_{do}\right).
\end{aligned}$$

Remember that $\gamma_{do}\equiv\dfrac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\kappa_{do}\chi\left(\kappa_{do}\right)}$. It is clear from the integral in the import relationship creation equation (19) that neither the finding rate for retailers, $\chi\left(\kappa_{do}\right)$, nor the tightness, κ_{do}, is a function of the productivity, φ. Given this, $M_{do}'\left(\varphi\right)$ and $\dfrac{\partial[p_{do}\left(q_{do}\right)q_{do}-t\left(q_{do},w_o,\tau_{do},\varphi\right)-f_{do}]}{\partial\varphi}$ will have the same sign. As long as flow profits without search frictions are strictly increasing in productivity, $M_{do}'\left(\varphi\right)>0$. Using the specific functional forms for $t\left(q_{do},w_o,\tau_{do},\varphi\right)+f_{do}$ used above, as well as the equilibrium values for n_{do}, p_{do}, and q_{do}, we can derive this result explicitly. In this case,

$$M_{do}(\varphi) = \alpha\gamma_{do}\left(\frac{1}{r+\lambda}\right)\left(\frac{\mu^{-\sigma}}{\sigma-1}\right)\left(w_o\tau_{do}\right)^{1-\sigma}\alpha C_dP_d^{\sigma-1}\varphi^{\sigma-1}-\gamma_{do}f_{do}+\gamma_{do}\left(l_{do}+\kappa_{do}\chi\left(\kappa_{do}\right)s_{do}\right).$$

Therefore the derivative is

$$\frac{\partial M_{do}\left(\varphi\right)}{\partial\varphi} = \alpha\gamma_{do}\left(\frac{1}{r+\lambda}\right)\mu^{-\sigma}\left(w_o\tau_{do}\right)^{1-\sigma}\alpha C_dP_d^{\sigma-1}\varphi^{\sigma-2}.$$

which is always positive.

As long as $M_{do}'(\varphi)>0$, we can demonstrate the way in which many other important quantities depend on the producer's productivity level, φ. From the surplus sharing rule (35) can be rewritten as

$$\beta M_{do}\left(\varphi\right)=(1-\beta)\left(X_{do}\left(\varphi\right)-U_{do}\left(\varphi\right)\right), \tag{45}$$

We know that in equilibrium, because $M_{do}'(\varphi)>0$, it must be that $X_{do}'\left(\varphi\right)-U_{do}'\left(\varphi\right)>0$. Differentiating both sides of equation (8) gives $rU_{do}'\left(\varphi\right)=\kappa_{do}\chi\left(\kappa_{do}\right)\left(X_{do}'\left(\varphi\right)-U_{do}'\left(\varphi\right)\right)>0$. We can combine these facts to show $X_{do}'\left(\varphi\right)>U_{do}'\left(\varphi\right)>0$. Using the definition of the joint surplus of a match $S_{do}\left(\varphi\right)=X_{do}\left(\varphi\right)+M_{do}\left(\varphi\right)-U_{do}\left(\varphi\right)-V_{do}$ we get $S_{do}'(\varphi)>0$. Likewise, the value of a relationship, $R_{do}\left(\varphi\right)=X_{do}\left(\varphi\right)+M_{do}\left(\varphi\right)$, has $R_{do}'(\varphi)>0$.

A.8 Proof of proposition 4: Market tightness and the cost of search

Let's first prove that $\kappa_{do} < \infty$ if $c_{do} > 0$. To do this, let's prove the contrapositive: assume that $c_{do} = 0$ and show that $\kappa_{do} = \infty$. Rearrange equation (19) slightly to get

$$0 = c_{do} = \chi\left(\kappa_{do}\right) \int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right) dG\left(\varphi\right).$$

We have shown that $M_{do}\left(\bar{\varphi}_{do}\right) \geq 0$ for any consummated match in equilibrium (Nash bargaining together with appendix A.6) and $M'_{do}\left(\varphi\right) > 0$ (appendix A.7). Therefore we know that $\int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right) dG\left(\varphi\right) > 0$. Thus, $\chi\left(\kappa_{do}\right)$ must be zero. Because $\chi'\left(\kappa_{do}\right) < 0$ this is true if and only if $\kappa_{do} = \infty$.

To prove that if $c_{do} > 0$ then $\kappa_{do} < \infty$, let's use equation (19) again. In particular, because $c_{do} > 0$ it must mean that $\chi\left(\kappa_{do}\right) \int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right) dG\left(\varphi\right) > 0$. As before, we know that $\int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right) dG\left(\varphi\right) > 0$ so it must be that $\chi\left(\kappa_{do}\right) > 0$ as well, which is true if and only if $\kappa_{do} < \infty$.

A.9 Producer and retailer existence

A.9.1 Retailing firms

Free entry implies that the ex-ante expected value from entering for a potential retailer equals the expected cost of entering. Assume for a moment that the potential retailers consider the value of becoming a retailer as defined by E^m_{do}. This value is characterized by the following Bellman equation

$$rE^m_{do} = -e^m_o + \left(V_{do} - E^m_{do}\right). \tag{46}$$

The potential retailer could sell the value E^m_{do} and invest the proceeds at the interest rate r getting flow payoff rE^m_{do} forever after. Alternatively, they could pay a cost e^m_o to become a retailer, at which point they will begin in the state of having a vacancy with value V_{do} (with certainty) and give up the value of being a potential retailer E^m_{do}. Free entry into becoming a retailer implies that $E^m_{do} = 0$ in equilibrium so that

$$\begin{aligned} 0 &= -e^m_o + V_{do} \\ e^m_o &= V_{do}. \end{aligned}$$

Hence, free entry into vacancies $V_{do} = 0$ implies $e^m_o = 0$ and we cannot have a sunk cost for entry into retailing. In other words, free entry into the search market along with assuming that one must post a vacancy before matching implies free entry into retailing.

Free entry into the search market subsumes free entry into retailing and so we only have one condition defined by free entry on the retailing side given by equation (19) and restated here

$$\frac{c_{do}}{\chi\left(\kappa_{do}\right)} = \int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right) dG\left(\varphi\right).$$

Remember this states that product vacancies continue being created until the expected cost of being an unmatched retailer, $c_{do}/\chi\left(\kappa_{do}\right)$, equals the expected benefit $\int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right) dG\left(\varphi\right)$. Because each potential retailer must post a product vacancy before forming a match, the expected cost of becoming a retailer (entering as a retailer) is the same as the expected cost of being an unmatched retailer. Likewise, the expected benefit of posting a vacancy and the expected benefit of becoming a retailer are also the same because we assume retailers must post a vacancy before matching.

A.9.2 Producing firms

Similar to the entry decision of retailers, the value of entry for producers, E^x_{do}, is defined by

$$\begin{aligned} rE^x_{do} &= -e^x_d + \int \max\left\{I_{do}\left(\varphi\right), U_{do}\left(\varphi\right)\right\} dG\left(\varphi\right) - E^x_{do} \\ &= -e^x_d + \int_1^{\bar{\varphi}_{do}} I_{do}\left(\varphi\right) dG\left(\varphi\right) + \int_{\bar{\varphi}_{do}}^{\infty} U_{do}\left(\varphi\right) dG\left(\varphi\right) - E^x_{do}. \end{aligned}$$

We assume that the potential producer must transit through the unmatched state before forming a match. After paying e^x_d and taking a productivity draw φ, the potential producer loses the value E^x_{do} with certainty and, depending on the drawn productivity, chooses between searching for a retailer and getting value $U_{do}\left(\varphi\right)$ or remaining idle and getting value $I_{do}\left(\varphi\right)$. If we assumed free entry into production, we would get $E^x_{do} = 0$ and that

$$e^x_d = \int_1^{\bar{\varphi}_{do}} I_{do}\left(\varphi\right) dG\left(\varphi\right) + \int_{\bar{\varphi}_{do}}^{\infty} U_{do}\left(\varphi\right) dG\left(\varphi\right). \tag{47}$$

which ensures the expected value of taking a productivity draw equals the expected cost.

Free entry into production, therefore, imposes another restriction on the equilibrium. We can use the facts that $X_{do}\left(\varphi\right) - U_{do}\left(\varphi\right) = \left(1-\beta\right) S_{do}\left(\varphi\right)$ and that $M_{do}\left(\varphi\right) = \beta S_{do}\left(\varphi\right)$ to write $X_{do}\left(\varphi\right) - U_{do}\left(\varphi\right) = \left(\frac{1-\beta}{\beta}\right) M_{do}\left(\varphi\right)$. Applying this to equation (8) gives

$$rU_{do}\left(\varphi\right) = -l_{do} + \kappa_{do}\chi\left(\kappa_{do}\right)\left(\left(\frac{1-\beta}{\beta}\right) M_{do}\left(\varphi\right) - s_{do}\right).$$

Computing the relevant integrals in equation (47) gives

$$\int_{\bar{\varphi}_{do}}^{\infty} U_{do}(\varphi)\, dG(\varphi) = -\left(\frac{l_{do} + s_{do}\kappa_{do}\chi(\kappa_{do})}{r}\right)(1 - G(\bar{\varphi}_{do})) + \frac{\kappa_{do}\chi(\kappa_{do})}{r}\left(\frac{1-\beta}{\beta}\right)\int_{\bar{\varphi}_{do}}^{\infty} M_{do}(\varphi)\, dG(\varphi).$$

Likewise, from (9) we have

$$\int_{1}^{\bar{\varphi}_{do}} I_{do}(\varphi)\, dG(\varphi) = \frac{h_{do}}{r} G(\bar{\varphi}_{do}).$$

Combining these with equation (47) gives

$$e_d^x = \frac{h_{do}}{r} G(\bar{\varphi}_{do}) - \left(\frac{l_{do} + s_{do}\kappa_{do}\chi(\kappa)}{r}\right)(1 - G(\bar{\varphi}_{do})) + \frac{\kappa_{do}\chi(\kappa_{do})}{r}\left(\frac{1-\beta}{\beta}\right)\int_{\bar{\varphi}_{do}}^{\infty} M_{do}(\varphi)\, dG(\bar{\varphi}_{do}). \quad (48)$$

which is the restriction that free entry into production for producers would place on equilibrium market tightness κ_{do}.

From equation (47), we can see that free entry into search for producers would require $\int_{\bar{\varphi}_{do}}^{\infty} U_{do}(\varphi)\, dG(\varphi) = 0$, in which case we would be left with

$$e_d^x = \frac{h_{do}}{r} G(\bar{\varphi}_{do}). \quad (49)$$

which still places a restriction on market tightness because $\bar{\varphi}_{do}$ from Proposition (17) includes κ_{do}.

Finally, we note that simultaneous combinations of free entry on both sides of the market are possible. Combining free entry into both existence and search for retailers from equation (19) with free entry into existence for producers from equation (48) gives

$$e_d^x = \frac{h_{do}}{r} G(\bar{\varphi}_{do}) - \left(\frac{l_{do} + s_{do}\kappa_{do}\chi(\kappa)}{r}\right)(1 - G(\bar{\varphi}_{do})) + \frac{c_{do}\kappa_{do}}{r}\left(\frac{1-\beta}{\beta}\right).$$

Likewise, allowing for free entry into both existence and search for retailers and producers would give the following system that defines κ_{do}

$$\begin{aligned} \frac{c_{do}}{\chi(\kappa_{do})} &= \int_{\bar{\varphi}_{do}} M_{do}(\varphi)\, dG(\varphi) \\ e_d^x &= \frac{h_{do}}{r} G(\bar{\varphi}_{do}). \end{aligned}$$

A.10 Aggregate resources

A.10.1 Number of producers

Similar to Chaney (2008), we assume that the number of producers in the origin market that take a draw from the productivity distribution is proportional to consumption expenditure in the economy, C_o. The basic intuition behind this is that larger economies have a larger stock of potential entrepreneurs. To make this explicit, we denote the total mass of potential entrants as $N_o^x = \xi_o C_o$, in which the proportionality constant $\xi_o \in [0, \infty)$ captures exogenous structural factors that affect the number of potential entrants in country k. Among others, these could include such factors as literacy levels and attitudes toward entrepreneurship. As discussed in section A.10.2, because the number of producers is fixed, the economy has profits. We assume that a global mutual fund collects worldwide profits and redistributes them as π dividends per share to each worker who owns w_o shares. We assume that $\xi_o = \dfrac{1}{1+\pi}$ so that

$$N_o^x = \frac{C}{(1+\pi)} \frac{C_o}{C} \tag{50}$$

in which we have multiplied and divided by global consumption, C.

A.10.2 Aggregate accounting and the global mutual fund

Our economy has profits because we restrict producer entry and the model features monopolistic competition. We define a global mutual fund that collects all profits in the economy and rebates them back to consumers. In order to calculate total profits, we first define variable profits earned in each market pair as

$$\Pi_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}} \pi_{do}(\varphi)\, dG(\varphi)$$

in which $\pi_{do}(\varphi) = p_{do}(\varphi)\, q_{do}(\varphi) - t_{do}(\varphi)$. Our functional form assumptions and the pricing rule in (16) ensure that profits are proportional to sales:
$p_{do}(\varphi)\, q_{do}(\varphi) - t_{do} = p_{do}(\varphi)\, q_{do}(\varphi)\, \sigma^{-1}$. Aggregating profits from each variety provides

$$\Pi_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}} p_{do}(\varphi)\, q_{do}(\varphi)\, \sigma^{-1} dG(\varphi) = \frac{C_{do}}{\sigma} \tag{51}$$

in which we define the value of total consumption in destination d of the differentiated good from origin o as

$$C_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}} p_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi).$$

This definition for the value of consumption is consistent with equation (1) in the main text.

The income that consumers in o earn and can spend on consumption, C_o, comes from three sources. The first two sources are labor income in the production and investment sectors of the economy, $w_d L_d$. The third source is dividends from the global mutual fund, which we assume owns all firms in all countries. Each country gets a share, π, of total global profits proportional to labor income in the economy. Explicitly GDP can be written as

$$Y_o = w_o L_o \left(1 + \pi\right),$$

in which

$$\pi = \Pi / \sum w_k L_k. \tag{52}$$

Notice that the dividend per unit value of labor π is proportional to the value of the global labor endowment and so also matches Chaney (2008) equation (6) in our model. Wage income is derived providing the fixed cost of production, the formation of relationships, creating new retailers and producers, and the variable cost of production:

$$w_o L_o = \sum_{k=1}^{D} \Phi_{ko}^{i} + \sum_{k=1}^{D} \Phi_{ko}^{e} + \sum_{k=1}^{D} \Phi_{ko}^{p} + w_o q_o\left(1\right)$$

in which

$$\begin{aligned}
\Phi_{do}^{i} &= \kappa_{od} u_{od} N_d^x c_{od} + u_{do} N_o^x \left(l_{do} + s_{do}\kappa_{do}\chi\left(\kappa_{do}\right)\right) + \left(1 - u_{do} - i_{do}\right) N_o^x f_{do} \\
\Phi_{do}^{e} &= N_o^x e_o^x \\
\Phi_{do}^{p} &= \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}} t_{do}\left(\varphi\right) dG\left(\varphi\right)
\end{aligned}$$

The production structures for the homogeneous good is undetermined because it is freely traded and has constant returns to scale production. Like Chaney (2008), we only consider equilibrium in which every country produces some of that good. In order to simplify and make accounting for resources in every country symmetric, we also assume that each country produces what it would like to consume itself, namely $q_o\left(1\right)$. While the good can be freely traded, in equilibrium there is no international trade of the homogeneous good. Despite no trade in this good, its price is the same in all countries because of a no-arbitrage condition. Each unit of the homogeneous good requires one unit of labor to produce so the cost of producing $q_o\left(1\right)$ units of the homogeneous good is given by $w_o q_o\left(1\right)$. Importantly, our definition of the income earned from labor used in producing the differentiated good, Φ_{do}^{p}, includes the iceberg transport costs so that labor is compensated for transporting goods.

Summing payments to labor across all countries of the world gives

$$\Phi^h = \sum_{k=1}^{O} w_k q_k\left(1\right), \quad \Phi^i = \sum_{k=1}^{O}\sum_{j=1}^{D} \Phi_{jk}^{i}, \quad \Phi^e = \sum_{k=1}^{O}\sum_{j=1}^{D} \Phi_{jk}^{e}, \quad \Phi^p = \sum_{k=1}^{O}\sum_{j=1}^{D} \Phi_{jk}^{p}. \tag{53}$$

Similarly, we can define global variable profits from operation in each market either as equation (51) or as

$$\Pi_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}} p_{do}(\varphi)\, q_{do}(\varphi) - t_{do}(\varphi)\, dG(\varphi) = C_{do} - \Phi_{do}^p \tag{54}$$

Summing variable profits throughout the world provides global profits

$$\Pi = \sum_{k=1}^{O} \sum_{j=1}^{D} \Pi_{jk} = \sum_{k=1}^{O} \sum_{j=1}^{D} C_{jk} - \Phi_{jk}^p = \sum_{k=1}^{O} \sum_{j=1}^{D} \frac{C_{jk}}{\sigma} = \frac{\alpha}{\sigma} C \tag{55}$$

The last two equalities come from our functional form assumptions. We can check that we have treated the global mutual fund correctly by ensuring that global income equals global expenditure. Start by defining investment in each market

$$I_{do} = \kappa_{od} u_{od} N_d^x c_{od} + u_{do} N_o^x \left(l_{do} + s_{do} \kappa_{do} \chi(\kappa_{do})\right) + \left(1 - u_{do} - i_{do}\right) N_o^x f_{do} + N_o^x e_o^x$$

in which it is also clear that $I_{do} = \Phi_{do}^i + \Phi_{do}^e$ and global investment is

$$I = \sum_{k=1}^{O} \sum_{j=1}^{D} I_{jk}.$$

Global consumption of both homogeneous and differentiated goods is

$$C = \sum_{k=1}^{O} C_k = \sum_{k=1}^{O} p_k(1)\, q_k(1) + \sum_{k=1}^{O} \sum_{j=1}^{D} C_{jk}$$

To check that we have everything correct, start with total resources available in the economy $Y_o = w_o L_o (1+\pi)$ and sum across economies

$$\begin{aligned}
\sum_{k=1}^{O} Y_k &= \sum_{k=1}^{O} w_k L_k (1+\pi) \\
Y &= (1+\pi) \sum_{k=1}^{O} w_k L_k \\
Y &= \left(1 + \frac{\Pi}{\sum_{k=1}^{O} w_k L_k}\right) \sum_{k=1}^{O} w_k L_k \\
Y &= \sum_{k=1}^{O} w_k L_k + \Pi \\
Y &= \Pi + \sum_{k=1}^{O} \left(\sum_{k=1}^{D} \Phi_{ko}^{i} + \sum_{k=1}^{D} \Phi_{ko}^{e} + \sum_{k=1}^{D} \Phi_{ko}^{p} + w_o q_o (1) \right) \\
Y &= \Pi + \Phi^i + \Phi^e + \Phi^p + \Phi^h
\end{aligned}$$

We can finish the proof by starting with the last line, which is the income approach to accounting, and showing that this expression also gives the expenditure approach

$$\begin{aligned}
Y &= \Pi + \Phi^h + \Phi^i + \Phi^e + \Phi^p \\
Y &= \Pi + \Phi^h + I + \Phi^p \\
Y &= \sum_{k=1}^{O} \sum_{j=1}^{D} \left(C_{jk} - \Phi_{jk}^{p} \right) + \Phi^h + I + \Phi^p \\
Y &= \sum_{k=1}^{O} \sum_{j=1}^{D} C_{jk} + \Phi^h + I \\
Y &= C + I
\end{aligned}$$

so that

$$Y = C + I, \tag{56}$$

and

$$C = Y - I. \tag{57}$$

Notice that we used

$$\Phi^h = \sum_{k=1}^{O} w_k q_k (1) = \sum_{k=1}^{O} p_k (1) \, q_k (1)$$

in the last line. Costless trading of the homogeneous good delivers a "no arbitrage condition," implying that its price must be the same in all countries, $p_k (1) = p (1)$. Because the homogeneous good is made with one unit of labor in each country, it must also be that $w_k = p_k(1) = p(1)$ pinning down the equilibrium wage in every country.

Finally, we point out that total resources in each economy are given by $Y_o = w_o L_o (1+\pi)$. Total resources are larger than the labor endowment because the definitions of payments to labor do not account for an existing mass of firms. With an existing mass of firms, the global economy is endowed not only with labor but also with that mass. This non-labor endowment is reflected in profits made by those firms. If there is a pre-existing mass of firms that does not make profits, the additional resources are paid to labor in the form of production costs. Without a pre-existing mass of firms, the cost of creating new firms is captured in the payments to labor, Φ^e, when creating those firms.

Notice that for one country, equation (56) can be written as

$$\begin{aligned}
Y_d &= C_d + I_d \\
&= p_d(1)\, q_d(1) + \sum_{k=1}^{O} \left(\frac{1 - u_{dk} - i_{dk}}{1 - i_{dk}} \right) N_k^x \int_{\bar{\varphi}_{dk}} p_{dk}(\varphi)\, q_{dk}(\varphi)\, dG(\varphi) \\
&+ N_d^x e_d^x + \sum_{k=1}^{O} \kappa_{dk} u_{dk} N_k^x c_{dk} + u_{kd} N_d^x \left(l_{kd} + s_{kd} \kappa_{kd} \chi(\kappa_{kd}) \right) + (1 - u_{kd} - i_{kd})\, N_d^x f_{kd},
\end{aligned}$$

which is equation (23) in the main text.

A.11 The ideal price index with our productivity distribution

A.11.1 Moving from an index to a distribution of goods

Melitz uses the following steps to move from index ω_{do} over a continuum of goods available to consume, Ω, which we assume has measure $M_{do} = |\Omega_{do}|$, to the cumulative distribution of productivity $G(\varphi)$ and the measure of goods available for consumption $(1 - i_{do}) M_{do}$.

The following steps keep the notation in Melitz's original work. Begin with the definition for the change of variables, also known as integration by substitution, which states

$$\int_a^b f(h(\varphi)) h'(\varphi) d\varphi = \int_{i(a)}^{i(b)} f(\omega) d\omega$$

Choose to index the goods ω with the indexing number $G(\varphi) M_{do}$ which is differentiable in φ such that $\omega = h(\varphi) = G(\varphi) M_{do}$. Then we can apply the rule from left to right to get

$$\int_0^\infty f(G(\varphi) M_{do}) \frac{\partial G(\varphi) M_{do}}{\partial \varphi} d\varphi = \int_{G(0) M_{do}}^{G(\infty) M_{do}} f(\omega) d\omega = \int_0^{M_{do}} f(\omega) d\omega = \int_{\omega \in \Omega_{do}} f(\omega) d\omega.$$

We choose $G(0) = 0$ and $G(\infty) = 1$ in our context because $G(\varphi)$ is a cumulative distribution function and it allows us to start the continuous indexing such that the upper bound of the integral is the measure of Ω_{do}. More generally, change of variables allows for any $G(\varphi)$ as long as $G(\varphi)$ is differentiable in φ.

Remember that in our context $f(\omega)$ is a function that simply indexes the continuum of goods ω so that $f(\omega)$ does not vary with ω even though $f(\varphi)$ will vary with φ. Therefore, we can reassign the indexing number $G(\varphi)M_{do}$ to φ to get

$$\int_0^\infty f(G(\varphi)M_{do})\frac{\partial G(\varphi)M_{do}}{\partial\varphi}d\varphi = \int_0^\infty f(\varphi)\frac{\partial G(\varphi)M_{do}}{\partial\varphi}d\varphi,$$

We often integrate over $[\bar{\varphi}_{do}, \infty)$ and not $[0, \infty)$ because some goods are not available in equilibrium. As long as $f(\varphi) = 0$ when $\varphi < \bar{\varphi}_{do}$, we can ignore those goods and

$$\int_0^\infty f(\varphi)\frac{\partial G(\varphi)M_{do}}{\partial\varphi}d\varphi = \int_{\bar{\varphi}_{do}}^\infty f(\varphi)\frac{\partial G(\varphi)M_{do}}{\partial\varphi}d\varphi$$

In order to relate this expression to economically meaningful concepts, it is helpful to rewrite this as

$$\int_{\bar{\varphi}_{do}}^\infty f(\varphi)\frac{\partial G(\varphi)M_{do}}{\partial\varphi}d\varphi = (1 - i_{do})\, M_{do}\int_{\bar{\varphi}_{do}}^\infty f(\varphi)\frac{g(\varphi)}{(1 - i_{do})}d\varphi$$

in which $i_{do} = G(\bar{\varphi}_{do})$, $g(\varphi) = \partial G(\varphi)/\partial\varphi$, and $g(\varphi)$ is a proper density because $1 = \int_{\bar{\varphi}_{do}}^\infty g(\varphi)\,(1 - i_{do})^{-1}\, d\varphi$. This implies that the measure of goods available to consume is $(1 - i_{do})\, M$ and the density of goods available to consume is given by $g(\varphi)\,(1 - i_{do})^{-1}$. The analogous measure of goods available to consume in our model is $(1 - u_{do} - i_{do})\, N_o^x$ and we have the same density of goods as Melitz because the unmatched fraction of products, u_{do}, is still available to consumers.

A.11.2 Differentiated goods price index

We are able to map from the price index defined using varieties, ω, in equation (3) to a price index in terms of firm productivities, φ, using the approach in appendix A.11.1 to obtain:

$$P_d = \left[\sum_{k=1}^{O}\left(\frac{1 - u_{dk} - i_{dk}}{1 - i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}}^\infty p_{dk}(\varphi)^{1-\sigma}\, dG(\varphi)\right]^{\frac{1}{1-\sigma}},$$

in which $G(\cdot)$ is a cumulative density function that is defined as Pareto distributed in section 2.1. With our assumptions about demand and the production structure in sections 2.2 and 2.1 we get equation (16), which is $p_{do}(\varphi) = \mu w_o \tau_{do}\varphi^{-1}$. Plugging this into the price index gives

$$P_d = \left[\sum_{k=1}^{O}\left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right)N_k^x\int_{\bar{\varphi}_{dk}}^{\infty}\left(\mu w_k\tau_{kd}\right)^{1-\sigma}dG\left(\varphi\right)\right]^{\frac{1}{1-\sigma}}$$

then we can use the moment $\int_{\bar{\varphi}_{dk}}^{\infty} z^{\sigma-1}dG\left(z\right) = \frac{\theta\bar{\varphi}_{dk}^{\sigma-\theta-1}}{\theta-\sigma+1}$ to get

$$P_d = \left[\sum_{k=1}^{O}\left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right)N_k^x\left(\mu w_k\tau_{kd}\right)^{1-\sigma}\frac{\theta\bar{\varphi}_{kd}^{\sigma-\theta-1}}{\theta-\sigma+1}\right]^{\frac{1}{1-\sigma}}$$

The threshold productivity is given in equation (18) in the main text, which is

$$\bar{\varphi}_{do} = \mu\left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}}\left(\frac{w_o\tau_{do}}{P_d}\right)\left(\frac{F_{do}\left(\kappa_{do}\right)}{C_d}\right)^{\frac{1}{\sigma-1}}$$

By substituting the threshold into the price index we get

$$P_d = \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{-\frac{1}{\theta}}\left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}}\mu C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}}\left(\sum_{k=1}^{O}\left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right)N_k^x\left(w_k\tau_{dk}\right)^{-\theta}F_{dk}\left(\kappa_{dk}\right)^{-\left[\frac{\theta}{\sigma-1}-1\right]}\right)^{-\frac{1}{\theta}}$$

Then we can employ our definition for the number of producers from section A.10.1 to derive

$$P_d = \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{-\frac{1}{\theta}}\left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}}\mu C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}}\left(\sum_{k=1}^{O}\left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right)\left(\frac{C}{1+\pi}\right)\frac{C_k}{C}\left(w_k\tau_{dk}\right)^{-\theta}F_{dk}\left(\kappa_{dk}\right)^{-\left[\frac{\theta}{\sigma-1}-1\right]}\right)^{-\frac{1}{\theta}}$$

Slightly rearranging terms and using the fact that $\left(1-\frac{u_{dk}}{1-i_{dk}}\right) = \frac{\kappa_{dk}\chi\left(\kappa_{dk}\right)}{\lambda+\kappa_{dk}\chi\left(\kappa_{dk}\right)}$ the price index becomes

$$\begin{aligned} P_d &= \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{-\frac{1}{\theta}}\left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}}\mu\left(\frac{C}{1+\pi}\right)^{-\frac{1}{\theta}} \\ &\times C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \\ &\times \left(\sum_{k=1}^{O}\frac{C_k}{C}\left(\frac{\kappa_{dk}\chi\left(\kappa_{dk}\right)}{\lambda+\kappa_{dk}\chi\left(\kappa_{do}\right)}\right)\left(w_k\tau_{dk}\right)^{-\theta}F_{dk}\left(\kappa_{dk}\right)^{-\left[\frac{\theta}{\sigma-1}-1\right]}\right)^{-\frac{1}{\theta}} \end{aligned}$$

The final expression of the differentiated goods price index is a simple function of three terms

$$P_d = \lambda_2 \times C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \times \rho_d \tag{58}$$

in which

$$\rho_d \equiv \left(\sum_{k=1}^{O} \frac{C_k}{C} \left(\frac{\kappa_{dk}\chi(\kappa_{dk})}{\lambda + \kappa_{dk}\chi(\kappa_{dk})} \right) (w_k \tau_{dk})^{-\theta} F_{dk} (\kappa_{dk})^{-\left[\frac{\theta}{\sigma-1}-1\right]} \right)^{-\frac{1}{\theta}},$$

and

$$\lambda_2 \equiv \left(\frac{\theta}{\theta - (\sigma - 1)} \right)^{-\frac{1}{\theta}} \left(\frac{\sigma}{\alpha} \right)^{\frac{1}{\sigma-1} - \frac{1}{\theta}} \mu \left(\frac{C}{1+\pi} \right)^{-\frac{1}{\theta}}.$$

The expression in (58) resembles the price index in Chaney (2008), equation (8) closely. We note that ρ_d, the "multilateral resistance term," in that model is an equilibrium object in wages and GDP, whereas now it's an equilibrium object in wages, total consumption expenditure, and market tightness.

A.12 Defining the equilibrium

The equilibrium reduces to these equations in the equilibrium variables:

1. The free entry condition for retailers, which pins down κ_{do}:

$$\frac{c_{do}}{\chi(\kappa_{do})} = \int_{\bar{\varphi}_{do}} M_{do}(\varphi)\, dG(\varphi)$$

   Notice that now there are d times o markets and each market has an associated tightness. With our functional form assumptions, this equation can be simplified. Remember that with $V_{do} = 0$

$$M_{do}(\varphi) = \frac{p_{do}q_{do} - n_{do}q_{do}}{r + \lambda}$$

   so that

$$\begin{aligned} \int_{\bar{\varphi}_{do}} M_{do}(\varphi)\, dG(\varphi) &= \left(\frac{1}{r+\lambda} \right) \int_{\bar{\varphi}_{do}} p_{do}q_{do} - n_{do}q_{do} dG(\varphi) \\ \Rightarrow &= \left(\frac{1}{r+\lambda} \right) \left(1 - \frac{u_{do}}{1 - i_{do}} \right)^{-1} \left(\frac{1}{N_o^x} \right) \Pi_{do}^m \end{aligned}$$

   in which Π_{do}^m is defined in equation (33) in the main text and we know that

$$\Pi_{do}^m = b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do})\, C_{do}$$

Using the equilibrium retailer entry condition gives

$$\begin{aligned}\frac{c_{do}}{\chi\left(\kappa_{do}\right)} &= \left(\frac{1}{r+\lambda}\right)\left(1-\frac{u_{do}}{1-i_{do}}\right)^{-1}\left(\frac{1}{N_o^x}\right)\Pi_{do}^m \\ \Rightarrow \kappa_{do} &= \left(\frac{1}{r+\lambda}\right)\left(\lambda+\kappa_{do}\chi\left(\kappa_{do}\right)\right)\frac{b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\left(1+\pi\right)}{c_{do}C_o}C_{do},\end{aligned}$$

in which we used $N_o^x = \frac{1}{1+\pi}C_o$ and $C_{do} = \alpha C_d$. In sum, this equilibrium condition can be written neatly as

$$\kappa_{do} = \left(\frac{1}{r+\lambda}\right)\left(\lambda+\kappa_{do}\chi\left(\kappa_{do}\right)\right)\frac{b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\left(1+\pi\right)}{c_{do}C_o}C_{do}.$$

2. The expression that equates variable profits with the effective entry cost, which pins down $\bar{\varphi}_{do}$:

$$\bar{\varphi}_{do} = \mu\left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}}\left(\frac{w_o\tau_{do}}{P_d}\right)C_d^{\frac{1}{1-\sigma}}F_{do}\left(\kappa_{do}\right)^{\frac{1}{\sigma-1}}.$$

in which

$$P_d = \lambda_2 \times C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \times \rho_d$$

and

$$\rho_d \equiv \left(\sum_{k=1}^{O}\frac{C_k}{C}\left(\frac{\kappa_{dk}\chi\left(\kappa_{dk}\right)}{\lambda+\kappa_{dk}\chi\left(\kappa_{dk}\right)}\right)\left(w_k\tau_{dk}\right)^{-\theta}F_{dk}\left(\kappa_{dk}\right)^{-\left[\frac{\theta}{\sigma-1}-1\right]}\right)^{-\frac{1}{\theta}}$$

and

$$\lambda_2 \equiv \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{-\frac{1}{\theta}}\left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}}\mu\left(\frac{C}{1+\pi}\right)^{-\frac{1}{\theta}}.$$

In this simplification we have used the assumption that

$$N_o^x = \frac{1}{1+\pi}C_o$$

3. National accounting/consumer's budget constraint pins down consumption C_d:

$$C_d = Y_d - I_d$$

in which

$$I_d = \sum_{k=1}^{O} I_{dk} = N_d^x e_d^x + \sum_{k=1}^{O}\kappa_{dk}u_{dk}N_k^x c_{dk} + u_{kd}N_d^x\left(l_{kd}+s_{kd}\kappa_{kd}\chi\left(\kappa_{kd}\right)\right) + \left(1-u_{kd}-i_{kd}\right)N_d^x f_{kd}$$

and

$$N_d^x = \frac{1}{1+\pi} C_d$$

and

$$Y_d = w_d L_d \left(1+\pi\right).$$

4. The global mutual fund pins down π:

$$\pi = \frac{\Pi}{\sum_k w_k L_k}$$

in which Π are the profits from the differentiated goods:

$$\begin{aligned}\Pi &= \sum_k \sum_j \Pi_{jk} = \sum_k \sum_j \left(1 - \frac{u_{jk}}{1-i_{jk}}\right) N_k^x \int_{\bar{\varphi}_{jk}} p_{jk}\left(\varphi\right) q_{jk}\left(\varphi\right) - t_{jk}\left(\varphi\right) dG\left(\varphi\right) \\ &= \alpha \frac{C_d}{\sigma}\end{aligned}$$

B Welfare, consumption elasticity, and import elasticity

B.1 Proof of proposition 5: Changes in welfare

We prove proposition 5 assuming monopolistic competition and following steps similar to those used to prove proposition 1 in Arkolakis, Costinot, and Rodríguez-Clare (2012). With the exception of the search frictions, our functional form assumptions allow us to relate the differentiated goods price index in our model from section 4 to the price index equation (A22) in Arkolakis, Costinot, and Rodríguez-Clare (2012, p. 123)

$$P_{do}^{1-\sigma} = \left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right)\left(P_{do}^{ACR}\right)^{1-\sigma}, \tag{59}$$

in which $\left(P_{do}^{ACR}\right)^{1-\sigma} = N_o^x\left(\mu w_o \tau_{do}\right)^{1-\sigma}\Psi_{do}$, and it will be useful to define the important one-sided moment $\Psi_{do} \equiv \int_{\bar{\varphi}_{do}}^{\infty} z^{\sigma-1} dG\left(z\right)$. Also define the elasticity of this integral with respect to the cutoff as $\psi_{do} \equiv \dfrac{\partial \ln\left(\Psi_{do}\right)}{\partial \ln\left(\bar{\varphi}_{do}\right)}$. A sufficient condition for $\psi_{do} \leq 0$ is $\sigma > 1$. The overall price index with both homogeneous and differentiated goods is

$$\Xi_d = \left(\frac{p_d\left(1\right)}{1-\alpha}\right)^{1-\alpha}\left(\frac{P_d}{\alpha}\right)^{\alpha},$$

in which $p_d\left(1\right)$ is the price of the freely traded homogeneous good and α is the share of consumption devoted to the differentiated goods bundle. In section A.11.2, we show that the price of the freely traded good in equilibrium is the same in all countries $p_d\left(1\right) = p\left(1\right)$. Lastly, it will be useful to denote the total derivative of the log of a variable x, as $d\ln x = \ln\left(x'/x\right) = \ln\left(\hat{x}\right)$ and so $\exp\left(d\ln x\right) = \hat{x}$.

B.1.1 Step 1: Small changes in welfare satisfy

$$d\ln\left(W_d\right) \quad = \quad d\ln\left(C_d\right) - \alpha d\ln\left(P_d\right). \tag{60}$$

Because the utility function is homogeneous of degree one, welfare is defined by real consumption expenditure $W_d = \dfrac{C_d}{\Xi_d}$. To derive equation (60) we use the definition of the price index to write

$$W_d \quad = \quad C_d\left(\frac{p\left(1\right)}{1-\alpha}\right)^{\alpha-1}\left(\frac{P_d}{\alpha}\right)^{-\alpha}$$

Taking logs gives

$$\ln(W_d) \quad = \quad \ln(C_d) + (\alpha - 1)\left[\ln(p(1)) - \ln(1-\alpha)\right] - \alpha\left[\ln(P_d) - \ln(\alpha)\right]$$

We define the price of the freely traded good as the numeraire, $p(1) = 1$, and then totally differentiate

$$d\ln(W_d) = d\ln(C_d) - \alpha d\ln(P_d)$$

Arkolakis, Costinot, and Rodríguez-Clare (2012) rely on two additional simplifications that remove consumption from this expression, which we cannot employ. First, they have that $C_d \propto Y_d$ with a proportionality constant that is only a function of exogenous parameters. We lack this simplification because investment in our setting is not exogenously proportional to output. Second, while Arkolakis, Costinot, and Rodríguez-Clare (2012) do not explicitly invoke restriction R2 here, they do rely on it to get that $\Pi_d \propto Y_d$ with a proportionality constant that is only a function of exogenous parameters. Because L_d is an exogenous endowment and w_d can be normalized, using R2 ensures that $w_d L_d + \Pi_d = Y_d \propto w_d L_d$ which ensures that $d\ln(w_d L_d) = 0$ implies $d\ln(C_d) = 0$ and welfare is determined soley by the price index.

B.1.2 Step 2: Small changes in the consumer price index satisfy

$$\begin{aligned} d\ln P_d \quad &= \quad \sum_{k=1}^{O} \frac{\lambda_{dk}}{\alpha\left(1-\sigma+\alpha^{-1}\psi_d\right)}\left[d\ln\left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right)\right. \\ &+ \quad (1-\sigma+\psi_{dk})\left(d\ln w_k + d\ln\tau_{dk}\right) + d\ln N_k^x + \psi_{dk}\left(\frac{1}{\sigma-1}\right)d\ln\left(F\left(\kappa_{dk}\right)\right) \\ &+ \quad \left.\psi_{dk}\left(\frac{1}{1-\sigma}\right)d\ln\left(C_d\right)\right], \end{aligned} \tag{61}$$

in which ψ_{do} is defined above and $\psi_d \equiv \sum_{k=1}^{O} \lambda_{dk}\psi_{dk}$.

Equation (61) is analogous to equation (A33) in Arkolakis, Costinot, and Rodríguez-Clare (2012, p. 125) (except there is a typo in their first multiplicative term because γ_{ij} should be γ_j). When the utility function has only differentiated goods ($\alpha = 1$) and there are no search frictions ($u_{do} = 0$), equations (A33) and (61) are the same. The signs on ψ_d and ψ_{do} differ between the models because our model is defined in terms of productivity, while theirs is defined in terms of marginal cost.

We derive equation (61) by starting with total consumption in destination country d for the differentiated goods bundle from origin country o by integrating over all varieties at final prices. Because CES preferences define the differentiated goods aggregate given in equation

(1), this integral is the value of CES demand for the bundle of country o products

$$C_{do} = \left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} p_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi) = \alpha \frac{P_{do}^{1-\sigma} C_d}{P_d^{1-\sigma}}.$$

This can be easily derived from equation (2). Define the consumption share λ_{do} as

$$\lambda_{do} \equiv \frac{C_{do}}{C_d} = \alpha \frac{P_{do}^{1-\sigma} C_d}{P_d^{1-\sigma}} \left(\frac{1}{C_d}\right) = \alpha \frac{P_{do}^{1-\sigma}}{P_d^{1-\sigma}}.$$

Our definition of the consumption share differs from Arkolakis, Costinot, and Rodríguez-Clare (2012) in two important ways. First, we use consumer expenditure instead of output because our model does not guarantee that income and consumption are proportional. Second, consumption, which is what matters for welfare, is measured at final sales prices, while the import share is measured at negotiated import prices. We will work with $P_d^{1-\sigma}$ using the definition of the price index for the differentiated good in the destination market d, given by

$$P_d = \left[\sum_{k=1}^{O} P_{dk}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

Take the log of this expression to get $(1-\sigma)\ln P_d = \ln \sum_{k=1}^{O} P_{dk}^{1-\sigma}$ and then totally differentiate to get

$$(1-\sigma)\, d\ln P_d = \sum_{k=1}^{O} \frac{P_{dk}^{1-\sigma}}{P_d^{1-\sigma}} \frac{dP_{dk}^{1-\sigma}}{P_{dk}^{1-\sigma}}.$$

Rearrange λ_{do} to get $\dfrac{\lambda_{do}}{\alpha P_{do}^{1-\sigma}} = \dfrac{1}{P_d^{1-\sigma}}$ and then use this to simplify

$$(1-\sigma)\, d\ln P_d = \sum_{k=1}^{O} \frac{\lambda_{dk}}{\alpha} d\ln P_{dk}^{1-\sigma}.$$

Taking logs of equation (59) and totally differentiating gives

$$d\ln P_{do}^{1-\sigma} = d\ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) + d\ln\left(P_{do}^{ACR}\right)^{1-\sigma}.$$

Employing our functional form assumptions, which gives $\left(P_{do}^{ACR}\right)^{1-\sigma} = N_o^x \left(\mu w_o \tau_{do}\right)^{1-\sigma} \Psi_{do}$, we can derive

$$d\ln\left(P_{do}^{ACR}\right)^{1-\sigma} = d\ln N_o^x + (1-\sigma)\left(d\ln w_o + d\ln \tau_{do}\right) + \psi_{do} d\ln\left(\bar{\varphi}_{do}\right),$$

in which we use the chain rule to get

$$d\ln\Psi_{do} = \psi_{do} d\ln\left(\bar{\varphi}_{do}\right).$$

Putting these parts together gives

$$(1-\sigma)\, d\ln P_d = \sum_{k=1}^{O} \frac{\lambda_{dk}}{\alpha}\left[d\ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) + (1-\sigma)\left(d\ln w_o + d\ln\tau_{do}\right) + d\ln N_o^x + \psi_{do} d\ln\left(\bar{\varphi}_{do}\right)\right]. \tag{62}$$

If we set $\alpha = 1$ and $c_{do} = 0$ so that $u_{do} = 0$ we match equation (A34) in Arkolakis, Costinot, and Rodríguez-Clare (2012, p. 125).

Next, take the log and total derivative of the cutoff expression from equation (18)

$$d\ln\left(\bar{\varphi}_{do}\right) = -d\ln\left(P_d\right) + d\ln\left(\tau_{do}\right) + \left(\frac{1}{\sigma-1}\right) d\ln\left(F\left(\kappa_{do}\right)\right) - \left(\frac{1}{\sigma-1}\right) d\ln\left(C_d\right) + d\ln\left(w_o\right), \tag{63}$$

which is the analog to equation (A36) in Arkolakis, Costinot, and Rodríguez-Clare (2012, p. 126). There are a few differences between equation (63) here and their equation (A36). First, the signs are reversed because we define everything in terms of productivity, while they use costs. Second, their term ξ_{ij} captures the fixed cost of entry like our term $F\left(\kappa_{do}\right)$ (see equation (A27) on page 124). And while their term ρ_{ij} allows for some foreign labor to be used to enter a foreign country, we do not. Making the same restriction in their model would require setting $h_{ij} = 1$ and hence $\rho_{ij} = 1$. Finally, our threshold expression includes total consumption.

Combining equations (62) and (63) gives equation (61).

B.1.3 Step 3: Small changes in the consumer price index satisfy

$$\begin{aligned}
d\ln P_d &= \sum_{k=1}^{O} \lambda_{dk}\left(\frac{d\ln\left(\lambda_{dk}\right) - d\ln\left(\lambda_{dd}\right)}{\alpha\left(1-\sigma+\alpha^{-1}\psi_d\right)}\right) + \frac{\left(\alpha\psi_{dd}-\psi_d\right) d\ln\left(\bar{\varphi}_{dd}\right)}{\alpha\left(1-\sigma+\alpha^{-1}\psi_d\right)} \\
&+ \frac{d\ln N_d^x}{\left(1-\sigma+\alpha^{-1}\psi_d\right)} \\
&+ \frac{\psi_d d\ln\left(F\left(\kappa_{dd}\right)\right)}{\left(\sigma-1\right)\alpha\left(1-\sigma+\alpha^{-1}\psi_d\right)} \\
&+ \frac{d\ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right)}{\left(1-\sigma+\alpha^{-1}\psi_d\right)} \\
&+ \frac{\psi_d d\ln\left(C_d\right)}{\left(1-\sigma\right)\alpha\left(1-\sigma+\alpha^{-1}\psi_d\right)}
\end{aligned} \tag{64}$$

If we set $\alpha = 1$ and $c_{do} = 0$ so that $u_{do} = 0$ and $l_{dd} = -h_{dd}$ so that $F\left(\kappa_{dd}\right)$ is a constant, then (64) becomes equation (A37) of Arkolakis, Costinot, and Rodríguez-Clare (2012, p.

126).

Start again with the consumption share $\lambda_{do} = \alpha \frac{P_{do}^{1-\sigma}}{P_d^{1-\sigma}}$ and form $\frac{\lambda_{do}}{\lambda_{dd}} = \frac{P_{do}^{1-\sigma}}{P_{dd}^{1-\sigma}}$. Substitute into the ratio $\frac{\lambda_{do}}{\lambda_{dd}}$ our functional form assumptions for the price index, take logs, and then totally differentiate to get

$$\begin{aligned} d\ln(\lambda_{do}) - d\ln(\lambda_{dd}) &= (1-\sigma)(d\ln w_o + d\ln \tau_{do}) + \psi_{do} d\ln(\bar{\varphi}_{do}) - \psi_{dd} d\ln(\bar{\varphi}_{dd}) \\ &+ d\ln N_o^x - d\ln N_d^x \\ &+ d\ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) - d\ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right). \end{aligned} \tag{65}$$

In obtaining this expression we have simplified terms by recalling that we are considering a foreign shock so that $d\ln \tau_{dd} = 0$ and that our normalization of the price of the freely traded good ensures that $d\ln w_o = d\ln w_d = 0$. We keep $d\ln w_o$ in the expression and ordered the terms as presented in Arkolakis, Costinot, and Rodríguez-Clare (2012, p. 126) to make comparing the expressions easier.

We can derive two cutoff expressions

$$d\ln(\bar{\varphi}_{do}) = -d\ln(P_d) + d\ln(\tau_{do}) + \frac{d\ln(F(\kappa_{do}))}{\sigma-1} + d\ln(w_o) - \left(\frac{1}{\sigma-1}\right) d\ln(C_d),$$

and also

$$d\ln(\bar{\varphi}_{dd}) = -d\ln(P_d) + \frac{d\ln(F(\kappa_{dd}))}{\sigma-1} - \left(\frac{1}{\sigma-1}\right) d\ln(C_d),$$

in which we again impose that $\tau_{dd} = 1$ and $d\ln w_d = 0$. Combining these two cutoff expressions gives

$$d\ln(\bar{\varphi}_{do}) = d\ln(\bar{\varphi}_{dd}) + d\ln(w_o) + d\ln(\tau_{do}) + \frac{d\ln(F(\kappa_{do}))}{\sigma-1} - \frac{d\ln(F(\kappa_{dd}))}{\sigma-1}, \tag{66}$$

which is akin to the last equation of Arkolakis, Costinot, and Rodríguez-Clare (2012, p. 126) with the exception that they have a typo because the equal sign should be a minus sign. In our model, it is not necessarily the case that $d\ln(F(\kappa_{dd})) = 0$ in response to a foreign shock because the effective entry cost, $F(\kappa_{dd})$, is an endogenous variable and not a parameter.

Combine expression (66) with (65) to get

$$\begin{aligned} d\ln(\lambda_{do}) - d\ln(\lambda_{dd}) &= (1-\sigma+\psi_{do})(d\ln w_o + d\ln\tau_{do}) \\ &+ \psi_{do}\left(\frac{d\ln(F(\kappa_{do}))}{\sigma-1} - \frac{d\ln(F(\kappa_{dd}))}{\sigma-1}\right) \\ &+ (\psi_{do}-\psi_{dd})\, d\ln(\bar{\varphi}_{dd}) + d\ln N_o^x - d\ln N_d^x \\ &+ d\ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) - d\ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right). \end{aligned} \tag{67}$$

Equation (67) is analogous to equation (A38) in Arkolakis, Costinot, and Rodríguez-Clare (2012, p. 127) which has a typo because α_{ij}^* should be α_{jj}^*. Substituting equation (67) into equation (61) and performing algebra gives equation (64).

B.1.4 Step 4: Small changes in the consumer price index satisfy

$$\begin{aligned} d\ln P_d &= \frac{d\ln(\lambda_{dd})}{\theta} \\ &- \frac{d\ln N_d^x}{\theta} \\ &- \frac{(\sigma-1-\theta)\, d\ln(F(\kappa_{dd}))}{(\sigma-1)\,\theta} \\ &- \frac{d\ln\left(\dfrac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right)}{\theta} \\ &- \frac{(\sigma-1-\theta)\, d\ln(C_d)}{(1-\sigma)\,\theta} \end{aligned} \tag{68}$$

We depart somewhat from the approach taken in step 4 of Arkolakis, Costinot, and Rodríguez-Clare (2012, p. 127) in simplifying equation (64) to derive equation (68). They invoke macro-level restriction number 3, ‘‘R3: The import demand system is such that for any importer j and any pair of exporters $i \neq j$ and $i' \neq j$, $\varepsilon_j^{ii'} = \varepsilon < 0$ if $i = i'$, and zero otherwise.’’ As they describe on page 103, this restriction imposes symmetry on the elasticity of the consumption ratio to changes in variable trade costs. That elasticity in our model in general is given by equation (74) and need not be symmetric across countries. A sufficient condition to derive equation (68), however, is that productivity distributions and consumer preferences are symmetric. For now, we impose those restrictions in the following steps but could likely relax them in future work.

The term we need to consider from equation (64) is $\psi_d \equiv \sum_{k=1}^{O} \lambda_{dk}\psi_{dk}$, which is the consumption share weighted average of the elasticity of the moment of the productivity distribution, in which $\psi_{do} = \dfrac{d\ln(\Psi_{do})}{d\ln(\bar{\varphi}_{do})}$. We assume that productivity $\varphi \in [1, +\infty)$ is Pareto distributed with CDF $G[\tilde{\varphi} < \varphi] = 1 - \varphi^{-\theta}$ and PDF $g(\varphi) = \theta\varphi^{-\theta-1}$ in which, as usual, $\theta > \sigma - 1$ in order to close the model. With this distribution, the moment $\Psi_{do} = \frac{\theta\bar{\varphi}_{do}^{\sigma-\theta-1}}{\theta - \sigma + 1}$ and the elasticity $\psi_{do} = -\frac{\bar{\varphi}_{do}^{\sigma}\theta\bar{\varphi}_{do}^{-\theta-1}}{\Psi_{do}} = -(\theta - \sigma + 1)$. Notice that the restriction that $\theta > \sigma - 1$ ensures $\psi_{do} < 0$ and $\psi_d < 0$. Also notice that $\psi_{do} = \psi_{dd}$ and the term we are actually interested in becomes

$$\psi_d \equiv \sum_{k=1}^{O} \lambda_{dk}\psi_{do} = \alpha(\sigma - 1 - \theta), \tag{69}$$

because by definition consumption shares $\alpha = \sum_{k=1}^{O} \lambda_{dk}$. Substituting equation (69) into (64) and also using the fact that Euler's homogeneous function theorem gives $\sum_{k=1}^{O} \lambda_{dk} d\ln(\lambda_{do}) = 0$ provides (68).

B.1.5 Step 5: Small changes in the number of producers

We cannot make the simplification $d\ln N_d^x = 0$ as done in step 5 of Arkolakis, Costinot, and Rodríguez-Clare (2012, p. 127) because we have assumed that $N_d^x = \frac{C}{(1+\pi)}\left(\frac{C_d}{C}\right)$ and both C_d and $\pi = \Pi / \sum_k w_k L_k$ are endogenous objects. Allowing free entry into the market for producers would be an alternative assumption but would then require an additional equation for determining equilbrium market tightness. We discuss that extension more in appendix A.9.

B.1.6 Combining step 1 to step 4 into the general welfare expression

Combining equation (60) with equation (68) provides the change in welfare in response to a foreign shock in our model

$$\begin{aligned}
d\ln(W_d) &= -\left(\frac{\alpha}{\theta}\right) d\ln(\lambda_{dd}) \\
&+ \left(1 + \left(\frac{\alpha}{\theta}\right)\left(1 - \frac{\theta}{\sigma - 1}\right)\right) d\ln(C_d) \\
&+ \left(\frac{\alpha}{\theta}\right) d\ln N_d^x \\
&+ \left(\frac{\alpha}{\theta}\right)\left(1 - \frac{\theta}{\sigma - 1}\right) d\ln(F(\kappa_{dd})) \\
&+ \left(\frac{\alpha}{\theta}\right) d\ln\left(\frac{1 - u_{dd} - i_{dd}}{1 - i_{dd}}\right)
\end{aligned} \tag{70}$$

We derive this by substituting the change in the price index from (68) into the welfare

expression from (60) and simplifying the algebra using, in particular,
$\frac{\alpha(\sigma-1-\theta)}{(\sigma-1)\theta} = \left(\frac{\alpha}{\theta}\right)\left(1-\frac{\theta}{\sigma-1}\right)$.

B.1.7 The change in welfare in proposition 5

We made an assumption that productivity, $\varphi \in [1, +\infty)$, follows a Pareto distribution with CDF $G[\tilde{\varphi} < \varphi] = 1 - \varphi^{-\theta}$ in appendix B.1.4 in order to derive the general equation (70). Two additional assumptions are needed to derive proposition 5 in the main text from the general welfare change in equation (70). The first of these assumptions is that the cost of remaining idle, $-h_{dd}$, is the same as the flow search costs that producers pay to find retailers, l_{dd}, so that $l_{dd} = -h_{dd}$. With this assumption, the effective entry costs become a function of exogenous parameters and $d\ln(F(\kappa_{dd})) = 0$. The second assumption is that the number of domestic producers does not respond to a foreign shock, $d\ln N_d^x = 0$. One could rationalize this assumption by assuming free entry into production or that N_d^x is exogenous.

Applying these two additional assumptions to the general welfare changes in equation (70) gives

$$d\ln(W_d) = -\left(\frac{\alpha}{\theta}\right) d\ln(\lambda_{dd}) + \left(1+\left(\frac{\alpha}{\theta}\right)\left(1-\frac{\theta}{\sigma-1}\right)\right) d\ln(C_d) + \left(\frac{\alpha}{\theta}\right) d\ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right), \quad (71)$$

which we can integrate to get the welfare response to any foreign shock in proposition 5 of the main text

$$\hat{W}_d = \hat{\lambda}_{dd}^{-\frac{\alpha}{\theta}} \left(1-\widehat{\frac{u_{dd}}{1-i_{dd}}}\right)^{\frac{\alpha}{\theta}} \hat{C}_d^{1+\frac{\alpha}{\theta}\left(1-\frac{\theta}{\sigma-1}\right)} \quad (72)$$

B.2 Proof of proposition 6: Consumption elasticity

B.2.1 Relating price indexes

To derive an analogous elasticity in our model, start with the functional form assumptions detailed in section 2. Because, with the exception of the search frictions, these functional form assumptions are the same as in Arkolakis, Costinot, and Rodríguez-Clare (2012), we can relate the price index in our model given in section 4 to the price index equation (A22) in Arkolakis, Costinot, and Rodríguez-Clare (2012, p. 123)

$$\left(P_{do}^{ACR}\right)^{1-\sigma} = N_o^x \left(\mu w_o \tau_{do}\right)^{1-\sigma} \Psi_{do},$$

in which it will be useful to define $\Psi_{do} = \int_{\bar{\varphi}_{do}}^{\infty} \varphi^{\sigma-1} dG(\varphi)$ and the elasticity of this integral with respect to the cutoff $\psi_{do} = \frac{\partial \ln(\Psi_{do})}{\partial \ln(\bar{\varphi}_{do})} \leq 0$ a sufficient condition for which is $\sigma > 1$.

B.2.2 Demand for a country's bundle of goods

We can derive total consumption in destination country d for the goods bundle from origin country o by integrating over all varieties at final prices. Because we have CES preferences, this integral is the value of CES demand for the bundle of country o products

$$C_{do} = \left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} p_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi) = \alpha \frac{P_{do}^{1-\sigma} C_d}{P_d^{1-\sigma}}.$$

Define the consumption share (which we note in our model is different from the observed trade share) as λ_{do}

$$\lambda_{do} = \frac{C_{do}}{C_d} = \alpha \frac{P_{do}^{1-\sigma} C_d}{P_d^{1-\sigma} C_d} = \alpha \frac{P_{do}^{1-\sigma}}{P_d^{1-\sigma}}.$$

We can also form relative consumption ratios, which is equivalent to Arkolakis, Costinot, and Rodríguez-Clare (2012) equation (21), page 110, and is just the ratio of the price indexes raised to a power

$$\frac{\lambda_{do}}{\lambda_{dd}} = \frac{C_{do}}{C_{dd}} = \frac{P_{do}^{1-\sigma}}{P_{dd}^{1-\sigma}}.$$

Using the country-specific price indexes given above we have

$$\frac{P_{do}^{1-\sigma}}{P_{dd}^{1-\sigma}} = \frac{\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right)\left(P_{do}^{ACR}\right)^{1-\sigma}}{\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right)\left(P_{dd}^{ACR}\right)^{1-\sigma}},$$

in which we used the definition of P_{do}^{ACR} . Taking the log of relative consumption ratios therefore gives

$$\ln\left(\frac{C_{do}}{C_{dd}}\right) = \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) + \ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) - \ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right). \tag{73}$$

B.2.3 Derivative of consumption ratio with respect to tariffs

The goal is to derive two derivatives. The first is the direct effect of a change in the tariffs τ_{do} on the consumption ratio

$$\frac{\partial \ln\left(C_{do}/C_{dd}\right)}{\partial \ln\left(\tau_{do}\right)} = \varepsilon_o^{dd}.$$

The second is the indirect effect, which documents how changing tariffs between a third country d' and the origin o changes relative consumption in country d

$$\frac{\partial \ln \left(C_{do}/C_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} = \varepsilon_{o}^{dd'}.$$

B.2.4 Direct effect of tariff changes ($d' = d$ case)

We begin by deriving $\frac{\partial \ln \left(C_{do}/C_{dd}\right)}{\partial \ln \left(\tau_{do}\right)} = \varepsilon_{o}^{dd}$ in the most general form and then apply a few restrictions to compare it to the elasticity in Arkolakis, Costinot, and Rodríguez-Clare (2012). Normalizing the price of the homogeneous good ensures that $\frac{\partial \ln \left(w_{d}\right)}{\partial \ln \left(\tau_{do}\right)} = 0$ and $\frac{\partial \ln \left(w_{o}\right)}{\partial \ln \left(\tau_{do}\right)} = 0$.

B.2.5 First and second terms of equation (73) ($d' = d$ case)

Differentiating and simplifying the first term of equation (73) gives

$$\frac{\partial}{\partial \ln \left(\tau_{do}\right)} \ln \left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) = \frac{\partial \ln \left(N_{o}^{x}\right)}{\partial \ln \left(\tau_{do}\right)} + (1-\sigma) + \psi_{do}\frac{\partial \ln \left(\bar{\varphi}_{do}\right)}{\partial \ln \left(\tau_{do}\right)},$$

and similarly

$$\frac{\partial}{\partial \ln \left(\tau_{do}\right)} \ln \left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) = \frac{\partial \ln \left(N_{d}^{x}\right)}{\partial \ln \left(\tau_{do}\right)} + \psi_{dd}\frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{do}\right)}.$$

Combining these gives

$$\begin{aligned}
\frac{\partial}{\partial \ln \left(\tau_{do}\right)} \ln \left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \frac{\partial}{\partial \ln \left(\tau_{do}\right)} \ln \left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) &= (1-\sigma) + \psi_{do}\frac{\partial \ln \left(\bar{\varphi}_{do}\right)}{\partial \ln \left(\tau_{do}\right)} - \psi_{dd}\frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{do}\right)} \\
&+ \frac{\partial \ln \left(N_{o}^{x}\right)}{\partial \ln \left(\tau_{do}\right)} - \frac{\partial \ln \left(N_{d}^{x}\right)}{\partial \ln \left(\tau_{do}\right)}.
\end{aligned}$$

The elasticities of the cutoffs $\bar{\varphi}_{do}$ and $\bar{\varphi}_{dd}$ are related because changing tariff τ_{do} changes the price index P_{d} which changes the cutoff $\bar{\varphi}_{dd}$. We can derive this relationship by differentiating the explicit expression for the cutoff given in equation (18)

$$\begin{aligned}
\frac{\partial \ln \left(\bar{\varphi}_{do}\right)}{\partial \ln \left(\tau_{do}\right)} &= 1 + \frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{do}\right)} \\
&+ \left(\frac{1}{\sigma-1}\right)\left[\frac{\partial \ln \left(F_{do}\right)}{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}\frac{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}{\partial \ln \left(\tau_{do}\right)} - \frac{\partial \ln \left(F_{dd}\right)}{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}\frac{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}{\partial \ln \left(\tau_{do}\right)}\right].
\end{aligned}$$

Substituting this into the elasticity of the general expression for the ratio of relative price indexes and simplifying gives

$$\frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) = (1-\sigma) + \psi_{do} + (\psi_{do} - \psi_{dd}) \frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{do})}$$
$$+ \left(\frac{\psi_{do}}{\sigma - 1}\right) \left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))} \frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{do})} - \frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))} \frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{do})}\right]$$
$$+ \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{do})} - \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{do})}.$$

The first line on the right is the same as equation (21) in Arkolakis, Costinot, and Rodríguez-Clare (2012) except that $\psi_{do} \leq 0$ in our case, while $\gamma_{ij} \geq 0$ in the Arkolakis et al.'s expressions because we define our model in terms of productivity, φ, while they define theirs in terms of marginal cost.

B.2.6 Elasticity of destination–origin market unmatched rate

Next, we calculate the elasticity of the destination–origin market unmatched producers' rate. Because we are studying a steady state, we use the definition $\frac{1 - u_{do} - i_{do}}{1 - i_{do}} = \frac{\kappa_{do}\chi(\kappa_{do})}{\lambda + \kappa_{do}\chi(\kappa_{do})}$ to derive

$$\frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\frac{1 - u_{do} - i_{do}}{1 - i_{do}}\right) = \left(\frac{u_{do}}{1 - i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})}\right),$$

in which we used the chain rule to write

$$\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})} = \frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \kappa_{do}\chi(\kappa_{do})} \frac{\partial \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})} = \frac{1}{\kappa_{do}\chi(\kappa_{do})}\left(\frac{\partial \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})}\right).$$

B.2.7 Elasticity of destination–destination market unmatched rate

Calculating the elasticity of the destination–destination market unmatched producers' rate with respect to τ_{do} also relies on the definition of $\frac{1 - u_{dd} - i_{dd}}{1 - i_{dd}} = \frac{\kappa_{dd}\chi(\kappa_{dd})}{\lambda + \kappa_{dd}\chi(\kappa_{dd})}$. The steps to derive this will be identical to the ones we took in calculating the destination–origin market unmatched rate with only the sub-indexes changing. The final derivative is

$$\frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\frac{1 - u_{dd} - i_{dd}}{1 - i_{dd}}\right) = \left(\frac{u_{dd}}{1 - i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{do})}\right),$$

in which we used the chain rule again to calculate

$$\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{do})} = \frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \kappa_{dd}\chi(\kappa_{dd})} \frac{\partial \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{do})} = \frac{1}{\kappa_{dd}\chi(\kappa_{dd})}\left(\frac{\partial \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{do})}\right).$$

B.2.8 General expression for $d' = d$ case

Here we try to write the most general possible expression only assuming that $\frac{\partial \ln (w_d)}{\partial \ln (\tau_{do})} = 0$ and $\frac{\partial \ln (w_o)}{\partial \ln (\tau_{do})} = 0$. Combining the general term expression in Arkolakis et al. with the elasticity of the finding rate with respect to tariffs gives

$$\begin{aligned} \frac{\partial}{\partial \ln (\tau_{do})} \ln \left(\frac{C_{do}}{C_{dd}} \right) &= (1-\sigma) + \psi_{do} + (\psi_{do} - \psi_{dd}) \frac{\partial \ln (\bar{\varphi}_{dd})}{\partial \ln (\tau_{do})} + \frac{\partial \ln (N_o^x)}{\partial \ln (\tau_{do})} - \frac{\partial \ln (N_d^x)}{\partial \ln (\tau_{do})} \\ &+ \left(\frac{u_{do}}{1-i_{do}} \right) \left(\frac{\partial \ln \kappa_{do} \chi (\kappa_{do})}{\partial \ln (\tau_{do})} \right) - \left(\frac{u_{dd}}{1-i_{dd}} \right) \left(\frac{\partial \ln \kappa_{dd} \chi (\kappa_{dd})}{\partial \ln (\tau_{do})} \right) \\ &+ \left(\frac{\psi_{do}}{\sigma - 1} \right) \left[\frac{\partial \ln (F_{do})}{\partial \ln (\kappa_{do} \chi (\kappa_{do}))} \frac{\partial \ln (\kappa_{do} \chi (\kappa_{do}))}{\partial \ln (\tau_{do})} - \frac{\partial \ln (F_{dd})}{\partial \ln (\kappa_{dd} \chi (\kappa_{dd}))} \frac{\partial \ln (\kappa_{dd} \chi (\kappa_{dd}))}{\partial \ln (\tau_{do})} \right]. \end{aligned}$$

B.2.9 Indirect effect of tariff changes ($d' \neq d$ case)

The second derivative is the indirect effect, which documents how changing tariffs between a third country d' and the origin o changes relative consumption in country d

$$\frac{\partial \ln (C_{do}/C_{dd})}{\partial \ln (\tau_{d'o})} = \varepsilon_o^{dd'}.$$

B.2.10 First and second terms of equation (73) ($d' \neq d$ case)

Following the general pattern used previously, we first derive the change in the price indexes in Arkolakis et al. as

$$\frac{\partial}{\partial \ln (\tau_{d'o})} \ln \left(\left(P_{do}^{ACR} \right)^{1-\sigma} \right) = \frac{\partial \ln (N_o^x)}{\partial \ln (\tau_{d'o})} + \psi_{do} \frac{\partial \ln (\bar{\varphi}_{do})}{\partial \ln (\tau_{d'o})},$$

and similarly

$$\frac{\partial}{\partial \ln (\tau_{d'o})} \ln \left(\left(P_{dd}^{ACR} \right)^{1-\sigma} \right) = \frac{\partial \ln (N_d^x)}{\partial \ln (\tau_{d'o})} + \psi_{dd} \frac{\partial \ln (\bar{\varphi}_{dd})}{\partial \ln (\tau_{d'o})}.$$

Combining these gives

$$\begin{aligned} \frac{\partial}{\partial \ln (\tau_{d'o})} \ln \left(\left(P_{do}^{ACR} \right)^{1-\sigma} \right) - \frac{\partial}{\partial \ln (\tau_{d'o})} \ln \left(\left(P_{dd}^{ACR} \right)^{1-\sigma} \right) &= \psi_{do} \frac{\partial \ln (\bar{\varphi}_{do})}{\partial \ln (\tau_{d'o})} - \psi_{dd} \frac{\partial \ln (\bar{\varphi}_{dd})}{\partial \ln (\tau_{d'o})} \\ &+ \frac{\partial \ln (N_o^x)}{\partial \ln (\tau_{d'o})} - \frac{\partial \ln (N_d^x)}{\partial \ln (\tau_{d'o})}. \end{aligned}$$

The elasticities of the cutoffs $\bar{\varphi}_{do}$ and $\bar{\varphi}_{dd}$ with respect to $\tau_{d'o}$ are also related because changing tariff $\tau_{d'o}$ changes the price index P, which changes the cutoff $\bar{\varphi}_{dd}$. We can derive this relationship by differentiating the explicit expression for the cutoff given in equation (18)

$$\frac{\partial \ln(\bar{\varphi}_{do})}{\partial \ln(\tau_{d'o})} = -\frac{\partial \ln(P)}{\partial \ln(\tau_{d'o})} + \left(\frac{1}{\sigma-1}\right)\frac{\partial \ln(F_{do})}{\partial \ln(\tau_{d'o})}$$

and symmetrically

$$\frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})} = -\frac{\partial \ln(P_d)}{\partial \ln(\tau_{d'o})} + \left(\frac{1}{\sigma-1}\right)\frac{\partial \ln(F_{dd})}{\partial \ln(\tau_{d'o})}.$$

So the relationship between the two cutoff elasticities is

$$\begin{aligned}\frac{\partial \ln(\bar{\varphi}_{do})}{\partial \ln(\tau_{d'o})} &= \frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})} \\ &+ \left(\frac{1}{\sigma-1}\right)\left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}\frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}\frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{d'o})}\right],\end{aligned}$$

and we use the chain rule to expand the derivatives with respect to the finding rate. Substituting the relationship between the cutoffs into the general expression for the ratio of relative prices and simplifying gives

$$\begin{aligned}\frac{\partial}{\partial \ln(\tau_{d'o})}\ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \frac{\partial}{\partial \ln(\tau_{d'o})}\ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) = (\psi_{do}-\psi_{dd})\frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})} + \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{d'o})} \\ + \left(\frac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}\frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}\frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{d'o})}\right].\end{aligned}$$

B.2.11 Elasticity of destination–origin market matched rate

We continue to follow the pattern used previously and calculate the elasticity of the destination–origin market matched producers' rate. Because we are studying a steady state, we use the definition $\dfrac{1-u_{do}-i_{do}}{1-i_{do}} = \dfrac{\kappa_{do}\chi(\kappa_{do})}{\lambda+\kappa_{do}\chi(\kappa_{do})}$ to derive

$$\frac{\partial}{\partial \ln(\tau_{d'o})}\ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) = \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{d'o})}\right),$$

in which we used the chain rule to write

$$\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{d'o})} = \frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \kappa_{do}\chi(\kappa_{do})}\frac{\partial \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{d'o})} = \frac{1}{\kappa_{do}\chi(\kappa_{do})}\frac{\partial \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{d'o})}.$$

For the record, the elasticity of the third term boils down to

$$\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) = \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{d'o})}\right).$$

The things that matter are the unmatched rate and the elasticity of the finding rate with respect to tariffs.

B.2.12 Elasticity of destination–destination market matched rate

The fourth term requires that we calculate

$$\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right).$$

The steps are identical to the ones we took in calculating the destination–origin market matched rate derivative with only the sub-indexes changing. The end result is

$$\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right) = \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{d'o})}\right),$$

in which we again used the chain rule to write

$$\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{d'o})} = \frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \kappa_{dd}\chi(\kappa_{dd})}\frac{\partial \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{d'o})} = \frac{1}{\kappa_{dd}\chi(\kappa_{dd})}\left(\frac{\partial \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{d'o})}\right).$$

B.2.13 General expression for $d' \neq d$ case

Here we try to write the most general possible expression only assuming that $\frac{\partial \ln(w_d)}{\partial \ln(\tau_{d'o})} = 0$ and $\frac{\partial \ln(w_o)}{\partial \ln(\tau_{d'o})} = 0$. The general term expression in Arkolakis et al. was

$$\begin{aligned}\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) &= (\psi_{do}-\psi_{dd})\frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})} + \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{d'o})} \\ &+ \left(\frac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}\frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}\frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{d'o})}\right].\end{aligned}$$

Combining these with the elasticity of unmatched rates gives

$$\begin{aligned}\frac{\partial \ln(C_{do}/C_{dd})}{\partial \ln(\tau_{d'o})} &= (\psi_{do}-\psi_{dd})\frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})} + \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{d'o})} \\ &+ \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{d'o})}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{d'o})}\right) \\ &+ \left(\frac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}\frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}\frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{d'o})}\right].\end{aligned}$$

B.2.14 Final general elasticity

The final expression is

$$\frac{\partial \ln \left(C_{do}/C_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} = \varepsilon_{o}^{dd'} = \begin{cases} \quad (1-\sigma) + \psi_{do} + (\psi_{do} - \psi_{dd}) \frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{do}\right)} + \frac{\partial \ln \left(N_{o}^{x}\right)}{\partial \ln \left(\tau_{do}\right)} - \frac{\partial \ln \left(N_{d}^{x}\right)}{\partial \ln \left(\tau_{do}\right)} \\ + \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{do}\right)}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln \left(\tau_{do}\right)}\right) \\ + \left(\frac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln \left(F_{do}\right)}{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)} \frac{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}{\partial \ln \left(\tau_{do}\right)} - \frac{\partial \ln \left(F_{dd}\right)}{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)} \frac{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}{\partial \ln \left(\tau_{do}\right)}\right] & if\ d' = d \\ \\ \quad (\psi_{do} - \psi_{dd}) \frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} + \frac{\partial \ln \left(N_{o}^{x}\right)}{\partial \ln \left(\tau_{d'o}\right)} - \frac{\partial \ln \left(N_{d}^{x}\right)}{\partial \ln \left(\tau_{d'o}\right)} \\ + \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right) \\ + \left(\frac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln \left(F_{do}\right)}{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)} \frac{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}{\partial \ln \left(\tau_{d'o}\right)} - \frac{\partial \ln \left(F_{dd}\right)}{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)} \frac{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}{\partial \ln \left(\tau_{d'o}\right)}\right] & if\ d' \neq d \end{cases} . \quad (74)$$

B.2.15 The elasticity in proposition 6

Three additional assumptions are needed to derive proposition 6 in the main text from the general elasticity equation (74). The first of these is that the cost of remaining idle, $-h_{do}$, is the same as the flow search costs that producers pay to find retailers, l_{do}, so that $l_{do} = -h_{do}\ \forall do$. With this assumption, the effective entry costs become a function of exogenous parameters and $\partial \ln \left(F_{do}\right)/\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right) = 0\ \forall do$. The second assumption is that $\frac{\partial \ln(N_{o}^{x})}{\partial \ln(\tau_{do})} = \frac{\partial \ln\left(N_{d}^{x}\right)}{\partial \ln(\tau_{do})}$ and $\frac{\partial \ln(N_{o}^{x})}{\partial \ln(\tau_{d'o})} = \frac{\partial \ln\left(N_{d}^{x}\right)}{\partial \ln(\tau_{d'o})}$. One could rationalize equality between these elasticities by studying symmetric equilibria or ensure that the elasticities are zero by either assuming free entry into production or that N_{d}^{x} and N_{o}^{x} are exogenous. The third assumption is that productivity, $\varphi \in [1, +\infty)$, follows a Pareto distribution with CDF $G\left[\tilde{\varphi} < \varphi\right] = 1 - \varphi^{-\theta}$. Appendix B.2.17 shows that this assumption implies that the terms in the elasticity that depend on moments of the productivity distribution simplify to

$$(1-\sigma) + \psi_{do} + (\psi_{do} - \psi_{dd}) \frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{do}\right)} = -\theta \qquad \text{and} \qquad (\psi_{do} - \psi_{dd}) \frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} = 0$$

for the $d' = d$ and $d' \neq d$ cases of the consumption elasticity, respectively.

Applying these three assumptions to the general elasticity equation (74) gives

$$\frac{\partial \ln \left(C_{do}/C_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} = \begin{cases} -\theta + \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{do}\right)}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln \left(\tau_{do}\right)}\right) & if\ d' = d \\ \\ \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right) & if\ d' \neq d \end{cases} , \quad (75)$$

which is the consumption share elasticity equation in proposition 6 of the main text.

B.2.16 Consumption elasticity as retailer search costs approach zero

As the search costs that retailers pay to find producers approaches zero in all destination-origin markets, $c_{do} \to 0 \; \forall do$, the following three things happen: 1) the fraction of unmatched searching producers goes to zero, $u_{do} \to 0 \; \forall do$, 2) the effective entry costs become a function of exogenous parameters, $\partial \ln (F_{do}) / \partial \ln (\kappa_{do} \chi (\kappa_{do})) \to 0 \; \forall do$, and 3) the value of imports converges to the value of consumption, $IM_{do} \to C_{do} \; \forall do$. These three facts together imply that the consumption elasticity converges to

$$\frac{\partial \ln (C_{do}/C_{dd})}{\partial \ln (\tau_{d'o})} = \varepsilon_o^{ACRdd'} = \begin{cases} (1-\sigma) + \psi_{do} + (\psi_{do} - \psi_{dd}) \dfrac{\partial \ln (\bar{\varphi}_{dd})}{\partial \ln (\tau_{do})} + \dfrac{\partial \ln (N_o^x)}{\partial \ln (\tau_{do})} - \dfrac{\partial \ln \left(N_d^x\right)}{\partial \ln (\tau_{do})} \; if \; d' = d \\ \\ (\psi_{do} - \psi_{dd}) \dfrac{\partial \ln (\bar{\varphi}_{dd})}{\partial \ln (\tau_{d'o})} + \dfrac{\partial \ln (N_o^x)}{\partial \ln (\tau_{d'o})} - \dfrac{\partial \ln \left(N_d^x\right)}{\partial \ln (\tau_{d'o})} \; if \; d' \neq d \end{cases} . \tag{76}$$

First, we highlight that this is the elasticity of imports with respect to variable trade costs that would result in a model that has the same structure but no search frictions. Second, if we are willing to assume that $\frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{do})} = \frac{\partial \ln\left(N_d^x\right)}{\partial \ln(\tau_{do})}$ and $\frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{d'o})} = \frac{\partial \ln\left(N_d^x\right)}{\partial \ln(\tau_{d'o})}$, then equation (76) becomes

$$\frac{\partial \ln (C_{do}/C_{dd})}{\partial \ln (\tau_{d'o})} = \frac{\partial \ln (IM_{do}/IM_{dd})}{\partial \ln (\tau_{d'o})} = \varepsilon_o^{ACRdd'} = \begin{cases} (1-\sigma) + \psi_{do} + (\psi_{do} - \psi_{dd}) \dfrac{\partial \ln (\bar{\varphi}_{dd})}{\partial \ln (\tau_{do})} & if \; d' = d \\ \\ (\psi_{do} - \psi_{dd}) \dfrac{\partial \ln (\bar{\varphi}_{dd})}{\partial \ln (\tau_{d'o})} & if \; d' \neq d \end{cases} , \tag{77}$$

in which $\psi_{do} = \dfrac{\partial \ln (\Psi_{do})}{\partial \ln (\bar{\varphi}_{do})} \geq 0$ and $\Psi_{do} = \int_{\bar{\varphi}_{do}}^{\infty} \varphi^{\sigma-1} dG(\varphi)$. Equation (77) is exactly the trade elasticity in the Melitz (2003) model as derived in Arkolakis, Costinot, and Rodríguez-Clare (2012), equation (21) except that $\psi_{do} \leq 0$ while $\gamma_{ij} \geq 0$. This sign difference occurs because we define our model in terms of productivity, while they define theirs in terms of marginal cost.

Our baseline calibration assumes that productivity, φ, follows a Pareto distribution with CDF $G[\tilde{\varphi} < \varphi] = 1 - \varphi^{-\theta}$. This assumption simplifies terms in equation (77) that depend on moments of the productivity distribution as shown in B.2.17 and leads to

$$\frac{\partial \ln (C_{do}/C_{dd})}{\partial \ln (\tau_{d'o})} = \frac{\partial \ln (IM_{do}/IM_{dd})}{\partial \ln (\tau_{d'o})} = \varepsilon_o^{ACRdd'} = \begin{cases} -\theta & if \; d' = d \\ \\ 0 & if \; d' \neq d \end{cases} . \tag{78}$$

Consumption and trade elasticities are equivalent in these models because trade and consumption are both evaluated at final sales prices. Equation (78), equation (27) in the main text, is the consumption and trade elasticity if $c_{do} \to 0 \; \forall do$ and productivity is Pareto distributed. This elasticity is identical to the Melitz (2003) model with the same productivity distribution. We compare the effects of search frictions on the consumption and trade elasticities from propositions 6 and 7 to standard trade models without search frictions in section 5.2 using our baseline calibration and equation (78).

B.2.17 Consumption elasticity with Pareto distributed productivity

The elasticity of the moment of the productivity distribution defined by $\psi_{do} = \frac{d \ln(\Psi_{do})}{d \ln(\bar{\varphi}_{do})}$ takes a particularly simple form if productivity $\varphi \in [1, +\infty)$ is Pareto distributed with CDF $G[\tilde{\varphi} < \varphi] = 1 - \varphi^{-\theta}$ and PDF $g(\varphi) = \theta \varphi^{-\theta-1}$. As usual, assume that $\theta > \sigma - 1$ in order to close the model, which also ensures that $\psi_{do} < 0$. With this distribution, the moment $\Psi_{do} \equiv \int_{\bar{\varphi}_{do}}^{\infty} z^{\sigma-1} dG(z) = \frac{\theta \bar{\varphi}_{do}^{\sigma-\theta-1}}{\theta - \sigma + 1}$ and the elasticity $\psi_{do} = -\frac{\bar{\varphi}_{do}^{\sigma} \theta \bar{\varphi}_{do}^{-\theta-1}}{\Psi_{do}} = \sigma - 1 - \theta$. Importantly, this implies that $\psi_{do} = \psi_{dd}$. The $d' = d$ case of the consumption elasticity therefore simplifies to

$$(1-\sigma) + \psi_{do} + (\psi_{do} - \psi_{dd}) \frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{do})} = (1-\sigma) + (\sigma - 1 - \theta) = -\theta, \tag{79}$$

and the $d' \neq d$ case of the consumption elasticity simplifies to

$$(\psi_{do} - \psi_{dd}) \frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})} = 0. \tag{80}$$

B.3 Proof of proposition 7: Trade elasticity

B.3.1 Relating the consumption and trade elasticities

We derive the trade elasticity by relating it to the consumption elasticity. Imports evaluated at negotiated prices and total sales evaluated at final prices are related through the gravity equation (31) and equation (32) as

$$IM_{do} = (1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do})) C_{do},$$

in which

$$(1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do})) = \left(1 - \frac{\gamma_{do}}{\sigma\theta}\left(\theta - \frac{\delta_{do}}{F_{do}}(\theta - (\sigma - 1))\right)\right).$$

This equation is not a general relationship and depends on the assumptions we have made about preferences, bargaining, and the productivity distribution. Forming the import ratio in markets do and dd in our model therefore gives

$$\frac{IM_{do}}{IM_{dd}} = \frac{(1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}))\, C_{do}}{(1 - b(\sigma, \theta, \gamma_{dd}, \delta_{dd}, F_{dd}))\, C_{dd}}.$$

It is straightfoward to see that the trade elasticity is related to the consumption elasticity according to

$$\frac{\partial \ln (IM_{do}/IM_{dd})}{\partial \ln (\tau_{d'o})} = \frac{\partial \ln (C_{do}/C_{dd})}{\partial \ln (\tau_{d'o})} + \frac{\partial \ln (1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}))}{\partial \ln (\tau_{d'o})} - \frac{\partial \ln (1 - b(\sigma, \theta, \gamma_{dd}, \delta_{dd}, F_{dd}))}{\partial \ln (\tau_{d'o})}.$$

In particular, the baseline trade elasticity presented in 6 of the main text is simply

$$\frac{\partial \ln (IM_{do}/IM_{dd})}{\partial \ln (\tau_{d'o})} = \begin{cases} -\theta + \left(\frac{u_{do}}{1 - i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln (\tau_{do})}\right) - \left(\frac{u_{dd}}{1 - i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln (\tau_{do})}\right) & \\ + \frac{\partial \ln (1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}))}{\partial \ln (\tau_{d'o})} - \frac{\partial \ln (1 - b(\sigma, \theta, \gamma_{dd}, \delta_{dd}, F_{dd}))}{\partial \ln (\tau_{d'o})} & if\ d' = d \\ \left(\frac{u_{do}}{1 - i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln (\tau_{d'o})}\right) - \left(\frac{u_{dd}}{1 - i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln (\tau_{d'o})}\right) & \\ + \frac{\partial \ln (1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}))}{\partial \ln (\tau_{d'o})} - \frac{\partial \ln (1 - b(\sigma, \theta, \gamma_{dd}, \delta_{dd}, F_{dd}))}{\partial \ln (\tau_{d'o})} & if\ d' \neq d \end{cases}, \quad (81)$$

The trade elasticity differs from the standard trade elasticity because it is affected by the endogenous markup change between the negotiated and final sales prices in addition to the change in the mass of unmatched varieties that also affect the final consumption elasticity as discussed in proposition 6.

B.3.2 Markup response to tariff changes in our baseline

The sign of the elasticity of the markup between consumption and imports with respect to iceberg costs,

$$\frac{\partial \ln (1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}))}{\partial \ln (\tau_{do})}$$

and

$$\frac{\partial \ln (1 - b(\sigma, \theta, \gamma_{dd}, \delta_{dd}, F_{dd}))}{\partial \ln (\tau_{do})},$$

respectively, only depend on market tightness κ_{do} because tariffs do not directly affect the $b(\cdot)$ term. The relevant partial derivative in the first case is

$$\frac{\partial \ln (1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}))}{\partial \ln (\tau_{do})} = -\frac{\partial b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do})}{\partial \kappa_{do}} \frac{\kappa_{do}}{(1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}))} \frac{\partial \ln (\kappa_{do})}{\partial \ln (\tau_{do})}$$

We showed that $\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})} \leq 0$, which implies that $\frac{\partial \ln \kappa_{do}}{\partial \ln(\tau_{do})} \leq 0$ as well. The term $\frac{\kappa_{do}}{(1-b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}))} \geq 0$ and so it remains to consider the sign of $\frac{\partial b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do})}{\partial \kappa_{do}}$. Our baseline calibration has $l_{do} = -h_{do}$ so that $F_{do}(\kappa_{do})$ is not a function of κ_{do},

$$F_{do} = f_{do} + h_{do} + \frac{(r+\lambda)}{\beta} s_{do}.$$

This assumption simplifies the desired derivative to

$$\frac{\partial b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do})}{\partial \kappa_{do}} = \frac{\gamma_{do}}{\sigma\theta}\left(\frac{\theta-(\sigma-1)}{F_{do}}\right)\left[\frac{\partial}{\partial \kappa_{do}}\kappa_{do}\chi(\kappa_{do})\, s_{do}\right] + \frac{b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do})}{\gamma_{do}}\frac{\partial}{\partial \kappa_{do}}\gamma_{do}$$

in which $\frac{\partial}{\partial \kappa_{do}}\kappa_{do}\chi(\kappa_{do}) \geq 0$ as mentioned above and $\frac{\partial \gamma_{do}}{\partial \kappa_{do}} \leq 0$ because $\gamma_{do} = \frac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\kappa_{do}\chi(\kappa_{do})}$ so that $\frac{\partial \gamma_{do}}{\partial \kappa_{do}} = -\frac{\gamma_{do}}{r+\lambda+\beta\kappa_{do}\chi(\kappa_{do})} \leq 0$ Even with our restriction $l_{do} = -h_{do}$, the sign is ambiguous. In particular, $\frac{\partial b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do})}{\partial \kappa_{do}}$ will be negative if $s_{do} = 0$ or if the first term is smaller than the second. Our baseline parameterization has that $\frac{\partial b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do})}{\partial \kappa_{do}} \leq 0$. This fact implies that

$$\frac{\partial \ln(1-b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}))}{\partial \ln(\tau_{do})} \leq 0$$

As tariffs increase, the aggregate markup term, $1-b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do})$, the difference between final sales prices and negotiated prices, declines. Similar logic applies for $\frac{\partial \ln(1-b(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd}))}{\partial \ln(\tau_{do})}$ because each term will have the same sign as before except that $\frac{\partial \ln(\kappa_{dd})}{\partial \ln(\tau_{do})} \geq 0$ so that

$$\begin{aligned}\frac{\partial \ln(1-b(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd}))}{\partial \ln(\tau_{do})} &= -\frac{\partial b(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd})}{\partial \kappa_{do}}\frac{\kappa_{dd}}{(1-b(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd}))}\frac{\partial \ln(\kappa_{dd})}{\partial \ln(\tau_{do})}\\ &\geq 0\end{aligned}$$

B.4 The gravity equation with search frictions

B.4.1 Proof of proposition 8: Deriving the gravity equation

The value of total imports will be

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} n_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi).$$

We need to integrate over the varieties to get the total value of imports going into the domestic market. Demand for a variety, φ, in the differentiated goods sector is given in equation (2): $q_{do}(\varphi) = p_{do}(\varphi)^{-\sigma} \frac{\alpha C_d}{P_d^{1-\sigma}}$. Given this demand, monopolistic competition, and constant returns-to-scale production imply that producers set optimal prices according to equation (16): $p_{do}(\varphi) = \mu w_o \tau_{do} \varphi^{-1}$. For notational simplicity, define $B_{do} \equiv \alpha (\mu w_o \tau_{do})^{-\sigma} C_d P_d^{\sigma-1}$ and combine the optimal price with the demand curve to get $q_{do}(\varphi) = B_{do}\varphi^{\sigma}$. Evaluated at final prices, the value of sales of each variety is $p_{do}(\varphi)\, q_{do}(\varphi) = \mu w_o \tau_{do} B_{do} \varphi^{\sigma-1}$ and the cost to produce $q_{do}(\varphi)$ units of this variety is $t_{do}(\varphi) + f_{do} = w_o \tau_{do} B_{do} \varphi^{\sigma-1} + f_{do}$. These expressions imply that total profits generated by each variety are $p_{do}(\varphi)\, q_{do}(\varphi) - t_{do}(\varphi) = A_{do}\varphi^{\sigma-1}$, in which it is also useful to define $A_{do} = w_o \tau_{do} B_{do} [\mu - 1]$. Using this profits expression, the productivity cutoff is $\bar{\varphi}_{do} = \left(\frac{F_{do}}{A_{do}}\right)^{\frac{1}{\sigma-1}}$, in which F_{do} is given in equation (18). The value of total imports from the negotiated price curve in equation (14) is

$$n(\varphi)\, q_{do}(\varphi) = [1 - \gamma_{do}]\, p_{do}(\varphi)\, q_{do}(\varphi) + \gamma_{do} [t_{do}(\varphi) + f_{do} - l_{do} - \kappa_{do}\chi(\kappa_{do})\, s_{do}].$$

Using the functional forms assumptions from above this becomes

$$n(\varphi)\, q_{do}(\varphi) = (\sigma - \gamma_{do})\, A_{do}\varphi^{\sigma-1} - \gamma_{do} [-f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})\, s_{do}].$$

Substituting the value of imports for a particular variety into the integral defining the value of total imports gives

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} (\sigma - \gamma_{do})\, A_{do}\varphi^{\sigma-1} - \gamma_{do} [-f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})\, s_{do}]\, dG(\varphi).$$

We assume productivity, φ, has a Pareto distribution over $[1, +\infty)$ with cumulative density function $G[\tilde{\varphi} < \varphi] = 1 - \varphi^{-\theta}$ and probability density function $g(\varphi) = \theta\varphi^{-\theta-1}$. The Pareto parameter and the elasticity of substitution are such that $\theta > \sigma - 1$, which ensures that the integral $\int_{\bar{\varphi}}^{\infty} z^{\sigma-1} dG(z)$ is bounded. Using these assumptions we can compute the integral to get

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \left[\left(\sigma - \gamma_{do}\right) A_{do} \frac{\theta \bar{\varphi}_{do}^{\sigma-\theta-1}}{\theta - \sigma + 1} - \gamma_{do} \left[-f_{do} + l_{do} + \kappa_{do}\chi\left(\kappa_{do}\right) s_{do}\right] \bar{\varphi}_{do}^{-\theta}\right],$$

in which we use the relevant moment of the productivity distribution $\int_{\bar{\varphi}_{do}}^{\infty} z^{\sigma-1} dG\left(z\right) = \frac{\theta \bar{\varphi}_{do}^{\sigma-\theta-1}}{\theta - \sigma + 1}$. Define $\delta_{do} \equiv f_{do} - l_{do} - \kappa_{do}\chi\left(\kappa_{do}\right) s_{do}$ to conserve on notation, substitute the export productivity threshold into this expression, and simplify to get

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \left[\left(\sigma - \gamma_{do}\right) \frac{\theta}{\theta - \sigma + 1} + \gamma_{do} \frac{\delta_{do}}{F_{do}}\right] F_{do}^{-\left(\frac{\theta}{\sigma-1} - 1\right)} A_{do}^{\frac{\theta}{\sigma-1}}.$$

Next, utilize the assumption that the number of producers in the origin market is proportional to output in that market $N_o^x = \left(\frac{C}{1+\pi}\right) \frac{C_o}{C}$ and the definition for $A_{do} = \mu^{-\sigma}\alpha \left(w_o \tau_{do}\right)^{1-\sigma} C_d P_d^{\sigma-1} \left[\mu - 1\right]$ to write

$$\begin{aligned} IM_{do} &= \left(1 - \frac{u_{do}}{1 - i_{do}}\right) \left(\frac{C}{1+\pi}\right) \frac{C_o}{C} \left[\left(\sigma - \gamma_{do}\right) \frac{\theta}{\theta - \sigma + 1} + \gamma_{do} \frac{\delta_{do}}{F_{do}}\right] \\ &\times F_{do}^{-\left(\frac{\theta}{\sigma-1} - 1\right)} \left(\mu^{-\sigma}\alpha \left(w_o \tau_{do}\right)^{1-\sigma} C_d P_d^{\sigma-1} \left[\mu - 1\right]\right)^{\frac{\theta}{\sigma-1}}. \end{aligned}$$

We presented the price index earlier as

$$P_d = \lambda_2 \times C_d^{\frac{1}{\theta} - \frac{1}{\sigma-1}} \times \rho_d.$$

Substituting that into our value of imports and simplifying gives

$$\begin{aligned} IM_{do} &= \left(1 - \frac{u_{do}}{1 - i_{do}}\right) \left[\left(\sigma - \gamma_{do}\right) \frac{\theta}{\theta - \sigma + 1} + \gamma_{do} \frac{\delta_{do}}{F_{do}}\right] \\ &\times \left(\mu^{-\sigma}\alpha \left[\mu - 1\right]\right)^{\frac{\theta}{\sigma-1}} \left(\frac{C}{1+\pi}\right) \lambda_2^{\theta} \left(\frac{C_o C_d}{C}\right) \left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1} - 1\right)}. \end{aligned}$$

In the price index section earlier we also define λ_2, which we can now substitute in here and then simplify to get

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) \left(1 - \frac{\gamma_{do}}{\sigma\theta} \left(\theta - \frac{\delta_{do}}{F_{do}} \left(\theta - \left(\sigma - 1\right)\right)\right)\right) \alpha \left(\frac{C_o C_d}{C}\right) \left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1} - 1\right)}. \qquad (82)$$

Define the bundle of search parameters

$$b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right) = \frac{\gamma_{do}}{\sigma\theta} \left(\theta - \frac{\delta_{do}}{F_{do}} \left(\theta - \left(\sigma - 1\right)\right)\right)$$

and substitute it into (82) in order to write the gravity equation more compactly as

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right)\left(1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)\right)\alpha\left(\frac{C_o C_d}{C}\right)\left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}.$$

which is equation (31) in proposition 8.

B.4.2 Search frictions reduce imports

Search costs reduce imports through the unmatched rate and difference between final and negotiated prices. In order to show this result, we show that the matched rate and the aggregate markup terms are weakly in the unit interval.

First, it is easy to see that

$$\left(1 - \frac{u_{do}}{1 - i_{do}}\right) = \left(\frac{\kappa_{do}\chi\left(\kappa_{do}\right)}{\lambda + \kappa_{do}\chi\left(\kappa_{do}\right)}\right) \in [0, 1],$$

because the finding and destruction rates must be finite in any model with positive search costs, $c_{do} > 0 \forall do$.

Second, proving the bundle of search parameters

$$1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right) \in [0, 1].$$

takes a few steps. Begin by proving that $\left(1 - \frac{\gamma_{do}}{\sigma\theta}\left(\theta - \frac{\delta_{do}}{F_{do}}\left(\theta - (\sigma - 1)\right)\right)\right) \leq 1$. We can prove this by noting that $\delta_{do} \equiv f_{do} - l_{do} - \kappa_{do}\chi\left(\kappa_{do}\right) s_{do}$ so it must be that $\delta_{do} \leq F_{do}$ and therefore $\delta_{do} F_{do}^{-1} \leq 1$. Also, the restriction that $\sigma > 1$, ensures $\theta - (\sigma - 1) < \theta$. Together, these ensure $\theta - \frac{\delta_{do}}{F_{do}}\left(\theta - (\sigma - 1)\right) \geq 0$. Combining this with the fact that $\gamma_{do} \in [0, 1]$ ensures that $1 - \frac{\gamma_{do}}{\sigma\theta}\left(\theta - \frac{\delta_{do}}{F_{do}}\left(\theta - (\sigma - 1)\right)\right) \leq 1$.

Next, show that $\left(1 - \frac{\gamma_{do}}{\sigma\theta}\left(\theta - \frac{\delta_{do}}{F_{do}}\left(\theta - (\sigma - 1)\right)\right)\right) \geq 0$ by showing that $1 \geq \frac{\gamma_{do}}{\sigma\theta}\left(\theta - \frac{\delta_{do}}{F_{do}}\left(\theta - (\sigma - 1)\right)\right)$. Because $\gamma_{do} \in [0, 1]$ and $\sigma > 1$ we know that $\frac{\gamma_{do}}{\sigma} < 1$. Likewise, $\sigma > 1$ ensures $\theta - (\sigma - 1) < \theta$ so that $\frac{\left(\theta - (\sigma - 1)\right)}{\theta} < 1$. We assume above that $\theta - (\sigma - 1) > 0$ in order to close the model. Together these imply that $\frac{\left(\theta - (\sigma - 1)\right)}{\theta} \in [0, 1]$. Finally, because $\delta_{do} F_{do}^{-1} \leq 1$ we have that $\frac{\delta_{do}}{F_{do}} \frac{\left(\theta - (\sigma - 1)\right)}{\theta} \leq 1$ and we have proved the result.

B.4.3 Consumption is imports evaluated at final sales prices

We could have also evaluated the quantity of goods imported at final sales prices $p_{do}(\varphi)$ instead of negotiated prices $n_{do}(\varphi)$. From equation (14), we can see $p_{do}(\varphi) = n_{do}(\varphi)$ if $\gamma_{do} = 0$. Setting $\gamma_{do} = 0$ in equation (82) then gives

$$C_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) \alpha \left(\frac{C_o C_d}{C}\right) \left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}.$$

We can obtain the same result by integrating

$$C_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} p_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi).$$

B.5 Implications for gravity equation estimation

The fact that introducing search frictions into a model of trade results in a scalar times the typical gravity equation has a few interesting implications for estimation.

First, if the fraction of matched exporters and the bundle of search friction parameters do not vary by destination–origin pairs, then their effect on trade would be lost in the constant term of a gravity regression. In this case, while estimates of the other coefficients in the model would be unbiased, search frictions could be a pervasive feature of international trade but would not be identifiable using the gravity equation.

Second, if the fraction of unmatched producers and the bundle of search frictions vary by importer–exporter pair, they may provide an additional rationale for why language, currency, common legal origin, historical colonial ties or other variables often included in gravity equations have an effect on aggregate trade flows. In particular, Rauch and Trindade (2002) argue that populations of ethnic Chinese within a country facilitate the flow of information, provide matching and referral services, and otherwise reduce informal barriers to trade. Their empirical specification matches the gravity equation with search that we have derived here if the destination–origin search frictions are a function of the ethnically Chinese population.

Third, any gravity regression that does not include adequate proxies for search frictions would suffer from omitted variable bias. In particular, suppose that a researcher omits search frictions, as measured by the matched rate, and estimates the following equation:

$$\ln IM_{do} = A_d + B_o + \beta_0 \ln \tau_{do} + \nu_{do}, \tag{83}$$

in which I_{do} are the imports from origin o to destination d, A_d is an importer-specific term, B_o is an exporter-specific term, β_0 is the partial elasticity of bilateral imports with respect to variable trade costs, and ν_{do} is an error term. Econometric theory suggests that the omitted variable, $Z_{do} = 1 - \dfrac{u_{do}}{1 - i_{do}}$, will introduce bias into the ordinary least squares estimate of $\hat{\beta}_0$, according to the well-known formula:

$$\mathbb{E}[\hat{\beta}_0|\mathbf{X}] = \beta_0 + \rho(\tau_{do}, Z_{do})\rho(I_{do}, Z_{do}), \tag{84}$$

in which $\mathbf{X}$ is a vector of all right-hand-side variables and $\rho\,(X, Y)$ is the correlation between X and Y. We know that $\rho(\tau_{do}, Z_{do}) < 0$ (higher variable trade costs, τ_{do}, raise the threshold productivity, $\bar{\varphi}_{do}$, increasing the fraction of idle firms, i_{do}, and lowering the matched rate) and $\rho(I_{do}, Z_{do}) > 0$ (increasing the matched rate increases trade flows) so the sign of the bias is negative.

Proposition 9. *Omitting the matched rate from a standard gravity equation implies that the estimate of trade elasticity with respect to variable trade costs is more negative than if one included the matched rate in the estimating equation.*

Proof. This result follows from the discussion in the text. □

B.6 Deriving aggregate welfare

Here we outline the steps to show that the indirect utility function (welfare) is C_d/P_d, in which C_d is total consumption expenditure, n is the vector of prices for each good, and P_d is the ideal price index. Assume that preferences are homothetic, which is defined in Mas-Colell, Whinston, and Green (1995), section 3.B.6, page 45. This means that they can be represented by a utility function that is homogeneous of degree one in quantities and that the corresponding indirect utility function is linear in total consumption expenditure. We can begin with the indirect utility function and then manipulate it as follows

$$\begin{aligned}
W_d\,(p, C_d) &= W_d\,(p, 1)\,C_d \\
W_d\,(p, e\,(p, u)) &= W_d\,(p, 1)\,e\,(p, u) \\
u &= W_d\,(p, 1)\,e\,(p, u) \\
1 &= W_d\,(p, 1)\,e\,(p, 1) \\
\frac{1}{e\,(p, 1)} &= W_d\,(p, 1)\,,
\end{aligned}$$

in which the first line comes from homothetic preferences; the second line follows by plugging in for consumption expenditure $C_d = e\,(p, u)$; the third line comes from equation (3.E.1) in MWG that says $W_d\,(p, e\,(p, u)) = u$ (also known as duality); and in the fourth line we plug in for utility level $u = 1$. The function $e\,(p, u)$ is the consumption expenditure function that solves the expenditure minimization problem. Using this result and the fact that the price index is defined as $e(p,1) \equiv P_d$ we can show that

$$W_d(p, C_d) = W_d(p,1)\,C_d = \frac{1}{e(p,1)} C_d = \frac{C_d}{P_d}.$$

Hence, as long as preferences are homothetic, we will always get welfare equal to consumption expenditure divided by the price index, $W_d(p, Y) = C_d/P_d$. The expenditure approach to accounting can be particularly useful for computing aggregate welfare in this setting because, $W_d(p, C_d) = \dfrac{C_d}{P_d} = \dfrac{Y - I_d}{P_d}$.

C Calibration appendix

C.1 Identifying importing retailers' search costs

In this section we describe how data on the fraction of exporters in uc and cu markets can be used to help inform the importing retailers' search cost parameters, c_{uc} and c_{cu}. In the model, the fraction of U.S. firms that export to Colombia is equal to the fraction of U.S. firms that export (anywhere) multiplied by the fraction of these firms that export to Colombia:

$$\frac{\left(1-\frac{u_{lu}}{1-i_{lu}}\right)N_u^x}{\left(1-\frac{u_{uu}}{1-i_{uu}}\right)N_u^x} \times \frac{\left(1-\frac{u_{cu}}{1-i_{cu}}\right)N_u^x}{\left(1-\frac{u_{lu}}{1-i_{lu}}\right)N_u^x} = \frac{\left(1-\frac{u_{cu}}{1-i_{cu}}\right)}{\left(1-\frac{u_{uu}}{1-i_{uu}}\right)},$$

in which we have used l to denote the U.S. destination market with the lowest threshold productivity. We also know that

$$\left(1-\frac{u_{cu}}{1-i_{cu}}\right) = \frac{\kappa_{cu}\chi\left(\kappa_{cu}\right)}{\lambda+\kappa_{cu}\chi\left(\kappa_{cu}\right)},$$

which is monotonically increasing in κ_{cu} because the producers' finding rate, $\kappa_{cu}\chi\left(\kappa_{cu}\right)$, is increasing in market tightness. From the free-entry condition (equation 19) we know that c_{cu} is important in determining equilibrium κ_{cu}. In particular, as the retailers' search cost rises, there is less entry into retailing and κ_{cu} falls. Therefore, we can use observed data on the fraction of U.S. firms that export to Colombia to pin down the c_{cu} parameter. Similarly, we can use the fraction of Colombian firms that export to the United States to pin down the c_{uc} parameter.

C.2 Identifying domestic retailers' search costs

In this section we describe how data on manufacturing capacity utilization can help identify the domestic retailers' search cost parameters, c_{uu} and c_{cc}. The U.S. Census Bureau surveys plants at a quarterly frequency to determine plant capacity utilization (Bureau of the Census, 2018). This utilization rate is largely determined from two questions. The first asks respondents to report the "market value of actual production for the quarter." The second asks respondents to report their "full production capability" for the quarter under assumptions of normal downtimes, fully available inputs, and with currently available machinery and equipment. Plant capacity utilization is computed as the ratio of these two valuations.

In our model the analogous capacity utilization concept for producers in country o is the value of all sales divided by the value of sales if there were no search frictions:

$$\frac{\sum_k IM_{ko}}{\sum_k IM_{ko}/\left(1-\frac{u_{ko}}{1-i_{ko}}\right)}.$$

In the main text we restrict our attention to capacity utilization in the domestic market only to make the exercise transparent. For the United States, domestic capacity utilization is defined as:

$$\frac{IM_{uu}}{IM_{uu}/\left(1-\frac{u_{uu}}{1-i_{uu}}\right)}=1-\frac{u_{uu}}{1-i_{uu}}.$$

As before, this quantity is monotonically increasing in κ_{uu}, which is negatively related to c_{uu}. Hence, we can use observed data on manufacturing capacity utilization to identify the domestic retailers' search costs, c_{uu}.