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We study long-run correlations between safe real interest rates in the United States and over 20 variables that have been hypothesized to influence real rates. The list of variables is motivated by the familiar intertermporal IS equation, by models of aggregate savings and investment, and by reduced form studies. We use annual data, mostly from 1890 to 2016. We find that safe real interest rates are correlated as expected with demographic measures. For example, the long-run correlation with labor force hours growth is positive, which is consistent with overlapping generations models. For another example, the long-run correlation with the proportion of 40- to 64-year-olds in the population is negative. This is consistent with standard theory where middle-aged workers are high-savers who drive down real interest rates. In contrast to standard theory, we do not find productivity to be positively correlated with real rates. Most other variables have a mixed relationship with the real rate, with long-run correlations that are statistically or economically large in some samples and by some measures but not in others.

Keywords: demographics, low frequency correlation, lowpass filtered correlation, natural rate of interest, productivity, steady state interest rate.

JEL codes: C32, E21, E22, E43.

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1. INTRODUCTION

It is well known that the safe real rate in the U.S. has declined over the last several decades. This decline poses difficulties for monetary policy because of the effective lower bound (Yellen (2016)), and may also signal secular changes in growth prospects (Summers (2016)). Hence it is vital to understand the reasons for this secular decline. In this paper we present some reduced form evidence on the decline, via estimation of long run correlations between the safe real interest rate in the U.S. and some variables that have been posited to move with safe rates.

There are broadly three approaches to thinking about long term movements in safe real interest rates. The approaches are not inconsistent, and can and do coexist in a given model. But they differ in which factors receive pride of place. One approach looks to secular movements in growth. Downward trends in real rates are tied to downward trends in growth (e.g., Laubach and Williams (2003), Yi and Zhang (2017)). Formally, the intertemporal IS equation, familiar from both asset pricing (e.g., Nason and Smith (2008)) and New Keynesian models (e.g., Galí (2011)), can be used to motivate this connection. A second approach looks to aggregate desired savings and investment. Outward shifts in the supply of savings or inward shifts in investment demand will result in lower real rates. Factors that have been posited to be important shifters include labor force (Baker et al. (2005), age distributions (e.g., Lisack et al. (2017)), the price of investment goods relative to consumption goods (e.g., Sajedi and Thwaites (2016)), flight to quality (e.g., del Negro et al. (2017)) and government saving or dissaving (e.g., Ball and Mankiw (1995)). Rachel and Smith (2015) provide a recent application of the aggregate supply and demand approach. Finally, some reduced form work has looked at some additional factors associated with the Mundell (1963)-Tobin (1965) effect. For example, Rapach and Wohar (2005) argue that regimes with higher inflation tend to have lower real rates.

We, too, take a reduced form approach. Using the literature outlined in the previous paragraph, we construct a list of over 20 variables hypothesized to be correlated with safe real rates. We call these variables “correlates.” We apply frequency and time domain techniques to estimate long run correlations between the safe real rate and each correlate. We also use our correlates to construct conditional and unconditional long horizon forecasts of the trend safe real rate. To estimate these long run correlations and long horizon forecasts, we collected long time samples of our correlates, most of which span 1890-2016. Because of the influence of the world wars, we also present results for a 1950-2016 subsample.

The estimates of long run correlations yield four notable results. First, growth in aggregate labor hours co-moves with the real rate as predicted by growth models or models of aggregate saving and investment. Second, demographic variables generally co-move with the real rate as predicted by overlapping generations models or models of aggregate saving and investment. For example, there is a negative long run correlation between the safe rate and the fraction of the population aged 40-64 and a positive long run correlation between the safe rate and the dependency ratio (percentage less than 20 or older than 65). These two results are consistent with much recent literature that points to working age population growth and age distributions as major factors in our current run of low safe real rates (Gagnon et al. (2016), Kara and von Thadden (2016)).

Third, we find a negative rather than a positive long run correlation between the safe rate and TFP growth and thus, presumably, between the safe rate and trend growth. To our knowledge this is a new result. It is inconsistent with earlier work that has emphasized on trend growth as a positive correlate of real rates (Laubach and Williams (2003, 2016), Yi and Zhang (2017)). It is also inconsistent with standard economic theory (Baker et al. (2005)). We do not attempt to explain this striking and puzzling result.

Fourth, other variables suggested by the three approaches listed above deliver a mixed picture. Examples include GDP growth, the current account and interest rate spreads (correlated with the real rate as expected in the 1950-2016 subsample but not in the 1890-2016 sample as a whole) and inflation and money growth (correlated with the real rate as expected in the 1890-2016 sample but not the 1950-2016 subsample).

Much current conventional wisdom views trend GDP growth as the primary driver of the secular trend in safe real rates (Laubach and Williams (2003, 2006), Fischer (2016, 2017)). However, the results in this paper and in research such as Leduc and Rudebusch (2014) and Hamilton et al. (2016) suggest that GDP growth and real rates do not show a reliably positive low frequency correlation. Now, GDP growth is driven by both productivity and labor hours growth. We find negative low frequency correlations between productivity growth and real rates, but positive low frequency correlations between labor growth and real rates. That is, labor growth shows a low frequency correlation with real rates that is reliably of the sign predicted by the economic models cited above. Hence, if forced to rely on a growth variable, labor hours growth seems preferable to GDP growth or TFP growth as a low frequency correlate of real rates.

Beyond the previous paragraph's decomposition of GDP growth into productivity growth and labor growth, we do not attempt to tease out reasons for mixed results noted in our fourth result. We do conclude that even with over a century of data, it is difficult to eliminate all but one or two variables as especially important correlates of safe real rates. This may be because of limited data span, or regime change, or of course it may be because in fact there are many important correlates.

We also use our correlates for forecasting. We execute a conditional forecasting experiment by imposing external forecasts for certain correlates for 2017-2026 (e.g., the Social Security forecast for the fraction of the population aged 40-64). We ask how the forecast of the 2017-2026 average value of the safe rate would change as hypothesized forecasts of the correlate change. (10 year averages are the time domain measure used in our analysis of low frequency correlation.) Plausible changes in hypothesized future values of the correlates imply economically large changes in the average safe real rate. This illustrates that the economic magnitude of the low frequency link between these correlates and the safe real rate is large, and underscores that there is substantial uncertainty about the future value of the safe real rate. Second, we produce a set of 10 year ahead forecasts of the 2017-2026 average value of the safe rate. Our forecasts typically are about 0.5%-1.1%. This can be contrasted with the current value of -1.3% (10 year average of the safe rate, 2007-2016). Thus the rate is forecast to rise well above current levels, though to settle down at a value well below the conventional 2% level.

A semantic note: we use "low frequency," "long run" and "trend" interchangeably.

Section 2 outlines the models that motivate our list of correlates. Section 3 describes our empirical methods, section 4 our data, section 5 empirical results, section 6 forecasting results. Section 7 concludes. The working paper contains two Appendices with some results omitted from the paper to save space.

2. UNDERLYING MODELS

Here we outline models that motivate the variables that we examine in our reduced form approach. We refer to these variables as potential "correlates" of the safe real interest rate r_t. The models that we are about to describe generally determine these variables jointly with r_t, and we do not attempt to rationalize causality from correlates to r_t. As well, some of these models relate r_t to the correlates not just at low frequencies but at each instant. But our interest in secular movements in r_t causes us to focus on low frequencies.

We draw on three approaches. The first relies on the familiar first order condition that relates consumption growth to the real interest rate–the intertemporal IS equation. The second involves an informal denumeration of factors affecting aggregate savings supply and aggregate investment demand. The third uses reduced form VARs. The three approaches are of course not inconsistent, and determinants suggested by one are often also suggested by another.

2.1 Intertemporal IS: Let $1+i_t$ = known gross return on nominally safe asset (issued in period t, payoff in period t+1), P_t = price level, P_{t+1}/P_t = stochastic gross inflation, C_{t+1} = stochastic consumption, β = discount factor, σ = coefficient of risk aversion / inverse of the intertemporal elasticity of substitution. The usual first order condition for purchase of a nominal one period bond is

$$(2.1) \qquad 1 = \beta E_t\left[\frac{1+i_t}{(P_{t+1}/P_t)}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \right]$$

A second order log linearization yields

$$(2.2) \qquad 0 \approx \ln(\beta) + \ln(1+i_t) - E_t\ln(P_{t+1}/P_t) - \sigma E_t\ln(C_{t+1}/C_t)$$
$$+ \frac{1}{2}\,[\mathrm{var}_t\ln(P_{t+1}/P_t) + \sigma^2\mathrm{var}_t\ln(C_{t+1}/C_t) + 2\sigma\mathrm{cov}_t[\ln(P_{t+1}/P_t), \ln(C_{t+1}/C_t)].$$

Let $\Delta c_{t+1} \equiv \ln(C_{t+1}/C_t)$ and $\pi_{t+1} \equiv \ln(P_{t+1}/P_t)$. Rearranging,

$$(2.3) \qquad \ln(1+i_t) - E_t\pi_{t+1} \approx -\ln(\beta) + \sigma E_t\Delta c_{t+1} - \frac{1}{2}\,[\sigma^2\mathrm{var}_t\Delta c_{t+1} + \mathrm{var}_t\pi_{t+1} + 2\sigma\mathrm{cov}_t(\pi_{t+1},\Delta c_{t+1})].$$

The left hand side variable–the real rate of interest on a nominally safe security–is our variable of interest. After approximating $\ln(1+i_t) \approx i_t$, our empirical counterpart to the left hand side of (2.3), which we call r_t, is constructed via

$$(2.4) \qquad r_t = i_t - E_t\pi_{t+1}.$$

We use rolling regressions used to construct $E_t\pi_{t+1}$ (details below).

The well known Laubach and Williams (2003, 2016) model for the natural rate of interest relies in large measure on (2.3). That model focuses on trend output growth as a determinant, with trend growth motivated by the $E_t\Delta c_{t+1}$ term. Trend output growth in turn suggests TFP growth as a determinant. Other research that has pointed to TFP growth as a long run determinant of real rates includes Yi and Zhang

(2017). The second order terms in (2.3) have received attention in Nason and Smith (2008).[1]

From this literature, we are motivated to consider the set of potential correlates listed in lines (1)-(3) in Table 1. "Aggregate growth" will be measured by per capita consumption growth (per (2.3)), per capita GDP growth, and TFP growth. The entries in the "expected sign" column come directly from (2.3): positive for aggregate growth, negative for second moments. Of course, (2.3) is an equilibrium relationship and the entries under "expected sign" unambiguously follow from (2.3) only if we hold all other variables constant. But here and throughout Table 1, we present the sign relevant if the variable is a dominant determinant of r and thus displays an unconditional correlation whose sign is consistent with the conditional correlation delivered when other variables are held constant.

2.2 Aggregate savings and investment Barro and Sala-i-Martin (1990) is an early example and Rachel and Smith (2015) is a recent example of research that considers trends in r when r is determined by the intersection of aggregate desired savings and aggregate desired investment. Factors that might shift the aggregate savings schedule include: demographics, such as the dependency ratio; inequality; government savings or dissavings; the emerging market savings glut; the spread between safe and risky rates. Factors that might shift the aggregate investment schedule include labor force growth and the falling relative price of capital goods.

Lines (4)-(10) in Table 1 list the correlates we consider. For brevity, we limit ourselves to one or two cites for each of our assertions.

•Baker et al. (2005) observe that in the steady state of certain overlapping generations models (and in the Solow model) interest rates are positively related to the rate of labor force growth.[2] Kara and von Thadden's (2016) numerical results illustrate that the positive relationship also obtains in Blanchard's (1985) and Gertler's (1999) multiperiod finite lived model. Those models typically have labor inelastically supplied. We use labor hours to allow for fluctuations in labor hours per individual.

•Define the dependency ratio as the percentage of the population younger than 20 or older than 64. An increase in the dependency ratio will shift the savings schedule in, thus raising r (Gagnon et al. (2016)); an increase of the fraction middle aged will work in the opposite direction. Geanakoplos et al. (2004) argue that changes in what they call the middle to young ratio will be positively correlated with r; we conclude from their study that the change in the fraction middle aged will also be positively correlated with r.

•Transitory decreases in government saving–i.e., increases in government purchases or decreases in taxes financed by borrowing–have been argued to push up real rates (Ball and Mankiw (1995)). If those transitory decreases happen every couple of decades, say because of wars, then there will be a low frequency link between government saving or dissaving and real rates. And even with lump sum taxes, in non-Ricardian models there can be a long run relation between government debt and real variables, with higher debt/GDP associated with higher interest rates (Gertler (1999)). Hence the "+" in line 6 (higher deficits and higher debt mean higher r).

•U.S. current account deficits have been argued in recent years to reflect an inflow of savings to the U.S. (Bernanke (2005)). If the deficit grows (becomes more negative), the real rate is also expected to move downwards. So there is a "+" in the "current account" line.

•A falling relative price of investment has ambiguous effects–a smaller expenditure on capital is needed to produce a given amount of output, but firms have an incentive to shift into capital. Eichengreen (2015), citing the IMF (2014), argues that the empirical evidence indicates that the sign is positive.

•Since higher income families have lower marginal propensities to consume (Dynan et al. (2004)), an increase in inequality will shift the saving schedule out, lowering *r*.

•Finally, an increase in spreads that results from a flight to quality will depress safe rates as savings are shifted from risky investment to Treasury debt (del Negro et al. (2017)).

<u>2.3 Mundell-Tobin effect</u> Here we add variables not directly suggested by the previous two literatures. In particular, some reduced form studies find that inflation regimes or inflation expectations regimes are correlated with *r* (Koedijk et al. (1994), Rapach and Wohar (2005)). So we add the rate of inflation and the rate of money growth to our list of potential correlates. Consistent with the Mundell (1963)-Tobin (1965) effect, the studies just cited find that higher inflation is associated with lower *r*. Hence we posit money growth and inflation will show a negative correlation with *r*.

<u>2.4 But what about DSGE models</u>? Our list of variables includes ones consistent with the logic of DSGE models that dominate monetary economics today. For example, a first order condition similar to (1) is ubiquitous in such models; even when (1) is generalized to allow features such as habit persistence there is still a link between trend growth and low frequency movement in *r* (see Hamilton et al. (2016)). Some such models include technology shocks that lead to a trend in the relative price of investment (e.g., Justiniano et al (2011)). On the other hand, such models typically do not have a life cycle component, nor, so far as we know, do they tie secular movements in real rates to movements in inflation or money. Hence our decision not to motivate our list from a DSGE model.

3. EMPIRICAL METHODS

Let x_t be one of our potential correlates of r_t. We do not perform tests for stationarity. We rely on the literature cited above to decide whether to difference a variable before relating it to r_t. For example, we use growth rates of TFP but levels of the relative price of investment goods. We generally use methods in which point estimates, though not confidence intervals and standard errors, are robust to the possible presence of unit roots.

For a given correlate, we measure the strength of the long run correlation with r_t via both frequency and time domain techniques. The frequency domain technique produces an estimate of the low frequency correlation between x_t and r_t. It also produces the R^2 of the low frequency band spectral regression of r_t on x_t, which we interpret as a measure of the strength of the correlation. The time domain technique simply averages x_t and r_t over long periods (with 10 years as our window) and computes a correlation using the averages as observations.

In the end, the two approaches yielded qualitatively similar results. Hence we will sometimes use "low frequency correlation" or "long run correlation" to encompass both types of correlations.

<u>3.1 Low frequency correlation</u> Assume (x_t, r_t) is stationary. Let S be the 2×2 long run variance of (x_t, r_t). Write

(3.1.1) $S = \begin{pmatrix} s_{xx} & s_{xr} \\ s_{xr} & s_{rr} \end{pmatrix}.$

Thus, s_{xx} is the long run variance of x_t and s_{rr} the long run variance of r_t. Müller and Watson (2017) propose the following measure of the low frequency relationship between x_t and r_t

(3.1.2) $\rho^{LP} \equiv s_{xr}/(s_{xx}s_{rr})^{½}$, $-1 \leq \rho_{xr} \leq 1$.

They call this the "long run correlation" between the two series.[3]

We rely on Müller and Watson (2017) to compute this from frequencies that are low but nonzero. We use the *LP* superscript to indicate that, in contrast to Müller and Watson (2017), we interpret our work as estimating not a correlation between series in which all but the zero frequency has been eliminated but between series in which a lowpass (LP) filter has been used to remove all but a set of low frequencies. That is, we interpret our statistic as an estimate of the correlation between two series put through a lowpass filter rather than as an estimate of the right hand side of (3.1.2). (Indeed, in some of our initial work we also computed correlations between series filtered using Christiano and Fitzgerald (2003) to remove all frequencies higher than 10 years. Those estimates were very close to the ones produced by Müller and Watson's (2017) procedure.) We fix the highest frequency used in the computation at 10 years rather than, as in Müller and Watson (2017), letting this frequency be lower for longer samples.

Confidence intervals are constructed as in Müller and Watson (2017) both under the assumption of stationarity ("ρ^{LP}-I(0)") and under an I(1) assumption ("ρ^{LP}-I(I)"). We caution the reader that Müller and Watson's (2017) results for both I(0) and I(1) data come from an asymptotic approximation in which the frequency cutoff gets lower as the sample size increases. As just noted, however, we kept the cutoff at 10 years in all samples.

We follow Müller and Watson (2017) to compute a band spectral regression of r_t on each correlate, and report the R^2. The frequencies used are the same as the ones used to estimate the correlation. We do not report the estimated slope coefficient, which of course has the same sign as that of the estimated correlation. We do not necessarily endorse R^2 as a measure of how much of r_t is explained by a given correlate. Rather, and recalling that in a bivariate regression such as ours R^2 is a monotonic function of the t-statistic on the correlate, R^2 supplements the confidence interval as an indicator of the statistical strength of the relationship.

<u>3.2 Correlations of 10 year moving averages</u>: Suppose we have annual data on x_t and r_t running from say 1890 to 2016. We compute 10 year moving averages 1890-1899, 1891-1900, ..., 2007-2016. We date these 1899, 1900, ..., 2016. Let x_t^{MA} and r_t^{MA} be the resulting series of 118 observations, 1899-2016, where "MA" is short for "moving average." We use these observations to estimate the correlation between x_t^{MA} and r_t^{MA},

(3.2.1) $\text{corr}(x_t^{MA}, r_t^{MA}) \equiv \rho^{MA}$.

In initial work, all our computations were repeated with non-overlapping 10 year samples, with very little change in point estimates. (That is, if we only use the 12 observations dated 1906, 1916, ..., 2016 to compute the correlation, the value is very close to that computed from all 118 observations.)

Confidence intervals are constructed from standard asymptotic theory, using Newey and West (1994).[4]

<u>3.3 Forecasts from horizon regressions</u> We use the long horizon regressions to forecast our time domain

measure low frequency of the real rate, and to quantify the magnitude of the low frequency link between a given correlate and the real rate. Define x_t^{MA} and r_t^{MA} as in (3.2.1). Consider the long horizon regression in which the 10 year average of interest rates and of a correlate are projected on data 10 years in the past:

(3.3.1a) $$r_{t+10}^{MA} = \alpha_r + \varphi_{11}r_t + \varphi_{12}r_{t-1} + \varphi_{13}x_t + \varphi_{14}x_{t-1} + u_{rt+10} \equiv Z_t'\beta_r + u_{rt+10},$$

(3.3.1b) $$x_{t+10}^{MA} = \alpha_x + \varphi_{21}r_t + \varphi_{22}r_{t-1} + \varphi_{23}x_t + \varphi_{24}x_{t-1} + u_{xt+10} \equiv Z_t'\beta_x + u_{xt+10}.$$

Here, Z_t consists of a constant, two lags of r_t and two lags of x_t.

We use (3.3.1) to make two types of forecasts. The first is the conventional "direct" forecast.[5] Let $\hat{\beta}_r$ and $\hat{\beta}_x$ be the least squares estimates of β_r and β_x. If T is the last observation in the sample (T=2016), we construct a forecast of r_{T+10}^{MA} in the usual way,

(3.3.2) $$\hat{r}_{2026}^{MA} = Z_{2016}'\hat{\beta}_r.$$

Each correlate produces a different forecast. We report the median along with the first and third quartiles of the forecasts. For example, we have 24 correlates in our 1890-2016 sample. We report the 6th (first quartile), median and 18th (third quartile) largest forecasts.

The second way we use (3.3.1) is to make a conditional forecast. For selected correlates, we make forecasts of the real rate conditional on a hypothesized future values of a correlate. We select the correlates partly on the basis of their performance in this paper and related literature, and partly on availability of external forecasts of the correlate 2017-2026. For example, we use the Social Security Administration's forecast for the percentage of the population that will be 40-64, 2017-2026, to forecast r_{2026}^{MA} conditional on this value. We do so following literature such as Clark and McCracken (2015). Let

(3.3.3) $$\hat{x}_{2026}^{MA} = Z_{2016}'\hat{\beta}_x$$

be our own forecast of the future 10 year average value of the change in the fraction of the population aged 40-64. We define the error $u_{x,2026}$ as the difference between the Social Security forecast–taken as truth–and our own forecast. We then use the correlation between u_{rt} and u_{xt} to refine our forecast of r_{2026}^{MA}. Specifically, we use that correlation and $u_{x,2026}$ to forecast $u_{r,2026}$, and use the forecast of $u_{r,2026}$ as part of our forecast of r_{2026}^{MA}. Confidence intervals are constructed in the usual way, accounting both for uncertainty in the estimate of β_r and uncertainty due to u_{rt}.

The conditional forecasts are partly of interest on their own. But we use them mainly to illustrate the economic (as opposed to statistical) magnitude of the low frequency link between correlates and the real rate, and to underscore uncertainty about future values of the safe real rate.

4. DATA

We describe construction of real rates and briefly describe other sources of data. We use annual data throughout. We aim to use samples that start in 1890, though some samples start later because of limited data. We use long samples because of the limited information available about low-frequency patterns or trends. For example, our 10 year moving averages yield only 12 non-overlapping observations between 1890 to 2016. Hence, even though we use up to 127 years of data, which is long by typical time series standards, the number of observations of the trend is limited.

We note that using long samples comes with a cost since longer samples have greater possibility of breaks and regime shifts. In particular, as we discuss below, the world wars have a large influence on the behavior of real rates. But the world wars may not be informative about the movement in real rates in the post-World War II decades. Hence, we also estimate long run correlations on a 1950-2016 sample.

<u>4.1 Real rates</u> For a nominally safe rate we use call money rates for 1870-1917, the discount rate for 1918-19, 3 to 6 month Treasury notes and certificates 1920-1933 and the three month T-bill for 1934-2016. These were obtained from the NBER Macro History Database and FRED. In the early years, we use call rates rather than commercial paper because Homer and Sylla (2005) indicate that call loans became more liquid in the late 19th century and because the call rates had lower average rates (suggesting greater safety). In each case the year t value was the monthly average of January rates in year t+1. We chose January in t+1 rather than December in t because of pronounced seasonality in call money rates. For inflation we use the GNP/GDP deflator, from Romer (1989) for 1870-1929 and from the BEA (line 1 of NIPA table 1.1.4) for 1930-2016. We set expected inflation to zero through 1914. (See Barsky (1987).) For 1914-present, we compute expected inflation from an AR(1) using rolling samples of 20 years, setting the AR(1) coefficient to 0.999 if it is estimated to exceed 1. In some initial work we experimented with constructing expected inflation from an AR(1) for 1890-1913. Results were hardly changed.

Figure 4.1A plots the resulting real rate. One can see that r_t is quite trendy, broadly trending down until the mid 1940s, then trending up until around 1980, and then trending down again. There is a very large negative spike in 1917 and another, not quite as large, in 1946. One can see in the plot in Figure 4.1B that this reflect a sharp positive spikes in expected inflation. Actual inflation (not plotted) rose from 1% (1914) to 20% (1917) and was still in double digits (13%) in 1920; it rose from 1% (1940) to 12% (1946) and remained elevated (5%) in 1948. (We comment on implications for trend real rates below.)

Figure 4.1C repeats the Figure 4.1A plot of r_t along with both of our trend measures of r_t. Because the 10 year moving average series (labeled r-MA in the figure) is a backward average and the low pass filtered series (r-LP) is two sided, the two trend measures, the moving average series is shifted forward relative to the low pass filtered series. But, that point aside, here and throughout almost all our analysis, the two trend measures are very similar. The two slide downward together through the mid-1940's. They then move upward until the 1980s, with the Müller and Watson (2017) filtered series (labeled r-LP in the figure) peaking a little earlier than does the 10 year moving average (r-MA). Finally, both move downward from the mid-1980s to the present.

Table 2 has basic statistics on the real rate, for a sample starting in 1890 along with the familiar postwar period (1950-). The real rate is volatile, though it became less volatile in the postwar period.

In the 1950-2016 column, the mean of 0.97% is below the conventionally presumed value of 2%. This partly reflects our choice of nominal interest rate, the 3 month T-bill. Over the period of overlap with the Federal Funds rate (1954-2016), the 3 month T-bill rate was 0.44% below Fed funds rate on average. As well, the beginning part of our 1950-2016 sample perhaps reflects financial repression that lingered on after the 1951 Treasury-Fed accord, and the end of our sample of course includes the period in which rates well below 2% inspired research such as ours. A shift in the interest rate measure and a focus on the 1954-2007 period would yield a figure of about 2%.

Figure 4.1A. Real rate

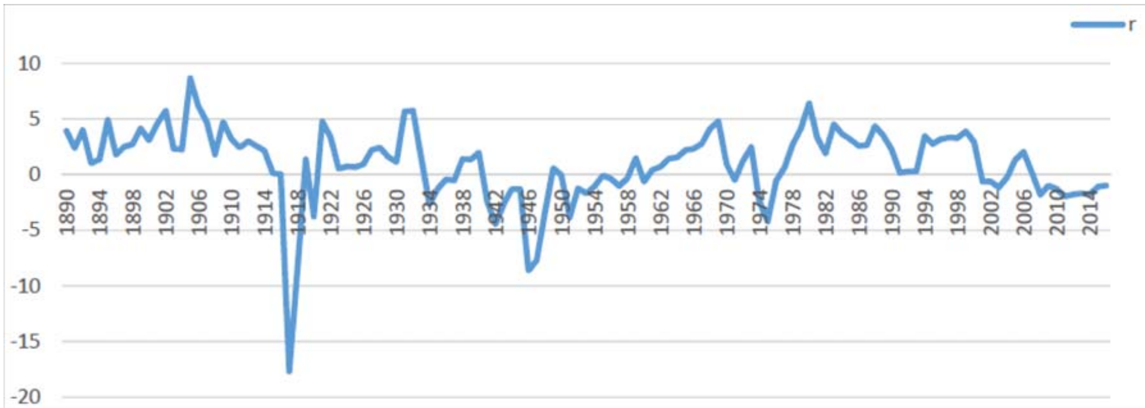


Figure 4.1B. Nominal rate i_t and expected inflation $E_t\pi_{t+1}$

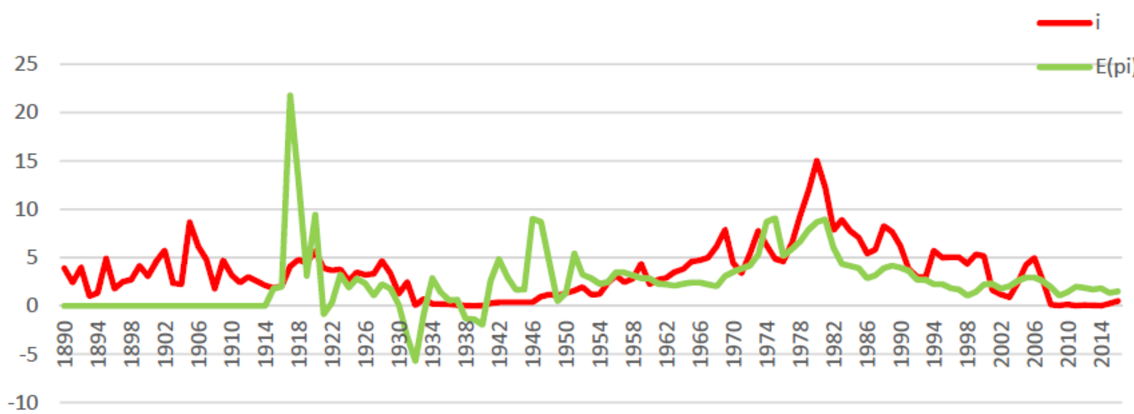
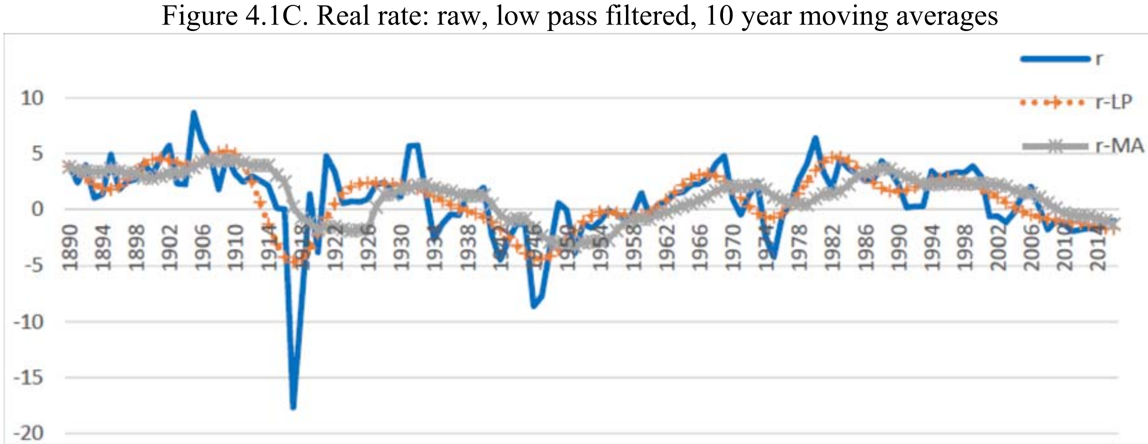


Figure 4.1C. Real rate: raw, low pass filtered, 10 year moving averages



A similar comment applies to the longer sample period in Table 2. Our choice of nominal interest rate (the call rate) was generally below a possible alternative, the commercial paper rate.

4.2 Correlates. For the most part, our correlates are constructed from U.S. data. When correlates were expressed per capita, the data source for population was Carter et al. (2006).

•Real GNP/GDP growth, per capita. (a)U.S.: Romer (1989) prior to 1929, BEA (line 1 of NIPA table 1.1.3) 1930-2016. Romer and the BEA were also the sources for nominal GDP used in the denominator of series described below that are expressed relative to GDP. (b)World: 23 countries, GDP measured at purchasing power parity rates.[6] Maddison Project 1890-2010, the IMF (GDP) and the UN (population) 2011-2015.

•Growth of real per capita consumption spending. (a)In the 1890- sample, we used total consumption spending: Kuznets (1961) prior to 1929, (line 2 of NIPA table 1.1.3) BEA 1930-2016. (b)In the 1950- sample, nondurables and services spending was available and hence was used (lines 5 and 6 of NIPA Table 1.1.5).

•Growth in total factor productivity. Gordon (2016) 1890-1948, FRED series MFPNFBS 1949-1987, the BLS 1988-2016.

•Conditional second moments in (2.2). Constructed from the variance-covariance matrix of the residuals of a bivariate VAR(1) in a measure of prices and a measure of aggregate growth. The VAR was estimated using 20 year rolling regressions.

•Growth in labor hours. We used hours in private business. Kendrick (1961) 1890-1947, BLS series PRS84006033 (average of quarterly figures) 1948-2016.

•Demographic measures. (a)US: 1890 and 1900-2000 from Carter et al. (2006); Haver Analytics for 2001-2016. 1891-1899 obtained by linear interpolation of the 1890 and 1900 data. (b)World, 1950-2015: The UN's *World Population Prospects*, using data for the several dozen countries defined as high income in 2015 (United Nations (2017a,p156)).

•Income inequality. Share of income that goes to the top 10% of the income distribution. From the World Wealth and Income Database, 1917-2015. As of this writing, 2016 data were not available.

•Relative price of investment. Relative to total consumption. Kuznets (1961) prior to 1929, BEA (lines 2 and 7 of NIPA table 1.1.4) 1930-2016.

•Government dissaving. (a)Federal government primary deficit relative to GDP. Carter et al. (2006) for 1869-1939; FRED series FYFSD for 1940-2016 (multiplied by -1 so that a positive value means deficit). (b)Federal debt/GDP. Carter et al. (2006) for 1869-1938; FRED series FYGFD for 1939-2016.

•Current account, expressed relative to GDP. Jordà et al. (2017) for 1870-1928; BEA (line 33 of NIPA table 4.1) for 1929-2016.

•Spread between public and private borrowing rates: BAA minus 10 year Treasuries. FRED series BAA

1920-2016, with Treasury rates from Homer and Sylla (2005) 1920-1953 and FRED series GS10 1954-2016.

•Money growth. M1 1916-1947 and M2 1869-1947 from Carter et al. (2006); both series 1948-1958 from Rasche (1987); both series 1959-2016 from FRED (series M1NS and M2NS), average of monthly figures.

To keep tables of manageable length, we put in the working paper Appendix results with correlates that in our view serve mainly to establish the robustness of the results presented here. Those correlates are: labor productivity growth (confirms results with TFP); the change of the U.S. and world dependency ratios and the level and difference of U.S. and world values of Geanakoplos's (2004) MY ratio (confirm results with the level and change of middle age ratios); Federal deficit/GDP and world debt/GDP (confirm results with Federal primary deficit/GDP and US debt/GDP).

The Appendix also contains the following plots for each of our correlates: scatterplots of 10 year moving averages of *r* vs. the correlate, one with all observations 1890-2016 and one with every 10th observation (the latter to give an uncluttered look at the progression of the relationship over time); bivariate time series plots of Müller and Watson (2017) filtered *r* and filtered correlate, 1890-2016 and 1950-2016 sample. We present some time series plots of 10 year moving average data in our discussion below.

5. EMPIRICAL RESULTS

5.1 Correlations: Overview Estimated correlations are in Table 3 (1890-2016) and Table 4 (1950-2016), with 68% confidence intervals. We present 68% confidence intervals because as discussed above we only have a small sample of observations on 10 year intervals. With a small sample, power is low. So a less stringent standard for rejecting a correlation of zero seems warranted. (See the discussion in Müller and Watson (2017).) Our working paper Appendix presents 90% confidence intervals.

As stated in notes to Table 3, due to data availability, a few series end in 2015 or (in Table 3) start later than 1890. Estimates whose confidence interval excludes zero are marked with a "*", and will be referred to as significant. The "(+)" and "(-)" in column (1) repeats, for convenience, entries in Table 1. In interpreting the estimated sign of a correlation, we refer to "expected" and "unexpected" signs, though, as noted above, the models described above generally make predictions about signs holding all other correlates constant rather than an unconditional prediction. We defer discussion of economic significance to our analysis of conditional forecasts in a subsequent section; we do note that that analysis suggests that correlations whose absolute values are 0.20 or larger can be economically significant. We use ρ^{MA}, ρ^{LP}-I(0) and ρ^{LP}-I(1) to denote the population values of the estimates in columns (2), (3a) and (4a).

Some general comments, before discussing specific entries. First, for a given correlate, the estimates of ρ^{MA}, ρ^{LP}-I(0) and ρ^{LP}-I(1) are similar. Roughly 70% of the rows have the same signs across all three measures. More precisely: the signs of the all three point estimates are the same in 17 of the 24 rows in Table 3 and 19 of the 27 rows in Table 4. (There are three more entries in Table 4 than in Table 3 because of world demographic variables.) When signs conflict across measures, it is generally the case that all three point estimates are insignificant.

The concordance across measures is especially high for ρ^{MA} and ρ^{LP}-I(0). This applies not only to sign (the two have same signs in 21 of 24 rows in Table 3 and 24 of 27 rows in Table 4) but to relative magnitude. We use rank order correlation to summarize concordance of relative magnitude:

(5.1) Rank order correlation of estimates	ρ^{MA} vs. ρ^{LP}-I(0)	ρ^{LP}-(0) vs. ρ^{LP}-I(1)	ρ^{MA} vs. ρ^{LP}-I(1)
1890-2016	0.94	0.83	0.67
1950-2016	0.94	0.88	0.83

The rank order correlation of estimates is above 0.9 for ρ^{MA} and ρ^{LP}-I(0), and more generally high across all possible pairs of measures and both sample periods. This means that a correlation that is relatively large in absolute value by one measure is also relative large in absolute value by the other two measures.

Thus our discussion for the most part will not have need to distinguish between measures of correlation, for either sign or relative magnitude.

5.2 Intertemporal IS In rows (1a) and (1b) of Table 3, we see that GDP and consumption growth have correlations with safe real rates that are small in magnitude and sometime negative over the 1890-2016 sample. However, the same rows in Table 4 show that GDP and consumption growth fare better over the 1950-2016 sample. Conversely, in row (1c) correlations between world GDP growth and safe rates are positive in the 1890-2016 sample, but have mixed signs and are relatively small in the 1950-2016 sample. These findings that positive correlations between economic growth and real rates are episodic corroborate earlier research (Leduc and Rudebusch (2014), Hamilton et al. (2016)). Further, in our view, the mixed picture of signs and significance is consistent with the large literature that finds the intertemporal IS equation wanting (e.g., Canzoneri et al. (2007)). As well, that the results are not in accord with the model seems a logical consequence of the fact that the real rate is trendy and aggregate growth variables generally are not (Neely and Rapach (2008)).

Figure 5.1A presents a time series plot of 10 year moving averages of *r* (identical the *r*-MA plot in Figure 4.1C) and of GDP growth. The positive correlation in the 1950-2016 sample is evident. That the two series generally did not move together prior to 1950 is also evident, with the two series moving in opposite directions almost every year from throughout the period from 1914 to 1945.

Figure 5.1B replaces GDP growth with TFP growth (again, 10 year moving averages), with the plot of trend *r* repeated.[7] It is patently obvious that the correlation is (unexpectedly) negative. The two series move in opposite directions not only during the Great Depression and World War II–when real rates were low and TFP growth was high–but more generally. For example, during period from the mid-1980's to the mid-2000's, trend TFP growth rose and trend *r* fell. This striking result is quantified in Tables 3 and 4 in row (1d): all six estimated correlations are unexpectedly negative (6 = 2 sample periods × three estimates of long run correlations). Moreover, these estimates are usually significant.

To explain persistently negative real rates following the 2008-09 recession, some economists have given pride of place to low productivity growth (e.g., Fischer (2016)). But the recent combination of low real rates and low TFP growth is not reflective of the overall historical pattern of low frequency movement between the two variables.

Figure 5.1A. Real rate r_t and per capita GDP growth, 10 year moving averages

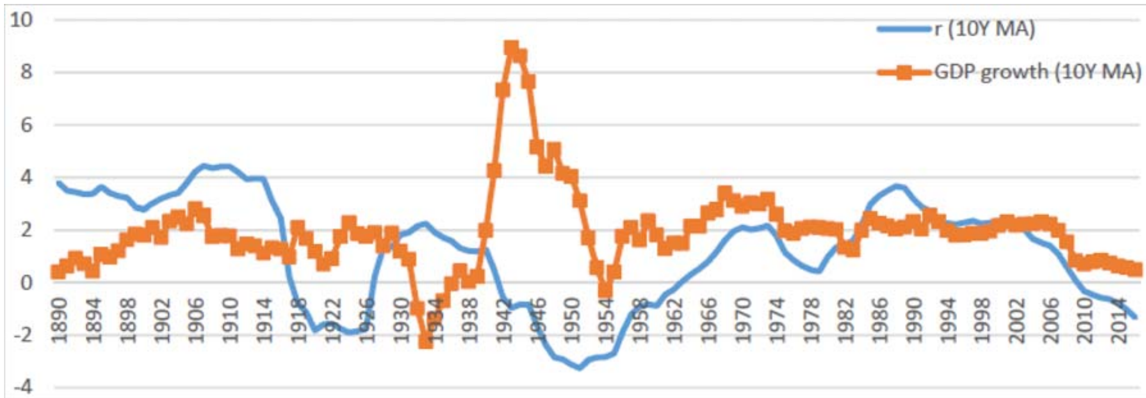
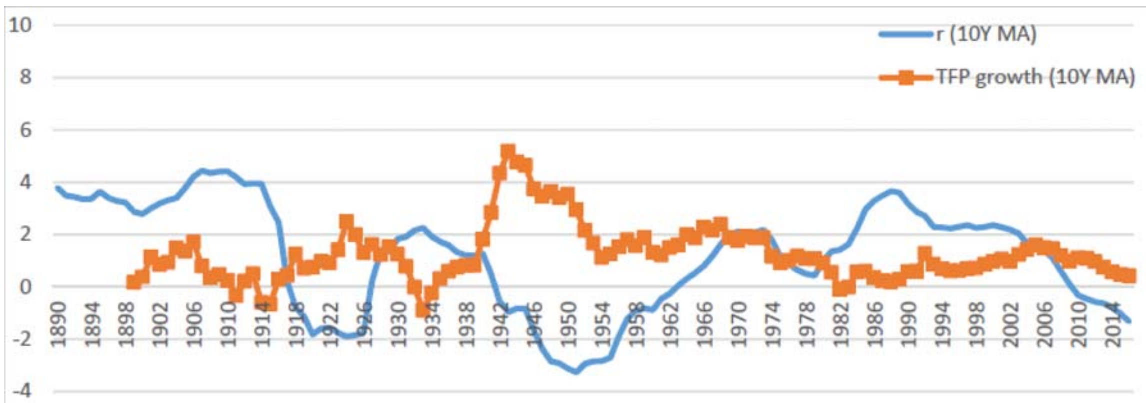


Figure 5.1B. Real rate r_t and TFP growth, 10 year moving averages



While the mixed results for economic growth and the negative results for TFP growth suggest that the intertemporal IS equation is wanting, the signs of the second moment variables in rows 2 and 3 are generally as expected in both samples. Further, by one or more measures of long run correlation the estimates are significant. This is consistent with the view that time varying second moments are an essential part of the intertemporal IS equation (Campbell and Cochrane (1999)).

For the intertemporal IS equation, then, the picture is mixed. For further insight, see the working paper Appendix where we depart from our focus on bivariate correlations an execute a series of low frequency multivariate regressions of r_t on correlates suggested by the intertemporal IS equation (2.3). In short, the results are not supportive of the intertemporal IS equation.

Figure 5.2A. Real rate r_t and labor hours growth, 10 year moving averages

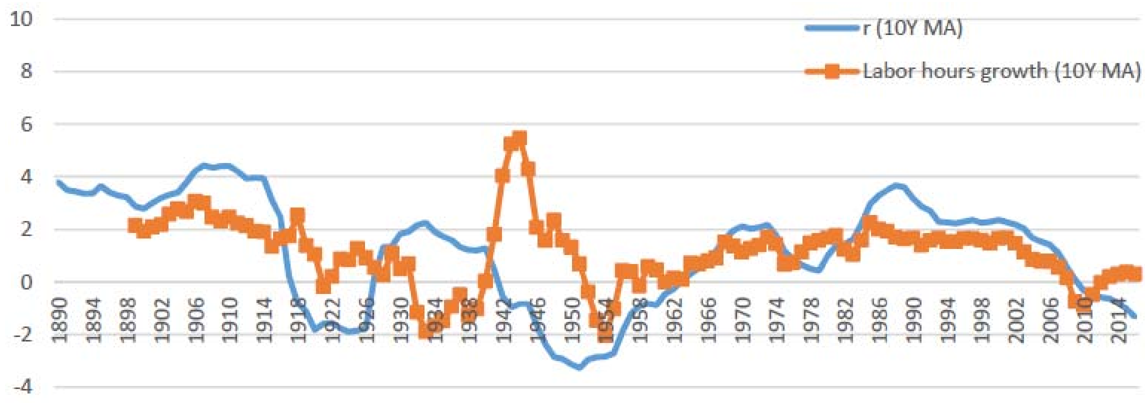
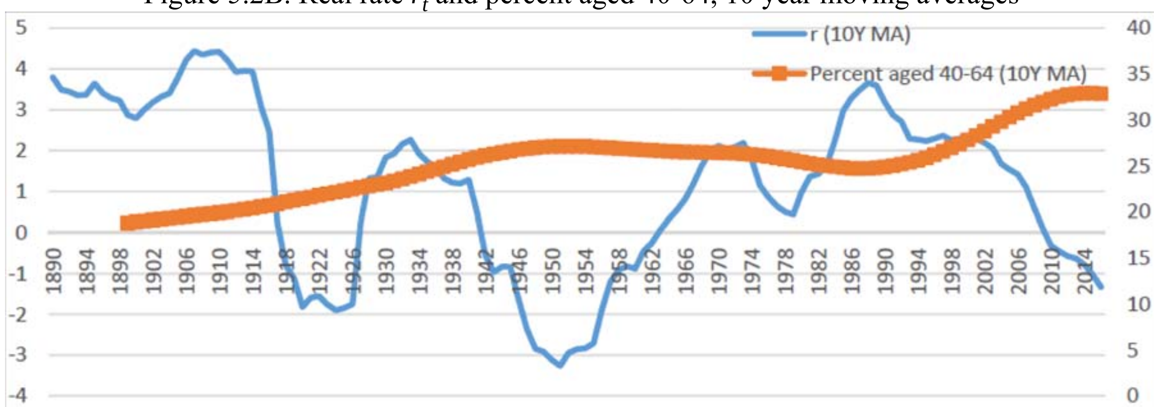


Figure 5.2B. Real rate r_t and percent aged 40-64, 10 year moving averages



5.3 Aggregate savings and investment We now move to correlates suggested by models of aggregate saving and investment. Across the two samples in Tables 3 and 4, the results in rows (4)-(10) can be divided into three categories: the variable produces correlations of expected sign, sometimes significant, in both sample periods (labor force growth, demographic variables); signs are sometimes as expected, sometimes not (current account, relative price of investment goods, top 10% income share, Baa-10 year spread, inflation/money growth); signs are not as expected (measures of government dissavings).

The first of several correlates to produce correlations of expected sign is labor hours growth in row (4). It is strongly positively correlated with *r* in both sample periods.[8] The strong positive correlation is evident in Figure 5.2A, which plots 10 year moving averages of *r* and of labor hours growth. In contrast to GDP growth or TFP growth, labor hours growth trends down with *r* in the last decades of the sample. Indeed, with the exception of the 1930's, trend labor hours growth tends to move in the same direction as trend *r* through the entire sample.

We are not aware of previous reduced form research that has quantified a link between labor hours growth and *r*. However, changes in trend employment growth associated with demographic change are important for understanding real interest rate trends in the structural models of Gagnon et al. (2016) and Kara and von Thadden (2016). As well, the informal calibration in Bullard (2017) uses trend labor force growth as one of the determinants of trend *r*.

Demographic variables comprise a second set of aggregate saving and investment correlates to produce estimated correlations with the expected sign. There are 27 entries for demographic variables in rows (5a)-(5c) in Table 3 and (5a)-(5f) in Table 4. 26 of these have expected sign, and most though not all are significant. In terms of support for demographic variables as correlates of real rates, these results fall roughly midway between the regression results of Poterba (2001) on the one hand, who finds modest support for demographic variables, and Geanakoplos et al. (2004) and Favero et al. (2016) on the other, who find exceptionally strong support.

Figure 5.2B plots 10 year moving averages of *r* and the percent aged 40-64. The expected negative correlations presented in line (5b) of Table 3 and line (5c) of Table 4 reflect long periods when the two moved in opposite directions: the mid-1900's-mid 1920's, 1930-1970, and the mid-1980's-2016. Note that the trend value of this correlate moves quite slowly. This perhaps suggests that one put more weight on the I(1) estimates or on the first difference estimates that are also in the tables. The dependency ratio is the only other correlate whose trend value appears equally slow moving.

The remaining aggregate saving and investment correlates do not consistently work as expected. We begin with measures of government dissaving–Federal deficits and debt, in rows (6a) and (6b). These correlates consistently yield estimates that have unexpected signs, and often are significant, especially in the 1890-2016 full sample. This seems to reflect in part the two world wars, periods in which *r* was quite low (indeed, negative) even though debt and deficits were high. As noted above, inflation and expected inflation rose during those wars. But nominal rates did not rise commensurately. This likely reflects financial repression as defined in Reinhart and Sbrancia (2015): nominal rates were kept low and government debt was sold in large part to a captive market.[9] In addition, our current bout of negative real rates came with large increases in federal deficits and debt following the 2008-09 recession. Thus our results suggest that government dissaving may be of second order importance for trends in real rates and that other factors, such as demographics, may dominate.

The final set of aggregate saving and investment correlates, in rows (7)-(10), display mixed results. Estimates for the current account, relative price of investment goods, top 10% income share, and Baa-10 year spread generally have the wrong signs for the 1890-2016 sample in Table 3, but the correct signs for the 1950-2016 sample in Table 4. However, even when the signs are correct, the point estimates are generally small and insignificant. Since our sample is unusually long, these results need not contradict the evidence in research that focuses on relatively recent years (e.g., Bernanke (2005), Sajedi and Thwaites (2016) or Del Negro et al. (2017)).

5.4 Mundell-Tobin effect The variables in rows (11) and (12) work as expected, and strongly so, in the 1890-2016 period. They do not work as expected in the 1950-2016 period. Over the longer sample, it seems the correlations are dominated by trend rises in the nominal variables in the two World Wars and the 1970s, periods in which the trend safe real rate was low. During the 1950-2016 period, behavior over the last four decades gets more weight: trend real rates and trend inflation have drifted down, while trend money growth has stayed more or less flat.

5.5 Summary

As noted in the introduction, much current conventional wisdom views trend GDP growth as the primary driver of the secular trend in safe real rates. The results reported here and in earlier research (e.g., Hamilton et al. (2016)) suggest that in the data the low frequency link with GDP growth is episodic, and the link with TFP growth is negative. A reduced form result that, so far as we know, is new to this paper is that labor hours is a strongly positive low frequency correlate of the safe real rate. Hence, if forced to rely on a growth variable, labor hours growth seems preferable to GDP growth as a low frequency correlate of real rates.

In the standard overlapping generations model, labor hours growth and TFP have a symmetric effect on the real rate in steady state. Hence, our finding of oppositely signed correlations for labor hours growth and TFP is unexpected. We suspect that labor hours works as expected because it partly reflects the age variables that are conventionally captured by our other demographic variables. Specifically, hours growth is the same as population growth in standard overlapping generations models. Thus, increases in aggregate investment that are needed to match capital with labor are met with equal increases in aggregate savings due to a larger population of savers. However empirically, hours growth also captures low frequency trends in labor force participation and in the length of the work week. This will cause low frequency fluctuations in the demand for capital without corresponding changes in population of savers, generating an extra source of fluctuation in real rates from labor hours growth.

Overall, labor hours growth and demographic variables seem to evidence the most reliable long run correlation with the safe rate, with estimated long run correlations that come with expected sign and generally are significant. Most other variables deliver a mixed picture in terms of such correlations. Because of this, we view trends in labor hours growth and in demographic variables as being most appealing if one is looking to use a small number of correlates to explain the trend the decline in safe real interest rates over the past four decades.

6. FORECASTS

In this section, we use the long horizon regressions (3.3.1) to make conditional and direct forecasts of the 10 year moving average of the real rate in 2026, i.e., the 10 year moving average 2017-2026. As explained below, the conditional forecasts serve in part to let us gauge the economic magnitude of the low frequency correlations presented above. We will take the median of the direct forecasts as a kind of summary over all correlates of how the correlates link to the real rate at the present time.

As in equation (3.3.1), denote the 10 year moving average 2017-2026 of the real rate and a given correlate x as r_{2026}^{MA} and x_{2026}^{MA}. Let us use $\hat{r}_{2026}^{MA}$ to denote a forecast of r_{2026}^{MA}. We rely on (3.3.1) to construct forecasts. (For correlates whose data end in 2015, we used analogues to (3.3.1) in which the left hand side variable was led 11 years instead of 10.) For the discussion to come, it may help to note that the 2016 trend value of r is $r_{2016}^{MA} = –1.3\%$ (that is, the average value of r_t, 2007-2016, is –1.3%). (Throughout this section, we use "trend value" to denote 10 year moving average.)

<u>6.1 Conditional forecasts</u> A conditional forecast of r_{2026}^{MA} is one conditioned on a hypothesized value of x_{2026}^{MA}. That is, it takes as truth a given forecast of x_{2026}^{MA}. We make conditional forecasts for four correlates, using the 1950-2016 sample. While these conditional forecasts are of some interest in themselves, we use them mainly for two other purposes. First, we use them to illustrate the economic

magnitude of the low frequency link between these correlates and the real rate. Second, we use them to emphasize the uncertainty about forecasts of the trend real rate–as we shall see, plausible variation in the future path of a correlate leads to wide movement in the forecast of r_{2026}^{MA}.

For concision, we report results for four correlates, which we think sufficient for the two purposes just described. These four are: labor force growth, change in percent 40-64 for the US and for the world, and the Baa-10 year Treasury spread. We chose labor force growth and change in percent 40-64 to further explore the positive results for the long run correlations discussed in the previous section. We chose the Baa-10 year Treasury spread to highlight the potentially large effects of a non-demographic variable. Further, these correlates have reasonable external forecasts, which are necessary for the conditional forecasting exercise.

Results are in Table 5. In each of the two panels, row (1) reports direct forecasts, which will be discussed for the entire set of correlates in Table 6. Column 1, rows (2) and (3) report alternative forecasts of r_{2026}^{MA} that condition on hypothesized forecasts for x_{2026}^{MA} for each of the four correlates. The various forecasts of r_{2026}^{MA} come with 68% confidence intervals, obtained from standard asymptotic theory.

To illustrate the calculation, consider the results for labor hours growth in the left half of Table 5A. Our bivariate VAR with labor hours growth predicts labor hours growth will average 0.88% over the 10 years 2017-2026, with 1.15% the comparable figure for the safe real rate. The CBO predicts 2017-2026 average employment growth of 0.59% (row (2), column (1c)), which we use as a forecast of average labor hours growth. We have seen above that lower trend labor hours growth is associated with a lower trend real rate. We see in Table 5A that if, indeed, the growth rate is forecast to be 0.59% instead of 0.88%, the implied forecast for the trend real rate is 0.73% rather than 1.15%. On the other hand, if labor growth were forecast to return to the 1985-2007 average of 1.22%, the forecast of the trend real rate would be 1.64%. Restating another way, the decline in labor growth from the 1985-2007 rate to the CBO's expectation for 2017-2026 is associated with 0.91% drop in the forecast of the trend safe real rate. Thus the elasticity of the forecast of the trend real rate with respect to trend labor hours growth is 1.44 (=(1.15-0.73)/(0.88-0.59)). We take these values to be economically large.

The demographic correlates in Table 5B also display economically large elasticities and effects on the forecast of the trend real rate. (We will return to the right half or Table 5A below.) For the U.S. (left half of Table 5B), the Social Security Administration forecasts a -0.24% average change in the percent aged 40-64 from 2017 to 2026. This corresponds to a real rate forecast of 1.47%. In contrast, if the percent aged 40-64 from 2017 to 2026 stays unchanged from 2017 to 2026, then the conditional real rate forecast is 2.68%. Hence, changes in the percent aged 40-64 are expected to be associated with a 1.21% drop in real rates relative to fixed demographics. For world demographics (right half of Table 5B), we see that using the UN's projection for populations lowers the forecast of r_{2026}^{MA} from 1.25% to 0.30%; a no-change forecast produces an intermediate value of $\hat{r}_{2026}^{MA}$ = 1.00%. Further, the elasticities for the changes in the percent aged 40-64 are 5.1 for U.S. data and 8.8 for world data.

To summarize, the conditional forecasts for aggregate hours growth and the percent aged 40-64 indicate that reasonable changes in these variables are associated with large changes in safe real rates. When paired with the consistently positive results of demographics in Section 5, the conditional forecasting results indicate that demographic change is a natural starting point for understanding trends in real rates. Further, these conditional forecast results suggest that demographic changes as forecasted by the Social Security Administration, the Census Bureau, and the UN are associated with low but positive real interest rates over the next 10 years.

We close our discussion of conditional forecasts with an important caveat: there is considerable uncertainty associated with these forecasts. This can be seen in several ways. First, there are large confidence intervals surrounding the conditional forecasts themselves. Second, reasonable changes in the forecasts of the demographic correlates have large effects on the forecasts of real rates. Third, this uncertainty can be seen by studying the conditional forecasts of other correlates. As an example, the right half of Table 5A shows the conditional forecasting results for the Baa-10 year Treasury spread. It indicates that the Baa-10 year Treasury spread returned forecasts of 2.40% for the trend value of the spread and 1.13% for $\hat{r}_{2026}^{MA}$. If the spread is forecast to stay at its 2010 to 2016 average of 2.81%, the trend real rate forecast falls to 0.59%. But if the spread is forecast to return to its average value from 1950-1998 of 1.54%, $\hat{r}_{2026}^{MA}$ rises to 2.26%. (We chose 1998 as the date in Alternative 1 because del Negro et al. (2017) identify the late 1990s as the period when a flight to quality took off.) That is, if risk premia recede as we move away from the 2008-09 recession, then, all else equal, we expect an increase in trend safe real rates that is as large as the decline predicted when one holds all else equal and focuses on demographics.

<u>6.2 Direct forecasts</u> We close with the direct forecasts. We make direct forecasts from bivariate VARs using, in sequence, each of the correlates in the tables above. We completed forecasts using both the longest possible span of data, and the postwar sample. The median across 24 (full sample) or 27 (postwar sample) correlates is given in Table 6, as are the forecasts at the 75th (Q3) and 25th (Q1) percentiles of the 24 or 27 forecasts. For example, the value of 0.72 for Q3 in column 1 indicates that of the 24 full sample forecasts, 18 were less than or equal to 0.72 in value.

Since our end of sample value is $r_{2016}^{MA} = -1.33$, the statistics on the forecasts in Table 6 clearly point towards an increase in this low frequency measure of the real rate. (Indeed, all of the 1890-2016 forecasts and all but two of the 1950-2016 forecasts yielded a rise in r^{MA}.) For the full sample, full mean reversion is not predicted: compare the 0.97 mean value for 1890-2016 in Table 2 with the values in column (2) in Table 6. For the postwar sample, mean reversion is more or less consistent with the 0.97 value for 1950-2016 in Table 2, though the values in Table 6 are lower than the conventional 2%. Taking the medians as the central tendency, and acknowledging the uncertainty documented above, we summarize the forecasts as: while there is considerable uncertainty, a reasonable range for the 2026 value of trend real rate is about 0.5%-1.1%,

7. CONCLUSIONS

Motivated by the decline in the safe real interest rate over the last several decades, we study long run correlations between the safe real rate and over 20 variables that have been posited to move with safe rates. We find that the safe real rate in the U.S. has statistically and economically important long run correlations with aggregate labor hours and demographic variables. For most other variables, we found substantive long run correlations in some samples and measures but not in others. Based on these results, we view demographic change as a reasonable starting point for understanding the recent secular decline in real rates. Further, we prefer labor hours growth to GDP growth or TFP growth for modeling trends in safe real rates. Finally, our conditional forecasts indicate that plausible paths for labor hours growth and demographic changes are associated with low but positive real rates over the next decade.

Our reduced form analysis did not attempt to provide a structural explanation for the results we found. One priority for future research is better understanding why some correlates work as expected while others do not.

FOOTNOTES

1. Some literature has extended the utility function to allow habit persistence. As explained in Hamilton et al. (2016), such an extension does not have important implications for trend real rates.

2. This is in contrast to the infinitely lived model underlying the intertemporal IS. In overlapping generations models, each generation faces the usual intertemporal condition (equation (2.1)) trading off first versus second period consumption. But higher labor force growth leads to lower capital per worker and a higher marginal product of capital (higher interest rate). See Romer (2012, ch. 1 and ch. 2).

3. For those familiar with the jargon of time series, it may help to note that the absolute value $|\rho^{LP}|$ is the coherence between the two series at frequency zero.

4. The lowpass correlation and the correlation of 10 year moving averages focus on similar aspects of the data. The lowpass filter zeroes out frequencies higher than 10 years, prior to computing correlations. Taking 10 year moving averages dampens rather than zeroes out higher frequencies. Hence it is not surprising the two yield similar results.

5. We use "direct" in the sense of the forecasting literature: making a multiperiod prediction from regression coefficients estimated with a multiperiod ahead left hand side variable.

6. The countries are: Australia, Austria, Belgium, Brazil, Canada, Chile, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sri Lanka, Sweden, Switzerland, United Kingdom, United States, Uruguay. These are the countries for which the Maddison data goes back to 1890.

7. The plot for TFP growth begins in 1899 because the underlying data start in 1890. Thus the first possible observation on 10 year moving averages is 1899.

8. The point estimates for 1950-2016 are, however, quite different for $\hat{\rho}^{MA}$ and $\hat{\rho}^{LP}$-I(0)–two measures that ordinarily yield very similar estimates. This appears to result from $\hat{\rho}^{LP}$-I(0) surprisingly (to us) producing a sharp rise in trend labor hours growth at the end of the sample, while $\hat{\rho}^{MA}$ produces a more plausible low estimate.

9. Unfortunately, there is no obvious quantitative measure of financial repression, so we have not included it as a correlate.

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Table 1

Possible low frequency correlates of *r*

	Variable	Expected sign of low frequency correlation with *r*
(1)	aggregate growth	+
(2)	volatility of aggregate growth	-
(3)	covariance between inflation and aggregate growth	-
(4)	labor hours	+
(5a)	dependency ratio (percent population <20 or >64)	+
(5b)	percent of population 40-64	-
(5c)	Δ percent of population 40-64	+
(6)	government dissaving	+
(7)	current account	+
(8)	relative price of investment goods	+
(9)	inequality	-
(10)	spread between public and private rates	-
(11)	inflation	-
(12)	money growth	-

Notes:

1. Variables in lines (1)-(3) are suggested by the intertemporal IS, in lines (4)-(10) by models of aggregate desired savings and investment, in lines (11)-(12) by reduced form studies. See text for references.

Table 2

Basic statistics on the annual real rate r_t

		1890-2016	1950-2016
(1)	Mean	0.97	0.97
(2)	Standard deviation	3.28	2.25
(3)	Median	1.38	0.71
(4)	First order autocorrelation	0.60	0.73
(5)	Maximum	8.65	6.37
(6)	Minimum	-17.7	-4.21

Notes:

1. Annual data, computed as nominal rate minus expected inflation: $r_t = i_t - E_t\pi_{t+1}$. The nominal rate i_t is the average of January rates in t+1: call rates 1890-1917, the discount rate 1918-19, three month treasury bills 1920-2016. Inflation π_t is measured by the GNP/GDP deflator, using Romer (1989) prior to 1929, BEA data 1930-present. Expected inflation $E_t\pi_{t+1}$ is set to zero 1890-1913. For 1914-2016, $E_t\pi_{t+1}$ is computed from an AR(1) in inflation using rolling samples of 20 years.

Table 3

Long run correlations, 1890-2016

	(1) Correlate (expected sign)	(2) 10Y moving avg. $\hat{\rho}^{MA}$	(3a) Lowpass filter–I(0) $\hat{\rho}^{LP}$	(3b) R^2	(4a) Lowpass filter–I(1) $\hat{\rho}^{LP}$	(4b) R^2
(1a)	GDP growth (+)	-0.15	0.01	0.01	0.20	0.20
		(-0.31,0.01)	(-0.19,0.20)		(-0.01,0.38)	
(1b)	Consumption growth (+)	0.06	0.04	0.00	-0.01	0.00
		(-0.11,0.23)	(-0.16,0.23)		(-0.21,0.19)	
(1c)	World GDP growth (+)	0.09	0.18	0.03	0.26*	0.07
		(-0.06,0.24)	(-0.02,0.36)		(0.06,0.43)	
(1d)	TFP growth (+)	-0.55*	-0.36*	0.13	-0.01	0.00
		(-0.64,-0.45)	(-0.52,-0.17)		(-0.21,0.19)	
(2a)	var_t(GDP growth) (-)	-0.62*	-0.49*	0.24	-0.23*	0.05
		(-0.78,-0.46)	(-0.62,-0.31)		(-0.41,-0.03)	
(2b)	var_t(C growth) (-)	-0.61*	-0.43*	0.18	-0.19	0.04
		(-0.76,-0.46)	(-0.58,-0.22)		(-0.39,0.03)	
(2c)	var_t(π_GDP) (-)	-0.17*	-0.04	0.00	0.26*	0.07
		(-0.29,-0.03)	(-0.24,0.16)		(0.06,0.43)	
(2d)	var_t(π_PCE) (-)	-0.17	-0.04	0.00	0.25*	0.06
		(-0.35,0.02)	(-0.25,0.18)		(0.03,0.44)	
(3a)	cov_t(π_GDP,GDP growth) (-)	-0.42*	-0.31*	0.10	0.10	0.01
		(-0.57,-0.26)	(-0.47,-0.11)		(-0.10,0.29)	
(3b)	cov_t(π_PCE,C growth) (-)	-0.66*	-0.49*	0.24	-0.13	0.02
		(-0.78,-0.55)	(-0.63,-0.29)		(-0.33,0.09)	
(4)	Labor hours growth (+)	0.31*	0.27*	0.07	0.24*	0.06
		(0.09,0.53)	(0.07,0.44)		(0.04,0.42)	
(5a)	Dependency ratio (+)	0.36*	0.38*	0.14	0.17	0.03
		(0.17,0.56)	(0.18,0.53)		(-0.04,0.35)	
(5b)	Percent aged 40-64 (-)	-0.43*	-0.43*	0.19	-0.24*	0.06
		(-0.61,-0.25)	(-0.58,-0.25)		(-0.42,-0.04)	
(5c)	ΔPercent aged 40-64 (+)	0.14	0.18	0.03	0.11	0.01
		(0.00,0.27)	(-0.02,0.36)		(-0.10,0.30)	
(6a)	Fed deficits/GDP (+)	-0.56*	-0.48*	0.23	-0.25*	0.06
		(-0.68,-0.43)	(-0.61, -0.30)		(-0.42,-0.05)	
(6b)	Fed debt/GDP (+)	-0.59*	-0.57*	0.32	-0.29*	0.08
		(-0.79,-0.39)	(-0.68,-0.40)		(-0.45,-0.09)	

Table continues on next page.

Table 3, continued

Long run correlations, 1890-2016

	(1) Correlate (expected sign)	(2) 10Y moving avg. $\hat{\rho}^{MA}$	(3a) Lowpass filter–I(0) $\hat{\rho}^{LP}$	(3b) R^2	(4a) Lowpass filter–I(1) $\hat{\rho}^{LP}$	(4b) R^2
(7)	Current account/GDP (+)	-0.15	-0.18	0.03	-0.47*	0.22
		(-0.35,0.05)	(-0.36,0.03)		(-0.61,-0.30)	
(8)	Relative price inv. goods (+)	-0.01	0.07	0.01	0.17	0.03
		(-0.17,0.15)	(-0.13,0.27)		(-0.03,0.36)	
(9)	Top 10% income share (-)	0.00	-0.05	0.00	0.16	0.03
		(-0.18,0.18)	(-0.27,0.18)		(-0.07,0.37)	
(10)	Baa-10 yr Treasury spread (-)	0.30*	0.21	0.04	0.22	0.05
		(0.11,0.48)	(-0.02,0.41)		(-0.01,0.43)	
(11a)	π_GDP (-)	-0.30*	-0.43*	0.19	-0.69*	0.48
		(-0.45,-0.16)	(-0.57,-0.24)		(-0.78,-0.56)	
(11b)	π_PCE (-)	-0.38*	-0.48*	0.23	-0.70*	0.49
		(-0.55,-0.22)	(-0.61,-0.30)		(-0.78,-0.58)	
(12a)	M1 growth (-)	-0.34*	-0.44*	0.20	-0.38*	0.14
		(-0.56,-0.12)	(-0.60,-0.24)		(-0.55,-0.16)	
(12b)	M2 growth (-)	-0.33*	-0.33*	0.11	-0.28*	0.08
		(-0.50,-0.16)	(-0.50,-0.14)		(-0.45,-0.08)	

Notes:

1. In lines (1)-(3): GDP and consumption are real and per capita. Consumption is total consumption. π_GDP and π_PCE are GDP and PCE inflation. World GDP is constructed from 23 countries given in a footnote in the text. The second moments in lines (2) and (3) are constructed from rolling samples of 20 years as described in the text. Variables in line (5) are defined in Table 1. Lines (6) and (7) are ratios of nominal variables. In (8), the GNP/GDP deflator for business fixed investment is expressed as a ratio to the deflator for total consumption. All data are annual.

2. "10Y moving avg." reports correlations of 10 year moving averages of r_t and the indicated variable. "Lowpass filter" uses Müller and Watson's (2017) lowpass filter to cut off frequencies higher than 10 years. The point estimate $\hat{\rho}^{LP}$ is the correlation between the filtered series, while the R^2 corresponds to a regression of filtered r_t on the filtered correlate (slope coefficient not reported). Column (3) applies Müller and Watson's (2017) assuming the data are I(0), while column (4) assumes the data are I(1). Asymptotic 68% confidence intervals in parentheses, with "*" indicating that the interval does not include zero. See section 3 for further discussion.

3. Data end in 2015 for the correlates in rows (1c) and (9). The sample start for the lowpass filter is 1890 except for the correlates in the following rows: (2a), (2c), (3a)–1891; (2b), (2d) and (3b)–1910; (6a) and (5b)–1891; (8)–1917; (10)–1920; (12a)–1916. The sample start is 9 years later r in the "10Y" column.

Table 4

Long run correlations, 1950-2016

	(1) Correlate (expected sign)	(2) 10Y moving avg. $\hat{\rho}^{MA}$	(3a) Lowpass filter–I(0) $\hat{\rho}^{LP}$–I(0)	(3b) R^2	(4a) Lowpass filter–I(1) $\hat{\rho}^{LP}$–I(1)	(4b) R^2
(1a)	GDP growth (+)	0.61*	0.23	0.05	-0.01	0.00
		(0.48,0.74)	(-0.06,0.46)		(-0.28,0.27)	
(1b)	Consumption growth (+)	0.56*	0.46*	0.21	0.20	0.04
		(0.40,0.72)	(0.19,0.64)		(-0.09,0.44)	
(1c)	World GDP growth (+)	0.05	0.00	0.00	-0.15	0.02
		(-0.23,0.33)	(-0.27,0.28)		(-0.40,0.14)	
(1d)	TFP growth (+)	-0.23*	-0.25	0.06	-0.38*	0.14
		(-0.46,-0.01)	(-0.49,0.04)		(-0.58,-0.10)	
(2a)	var_t(GDP growth) (-)	-0.35*	-0.38*	0.14	-0.42*	0.17
		(-0.59,-0.10)	(-0.58,-0.10)		(-0.61,-0.14)	
(2b)	var_t(C growth) (-)	-0.22	-0.40*	0.16	-0.54*	0.29
		(-0.48,0.03)	(-0.60,-0.13)		(-0.70,-0.29)	
(2c)	var_t(π_GDP) (-)	-0.28*	-0.33*	0.11	-0.28	0.08
		(-0.54,-0.02)	(-0.54,-0.05)		(-0.50,0.01)	
(2d)	var_t(π_NDS) (-)	-0.16	-0.31*	0.10	-0.20	0.04
		(-0.43,0.12)	(-0.53,-0.03)		(-0.45,0.08)	
(3a)	cov_t(π_GDP,GDP growth) (-)	-0.24*	-0.36*	0.13	-0.06	0.00
		(-0.42,-0.06)	(-0.57,-0.08)		(-0.33,0.22)	
(3b)	cov_t(π_NDS,C growth) (-)	-0.76*	-0.64*	0.41	-0.37*	0.14
		(-0.83,-0.69)	(-0.77,-0.42)		(-0.58,-0.09)	
(4)	Labor hours growth (+)	0.81*	0.38*	0.15	0.11	0.01
		(0.76,0.87)	(0.10,0.58)		(-0.18,0.37)	
(5a)	Dependency ratio (+)	-0.04	0.10	0.01	0.13	0.02
		(-0.30,0.23)	(-0.19,0.36)		(-0.16,0.38)	
(5b)	Dependency ratio, world (+)	0.00	0.08	0.01	0.08	0.01
		(-0.27,0.26)	(-0.21,0.34)		(-0.21,0.34)	
(5c)	Percent aged 40-64 (-)	-0.62*	-0.54*	0.29	-0.25	0.06
		(-0.79,-0.45)	(-0.70,-0.29)		(-0.48,0.04)	
(5d)	Percent aged 40-64,world (-)	-0.41*	-0.40*	0.16	-0.18	0.03
		(-0.64,-0.18)	(-0.60,-0.12)		(-0.43,0.10)	
(5e)	ΔPercent aged 40-64 (+)	0.18	0.33*	0.11	0.32*	0.11
		(-0.01,0.36)	(0.05,0.55)		(0.04,0.54)	
(5f)	ΔPercent aged 40-64, world (+)	0.20	0.24	0.06	0.14	0.02
		(-0.01,0.42)	(-0.04,0.48)		(-0.15,0.39)	
(6a)	Fed deficits/GDP (+)	-0.27	-0.26	0.07	-0.16	0.03
		(-0.57,0.04)	(-0.49,0.03)		(-0.41,0.12)	
(6b)	Fed debt/GDP (+)	-0.58*	-0.62*	0.39	-0.28	0.08
		(-0.76,-0.40)	(-0.76,-0.40)		(-0.51,0.00)	

Table continues on next page.

Table 4, continued

Long run correlations, 1950-2016

	(1) Correlate (expected sign)	(2) 10Y moving avg. $\hat{\rho}^{MA}$	(3a) Lowpass filter–I(0) $\hat{\rho}^{LP}$	(3b) R^2	(4a) Lowpass filter–I(1) $\hat{\rho}^{LP}$	(4b) R^2
(7)	Current account/GDP (+)	0.18	0.12	0.01	0.07	0.00
		(-0.09,0.44)	(-0.16,0.38)		(-0.21,0.33)	
(8)	Relative price inv. goods (+)	0.22	0.20	0.04	0.10	0.01
		(-0.06,0.50)	(-0.09,0.44)		(-0.19,0.36)	
(9)	Top 10% income share (-)	-0.25	-0.29*	0.08	-0.18	0.03
		(-0.50,0.00)	(-0.51,-0.00)		(-0.43,0.11)	
(10)	Baa-10 yr Treasury spread (-)	-0.04	-0.07	0.00	-0.16	0.03
		(-0.37,0.29)	(-0.33,0.22)		(-0.41,0.13)	
(11a)	π_GDP (-)	0.33*	0.26	0.07	-0.10	0.01
		(0.15,0.51)	(-0.02,0.50)		(-0.36,0.18)	
(11b)	π_NDS (-)	0.39*	0.31*	0.09	-0.16	0.03
		(0.21,0.56)	(0.02,0.53)		(-0.41,0.13)	
(12a)	M1 growth (-)	0.22	0.00	0.00	-0.03	0.00
		(-0.08,0.51)	(-0.28,0.28)		(-0.30,0.25)	
(12b)	M2 growth (-)	0.11	0.15	0.02	-0.02	0.00
		(-0.12,0.34)	(-0.14,0.40)		(-0.30,0.26)	

Notes:

1. See notes to Table 3. Consumption is nondurables and services, with π_NDS the corresponding inflation rate. In (4b) and (4d), world demographic variables are constructed from the several dozen countries that are labeled high income by the UN in 2016 (United Nations (2017a,p156)).

2. For the lowpass filter correlation, the sample period is 1950-2016. For 10Y moving averages, the sample period is 1959-2016. Data end in 2015 for the correlates in rows (1c), (4b), (4d) and (8). Data start in 1951 for the correlates in rows (2d), (3b), (4b) and (4d).

Table 5

Conditional forecasts

A. Conditional on exogenous forecasts of labor hours growth or Baa spread

		(1a)	(1b)	(1c)	(2a)	(2b)	(2c)
		Labor hours Growth			Baa-10yr Treasury spread		
		$\hat{r}^{MA}_{2026}$	$\hat{x}^{MA}_{2026}$	$\mathring{x}^{MA}_{2026}$	$\hat{r}^{MA}_{2026}$	$\hat{x}^{MA}_{2026}$	$\mathring{x}^{MA}_{2026}$
(1)	Baseline	1.15	0.88		1.13	2.40	
		(-0.20,2.49)			(-0.45,2.71)		
(2)	Alternative 1	0.73		0.59	2.26		1.54
		(0.01,1.44)			(1.02,3.50)		
(3)	Alternative 2	1.64		1.22	0.59		2.81
		(0.92,2.35)			(-0.99,2.17)		

B. Conditional on exogenous forecasts of Δpercent aged 40-64

		(1a)	(1b)	(1c)	(2a)	(2b)	(2c)
		ΔPercent aged 40-64			ΔPercent aged 40-64, world		
		$\hat{r}^{MA}_{2026}$	$\hat{x}^{MA}_{2026}$	$\mathring{x}^{MA}_{2026}$	$\hat{r}^{MA}_{2026}$	$\hat{x}^{MA}_{2026}$	$\mathring{x}^{MA}_{2026}$
(1)	Baseline	1.74	-0.19		1.25	0.03	
		(0.69,2.79)			(0.29,2.22)		
(2)	Alternative 1	1.47		-0.24	0.30		-0.08
		(0.73,2.20)			(-0.26,0.85)		
(3)	Alternative 2	2.68		0.00	1.00		0.00
		(1.91,3.45)			(0.45,1.54)		

Notes:

1. $\hat{r}^{MA}_{2026}$ and $\hat{x}^{MA}_{2026}$ are forecasts of the average values of the real rate and of the indicated correlate, 2017-2026. The baseline forecast in line (1) is constructed from the bivariate VAR (3.3.1), with the left hand side variable advanced one year when the data on the correlate end in 2015 (right half of panel B). In rows (2) and (3), the forecast $\hat{r}^{MA}_{2026}$ is conditional on the forecast of x^{MA}_{2026} taking the value stated in columns (1c) or (2c). See text for details on how the conditional forecast was computed.

2. All regressions use the postwar sample. 68% confidence intervals in parentheses.

3. Sources of forecasts $\mathring{x}^{MA}_{2026}$ (row (2) / row (3)): Labor hours growth: CBO (2017) (June 2017 edition) / average hours growth from 1985 to 2007. Baa-10 year Treasury spread: average Baa spread from 1950 to 1998 / average Baa spread from 2010 to 2016. ΔPercent aged 40-64: Social Security Administration (2017) / no change 2016-2026. ΔPercent aged 40-64, world: United Nations (2017b), “Annual Population by Age Groups - Both Sexes” / no change 2015-2026.

Table 6

Forecasts of r_{2026}^{MA}

(1) Quartiles of forecasts (across correlates)		(2) $\hat{r}_{2026}^{MA}$ full sample	(3) postwar sample
(1)	Q3	0.72	1.27
(2)	Median	0.45	1.13
(3)	Q1	-0.13	0.35

Notes:

1. See notes to previous table.

2. Using the direct forecasting method, bivariate models were used to construct 24 (full sample) or 27 (postwar sample) forecasts of the average value of the safe rate, 2017-2026. See equation (3.3.1). The two variables on the right hand side in the model were the real rate and a correlate; the variables on the left hand side were 10 year moving averages led 10 years (led 11 years for correlates whose data end in 2015). Q3, median and Q1 presents the 75 percentile, median and 25 percentile across these forecasts. For example, the 0.72 in row 1, column 1, indicates that of the 24 forecasts, 18 were less than or equal to 0.72.