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Natural Monopoly**

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Costly Information Intermediation as a Natural Monopoly

Daniel Monte and Roberto Pinheiro

In this paper, we show that information trade has similar characteristics to a natural monopoly, where competition may be detrimental to efficiency due either to the duplication of direct costs or the slowing down of information dissemination. We present a model with two large populations in which consumers are randomly matched to providers on a period-by-period basis. Despite a moral hazard problem, cooperation can be sustained through an institution that gives incentives to information exchange. We consider different information pricing mechanisms (membership vs. buy and sell) and different competitive environments. In equilibrium, both pricing and competitive schemes affect the direct and indirect costs of information transmission, represented by directed fees paid by consumers and the expected loss due to imperfect information, respectively.

Keywords: Costly Information trade; Market Structure, Natural Monopoly

JEL D47, D83, D85

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1 Introduction

Many important market interactions are infrequent in that each consumer only requires services sporadically and the parties involved do not engage in long-term relationships. In this environments, the expected reputational cost of providing bad service or defaulting on small loans is quite small, in particular when the market size is large (see Kandori [1992]). For example, the cost of suing a plumber for bad service, a store for poorly provided services, or a debtor that has not paid a small loan may be prohibitive, in particular once we introduce the opportunity cost of going through the legal process (see Schmitz [2016]). In these markets, investing in reputation is important only insofar it might help spread the word to other customers. While word-of-mouth is important to sustain cooperation in many markets, in others, for example in large and mobile populations, it might serve less well. In many of these situations, markets develop information sharing and reporting systems that allow the harmed party to disclose information. This information can be used by other agents in the future in order to avoid trading with a dishonest (or simply incompetent) counterpart. This reporting system provides not only an incentive to agents in developing a reputation of good providers or debtors, but also a mechanism that allows consumers and creditors to reduce the likelihood that they will be harmed by faulty service.

In this paper, we show that the trade of costly information in a market with atomistic agents has similar characteristics to a natural monopoly. First of all, the direct cost of information acquisition can be seen as a high fixed cost that may be duplicated due to competition among information intermediaries. Second, even in cases where the information intermediary designs an information pricing mechanism that attempts to reduce direct costs of information acquisition, competition may have a negative effect. These alternative pricing schemes reduce the direct cost of information acquisition by obtaining only a fraction of the available information, while learning the remaining information over time. Hence, these pricing schemes introduce an indirect cost to informed agents through an expected loss due to default in the short-run, as the acquired information is incomplete. Competition among information intermediaries increase these indirect costs by slowing down the process of information dissemination. When information intermediaries with incomplete information compete, the average number of agents that transact with each bureau goes down. Consequently, the number of agents that learn about a default at each round is smaller, slowing down learning.

We model the introduction of an information sharing bureau in a random matching set up in which the population is large enough that a folk theorem as the one presented in Kandori [1992] and Ellison [1994] is not possible.¹ Specifically, we consider two large populations of consumers and providers (with a continuum mass) in an infinite horizon economy. At the start of each period, each consumer is randomly matched to a single service provider and they play a sequential game of perfect information

¹This relates our benchmark model with the literature on community enforcement starting with Okuno-Fujiwara and Postlewaite [1995], where they showed that the Folk Theorem holds in a random matching game. Recent papers on community enforcement by Takahashi [2010], Awaya [2014], and Deb [2008] also relates to our benchmark model.

in which the provider has a short-run incentive to shirk. Agents are assumed to be risk-neutral and forward looking.² We assume that public information revelation by consumer or creditor is costly, so in an equilibrium without a bureau, no messages are sent, even when costs are arbitrarily small. As a consequence, without an information intermediary, market for services collapses and services are never hired. Our focus is to study the implications of the introduction of a bureau in this environment.

We consider the introduction of a bureau through different market structures: A non-profit bureau that aims to maximize social welfare while subject to a balanced-budget restriction and where participation is voluntary; a for-profit bureau that acts as a monopolist, and for-profit bureaus in a competitive market. We also consider different pricing methods: a.) membership fees, that require that members provide information whenever they receive the service from a provider, b.) buy and sell information, where the bureau pays a fee f_1 for reports of services provided, while charging a fee f_2 whenever a consumer or creditor would like to access their database. In the case of buy and sell information, agents that sell information are not required to buy information or vice-versa. We consider the case in which all fees are paid by the side of the market that face potential harm (consumers or creditors). Our goal is normative: we see the problem through the lenses of a policy maker that equally values the well-being of consumers or creditors and providers or debtors and evaluates the social welfare provided by each market structure. Consequently, we are concerned with how the different market structures affect the number of informed consumers or creditors in the economy, the number of providers or debtors that default in equilibrium, and the cost of implementing the different policies. We assume that all providers are rational and only earnings enter the utility function. Hence, providers face the loss of future earnings as their only motivation for good behavior.

The results from our benchmark model show that a non-profit bureau that offers memberships would provide the highest welfare, as well as the highest number of informed consumers or creditors and lowest number of defaulting providers or debtors in an equilibrium with risk-neutral agents. A non-profit bureau with a buy and sell pricing framework would be a second-best among the considered mechanisms. In the polar opposite, a monopolistic bureau would deliver the lowest social welfare, regardless the pricing mechanism. However, the monopolistic bureau would prefer a membership fee instead of buy and sell.

Once we introduce competition among for-profit bureaus, we see that, while the number of informed consumers and defaulting providers is the same as in a non-profit bureau buying and selling information, the social welfare is lower due to duplications in the cost of acquiring information. Moreover, once we allow for a membership pricing, we see that the equilibrium number of providers that default is the same as the one in the monopoly case. Consequently, the case of competing bureaus with membership is in equilibrium as bad as a monopoly in terms of social welfare.

It is important to highlight that the different pricing and competition mechanisms not only induce

²We focus on the case of an economy with risk neutral agents in order to avoid issues of insurance against negative shocks that may complicate the analysis.

equilibria with a different measure of informed and uninformed consumers, but also differ in terms of the quality or completeness of the information that informed consumers obtain. For example, let's consider the case of a non-profit bureau. While more consumers are informed in an equilibrium with membership than in buy and sell, the quality of the information held by members is worse. In particular, members have no information about poor service delivered to non-members, so the knowledge about bad providers builds slowly over time in a membership equilibrium, as members interact with providers. Differently, in a buy and sell equilibrium, since all consumers sell information, the bureau has all information about previous deviations at once, so the quality of information is high from the beginning.

The counterpart of the differences in information quality is the cost of information acquisition. There are two components to the cost of information. First, there is a direct cost, which is captured by the membership fee or the information buying fee. This direct cost is strictly increasing in information quality, since the quality depends on the number of consumer that must be compensated for reporting a transaction with a provider. Hence, the buy and sell arrangement naturally implies a higher direct cost. We should also emphasize that the direct cost does not fluctuate much over time, although it shrinks a bit over time in the membership case, as members learn about defaulting providers and stop buying their services. On the other side, the indirect cost is given by the expected loss due to faulty service that informed consumers may face. Notice that this cost is strictly decreasing in the information's quality. In the case of the buy and sell arrangement, the indirect cost is always zero. Differently, in membership the indirect cost is strictly positive, while approaches zero as time passes and the bureau learns over time. However, the speed at which the bureau's quality improves varies with the bureau's size. Large bureaus learn faster, so indirect costs converge to zero at a much faster rate. Consequently, competition among membership bureaus, by decreasing the individual bureau's size, reduces the speed at which both indirect and direct costs shrink over time.

In summary, the choice between non-profit bureau and competition, as well as different pricing mechanisms – buy and sell vs. membership – turns out to generate a trade off between information quality and information costs. Even more, different mechanisms also imply a trade off between direct and indirect costs and the speed at which indirect costs diminish over time. Our results for the case of risk neutral agents, i.e. a case that disregards the need for insuring consumers against consumption volatilities, show that trading information may present features similar to a natural monopoly, as competition imply either a duplication of information acquisition costs or no improvement over a pure monopolistic case. Moreover, the fact that in the case of a non-profit altruistic bureau would prefer membership to buy and sell indicates that if a bureau is large enough and there are no concerns of insuring consumers, the reduction in direct costs more than compensate the expected losses through indirect costs, as long the bureau is able to learn fast enough, reducing quickly indirect costs.

Our paper is related to the literature of information sharing in credit markets, as presented by Pagano and Jappelli [1993], Jappelli and Pagano [2002], and Brown et al. [2009]. A few papers have studied how markets that depend on permanent reputations are affected by different information sharing

mechanisms. Some important papers on this area that link to ours are Vercammen [1995], Ekmekci [2011], Liu and Skrzypacz [2014], Elul and Gottardi [2015], and Kovbasyuk and Spagnolo [2017]. Finally, our paper is related to mechanisms that were developed by society throughout history in order to overcome the lack of community enforcement in large societies, as pointed out by Milgrom et al. [1990], Araujo [2004], and Araujo and Minetti [2011], among others.

Our paper is also related to a policy-relevant topic that in our opinion has not received enough attention from the theoretical literature. Empirically, we observe that implementation of information reporting systems has taken different forms. Different market reporting systems have evolved across markets and countries. For example, in many countries, credit reporting systems for both consumer credit and business credit are done through public credit registries (see Miller [2000]) with mandatory participation. In this case, not only all information is available - once counterparts are obliged to report - but also the information quality is high due to potential legal penalties. In the US, credit report system for both consumer credit and business credit are done through the private sector. However, the sectors' structures are quite different. While the consumer credit reporting sector has three major players - Experian, Equifax, and TransUnion - the business credit report system is concentrated in only one firm - Dun & Bradstreet. Similarly, distinct market structures are seen in the consumer protection side, where we have Better Business Bureau, a non-profit that provides information to consumers about the quality of services received by previous customers, and Angie's List, a for-profit organization that earns its income through memberships and advertising. Moreover, the way these bureaus charge customers for the services provided also vary considerably. While most privately owned credit bureaus provide services either through membership fees or a fee per credit report requested, services like Angie's list have until recently been funded mostly through membership fees. Finally, the Better Business Bureau charges membership fees from the business that integrate the bureau, so focusing its revenue stream on the other side of the market. Consequently, evaluating the social welfare impact of these different market designs becomes a relevant question.

The paper is divided in 6 sections. Section 2 introduces the environment without a information bureau. Section 3 considers the case in which a non-profit altruistic bureau is introduced. Section 4 discusses the case of a for profit monopolistic bureau, while section 5 introduces competition among for profit bureaus. Finally, section 6 concludes the paper. All proofs are presented in the appendix.

2 Basic Model

Consider two populations indexed by i , where i lies in I_1 and I_2 and $I_i = [0, 1]$, $\forall i \in \{1, 2\}$. We will assume that $x \in I_1$ is a consumer and $y \in I_2$ is a provider and we assume that the agents are distributed according to Lebesgue measure. In each period $t = 1, 2, \dots$, each consumer is randomly matched to a provider to play a stage game Γ . We assume that the probability distribution over opponents in each period is uniform regardless of the past history of matching. Therefore, the probability that a currently

matched pair of players will match again is zero.

The stage game Γ is represented by the game tree in figure 1. Consumers initially decide if they should hire the provider or not. Not hiring the services generates a payoff of zero to both parts. Hiring the provider's services implies that the consumer must pay w to the provider, irrespective the quality of the service. If the provider is hired, then the provider must decide if he puts effort in the service or not. If the provider puts effort, the service is high quality, inducing a payoff of $P > w$ to the consumer. If no effort is exercised, the service turns out to be low quality, generating no benefit to the consumer. Putting effort in a task generates a disutility e to the provider. We assume that $0 < e < w$, but $P - e > 0$, so hiring the service and exercising effort would be the socially optimal equilibrium. Effort is verifiable but the expected cost of a lawsuit is too high to be used as a credible threat. After the service is provided, the consumer must decide if she sends a message to others about the quality of the service received or not. These messages consist of saying if the provider made effort or not. We must be aware that this is not related to any intrinsic quality of the provider, only with his immediate previous action (all providers are *ex ante* identical, acting rationally to maximize their payoffs). These messages can be public or private signals and, no matter if they are public or private, the consumer incurs on the same cost $c > 0$ sending them.³

We assume that this is an infinitely repeated game in which each agent discounts future periods by the same rate $\delta \in (0, 1)$. Histories are private and a history observed by an agent i at time $\tau > 1$, denoted by $h_{i,\tau}$, is a sequence of private interactions that this agent i observed from periods $1, 2, \dots, \tau - 1$. That is, $h_{i,\tau} = \{(j_t, a_{i,j,t}, \eta_{i,j,t})\}_{t=1}^{\tau-1}$, where by j_t we mean the agent matched to i at time t , $a_{i,j,t}$ represents the action of hiring or not hiring the service of the provider at time t and $\eta_{i,j,t}$ represents the empty set if the provider was not hired and an element of $\{\text{effort}, \text{no-effort}\}$ if the service was hired. The set of all (private histories) at time t is denoted by \mathcal{H}_t and the set of all histories by $\mathcal{H} = \cup_{t=1}^{\infty} \mathcal{H}_t$. Moreover, we must consider the possibility that the consumer receives a signal about the matched provider's past behavior – either good or bad. Denote $\sigma = \{\sigma_t\}_{t=1}^{\infty}$ the consumer's behavior strategy. It encompasses first her decision of whether hire not the provider, conditional on the information set at that period t , characterized by her pure strategy $s_{h,i} : \mathcal{H} \times \{\text{good}, \text{bad}, \emptyset\} \rightarrow \{\text{hire}, \text{don't hire}\}$. In her next decision node in the stage game, she must decide whether or not send a costly message to inform others about the received service: $s_{m,i} : \mathcal{H} \times \{\text{good}, \text{bad}, \emptyset\} \times \{\text{hire}, \text{don't hire}\} \times \{\text{effort}, \text{no-effort}\} \rightarrow \{\text{good}, \text{bad}, \emptyset\}$. A pure strategy for the consumer can be denoted simply as $s_i = (s_{h,i}, s_{m,i})$. Similarly, a behavior strategy for the provider is denoted by $\sigma_i : \mathcal{H} \times \{\text{hire}, \text{don't hire}\} \rightarrow \{\text{effort}, \text{no-effort}\}$. A strategy profile σ^* is an *equilibrium* if, for every $t \geq 1$, every $h \in \mathcal{H}$, every pair (i, j) , every play of the stage game and every σ it holds that: $U_t(\sigma^*|h_t) \geq U_t(\sigma_i, \sigma_j^*|h_t)$, where $U_t(\cdot)$ represents the expected continuation payoff of the repeated game.

Given that there is a continuum of agents, there is a zero probability of rematching with a former

³For example, we can imagine that these messages are sent by e-mail, blog post, or review on line and there is no cost difference between sending an e-mail to one person or sending the same e-mail to the entire mail list.

partner. Thus, there is no incentive to insure yourself of former deviations or obtain gains punishing former deviator by sending messages. Therefore, since messages are costly, no consumer would send a message. Hence, there is no way to punish a former deviator. Even if we try to apply Kandori [1992]’s contagious equilibrium, it would fail because of the continuum of agents hypothesis. Formally, suppose, by contradiction that there exists a history in which a provider is hired with positive probability. This means that the consumer expects that the provider will exert effort with positive probability. At this history, if this provider shirks, he has a higher current payoff than if he exerts effort (by avoiding the cost of effort). Moreover, his expected continuation payoff is given by the payoff that he expects to get by matching to each consumer in each future period. Given that actions are private to the interaction, the maximum number of players that have been exposed to the defection after t periods is at most 2^{t-1} . In particular, there is a countable number of such agents, and the measure of the union of these agents is zero. Thus, if f is the probability density function (henceforth p.d.f.) over all possible consumers and f_t^* is the p.d.f. over consumers who have not been exposed by the original defector at time t , then f and f_t^* are equal a.e. for any t . Simply put, the chance that one such consumer will meet the original defector again is zero for any time t , so that there is no incentive for cooperation. Therefore, the only equilibrium would be the infinite repetition of the stage game Nash equilibrium. We state the result in the following proposition.

Proposition 1 (No Market) *The only equilibrium in this game is one in which there is no market: On-equilibrium path providers are never hired and off-equilibrium path providers shirk and consumers do not send messages.*

In summary, the market collapses in the absence of an information sharing bureau, regardless of how small the cost of providing information is. In this sense, the introduction of an information sharing bureau is likely to be social welfare improving. The question becomes what bureau design would be socially optimal and also whether a profit-seeking bureau could improve welfare. We focus on two forms that the bureau might take. First, we consider a bureau that posts a price to buy information from consumers that have experienced the product and sells verification information to whoever wants to buy it. Second, we consider a bureau that operates with a membership system: members must pay a membership fee, and then they can access information at no additional costs, moreover, they are compensated for sending information to their bureaus. To carry on our analysis, we assume that the bureau can keep track of individual providers and their past behaviors. Formally, this means that a bureau can distinguish two distributions even if they differ by measure zero.⁴

⁴Technically, we generally assume that two random variables are equal if they are equal almost everywhere (a.e.), using some notion of measure zero. Here we assume that two random variables are equal only if they are equal everywhere. A similar assumption is present in Kocherlakota [1998].

3 Information Sharing Bureau: Non-profit altruistic bureau

We now extend our basic model by introducing an information sharing bureau. Throughout the paper we focus on stationary equilibria. These are equilibria in which agents play a stationary strategy. In these equilibria, stationary strategies are best responses when all other agents are playing stationary strategies. Keep in mind that in this class of equilibria, there is always a trivial equilibrium in which every provider defaults if hired, but none are hired on-equilibrium path. We focus on the non-trivial equilibrium in stationary strategies.

We look at this problem from the view of a policy maker that is trying to maximize the social welfare of consumers and providers using a equally weighted social welfare function. In order to do that, the policy maker can design the features of the market for information sharing bureaus, granting permits to potential competitors, fostering competition, creating a government-run, non-profit bureau, etc. However, the policy maker cannot force consumers to participate in the information exchange, i.e., bureau participation must be voluntary. Moreover, in the case presented in this section in which the policy maker establishes a government-run bureau, we consider that the bureau has an initial endowment of \mathcal{F} , but apart from the endowment the bureau must be self-sustained, i.e., it must raise enough money to generate at least zero profits every period. We consider two pricing mechanisms: buy and sell information and membership. We call the government-run bureau an altruistic bureau, since its goal is to maximize social welfare.

3.1 Buying and Selling information

We assume that the altruistic agency pays $f_1 \geq c$ for reported information and asks a fee of f_2 to disclose information about any given provider. If the agency doesn't have the asked information, the consumer pays nothing. Without loss of generality, we impose that if a consumer is indifferent between selling information or not, she sells it. Fees charged by the bureau are known by all agents in the economy and the bureau can credibly commit to posted fees. In this sense, a functioning bureau must set $f_1 \geq c$, implying that all consumers that hire the provider's services sell information to the bureau in equilibrium. As presented in figure 2 in the appendix, the extended game tree has an additional decision node at the beginning of the tree, in which the consumer decides if she purchases information from the bureau or not. After this node, all the remaining tree is identical to the one presented in figure 1 apart from the payoffs in the terminal nodes, where we must include the paid and received fees. Therefore, if the consumer decides to purchase information from the bureau, we subtract f_2 from her final payoff. Similarly, if the consumer decides to sell information to the bureau, we must add the received fee f_1 to her payoff. No changes are needed for the provider's payoffs or decisions.⁵

⁵The game presented in Figure 2 has some abuse of notation, considering that we assumed that the bureau would not charge the consumer if there was no information about the matched provided. As we will see, the bureau's information is complete in the case of buying and selling, so the abuse of notation is without loss of generality.

Providers' problem

Assume that a fraction $X_{A, buy}$ of consumers buy the information from the bureau once it is established. Informed consumers only buy services from providers with no history of default. Let's also assume that uninformed consumers hire any provider with whom they match, free-riding on the discipline imposed by informed consumers. We must show that in equilibrium it is optimal for uninformed consumers to hire the service.

Let's consider the decision problem of a provider that has never defaulted before. His only possible stage game action is $\eta = \{effort, no-effort\}$. As previously mentioned, consumers that buy information never hire the service of providers that have previously defaulted and all customers sell information. Consequently, providers know that after putting no effort once, no informed consumer will hire their services henceforth. As a result, we can focus on the once for all decision of effort or not. As a result, a provider prefers putting no effort if:

$$(1 - \delta)w + \delta [(1 - X_{A, buy}) \times w + X_{A, buy} \times 0] > w - e \quad (1)$$

where the left hand side (henceforth, LHS) of equation (1) is the payoff of always putting no effort, while the expression on the right hand side (henceforth, RHS) is the payoff of always delivering high-quality service. Then, simplifying the expression in equation (1), we have:

$$\delta < \frac{e}{wX_{A, buy}} \quad (2)$$

So, if the fraction of informed consumers $X_{A, buy}$ is high enough, a provider always puts effort. However, this would kill the incentive to buy information in the first place. Therefore, given that providers are *ex ante* identical, there is no equilibrium in which all providers follow the same pure strategy and a positive fraction of consumers buy information. Consequently, if in equilibrium a fraction of providers puts no effort, while the remainder delivers high-quality services, we must have that all providers are indifferent between putting effort or not. Therefore, from equation (2), the measure of informed consumers in equilibrium is:

$$X_{A, buy} = \frac{e}{\delta w} \quad (3)$$

Notice from equation (3) that the more costly the effort, the higher the measure of informed consumers in order to keep providers indifferent between delivering high-quality service or not. Differently, the more costly it is to lose business – the higher the w – and the more patient providers are – the higher the δ – the smaller is the needed fraction of informed consumers.

Consumers' problem

We now consider the consumer's decision. Keep in mind that in the stage game, the consumer has three decision nodes. First, she decides if she buys information from the bureau, paying a fee f_2 . Then, based on the information in hand, the consumer must decide to hire the providers or not. Finally, if the consumer hires the provider, she must decide to sell the information to the bureau or not.

We focus on the equilibria in which a bureau is sustained in equilibrium. Consequently, we must have an equilibrium in which consumers sell information and uninformed consumers hire providers. Let's start with the decision of selling information. As we mentioned before, we assume that all consumer sell information if they are indifferent between selling information or not. Consequently, a consumer sells information as long as $f_1 \geq c$. Let's then consider the decision of an uninformed consumer in purchasing the provider's services. Denote $Y_{A,buy}$ the fraction of providers that put no effort in equilibrium once the bureau is installed. Then, a consumer that buys no information would still prefer hiring a provider if:

$$P(1 - Y_{A,buy}) - w + f_1 - c \geq 0 \quad (4)$$

Rearranging this inequality, we have that:

$$Y_{A,buy} \leq \frac{P - w + f_1 - c}{P} \quad (5)$$

Consequently, as long the fraction of providers that put no effort is below the threshold presented in equation (5), uninformed consumers hire the matched providers. Notice that if this restriction is not satisfied, the market unravels. First of all, even after the announcement that a bureau will be installed, no agent will buy services in the first place. Consequently, no information is aggregated by the bureau. Second, if in equilibrium uninformed consumers decide not to hire the service and informed ones only purchase services from providers that always put effort, there is no incentive for providers to put no effort, but that eliminates the incentive to buy information in the first place.

Then, assuming that equation (5) and $f_1 \geq c$ are satisfied, we move towards the decision of buying information or not. Since we don't have any punishment if the consumer does not buy information in a given period, we just need to compare the payoff of buying and selling information with the payoff of just selling it. Since the problem presents a recursive environment, we don't need to evaluate strategies in which the agent presents a mixture of both. So, the payoff of buying and selling information is given by:

$$[-f_2 Y_{A,buy} + (P - w + f_1 - f_2 - c)(1 - Y_{A,buy})] \quad (6)$$

while the payoff of just selling information is given by:

$$(-w + f_1 - c) Y_{A,buy} + (P - w + f_1 - c)(1 - Y_{A,buy}) \quad (7)$$

Therefore, the consumer is indifferent between buying information or not is given by:

$$Y_{A,buy} = \frac{f_2}{w - f_1 + c} \quad (8)$$

Altruistic Bureau's Problem

The Bureau's objective is to maximize the social welfare of consumers and providers. In particular we consider an egalitarian social welfare function, that weights equally consumers and providers. The two populations are equally weighted and normalized to 1 (i.e., $I_1 = I_2 \equiv 1$). Consequently, the social welfare in period t is given by:

$$SW_t = \frac{1}{2} \{U_{consumer}(t) + U_{providers}(t)\} \quad (9)$$

Once we are focusing on the set of equilibria that has a functioning bureau, we consider the equilibria in which providers are indifferent between putting effort or not and consumers are indifferent between buying information or not. Consequently, we have that:

$$U_{consumer}(t) = (1 - Y_{A,buy})P - w + f_1 - c \quad \text{and} \quad U_{provider}(t) = w - e, \quad \forall t > 0 \quad (10)$$

where we set $U_{consumer}$ equal to the uninformed consumer's utility, while $U_{provider}$ is set to equal the utility earned by a provider that puts effort in equilibrium. Substituting back into equation (9) and rearranging:

$$SW_t = \frac{1}{2} \{(1 - Y_{A,buy})P + f_1 - c - e\} \quad (11)$$

Consequently, the bureau's problem is given by:

$$\mathbf{SW}_{A,buy} \equiv \max_{f_1, f_2} \frac{1}{2} \sum_{t=0}^{\infty} \delta^t SW_t \quad (12)$$

subject to:

$$\frac{f_2 \delta X_{A,buy} - f_1 [1 - \delta X_{A,buy} Y_{A,buy}]}{1 - \delta} \geq 0 \quad (C.1)$$

$$Y_{A,buy} \leq \frac{P - w + f_1 - c}{P} \quad (C.2)$$

$$X_{A,buy} \in [0, 1] \quad (C.3)$$

$$Y_{A,buy} \in [0, 1] \quad (C.4)$$

$$X_{A,buy} = \frac{e}{\delta w} \quad (C.5)$$

$$Y_{A,buy} = \frac{f_2}{w - f_1 + c} \quad (C.6)$$

$$f_1 \geq c \quad (C.7)$$

where (C.1) is the break-even condition, implying that the bureau must be self-funded once established. Since all informed consumers matched to providers that put no effort do not hire the services, the bureau buys information from a fraction $1 - X_{A,buy} Y_{A,buy}$ of informed consumers each period. Restriction (C.2)

says that uninformed consumers must still buy information in equilibrium, as presented in equation (5). Then, simplifying the bureau's problem we have:

$$\max_{f_1, f_2} \frac{1}{2(1-\delta)} \left\{ \left[1 - \frac{f_2}{w+c-f_1} \right] P - f_1 + c - e \right\}$$

subject to:

$$\frac{e}{w} f_2 - f_1 \left[\frac{1 - \frac{ef_2}{w(w+c-f_1)}}{1-\delta} \right] \geq 0 \quad (\text{C.1}')$$

$$f_1 \geq c \quad (\text{C.2}')$$

$$0 \leq \frac{f_2}{w+c-f_1} \leq \min \left\{ \frac{P+f_1-(w+c)}{P}, 1 \right\} \quad (\text{C.3}')$$

Manipulating (C.1'), we obtain:

$$f_2 \geq \frac{w}{e} f_1 \left[\frac{w+c-f_1}{w+c} \right] \quad (\text{C.1}'')$$

We can now show the following result:

Lemma 1 *In an economy in which it is optimal to establish information trade, the Bureau's optimal paying fee is c , i.e., $f_1 = c$*

Based on the proof of lemma 1, we observe that in the case in which we have an operating market, at the optimum we must have (C.2') and (C.1'') satisfied with equality. Therefore, we have that:

$$f_1 = c \quad \text{and} \quad f_2 = \frac{w^2 c}{e(w+c)} \quad (13)$$

while the social welfare is given by:

$$\mathbf{SW}_{\mathbf{A}, \text{buy}} = \frac{1}{2(1-\delta)} \left\{ \left[1 - \frac{wc}{e(w+c)} \right] P - e \right\} \quad (14)$$

Notice that we must still satisfy restriction (C.3'). Consequently, we must have:

$$\frac{wc}{e(w+c)} \leq \frac{P-w}{P} \quad (15)$$

If this restriction is not satisfied, we have that (C.3') is binding and $f_2 = \frac{P-w}{P} w$. However, based on the proof of lemma 1, we can show the following lemma:

Lemma 2 *In the cases that an operating market is possible, the inequality (15) is trivially satisfied. In other words, whenever there is an active bureau and a functioning market, (C.3') is non-binding.*

In order to simplify the presentation, corollary 1 collects the parameter restrictions obtained in the proof of lemma 1 that allows us to obtain a functioning provider's market with an active information bureau.

Corollary 1 *All restrictions (C.1) – (C.7) of the bureau’s problem presented in equation (12) can be jointly satisfied if:*

$$c \leq \frac{ew(P - w)}{P(w - e) + ew} \quad (16)$$

otherwise, a buy-and-sell bureau cannot be installed and the market collapses.

Consequently, from lemma 2 and corollary 1 we can conclude that, for the relevant set of parameters – i.e., the ones in which it would be optimal for the policy maker to establish an information bureau – uninformed consumers prefer hiring the provider. Moreover, unless (16) is satisfied with equality, consumers strictly prefer hiring the services and are better off if the market for providers exists.

3.2 Membership

Suppose that the bureau requires that consumers pay a membership fee before they can either access or sell information. The bureau commits to a sequence of fees, which we will denote by $\{f_{ee}^t\}_{t=1}^{\infty}$, where by f_{ee}^{τ} we mean the fee that is required to join the bureau at time τ . We assume that the membership fee is characterized as a per-period fee in order to avoid additional present value costs for up-front fees. However, results presented in the paper are robust to an up-front membership fee unless otherwise mentioned. Bureau members always receive a compensation for giving information, just enough to cover their costs of sending the information. *Only members can receive or report information.* Assume that there are two technological constraints in the environment: (1) it is impossible for the bureau to credibly reveal to the service provider who is a member and who is not. Indeed, think of this as a rating system where the service provider cannot really tell where the customer got her information from; and (2) it is not possible to make information market-wide public.

Let $Y_{A,member}$ be the fraction of providers who default in equilibrium and $X_{A,member}$ be the fraction of consumers who buy membership. If a consumer is a member, her period payoff depends on the fraction of providers who put effort ($1 - Y_{A,member}$), the fraction of providers who default every period and have at least once served to a bureau’s member, and the fraction of providers who default, but have never previously served a bureau’s member. The latter measure is the source of an indirect cost for informed consumers in the membership case. Differently from the buy and sell case, bureau members have incomplete information, facing the possibility of default even after acquiring information. As a counterpart, this reduction of information quality induces a lower membership fee. Moreover, the likelihood of members facing default decreases over time. As providers that default eventually meet a bureau’s member, the information becomes available to all other members, reducing the likelihood of a member facing default in the future.

Consumers We initially focus on stationary equilibrium in which all consumers that join the membership do so on period 1 – we show that this is without loss of generality later in this section. Thus, in the first period, there is a $1 - Y_{A,member}$ chance that the consumer faces a provider who does

not default and a $Y_{A,member}$ chance that the matched provider defaults. Given that this is the first period, no consumer knows which provider she's facing. On the second period, assuming that a fraction $X_{A,member}$ has bought the membership, there is a $1 - Y_{A,member}$ chance of facing a provider that does not default, a $X_{A,member}Y_{A,member}$ chance of facing a provider who is known to default – therefore, the bureau's member will not hire in his services – and a $(1 - X_{A,member})Y_{A,member}$ chance of facing a provider who defaults, but was not caught in the previous period. Summing up for all periods, this means that the payoff of the consumer joining the bureau in period 1 is:

$$\begin{aligned} & (1 - Y_{A,member})(P - w) - f_{ee}^1 + (1 - \delta) \sum_{t=0}^{\infty} \delta^t (1 - X_{A,member})^t Y_{A,member} (-w) \\ & = (1 - Y_{A,member})(P - w) - f_{ee}^1 - (1 - \delta) \frac{Y_{A,member}w}{1 - \delta(1 - X_{A,member})} \end{aligned} \quad (17)$$

Note that the membership becomes more attractive over time (there is more information about providers), so the fees must account for that, or else consumers would wait. If a consumer does not buy a membership, her payoff when hiring is:

$$(1 - Y_{A,member})(P - w) + Y_{A,member}(-w) = (1 - Y_{A,member})P - w. \quad (18)$$

Therefore, if the consumer is indifferent between joining the bureau or not in period 1, it must be the case that:

$$(1 - Y_{A,member})(P - w) - f_{ee}^1 - \frac{(1 - \delta)Y_{A,member}w}{1 - \delta(1 - X_{A,member})} = (1 - Y_{A,member})P - w,$$

which leads us to the following condition:

$$Y_{A,member}w \frac{\delta X_{A,member}}{1 - \delta(1 - X_{A,member})} = f_{ee}^1. \quad (19)$$

Suppose that the consumer is deciding on whether or not to join in some random time τ . If she does not join, her payoff is $(1 - Y_{A,member})P - w$. If she does join, her payoff is:

$$\begin{aligned} & (1 - Y_{A,member})(P - w) - f_{ee}^\tau + (1 - \delta) Y_{A,member} (-w) \left[1 + \sum_{t=1}^{\infty} \delta^t (1 - X_{A,member})^{t+\tau-1} \right] \\ & = (1 - Y_{A,member})(P - w) - f_{ee}^\tau - (1 - \delta) Y_{A,member} w \left[1 + \frac{\delta(1 - X_{A,member})^\tau}{1 - \delta(1 - X_{A,member})} \right]. \end{aligned} \quad (20)$$

Indifference between joining at time τ and not joining requires that:

$$\left\{ \begin{array}{l} (1 - Y_{A,member})(P - w) - f_{ee}^\tau \\ -(1 - \delta) Y_{A,member} w \left[1 + \frac{\delta(1 - X_{A,member})^\tau}{1 - \delta(1 - X_{A,member})} \right] \end{array} \right\} = (1 - Y_{A,member})P - w \quad (21)$$

Therefore, if the bureau announces (and credibly commits to) a sequence of fees $\{f_{ee}^t\}_{t=1}^{\infty}$, in this

stationary equilibrium that we are looking at, they must satisfy:

$$f_{ee}^\tau = w\delta Y_{A,member} \left[1 - \frac{(1-\delta)(1-X_{A,member})^\tau}{1-\delta(1-X_{A,member})} \right] \quad (22)$$

Consequently, f_{ee}^τ is strictly increasing with τ and $\lim_{\tau \rightarrow \infty} f_{ee}^\tau = w\delta Y_{A,member}$.

Lemma 3 *Consumers are indifferent between enrolling into the bureau in period τ instead of $\tau + 1$.*

So the discounted change in fees is just enough to absorb all the delay's potential benefit or loss. Since the consumer is indifferent about when to enroll in the bureau, we focus on enrollments in period 1.

We now restrict the parameters to the case in which non-members as well as member purchase the service in the period before the membership kicks in. Notice that the expected payoff of hiring a service with no record on the provider is $(1 - Y_{A,member})P - w$. Thus, in the equilibrium that we are considering, $(1 - Y_{A,member})P - w \geq 0$, i.e.:

$$Y_{A,member} \leq 1 - \frac{w}{P}. \quad (23)$$

Providers The provider chooses to default or not. If he chooses not to default, he gets a payoff of $w - e$ for every period (recall that in the equilibrium that we focus, non-members also hire every period). If he decides to default in any period t , he may default against either a member or a non-member. If he defaults against a member, he will never be hired by members again. Consequently, the payoff of defaulting can be obtained using the following recursive equation:

$$U_{Default} = w + \delta \left\{ X_{A,member} \frac{(1 - X_{A,member})w}{1 - \delta} + (1 - X_{A,member})U_{Default} \right\} \quad (24)$$

therefore, the provider is indifferent between defaulting and not defaulting if:

$$\frac{(1 - \delta)w}{1 - \delta(1 - X_{A,member})} + \delta \frac{X_{A,member}(1 - X_{A,member})w}{(1 - \delta(1 - X_{A,member}))} = w - e. \quad (25)$$

Simplifying it, we get:

$$\frac{\delta X_{A,member}^2 w}{1 - \delta(1 - X_{A,member})} = e \quad (26)$$

Solving the equation for $X_{A,member}$ and keeping in mind that $X_{A,member} \in [0, 1]$, we have

$$X_{A,member} = \frac{e\delta + \sqrt{e^2\delta^2 + 4\delta(1-\delta)we}}{2\delta w}. \quad (27)$$

Note that given the parameters e, δ, w there is only one value of $X_{A,member}$ that is consistent with a stationary equilibrium. Moreover, in a stationary equilibrium, conditions (19) and (23) must also hold.

Combining both conditions gives us:

$$f_{ee}^1 \leq \left(1 - \frac{w}{P}\right) \frac{w\delta X_{A,member}}{1 - \delta(1 - X_{A,member})}, \quad (28)$$

with equality when $Y_{A,member} = \left(1 - \frac{w}{P}\right)$.

Altruistic Bureau's Problem

As in section 3.1, the altruistic bureau's problem is to maximize an egalitarian social welfare function that equally weights consumers and providers utilities, conditional to some restrictions that include a break-even condition, i.e., that the bureau must be self-funded. Consequently, let's start looking at the bureau's profit function:

$$\Pi_{A,member} = X_{A,member} \left\{ \frac{f_{ee}^1}{1-\delta} - c - \frac{\delta(1-Y_{A,member})c}{1-\delta} - Y_{A,member}c \sum_{t=1}^{\infty} \delta^t (1 - X_{A,member})^t \right\} \quad (29)$$

from equation (19), we have that, after a few simplifications:

$$\Pi_{A,member} = \frac{X_{A,member}}{1 - \delta} \left\{ f_{ee}^1 \frac{c + w}{w} - c \right\} \quad (30)$$

where $X_{A,member}$ in equilibrium does not depend on f_{ee}^1 , so in order to keep the expression simple, we are not going to substitute it here.

Then, moving to the social welfare function, we have that the per period social welfare function is given by

$$SW_{A,member}(t) = \left\{ \frac{1}{2} \left[\begin{array}{l} (1 - Y_{A,member})(P - w) \\ + Y_{A,member}(-w) \\ + \frac{1}{2}(w - e) \end{array} \right] \right\} = \frac{1}{2} [(1 - Y_{A,member})P - e] \quad (31)$$

Consequently, the altruistic bureau's problem in the case of membership is given by:

$$\mathbf{SW}_{\mathbf{A},\mathbf{member}} \equiv \max_{f_{ee}^1} \frac{1}{2(1 - \delta)} [(1 - Y_{A,member})P - e] \quad (32)$$

subject to:

$$\frac{X_{A,member}}{1-\delta} \left\{ f_{ee}^1 \frac{c+w}{w} - c \right\} \geq 0 \quad (\text{C.1})$$

$$0 \leq Y_{A,member} \leq \frac{P-w}{P} \quad (\text{C.2})$$

$$Y_{A,member} = \frac{[1-\delta(1-X_{A,member})]}{\delta X_{A,member} w} f_{ee}^1 \quad (\text{C.3})$$

where again (C.1) is the break even constraint and the second constraint is obtained by a combination of $Y_{A,member} \in [0, 1]$ and equation (23). Restriction (C.3) is given by equation (19). Substituting (C.3) into (C.2) and the objective function, we can see that the objective function is linearly decreasing in

f_{ee}^1 . Therefore, at the optimum (C.1) must be binding:

$$f_{ee}^1 = \frac{cw}{w+c} \quad (33)$$

we are now able to show the following proposition:

Proposition 2 *Assume that the restriction presented in corollary 1 is satisfied with inequality. The policy maker would prefer a pricing mechanism based on membership instead of buy and sell. Moreover, the fraction of providers offering high-quality services and the fraction of informed consumers are both higher with membership.*

Moreover, from the proof of proposition 2, jointly with lemma 2 and corollary 1, we can easily show the following corollary:

Corollary 2 *In equilibrium, the constraint that guarantees that uninformed consumers buy the providers' services is non-binding in a membership set-up.*

Consequently, the policy-maker strictly prefers membership. Notice, however, that there is a clear trade-off between membership and buy-and-sell schemes. By creating a bureau based on membership, the policy maker avoids spending too much money by purchasing the information of non-members. While the bureau's informational content is not as deep as in the case of buy and sell information, the fact that it is cheaper implies that in equilibrium more consumers will become informed and the fraction of times that providers that default will not be hire actually goes up. However, since with membership not all information is aggregated by the bureau, even members face default in equilibrium. However, given that we have only one large bureau, the information propagates relatively fast, as we can observe in graph 1.

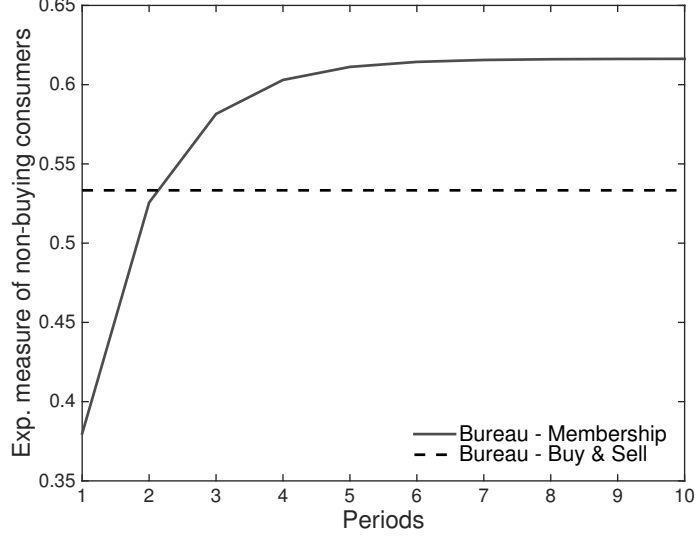
4 Profit-oriented Bureau: Monopoly case

The consumer's and provider's problems only depend on the pricing mechanism and actual fees charged by the bureau (and how these features affect the number of informed consumers and providers that put effort in equilibrium). Therefore, nothing that was presented in sections 3.1 and 3.2 about the consumers' and providers' problem need to be changed. Consequently, we just need to focus on the bureau's profit maximization and social welfare issues.

4.1 Buying and Selling information

4.1.1 Bureau's Profit Maximization

The bureau's profit maximization problem is given by:



Graph 1: Speed of learning with membership

$$\Pi_{M,buy} = -f_1 + \sum_{t=1}^{\infty} \delta^t \{X_{M,buy} [f_2 - (1 - Y_{M,buy})f_1] + (1 - X_{M,buy})(-f_1)\} \quad (34)$$

subject to:

$$0 \leq \frac{f_2}{w+c-f_1} \leq \min \left\{ \frac{P+f_1-(w+c)}{P}, 1 \right\} \quad (C.1)$$

$$f_1 \geq c \quad (C.2)$$

$$X_{M,buy} = \frac{e}{\delta w} \quad (C.3)$$

The bureau's profit maximization takes into account that it must buy information from all consumers in period 0. From period 1 on, it must buy information only from consumers that hire the service, corresponding to all uninformed consumers as well as all informed consumers matched with a provider that has not yet defaulted, i.e. $X_{M,buy} \times (1 - Y_{M,buy})$. The restrictions (C.1)–(C.3) are just a simplified version of the restrictions (C.2)–(C.7) presented at the altruistic bureau's problem, i.e., the same restrictions but the break-even condition. Rearranging the profit function, we have:

$$\Pi_{M,buy} = \frac{\delta X_{M,buy} f_2 - f_1 [1 - \delta X_{M,buy} Y_{M,buy}]}{1 - \delta} \quad (35)$$

We can then show the following lemma:

Lemma 4 *Assume that the parameter constraint defined in corollary 1 holds. A monopolist buy and sell bureau would optimally choose $f_1 = c$.*

Therefore, once we substitute f_1 , $X_{M,buy}$, and $Y_{M,buy}$, the bureau's problem becomes:

$$\max_{f_2} \frac{\frac{e}{w} [f_2 + (1 - \frac{w}{P}) c] - c}{1 - \delta} \quad (29')$$

subject to:

$$f_2 \leq \frac{(P-w)w}{P} \quad (\text{C.1}')$$

Given the linearity of the profit function, the constraint should be binding, i.e. $f_2 = \frac{(P-w)w}{P}$. Consequently, in equilibrium, we have:

$$Y_{M, \text{buy}} = 1 - \frac{w}{P}, \quad X_{M, \text{buy}} = \frac{e}{\delta w}, \quad f_1 = c, \quad f_2 = \frac{(P-w)w}{P}. \quad (36)$$

While the bureau's profit is:

$$\Pi_{M, \text{buy}} = \frac{1}{1-\delta} \left\{ \left[\frac{(w+c)(P-w)}{Pw} \right] e - c \right\} \quad (37)$$

Social Welfare

Let's now consider the social welfare function. Apart from the measure of providers that default in equilibrium $Y_{M, \text{buy}}$, the social welfare function in the monopoly case with buy and sell is the same as the one presented for the altruistic case in equations (9) to (11). Consequently, we have that

$$\mathbf{SW}_{M, \text{buy}} = \frac{1}{2(1-\delta)} \{ (1 - Y_{M, \text{buy}})P + f_1 - c - e \} \quad (38)$$

Substituting f_1 and $Y_{M, \text{buy}}$ using equation (36), we have:

$$\mathbf{SW}_{M, \text{buy}} = \frac{1}{2(1-\delta)} (w - e) \quad (39)$$

4.2 Membership

Bureau's Profit Maximization

The bureau's profit maximization problem in the case of membership is given by⁶:

$$\Pi_{M, \text{member}} \equiv \max_{f_{ee}^M} X_{M, \text{member}} \left[\frac{f_{ee}^M}{1-\delta} - c - \sum_{t=1}^{\infty} \delta^t \{ (1 - Y_{M, \text{member}}) + Y_{M, \text{member}} (1 - X_{M, \text{member}})^t \} c \right] \quad (40)$$

subject to:

$$f_{ee}^M \leq \left(1 - \frac{w}{P}\right) \frac{w\delta X_{M, \text{member}}}{1-\delta(1-X_{M, \text{member}})} \quad (\text{C.1})$$

The profit function takes into account the fact that, while all members must pay the membership fee, only the ones that hire the provider's service must be compensated for sending information to the bureau. Consequently, the only members that send information to the bureau at time t are the ones matched

⁶Since we already showed that the consumer is indifferent between joining the bureau in period 1 and period $t > 1$, we focus on membership at period 1 here

to providers with no registered history of default. There are two types of providers with a clean history at time t : Providers that always offer good services $(1 - Y_{M,member})$ and providers that provide bad service but have not matched with bureau members before, i.e., $Y_{M,member}(1 - X_{M,member})^t$. Moreover, the constraint is just a combined version of constraints (C.2) and (C.3) for altruistic bureau's problem presented in equation (32). Then simplifying and substituting equation (19), the bureau's problem becomes:

$$\Pi_{M,member} \equiv \max_{f_{ee}^M} X_{M,member} \left\{ \left(\frac{w+c}{w} \right) f_{ee}^M - \frac{c}{1-\delta} \right\} \quad (36')$$

subject to:

$$f_{ee}^M \leq \left(1 - \frac{w}{P}\right) \frac{w\delta X_{M,member}}{1-\delta(1-X_{M,member})} \quad (C.1)$$

Since $X_{M,member}$ does not depend on f_{ee}^M , we can see that $\Pi_{M,member}$ is linearly increasing in f_{ee}^M . Consequently, the restriction is binding and we have that: a

$$f_{ee}^M = \left(1 - \frac{w}{P}\right) \frac{w\delta X_{M,member}}{1-\delta(1-X_{M,member})} \quad (41)$$

Substituting equation (26) and manipulating, we obtain:

$$f_{ee}^M = \left(1 - \frac{w}{P}\right) \frac{e}{X_{M,member}} \quad (42)$$

and the profit of the monopolistic bureau that provides membership is then given by:

$$\Pi_{M,member} = \frac{1}{1-\delta} \left\{ \left[\frac{(w+c)(P-w)}{Pw} \right] e - cX_{M,member} \right\} \quad (43)$$

where $X_{M,member} = \frac{e\delta + \sqrt{e^2\delta^2 + 4\delta(1-\delta)we}}{2\delta w}$.

Remark: We assume that $X_{M,member} \in (0, 1)$ which is guaranteed by the assumption that $\delta > \frac{2e}{e+2w}$.

Comparing this expression with the profit of a monopolistic bureau buying and selling information, we can easily show the following result:

Proposition 3 *A monopolistic bureau prefers offering membership to buying and selling information.*

Social Welfare

Let's now consider the social welfare function. Apart from the measure of providers that default in equilibrium, the social welfare function in the monopoly case with membership is the same as the one presented for the altruistic case presented in equation (31). Consequently, we have that:

$$\mathbf{SW}_{M,member} = \frac{1}{2(1-\delta)} [(1 - Y_{M,member})P - e] = \frac{1}{2(1-\delta)} (w - e) \quad (44)$$

In this case, since $Y_{M,buy} = Y_{M,member}$, we have that $\mathbf{SW}_{M,buy} = \mathbf{SW}_{M,member}$. Moreover, both components, i.e. consumer and provider's surpluses are identical in both cases, even though the number of bureau members in the membership equilibrium is different from the number of informed consumers in the buy-sell information equilibrium. The reason for that it is that members have partial information while information buyers have full information. Consequently, in equilibrium, members may still face default, while information buyers will never face default.

5 Competitive Bureaus

5.1 Buy and Sell

Let's consider that there are two competing bureaus. Define f_1^i as the fee paid by bureau i in order to acquire consumers' information. Similarly, f_2^i is the fee charged by bureau i in order to sell information. Both fees are endogenously pinned down in equilibrium. We assume that bureaus cannot deny buying information from any given consumer, or that consumers can punish bureaus by only buying information from bureaus that also acquire information.⁷

We assume that information is non-rival, i.e., consumers can sell the information for multiple bureaus at the same time. Moreover, as before bureaus can verify that services were purchased and consequently only customers that bought the service are allowed to sell information.

We restrict our analysis here to the case in which all consumers sell information to all bureaus but only buy from one bureau. We assume they equally randomize across all bureaus, so they equally share the demand. Since each bureau buys information from all customers, we have that the information is the same, irrespective the bureau an agent buy information from. Therefore, the restriction that consumers only buy information from at most one bureau is without loss of generality. Moreover, notice that the consumer's and provider's problems are still exactly the same, apart from the fact that consumers buy information from the cheapest bureau. Therefore, we have:

$$X_{C,buy} = \frac{e}{\delta w} \quad \text{and} \quad Y_{C,buy} = \frac{\min\{f_2^i, f_2^{-i}\}}{w + c - \max\{f_1^i, f_1^{-i}\}} \quad (45)$$

Consequently, the main change is in the bureau's profit maximization. Then, we have that the

⁷We consider that consumers sell information before buying it, so they can observe deviations and punish appropriately.

bureau 1's demand for service can be given by Bertrand competition, i.e.:

$$D_1(f_2^1, f_2^2; f_1^1, f_1^2) = \begin{cases} X_{C,buy} & \text{if } f_2^1 < f_2^2 \text{ and } f_1^1 \geq c, f_1^2 \geq c \\ X_{C,buy} & \text{if } f_1^2 < c \text{ and } f_1^1 \geq c \\ \frac{X_{C,buy}}{2} & \text{if } f_2^1 = f_2^2 \text{ and } f_1^1 \geq c, f_1^2 \geq c \\ 0 & \text{if } f_2^1 > f_2^2 \text{ and } f_1^1 \geq c, f_1^2 \geq c \\ 0 & \text{if } f_1^1 < c \end{cases} \quad (46)$$

Similarly, for bureau 2, we have:

$$D_2(f_2^1, f_2^2; f_1^1, f_1^2) = \begin{cases} X_{C,buy} & \text{if } f_2^1 > f_2^2 \text{ and } f_1^1 \geq c, f_1^2 \geq c \\ X_{C,buy} & \text{if } f_1^1 < c \text{ and } f_1^2 \geq c \\ \frac{X_{C,buy}}{2} & \text{if } f_2^1 = f_2^2 \text{ and } f_1^1 \geq c, f_1^2 \geq c \\ 0 & \text{if } f_2^1 < f_2^2 \text{ and } f_1^1 \geq c, f_1^2 \geq c \\ 0 & \text{if } f_1^2 < c \end{cases} \quad (47)$$

Therefore, D_i measures the fraction of consumers that decide to buy information in equilibrium that buy the information from bureau i . Moreover, notice that D_i gives us additional information. It shows that consumers would not buy information from a bureau that does not pay enough to acquire information, i.e., $D_i(f_2^1, f_2^2; f_1^1, f_1^2) = 0$ if $f_1^i < c$. We consider the simplest sharing rule that if both firms ask the same fee f_2 , they evenly split the market. Hoernig [2007] presents alternative sharing rules. Given the fact that the consumers' problem is still the same and consequently $X_{C,buy} = \frac{e}{\delta w}$, the demand functions become:

$$D_1(f_2^1, f_2^2; f_1^1, f_1^2) = \begin{cases} \frac{e}{\delta w} & \text{if } f_2^1 < f_2^2 \text{ and } f_1^1 \geq c, f_1^2 \geq c \\ \frac{e}{\delta w} & \text{if } f_1^2 < c \text{ and } f_1^1 \geq c \\ \frac{e}{2\delta w} & \text{if } f_2^1 = f_2^2 \text{ and } f_1^1 \geq c, f_1^2 \geq c \\ 0 & \text{if } f_2^1 > f_2^2 \text{ and } f_1^1 \geq c, f_1^2 \geq c \\ 0 & \text{if } f_1^1 < c \end{cases} \quad (48)$$

Similarly, for bureau 2, we have:

$$D_2(f_2^1, f_2^2; f_1^1, f_1^2) = \begin{cases} \frac{e}{\delta w} & \text{if } f_2^1 > f_2^2 \text{ and } f_1^1 \geq c, f_1^2 \geq c \\ \frac{e}{\delta w} & \text{if } f_1^1 < c \text{ and } f_1^2 \geq c \\ \frac{e}{2\delta w} & \text{if } f_2^1 = f_2^2 \text{ and } f_1^1 \geq c, f_1^2 \geq c \\ 0 & \text{if } f_2^1 < f_2^2 \text{ and } f_1^1 \geq c, f_1^2 \geq c \\ 0 & \text{if } f_1^2 < c \end{cases} \quad (49)$$

Then, we have that the profit maximization problem for bureau i is given by:

$$\Pi_{C,buy} = \max_{f_1^i, f_2^i} \frac{\delta D_i(f_2^i, f_2^{-i}; f_1^i, f_1^{-i}) f_2^i - f_1^i (1 - \delta X_{C,buy} Y_{C,buy})}{1 - \delta} \quad (50)$$

subject to:

$$0 \leq \frac{f_2^i}{w+c-f_1^i} \leq \min \left\{ \frac{P+f_1^i-(w+c)}{P}, 1 \right\} \quad (C.1)$$

$$f_1^i \geq c \quad (C.2)$$

where the constraints are the same as in the monopoly case. Notice that if (C.2) is not satisfied for bureau i , we trivially have $\Pi_{buy}^i = 0$, regardless the other fees. Therefore, the easiest way for a bureau to shut down it is to establish a buying fee below cost c . In this case, its competitor becomes a monopoly. Consequently, we have that:

$$\Pi_{C,buy}^i = \frac{e(P-w)(w+c)}{Pw(1-\delta)} - \frac{c}{1-\delta}, \quad \text{if } f_1^{-i} < c \text{ and } f_1^i \geq c. \quad (51)$$

Then, if $f_1^{-i} \geq c$ and $f_1^i \geq c$, we have:

$$\Pi_{C,buy}^i = \begin{cases} - \left(1 - \frac{ef_2^{-i}}{w(w+c-\max\{f_1^i, f_1^{-i}\})} \right) \frac{f_1^i}{1-\delta} & \text{if } f_2^i > f_2^{-i} \\ \frac{1}{1-\delta} \left[\frac{e}{2w} f_2^i - \left(1 - \frac{ef_2^i}{w(w+c-\max\{f_1^i, f_1^{-i}\})} \right) f_1^i \right] & \text{if } f_2^i = f_2^{-i} \\ \frac{1}{1-\delta} \left[\frac{e}{w} f_2^i - \left(1 - \frac{ef_2^i}{w(w+c-\max\{f_1^i, f_1^{-i}\})} \right) f_1^i \right] & \text{if } f_2^i < f_2^{-i} \end{cases} \quad (52)$$

As long as $f_1^i \geq c$ is satisfied, f_1^i only affects the cost of acquiring information. Therefore, following a similar argument presented in the proof of lemma 4, we can show that it is optimal for the bureaus to

set $f_1 = c$:

$$\Pi_{C, buy}^i = \begin{cases} -\left(1 - \frac{ef_2^{-i}}{w^2}\right) \frac{c}{1-\delta} & \text{if } f_2^i > f_2^{-i} \\ \frac{1}{1-\delta} \left[\frac{e}{2w} f_2^i - \left(1 - \frac{ef_2^i}{w^2}\right) c \right] & \text{if } f_2^i = f_2^{-i} \\ \frac{1}{1-\delta} \left[\frac{e}{w} f_2^i - \left(1 - \frac{ef_2^i}{w^2}\right) c \right] & \text{if } f_2^i < f_2^{-i} \end{cases} \quad (53)$$

Then bureau i 's reaction correspondence is given by:

$$R^i(f_1^{-i}, f_2^{-i}) = \begin{cases} f_2^i = \frac{(P-w)w}{P} \text{ and } f_1^i = c & \text{if } f_1^{-i} < c; \\ f_2^i = \frac{(P-w)w}{P} \text{ and } f_1^i = c & \text{if } f_1^{-i} \geq c \text{ and } f_2^{-i} > \frac{(P-w)w}{P}; \\ f_2^i = f_2^{-i} - \varepsilon \text{ and } f_1^i = c & \text{if } f_1^{-i} \geq c \text{ and } \frac{2cw^2}{e(w+2c)} < f_2^{-i} \leq \frac{(P-w)w}{P} \\ f_2^i = \frac{2cw^2}{e(w+2c)} \text{ and } f_1^i = c & \text{if } f_1^{-i} \geq c \text{ and } f_2^{-i} = \frac{2cw^2}{e(w+2c)} \\ f_2^i \in \mathbb{R}_+ \text{ and } f_1^i < c & \text{if } f_1^{-i} \geq c \text{ and } f_2^{-i} < \frac{2cw^2}{e(w+2c)} \end{cases} \quad (54)$$

where ε is arbitrarily small. Notice that a bureau decides to leave the market by placing a purchase fee $f_1 < c$. Consequently, in an equilibrium in which we have two active bureaus, we have:

$$f_2^1 = f_2^2 = \frac{2cw^2}{e(w+2c)} \text{ and } f_1^1 = f_1^2 = c \quad (55)$$

with

$$\Pi_{buy}^1 = \Pi_{buy}^2 = 0 \text{ and } X_{buy} = \frac{e}{\delta w}, Y_{buy} = \frac{2wc}{e(w+2c)} \quad (56)$$

Remark: In order for (C.1) in equation (50) not to bind, we must have:

$$\frac{2cw}{e(w+2c)} < \frac{P-w}{P} \quad (57)$$

Notice that, since $\frac{2cw}{e(w+2c)} > \frac{cw}{e(w+c)}$, we have that if the restriction in equation (15) is satisfied, (57) must be satisfied as well. Consequently, given that the parameter restriction presented in corollary 1 is satisfied, the inequality presented in equation (57) is satisfied as well.

Social Welfare

Let's now consider the social welfare function. Apart from the measure of providers that default in equilibrium $Y_{C, buy}$, the social welfare function in the competitive case with buy and sell it is the same as the one presented for the altruistic case in equations (9) to (11). Consequently, we have that, given $f_1^1 = f_1^2 = c$:

$$SW_{C, buy} = \frac{1}{2(1-\delta)} \{(1 - Y_{C, buy})P - e\} \quad (58)$$

Substituting $Y_{C,buy}$ using equation (56), we have:

$$\mathbf{SW}_{C,buy} = \frac{1}{2(1-\delta)} \left\{ \left[1 - \frac{2wc}{e(w+2c)} \right] P - e \right\} \quad (59)$$

Then, we are ready to show the following proposition:

Proposition 4 *Assume that the parameter restriction presented in corollary 1 is satisfied with inequality. Then, we have:*

$$\mathbf{SW}_{A,member} > \mathbf{SW}_{A,buy} > \mathbf{SW}_{C,buy} > \mathbf{SW}_{M,buy} = \mathbf{SW}_{M,member}$$

Therefore, we can rank the different bureau designs and pricing mechanisms, with an altruistic bureau with membership at the top.

5.2 Membership

In this session we develop a model of competition between bureaus with a membership pricing mechanism. Specifically, we consider two bureaus A and B where bureau i charges f_i for the membership. In the stationary equilibria that we consider, each bureau has a consumer base X_i ($X_A + X_B \leq 1$) and there is a fraction $Y_{C,member}$ of providers that choose to default every period. The timing of this game is the following: first, the bureaus post their membership fees simultaneously, then the consumers and providers play an infinitely repeated game with private histories given the fees that were posted.⁸ We look for the subgame perfect equilibria of this repeated game.

For ease of exposition, we will construct the equilibria by considering each group of agents' incentives separately. We start with the incentives of the providers.

Providers Let us now have a closer look at the providers' incentives. We introduce the following notation: U^0 is the expected continuation utility of a provider that decides to default and: (i) either has never defaulted before or (ii) has defaulted but has never been caught by an agent belonging to a membership; U^i is a provider that has been caught by at least a member of bureau i , but has not interacted with members of bureau $j \neq i$, and U^{AB} is the expected continuation payoff of a bureau that has been caught by at least a member of A and by at least a member of B . These expected continuation payoffs can be written in recursive form as follows:

$$\begin{aligned} U^0 &= w + \delta (X_A U^A + X_B U^B + (1 - X_A - X_B) U^0) \\ U^A &= X_A \delta U^A + X_B (w + \delta U^{AB}) + (1 - X_A - X_B) (w + \delta U^A) \\ U^B &= X_A (w + \delta U^{AB}) + X_B \delta U^B + (1 - X_A - X_B) (w + \delta U^B) \\ U^{AB} &= X_A \delta U^{AB} + X_B \delta U^{AB} + (1 - X_A - X_B) (w + \delta U^{AB}) \end{aligned}$$

⁸For simplicity, we assume here that all membership affiliations are decided at this initial time.

We can solve this system of linear equations starting from U^{AB} :

$$U^{AB} = \frac{(1 - X_A - X_B) w}{1 - \delta} \quad (60)$$

Thus, we can write U^A as:

$$U^A = \frac{w(1 - X_A)}{1 - \delta(1 - X_B)} + X_B \delta \frac{(1 - X_A - X_B) w}{(1 - \delta)(1 - \delta(1 - X_B))} \quad (61)$$

Similarly,

$$U^B = \frac{w(1 - X_B)}{1 - \delta(1 - X_A)} + X_A \delta \frac{(1 - X_A - X_B) w}{(1 - \delta)(1 - \delta(1 - X_A))} \quad (62)$$

We can finally solve for U^0 , and we have that:

$$U^0 = \frac{w}{1 - \delta(1 - X_A - X_B)} + \delta \frac{w}{1 - \delta(1 - X_A - X_B)} \left(\frac{X_A(1 - X_A)}{1 - \delta(1 - X_B)} + X_A X_B \delta \frac{(1 - X_A - X_B)}{(1 - \delta)(1 - \delta(1 - X_B))} + \frac{X_B(1 - X_B)}{1 - \delta(1 - X_A)} + X_B X_A \delta \frac{(1 - X_A - X_B)}{(1 - \delta)(1 - \delta(1 - X_A))} \right) \quad (63)$$

In the first period, providers are indifferent between a default and providing effort when hired if $U^0 = w - e$. This leads us to our first stationary equilibrium condition:

$$\frac{w}{1 - \delta(1 - X_A - X_B)} \left\{ 1 + \delta \left(\frac{X_A(1 - X_A)}{1 - \delta(1 - X_B)} + X_A X_B \delta \frac{(1 - X_A - X_B)}{(1 - \delta)(1 - \delta(1 - X_B))} + \frac{X_B(1 - X_B)}{1 - \delta(1 - X_A)} + X_B X_A \delta \frac{(1 - X_A - X_B)}{(1 - \delta)(1 - \delta(1 - X_A))} \right) \right\} = \frac{w - e}{1 - \delta} \quad (64)$$

Let us define the differentiable function $F : [0, 1] \times [0, 1] \rightarrow \mathbb{R}$ as

$$F(X_A, X_B) = \frac{w}{1 - \delta(1 - X_A - X_B)} \left\{ 1 + \delta \left(\frac{X_A(1 - X_A)}{1 - \delta(1 - X_B)} + X_A X_B \delta \frac{(1 - X_A - X_B)}{(1 - \delta)(1 - \delta(1 - X_B))} + \frac{X_B(1 - X_B)}{1 - \delta(1 - X_A)} + X_B X_A \delta \frac{(1 - X_A - X_B)}{(1 - \delta)(1 - \delta(1 - X_A))} \right) \right\} - \frac{w - e}{1 - \delta}$$

and denote the level set of F corresponding to 0 by the set $\{(X_A, X_B) : X_A + X_B \leq 1 \text{ and } F(X_A, X_B) = 0\}$.

In words, these are the possible combinations of each pair of consumer bases that makes the providers indifferent between exerting effort or not. Note that $F(X_A, 0) = F(0, X_B)$ and x that solves $F(x, 0) = F(0, x) = 0$ is $X_A = \frac{e\delta + \sqrt{e^2\delta^2 + 4\delta(1-\delta)we}}{2\delta w} < 1$. Moreover, we have the following result relating the consumer basis of the two bureaus in the stationary equilibria with competitive bureaus.

Lemma 5 *Equation (64) defines a strictly decreasing relationship between X_A and X_B .*

Consumers Whenever there are two bureaus operating in equilibrium, that is, with positive consumer bases, the utility of the consumers from buying from bureau A must be the same as buying from bureau

B and the same as not buying at all. This lead us to the following indifference conditions:

$$(1 - Y_{C,member})(P - w) - f_i + (1 - \delta) \sum_{t=0}^{\infty} \delta^t (1 - X_i)^t Y_{C,member}(-w) = (1 - Y_{C,member})P - w,$$

where the LHS is the consumer's payoff from joining bureau i and the RHS is the payoff of not joining any bureau. Given that the indifference must hold for both bureaus, we have the following two equations

$$f_i = Y_{C,member} \frac{w\delta X_i}{1 - \delta(1 - X_i)}, \forall i = A, B \quad (65)$$

The ratio of fees is given by:

$$\frac{f_A}{f_B} = \frac{X_A(1 - \delta) + \delta X_A X_B}{X_B(1 - \delta) + \delta X_A X_B} \quad (66)$$

Proposition 5 *For any pair of fees (f_A, f_B) , there is at most one stationary equilibrium with two operating bureaus ($X_A > 0$ and $X_B > 0$) in the continuation game.*

Implicit in the statement of the proposition above is the fact that there might be multiple equilibria in the continuation game in which either only one bureau operates (that is, $X_i > 0$ and $X_j = 0$) or in which there is no bureau operating, so essentially no market ($X_A = X_B = 0$).

In any stationary equilibria, we can partition the set of consumers in two subsets: consumers who join at least one bureau and consumers who do not join any bureau. The last condition that we need to construct a stationary equilibrium with two operating bureaus is the condition that the consumers who do not join any bureau will also find it profitable to hire providers, despite the fact that they will not have any information on the provider's past behavior. This gives us our last equilibrium condition:

$$Y_{C,member} \leq \frac{P - w}{P} \quad (67)$$

It will be convenient to define a feasible set for the providers' fees. Let us note that there is an upper bound for a fee of a bureau that operates in equilibrium. We can compute this upper bound using (65) and (67) and the fact that X_i has an upper bound which is given by $X_i \leq \frac{e\delta + \sqrt{e^2\delta^2 + 4\delta(1-\delta)we}}{2\delta w}$. A fee f_i will be said to be *feasible* if it is below this upper bound, that is, if there exists a stationary equilibrium in which consumers buy from firm i at this fee f_i :

$$f_i \leq \frac{P - w}{P} w \frac{\delta \frac{e\delta + \sqrt{e^2\delta^2 + 4\delta(1-\delta)we}}{2\delta w}}{1 - \delta \left(1 - \frac{e\delta + \sqrt{e^2\delta^2 + 4\delta(1-\delta)we}}{2\delta w} \right)}. \quad (68)$$

Before we proceed with our full equilibrium analysis, let us discuss our equilibrium refinement. Starting from a given pair (f_i, f_j) , a deviation by one of the two bureaus, say bureau i , leads to a new pair (f'_i, f_j) in which it is possible to construct a stationary equilibrium of the continuation game

in which (i) both bureaus still operate; (ii) only bureau i operates; (iii) only bureau j operates (iv) neither one of the two bureaus operates. Our refinement will be to consider equilibria in which the equilibrium after the deviation is the one in which both bureaus operate, if such an equilibrium exists. If, after a deviation, there is no equilibrium with two operating bureaus, then we will consider only the equilibrium in which only the cheapest bureau operates.⁹

Given that each bureau's profit is a direct function of $fee * X$, any deviation that increases f sounds like a good deviation. But can the bureau increase its fee without bounds? Note that if $f_A > Y_{C,member} \frac{w\delta X_A}{1-\delta(1-X_A)}$, then nobody buys from bureau A . Therefore, certainly (68) imposes an upper bound for a fee of an operating bureau. This imposes a non-tight bound.

Consider a case in which both bureaus set the same fee and suppose that

$$f_A = f_B = \frac{P-w}{P} \frac{w\delta X}{1-\delta(1-X)},$$

where X solves

$$\frac{(1-\delta)w}{1-\delta(1-2X)} + \delta 2 \frac{wX}{(1-\delta(1-2X))(1-\delta(1-X))} ((1-\delta)(1-X) + X\delta(1-2X)) = w - e \quad (69)$$

Denote the X that solves the above equation by X^{sym} . Thus, consider a situation in which both firms charge

$$\bar{f} = \frac{P-w}{P} \frac{w\delta X^{sym}}{1-\delta(1-X^{sym})} \quad (70)$$

Before we prove the main result in this section, the next lemma will be useful.

Lemma 6 *For any given pair (f_i, f_j) with $f_i \leq f_j \leq \frac{P-w}{P} \frac{w\delta X^{sym}}{1-\delta(1-X^{sym})}$ there exists an equilibrium with two operating bureaus (unique in this class) in the continuation game.*

Now we are ready to prove the main result of this section.

Proposition 6 (Unique Stationary Competitive Equilibrium) *There is a unique stationary equilibrium in which both bureaus operate. In this equilibrium, $f_i = f_j = \frac{P-w}{P} \frac{w\delta X^{sym}}{1-\delta(1-X^{sym})}$, with symmetric consumer bases $X_i = X_j = X^{sym}$ and $Y_{C,member} = \frac{P-w}{P}$.*

Therefore, there is a unique stationary equilibrium in the duopoly competition game where the firms set fee simultaneously and the consumers and providers play an infinitely repeated game following the chosen fees. In this equilibrium, the fraction of providers who default is $Y_{C,member} = \frac{P-w}{P}$, which is the lowest possible fraction that sustains a stationary equilibrium.

⁹Suppose, instead, that we consider a refinement in which whenever there is no equilibrium with two bureaus, the consumers buy only from the most expensive bureau. Then, there are two equilibria only, one in which bureau i is the monopolist and one in which j is the monopolist. For i monopolist, the fee is given by $f_i = \frac{P-w}{P} w \frac{\delta X_i}{1-\delta(1-X_i)}$, $X_i = \frac{e\delta + \sqrt{e^2\delta^2 + 4\delta(1-\delta)we}}{2\delta w}$ and $Y_{C,member} = \frac{P-w}{P}$, $f_j \geq f_i$ and $X_j = 0$.

5.2.1 Social Welfare

Recall that the social welfare is given by:

$$SW_t = \frac{1}{2} \{U_{consumer} + U_{providers}\} \quad (71)$$

We are focusing on equilibria in which consumers are indifferent between buying a membership from bureau i, j , or not buying at all, but hiring nonetheless. Thus, we have that:

$$U_{consumer} = (1 - Y_{C,member})P - w \quad \text{and} \quad U_{provider} = w - e \quad (72)$$

where $Y_{C,member} = \frac{P-w}{P}$, so that $U_{consumer} = (1 - \frac{P-w}{P})P - w = 0$. The social welfare becomes

$$SW_{C,member} = \frac{1}{2(1-\delta)} \{w - e\} \quad (73)$$

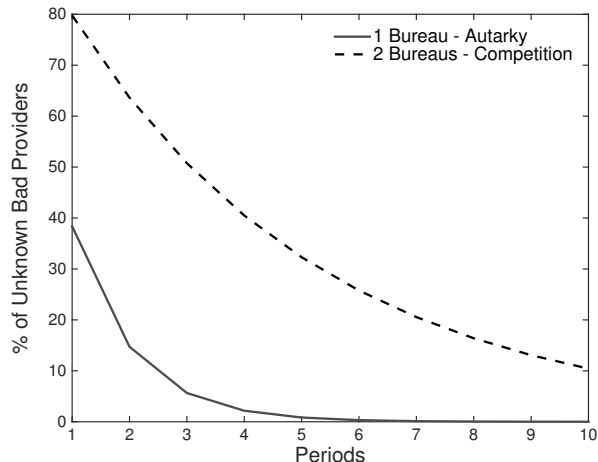
Given that $SW_{M,buy} = \frac{1}{2(1-\delta)}(w - e)$, we have: $SW_{C,member} = SW_{M,buy}$. Thus, assuming that the parameter restriction in corollary 1 is satisfied with inequality, we have:

$$SW_{A,member} > SW_{A,buy} > SW_{C,buy} > SW_{M,buy} = SW_{M,member} = SW_{C,member}$$

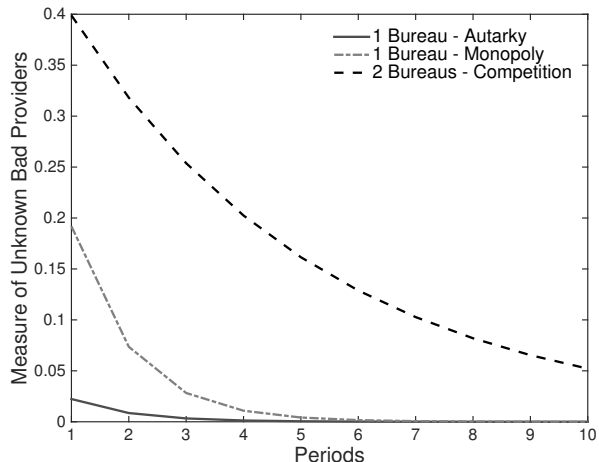
Consequently, competition, while possibly improving in comparison with monopoly, it is significantly worse than either case of the altruistic bureau. There are several reasons for this. First of all, as mentioned in section 5.1, the presence of competition in a buy and sell set up generates a duplication of direct costs, significantly decreasing the benefits of the bureau introduction. Moreover, in the case of membership, the indirect costs become significantly higher, not only because bureaus are smaller, but also because the measure of defaulting providers must be higher in equilibrium in order to sustain multiple bureaus. In summary, not only learning is slowed down, as we see in graph 2a, but also the indirect costs through facing default while informed are consistently higher throughout, as we can see in graph 2b, where competition seems to consistently under-perform even compared to a monopolist. In this sense, information trade in many ways present the same characteristics as natural monopolies, where in order to avoid the duplication of costs and harvest the benefits of economy of scale, the optimal number of producers (or in this case information brokers) is one, while regulated in order to avoid a concentration of market power.

6 Conclusion

In this paper, we show that not only the availability of information matters for a well-functioning market, but also how information is negotiated. The pricing and selling mechanisms, as well as the number of information brokers in the market are important to determine not only how many agents choose to



Graph 2a: Speed of learning with membership



Graph 2b: Normalized indirect costs

become informed, but also the quality of information available to them. At the end, these features will pin down how much discipline the information trade imposes in both sides of the market, affecting provider's incentives and ultimately the social welfare in the economy. These results are true even in an environment in which we disregard insurance issues. In particular, we consider different information pricing mechanisms – membership vs. buy and sell information – and competitive environment – non-profit, monopoly, and competitive – in an economy with random matching between a large population of consumers and providers. We show that both dimensions affect direct and indirect costs, represented by fees and expected loss due to default while informed, respectively. In particular, we show that information trade has similar characteristics to a natural monopoly, where competition may hurt due to duplication of costs as well as by slowing down the information aggregation by each individual information broker. Moreover, we show that there is a trade-off between information quality and cost. In particular, in a world with only one non-profit information bureau, a membership set up, while having lower information quality, induces low enough direct costs through fees that more than compensate the initially high indirect costs. However, this is only true due to the fact that the bureau is large enough to quickly learn and reduce indirect costs.

Finally, we would like to emphasize that risk aversion may significantly change our results, since the bureau may be able to provide insurance against losses through default by paying more for the reported information. However, this introduces an additional trade-off between insurance and the incentive to buy information, that may also influence the provider's incentives of exercising effort. Additional research is needed in order to disentangle these additional complications.

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Appendix - Proofs

Proof of Lemma 1:

Proof. Initially, let's consider the case in which (C.3') is not binding. In this case, we focus on (C.1') and (C.2'). Then, notice that:

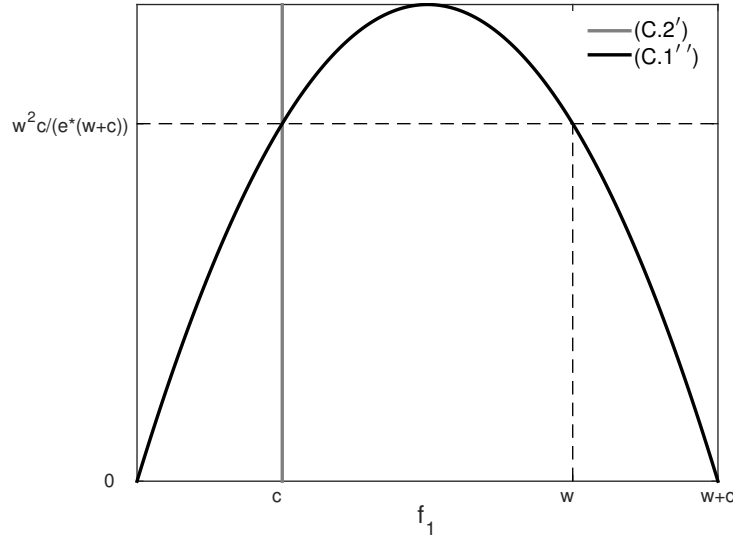
$$\frac{\partial \mathbf{SW}_{\mathbf{A},\text{buy}}}{\partial f_2} = -\frac{P}{w+c-f_1} < 0$$

and

$$\frac{\partial \mathbf{SW}_{\mathbf{A},\text{buy}}}{\partial f_1} = -\frac{f_2}{[w+c-f_1]^2} P - 1 < 0$$

So $\mathbf{SW}_{\mathbf{A},\text{buy}}$ is strictly decreasing in both f_2 and f_1 . Moreover, from (C.1''), we have that RHS(C.1'') is strictly concave and it has roots at $f_1 = 0$ and $f_1 = w+c$. Moreover, since $\mathbf{SW}_{\mathbf{A},\text{buy}}$ is strictly decreasing in f_2 , in order to maximize social welfare, (C.1'') must be satisfied with equality. Then, let's consider two cases:

Case 1: $w \geq c$: In this case, constraints (C.1'') and (C.2') can be graphically represented as:



Notice that, while $f_1 < c$ does not satisfy (C.2'), we also have that for $f_1 \in (c, w)$, in order to satisfy (C.1'') we must have that both $f_1 > c$ and $f_2 > \frac{w^2c}{e(w+c)}$, implying a lower $\mathbf{SW}_{\mathbf{A},\text{buy}}$. Similarly, notice that

$$\mathbf{SW}_{\mathbf{A},\text{buy}}(f_1 = c) - \mathbf{SW}_{\mathbf{A},\text{buy}}(f_1 = w) = \frac{w-c}{2(1-\delta)} \left\{ \frac{w}{e(w+c)} P + 1 \right\} > 0.$$

Then, we just need to evaluate the cases in which $f_1 = w + \alpha c$, with $\alpha \in (0, 1)$. In this case, we have:

$$\mathbf{SW}_{\mathbf{A},\text{buy}}(f_1 = c) - \mathbf{SW}_{\mathbf{A},\text{buy}}(f_1 = w + \alpha c) = \frac{w - (1-\alpha)c}{2(1-\delta)} \left\{ \frac{w}{e(w+c)} P + 1 \right\} > 0.$$

Consequently, we are unable to increase $\mathbf{SW}_{\mathbf{A},\text{buy}}$ by setting $f_1 > c$.

Case 2: $w < c$: In this case, we just need to consider setting $f_1 = c + \alpha w$ with $\alpha \in (0, 1)$. In this case, we have:

$$\mathbf{SW}_{\mathbf{A},\text{buy}}(f_1 = c) - \mathbf{SW}_{\mathbf{A},\text{buy}}(f_1 = c + \alpha w) = \frac{\alpha w}{2(1 - \delta)} \left\{ \frac{w}{e(c + w)} P + 1 \right\} > 0.$$

Consequently, again we are unable to increase $\mathbf{SW}_{\mathbf{A},\text{buy}}$ by setting $f_1 > c$.

Now, let's consider the possibility that (C.3') is binding. First of all, from (C.1'') and (C.3') notice that, in order to satisfy both constraints, we must have that:

$$\frac{w}{e} f_1 \left[\frac{w + c - f_1}{w + c} \right] \leq f_2 \leq \left[\frac{P - (w + c - f_1)}{P} \right] (w + c - f_1)$$

In order for this restriction to be satisfied, we must have:

$$\text{Gap} = \left[\frac{P - (w + c - f_1)}{P} \right] (w + c - f_1) - \frac{w}{e} f_1 \left[\frac{w + c - f_1}{w + c} \right] \geq 0$$

Rearranging it:

$$\text{Gap} = \left[\frac{P - (w + c - f_1)}{P} - \frac{w}{e(w + c)} f_1 \right] (w + c - f_1) \geq 0$$

or

$$\text{Gap} = \left[\frac{Pw - e(w + c)}{Pe(w + c)} \right] f_1^2 + \left[\frac{2e(w + c) - P(w + c)}{Pe} \right] f_1 + (w + c) \left[\frac{P - (w + c)}{P} \right] \geq 0$$

that has roots: $\underline{f}_1 = \left(\frac{P - (w + c)}{P - \frac{e}{w}(w + c)} \right) e \left(\frac{w + c}{w} \right)$ and $\bar{f}_1 = w + c$. Then, notice that the range in which the solution is satisfied depends on the concavity of Gap. Let's consider the different cases:

case a: $\left[\frac{Pw - e(w + c)}{Pe(w + c)} \right] > 0$. In this case, Gap is strictly convex. In order to satisfy both constraints, we must have $f_1 < \underline{f}_1$. But then, as long as $\underline{f}_1 \geq c$, we are back to the previous cases and the optimal $f_1 = c$. Otherwise, if $\underline{f}_1 < c$ there is no solution that satisfies all constraints and the market collapses.

case b: $\left[\frac{Pw - e(w + c)}{Pe(w + c)} \right] < 0$. In this case, Gap is strictly concave. We can in principle eliminate this case through the following reasoning. If $\left[\frac{Pw - e(w + c)}{Pe(w + c)} \right] < 0 \Rightarrow P < (w + c)$, since $e < w$. Then, given that we must have $f_1 < w + c$ - otherwise restrictions constraints (C.1), (C.4), and (C.6) cannot be jointly satisfied - we have that restrictions (C.2) and (C.4) cannot be jointly satisfied. Therefore in this case the market collapses.

For the same reasons presented in case b, if $\left[\frac{Pw - e(w + c)}{Pe(w + c)} \right] = 0$ the market collapses.

Consequently, $f_1 = c$ is optimal for the bureau. ■

Proof of Lemma 2

Proof.

From the proof of lemma 1, we showed that, in order for both restrictions (C.1'') and (C.3') to be

satisfied, we must have:

$$\left(\frac{P - (w + c)}{P - \frac{e}{w}(w + c)} \right) e \left(\frac{w + c}{w} \right) \geq c$$

Rearranging it:

$$\frac{P - (w + c)}{P - \frac{e}{w}(w + c)} \geq \frac{wc}{e(w + c)} \quad (74)$$

Rearranging and manipulating it, we have:

$$\frac{wc}{e(w + c)} \leq \frac{P - w}{P}$$

Consequently, for the cases in which we have a functioning market, (15) is satisfied and (C.3') is non-binding. ■

Proof of Corollary 1

Proof. From the proof of lemma 1, we showed that, in order for both restrictions (C.1'') and (C.3') to be satisfied, we must have:

$$\left(\frac{P - (w + c)}{P - \frac{e}{w}(w + c)} \right) e \left(\frac{w + c}{w} \right) \geq c$$

Rearranging and manipulating the above equation, we obtain:

$$c \leq \frac{ew(P - w)}{P(w - e) + ew} \quad (75)$$

Similarly, from the proof of lemma 1, case b, we showed that in order to satisfy jointly restrictions (C.2) and (C.4), we also needed:

$$Pw - e(w + c) > 0$$

Rearranging, we have:

$$c < \frac{Pw - ew}{e} \quad (76)$$

Now, let's comparing the RHS((75)) against RHS((76)). Then notice that:

$$\frac{Pw - ew}{e} - \frac{ew(P - w)}{P(w - e) + ew} = \frac{P^2w(w - e)}{e[P(w - e) + ew]} > 0$$

Therefore, whenever (75) is satisfied, (76) is also satisfied, concluding that our only constraint is (75). ■

Proof of Lemma 3

Proof. The one shot deviation from delaying membership by one period is suboptimal if:

$$\left\{ \begin{array}{l} (1 - Y_{A,member})(P - w) - f_{ee}^\tau \\ -(1 - \delta)Y_{A,member}w \left[1 + \frac{\delta(1 - X_{A,member})^\tau}{1 - \delta(1 - X_{A,member})} \right] \end{array} \right\} \geq \left\{ \begin{array}{l} (1 - Y_{A,member})(P - w) - \delta f_{ee}^{\tau+1} \\ -(1 - \delta)Y_{A,member}w \left[1 + \delta + \frac{\delta^2(1 - X_{A,member})^{\tau+1}}{1 - \delta(1 - X_{A,member})} \right] \end{array} \right\}$$

Simplifying it:

$$\delta f_{ee}^{\tau+1} - f_{ee}^{\tau} \geq -(1-\delta)Y_{A,member}w\delta [1 - (1 - X_{A,member})^{\tau}]$$

Finally, substituting $f_{ee}^{\tau+1}$ and f_{ee}^{τ} and simplifying, we have:

$$-Y_{A,member}w\delta [1 - (1 - X_{A,member})^{\tau}] \geq -Y_{A,member}w\delta [1 - (1 - X_{A,member})^{\tau}]$$

■

Proof of Proposition 2

Proof. Substituting equation (26) into restriction (C.3) in the bureau's problem shown in (32), we have:

$$Y_{A,member} = \frac{[1 - \delta(1 - X_{A,member})]}{\delta X_{A,member}w} f_{ee} = \frac{X_{A,member}}{e} f_{ee}$$

Substituting f_{ee} :

$$Y_{A,member} = \frac{X_{A,member}}{e} \frac{cw}{(w+c)} = X_{A,member} \frac{cw}{e(w+c)} = X_{A,member} Y_{A,buy}$$

Consequently $Y_{A,member} < Y_{A,buy} \Rightarrow SW_{A,member} > SW_{A,buy}$.

Finally, we just need to show that the fraction of informed consumers is higher with membership, i.e., $X_{A,member} > X_{A,buy}$. In order to show that, first notice that the LHS of (1) is strictly decreasing in $X_{A,buy}$. Moreover, notice that $LHS(1) = w$ if $X_{A,buy} = 0$ and $LHS(1) = (1-\delta)w$ if $X_{A,buy} = 1$. Differently, rearranging equation (25), we have:

$$w \left\{ 1 - \frac{\delta X_{A,member}^2}{1 - \delta [1 - X_{A,member}]} \right\} = w - e \quad (24')$$

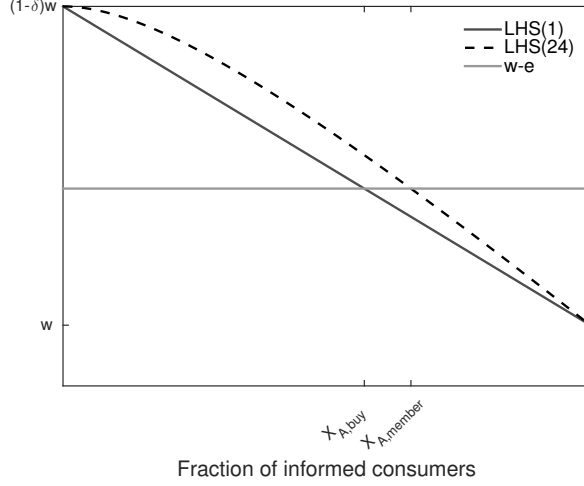
Again, notice that $LHS(24') = w$ if $X_{A,buy} = 0$ and $LHS(24') = (1-\delta)w$ if $X_{A,buy} = 1$. Moreover, taking the derivative of $LHS(24')$ with respect to $X_{A,member}$, we have:

$$\frac{dLHS(24')}{dX_{A,member}} = -\delta w \left\{ \frac{\delta X_{A,member}^2 + 2X_{A,member}(1-\delta)}{[1 - \delta(1 - X_{A,member})]^2} \right\} < 0. \quad (77)$$

Moreover:

$$\frac{d^2LHS(24')}{dX_{A,member}^2} = -\frac{2\delta(1-\delta)w}{[1 - \delta(1 - X_{A,member})]^3} < 0 \quad (78)$$

Consequently, $LHS(24')$ is strictly decreasing and concave in $X_{A,member}$. Then, given that $RHS(1) = RHS(24')$, we must have that $X_{A,member} > X_{A,buy}$. Graph 3 below illustrates the result.



Graph 3: Comparison $X_{A,buy}$ and $X_{A,member}$

■

Proof of Lemma 4

Proof. Let's consider that the monopolist sets $f_1 = c + \gamma$, with $\gamma \in [0, w)$. Then, we have that:

$$\Pi_{M,buy} = \frac{1}{1-\delta} \left\{ \frac{e}{w} f_2 \left(\frac{w+c}{w-\gamma} \right) - w - \gamma \right\}$$

subject to:

$$\frac{f_2}{w-\gamma} \leq \frac{P+\gamma-w}{P} \quad (C.1')$$

Since the objective function is linearly increasing in f_2 , (C.1') must be satisfied with equality, i.e., $f_2 = \frac{P+\gamma-w}{P}$. Consequently, the profit function becomes:

$$\Pi_{M,buy} = \frac{1}{1-\delta} \left\{ \frac{e}{w} \left(\frac{P+\gamma-w}{P} \right) (w+c) - \gamma - w \right\}$$

Notice that:

$$\frac{\partial \Pi_{M,buy}}{\partial \gamma} = \frac{1}{1-\delta} \left\{ \frac{e}{w} \left(\frac{w+c}{P} \right) - 1 \right\}$$

Given that the constraint in corollary 1 is satisfied, as we showed in Lemma 1, we have that $P - \frac{e}{w}(w+c) > 0$. Rearranging it, we have that $\frac{e}{w} \left(\frac{w+c}{P} \right) - 1 < 0$. Consequently, $\frac{\partial \Pi_{M,buy}}{\partial \gamma} < 0$ and it's optimal to minimize γ , setting $\gamma = 0$. ■

Proof of Proposition 3

Proof. Comparing equations (43) and (37), we have that, since $X_{M,member} < 1$, $\Pi_{M,member} > \Pi_{M,buy}$.

■

Proof of Proposition 4

Proof. First of all, notice that proposition 2 and corollary 2 showed the first inequality in the left, while we showed the last equality in section 4. Therefore, we just need to show the two inequalities in the middle. In order to show that $SW_{A,buy} \geq SW_{C,buy}$, notice that:

$$Y_{C,buy} - Y_{A,buy} = \frac{2wc}{e(w+2c)} - \frac{wc}{e(w+c)} = \frac{wc}{e} \left[\frac{2}{w+2c} - \frac{1}{w+c} \right] = \frac{w^2c}{e(w+c)(w+2c)} > 0$$

Since $SW = \frac{1}{2\delta}(1-Y)P - e$, once $Y_{A,buy} < Y_{C,buy}$, we must have $SW_{A,buy} > SW_{C,buy}$. The equality occurs if we have that the constraint on uninformed consumers having to buy information is binding. However, as we mentioned in the last remark, this is only binding for $Y_{A,buy}$ if it is also binding for $Y_{C,buy}$. Finally let's look at $SW_{C,buy} \geq SW_{M,buy}$. Notice that if (57) is satisfied, then $Y_{C,buy} < Y_{M,buy}$ and $SW_{C,buy} > SW_{M,buy}$. If (57) is not satisfied, the constraint on uninformed consumers hiring providers is binding and we have that $SW_{C,buy} = SW_{M,buy}$, concluding our proof. ■

Proof of Lemma 5:

Proof. Equation (9) gives us the following result:

$$\frac{w}{1-\delta(1-X_A-X_B)} + \delta \frac{w}{1-\delta(1-X_A-X_B)} \left(\frac{X_A(1-X_A)}{1-\delta(1-X_B)} + X_A X_B \delta \frac{(1-X_A-X_B)}{(1-\delta)(1-\delta(1-X_B))} + \frac{X_B(1-X_B)}{1-\delta(1-X_A)} + X_B X_A \delta \frac{(1-X_A-X_B)}{(1-\delta)(1-\delta(1-X_A))} \right) = \frac{w-e}{1-\delta}$$

Rearranging it, we have:

$$\times \left\{ \frac{w}{1-\delta(1-X_A-X_B)} \times \frac{1}{(1-\delta)(1-\delta+\delta X_A)(1-\delta+\delta X_B)} \times \begin{aligned} &(1-\delta)(1-\delta+\delta X_A)(1-\delta+\delta X_B)+ \\ &\delta(1-\delta)X_A(1-X_A)(1-\delta+\delta X_A)+ \\ &+\delta^2 X_A X_B(1-X_A-X_B)(1-\delta+\delta X_A)+ \\ &\delta(1-\delta)X_B(1-X_B)(1-\delta+\delta X_B)+ \\ &\delta^2 X_B X_A(1-X_A-X_B)(1-\delta+\delta X_B) \end{aligned} \right\} = \frac{w-e}{1-\delta}$$

Notice then that:

$$\left\{ \begin{aligned} &(1-\delta)(1-\delta+\delta X_A)(1-\delta+\delta X_B)+ \\ &\delta(1-\delta)X_A(1-X_A)(1-\delta+\delta X_A)+ \\ &+\delta^2 X_A X_B(1-X_A-X_B)(1-\delta+\delta X_A)+ \\ &\delta(1-\delta)X_B(1-X_B)(1-\delta+\delta X_B)+ \\ &\delta^2 X_B X_A(1-X_A-X_B)(1-\delta+\delta X_B) \end{aligned} \right\} = \left\{ \begin{aligned} &-\delta^2 X_A^2 X_B - \delta(1-\delta)X_A^2 - \delta^2 X_A X_B^2 + \\ &+\delta^2 X_A X_B + \delta(1-\delta)X_A - \delta(1-\delta)X_B^2 \\ &+\delta(1-\delta)X_B + (1-\delta)^2 \end{aligned} \right\}$$

Substituting back and rearranging, we have:

$$\frac{1}{(1-\delta+\delta X_A)(1-\delta+\delta X_B)} \left\{ \begin{array}{l} -\delta^2 X_A^2 X_B - \delta(1-\delta)X_A^2 - \delta^2 X_A X_B^2 + \\ +\delta^2 X_A X_B + \delta(1-\delta)X_A - \delta(1-\delta)X_B^2 \\ +\delta(1-\delta)X_B + (1-\delta)^2 \end{array} \right\} = \frac{w-e}{w}$$

Then, notice that:

$$\left\{ \begin{array}{l} -\delta^2 X_A^2 X_B - \delta(1-\delta)X_A^2 - \delta^2 X_A X_B^2 + \\ +\delta^2 X_A X_B + \delta(1-\delta)X_A - \delta(1-\delta)X_B^2 \\ +\delta(1-\delta)X_B + (1-\delta)^2 \end{array} \right\} = \left\{ \begin{array}{l} -\delta X_A^2 (1-\delta+\delta X_B) \\ -\delta X_B^2 (1-\delta+\delta X_A) \\ +(1-\delta+\delta X_A)(1-\delta+\delta X_B) \end{array} \right\}$$

Substituting it back and rearranging, we have:

$$\frac{e}{w} - \frac{\delta X_A^2}{1-\delta+\delta X_A} - \frac{\delta X_B^2}{1-\delta+\delta X_B} = 0 \quad (\star)$$

Then (\star) defines a functional F . Notice that:

$$F_A = -\frac{\delta X_A(2(1-\delta)+\delta X_A)}{(1-\delta+\delta X_A)^2} < 0$$

and

$$F_B = -\frac{\delta X_B(2(1-\delta)+\delta X_B)}{(1-\delta+\delta X_B)^2} < 0$$

Since (\star) implicitly defines X_B as a function of X_A , from the implicit function theorem, we have:

$$\frac{dX_B}{dX_A} = -\frac{F_A}{F_B} = -\left(\frac{-\frac{\delta X_A(2(1-\delta)+\delta X_A)}{(1-\delta+\delta X_A)^2}}{-\frac{\delta X_B(2(1-\delta)+\delta X_B)}{(1-\delta+\delta X_B)^2}} \right)$$

Simplifying it, we have:

$$\frac{dX_B}{dX_A} = -\frac{(1-\delta+\delta X_B)^2 X_A(2(1-\delta)+\delta X_A)}{(1-\delta+\delta X_A)^2 X_B(2(1-\delta)+\delta X_B)} < 0$$

■

Proof of Proposition 5

Proof. We will show that for each given pair of fees f_A and f_B , there is a unique pair (X_A, X_B) that simultaneously solve equations (64) and (66). First let us show that for each given pair of fees f_A and f_B , equation (66) above defines a strictly increasing function $X_B(X_A)$. For convenience, let us define $\frac{f_A}{f_B} = \frac{1}{l}$.

$$\frac{X_A(1-\delta)+\delta X_A X_B}{X_B(1-\delta)+\delta X_A X_B} = \frac{1}{l}$$

Therefore, for each given pair of fees, the indifference condition of the consumers defines a relation between the consumer bases of the bureaus, and we can write X_B as an explicit function of X_A :

$$X_B = \frac{lX_A(1-\delta)}{(1-\delta) + \delta X_A(1-l)} \quad (79)$$

Thus, the function is increasing X , but it is discontinuous:

$$\frac{\partial X_B(X_A)}{\partial X_A} = \frac{l(1-\delta)^2}{((1-\delta) + \delta X_A(1-l))^2} > 0, \forall l > 0$$

The shape of this function depends on l . Let us look at this function for each of the possible three cases.

(1) If $f_A > f_B$ ($l < 1$), then $X_A > X_B > 0$ and $X_B(X_A)$ is a continuous and concave function;

(2) If $l = 1$, then $X_B = X_A$;

(3) Finally, if $f_A < f_B$ ($l > 1$) then $X_A < X_B$. Moreover, the function is discontinuous at $(1-\delta) + \delta X_A(1-l) = 0$, that is, there is a value $\hat{X}_A > 0$, given by

$$\hat{X}_A = \frac{(1-\delta)}{\delta(l-1)},$$

where if $X_A < \hat{X}_A$ then X_B that solves (79) is an increasing function from zero and increasing asymptotically to ∞ as X_A approaches \hat{X}_A .

Lemma (5) completes the proof, i.e., the pair (X_A, X_B) that solves (64) is a strictly decreasing function with both X_A and X_B positives, so there is a unique point in which this decreasing function crosses the curve $X_B(X_A)$ defined by (79). ■

Proof of Lemma 6

Proof. First, note that (f_i, f_j) with $f_i = f_j = \frac{P-w}{P} \frac{\delta w X^{sym}}{1-\delta(1-X^{sym})}$ with $X_i = X_j = X^{sym}$ and $Y = \frac{P-w}{P}$ is an equilibrium. Second, let us look at the case where $f_i < f_j = \frac{P-w}{P} \frac{\delta w X^{sym}}{1-\delta(1-X^{sym})}$. There is a unique pair (X_i, X_j) that solves both conditions (64) and (66). This pair is such that $X_i < X^{sym} < X_j$. Also, let

$$Y_{C,member} = \bar{f} \frac{1-\delta(1-X_j)}{\delta w X_j},$$

where $Y_{C,member} = \frac{P-w}{P} \frac{\delta X^{sym}}{1-\delta(1-X^{sym})} \frac{1-\delta(1-X_j)}{\delta X_j} < \frac{P-w}{P}$, since $\frac{\delta X^{sym}}{1-\delta(1-X^{sym})} < \frac{\delta X_j}{1-\delta(1-X_j)}$, so condition (67) is also satisfied.

Now suppose that $f_i = f_j < \frac{P-w}{P} \frac{\delta w X^{sym}}{1-\delta(1-X^{sym})}$. Then, let $X_i = X_j = X^{sym}$ and Y be given by

$$\begin{aligned} Y_{C,member} &= f_i \frac{1 - \delta(1 - X^{sym})}{\delta w X^{sym}} \\ &< \frac{P-w}{P} \frac{\delta w X^{sym}}{1 - \delta(1 - X^{sym})} \frac{1 - \delta(1 - X^{sym})}{\delta w X^{sym}} \\ &= \frac{P-w}{P}, \end{aligned}$$

so, again, condition (67) is satisfied. Finally, let $f_i < f_j < \frac{P-w}{P} \frac{\delta w X^{sym}}{1-\delta(1-X^{sym})}$. Again, there is a unique pair (X_i, X_j) that solves both conditions (64) and (66). This pair is such that $X_i < X^{sym} < X_j$. Also, let

$$\begin{aligned} Y_{C,member} &= f_j \frac{1 - \delta(1 - X_j)}{\delta w X_j} \\ &< \frac{P-w}{P} \frac{\delta X^{sym}}{1 - \delta(1 - X^{sym})} \frac{1 - \delta(1 - X_j)}{\delta X_j} \\ &< \frac{P-w}{P}. \end{aligned}$$

■

Proof of Proposition 6

Proof. Suppose that this is not an equilibrium. First, assume that there is a profitable deviation for A in which A increases its fee. Thus, $f_A > f_B = \bar{f}$. In this case, to satisfy both conditions (64) and (65), we need a higher X_A and smaller X_B . However, note that for firm B to operate in a market with such fees, we need condition (65) to be satisfied. Given that $f_B = \bar{f}$ and that we require a smaller X_B , we need that the new mass of providers buying in equilibrium must be higher, that is, $Y' > Y = \frac{P-w}{P}$, but this cannot be an equilibrium, since it violates (67). There are only two equilibria following such deviation: one in which A is a monopolist and one in which B is a monopolist. Given our refinement, we will assume that following such deviation, only bureau B (because it has lower fee) will operate, a contradiction.

Suppose that the profitable deviation is one in which A decreases its fee. Thus, $f_A < f_B = \bar{f}$. From lemma (6) we know that there exists a unique equilibrium with two operating bureaus. Now, to satisfy both conditions (64) and (65), we need a lower X_A and a higher X_B . A lower X_A together with a lower f_A imply that A has decreased its profit, so this is not a profitable deviation either. This proves that the proposed candidate is indeed a stationary competitive equilibrium. Below, we prove that it is unique.

Suppose that bureau j is a monopolist and is charging a feasible fee f_j . Then, any fee $0 < f_i < f_j$ is a profitable deviation for firm i , since it either accommodates two bureaus in the continuation game or it shifts the monopoly to firm i , in either case firm i will make positive profits.

Now suppose that two firms are operating and $f_i < f_j$. If i increases its fee (but such that it is still lower than f_j) the new pair (f'_i, f_j) will increase the consumer basis of firm i , which, together with the higher fee increases its profit. This proves that a stationary competitive equilibrium must be symmetric. Finally, suppose that $f_i = f_j < \bar{f}$. Then, $X_i = X_j = X^{sym}$, and $Y < \frac{P-w}{P}$. Consider a deviation in which firm i increases f_i such that both firms still operate (otherwise, given our refinement, only j will operate). We know from lemma (6) that such a deviation in which the equilibrium in the continuation game has two operating bureaus exists. Then, the new consumer basis of firm i must increase $X'_i > X^{sym} > X'_j$. Given that this is a profitable deviation for firm i , such an equilibrium cannot exist either. Therefore, the symmetric equilibrium with $f_i = f_j = \bar{f}$ is the unique equilibrium in which two bureaus operate. ■

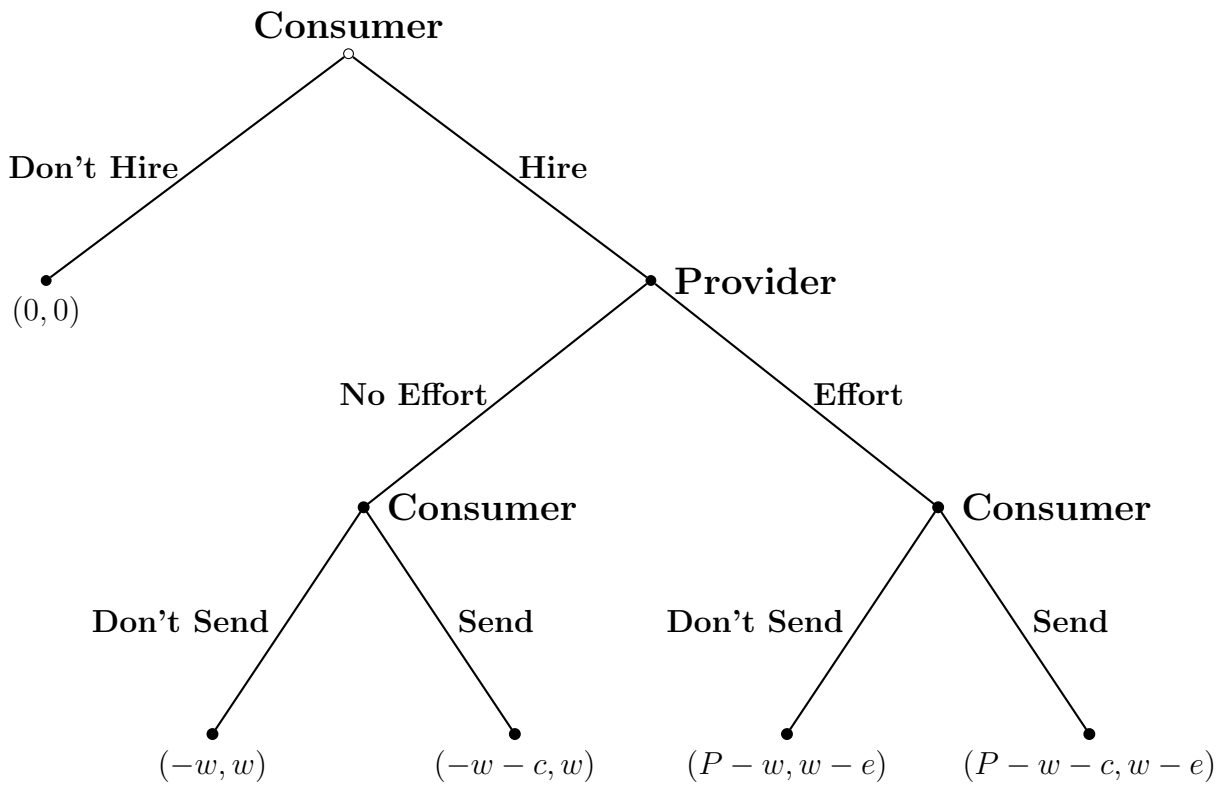


Figure 1: Game Tree – No bureau Case

Figure 2: Extended Game Tree – Bureau that buys and sells information

