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# Macro Credit Policy and the Financial Accelerator 

Charles T. Carlstrom and Timothy S. Fuerst

This paper studies macro credit policies within the celebrated financial accelerator model of Bernanke, Gertler and Gilchrist (1999). The focus is on borrower-based restrictions on lending such as loan-to-value (LTV) ratios. We find that the efficacy of cyclical taxes on LTV ratios depends upon the nature of the underlying loan contract. If the loan contract contains equity-like features such as indexation to aggregate conditions, then there is little role for cyclical taxation. But if the loan contract is not indexed to aggregate conditions, then there are substantial gains to procyclical taxes on LTV ratios.

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The recent financial crisis has spawned a great deal of interest in financial regulation generally and macro-prudential policy more specifically. Much of the focus has been on the efficacy of lenderbased restrictions such as countercyclical bank capital requirements, a prominent issue in the Dodd-Frank legislation. There is an interesting complementary question left largely unexamined: is there a role for cyclical borrower-based credit standards? For example, Claessens (2014) provides evidence that restrictions on loan-to-value (LTV) ratios are one of the most commonly used macro-prudential tools in emerging markets and developed countries.

Any theory of regulation begins with an assessment of market imperfections that would motivate government action. Such an assessment requires the use of a model. To examine the cyclical nature of credit policies we need a model where these frictions also feed into the macro-economy. We use the financial accelerator model of Bernanke, Gertler, and Gilchrist (1999), hereafter BGG, because it is widely used as a convenient mechanism for integrating financial factors into DSGE models, and the agency friction in BGG arises from private information on the borrower side.

A principal result of the analysis is that there is a pecuniary externality present in the BGG model. ${ }^{1}$ Individual agents do not internalize the effect their actions have on the price of capital, and this price has a first order effect on welfare. The pecuniary externality arises because the price of capital determines the borrower's net worth, and thus their ability to finance activity. This is a familiar source of pecuniary externalities in models with borrowing constraints, eg., Bianchi (2012) and Jeanne and Korinek (2012). One novelty in the present analysis is that the asset price also affects the allocation of consumption between borrowers and lenders. Since the marginal consumption utilities of these two

[^0]agents are typically different, this allocation effect has a first order effect on welfare. Consequently, the competitive equilibrium price of capital is sub-optimal from the planner's perspective. This is true for both the steady state asset price and the dynamic behavior of capital prices in the wake of business cycle shocks.

There are many ways to decentralize the planner's choices. Motivated by the literature on macroprudential regulation, we consider taxes (or subsidies) on leveraged-lending. The leveraged-lending tax is a tax levied on lending, but where the tax rate is proportional to the degree of borrower leverage, i.e., the LTV ratio. We find that the efficacy of such a tax depends upon the underlying financial contract. Carlstrom, Fuerst and Paustian (2015), hereafter CFP, show that the financial contract imposed by BGG is not the privately optimal contract implied by the model. Instead, the privately optimal contract (POC) is a debt contract with equity-like features in which the promised repayment varies with observable macro variables. We consider both the BGG and POC contracts in the analysis below. For the case of the BGG contract, we find that there are significant welfare gains to a macro-prudential policy in which the lending tax is strongly pro-cyclical. In contrast, for the case of the POC contract, the contract is already indexed to macro variables so there is little role for cyclicality of the tax.

The paper proceeds as follows. The next section outlines the competitive equilibrium of the model for both the POC and BGG contracts. Section II introduces the planner and focuses on the pecuniary externality. Section III shows how to decentralize the planner's allocation. The quantitative analysis, including welfare implications, is carried out in Section IV. Concluding comments are provided in Section V.

## I. The Model.

## A. Households

The typical household consumes the final good $C_{t}$ and sells labor input $L_{t}$ to the firm at real wage $w_{t}$. Preferences are given by

$$
U\left(C_{t}, L_{t}\right) \equiv \frac{C_{t}^{1-\sigma}}{1-\sigma}-B \frac{L_{t}^{1+\eta}}{1+\eta} .
$$

The household budget constraint is given by

$$
C_{t}+D_{t} \leq w_{t} L_{t}+R_{t}^{d} D_{t-1}+\Pi_{t}
$$

The household chooses the level of deposits $D_{t}$ which are then used by the lender to fund the capitalists (more details below). As developed below, the lender's return on its portfolio of loans is realized at timet , and this return is passed on one-for-one to the depositors. Hence, the gross real return on time $t-1$ deposits $\left(R_{t}^{d}\right)$ is realized at time-t is conditional on aggregate shocks. ${ }^{2}$ The household owns shares in the final goods firms, capital-producing firms, and the lender. Only the capital-producing firms will generate profits $\left(\Pi_{t}\right)$ in equilibrium. The household's optimization conditions are given by:

$$
\begin{align*}
& -U_{L}(t) / U_{c}(t)=w_{t}  \tag{1}\\
& 1=E_{t} M_{t+1} R_{t+1}^{d} \tag{2}
\end{align*}
$$

where $M_{t+1} \equiv \beta \frac{U^{\prime}\left(c_{t+1}\right)}{U^{\prime}\left(c_{t}\right)}$, which is the pricing kernel.

## B. Final goods firms

Final goods are produced by competitive firms who hire labor and rent capital in competitive factor markets at real wage $w_{t}$ and rental rate $r_{t}$. The production function is Cobb-Douglass where $A_{t}$ is the random level of total factor productivity:

$$
\begin{equation*}
Y_{t}=A_{t}\left(K_{t}^{f}\right)^{\alpha}\left(L_{t}\right)^{1-\alpha} \tag{3}
\end{equation*}
$$

[^1]The realization of total factor productivity is publicly observed at the beginning of time-t. The variable $K_{t}^{f}$ denotes the amount of capital available for time-t production. This is different than the amount of capital at the end of the previous period since some is lost because of monitoring costs. The optimization conditions are:

$$
\begin{align*}
& m p l_{t}=w_{t}  \tag{4}\\
& m p k_{t}=r_{t} \tag{5}
\end{align*}
$$

where $m p l_{t}$ and $m p k_{t}$ denote the marginal products of labor and capital, respectively.

## C. New Capital Producers

The production of new capital is subject to adjustment costs. In particular, investment firms take $I_{t}$ consumption goods and transform them into $I_{t} \vartheta\left(\frac{I_{t}}{I^{*}}\right)$ new capital goods that are sold at price $Q_{t}$, where the function $\vartheta$ is concave. Variations in investment lead to variations in the price of capital, which is the key to the financial accelerator mechanism.

## D. Lenders

The representative lender accepts deposits from households and provides loans to the continuum of capitalists. These loans are intertemporal, with the loans made at the end of time $t$ being paid back in time $t+1$. Each individual loan is subject to idiosyncratic and aggregate risk, but since the lender holds an entire portfolio of loans only aggregate risk remains. We assume free entry into the lending market so that lenders make zero profits. This implies that the gross real returns on deposits $\left(R_{t+1}^{d}\right)$ must equal the gross real return to the lender's loan portfolio $\left(R_{t+1}^{l}\right)$. In some sense, the lender is merely a passive passthrough entity, whose primary function is to hold on behalf of depositors a diversified portfolio of loans across the many capitalists.

## E. Capitalists and the Loan Contract

There are a continuum of risk-neutral capitalists who discount the future at rate $\beta$, and are the sole intertemporal holders of physical capital. As demonstrated in CFP (2015), the linearity in preferences and technologies implies that the decisions of capitalists will aggregate so we need only track a representative capitalist. As in BGG we assume all capital must be liquidated and all capital repurchased each period. The market price of capital is $Q_{t}$. The sale of capital generates net worth $N_{t}$. All of this capital is then immediately re-purchased, along with any net additions to the capital stock, by the collection of capitalists. The time $t$ purchase of capital is given by $K_{t+1}$. This purchase is financed with capitalist net worth and external financing from a lender. This external finance takes the form of an intertemporal loan with repayment occurring in time $t+1$.

The capitalist's ability to repay the loan will be dependent upon the intertemporal return to capital. This return is a product of two factors, the aggregate return to capital and the idiosyncratic return of each capitalist. The aggregate return to capital $\left(R_{t+1}^{k}\right)$ is publicly observed and is given by:

$$
\begin{equation*}
R_{t+1}^{k} \equiv \frac{r_{t+1}+(1-\delta) Q_{t+1}}{Q_{t}} . \tag{6}
\end{equation*}
$$

where $r_{t+1}$ is the rental rate and $\delta$ is the capital depreciation rate. That is, a unit of capital costs $Q_{t}$ at the end of time $t$, while a unit of capital generates rental rate $r_{t+1}$ and re-sale value $(1-\delta) Q_{t+1}$ in period $t+1$. As for the idiosyncratic return, one unit of capital purchased at the end of time-t is transformed into $\omega_{t+1}$ units of capital in time $t+1$, where $\omega_{t+1}$ is an idiosyncratic random variable with density $\phi(\omega)$, cumulative distribution $\Phi(\omega)$, and a mean of one. We assume that $\omega_{t+1}$ is uncorrelated with $R_{t+1}^{k}$. The total return on the capital project is thus a product of two independent random variables, $\omega_{t+1} R_{t+1}^{k}$.

In contrast to the common aggregate return to capital, the idiosyncratic realization of $\omega_{t+1}$ is directly observed only by the capitalist. The lender can observe the realization only if a costly monitoring occurs, a cost that destroys part of the capital produced by the project. We assume that this monitoring
cost is linear in the project outcome, $\mu \omega_{t+1} R_{t+1}^{k} K_{t+1}$. In this costly state verification environment, the optimal contract between the capitalist and lender is risky debt in which monitoring only occurs if the promised payoff is not forthcoming. ${ }^{3}$ The debt contract specifies a promised gross loan rate of $Z_{t+1}$, and is risky because of the possibility of default. The contract is characterized by a reservation value of the idiosyncratic shock that separates repayment from default. Debt repayment does not occur (i.e., "bankruptcy") for sufficiently low values of the idiosyncratic shock, $\omega_{t+1} \leq \varpi_{t+1}$. Note that $\varpi_{t+1}$ is realized in time $t+1$ and thus can be contingent on the observed aggregate shock $R_{t+1}^{k}$. The relationship between the promised repayment rate and this reservation value is given by

$$
\begin{equation*}
Z_{t+1}\left(Q_{t} K_{t+1}-N_{t}\right) \equiv \varpi_{t+1} R_{t+1}^{k} Q_{t} K_{t+1} . \tag{7}
\end{equation*}
$$

We find it convenient to express this in terms of the borrower's leverage ratio $\kappa_{t} \equiv\left(\frac{Q_{t} K_{t+1}}{N_{t}}\right)$ such that (7) becomes

$$
\begin{equation*}
Z_{t+1} \equiv \varpi_{t+1} R_{t+1}^{k} \frac{\kappa_{t}}{\kappa_{t}-1} \tag{8}
\end{equation*}
$$

Let $f\left(\varpi_{t+1}\right)$ and $g\left(\varpi_{t+1}\right)$ denote the expected shares of the project outcome being earned by, respectively, the capitalist and lender:

$$
\begin{align*}
& f(\varpi) \equiv \int_{\varpi}^{\infty} \omega \phi(\omega) d \omega-[1-\Phi(\varpi)] \varpi  \tag{9}\\
& g(\varpi) \equiv[1-\Phi(\varpi)] \varpi+(1-\mu) \int_{0}^{\varpi} \omega \phi(\omega) d \omega . \tag{10}
\end{align*}
$$

Conditional on the aggregate return on capital $R_{t+1}^{k}$, the expected capitalist's payoff and lender return are thus given by:

$$
\begin{equation*}
\text { Capitalist payoff }=R_{t+1}^{k} Q_{t} K_{t+1} f\left(\varpi_{t+1}\right)=R_{t+1}^{k} f\left(\varpi_{t+1}\right) \kappa_{t} N_{t} \tag{11}
\end{equation*}
$$

[^2]\[

$$
\begin{equation*}
\text { Lender return }=R_{t+1}^{l}=\frac{R_{t+1}^{k} g\left(w_{t+1}\right) Q_{t} K_{t+1}}{\left(Q_{t} K_{t+1}-N_{t}\right)}=R_{t+1}^{k} g\left(\varpi_{t+1}\right) \frac{\kappa_{t}}{\kappa_{t}-1} . \tag{12}
\end{equation*}
$$

\]

In the neighborhood of the steady state, $E_{t} \beta R_{t+1}^{k} f\left(\varpi_{t+1}\right) \kappa_{t}>1$, so that each capitalist postpones consumption indefinitely.

To avoid self-financing in the long run, we assume that each capitalist faces probability (1- $\gamma$ ) of death each period. Capitalists receive the news at the beginning of the period whether they will die at the end of the period. Dying capitalists will thus choose to consume all of their net worth before exiting the economy. The dead are then replaced by an equal number of new capitalists. New capitalists need a trivial amount of initial net worth to begin activity. We assume that this comes from a lump sum transfer from the existing capitalists. Since this transfer can be arbitrarily small, and since only aggregate net worth matters in this setting, we neglect these transfers in what follows.

In summary, a typical capitalist sets $C_{t}^{k}=N_{t}$ with probability (1- $\gamma$ ), or with probability $\gamma$ consumes nothing and uses all his net worth to finance capital purchases so that $N_{t+1}=R_{t+1}^{k} f\left(\varpi_{t+1}\right) \kappa_{t} N_{t}$. Following CFP (2015), the Bellman equation is given by:

$$
\begin{equation*}
V_{t} N_{t}=(1-\gamma) C_{t}^{k}+\beta \gamma \max _{\kappa_{t}, \omega_{t+1}} E_{t} V_{t+1} N_{t+1} \tag{13}
\end{equation*}
$$

where the maximization is subject to the lender's participation constraint (equation (16) below). Substituting in the consumption decision of the dying capitalists, $C_{t}^{k}=N_{t}$, and the savings decision of the surviving capitalists, $N_{t+1}=R_{t+1}^{k} f\left(\varpi_{t+1}\right) \kappa_{t} N_{t}$, the Bellman equation can be rewritten as

$$
\begin{equation*}
V_{t}=(1-\gamma)+\max _{\kappa_{t}, \omega_{t+1}} \beta \gamma E_{t} V_{t+1} R_{t+1}^{k} f\left(\varpi_{t+1}\right) \kappa_{t} \tag{14}
\end{equation*}
$$

The optimal contract maximizes the return to the capitalist subject to the lender's return being equal to the deposit rate.

As discussed by CFP, BGG do not analyze the optimal contract but instead impose a contract that creates a financial accelerator. We first present the optimal contract between the lenders and capitalists, and then turn to the contract imposed by BGG. The end of time-t contracting problem is given by:

$$
\begin{equation*}
\max _{\kappa_{t}, \varpi_{t+1}} \beta \gamma E_{t} V_{t+1} R_{t+1}^{k} f\left(\varpi_{t+1}\right) \kappa_{t} \tag{15}
\end{equation*}
$$

subject to

$$
\begin{equation*}
E_{t} M_{t+1} R_{t+1}^{k} g\left(\varpi_{t+1}\right) \kappa_{t} \geq\left(\kappa_{t}-1\right) \tag{16}
\end{equation*}
$$

The lender's participation constraint (16) comes from combining the definition of the lender's return (12), the household's pricing kernel (2), and $R_{t+1}^{l}=R_{t+1}^{d}$. The first order conditions to this problem are given by:

$$
\begin{align*}
& \beta \gamma V_{t+1} f^{\prime}\left(\varpi_{t+1}\right)+\Lambda_{t} M_{t+1} g^{\prime}\left(\varpi_{t+1}\right)=0  \tag{17}\\
& \beta \gamma E_{t} V_{t+1} R_{t+1}^{k} f\left(\varpi_{t+1}\right)+\Lambda_{t}\left[E_{t} M_{t+1} R_{t+1}^{k} g\left(\varpi_{t+1}\right)-1\right]=0  \tag{18}\\
& E_{t} R_{t+1}^{k} M_{t+1} g\left(\varpi_{t+1}\right) \frac{\kappa_{t}}{\kappa_{t}-1}=1 \tag{19}
\end{align*}
$$

where $\Lambda_{t}$ denotes the multiplier on the constraint (16). The privately optimal contract (POC) is thus described by the $\varpi_{t+1}$ that satisfies:

$$
\begin{equation*}
\frac{\Lambda_{t} M_{t+1}}{\beta \gamma\left[1-\gamma+\Lambda_{t+1}\right]}=\frac{-f^{\prime}\left(\varpi_{t+1}\right)}{g^{\prime}\left(\varpi_{t+1}\right)} \equiv F\left(\varpi_{t+1}\right), \tag{20}
\end{equation*}
$$

where we have used (18) and (14) to link the value function and $\Lambda_{t}$. The second order condition for a maximum implies $F^{\prime}\left(\varpi_{t+1}\right)>0$. From the perspective of time $t$, the conditional mean behavior of $\varpi_{t+1}$ is constrained by (18)-(19). But (20) indicates that the default cut-off is indexed to time $t+1$ variables in a natural way. The promised repayment rate is given by $Z_{t+1} \equiv \varpi_{t+1} R_{t+1}^{k} \frac{\kappa_{t}}{\kappa_{t}-1}$, so that state-dependence in the cut-off rate implies state-dependence in the repayment amount. When $C_{t+1}$ is low ( $M_{t+1}$ is high), the optimal $\varpi_{t+1}$ and thus $Z_{t+1}$ increase as a form of consumption insurance to the household. Similarly, when the cost of external finance is high $\left(\Lambda_{t+1}\right.$ is high), the contract calls for a lower $\varpi_{t+1}$ and $Z_{t+1}$ such that the capitalist holds on to more net worth.

In contrast, BGG assumed that the lender's return is pre-determined, ie., constraint (19) is assumed to hold state-by-state. The BGG contract is thus given by:

$$
\begin{align*}
& \beta \gamma V_{t+1} f^{\prime}\left(\varpi_{t+1}\right)+\Lambda_{t+1} M_{t+1} g^{\prime}\left(\varpi_{t+1}\right)=0  \tag{21}\\
& \beta \gamma E_{t} V_{t+1} R_{t+1}^{k} f\left(\varpi_{t+1}\right)+E_{t} \Lambda_{t+1}\left[M_{t+1} R_{t+1}^{k} g\left(\varpi_{t+1}\right)-1\right]=0  \tag{22}\\
& R_{t+1}^{k} g\left(\varpi_{t+1}\right) \frac{\kappa_{t}}{\kappa_{t}-1}=R_{t}^{d} \tag{23}
\end{align*}
$$

From (23), the BGG contract has the default cut-off $\varpi_{t+1}$ independent of all innovations in aggregate variables except for the return to capital $R_{t+1}^{k}$.

The differences in the two contracts are transparent if we look at the log-linear approximation to the promised repayment rate:

$$
\begin{align*}
z_{t}^{P O C}= & E_{t-1} r_{t}^{l, P O C}+\frac{\left(1-\Theta_{\mathrm{g}}\right)\left[1-v\left(\kappa_{s s}-1\right)\right]}{\Theta_{\mathrm{g}}\left(\kappa_{s s}-1\right)} \tilde{\kappa}_{t-1}+\left(r_{t}^{k}-E_{t-1} r_{t}^{k}\right)+\frac{1}{\Psi}\left(m_{t}-E_{t-1} m_{t}\right)-\frac{\beta}{\Psi}\left(\lambda_{t}-E_{t-1} \lambda_{t}\right)  \tag{24}\\
& z_{t}^{B G G}=r_{t-1}^{l, B G G}+\frac{\left(1-\Theta_{\mathrm{g}}\right)\left[1-v\left(\kappa_{s s}-1\right)\right]}{\Theta_{\mathrm{g}}\left(\kappa_{s s}-1\right)} \tilde{\kappa}_{t-1}+\left(\frac{\Theta_{\mathrm{g}}-1}{\Theta_{\mathrm{g}}}\right)\left(r_{t}^{k}-E_{t-1} r_{t}^{k}\right) \tag{25}
\end{align*}
$$

where $\Psi \equiv \frac{\varpi_{s s} F^{\prime}\left(\varpi_{s s}\right)}{F\left(\varpi_{s s}\right)}>0, \quad \Theta_{\mathrm{g}} \equiv \frac{\varpi_{s s} g \prime\left(\varpi_{s s}\right)}{g\left(\varpi_{s s}\right)}, 0<\Theta_{\mathrm{g}}<1, \Theta_{\mathrm{f}} \equiv \frac{\varpi_{s s} f \prime\left(\varpi_{s s}\right)}{f\left(\varpi_{s s}\right)}<0$, and $v \equiv\left[\frac{\Psi}{\left(\kappa_{s s}-1\right) \Psi-\kappa \theta_{f}}\right]$. The lower case letters denote log deviations of the corresponding endogenous variables, and $\tilde{\kappa}_{t}$ denotes the log deviation of $\kappa_{t}$.

Since the POC and BGG contract differ only by (19) and (23), the linearized repayment rates differ only by innovations. The innovations in the POC are a form of indexation to aggregate shocks. First, the promised repayment is scaled one-for-one by innovations in $r_{t}^{k}$ such that the default cut-off is sterilized from these innovations. Indexing the promised repayment to the return to capital is quite natural. There are two sources of uncertainty within the underlying CSV problem: unobserved idiosyncratic shocks, and the observed aggregate return on capital. Bankruptcy and costly monitoring are
part of the optimal debt contract as the mechanism to ensure truthful revelation of the idiosyncratic shock. But there is no need for such a deterrent for observed aggregate shocks. A second key feature of the POC is that it provides consumption insurance to the household in that the repayment rate is increasing when the marginal utility of consumption is unexpectedly high ( $m_{t}$ is high). The higher lender return is then passed on to the household via increases in the return on deposits. Third, the POC provides for a hedge to the capitalist in that when the return to internal funds is high ( $\lambda_{t}$ is high), the repayment to the lender declines so that the capitalist can build up net worth.

In sharp contrast, the BGG repayment rate (25) depends only upon innovations in $r_{t}^{k}$. For typical calibrations $\Theta_{\mathrm{g}}<1$, so that the BGG repayment rate falls with innovations in $r_{t}^{k}$. This is a natural implication of the BGG assumption that the lender's return is pre-determined. All else equal, a positive innovation in the return on capital lowers the default rate, so that a pre-determined lender return is possible only if the promised repayment rate declines. The previous discussion suggests this is peculiar for two reasons. First, the innovation in the return to capital is publicly observed, so there is no reason for the CSV contract to respond to these movements. Second, the BGG contract is missing the household and capitalist hedging motives of the POC.

Although BGG and POC differ only by innovations, the inertial dynamics of net worth imply that these differences will have persistent consequences. The evolution of aggregate net worth is given by

$$
\begin{equation*}
N W_{t+1}=\gamma R_{t+1}^{k} f\left(\varpi_{t+1}\right) \kappa_{t} N W_{t} . \tag{26}
\end{equation*}
$$

In response to an aggregate shock, the behavior of repayment rates and thus bankruptcy cut-off rates $\varpi_{t+1}$ differ by innovations, but these differences persist for a long time.

## F. Market Clearing and Equilibrium

In equilibrium household deposits fund the capitalists' projects, $D_{t}=Q_{t} K_{t+1}-N W_{t}$. Net of monitoring costs, the amount of capital available for production is given by $K_{t}^{f}=h\left(\varpi_{t}\right) K_{t}$, where
$h\left(\varpi_{t}\right) \equiv f\left(\varpi_{t}\right)+g\left(\varpi_{t}\right)=1-\mu \int_{0}^{\omega_{t}} x \phi(x) d x$. As noted earlier, the deposit rate is tied to the return on loans, such that $R_{t}^{l}=R_{t}^{d}$. The POC is defined by the variables $\left\{C_{t}, L_{t}, I_{t}, K_{t+1}, \varpi_{t}, \Lambda_{t}, \kappa_{t}, C_{t}^{k}, Q_{t}\right\}$ that satisfy

$$
\begin{align*}
& -U_{L}(t) / U_{c}(t)=m p l_{t}  \tag{27}\\
& E_{t} M_{t+1} R_{t+1}^{k} g\left(\varpi_{t+1}\right) \frac{\kappa_{t}}{\left(\kappa_{t}-1\right)}=1  \tag{28}\\
& \frac{\Lambda_{t-1} M_{t}}{\beta \gamma\left[1-\gamma+\Lambda_{t}\right]}=F\left(\varpi_{t}\right)  \tag{29}\\
& \Lambda_{t}=\beta \gamma E_{t}\left[(1-\gamma)+\Lambda_{t+1}\right] R_{t+1}^{k} f\left(\varpi_{t+1}\right) \kappa_{t}  \tag{30}\\
& Q_{t} K_{t+1}=\gamma\left[Q_{t}(1-\delta)+m p k_{t}\right] f\left(\varpi_{t}\right) K_{t} \kappa_{t}  \tag{31}\\
& K_{t+1}=(1-\delta) h\left(\varpi_{t}\right) K_{t}+I_{t} \vartheta\left(\frac{I_{t}}{I^{*}}\right)  \tag{32}\\
& C_{t}+I_{t}+C_{t}^{k}=A_{t}\left(h\left(\varpi_{t}\right) K_{t}\right)^{\alpha}\left(L_{t}\right)^{1-\alpha}  \tag{33}\\
& C_{t}^{k}=(1-\gamma)\left[Q_{t}(1-\delta)+m p k_{t}\right] f\left(\varpi_{t}\right) K_{t}  \tag{34}\\
& Q_{t}=\left[\vartheta\left(\frac{I_{t}}{I^{*}}\right)+\frac{I_{t}}{I^{*}} \vartheta^{\prime}\left(\frac{I_{t}}{I^{*}}\right)\right]^{-1} \tag{35}
\end{align*}
$$

where $M_{t+1} \equiv \beta \frac{U^{\prime}\left(c_{t+1}\right)}{U^{\prime}\left(c_{t}\right)}, \kappa_{t} \equiv\left(\frac{Q_{t} K_{t+1}}{N W_{t}}\right), F\left(\varpi_{t}\right) \equiv \frac{-f^{\prime}\left(\varpi_{t}\right)}{g^{\prime}\left(\varpi_{t}\right)}$, and $R_{t+1}^{k} \equiv \frac{m p k_{t+1}+(1-\delta) Q_{t+1}}{Q_{t}}$. The marginal products are defined as $m p l_{t} \equiv(1-\alpha) Y_{t} / L_{t} \quad$, and $m p k_{t} \equiv a Y_{t} /\left(h\left(\varpi_{t}\right) K_{t}\right) \quad$, with $Y_{t} \equiv A_{t}\left(h\left(\varpi_{t}\right) K_{t}\right)^{\alpha}\left(L_{t}\right)^{1-\alpha}$. The BGG equilibrium is similar, but with the relevant change in (28) and equation (29) is replaced with (23).

## II. The Constrained Social Planner.

In this section we consider the social planner's problem and compare it to the BGG and POC equilibria. The planner is assumed to maximize a weighted sum of the lifetime utility flow of the representative household and capitalist. The linearity in the model implies that we can aggregate capitalist consumption. With a utility weight of $\epsilon$ on the aggregate consumption of capitalists, the planner maximizes:

$$
\begin{equation*}
E_{t} \sum_{j=0}^{\infty} \beta^{j}\left[U\left(C_{t+j}, L_{t+j}\right)+\epsilon C_{t+j}^{k}\right] \tag{36}
\end{equation*}
$$

subject to the resource constraints and private optimality. ${ }^{4}$ We assume that the planner is constrained by the social resource constraints (32)-(33), and must respect the private information barrier on observing capitalist payoffs. In particular, the planner is able to redistribute consumption only by varying the terms in the debt contract that links households and capitalists. These terms are entirely summarized by the bankruptcy cut-off $\varpi_{t}$, and its effect on the allocation of consumption. Hence, the planner is also constrained by (34). The presence of the price of capital in (34) is a manifestation of the pecuniary externality in the model. The planner will internalize the effect of his choices on this price, an internalization that is absent in the competitive equilibrium of BGG and POC.

The planner's problem is thus to maximize (36) subject to (32)-(34). Let $\Lambda_{1 t}, \Lambda_{2 t}$, and $\Lambda_{3 t}$, denote the multipliers on (32)-(34), respectively. Equation (35) implicitly is a constraint, but we treat the price of capital parametrically as defined by (35) so that $Q_{I}(t)$ denotes the response of the price of capital to investment. The FOC to the planner's problem are given by:

$$
\begin{align*}
& U_{c}(t)=\Lambda_{2 t}  \tag{37}\\
& \Lambda_{2 t}=\frac{\Lambda_{1 t}}{Q_{t}}-\Lambda_{3 t}(1-\delta)(1-\gamma) f\left(\varpi_{t}\right) K_{t} Q_{I}(t)  \tag{38}\\
& \Lambda_{3 t}=\Lambda_{2 t}-\epsilon \tag{39}
\end{align*}
$$

[^3]\[

$$
\begin{align*}
& -U_{L}(t)=\Lambda_{2 t} m p l_{t}-\Lambda_{3 t} \alpha m p l_{t} x_{t}  \tag{40}\\
& \Lambda_{1 t}=\beta E_{t} h\left(\varpi_{t+1}\right)\left\{\begin{array}{c}
\Lambda_{1 t+1}(1-\delta)+\Lambda_{2 t+1} m p k_{t+1} \\
-\Lambda_{3 t+1} x_{t+1}\left[\alpha m p k_{t+1}+(1-\delta) Q_{t+1}\right]
\end{array}\right\}  \tag{41}\\
& \frac{h^{\prime}\left(\varpi_{t}\right)}{f^{\prime}\left(\varpi_{t}\right)}=\frac{\Lambda_{3 t}(1-\gamma)\left[Q_{t}(1-\delta)+m p k_{t}\right]}{\left[\Lambda_{1 t}(1-\delta)+\Lambda_{2 t} m p k_{t}+\Lambda_{3 t} x_{t}(1-\alpha) m p k_{t}\right]} \tag{42}
\end{align*}
$$
\]

where we define $x_{t} \equiv(1-\gamma) \frac{f\left(\omega_{t}\right)}{h\left(\omega_{t}\right)}$. From (39), the multiplier $\Lambda_{3 t}$ denotes the difference in the marginal utilities between the capitalist and the household. The planner wants to equate these two (and thus set $\Lambda_{3 t}$ $=0)$ by redistributing consumption. The agency problem, however, constrains the planner as these transfers can only be carried out through the debt contract. Since $f^{\prime}\left(\varpi_{t}\right)$ and $h^{\prime}\left(\varpi_{t}\right)$ are both negative, (42) implies that $\Lambda_{3 t}$ is positive (assuming an interior solution). ${ }^{5}$ The planner thus tolerates the deadweight loss of positive bankruptcy rates only because on the margin he desires to transfer consumption units from the capitalist back to the household. The positive monitoring costs imply that the planner is ultimately frustrated and does not achieve equal marginal utilities $\left(U_{c}(t)>\epsilon\right)$.

This incomplete consumption redistribution illuminates the remaining differences between the planner and the competitive equilibrium. The total differential of capitalist consumption is given by:

$$
\begin{equation*}
\Delta \mathrm{C}_{\mathrm{t}}^{\mathrm{k}}=\frac{\partial C_{t}^{k}}{\partial I_{t}} d I_{t}+\frac{\partial C_{t}^{k}}{\partial L_{t}} d L_{t}+\frac{\partial C_{t}^{k}}{\partial K_{t}} d K_{t} \tag{43}
\end{equation*}
$$

where

$$
\begin{align*}
& \frac{\partial c_{t}^{k}}{\partial L_{t}}=(1-\gamma) Q_{I}(t)(1-\delta) f\left(\varpi_{t}\right) K_{t}  \tag{44}\\
& \frac{\partial C_{t}^{k}}{\partial L_{t}}=(1-\gamma) \frac{d m p k_{t}}{d L_{t}} f\left(\varpi_{t}\right) K_{t}=\alpha m p l_{t} x_{t}  \tag{45}\\
& \frac{\partial C_{t}^{k}}{\partial K_{t}}=(1-\gamma) f\left(\varpi_{t}\right)\left[Q_{t}(1-\delta)+m p k_{t}+\frac{d m p k_{t}}{d K_{t}} K_{t}\right] \tag{46}
\end{align*}
$$

[^4]$$
=h\left(\varpi_{t}\right) x_{t}\left[Q_{t}(1-\delta)+\alpha m p k_{t}\right]
$$

The first term arises because changes in investment alter the price of capital and thus the consumption of capitalists. This term enters into the planner's investment choice (38). Since changes in the price of capital lead directly to a redistribution from the capitalist to the household, the planner typically prefers a different capital price than that implied by the competitive equilibrium. The remaining two terms come from the planner’s desire to change the marginal product of capital (and thus the rental rate). Again, since all capital income flows to the capitalist, the planner internalizes the effect of labor choice (equation (40) and capital accumulation on the rental rate (equation (41)).

In summary, the planner's allocations differ from the competitive equilibrium because the planner internalizes the effect of the household's decisions on the price of capital and the rental rate of capital. These prices directly affect the distribution of consumption between the agents. These are pecuniary externalities: the planner prefers a different rental rate and price of capital than those implied by the competitive equilibrium.

## III. Decentralizing the planner allocation.

After accounting for the three Lagrange multipliers in (37)-(42), the planner needs three instruments to decentralize the desired allocation. Here we demonstrate a set of distortionary taxes (with proceeds redistributed to households in a lump-sum manner) that can be used to achieve this end. There are many ways of decentralizing the allocation. Our focus will be on choosing taxes that when possible can be interpreted as a borrower-based macro-prudential policy.

We assume that the planner has access to the following set of taxes: (i) a tax of $\tau_{t}^{\text {lev }}$ on lending, (ii) a tax of $\tau_{t}^{\text {def }}$ on the average level of defaults on the lender's loans, and (iii) a tax of of $\tau_{t}^{L}$ on household labor income. The labor tax alters the labor margin:

$$
\begin{equation*}
-U_{L}(t) / U_{c}(t)=m p l_{t}\left(1-\tau_{t}^{L}\right) \tag{47}
\end{equation*}
$$

The planner sets $\tau_{t}^{L}$ to coincide with (40). The taxes on lending and default affect the return to savings and the default decision which is necessary from (41) and (42). With these taxes the lender's payoff on its loan portfolio is given by:

$$
\begin{equation*}
\text { Lender payoff }=\tilde{g}\left(\varpi_{t+1}\right) Q_{t} R_{t+1}^{k} K_{t+1}-\tau_{t}^{l e v} \kappa_{t}\left(Q_{t} K_{t+1}-N W_{t}\right) \tag{48}
\end{equation*}
$$

where

$$
\begin{equation*}
\tilde{g}\left(\varpi_{t+1}\right) \equiv g\left(\varpi_{t+1}\right)-\tau_{t+1}^{d e f} \Phi\left(\varpi_{t+1}\right) \tag{49}
\end{equation*}
$$

The default tax/subsidy $\left(\tau_{t+1}^{\text {def }}\right.$ ) is a fee paid (or rebate received) by the lender based upon the average default rate of its loan portfolio. The leveraged-lending tax $\tau_{t}^{l e v}$ is a tax on loan size $\left(Q_{t} K_{t+1}-N W_{t}\right)$, but scaled by the level of borrower leverage $\kappa_{t}$. We divide the lender's payoff by loan size to convert this to a return:

$$
\begin{equation*}
R_{t+1}^{L}=\frac{\tilde{g}\left(\varpi_{t+1}\right) Q_{t} R_{t+1}^{k} K_{t+1}}{\left(Q_{t} K_{t+1}-N W_{t}\right)}-\tau_{t}^{\kappa} \kappa_{t}=R_{t+1}^{k} \tilde{g}\left(\varpi_{t+1}\right) \frac{\kappa_{t}}{\kappa_{t}-1}-\tau_{t}^{l e v} \kappa_{t} \tag{50}
\end{equation*}
$$

The leverage tax can be equivalently interpreted as a tax on the loan-to-value (LTV) ratio which is given by $L T V \equiv\left(\frac{Q_{t} K_{t+1}-N W_{t}}{Q_{t} K_{t+1}}\right)=\frac{\kappa_{t}-1}{\kappa_{t}}$. With these taxes, the POC contract optimization conditions are now given by:

$$
\begin{align*}
& \beta \gamma V_{t+1} f^{\prime}\left(\varpi_{t+1}\right)+\Lambda_{t} M_{t+1} \tilde{g}^{\prime}\left(\varpi_{t+1}\right)=0  \tag{51}\\
& \beta \gamma E_{t} V_{t+1} R_{t+1}^{k} f\left(\varpi_{t+1}\right)+\Lambda_{t}\left\{E_{t} M_{t+1}\left[R_{t+1}^{k} \tilde{g}\left(\varpi_{t+1}\right)-\tau_{t}^{l e v}\left(2 \kappa_{t}-1\right)\right]-1\right\}=0  \tag{52}\\
& E_{t} M_{t+1}\left[R_{t+1}^{k} \tilde{g}\left(\varpi_{t+1}\right) \frac{\kappa_{t}}{\kappa_{t}-1}-\tau_{t}^{l e v} \kappa_{t}\right]=1 \tag{53}
\end{align*}
$$

The planner uses the default tax $\tau_{t+1}^{d e f}$ to achieve the desired default cut-off in (42). The leveraged-lending tax alters the return on savings and thus achieves the planner's desired level of investment and capital accumulation implied by (41). Recall that the BGG and POC contracts differ only by innovations. Since the leveraged lending tax is assessed at the time of the loan, it does not respond to subsequent innovations so that the supporting lending tax is identical for BGG and POC in the impulse response functions below.

To provide a quantitative sense of the importance of the two taxes on lending, it is instructive to express the lender's return on its loans as a function of the promised repayment rate. Tedious algebra implies the following:

$$
\begin{equation*}
\text { Lender's return }=\left\{\left[1-\Phi\left(\varpi_{t+1}\right)\right] Z_{t+1}-\tau_{t}^{l e v} \kappa_{t}\right\}+\frac{Z_{t+1}}{\varpi_{t+1}}\left\{(1-\mu) \int_{0}^{\infty} \omega \phi(\omega) d \omega-\tau_{t+1}^{d e f} \Phi\left(\varpi_{t+1}\right)\right\} \tag{54}
\end{equation*}
$$

The steady-state probability of default is small, so that quantitatively the default tax has a trivial effect on the lender's return. In contrast, the tax on leveraged-lending, $\tau_{t}^{l e v} \kappa_{t}$, reduces the return one-for one. In the steady-state the lender's return is given by $1 / \beta$, so that a higher leveraged-lending tax maps one-forone into an increase in the required loan repayment, thus dampening capital accumulation.

It is important to emphasize the taxes to which the planner does not have access. If the planner could choose the price of capital directly, she could achieve an efficient consumption allocation $\left(U_{c}(t)=\right.$ $\epsilon$ ) with no resource cost, ie., $\Phi\left(\varpi_{t}\right)=0$, and the CSV problem would disappear from the model. It is for this reason that we assume the planner cannot levy a tax on the sale of new capital for this would allow the planner to choose the price of capital independently of the level of investment (a similar argument applies to the capital rental rate). For example, suppose the new-capital producer maximized:

$$
\begin{equation*}
Q_{t}\left(1-\tau_{t}^{q}\right) \vartheta\left(\frac{I_{t}}{I^{*}}\right) I_{t}-I_{t} \tag{55}
\end{equation*}
$$

where $\tau_{t}^{q}$ is a tax on new capital levied on the seller. This implies the following price of capital:

$$
\begin{equation*}
Q_{t}=\left(1-\tau_{t}^{q}\right)^{-1}\left[\vartheta\left(\frac{I_{t}}{I^{*}}\right)+\frac{I_{t}}{I^{*}} \vartheta^{\prime}\left(\frac{I_{t}}{I^{*}}\right)\right]^{-1} \tag{56}
\end{equation*}
$$

By varying $\tau_{t}^{q}$, the planner could achieve any capital price that is desired, and thus, via (34), any desired level of capitalist consumption. This means that (34) will no longer be a constraint, $\Lambda_{3 t}=0$, and the planner sets $\Phi\left(\varpi_{t}\right)=0$. As noted, this is the (uninteresting) case of perfect consumption sharing, and the informational friction drops from the model. A similar result holds in Jeanne and Korinek (2010) and Bianchi (2011): a time-varying subsidy on assets (Jeanne and Korinek) or non-tradeable consumption (Bianchi) can eliminate the borrowing constraint and achieve the frictionless allocation. We ignore such a capital tax because it makes things too simple for the planner. That is, since capitalists are inelastic savers
and are the only holders of capital in the BGG model, a capital tax effectively gives the planner a lump sum means of redistributing consumption. ${ }^{6}$

## IV. Quantitative Analysis

We set parameters to values familiar from the literature. The discount factor $\beta$ is set to 0.99 . Utility is assumed to be logarithmic in consumption ( $\sigma=1$ ), and the elasticity of labor is assumed to be 3 ( $\eta=1 / 3$ ). We choose the constant $B$ to normalize steady-state labor in the competitive equilibrium to unity. The production function parameters include $\alpha=1 / 3$, and quarterly depreciation is $\delta=.02$. The investment adjustment cost function is given by

$$
\vartheta\left(\frac{I_{t}}{I^{*}}\right) \equiv X_{t}\left[1-\frac{\psi}{2}\left(\frac{I_{t}}{I^{*}}-1\right)^{2}\right]
$$

where $X_{t}$ is an exogenous shock to the marginal efficiency of investment (MEI). We set $\psi=0.50$, and choose $I^{*}$ such that in the steady state the price of capital in the competitive equilibrium is equal to unity.

As for the credit-related parameters, we calibrate the model so that the steady state of the competitive equilibrium is consistent with: (i) a spread between $Z$ and $R^{d}$ of 200 bp (annualized), (ii) a quarterly bankruptcy rate of $.75 \%$, and (iii) a borrower leverage ratio of $\kappa=2$. These values imply a survival rate of $\gamma=0.94$, a standard deviation of the idiosyncratic productivity shock of 0.28 , and a monitoring cost of $\mu=0.63$. Since the BGG and POC contracts differ only by innovations their steady states are identical. However, the planner's steady state will depend upon the welfare weight placed on capitalist consumption. We will conduct sensitivity analysis on this variable.

[^5]
## Steady state analysis.

Given the calibration outlined above, Figures 1-3 plot the planner's allocation as a function of the utility weight placed on capitalists. We restrict ourselves to $\epsilon \leq 0.72$, at $\epsilon=0.72$ the marginal utility of consumption for the household and capitalist are equalized so that the planner chooses a bankruptcy rate of zero (see Figure 3).

The size of the capital stock and the level of lending can be above or below the competitive equilibrium level depending upon the welfare weight. Since capitalist consumption is proportional to the capital stock, the level of the capital stock chosen by the planner is strictly increasing in the welfare weight. For $\epsilon<0.26$, the planner prefers less capital than that implied by the competitive equilibrium. Capital accumulation is partly funded by lending so that the level of lending chosen by the planner is increasing in $\epsilon$. The planner's ratio of lending-to-GDP crosses the competitive equilibrium level at $\epsilon=0.47$. In summary, if the weight on capitalists is small (large) enough, the competitive equilibrium has too much (little) capital and too much (little) lending. For all values of $\epsilon$, the planner prefers a lower level of borrower leverage than that implied by the competitive equilibrium, ie., LTV ratios are too high in the competitive equilibrium.

Figures 2-3 show how other features of the credit market are altered by the planner's weight on capitalists. The annual default rate is always below the level implied the competitive equilibrium (3\%), and is decreasing in the welfare weight. Bankruptcy involves a deadweight loss of resources so it is not surprising that the competitive equilibrium has too much default. Further, lower levels of default increase the consumption levels of capitalists so that the default rate is strictly decreasing in $\epsilon$. Through the link between $\varpi_{t}$ and $Z_{t}$, these lower levels of default map into a smaller risk premium, $Z_{s s}-\frac{1}{\beta}$. For $\epsilon>0.3$, the risk premium actually becomes negative. Figure 4 plots the leveraged-lending tax that supports this planner behavior (the figure plots $\tau_{t}^{\text {lev }} \bar{\kappa}_{t}$ ). For small levels of $\epsilon$, the planner taxes loans to lower lending and the capital stock below the levels implied by the competitive equilibrium. As the planner weight increases, this gives way to a subsidy that compensates the lenders for the negative risk premium.

## Dynamic analysis.

We consider the dynamic response to four shocks that are common in the business cycle literature: (i) TFP shocks, (ii) marginal efficiency of investment (MEI) shocks, (iii) idiosyncratic variance shocks, and (iv) net worth shocks (shocks to the death rate of capitalists). These four shocks follow the following laws of motion:

$$
\begin{aligned}
& \log \left(A_{t}\right)=\rho_{A} \log \left(A_{t-1}\right)+\varepsilon_{t}^{A} \\
& \log \left(\sigma_{t}\right)=\left(1-\rho_{\sigma}\right) \log \left(\sigma_{s s}\right)+\rho_{\sigma} \log \left(\sigma_{t-1}\right)+\varepsilon_{t}^{\sigma} \\
& \log \left(X_{t}\right)=\rho_{X} \log \left(X_{t-1}\right)+\varepsilon_{t}^{X} \\
& \log \left(\gamma_{t}\right)=\left(1-\rho_{\gamma}\right) \log \left(\gamma_{s s}\right)+\rho_{\gamma} \log \left(\gamma_{t-1}\right)+\varepsilon_{t}^{\gamma}
\end{aligned}
$$

We use the estimates of Christiano, Motto and Mostagno (2014) for the standard deviation and autocorrelation of these shocks:
$\begin{array}{ll}\text { TFP: } & \rho_{A}=0.81, S D=0.46 \\ \text { MEI: } & \rho_{X}=0.91, S D=5.5 \\ \text { Variance: } & \rho_{\sigma}=0.97, S D=7.0\end{array}$

Net worth: $\quad \rho_{\gamma}=0, S D=0.81$.

The autocorrelation of net worth in Christiano et al. (2014) is assumed to be zero, $\rho_{\gamma}=0$. Christiano et al. (2014) have both unanticipated and anticipated shocks to idiosyncratic variance. We use their estimates for the unanticipated shock as these are the only shocks in the model presented here.

We consider three different models: POC, BGG, and the planner. For the planner we set $\epsilon=$ $0.20 .{ }^{7}$ As noted earlier, this implies that the planner will choose different steady-state levels of capital, labor, bankruptcy, etc., than BGG and POC. To focus on dynamic issues, we choose steady-state taxes in POC and BGG so that the non-stochastic steady-state of all three models are identical. These choices imply a labor tax of 0.007 , a leveraged-lending tax of 0.0033 , and a bankruptcy tax of 0.062 . As suggested earlier, the leverage-lending tax is the most important of these three taxes. With a leverage rate of about 2 , this implies an annualized tax on interest income of 260 bp .

Table 1 reports the variance decomposition for the three models (planner, POC, BGG). Consistent with Christiano et al. (2014) who assume the BGG contract, the risk and MEI shocks are the primary drivers of real activity for the BGG contract. ${ }^{8}$ With the POC contract, risk shocks are not that important. With the BGG contract, 32 percent of the variance of output is explained by risk shocks. Consistent with the muted financial accelerator, this drops to 7 percent with POC.

Figures 5-8 report IRFs to the four shocks. The financial accelerator with the BGG contract is most clearly seen in the sharp movements in net worth in response to all four shocks. The equity-type indexation in the POC contract disrupts this accelerator so that there are only modest movements in net worth. Consider, for example, the iid net worth shock. The BGG financial accelerator implies a virtuous feedback loop between net worth and the price of capital so that both rise sharply and persistently to this iid innovation. In contrast, under the POC, net worth moves by only a trivial amount as the optimal contract has the observed net worth shock shared between the lender and the borrowers. In comparison to POC, the presence of the financial accelerator in BGG magnifies the output and investment response to all shocks, with the exception being in the MEI shock. A MEI shock causes a decrease in the price of

[^6]capital which under BGG decreases net worth and thus dampens the expansionary effect of the shock on investment spending. But under the POC, the MEI shock has a large expansionary effect on investment. Recall that leverage is given by $\kappa_{t} \equiv\left(\frac{Q_{t} K_{t+1}}{N_{t}}\right)$. The equity-like indexation under the POC dampens movements in leverage and net worth so that the drop in $Q_{t}$ has to be matched by an increase in capital and thus a sharp increase in investment.

There are two striking features of the planner. First for risk and net worth shocks the planner limits movements in investment. This not too surprising. From the planner's perspective both shocks transfer resources to the entrepreneur. The net worth shock does this directly, while the risk shock increases $f\left(\varpi_{t}\right)$ and decreases $g\left(\varpi_{t}\right)$. The planner prefers to preempt this redistribution by limiting movements in investment and thus the price of capital.

The second notable feature of the planner is the trivial movement in default rates. The planner limits these movements because of the deadweight loss of bankruptcy, which is convex in the probability of default. Instead the planner uses the leveraged-lending tax to move the risk-premium $\left(Z_{t}-R_{t}^{d}\right)$ to the desired level while having only a trivial effect on the default rate. The mechanism is via movements in the return on capital. Suppose we combine equation (8) and (50):

$$
R_{t+1}^{L}=R_{t+1}^{k} \tilde{g}\left(\varpi_{t+1}\right) \frac{\overline{\kappa_{t}}}{\overline{\kappa_{t}}-1}-\tau_{t}^{l e v}{\overline{\kappa_{t}}}_{t}=\frac{z_{t+1}}{\omega_{t+1}} \tilde{g}\left(\varpi_{t+1}\right)-\tau_{t}^{l e v} \overline{\kappa_{t}}
$$

For a given required lender return and bankruptcy cut-off, movements in the leveraged-lending tax map directly into movements of $Z_{t+1}$. Hence, the time-path of the leverage tax closely mirrors the planner's choice for the risk spread. Note that the risk premium reported in the graphs is the expected repayment spread for loans taken out in time-t, $E_{t}\left(Z_{t+1}-R_{t+1}^{d}\right)$, and thus does not include the contemporaneous movement of the repayment rate on loans from the previous period. Again, this behavior is easily seen for the iid net worth shock. The planner does not vary bankruptcy (so that net worth increases
contemporaneously), but uses the lending tax and subsequent increase in the lending rate to transfer net worth back to steady state levels.

## Welfare costs.

Using the calibrated parameter and shock processes, we compute a second-order approximation to the model to compute lifetime utility for households, capitalists, and the planner (with $\epsilon=0.20$ ), under the three allocations. Table 2 reports the welfare losses for individual shocks. Since household utility is logarithmic, the differences in total welfare can be interpreted as the perpetual percentage increase in household consumption needed to equate planner utility under the three different allocations. The thing that jumps out is how disastrous risk shocks are in the BGG contract. The welfare cost of the BGG contract is $1.35 \%$ into perpetuity versus $0.10 \%$ for the POC contract. This was anticipated in the impulse response functions. For risk shocks there is a strong financial accelerator mechanism present that is nearly absent in the POC contract. The welfare cost of net worth shocks is not as dramatic simply because they are assumed to be i.i.d. and have a small calibrated standard deviation.

Table 3 reports the results with all the shocks present. The total welfare gains of the planner over POC and BGG are substantial, e.g., the welfare cost of the BGG contract is $1.86 \%$ of consumption into perpetuity and $0.34 \%$ for POC. Neither the POC nor the planner are Pareto improvements over BGG. In fact, capitalists strongly prefer the BGG contract for all four shocks because of its effect on average capitalist consumption. This preference is especially strong for risk and MEI shocks, quantitatively the two most important shocks. The reason for this is because consumption for the capitalist (equation 34) consists of the product of the borrower's share, $f\left(\varpi_{t}\right)$ and the return to capital, $\left[Q_{t}(1-\delta)+m p k_{t}\right]$. This correlation is reported in Table 3. This correlation is positive for BGG, negative for the planner, and essentially zero for POC. In BGG, shocks that increase the price of capital set off a virtuous circle in which the higher price of capital increases net worth, which in turn increases the price of capital, etc. (this
is just the financial accelerator). Since the BGG repayment rate is not indexed to aggregate shocks, this surge in net worth leads to a persistent decline in $\varpi_{t}$ and persistent increase in $f\left(\varpi_{t}\right)$.

## A Cyclical credit policy.

Arguably the leveraged-lending tax is the easiest for a macroprudential policy to implement. Here we continue to assume that the three supporting taxes are set to achieve the planner's steady-state, but the only cyclical instrument is the leveraged-lending tax. Furthermore, we assume a policy where the regulator cannot observe the individual shocks and instead introduces a simple policy rule for the lending tax that responds to aggregate output. Suppose the leveraged-lending tax is given by:

$$
\tau_{t}^{l e v}=\tau_{s s}^{l e v}+\phi_{l e v} * \log \left(\frac{Y_{t}}{Y_{s s}}\right) .
$$

Figure 9 plots the total welfare loss (compared to the planner) as a function of $\phi_{\text {lev }}$. For the BGG contract, there are substantial gains to making the leverage tax pro-cyclical. For example, at $\phi_{\text {lev }}=1$, the improvement is substantial. The welfare loss drops from $1.86 \%$ to $1.29 \%$. Such a tax policy is preferred because it dampens the financial accelerator and over-response of investment under the BGG contract. In contrast, there is only a trivial financial accelerator under the POC contract, so that there is a much smaller advantage to such a tax policy. The optimal $\phi_{\text {lev }}$ for the POC contract is around 0.1 with welfare losses dropping from $0.34 \%$ to $0.25 \%$.

This also illustrates the importance of knowing whether the financial structure is closer to the BGG benchmark or the POC benchmark. If $\phi_{\text {lev }}$ is set to unity thinking the world is BGG the welfare loss drops from $1.89 \%$ to $1.29 \%$, but with the POC contract the welfare loss would increase substantially from $0.34 \%$ to $0.93 \%$. A more robust level of $\phi_{\text {lev }}$ would be 0.25 , the welfare loss of the POC contract would be nearly identical ( 0.35 versus 0.34 ) but there would be a sizeable improvement in the BGG contract from $1.86 \%$ to $1.4 \%$.

## V. Conclusion

This paper has investigated an issue that has been relatively unexplored in the burgeoning macro prudential literature. This is the role for potential regulations to affect the borrower's LTV ratios in macroprudential policy. For example, this analysis suggests that risk weights in a financial intermediary's capital structure depend on the LTV ratio of the firms who are borrowing the money. Furthermore these weights should vary cyclically. This is especially true for the model with BGG contracts.

Left unanswered is whether we should view the contracting structure as BGG or POC. One potential way to look at this question is to observe the sophistication of the intermediary's financial structure. For example, one may interpret the prevalence of hedging as a proxy for the sophistication of the financial structure, suggesting there is little role for cyclical macro-prudential policy. However, if the financial structure is more primitive there might be a sizable role for cyclical policies. There is indirect evidence suggesting that something similar to a POC contract is more empirically relevant than one might initially think. Carlstrom, Fuerst, Ortiz, and Paustian (2014) use familiar Bayesian methods to estimate a medium-scale DSGE model with a BGG financial structure and an exogenous level of contract indexation. The empirical estimate of this indexation parameter is large and significant, and essentially eliminates the financial accelerator.

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## Table 1: Variance Decomposition

|  | TFP | NW | Risk | MEI |
| :--- | :--- | :--- | :--- | :--- |
| Output | $5,4,9$ | $0,0,3$ | $8,7,32$ | $87,89,57$ |
| Investment | $0,1,2$ | $0,0,2$ | $17,3,50$ | $83,96,47$ |
| Default | $0,0,0$ | $0,2,1$ | $94,89,82$ | $6,9,16$ |
| Ex ante <br> risk <br> premium | $1,4,1$ | $35,0,2$ | $9,43,58$ | $54,53,38$ |
| (Planner, POC, BGG) |  |  |  |  |

## Table 2: Welfare Analysis, Individual Shocks

| Shock | Welfare | Planner | BGG | POC | Planner-BGG | Planner-POC | POC-BGG |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| TFP |  |  |  |  | \%HH C | \%HH C | \%HH C |
|  | Total | -11.837 | -11.848 | -11.839 | 0.011 | 0.002 | 0.009 |
|  | HHs | -19.974 | -19.986 | -19.975 | 0.012 | 0.002 | 0.010 |
|  | Capitalists | 40.684 | 40.688 | 40.683 | -0.004 | 0.001 | -0.005 |
| MEI |  |  |  |  |  |  |  |
|  | Total | -11.653 | -12.016 | -11.888 | 0.363 | 0.236 | 0.127 |
|  | HHs | -19.867 | -20.309 | -20.015 | 0.442 | 0.148 | 0.294 |
|  | Capitalists | 41.070 | 41.466 | 40.632 | -0.396 | 0.438 | -0.834 |
| Risk |  |  |  |  |  |  |  |
|  | Total | -11.888 | -13.331 | -11.984 | 1.443 | 0.096 | 1.346 |
|  | HHs | -20.042 | -21.625 | -20.121 | 1.583 | 0.078 | 1.504 |
|  | Capitalists | 40.772 | 41.471 | 40.683 | -0.700 | 0.089 | -0.788 |
| NW |  |  |  |  |  |  |  |
|  | Total | -11.837 | -11.880 | -11.840 | 0.043 | 0.004 | 0.040 |
|  | HHs | -19.974 | -20.040 | -19.977 | 0.066 | 0.004 | 0.062 |
|  | Capitalists | 40.685 | 40.797 | 40.685 | -0.112 | 0.000 | -0.112 |

The first three columns report expected lifetime utility evaluated at the non-stochastic steady-state. Total welfare is the sum of lifetime utility of households and capitalists, with the latter weighted by $\epsilon$. The final three columns report the differences in lifetime utility.

## Table 3: Welfare Analysis, All Shocks

|  | Planner | BGG | POC | Planner- <br> BGG | Planner- <br> POC | POC-BGG |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Total welfare | -11.706 | -13.566 | -12.043 | 1.860 | 0.337 | 1.523 |
| Households | -19.938 | -22.040 | -20.170 | 2.103 | 0.232 | 1.871 |
| Capitalists | 41.160 | 42.371 | 40.632 | -1.212 | 0.528 | -1.739 |
| Mean $\boldsymbol{C}_{\boldsymbol{t}}^{\boldsymbol{k}}$ | 0.4128 | 0.4260 | 0.4048 |  |  |  |
| Correlation <br> $\left\{\left[\alpha_{t}(1-\boldsymbol{\delta})\right.\right.$ <br> $\left.\left.+m p k_{t}\right], f\left(\omega_{t}\right)\right\}$ | -0.1622 | 0.4453 | -0.0277 |  |  |  |
| Mean $\boldsymbol{K}_{\boldsymbol{t}}$ | 13.479 | 12.655 | 13.002 |  |  |  |

The first three columns report expected lifetime utility evaluated at the non-stochastic steady-state. Total welfare is the sum of lifetime utility of households and capitalists, with the latter weighted by $\epsilon$. The final three columns report the differences in lifetime utility.

Figure 1: Planner Steady State Relative to CE

$$
\simeq \text { capital } \simeq \text { Loans/GDP } \simeq \text { leverage }
$$



Figure 2: Planner Steady State Annual Risk Premium (\%)


Figure 3: Planner Steady State Annual Default Rate (\%)


Figure 4: Planner Steady State Tax on Leveraged Lending (\%)


Figure 5: TFP Shocks


## Figure 6: NW Shocks



Figure 7: MEI Shocks


## Figure 8: Risk Shocks



FIGURE 9: WELFARE COST


## APPENDIX.

## The Planner at the Corner.

Suppose $\epsilon=\bar{\epsilon}$ so that $\Lambda_{3 s s}=U_{c}-\epsilon=0$. Here we will characterize the planner's allocation. This implies that $\Phi\left(\varpi_{s s}\right)=0$, and $h\left(\varpi_{s s}\right)=1$.

$$
\begin{align*}
& \delta K_{s s}=\vartheta\left(\frac{I_{s s}}{I^{*}}\right) I_{s s}  \tag{A1}\\
& C_{s s}+I_{s s}+\mathrm{C}_{s s}^{\mathrm{e}}=\left(K_{s s}\right)^{\alpha}\left(L_{s s}\right)^{1-\alpha}  \tag{A2}\\
& (1-\gamma)\left[(1-\delta)+m p k_{s s}\right] K_{s s}=C_{s s}^{e}  \tag{A3}\\
& Q_{s s}=\left[\vartheta\left(\frac{I_{s s}}{I^{*}}\right)+\frac{I_{S s}}{I^{*}} \vartheta^{\prime}\left(\frac{I_{s s}}{I^{*}}\right)\right]^{-1}  \tag{A4}\\
& -U_{L}=U_{c} m p l_{s s}  \tag{A5}\\
& Q_{s s}=\beta\left[Q_{s s}(1-\delta)+m p k_{s s}\right] \tag{A6}
\end{align*}
$$

Note that there is something of an income effect on labor supply as capitalist consumption lowers household consumption. For our benchmark calibration, the corner is given by $\epsilon=0.720$. At that value the planner chooses $\varpi_{s s}=0.17, L_{s s}=1.67, K_{s S}=22.13, Q_{s s}=1.98, \kappa=1.26$.


[^0]:    1 "A pecuniary externality is an externality that operates through prices rather than through real resource effects.... Under complete markets pecuniary externalities offset each other....However, when markets are incomplete or constrained, then pecuniary externalities are relevant for Pareto efficiency... [as] the welfare effects of a price movement on consumers and producers do not generally offset each other.... When some agents are subject to financial constraints, then changes in their net worth or collateral that result from pecuniary externalities may have first order welfare implications....This is an important welfare-theoretic justification for macroprudential regulation.." Wikipedia, 2015.

[^1]:    ${ }^{2}$ This is isomorphic to assuming that the deposit rate is pre-determined, and the dividend flow from the lenders is conditional on aggregate shocks.

[^2]:    ${ }^{3}$ See Townsend (1979).

[^3]:    ${ }^{4}$ An equivalent formation of the problem is to assume that the planner weights the two utilities equally, but the capitalist has linear preferences given by $\epsilon C_{t}^{k}$.

[^4]:    ${ }^{5}$ The appendix discusses the case in $\epsilon$ is sufficiently large so that the planner is pushed to the corner and sets bankruptcy equal to zero.

[^5]:    ${ }^{6}$ In a model such as Carlstrom and Fuerst (1997), where both households and capitalists hold capital, the capital tax could not be used so effectively. More generally, in more elaborate models it is surely not the case that a capital tax is isomorphic to a lump sum tax.

[^6]:    ${ }^{7}$ We have conducted sensitivity analysis on the IRFs to the planner for $\epsilon=0.10$, and $\epsilon=0.30$. The results are only modestly affected quantitatively.
    ${ }^{8}$ Christiano et al. (2014) report the dominance of risk shocks over MEI shocks only if the risk shocks include anticipated or "news" shocks.

