working paper

15 18

Intermediation in Networks

Jan-Peter Siedlarek

FEDERAL RESERVE BANK OF CLEVELAND

Working Paper 15-18 October 2015*

Intermediation in Networks

Jan-Peter Siedlarek

I study intermediation in networked markets using a stochastic model of multilateral bargaining in which players compete on different routes through the network. I characterize stationary equilibrium payoffs as the fixed point of a set of intuitive value function equations and study efficiency and the impact of network structure on payoffs. There is never too little trade but there may be an inefficiency through too much trade in states where delay would be efficient. With homogeneous trade surplus the payoffs for players that are not essential to a trade opportunity go to zero as trade frictions vanish.

JEL Classification: C73, C78, L14.

Keywords: bargaining, financial networks, intermediation, matching, middlemen, networks, over-the-counter markets, stochastic games

Suggested citation: Siedlarek, Jan-Peter, 2015. “Intermediation in Networks,” Federal Reserve Bank of Cleveland, working paper no. 15-18.

Jan-Peter Siedlarek is at the Federal Reserve Bank of Cleveland (jan-peter.siedlarek@ clev.frb.org. He thanks Fernando Vega-Redondo for extensive discussions and advice throughout and Douglas Gale for his guidance on this project. He also thanks Heski Bar-Isaac, Francis Bloch, Edoardo Gallo, Piero Gottardi, Francesco Nava, Volker Nocke, Debraj Ray, Tomás Rodríguez Barraquer, Matan Tsur for helpful comments and suggestions, as well as seminar participants at the EUI, Microsoft Research Cambridge, NYU, CTN2012, GAMES2012 and SAET2013. The author gratefully acknowledges financial support from the DAAD (German Academic Exchange Service) and through DFG grant SFB-TR15.

*First version: November 14, 2011. This version: September 23, 2015.A different version of this paper has been circulated under the title “Exchange with Intermediation in Networks.”

1. Introduction

This paper studies a network model of intermediation in markets. The network perspective, which puts the structure of connections between trading parties at the heart of the analysis, is particularly appropriate for the study of markets in which existing relationships matter for the interaction of economic agents. Many settings can be usefully thought of as networked markets, including markets explicitly relying on transport networks (pipelines, rail networks, ports) as well as markets where the connections are less tangible such as financial markets, in particular when traded over-the-counter (OTC), international trade and complex consumer goods including for example real-estate and insurance. In the latter markets connections take the form of relationships built on trust, a history of previous interaction or having sufficient information about trading partners. In the financial markets setting a relationship helps traders to manage their counterparty risk exposure, overcome reputational concerns or ensure that collateral provisions are in place.

In these relationship-based markets we often find intermediaries in the form of dealers, brokers and market makers that provide intermediation services for actors that do not trade directly with each other. The need for such intermediation arises naturally in network settings whenever there are opportunities for trade involving two parties that do not have a direct relationship, preventing them from direct interaction. They may then nonetheless exploit their opportunities for mutual trade by engaging indirectly, involving one or more intermediaries that provide the necessary chain of relationships that makes the trade feasible.

In this paper I employ a modeling approach that explicitly incorporates a network perspective on intermediation activity. The approach brings into focus the role and value of relationships used by third parties to facilitate transactions between players that otherwise might lack the opportunity to conduct trade directly. Specifically, I present a dynamic model of multilateral bargaining and exchange in a network setting with intermediation. Each period, a random matching process selects a *route*, that is, a group of players connecting two trading parties via connecting intermediaries in the network. One – randomly selected – player on the route can make a proposal to the other players. If it is accepted, the trade is implemented. If at least one player on the route rejects, a new route and proposer are drawn. I show that the model has a stationary subgame perfect equilibrium in which payoffs are characterized through an intuitive set of value function equations and use this to study efficiency and the sharing of surplus between parties. The equilibrium payoffs illustrate the effect that competition between intermediation routes has on the payoffs traders can expect. Efficiency considerations come into play when different routes may offer different levels of surplus, resulting for example from variation in buyer valuations or trade costs. The question is then to find the correct routes to trade on. I show that whilst in equilibrium players never unduly delay trade, there can exist instances where players agree to trade at times when delay would be efficient. The inefficiency arises from the strategic advantage for players that can trade across multiple routes with alternative players. They can increase their own payoffs relative to those who are in competition with each other. Players thus have an incentive to keep in play multiple routes, even if not all of them are efficient for trade. The same reasoning also suggests that traders in these markets have an incentive to (over)invest in creating competing routes (see also Elliott, 2014).

That markets for financial assets may be thought of as networks is startlingly exposed by looking at the data on trades in such markets. Early work in this direction includes Upper and Worms (2004) and Craig and von Peter (2014) who analyze the German interbank market. Their data reveal a network in a core-periphery structure with many peripheral banks that do not trade directly with others but only through the well-connected intermediaries at the center of the network. The model in this paper can be usefully seen to capture a market with such a core-periphery configuration: The seller in the model represents a bank in the periphery trying to access another periphery bank acting as the buyer. As no direct connections exist between banks in the periphery, intermediaries from the core of the network are required to facilitate the trade. The model then offers useful predictions concerning the trade pattern across the network as well as incentives for the banks to position themselves in the network.

I study a model with a single trade opportunity specific to a given seller, reflecting the notion of a thin market. Once trade concludes, the game is over and there is no replacement. This assumption approximates trade in highly customized products such as the complex financial securities commonly traded in OTC markets. This is in contrast to markets of more generic assets such as commodities or standard financial products where there may be many buyers and sellers in the market at the same time.

Note also that whilst I refer to buyers and sellers throughout the paper, the model may usefully be applied to study other value adding interactions between two parties, such as liquidity provision between banks, R & D cooperation between firms, the formation of joint companies by multiple entrepreneurs, coalition formation in political economy settings, etc.

The paper is structured as follows. The next Section 2 provides the literature context for the research questions investigated. Section 3 sets out the model and Section 4 characterizes equilibrium payoffs. An analysis and key results of the paper concerning efficiency and the relationship between structural features and payoffs are presented in Sections 5 and 6. Section 7 concludes.

2. Literature Context

This paper presents a contribution to the fast-growing literature on trade in networks and in particular the analysis of intermediation in such networks.

The provision of intermediation services and middlemen activities which this paper investigates in a network setting has been investigated in other non-structural frameworks by several authors, with overviews provided in Bose (2001) and Spulber (1999). Intermediaries have been credited with a number of different functions, including the provision of immediacy (Demsetz, 1968) or acting as a screening device between different types of traders that might be prevented from engaging directly with each other as in Bose and Pingle (1995) or Brusco and Jackson (1999). In the latter, an intermediary arises endogenously to overcome inefficiencies in trade across competitive markets. A seminal paper in this literature is Rubinstein and Wolinsky (1987). They investigate a setting with three types of players: buyers, sellers and middlemen. Trade is conducted on the basis of stochastic pairwise matching and a steady state equilibrium is derived.[1] A key insight of that paper is that the outcome of trade and the terms of trade depend on whether the middleman takes ownership of the good from sellers or work on a consignment basis. In the first case, the market is biased in favor of buyers, whereas in the second case symmetry between parties is restored. Duffie et al. (2005) study a search and matching model for OTC markets. They analyze a model in which trading opportunities arise endogenously and study amongst others the implications of greater competition for intermediation services. As in Rubinstein and Wolinsky (1987), the model does not capture heterogeneity in the connections that traders may have to the intermediaries and amongst the intermediaries itself.

In contrast to the work cited above, structural features are at the core of a fast-growing literature on exchange in networks with numerous recent contributions. Seminal early works in this field include Corominas-Bosch (2004) on bargaining in networks and the exchange model in Kranton and Minehart (2001). Both adopt a bipartite networks approach, precluding an analysis of intermediation. More recent contributions in this direction include Manea (2011), Elliott (2014), Polanski (2007) and Polanski and Vega-Redondo (2013). Models which allow for multiple steps in trading come in two distinct flavors. Gale and Kariv (2007), Manea (2013) and Gofman (2011) all consider a trading protocol in which the good travels from seller to buyer in a step-wise fashion, with traders interacting bilaterally at each step. The paper by Nava (2015), which studies quantity competition instead of an explicit bargaining setting, arguably also falls into this category as intermediaries benefit from double marginalization.

[1]In steady state equilibrium the outflow of pairs of traders who conclude a trade is exactly balanced by an exogenously given inflows of players.

In contrast Blume et al. (2009), Polanski and Lazarova (2014) and Nguyen (2012) allow for simultaneous multilateral interaction, which is also the approach I adopt in this paper. The key distinction of the current work is that contrary to Blume et al. (2009) I consider an explicit bargaining protocol whereas they consider price-setting intermediaries (whom they call "traders"). Furthermore, contrary to Nguyen (2012) and Polanski and Lazarova (2014) I focus on a setting without replacement, that is, an environment where parties that conclude a trade are not replaced by replica players. My model is therefore more suitable to study markets where "trade opportunities" are just that: opportunities that ought to be taken and that carry an opportunity cost via the risk of missing out as players cannot expect to get the same opportunity again.[2] The model thus offers a better match for real world markets where trade opportunities are not limitless, which arguably is the case in many relationship based markets, including for financial and non-financial assets as well as interactions in which players collaborate to conduct a joint project, e.g. an R & D joint venture. The assumption of no replacement has significant implications on equilibrium predictions. For example competition between multiple intermediaries is significantly tougher than in model with replacement.

The literature on financial networks employs network tools to analyze various aspects of financial markets, including risk sharing and contagion amongst financial institutions. An overview is provided in Allen and Babus (2009). Recent contributions in Babus (2012) and Farboodi (2014) provide a network perspective to OTC trading and investigate the incentives for financial institutions seeking to exchange assets to form relationships for trading and intermediation.

Finally, at a technical level, this paper employs the framework of stochastic bargaining games with perfect information analyzed in detail in Merlo and Wilson (1995, 1998) and extends it for use in analyzing games on networks. One contribution of my paper to this literature is to identify a new source of inefficiency in such stochastic bargaining settings, which does not arise in the setting of Merlo and Wilson (1995, 1998) as their model does not allow for the set of players bargaining changing each period. These changes are crucial in the network setting I study as they correspond to different routes and also introduce the notion of players being excluded from the bargaining table.

3. Model

This section presents a model in which players bargain over a surplus on a network. We consider a setting in which a network of relationships describes the possibilities for players to interact.

[2]Even if a new opportunity were to arise, the opportunity cost applies as long as players are not prevented from taking part in more than a single trade.

Players have access to an opportunity that generates surplus, e.g. generated by transferring an asset from a seller to a buyer. Players are matched along the network of existing relationships and bargain over the allocation of the available surplus within groups that form feasible trade routes. The bargaining protocol allows for the random selection of trade routes as well as the identity of proposer, incorporating the notion of competition between different alternative trade routes.

The model I present here includes a number of stark simplifying assumptions, e.g. concerning the underlying matching and bargaining protocol. These have been imposed in order to make the exposition as clean and transparent as possible. The same general insights would remain valid under less restrictive assumptions on many elements of the model.

Players: Players are denoted by the set $N = \{1, 2, ..., n\}$. There is one player $A \in N$ – the seller – who holds a single, indivisible good that she can sell to each of a set of m buyers $B = \{B_1, B_2, \dots B_m\}$ and $B_i \neq a$.[3]

Network: Players interact according to an undirected network denoted by $g = (N, E)$ where the set of edges $E \subset \{(i, j) : i \neq j \in N\}$ describes the set of feasible bilateral interactions. A group of players can trade with each other if and only if there exists a path in g between them. As will be described in greater detail below, trade between two nodes that are only indirectly connected is feasible through intermediaries if there exists at least one path between them. I assume that the network is connected.[4]

Routes: A path $R \subseteq N$ between a pair of nodes i and j is a sequence of nodes $(i_1, \dots, i_K)$ with $(i_k, i_{k+1}) \in E \; \forall \; k = 1, 2, \dots, K$, $i_1 = i$, $i_K = j$ and each node in the sequence distinct. A path is therefore acyclic. As the network is connected there exists at least one path in the network g between each buyer/seller pair. We call such an acyclic path connecting A and a given buyer B_i a *route*. Each route R_j has a surplus v_j attached to it reflecting buyer valuation less any costs. Depending on the network g for each given buyer-seller pair there may be multiple routes.[5]

Matching and bargaining protocol: The model operates in discrete time. In each period traders are matched and bargain under a stochastic route selection and bargaining protocol building on Merlo and Wilson (1995) as follows.

[3]The labels of buyers and sellers can be reversed without consequence for further analysis. The key simplification of the model is that there is just one trade opportunity and one node is involved in all possible coalitions that can realize the opportunity.
[4]This assumption is without loss of generality here as disconnected players simply cannot trade.
[5]One may restrict attention to shortest paths or geodesics only, but this restriction is not essential for the analysis.

Each period one trade route is activated and an order of play for players on this route is randomly determined. Based on this draw, players that are on the route bargain according to the order prescribed within the state, with the first acting as proposer.

Formally, in each period a state s from finite state space S is selected by a Markov process $\sigma = (\sigma_0, \sigma_1, \sigma_2, \ldots)$. A state s contains information about three elements of the model:

i. The active buyer $B(s) \in B$.
ii. The route $R(s) \subseteq N$ connecting the pair of players who have the trade opportunity with associated valuation $v(s)$, representing the surplus available in state s if there is agreement.
iii. A permutation $\rho(s)$ on $R(s)$ which denotes the order in which players move through the bargaining protocol. $\rho_i(s) \in N$ denotes the player moving in ith position. Following Merlo and Wilson (1995) we denote by $\kappa(s) \equiv \rho_1(s)$ the first mover in the order.

We take the set of states S to span all feasible trade routes in g as well as for each route all permutations of players on that route. Furthermore, to simplify the exposition I assume that σ is time homogeneous, such that $\sigma_t = \sigma_{t'}\ \forall\ t, t'$ and each period's draw is independent of the previous period's state. The independent ex ante probability of state s is denoted $\pi(s)$. Finally, we assume each $s \in S$ is drawn with strictly positive probability. Thus, every route is selected and every player is called upon as proposer with positive probability.[6]

On realization of state s, trader $\kappa(s)$ may propose an allocation or pass. If a proposal is made, this takes the form of a vector $x \in \mathbb{R}^n$ such that $x_i \geq 0$ and $\sum_{i \in N} x_i \leq v(s)$. x thus represents a split of available surplus amongst all players, allocating a nonnegative share x_i of the surplus to each trader in N. The other traders on the route then respond sequentially in order given by $\rho(s)$ by accepting or rejecting the proposal. This process continues until either (i) one player rejects proposal x or (ii) all players in $R(s)$ have accepted it.

If all responders accept x, the proposed split is implemented and the game ends. If the proposer passes or at least one responder rejects the proposed split, the bargaining round ends and the game moves to the next period in which a new state s' consisting of both a route $R(s')$ and a new order of play $\rho(s')$ is drawn and the bargaining process is repeated. This sequence is continued until an allocation is accepted by all players.

Information Structure: All players observe the realized states and all actions taken by other players.

[6]The assumption of independence allows me to dispense with conditioning on the current state whenever expectations about future realizations are formed and follows standard random proposer bargaining games. However, a general Markov process would leave general results unaffected as long as it is ergodic.

Payoffs: Payoffs are linear in the share of surplus allocated, with common discount factor $\delta \in (0, 1)$. If proposal x is accepted in period t, player i receives utility:

$$u_i(x) = \delta^t x_i$$

We assume that the surplus to be allocated is bounded above such that $u_i(x) \to 0$ as as agreement time $t \to \infty$.

The model forms an infinite horizon dynamic game of complete information. Players take a decision in two distinct roles: as proposer and as responder. As proposer, a player either passes or suggests a split of surplus on a given route conditional on the route selected and being selected as proposer. As responder, players have to decide whether to accept or reject a proposed surplus division. A responder's decision is conditioned on the selected route and proposer as well as the surplus division on the table.

A history is defined by a sequence of realized states and actions taken by players. A strategy specifies a feasible action at every possible history when a player must act.

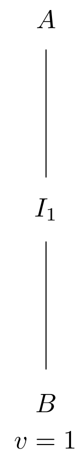
Note that bargaining in the model is multilateral and follows a unanimity rule: the good remains with the seller unless agreement with all intermediaries *on the selected route* to the buyer has been reached. Thus the model is applicable to markets in which intermediators act as a "broker" rather than ones in which they take possession of the good and act as a "market-maker".[7] Considerations which arise in markets described by a good "traveling" along the route, with intermediaries assuming ownership, such as questions of hold-up (intermediaries being in possession of the good but not intrinsically valuing it) or counterparty risk associated with disappearing resale opportunities, thus remain outside the model.[8]

Example State Space. To illustrate the model and in particular the workings of the matching and bargaining protocol, consider the network displayed in Figure 1. There is just one feasible trade routes generating a surplus of 1. The trade route consists of the seller A, one intermediary I and one buyer B. There are six feasible permutations of the three players on the route. In total, there are thus six states as enumerated in the adjacent table.

---

[7] Reporting of corporate bond markets suggests that in the wake of the 2008 financial crisis brokers increasingly showed the behavior implied in the model: *"In the wake of the financial crisis and ahead of tighter regulatory constraints, large Wall Street dealers have become far less willing to hold the risk of owning corporate bonds, known in market parlance as 'inventory,' in order to facilitate trading for their clients. Instead, they are increasingly trying to match buyers and sellers, acting more as a pure intermediary, rather than stockpiling bonds and encouraging a liquid market for secondary trading."* Source: Financial Times, November 8, 2011.

[8] See the discussion in Rubinstein and Wolinsky (1987) concerning the difference between middlemen taking ownership of the good and acting on consignment. Models exploring trade in networks in which the good travels on a bilateral basis from seller to buyer are analyzed in Gofman (2011) and Condorelli and Galeotti (2012).



s	$\pi(s)$	$R(s)$	$\rho_1(s)$	$\rho_2(s)$	$\rho_3(s)$	$\kappa(s)$
1	1/6	$\{A, I, B\}$	A	I	B	A
2	1/6	$\{A, I, B\}$	A	B	I	A
3	1/6	$\{A, I, B\}$	I	A	B	I
4	1/6	$\{A, I, B\}$	I	B	A	I
5	1/6	$\{A, I, B\}$	B	A	I	B
6	1/6	$\{A, I, B\}$	B	I	A	B

FIGURE 1. Example Network and State Space with a Single Trade Route

4. EQUILIBRIUM PAYOFFS

This section develops the equilibrium analysis of the model. We restrict attention to stationary subgame perfect equilibria (SSPE), that is, subgame perfect equilibria consisting of strategies which condition on payoff relevant histories only: the state (selected route and order of proposals), and the offer on the table in the given period.

Stationary equilibrium payoffs are characterized as a fixed point to an intuitive set of recursive equations using results derived in Merlo and Wilson (1998) and extending the analysis to the setting of networked markets. All proofs in this as well as subsequent sections are collected in the appendix.

Let f be an expected payoff where $f(s) \in \mathbb{R}^n$ denotes the vector of expected payoffs for players in state s. Define an operator $\mathbb{A}$ on payoff f which maps from $\mathbb{R}_+^{n\cdot|S|}$ to $\mathbb{R}_+^{n\cdot|S|}$ such that:

a. If $v(s) > \delta \sum_{j\in R(s)} E\left[f_j(s')\right]$ (Agreement):

$$\mathbb{A}_i(f)(s) = \begin{cases} v(s) - \delta E\left[\sum_{j\in R(s)\setminus i} f_j(s')\right] & \text{for Proposer } i = \kappa(s) \\ \delta E\left[f_i(s')\right] & \text{for Responder } i \in R(s) \setminus \kappa(s) \\ 0 & \text{for Excluded } i \notin R(s) \end{cases}$$

b. If $v(s) < \delta \sum_{j\in R(s)} E\left[f_j(s')\right]$ (Delay):

$$\mathbb{A}_i(f)(s) = \delta E\left[f_i(s')\right] \;\; \forall\, i \in N$$

c. If $v(s) = \delta \sum_{j \in R(s)} E\left[f_j(s')\right]$ (Mixing):

$$\mathbb{A}_i(f)(s) = \begin{cases} \delta E\left[f_i(s')\right] & \forall\ i \in R(s) \\ \phi(s)\delta E\left[f_i(s')\right] \text{ with } \phi(s) \in [0,1] & \forall\ i \notin R(s) \end{cases}$$

where $\phi(s)$ is the probability of disagreement in state s.

The payoff operator $\mathbb{A}(f)$ distinguishes three cases depending on $v(s)$, the surplus in state s. These can be interpreted as follows:

a. **(Agreement)** If the available surplus $v(s)$ exceeds the total expected value of moving to the next stage for players on the selected route ($\delta E\left[\sum_{j \in R(s) \setminus i} f_j(s')\right]$), then $\mathbb{A}(f)$ assigns to the proposer a payoff that extracts from responding parties on the selected route all surplus over and above their endogenously determined outside option value given by $\delta E\left[f(s')\right]$, leaving zero to traders not included on the route.
b. **(Delay)**If the available surplus $v(s)$ is less than the expected value of moving to the next stage for players on the selected route, then $\mathbb{A}(f)$ assigns that payoff to each player.
c. **(Mixing)** If the available surplus $v(s)$ is equal to the expected value of moving to the next stage for players on the selected route, $\mathbb{A}(f)$ for players on the route is equal to their outside option. For excluded players the payoff is between zero and their outside option. Their exact payoff is a share of their outside option equal to the probability of disagreement in the state.

A stationary equilibrium payoff of the bargaining game is a fixed point of this correspondence. The proof follows standard approaches and is presented in the appendix.

Proposition 1. *There exists an SSPE payoff f. f is an SSPE payoff if and only if $\mathbb{A}(f) = f$.*

The equilibrium payoff is supported by a strategy profile in which every player adopts a strategy with the following standard properties. When responding a player accepts any offer which gives her at least the discounted expected next period payoff and reject otherwise. If proposing, she offers every responder their outside option if the residual amount is strictly larger than the proposer's discounted expected next period payoff. If the residual is strictly less, the proposer passes with probability one. In case of indifference the proposer makes an offer as above with probability between zero and one. We discuss the role of such "mixed agreement" states further below. Here it suffices to note that the agreement probabilities may not be uniquely pinned down for each state as different combinations of agreement probabilities may support the same vector of expected equilibrium payoffs.

Proposition 1 allows the analysis of equilibrium outcomes and payoffs for all possible trade networks and buyer valuations on the basis of a set of equations describing value functions in a recursive manner. We will exploit the characterization to study efficiency and the impact of network structure on equilibrium outcomes in subsequent sections. At this point it is worthwhile to emphasize the implications of the "no replacement" assumption on equilibrium payoffs. Proposition 1 implies that excluded players receive a zero payoff in states of agreement whilst they can have a positive expected payoff in states of disagreement. This reflects the fact that they may be included in successful negotiations in a future period. The zero payoff for excluded players in case of agreement presents a significant difference to models with replacement (e.g. Nguyen (2012) and Polanski and Lazarova (2014)) in which players who do not take part in a trade that is concluded simply wait for the next period to be offered an essentially unchanged environment opportunity. It significantly intensifies the competition between different trading routes as they vie to be included in the group that reaches agreement. Section 6 provides further analysis on this topic.

Example Equilibrium Payoffs. To illustrate the equilibrium payoff characterization of Proposition 1, we return to the example in Figure 1. First given in every state the available surplus is 1, we conjecture that agreement will take place in every state, compute the resulting payoffs and verify the agreement decision later. Under the conjecture buyer A will receive a "responder" payoff of $\delta E\left[f_A(s')\right]$ in four out of six states $(3-6)$. Thus, $f_A(s) = \delta E\left[f_A(s')\right]$ for $s \in \{3,4,5,6\}$. When proposing, A will receive the residual surplus after offering just enough to I and B to make them accept. Thus $f_A(1) = f_A(2) = 1 - \delta E\left[f_I(s')\right] - \delta E\left[f_B(s')\right]$. Plugging these expressions into the expansion of $E\left[f_A(s')\right]$ yields:

$$E\left[f_A(s')\right] = \frac{4}{6}\delta E\left[f_A(s')\right] + \frac{2}{6}\left\{1 - \delta E\left[f_I(s')\right] - \delta E\left[f_B(s')\right]\right\} \tag{1}$$

By symmetry, identical expressions characterize $E\left[f_I(s')\right]$ and $E\left[f_B(s')\right]$ and in equilibrium all three players receive the same payoff. Thus we can solve Equation 1 for $E\left[f_A(s')\right] = \frac{1}{3}$. Finally, the solution is consistent with our conjecture about agreement behavior: $\sum_{i\in R(s)} \delta E\left[f_i(s')\right] = \delta < 1 \;\forall\; s \in S$ and thus agreement in all states is indeed optimal.

5. Efficiency

This section discusses the efficiency properties of the equilibrium of the bargaining game. Efficiency is achieved by adopting an optimal stopping rule which implements agreement in states which offer sufficiently high surplus and delays otherwise.

Let $\phi(s) : S \to [0,1]$ describe a function that for each state $s \in S$ denotes the probability of "stopping". Stopping implies that the surplus $v(s)$ is collected and the game end. Not stopping implies that one period passes and a new state is drawn. Given independence of the realizations of s across time, the total surplus $w(\phi)$ associated with a stopping rule ϕ is computed recursively by the expression

$$w(\phi) = \sum_{s \in S} \pi(s) \left\{ \phi(s) v(s) + [1 - \phi(s)] \, \delta w(\phi) \right\}$$

The optimal stopping rule ϕ^* is defined as:

$$\phi^* = \arg\max_{\phi} w(\phi)$$

Denote w^* the ex ante expected total surplus that can be derived under the optimal stopping rule ϕ^*. By the principle of optimality the efficient stopping rule ϕ^* satisfies a threshold rule for all $s \in S$ that collects the available surplus $v(s)$ if it is larger than w^* and passes otherwise:

$$\tilde{\phi}(s) = \begin{cases} 1 & \text{if } v(s) > \delta w^* \\ \phi \in [0, 1] & \text{if } v(s) = \delta w^* \\ 0 & \text{if } v(s) < \delta w^* \end{cases}$$

The efficiency benchmark suggests two possible sources of inefficiency: there may be too much trade or too little. Too much trade is conducted if the parties involved in bargaining on a route agree to an allocation in a state in which it would be efficient to delay. There is too little trade if the parties do not agree on an allocation in a state where trade would be strictly efficient in the sense that available surplus strictly exceeds what could be gained from waiting. I will show that the SSPE of the game specified does not exhibit the latter type of inefficiency but is subject to the former.

Proposition 2. *In any SSPE players reach agreement with probability one in all states in which agreement is strictly efficient.*

Proposition 2 implies a corollary for the baseline case where all feasible routes generate the same surplus v. In this case, $w^* = v$ and thus efficiency demands that trade be concluded immediately without delay.

Corollary 3. *If $v(s) = v \; \forall \; s \in S$, in any SSPE trade is conducted immediately and the equilibrium outcome is efficient.*

A necessary condition for delay in this model is thus the heterogeneity of surplus across different routes.

Proposition 2 also implies that trade is concluded even along intermediation routes which may involve relatively large numbers of intermediaries when shorter, more direct routes are available.

Thus, an intuitive prediction that it might be better for buyer and seller to delay trade in such situations to avoid splitting the surplus with additional parties does not hold. This is due to the fact that payoffs for intermediaries on the longer route are endogenously adjusted downwards in equilibrium, reflecting the constraint exerted by the presence of the shorter route. Thus, in this model there is no "strategic" cost from additional intermediaries per se. What matters for whether a route is actively traded over is the surplus it generates. This feature is an important implication of the model which recently has received experimental support in Choi et al. (2014).

Can trade occur too early in equilibrium? Yes, as long as $\delta < 1$ as I will illustrate in a variation of the example seen above. Consider the setting with a single seller and two possible routes, each with one intermediary and one buyer, illustrated in Figure 2. The low valuation route generates a surplus of 1 whilst the high valuation route generates a surplus of $v \geq 1$. Assume as above a uniform stochastic process such that each route is selected with probability $\frac{1}{2}$ and along each route each player is selected with equal probability. Thus, each route is played half of the time and conditional on a route being selected each of the three players is proposing with equal probability.

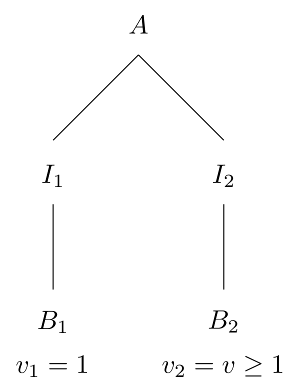


FIGURE 2. Network with Two Asymmetric Intermediation Routes

The efficient outcome in this case involves either trade along both routes or trade along the high value route with valuation v only, depending on the discount factor δ. Specifically, comparing expected total payoffs we can derive a critical discount factor of $\delta^* = \frac{2}{1+v}$ at which delay and agreement on the low value route generate the same payoff. For $\delta > \delta^*$ efficiency requires trade to take place only along the high value route.

In contrast, the vector of equilibrium payoffs is such that agreement takes place in low value states with positive probability for a range of $\delta > \delta^*$. To see why consider payoffs in a hypothetical equilibrium in which indeed trade takes place with the low valuation buyer with probability zero. In this case, $E\left[f_{B_1}(s)\right] = E\left[f_{I_1}(s)\right] = 0$ as this route would never be involved in trade agreement.

For the players on the high value route (seller as well as the buyer and intermediary) the payoff equations would then be symmetric similar to the example in Figure 1 and can be solved for $E[f_A(s)] = E[f_{B_1}(s)] = E[f_{I_1}(s)] = \frac{v}{6-3\delta}$. However, for $\delta < \tilde{\delta} = \frac{6}{3+v}$ this solution would imply $\delta E[f_S(s)] < 1$. The seller would have a profitable deviation to offer some $\epsilon > 0$ to the other players on the low value route (who would accept it). Thus for $\delta < \frac{6}{3+v}$ there is no stationary equilibrium in which trade occurs on the low value route with zero probability.

Note that $\delta^* < \tilde{\delta}$ and thus there is an interval of discount factors δ with strictly positive measure in which equilibrium payoffs will be such that they imply trade with positive probability with the low valuation buyer – despite this being inefficient. Indeed as δ increases within $[\delta^*, \tilde{\delta}]$ we observe that starting from $\delta > \frac{2}{5}\left(5 - \sqrt{10}\right) \approx 0.735$ the equilibrium involves mixed strategies such that trade occurs on the low value route with a probability that is positive but strictly less than one. We can interpret this equilibrium as the seller keeping the low value route in play in order to maintain her strategic advantage relative to the high value route.

Two following two figures summarize the workings of the example with $v = 4$ by plotting equilibrium expected payoffs for all players (Figure 3), the probability of agreement in low valuation states (Figure 4) and the total surplus (Figure 5). The critical discount factor δ^* above which trade on the low value route become inefficient is $\frac{2}{5}$ in this case. As Figure 4 illustrates, in equilibrium trade occurs with probability one for an interval above this and then declines smoothly towards zero, hitting zero at $\frac{6}{7}$. In between these two values, the expected total surplus realized in equilibrium is below the efficient one (Figure 5).

Two further points are worth noting about the expected payoffs of the seller and the downstream players (buyer and intermediary). First, for $\delta > \frac{6}{7}$ we see the payoffs for the seller and the downstream traders on the high value route overlapping, reflecting the "strategic symmetry" of the three players whenever only the high value route is traded on. Second, for $\frac{2}{5} < \delta < \frac{6}{7}$ the chart shows higher payoffs for the seller, which illustrates the "strategic asymmetry" that results from the seller making active use of her outside option of trading on the low value route.

The source of the "too much trade" inefficiency identified here is a hold-up problem: from an efficiency perspective the seller should "invest" by delaying in the low surplus state, accessing the surplus of higher expected valuation. However, in the resulting configuration the symmetry between the seller and the high valuation buyer would result in equal payoffs for both players which leaves the seller worse off. Efficiency could be restored were the high valuation buyer able to commit to compensate the seller for the delay decision by promising a higher share of the surplus in the high value states. However, an SSPE does not permit strategies implementing such promises.

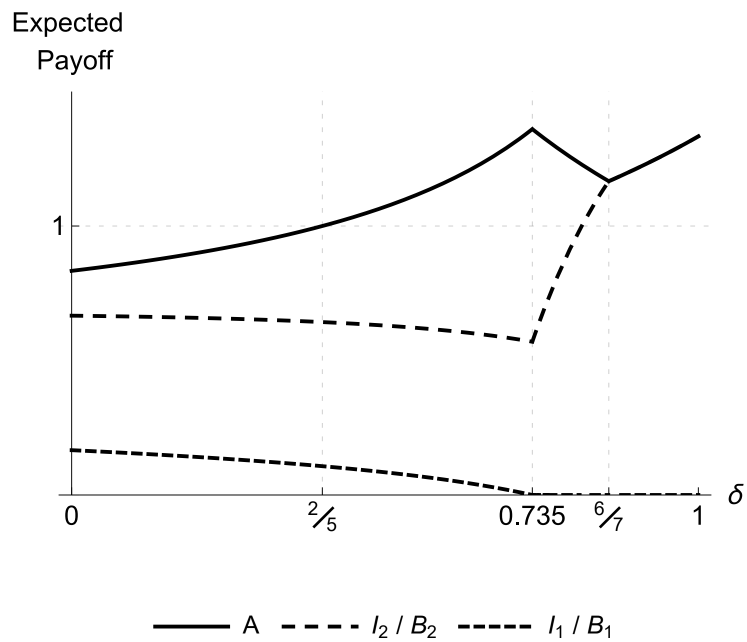


FIGURE 3. Example – Expected Payoffs

Looked at from another perspective, the inefficiency can be regarded as the result of the seller's privileged position and her unwillingness to give up the payoff benefits that result from having alternative sources of supply. If there were only a single buyer, then the trading outcome would be efficient, even if we hold constant at one half the probability of the high valuation route being activated in each period. Thus the addition of a trade route, a "thickening" of the market, can lead to a less efficient outcome. Even worse, the equilibrium payoffs are such that there are incentives for the seller to create such connections to additional buyers, even if these have a lower valuation and lead to a lower total surplus in equilibrium. The bargaining model thus exhibits incentives for over-investment in connections.

Finally note that as $\delta \to 1$, the equilibrium outcome realigns with efficiency as trade takes place along routes other than those with the highest value with probability zero. However, the incentives to over-invest by connecting to lower valuation routes may remain in place as players still gain from creating a strategic alternative for themselves and appropriating a larger share of the surplus. If such alternatives have a cost $\epsilon > 0$ attached to them, such investment would be wasteful even if in equilibrium trade occurred only on the efficient route.

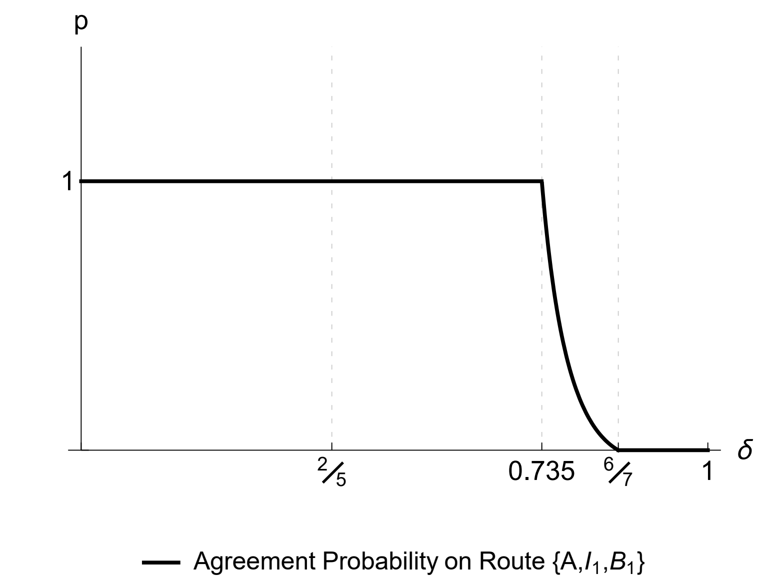


FIGURE 4. Example – Agreement Probability

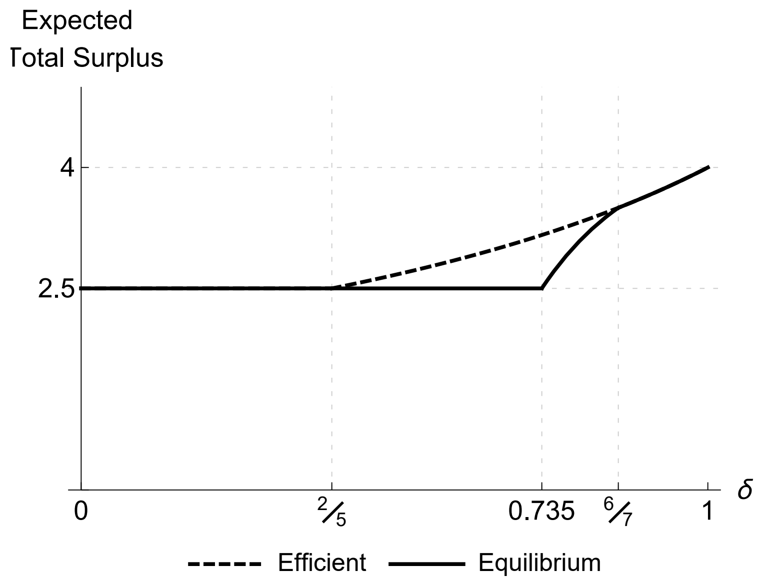


FIGURE 5. Example – Total Surplus

6. Network Structure and Equilibrium Payoffs

This section considers the relationship between structural features of the trade network and equilibrium payoffs. One implication of Proposition 1 is that players excluded in a state where

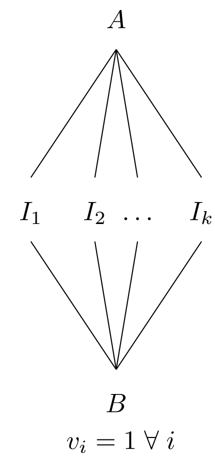


FIGURE 6. A setting with k intermediaries

agreement is struck receive a zero payoff. As a consequence, players who find themselves in such situations may be expected to have their bargaining power reduced. I investigate this question first by considering the way in which payoffs change as the number of competing intermediaries increases before deriving a more general result by considering the impact of being "essential" to a trade on payoffs. I restrict attention in the following to a setting in which all routes generate the same surplus in all states such that $v(s) = 1 \; \forall \; s$ to focus attention on the strategic competition between otherwise comparable routes.

6.1. **Additional Intermediation Routes.** To investigate the impact the number of intermediaries has on payoffs, consider first a simple setting with a single buyer and a set of k intermediaries that directly link to both the seller and the single buyer for the asset (see Figure 6), each generating a surplus of 1. Expected equilibrium payoffs for the end-nodes A and B and any intermediary I_i are then given by $E[f_A]$, $E[f_B]$ and $E[f_{I_i}]$, respectively:

$$E[f_A] = E[f_B] = \frac{k-\delta}{k(3-\delta)-2\delta}$$
$$E[f_{I_i}] = \frac{1-\delta}{k(3-\delta)-2\delta}$$

As expected, payoffs for end-nodes increase with the entry of additional intermediaries. Also as $\delta \to 1$, payoffs for intermediaries go to zero. The ratio of the payoffs is given by $\frac{f_B}{f_I} = 1 + \frac{k-1}{1-\delta}$. At $k = 1$, the relative shares are equal and as k increases the ratio increases linearly at rate $\frac{1}{1-\delta}$.

6.2. **Limit Payoffs on a Network with Competing Routes.** The analysis in the previous section illustrates the impact of competition in a simple setting with single-step, competing intermediaries.

One result of this analysis is that as trade frictions vanish in the limit intermediaries receive an expected payoff of zero. This section shows how the intuition derived from this simple example carries through to general structures.

Definition 1. *A player i is essential to a trade opportunity if $i \in R(s)\ \forall\ s \in S$.*

The definition reflects the approach adopted in Goyal and Vega-Redondo (2007) applied to the present model. Structurally speaking, a player is essential if he is located on all possible trade routes between the buyer and the seller of the good. As such, non-essential traders are competing for the business of intermediating the trade opportunity.

Proposition 4. *In an SSPE of the game with equal surplus in all states, the limit payoff of trader i as $\delta \to 1$ is strictly greater than zero if and only if the trader is essential.*

Intuitively, the key distinction between essential and non-essential players is that the latter have a positive probability of being excluded. This means that in the limit their implicit discount factor remains strictly below one whilst for essential players it converges to one.

Proposition 4 provides microfoundations for an analysis of competing intermediaries on networks and maps the intuitive Bertrand outcome into the bargaining setting investigated here. As such it provides a justification for the payoff structure used in Goyal and Vega-Redondo (2007), who investigate incentives for network formation in a setting with intermediation rents. Whilst they assume that non-essential players receive zero payoff, justifying it as the kernel and core in a cooperative bargaining setup, the present analysis may provide some grounding for this assumption in a non-cooperative bargaining setting.

7. Conclusion

In this paper, I study a model of bargaining and exchange with intermediation on networks, extending the Merlo and Wilson (1995) framework as a tool to analyze stochastic bargaining games into a network setting. I characterize payoffs with a simple set of value function equations allowing the analysis of efficiency and the impact of structure on payoffs in equilibrium outcomes. I find that trade in settings with homogeneous valuations across all routes, trade is efficient. However, with heterogeneity of surplus across routes, there can be too much trade in the shape of inefficiently early agreement in equilibrium, arising from a potential hold-up problem. Competition between intermediaries is shown to reduce payoffs for this type of player. In the limit as bargaining frictions disappear, all players who are not essential to a trade opportunity receive equilibrium payoffs of zero. I have imposed a number of simplifying assumptions to offer a clean and transparent exposition of the effects in my model. The same insights would remain if the model were generalized in a number of possible directions, including a more general stochastic process of selecting routes and proposers.

The present analysis suggests there is scope for future research in a number of directions. These include in particular a more explicit study of the implications of the bargaining model for network formation identifying the incentives for players to invest in connections. The resulting predictions can then be compared to those in models with different payoffs structures including for example Babus (2012) and Goyal and Vega-Redondo (2007).

8. APPENDIX

8.1. Proof of Proposition 1.

Characterization. This section presents the proof of Proposition 1. The approach taken employs a standard argument adapted from Merlo and Wilson (1998). The proof of the proposition requires demonstrating that f is an SSPE payoff *if and only if* $\mathbb{A}(f) = f$.

Proof. $\Rightarrow$ *"f is an SSPE payoff" implies "$\mathbb{A}(f) = f$"*

Consider an SSPE payoff f and fix a state s with $i = \kappa(s)$. Given f, it is a best reply for responder j to a given proposal x to reject if $x_j < \delta E\left[f_j(s')\right]$ and to accept if $x_j > \delta E\left[f_j(s')\right]$. This implies that i can earn $v(s) - \delta E\left[\sum_{j\in R(s), j\neq i} f_j(s')\right]$ from making a proposal that is accepted and $E\left[f_i(s')\right]$ from passing. Thus, if $v(s) < \delta E\left[\sum_{j\in R(s)} f_j(s')\right]$, the proposer will pass in a SSPE and $f_i(s) = \delta E\left[f(s')\right] \;\forall\; i$. If $v(s) > \delta E\left[\sum_{j\in R(s)} f_j(s')\right]$, i will make a proposal in an SSPE that is accepted, earning:

$$v(s) - \delta E\left[\sum_{j\in R(s), j\neq i} f_j(s')\right] \text{ for } i$$

$$\delta E\left[f_j(s')\right] \text{ for } j \in R(s)\setminus i$$

$$0 \text{ for } k \notin R(s)$$

If $v(s) = \delta E\left[\sum_{j\in R(s)} f_j(s')\right]$, the proposer is indifferent with $f(s) = \delta E\left[f(s')\right]$ again. This implies that in an SSPE an agreement can be reached with any probability between zero and one, which implies payoffs for any excluded player k that are in $[0, \delta E\left[f_k(s')\right]]$. Thus $\mathbb{A}(f) = f$.

$\Leftarrow$ *"$\mathbb{A}(f) = f$" implies "f is an SSPE payoff"*

Assume $\mathbb{A}(f) = f$. We show that f is an SSPE payoff by defining a suitable strategy profile and demonstrating that no player can be better off by unilaterally deviating. The strategy profile instructs proposers to pass unless $v(s) < \delta E\left[\sum_{j\in R(s)} f_j(s')\right]$ in which case the proposer offers each responder j the $\left[f_j(s')\right]$. Responders will then accept, which yields $\delta E\left[f_i(s')\right]$. Now, given payoffs f there is no incentive for any $j \in R(s)\setminus i$ to deviate and reject. For player i, there is no incentive to deviate as $f_i(s) \geq \delta E\left[f_i(s')\right]$. Finally, for $k \notin R(s)$, the rules are such that no action is taken and thus there no possibility for deviation. Similarly, if $v(s) > \delta E\left[\sum_{j\in R(s)} f_j(s')\right]$ given decision rules by responders, proposer i cannot benefit from deviating to a proposal that is accepted with positive probability. Finally, if $v(s) = \delta E\left[\sum_{j\in R(s)} f_j(s')\right]$ the strategy profile instructs the proposer to make an acceptable proposal with positive probability $\phi(s)$ such that for excluded players k $\phi(s)\cdot E\left[f_k(s')\right] = f_k(s)$ as required.

□

Equilibrium Existence. We prove existence of equilibrium by showing the existence of a fixed point of the correspondence $\mathbb{A}$. The argument is standard and makes use of Kakutani's fixed point theorem.

Proof. $\mathbb{A}$ is a self mapping on the space of payoffs which is a subspace $X \subseteq \mathbb{R}^{n \cdot |S|}$. X is non-empty, closed, bounded and convex. Boundedness can be seen by recognizing that the maximum payoff of any player in any state is the maximum valuation across all states.

Now, $\mathbb{A}$ is single valued for most of its domain. It is set valued for excluded players where payoffs for active players are equal for agreement and delay. In those instances the correspondence maps into a closed interval which implies that the correspondence is convex. Finally end-points of the interval are such that $\mathbb{A}$ has a closed graph.

Then by Kakutani's Fixed Point Theorem $\mathbb{A}()$ has a fixed point. □

8.2. **Proof of Proposition 2.** We proof by contradiction. Assume $\exists\ \tilde{s}$ s.t. $v(\tilde{s}) > \delta v^*$ so that delay is not efficient and no agreement is struck. Then by Proposition 1:

$$v(\tilde{s}) \leq \delta \sum_{i \in R(\tilde{s})} E\left[f_i(s')\right]$$

As v^* refers to the total expected payoff and thus the maximum that all players can jointly achieve, we have:

$$\begin{aligned} &\sum_{i \in R(\tilde{s})} E\left[f_i(s')\right] \\ \leq &\sum_{i \in N)} E\left[f_i(s')\right] \\ \leq & v^* \end{aligned}$$

Combining these terms we get:

$$\begin{aligned} v(\tilde{s}) &\leq \delta \sum_{j \in R(\tilde{s})} E\left[f_j(s')\right] \\ &\leq \delta v^* \end{aligned}$$

where the final step establishes the contradiction. □

8.3. **Proof of Proposition 4.** Consider first payoffs of essential players as $\delta \to 1$. Let i be essential, then by Proposition 1, for states s in which i is responding, $f_i(s) \to E\left[f_i(s')\right]$. Adding across states and noting that by being essential i is either proposing or responding, this implies equalization of payoffs across states, i.e. $f_i(\tilde{s}) \to E\left[f_i(s')\right]$ for states $\tilde{s}$ in which i is proposing.

Now consider a non-essential player k involved in two states s and $\tilde{s}$ that share the same route such that $R(s) = R(\tilde{s}) = R$ and $k \in R$. Furthermore, let $k = \kappa(s)$ and $i = \kappa(\tilde{s})$ with i essential. Then as $\delta \to 1$, payoffs for i tend to the same amount across s and $\tilde{s}$. All other responding players will receive equal payoff on the route by Proposition 1. This implies that also for k payoffs will be equal, i.e. $f_k(s) \to E\left[f_k(s')\right]$ and $f_k(\tilde{s}) \to E\left[f_k(s')\right]$.

Finally, by Proposition 1 $f_k(s) = 0$ for s in which k is excluded. As such states arrive with positive probability, we deduce $E\left[f_k(s')\right] = 0$ as required. □

References

Allen, F. and A. Babus (2009). Networks in Finance. In P. R. Kleindorfer and J. Wind (Eds.), *The Network Challenge*, pp. 367–382. Prentice Hall.

Babus, A. (2012). Endogenous Intermediation in Over-the-Counter Markets. Working Paper.

Blume, L., D. Easley, J. Kleinberg, and E. Tardos (2009). Trading Networks with Price-setting Agents. *Games and Economic Behavior 67*(1), 36–50.

Bose, G. (2001). Dealers, Markets, and Exchanges: A Study of Intermediation. In A. Bose, D. Ray, and A. Sarkar (Eds.), *Contemporary Macroeconomics*. Oxford University Press.

Bose, G. and M. Pingle (1995). Stores. *Economic Theory 6*(2), 251–262.

Brusco, S. and M. Jackson (1999). The Optimal Design of a Market. *Journal of Economic Theory 88*(1), 1–39.

Choi, S., A. Galeotti, and S. Goyal (2014). Trading in Networks: Theory and Experiments. *Cambridge-INET Working Paper 8*.

Condorelli, D. and A. Galeotti (2012). Bilateral Trading in Networks. Working Paper.

Corominas-Bosch, M. (2004). Bargaining in a Network of Buyers and Sellers. *Journal of Economic Theory 115*(1), 35 – 77.

Craig, B. and G. von Peter (2014). Interbank Tiering and Money Center Banks. *Journal of Financial Intermediation 23*(3), 322 – 347.

Demsetz, H. (1968). The Cost of Transacting. *The Quarterly Journal of Economics 82*(1), 33–53.

Duffie, D., N. Gârleanu, and L. Pedersen (2005). Over-the-Counter Markets. *Econometrica 73*(6), 1815–1847.

Elliott, M. (2014). Inefficiencies in Networked Markets. Working Paper. Forthcoming in American Economic Journal, Microeconomics.

Farboodi, M. (2014). Intermediation and Voluntary Exposure to Counterparty Risk. Working Paper. Available at SSRN 2535900.

Gale, D. and S. Kariv (2007). Financial networks. *The American Economic Review 97*(2), 99–103.

Gofman, M. (2011). A Network-Based Analysis of Over-the-Counter Markets. Working Paper.

Goyal, S. and F. Vega-Redondo (2007). Structural Holes in Social Networks. *Journal of Economic Theory 137*(1), 460–492.

Kranton, R. and D. Minehart (2001). A Theory of Buyer-Seller Networks. *The American Economic Review 91*(3), 485–508.

Manea, M. (2011). Bargaining in Stationary Networks. *The American Economic Review 101*(5), 2042–80.

Manea, M. (2013). Intermediation in networks. Working Paper.

Merlo, A. and C. Wilson (1995). A Stochastic Model of Sequential Bargaining with Complete Information. *Econometrica 63*(2), 371–399.

Merlo, A. and C. Wilson (1998). Efficient Delays in a Stochastic Model of Bargaining. *Economic Theory 11*(1), 39–55.

Nava, F. (2015). Efficiency in Decentralized Oligopolistic Markets. *Journal of Economic Theory 157*(0), 315 – 348.

Nguyen, T. (2012). Coalitional Bargaining in Networks. In *Proceedings of the 13th ACM Conference on Electronic Commerce*, pp. 772–772. ACM.

Polanski, A. (2007). Bilateral Bargaining in Networks. *Journal of Economic Theory 134*(1), 557–565.

Polanski, A. and E. Lazarova (2014). Dynamic Multilateral Markets. *International Journal of Game Theory*, 1–19.

Polanski, A. and F. Vega-Redondo (2013). Markets, Bargaining, and Networks with Heterogeneous Agents.

Rubinstein, A. and A. Wolinsky (1987). Middlemen. *The Quarterly Journal of Economics 102*(3), 581–594.

Spulber, D. (1999). *Market Microstructure: Intermediaries and the Theory of the Firm*. Cambridge University Press.

Upper, C. and A. Worms (2004). Estimating Bilateral Exposures in the German Interbank Market: Is there a Danger of Contagion? *European Economic Review 48*(4), 827–849.