The Piketty Transition

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We study the effects on inequality of a "Piketty transition" to zero growth. In a model with a worker-capitalist dichotomy, we show first that the relationship between inequality (measured as a ratio of incomes for the two types) and growth is complicated; zero growth can raise or lower inequality, depending on parameters. Extending our model to include idiosyncratic wage risk we show that growth has quantitatively negligible effects on inequality, and the effect is negative. Finally, following Piketty's thought experiment, we study how the transition might occur without declining returns; here, we find inequality decreases substantially if financial innovation acts to reduce idiosyncratic return risk, and does not change much at all if it acts to increase capital's share of income.

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1 Introduction

Thomas Piketty's *Capital in the Twenty-First Century* attacks the question of wealth inequality from two perspectives. The first is a monumental study of historical data, going back hundreds of years, that documents the dynamics of wealth inequality across several countries. There is no doubt that this data will be a fruitful source of material, and Piketty has graciously made the entire set publicly available for researchers to mine.

The second part is a rough sketch of an economic model that details the disastrous effects (as Piketty sees them) of low productivity growth, in terms of ever-expanding inequality. Crudely put, Piketty's model has two groups of households, workers and capitalists, who derive all of their income from a single source (labor and capital, respectively). Under some unusual assumptions about the form of the production function and the savings behavior of capitalists (see Krusell and Smith 2014 for thorough discussions of these points), Piketty arrives at the conclusion that inequality – measured as the share of national income that accrues to capital – will increase explosively as growth falls to zero.

Our goal is to shed light on this assertion using (fairly) standard macroeconomic tools. The basic model of macroeconomic inequality is Aiyagari (1994) (with predecessors Bewley 1986, İmrohoroğlu 1989, and Huggett 1993), where ex ante identical households experience different realizations of their labor productivity and, as a result, accumulate differing amounts of wealth. This model has been successful at matching a large number of facts about US inequality, at least when extended in appropriate ways (Krusell and Smith 1998, Carroll 2001, Castañeda, Díaz-Giménez, and Ríos-Rull 2003). We use a variant of this model, extended to include a capitalist-worker dichotomy, to study how inequality would be expected to respond in the presence of declining growth.

Our basic model has the following ingredients. Some households, called capitalists, own claims to the productive technology while other ones, called laborers, do not; both types have an endowment of time that can be rented to firms in return for labor income. We first study a version of this model where workers cannot access financial markets at all and idiosyncratic risk is absent. In this model, we can analytically characterize the relationship between growth and inequality (measured as the ratio of capitalist income to laborer income), and this expression is tractable if capitalists do not work. If capitalists work, the expression is too difficult to analyze

and we use numerical examples to show the following facts.

We find that, in general, inequality declines with growth, as Piketty maintains. When growth is low, the capitalist discounts the future at a lower rate, and thus accumulates more capital leading to higher wages – one can think of this effect as a rightward shift in the long-run inelastic supply curve for capital. Provided capital and labor are substitutes (but not too strongly), the resulting wage growth leads the capitalist to supply more labor. The combination leads to higher inequality; we can show that around $g = 0$ the hours effect dominates, meaning that inequality is driven by the labor supply effect on the capitalists and not the savings rate. If capital and labor are too easily substituted, then the capitalist does not work around the $g = 0$ steady state and inequality will rise due to a strong substitution into capital.

We then turn to a more elaborate environment in which (i) workers can save via a return-dominated asset (money or stored consumption) and (ii) labor productivity is stochastic for both types. Thus, our model is a two-type version of the one used in Krusell and Smith (2014), and we think it captures better the features that Piketty seems to have in mind. In this model we are able to measure inequality using standard concepts (in particular, Lorenz curves and Gini coefficients), and find that low growth involves small *decreases* in inequality. The mechanism is the same one identified in the simpler environment. Similar negative results are found in Condie, Evans, and Phillips (2014) in a very different model.

The transition our model produces implies that returns to capital will fall over time as the capital-labor ratio rises with falling growth. Piketty maintains in his model that returns do not fall, so to assess his transition's effects we need a mechanism for preventing a decline in returns. The basic model has little "room" for preventing declines in returns, because the wedge between the discount factor and the return to savings is small whenever idiosyncratic risk is unimportant (which is the case in our model if capitalists are very wealthy and do not work). We consider two possibilities here, namely financial innovation that eliminates idiosyncratic return risk for capitalists and capital-biased technical change. The first feature opens a wider gap between the return to saving and the time rate of preference by increasing the amount of precautionary savings. The second feature shifts the demand for capital to the right, counteracting the decline in returns associated with rising capital. The combination could generate a muted decline in

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returns, but does not; the problem is that asset supply is too elastic near the equilibrium return[1]

Our conclusion from these exercises is that there is little scope for a standard model to generate transitions without substantial declines in returns, and modifications along the lines suggested by Piketty do not resolve that issue except under specific circumstances. Furthermore, the effects of growth on conventionally-measured inequality are small – any effect of a "Piketty transtion" on inequality will come through technological progress and it will reduce, not increase, inequality.

2 Model

The model economy is populated by two groups, called capitalists and workers, who are situated in dynasties that live forever and value the utility of descendents. Both groups face uninsurable random movements in the productivity of their labor effort; we remain agnostic as to the sources of these fluctuations (losing one job and finding one that pays less money, promotions, changes in ability across generations, etc.). Both groups also have identical preferences over consumption and leisure (non-work time), so that capitalists are not just "patient" people who got rich because they were thrifty. Our main assumption is that there is no mobility across groups – at some point in the infinite past, some dynasties were lucky enough to get granted access to a productive asset called capital, and some were not.

We can represent the dynamic problem of a typical capitalist as

$$v\left(k,e\right) = \max_{k',h,c} \left\{ \frac{\left(c\left(1-h\right)^{\theta}\right)^{1-\sigma}}{1-\sigma} + \beta\left(1+g\right)^{1-\sigma} E\left[v\left(k',e'\right)\right] \right\} \tag{1}$$

$$c + \left(1+g\right)k' \leq \left(r+1-\delta\right)k + weh \tag{2}$$

$$k' \geq 0$$

$$h \geq 0$$

$$c \geq 0.$$

That is, the capitalist chooses consumption c, work effort h, and capital holdings k' to maximize lifetime utility; we have already incorporated growth in labor productivity g in the usual method to ensure the (normalized) wealth of the capitalist remains bounded over time (see King, Plosser,

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[1]The reasons that the asset supply is very elastic are well known; they are elaborated formally in Aiyagari (1994).

and Rebelo 1988 for details on how this normalization is done). Note the absence of insurance claims against e, the productivity of labor. As a result, there is a "precautionary saving" motive that leads capitalists to accumulate more capital than they normally would; however, this motive can disappear as the capitalist can choose to completely eliminate the risk by setting $h = 0$.

The dynamic problem of a typical worker is

$$V(m, s) = \max_{m', l, x} \left\{ \frac{\left(x (1 - l)^{\theta} \right)^{1-\sigma}}{1 - \sigma} + \beta (1 + g)^{1-\sigma} E \left[V(m', s') \right] \right\} \tag{3}$$

$$x + (1 + g) m' \leq \frac{m}{1 + \pi} + wsl \tag{4}$$

$$m' \geq 0$$

$$l \geq 0$$

$$x \geq 0.$$

Note here the key difference: the return to the worker saving is, on net, negative (we suppose $\pi \geq 0$); while capitalists can earn a positive return by renting capital to firms, workers can only "store" their savings as money and thus lose purchasing power over time via inflation. One could just as easily imagine the workers saving in the form of inventories of goods that rot slowly over time.

We can obtain the aggregate capital stock and labor input by summing over all individuals. Let $\Gamma(k, e)$ be the density of capitalists across different levels of capital and productivity, and $\Upsilon(m, s)$ be the density of workers over money and productivity.

$$K = \int_k \int_e k \Gamma(k, e) \tag{5}$$

$$N = \int_k \int_e eh(k, e) \Gamma(k, e) + \int_m \int_s sl(m, s) \Upsilon(m, s). \tag{6}$$

Note the asymmetry – capitalists supply all the capital, but labor is (at least in principle) supplied by both; note also that aggregate labor input is in terms of "effective" units of labor (hours weighted by productivity). The wage index w is then to be interpreted in the same way – one effective unit of labor earns w units of wage as compensation.

The supply side of our economy consists of a single firm employing a constant returns to scale production technology (nothing would change if we had a large number of identical firms, except

notation would be more tedious):

$$Y = \left(\alpha K^\nu + (1 - \alpha) N^\nu \right)^{\frac{1}{\nu}}, \tag{7}$$

where $\alpha \in (0, 1)$ is the "share" of capital in production and $\nu \leq 1$ governs the elasticity of substitution. If $\nu = 1$, capital and labor are perfectly substitutable, so that the firm will employ only the cheaper factor. If $\nu = -\infty$, capital and labor are perfect complements, and therefore will be employed in fixed ratios (given by $\frac{\alpha}{1-\alpha}$). If $\nu = 0$, we get the Cobb-Douglas case where the shares of capital and labor income in total income will be fixed at α and $1 - \alpha$. Profit maximization yields

$$r = \alpha \left(\alpha + (1 - \alpha) \left(\frac{K}{N} \right)^{-\nu} \right)^{\frac{1-\nu}{\nu}} \tag{8}$$

$$w = (1 - \alpha) \left(\alpha \left(\frac{K}{N} \right)^{\nu} + 1 - \alpha \right)^{\frac{1-\nu}{\nu}}. \tag{9}$$

Note that both the rental rate and the wage rate are related to the capital-labor ratio, but not to the levels of capital and labor.

Finally, there are aggregate conditions that relate supply and demand in each of three markets – the markets for capital, labor, and "goods". First, the firm must hire all the capital and labor supplied by households (these conditions are ensured by variations in r and w). Second, the supply of goods must be sufficient to cover the consumption of capitalists, the consumption of workers, and the investment by capitalists into new capital:

$$\int_k \int_e c(k, e) \Gamma(k, e) + \int_m \int_s x(m, s) \Upsilon(m, s) + G + \int_k \int_e \left(k'(k, e) - (1 - \delta) k \right) \Gamma(k, e) = Y. \tag{10}$$

The term G denotes the loss of resources associated with worker saving (since $\pi \geq 0$, $G \geq 0$ can be interpreted as government consumption that is financed by seigniorage or as inventory adjustments depending on how one views the worker savings instrument). Walras's law ensures that the goods market condition will be satisfied provided both the labor and capital markets clear.

3 The Model without Idiosyncratic Risk

We first discuss a simplified version of the main model that can be analyzed without using numerical methods, in order to shed light on the role of various parameters. The model environment

will be just like that from the main model with two exceptions – there are no shocks to labor productivity and workers do not have access to financial markets. Denote by e and s the respective constant labor productivities.

3.1 Household Problems

3.1.1 Laborers

In the absence of risk, a laborer has no incentive to hold an asset which pays a rate of return strictly below the rate of time preference. Therefore money will not be held, and in each period the laborers will consume their earnings. A typical laborer's problem then is reduced to a static choice of how many hours, l, to supply at a given wage w:

$$V = \max_l \left\{ \frac{\left[wls \left(1 - l\right)^\theta \right]^{1-\sigma}}{1 - \sigma} \right\}. \tag{11}$$

The solution is

$$l^* = \frac{1}{1 + \theta} \tag{12}$$

$$x^* = \frac{ws}{1 + \theta}. \tag{13}$$

3.1.2 Capitalists

A typical capitalist chooses consumption, hours, and savings to solve the dynamic program

$$v\left(k\right) = \max_{k',h,c} \left\{ \frac{\left[c \left(1 - h\right)^\theta \right]^{1-\sigma}}{1 - \sigma} + \beta \left(1 + g\right)^{1-\sigma} v\left(k'\right) \right\} \tag{14}$$

subject to

$$c + \left(1 + g\right) k' \leq whe + \left(r + 1 - \delta\right) k \tag{15}$$

$$k' \geq 0, c \geq 0, h \in [0, 1).$$

Since the first two boundary conditions will never bind, we ignore them going forward. Taking the first-order conditions and applying the envelope condition produces three conditions:

$$\left[c \left(1 - h\right)^\theta \right]^{-\sigma} = \beta \left(1 + g\right)^{-\sigma} \left[c' \left(1 - h'\right)^\theta \right]^{-\sigma} \left(r' + 1 - \delta\right) \tag{16}$$

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$$\left[c\left(1-h\right)^\theta\right]^{-\sigma}\left[we\left(1-h\right)^\theta-\theta c\left(1-h\right)^{\theta-1}\right]\leq 0 \tag{17}$$

$$c+(1+g)\,k'=whe+(r+1-\delta)\,k; \tag{18}$$

the second condition holds with equality if $h>0$.

3.1.3   General Equilibrium

A recursive competitive equilibrium is a set of household functions $\{v\left(k,K\right),h\left(k,K\right),c\left(k,K\right),k'\left(k,K\right),l\left(k,K\right),$ price functions $r\left(K\right)$ and $w\left(K\right)$, and aggregate labor $N\left(K\right)$ such that

1. Given pricing functions, $\{v\left(k,K\right),h\left(k,K\right),c\left(k,K\right),k'\left(k,K\right),l\left(k,K\right),x\left(k,K\right)\}$ solve the capitalist and laborer household problems;

2. Given pricing functions the firm maximizes profit by demanding K and $N\left(K\right)$;

3. Markets clear:
$$k=\mu K$$

$$N\left(K\right)=\mu h\left(K,K\right)e+\left(1-\mu\right)l\left(K,K\right)s$$

$$Y\left(K\right)=\mu c\left(K,K\right)+\left(1-\mu\right)x\left(K,K\right)+\mu\left(1+g\right)k'\left(K,K\right)-\left(1-\delta\right)\mu K.$$

3.2   Steady State

The balanced growth path is characterized by the system of equations

$$1=\beta\left(1+g\right)^{-\sigma}\left(r+1-\delta\right) \tag{19}$$

$$h=\max\left\{\frac{we-\left(r-g-\delta\right)k}{1+\theta},0\right\} \tag{20}$$

$$c=whe+\left(r-g-\delta\right)k \tag{21}$$

$$l=\frac{1}{1+\theta} \tag{22}$$

$$x=w\left(r\right)ls \tag{23}$$

$$w\left(r\right)=\left(1-\alpha\right)^{\frac{1}{\nu}}\frac{r}{\alpha}\left[\left(\frac{r}{\alpha}\right)^{\frac{\nu}{1-\nu}}-\alpha\right]^{\frac{\nu-1}{\nu}}. \tag{24}$$

The steady state Euler equation pins down r,

$$r = \frac{(1+g)^\sigma - \beta(1-\delta)}{\beta}, \tag{25}$$

as well as the steady state wage

$$w = (1-\alpha)^{\frac{1}{\nu}} \frac{r}{\alpha} \left[\left(\frac{r}{\alpha} \right)^{\frac{\nu}{1-\nu}} - \alpha \right]^{\frac{\nu-1}{\nu}}. \tag{26}$$

Notice that for $\sigma > 0$, the steady state interest rate is increasing in g. If we restrict attention to non-negative growth rates, the interest rate attains its minimum and the wage rate its maximum when $g = 0$, where the interest rate is

$$r_{\min} = \frac{1 - \beta(1-\delta)}{\beta}$$

and the wage is

$$w_{\max} = (1-\alpha)^{\frac{1}{\nu}} \frac{r_{\min}}{\alpha} \left[\left(\frac{r_{\min}}{\alpha} \right)^{\frac{\nu}{1-\nu}} - \alpha \right]^{\frac{\nu-1}{\nu}}. \tag{27}$$

Notice that while $r_{\min}$ is determined only by preferences and depreciation, $w_{\max}$ also depends upon capital share in production, α, and elasticity of substitution parameter, ν. Figure 1 plots the steady state wage when $g = 0$. For higher values of ν, the steady state wage increases exponentially, and the slope is increasing in α. Not all combinations of α and ν are permissible since

$$\alpha^{\frac{1}{1-\nu}} < r_{\min}^{\frac{\nu}{1-\nu}} \tag{28}$$

must hold for wages to be real numbers. Given (α, β, δ), the upper bound on ν is $\nu_{\max} = \frac{\log(\alpha)}{\log(r_{\min})}$. Under the baseline calibration (see below), $r_{\min} \approx 0.0351$. Thus, $\nu_{\max} \approx 0.350$ and it falls to 0.238 if $\alpha = 0.45$. Thus, balanced growth puts a restriction on the degree to which capital and labor are substitutable.

Under appropriate conditions for α and ν, we can find K by imposing the the capital market clearing condition at r:

$$K = \left\{ \frac{\left[\frac{r}{\alpha} \right]^{\frac{\nu}{1-\nu}} - \alpha}{1 - \alpha} \right\}^{-\frac{1}{\nu}} N$$

$$= \varphi N \tag{29}$$

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where

$$N = \mu h e + (1 - \mu) \frac{1}{1 + \theta} s \tag{30}$$

Since under the restrictions on α and ν

$$\frac{d\varphi}{dr} = -\frac{1}{\alpha \ (1 - \alpha) \ (1 - \nu)} \left(\frac{\left(\frac{r}{\alpha}\right)^{\frac{\nu}{1-\nu}} - \alpha}{1 - \alpha} \right)^{-\frac{1+\nu}{\nu}} \left(\frac{\alpha}{r} \right)^{\frac{1-2\nu}{1-\nu}} < 0 \tag{31}$$

and $\frac{dr}{dg} < 0$, φ rises as g falls. Thus, the capital-labor ratio will be higher in a low-growth economy.

Aggregate effective labor will be a function of K because the capitalist hours decision depends upon wealth. If wealth is sufficiently high, then the non-negativity constraint on hours will bind. We consider both the binding and nonbinding cases below.

3.2.1 Case 1: Capitalists do not work

The solution is simpler if capitalists do not work. When $h = 0$, N is fixed at $\frac{1-\mu}{1+\theta} s$ and therefore

$$K = \varphi \left(\frac{1 - \mu}{1 + \theta} s \right) \tag{32}$$

where

$$\varphi = (1 - \alpha)^{\frac{1}{\nu}} (\alpha\beta)^{\frac{1}{1-\nu}} \left\{ [(1 + g)^{\sigma} - \beta (1 - \delta)]^{\frac{\nu}{1-\nu}} - \alpha (\alpha\beta)^{\frac{\nu}{1-\nu}} \right\}^{-\frac{1}{\nu}}$$

$$= (1 - \alpha)^{\frac{1}{\nu}} \alpha^{\frac{1}{1-\nu}} \left\{ [\beta^{-1} (1 + g)^{\sigma} - 1 + \delta]^{\frac{\nu}{1-\nu}} - \alpha^{\frac{1}{1-\nu}} \right\}^{-\frac{1}{\nu}} . \tag{33}$$

Note that $k = \frac{K}{\mu}$. With this result, we can derive a measure of steady state income inequality, ζ, measured by the ratio of capitalist's income, $y = (r - g - \delta) k$, to laborer's income, $q = \frac{ws}{1+\theta}$:

$$\zeta = \frac{1 - \mu}{\mu} \frac{\left(\frac{\psi_1}{\psi_1 - \alpha} \right)^{\frac{\nu - 1}{\nu}} \psi_2}{(1 - \alpha)^{\frac{1}{\nu}} \left(\frac{\psi_1 - \alpha}{1 - \alpha} \right)^{\frac{1}{\nu}}}$$

where

$$\psi_1 = \left(\frac{\delta + \frac{(1+g)^{\sigma}}{\beta} - 1}{\alpha} \right)^{\frac{\nu}{1-\nu}} = \left(\frac{r}{\alpha} \right)^{\frac{\nu}{1-\nu}} \tag{34}$$

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and

$$\psi_2 = \frac{(1+g)^\sigma}{\beta} - g - 1 = r - g - \delta. \tag{35}$$

Notice that inequality increases as the measure of capitalists, μ, decreases. Holding all other parameters constant, the steady state Euler equation implies a unique capital-to-effective labor input ratio, and consequently w and r are invariant to μ. Because laborer's hours are constant, lower μ necessarily results in higher effective labor supply. Therefore, K must rise proportionally to N. Because factor prices do not change, q does not change with μ, but y will increase because $\frac{K}{\mu}$ rises.

It is not obvious how inequality behaves near a zero growth steady state. Remember that r falls to $r_{\min}$ as g goes to zero. Since N is fixed, equilibrium K, and likewise k, must rise. Therefore capitalists will be wealthier in a no-growth steady state. The steady state net return on capital $(r - g - \delta)$ falls as long as

$$\sigma (1+g)^{\sigma-1} > \beta,$$

which, near $g = 0$, requires $\sigma \geq 1$.[2] Because the return on capital is falling while the stock is rising, it is not clear whether total income $(r - g - \delta)\,k$ will be higher or lower in a zero growth steady state. In addition, even knowing whether income rises or falls does not identify whether inequality is higher or lower since $w = w_{\max}$ when $g = 0$. Even if $(r - g - \delta)\,k$ rises, laborer's income may rise by a greater proportion. We will use numerical methods to study this question, after presenting the case where capitalists supply positive labor in the steady state.

Note that if we specialize to $\sigma = 1$ (logarithmic preferences), we can analytically characterize the entire transition and not simply the steady state; as noted by Moll (2014), this model is isomorphic to the standard Solow growth model with a capital evolution equation

$$K' = \frac{\alpha\beta}{1+g} K^\alpha \left(\frac{1-\mu}{1+\theta} s \right)^{1-\alpha} + \left(1 - \frac{\beta}{1+g}(1-\delta) \right) K$$
$$= \widehat{\beta} Y + \left(1 - \widehat{\delta} \right) K.$$

The dynamics of this model are well-known and discussed in Krusell and Smith (2014), so we omit a thorough discussion here.

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[2]When this condition holds, both ψ_1 and ψ_2 are decreasing in g.

3.2.2 Case 2: Capitalists work

When capitalist households work, aggregate effective labor input responds to factor prices through h. The resulting expressions for K, k, h, and ζ are significantly more complicated, but still available in closed form:

$$K = \left\{ \frac{\left[\frac{r}{\alpha}\right]^{\frac{\nu}{1-\nu}} - \alpha}{1 - \alpha} \right\}^{-\frac{1}{\nu}} \left[\mu e \left(\frac{we - (r - g - \delta)\,k}{1 + \theta} \right) + (1 - \mu)\left(\frac{1}{1 + \theta} \right) s \right]$$

$$= \frac{s\,(1 - \mu) + \frac{e^2\,\mu\,(1-\alpha)^{\frac{1}{\nu}}}{\left(1 + \frac{\alpha}{\psi_1 - \alpha}\right)^{\frac{\nu-1}{\nu}}}}{\left(\frac{\psi_1 - \alpha}{1 - \alpha}\right)^{\frac{1}{\nu}}\left(1 + \theta + \frac{\varepsilon\,\psi_2}{\left(\frac{\psi_1 - \alpha}{1 - \alpha}\right)^{\frac{1}{\nu}}}\right)} \tag{36}$$

Again $k = \frac{K}{\mu}$, which we can substitute to find

$$h = \frac{we - (r - g - \delta)\,k}{1 + \theta}$$

$$= \left(\frac{1}{1 + \theta}\right)\left\{ \frac{(1-\alpha)^{\frac{1}{\nu}} e}{\left(1 + \frac{\alpha}{\psi_1 - \alpha}\right)^{\frac{\nu-1}{\nu}}} + \frac{\left(s\,(1-\mu) + \frac{e^2\,\mu\,(1-\alpha)^{\frac{1}{\nu}}}{\left(1 + \frac{\alpha}{\psi_1 - \alpha}\right)^{\frac{\nu-1}{\nu}}}\right)\psi_2}{\mu\left(\frac{\psi_1 - \alpha}{1 - \alpha}\right)^{\frac{1}{\nu}}\left(1 + \theta + \frac{\psi_2}{\left(\frac{\psi_1 - \alpha}{1 - \alpha}\right)^{\frac{1}{\nu}}} e\right)} \right\} \tag{37}$$

and

$$\zeta = \frac{(1 + \theta)\left(1 + \frac{\alpha}{\psi_1 - \alpha}\right)^{\frac{\nu-1}{\nu}}}{s\,(1 - \alpha)^{\frac{1}{\nu}}}\left(\frac{\left(s\,(1-\mu) + \frac{e^2\,\mu\,(1-\alpha)^{\frac{1}{\nu}}}{\left(1 + \frac{\alpha}{\psi_1 - \alpha}\right)^{\frac{\nu-1}{\nu}}}\right)\psi_2}{\mu\left(\frac{\psi_1 - \alpha}{1 - \alpha}\right)^{\frac{1}{\nu}}\left(1 + \theta + \frac{e\,\psi_2}{\left(\frac{\psi_1 - \alpha}{1 - \alpha}\right)^{\frac{1}{\nu}}}\right)} \right.$$

$$\left. + \frac{e\left(\frac{e\,(1-\alpha)^{\frac{1}{\nu}}}{\left(1 + \frac{\alpha}{\psi_1 - \alpha}\right)^{\frac{\nu-1}{\nu}}} + \frac{\left(s\,(\mu-1) - \frac{e^2\,\mu\,(1-\alpha)^{\frac{1}{\nu}}}{\left(1 + \frac{\alpha}{\psi_1 - \alpha}\right)^{\frac{\nu-1}{\nu}}}\right)\psi_2}{\mu\left(\frac{\psi_1 - \alpha}{1 - \alpha}\right)^{\frac{1}{\nu}}\left(1 + \theta + \frac{e\,\psi_2}{\left(\frac{\psi_1 - \alpha}{1 - \alpha}\right)^{\frac{1}{\nu}}}\right)}\right)(1-\alpha)^{\frac{1}{\nu}}}{(1 + \theta)\left(1 + \frac{\alpha}{\psi_1 - \alpha}\right)^{\frac{\nu-1}{\nu}}} \right). \tag{38}$$

Here it is very difficult to disentangle how growth affects a capitalist household's behavior or long run inequality, given the formidable nature of this expression; we will study it using numerical tools.

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3.2.3   How income inequality changes with growth

Because the closed-form expressions for steady state inequality are too complicated to yield unambiguous results, we use a computer to evaluate the expressions numerically and plot the results for long run growth rates between 0 and 10 percent. Figures 2-3 show the steady state ratio of capitalist income to laborer income (i.e., inequality) for growth rates between 0 and 10 percent for the baseline capital share of income in production and for a higher value. Generally, inequality falls as steady state growth increases. In fact, for a wide range of combinations of α and ν, $\frac{d\zeta}{dg} < 0$ for all growth rates considered.

It is possible for inequality to increase with growth. When $\alpha = 0.45$ (the most favorable case for generating a positive derivative), the first instance occurs when $\nu < -0.11$ at a growth rate of almost 10 percent. Even when α is at 0.45, ν must be as low as -1.7 before the derivative is positive at a steady state growth rate as low as 2 percent. Within the relevant region of the parameter space for the questions posed in this paper, inequality always rises as the long run growth rate goes to zero; that is, the cases where low growth leads to low inequality are those with complementarity between capital and labor.

To better understand why inequality rises near $g = 0$, it is useful to rewrite the definition of inequality as the sum of two ratios: the ratio of capitalist-to-laborer effective hours (the effective hours ratio) and the ratio of capital income to laborer income

$$
\begin{aligned}
\zeta &= \frac{whe + (r - g - \delta)\,k}{w\frac{s}{1+\theta}} \\
&= \frac{he}{ls} + \frac{(r - g - \delta)\,k}{q};
\end{aligned}
\tag{39}
$$

notice we use the word "capital" and not "capitalist" since the capitalist household may have labor income as well.

The effective labor ratio depends solely upon h because laborer's hours are invariant to g. All else equal, if capitalists work more hours, inequality will rise. The second ratio is capital income relative to laborer's income which can also be expressed as a multiple of the product between relative net factor prices and wealth

$$
\zeta = \frac{he}{ls} + \frac{1 + \theta}{s}\frac{r - g - \delta}{w}k.
\tag{40}
$$

A rise in wealth or in the return to capital relative to that of labor increases inequality.

Since $w(r)$ is decreasing in r,

$$\frac{dw}{dg} < 0.$$

Therefore when the long-run growth rate is close to zero, the return to capital falls relative to labor. On its own, this result would decrease inequality; however, capitalist hours and wealth also respond to g. The general equilibrium interaction of the hours, wealth, and factor prices must be jointly determined.

For a wide range around the baseline parameter values, wealth decreases with growth. Figures 4-5 plot $k(g)$ for several values of ν and of α. As ν moves toward 0.2, the level of capital in zero-growth steady state is very large, especially when capital's share, α, is high. Unless ν and α are high, $\frac{r-g-\delta}{w}$, which falls as g goes to zero, and k, which rises, nearly offset each other. Under a wide range around the baseline parametrization the capital income to laborer income ratio declines in the neighborhood of zero growth. The reason inequality is higher when $g = 0$ is because capitalist are induced by the higher wage to work more hours.

When hours are positive,

$$\begin{aligned} h &= \frac{we - (r - g - \delta)k}{1 + \theta} \\ &= \frac{we}{1 + \theta} - \frac{(r - g - \delta)k}{1 + \theta}. \end{aligned} \tag{41}$$

The first term, through w, increases as g decreases and encourages capitalists to work more hours. The second term decreases sharply as g goes to zero, but it generally has a weaker magnitude than the first term and so does little to slow the increase in hours. Figures 6-7 plot $h(g)$. Notice that hours are higher in low growth steady states except when ν is near its upper bound. When α is also high, hours are zero.

To analyze the model numerically, we need to assign values to the structural parameters of the model. Here, we pick a reasonable set of values for some parameters, where reasonable means "gives rise to aggregates roughly consistent with US post-war averages." These numbers are $\beta = 0.99$, $\sigma = 2$, $\alpha = 0.36$, $\delta = 0.025$, $\theta = 1.25$, $g = 0.02$, and $\pi = 0.02$ (this parameter is not relevant in this section but plays a role later); our capital/output ratio is 6, higher than the usual value but consistent with not only Piketty's measurement (that uses financial wealth as capital) but also the work by McGrattan and Prescott (2013) that values intangible capital

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(which presumably is what the extra financial capital Piketty finds is backed by). Finally, to give Piketty's argument a stronger case, we set $\nu = 0.1$, so that capital and labor are more substitutable than the usual Cobb-Douglas case (the elasticity is given by $\frac{1}{1-\nu} = 1.1$); this value of ν satisfies the restrictions needed to have a steady state growth path.[3]

Figure 8 plots hours under the baseline parameter values along with the first and second components (summing the two effects produces $h(g)$). Both components increase rapidly in magnitude at low growth levels. Under the baseline, the wage effect is stronger and hours increase substantially. When ν and α are high, the second effect can dominate at low rates of growth, generating a hump shape in the long run hours curve. At low g, capitalists have very high wealth and work few or zero hours. As g rises, steady state wealth falls rapidly, pushing hours up. As g rises further, hours decline again. In this region, the wage has dropped sufficiently so that capitalists once again reduce hours despite having low levels of wealth.

Depending upon how all these forces interact, long run inequality may be higher or lower, even if capitalists do not work. Returning to figure 2, we can now understand why inequality does not increase monotonically across ν. When ν is close to the baseline, capitalists supply positive hours when $g = 0$, and inequality is highest at zero growth. As ν increases, capitalists stop working which, combined with a rise in w, causes inequality to be lower. As ν increases further towards its upper bound, capitalists continue not to work, and the wage continues to rise, but inequality rises again because wealth increases more rapidly than the factor price ratio falls.

We can use the decomposition from Equation (40) to uncover the cause of rising equality as g approaches zero. Around the baseline parameter values, inequality is higher for low g because the effective hours ratio increases sharply in that region. Figures 9 and 10 plot the decomposition of long run inequality. While the capital-laborer income ratio is typically about three quarters of total inequality, this share declines as long run growth falls. When $g = 0$, the effective hours ratio accounts for 50 percent of total inequality. Moreover, the rise in inequality near $g = 0$ is entirely due to capitalists working more hours since $\frac{r-g-\delta}{w}k$ falls in that region. If we consider a much higher capital share of income, the story remains the same qualitatively. Quantitatively,

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[3]Thus, our productivity growth should be interpreted as purely labor-augmenting (see King, Plosser, and Rebelo 1988); with Cobb-Douglas it does not matter whether the productivity growth affects capital, labor, or both. Rognlie (2014) and Semieniuk (2014) argue that Piketty overstates the elasticity of substitution.

the capital-laborer income ratio does account for a higher share of total inequality regardless of g, but once again, the increase in inequality as g falls is due to the rise in the effective hours of capitalists.

In the two previous cases, capitalists always worked positive hours. In addition, an increasing wage as g goes to zero, resulted in capitalists working more hours. When we increase ν, the relationship changes. Figures 11 and 12 plot the same inequality decomposition for higher ν values. When α is at its baseline value and $\nu = 0.25$, a higher wage causes capitalists to decrease their hours. The effective hours ratio plays little to no role in long run inequality, especially at low level of g where the non-negativity constraint on hours binds. In this region, long run inequality is completely due to the factor price ratio $\frac{r-g-\delta}{w}$ and the level of wealth. On net as g declines, capitalists' wealth rises more quickly than the ratio in factor price declines, and total inequality increases. The story holds for higher α as well. The non-negativity constraint on hours binds at higher values of g, and inequality is greater at the $g = 0$ steady state.

3.3 Takeaways

This simplified two-household model has shown that the parameters that primarily govern the behavior of inequality in a zero growth steady state are related to production. The capital share and the elasticity of substitution between capital and labor control how quickly both the steady state wage rate and wealth rise as g nears zero. In addition, they also change the response of hours. In general, steady state hours are higher when $g = 0$, but if both α and ν are sufficiently high, hours are lower (perhaps zero) in low growth steady states and rise as g increases.

In all cases, steady state inequality is higher when $g = 0$ than it is under a positive growth rate. The cause for the rise, however, depends upon the parameters. Generally, it is the result of capitalists supplying more hours. In fact, capitalist's income from wealth relative to laborer's income declines as growth nears zero. Only when capital's share of production and the elasticity of substitution are high does a rise in capital income relative to laborer's income account for high inequality.

In our view, though, this model provides a view of inequality that is likely too simple – within group inequality is also important. For evidence, we point to the fact that capital income as a share of total income varies substantially across individuals and capital and labor income are

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positively correlated (see Table 1 in Carroll and Young 2009 or Budría Rodríguez *et al.* 2002). Furthermore, there is substantial mobility in income and wealth (see Budría Rodriguez *et al.* 2002 or Carroll, Dolmas, and Young 2014). To accommodate these features, we move to study the model with idiosyncratic risk.

4   The Model with Idiosyncratic Risk

We now suppose that e and s follow identical, highly persistent AR(1) processes in logs:

$$\log\left(e'\right) = 0.95 \log\left(e\right) + 0.1\eta'$$

where η' is a standard normal random variable. The definition of equilibrium for this model is a straightforward extension of the model without idiosyncratic risk and is omitted.

Due to the special relationship between r and w we can solve this model by finding a single number, namely the rental rate r, such that at that given rate the household's supply of capital and labor, if hired entirely by the firm, lead to a marginal product of capital equal to r itself (that is simply Equation 8). We can draw a picture of the steady state as the intersection of the "demand curve" corresponding to the right-hand-side of Equation 8 and a "supply curve" that links the aggregate capital/labor ratio (as chosen by households) to the return; see Figure 13.[4]

Inequality arises in this model through two forces. First, there is "luck". In Figure 14 we show how the typical capitalist saves, and Figure 15 shows the typical worker. When e is high (the capitalist is currently "lucky"), next period's capital lies above current capital, so this capitalist is "saving" or accumulating. Similarly, when e is low (the capitalist is currently "unlucky"), there will be decumulation. Furthermore, the "gaps" are not symmetric; accumulation occurs much more quickly than decumulation, due to decreasing marginal utility. Thus, as e bounces around the capitalists cycle through various different wealth levels; there are upper and lower bounds on these levels in equilibrium.

Second, there is a fixed component to inequality. r declines as K increases, holding N fixed. Thus, as capitalists save more they reduce the "return gap" between themselves and workers.

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[4]We use standard numerical methods to solve for the steady state and the transitional dynamics; a technical appendix outlines the details and is available upon request.

But as noted above there is an upper limit to any capitalist's saving, so there is an upper limit to K. It turns out that one can show easily that, at that upper limit, $r - \delta > 0$, because the labor supply of the capitalist eventually goes to zero (see Figure 17); that is, there will always be a return gap if $\pi \geq 0$. As a result, the "steeper" slope of the capitalist's savings function acts to expand inequality.

We focus on measuring inequality using Lorenz curves and the Gini coefficient. We present in Figure 17 the Lorenz curves both for our model and for the recent US, using the Survey of Consumer Finances 2007 sample. Our model does a reasonable job of fitting the US Lorenz curve (see Figure 16); the middle part would be matched better if we allowed occasional transits between capitalists and workers. We explore a setting below with idiosyncratic return risk that will fit the upper tail better.

Making comparisons across Lorenz curves is difficult, since they could cross multiple times. When making comparisons we will use the Gini coefficient, which is obtained by integrating the area between the perfect-equality line and the actual Lorenz curve. Larger Gini coefficients translate into more unequal distributions. In terms of Gini coefficients, our model does a reasonable job reproducing the extreme inequality observed in the US – our Gini coefficient is even slightly larger, at 0.84, than the US at 0.8. Comparing the two model curves we see that inequality actually drops as g goes to zero, at least in the long run, but not much – the dashed-line curve lies everywhere above the solid one but they are quite close together (the new Gini coefficient is 0.83). Thus, Piketty's prediction of explosive inequality, at least if measured in the conventional way, is not consistent with our model. However, the fact that r drops significantly is also not consistent with Piketty's maintained hypothesis, and may play an important role. r drops because the capital-output ratio roughly doubles while aggregate labor input remains roughly constant, leading to a large increase in the capital/labor ratio and a concomitant decline in returns.[5]

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[5]Krusell and Smith (2014) also find small effects of g on inequality, although their model features only one type of household and shocks to household discount factors drive much of the inequality. The underlying reasons are the same as ours, though.

4.1 Transition to Zero Growth

Because the long run comparisons can be misleading, we explicitly compute the transition path as the economy moves from the initial growth path with $g = 0.02$ to the one with $g = 0$. We focus on four variables because these are the ones Piketty highlights, namely the capital/output ratio, the return to capital, capital's share of income, and the Gini coefficient on wealth. As seen in Figure 18, the transition takes over 100 years to complete for the mean capital stock (which is all that matters for r and $\frac{K}{Y}$), and takes even longer for the Gini coefficient (these last two results are manifestations of the approximate aggregation property of this model, as discussed in Krusell and Smith 1998, namely that higher moments of the distribution of wealth do not materially affect prices). The Gini coefficient first drops a bit, then recovers, but the quantitative size of the movements are small, meaning that the steady state is not hiding substantial inequality dynamics.

5 Alternative Models

Clearly, our model does not reproduce the transition that Piketty envisions – while $\frac{K}{Y}$ rises significantly, r falls and therefore $\frac{rK}{Y}$ increases but not substantially. In contrast, Piketty maintains that r will not fall, meaning that the increase in $\frac{K}{Y}$ will translate directly into an increase in $\frac{rK}{Y}$. It is clear from inspecting Figure 13 that the model cannot reconcile a decline in g with a constant r, since there is no "room" between r and the effective discount factor of the capitalists $(\beta(1+g)^{1-\sigma})$ – if g drops by a nontrivial amount, r must fall in the new steady state. We therefore consider transitions that involve a combination of declining growth and some force that works to prevent returns from falling.

We also study in this section alternative methods for producing the return gap between capitalists and workers; none of these extensions affect our results in any material way, so we only briefly describe them here.

5.1 Financial Innovation/Capital-Biased Technical Change

Piketty suggest a number of possible mechanisms that would prevent r from falling (or even cause it to increase). We pick one of these proposed mechanisms here – an improvement in financial innovation. Specifically, we assume that, in the initial steady state, the capitalist is exposed to

an iid idiosyncratic shock to end-of-period wealth u, changing his program to

$$v\left(k,e\right) = \max_{k',h,c} \left\{ \frac{\left(c\left(1-h\right)^{\theta}\right)^{1-\sigma}}{1-\sigma} + \beta\left(1+g\right)^{1-\sigma} E\left[v\left(u'k',e'\right)\right] \right\}$$

$$c + \left(1+g\right)k' \leq \left(r+1-\delta\right)k + weh$$

$$k' \geq 0$$

$$h \geq 0$$

$$c \geq 0.$$

Figure 19 shows that the initial steady state now has a substantially lower r and a larger gap between the discount factor and the return (a symptom of market incompleteness). As discussed in Mendoza, Quadrini, and Ríos-Rull (2009), a decline in idiosyncratic risk will make the asset supply curve shift to the left as precautionary motives are blunted, a force which will increase r and work against the decline in g.[6]

The transition with financial innovation looks very similar qualitatively to the benchmark model, with one clear exception – there is now a substantial *decrease* in the Gini coefficient on wealth. Thus, inequality as measured by Piketty ($\frac{rK}{Y}$) and by standard measures (Gini) move in opposite directions. Figures 20 and 21 show the Lorenz curves and transitional dynamics; the decrease in inequality occurs because there is less variance in wealth for the capitalists.[7]

We also consider an increase in α, a form of capital-biased technical change; specifically, we consider what happens if α increases to 0.45 at the same time the variance of u goes to zero and g goes to zero.[8] Figure 22 shows the rightward shift in the demand curve that a rise in α generates. Thus, a combination of the two forces – financial innovation and capital-biased technical change – could result in a small (or even zero) decline in r; however, as noted earlier, the near-infinite

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[6]This setup is not isomorphic to an entrepreneurial economy with persistent productivity shocks, such as the model used by Cagetti and DeNardi (2008) to study wealth inequality, but it captures enough of the critical details to make our point here. Computing a transition in the Cagetti and DeNardi (2008) model is computationally more demanding due to a nontrivial market clearing condition for labor.

[7]The "blip" in the dynamic path for the Gini coefficient is not a numerical artifact – it is an upward jump followed by a quick but continuous decline back to the previous transition path. We have not been able to figure out where this blip comes from, but it clearly does not affect the results we emphasize.

[8]Remember that the size of α is restricted, so 0.45 is almost as large as we can make it.

elasticity of asset supply near the equilibrium return makes a decline in r inevitable; the result is that changes in α have little effect.

5.2 Alternative Saving Vehicles for Workers

Here we present some alternative possibilities for the savings of workers. First, we suppose there are two technologies, one accessible only by each type of household, given by

$$Y_1 = (\alpha K_1^\nu + (1 - \alpha) N_1^\nu)^{\frac{1}{\nu}}$$

$$Y_2 = (\eta K_2^\nu + (1 - \eta) N_2^\nu)^{\frac{1}{\nu}}$$

with $\eta < \alpha$, where

$$K_1 = \int_k \int_e k\Gamma(k, e)$$

$$K_2 = \int_m \int_s m\Upsilon(m, s)$$

$$N_1 + N_2 = \int_k \int_e eh\Gamma(k, e) + \int_m \int_s sl\Upsilon(m, s).$$

That is, labor is mobile, capital is not, and workers have access to an inferior savings vehicle. The equilibrium requires that wages be equal across "sectors",

$$(\alpha K_1^\nu + (1 - \alpha) N_1^\nu)^{\frac{1-\nu}{\nu}} (1 - \alpha) N_1^{\nu-1} = (\eta K_2^\nu + (1 - \eta) N_2^\nu)^{\frac{1-\nu}{\nu}} (1 - \eta) N_2^{\nu-1},$$

but returns are not equalized.

Second, we suppose that workers simply have access to an inferior capital stock. In this case, total capital is given by

$$K = \frac{1}{1 + \omega} \int_k \int_e k\Gamma(k, e) + \frac{\omega}{1 + \omega} \int_m \int_s m\Upsilon(m, s)$$

for $\omega < 1$. Now both types of households are connected through common movements in r; the only difference is that workers effectively transfer part of their returns to the capitalists.

Our results are qualitatively and quantitatively unchanged by the nature of the worker saving vehicle, provided it is sufficiently inferior to the one the capitalist uses.

6   Conclusion

We have not engaged Piketty's policy suggestions in this paper. He suggests that a tax on wealth, particularly inherited wealth, will be needed to defend society against the corrupting influence of the explosion in inequality. Recently optimal tax theorists have studied the nature of optimal taxation when redistribution is a concern and growth is small (see the references in Farhi and Werning 2014), finding that the case for a progressive wealth tax relies on very specific assumptions about how individuals value their children. We leave (as Farhi and Werning themselves do) to others careful scrutiny of those assumptions empirically, merely noting them here so the reader can engage that literature more easily.

One could easily think about optimal allocations for our model. In our setup, the reasons that households cannot borrow and cannot buy contingent claims are not specified explicitly. If we assume that, whatever these factors are, they apply also to the government, then the government cannot transfer resources across individuals or over time (no insurance and no debt), as in Davila *et al.* (2012). In that paper, which does not feature either labor effort nor differentiated access to capital markets, ever-increasing inequality and an enormous increase in $\frac{K}{Y}$ turns out to be optimal, because the increase in capital supports the wages of the poor. Clearly, this setup does not capture Piketty's policy prescriptions, which involve large transfers to the workers financed by capital taxation, but it can be extended by permitting lump-sum transfers from capitalists to workers, with no transfers within those groups.

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Figure 1: Equilibrium Wage in $g = 0$ Steady State
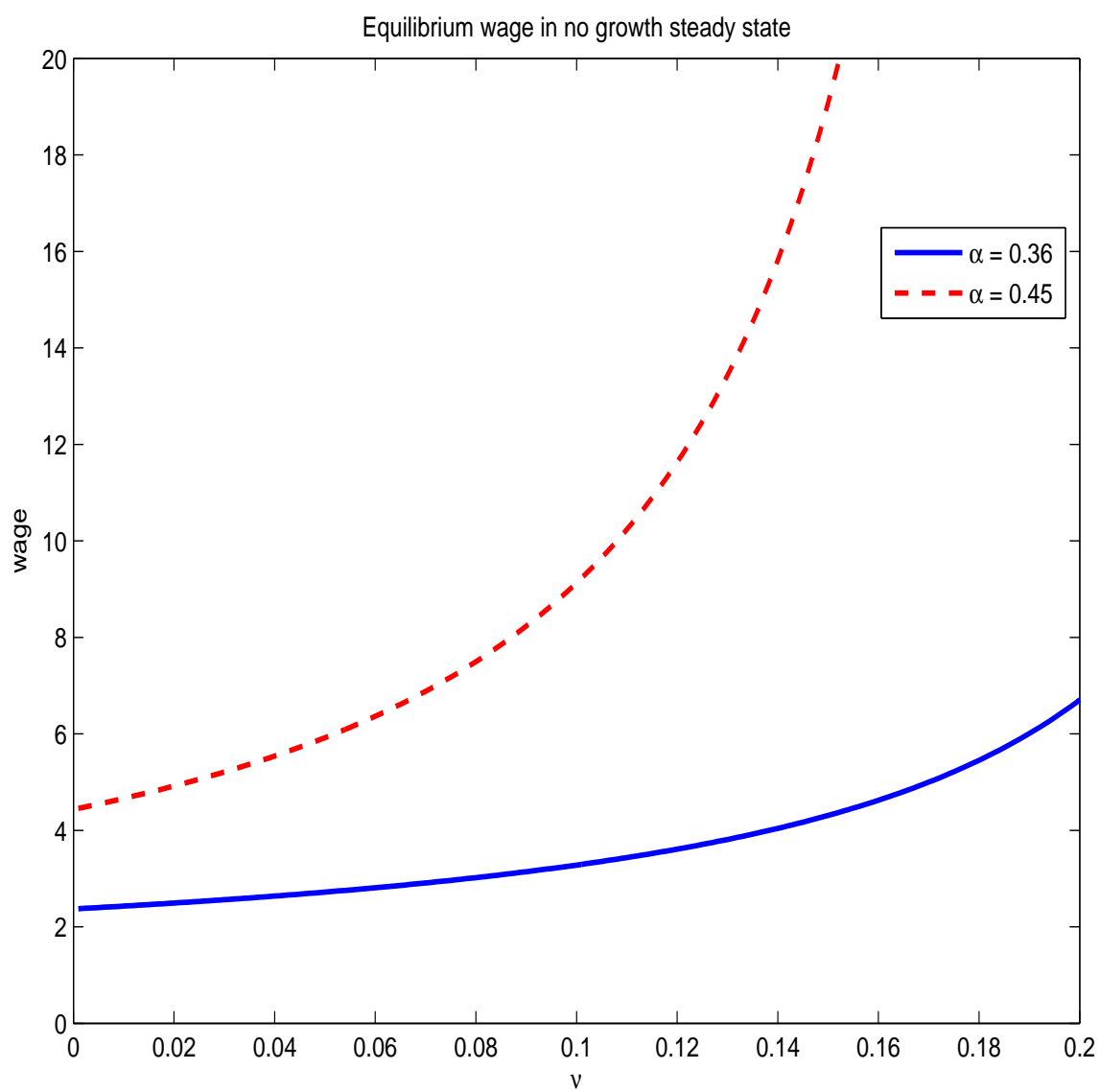
Steady state inequality

Steady state inequality

Steady state wealth

α = 0.36

ν = 0.25
ν = 0.20
ν = 0.10
ν = 0.00
ν = −0.20
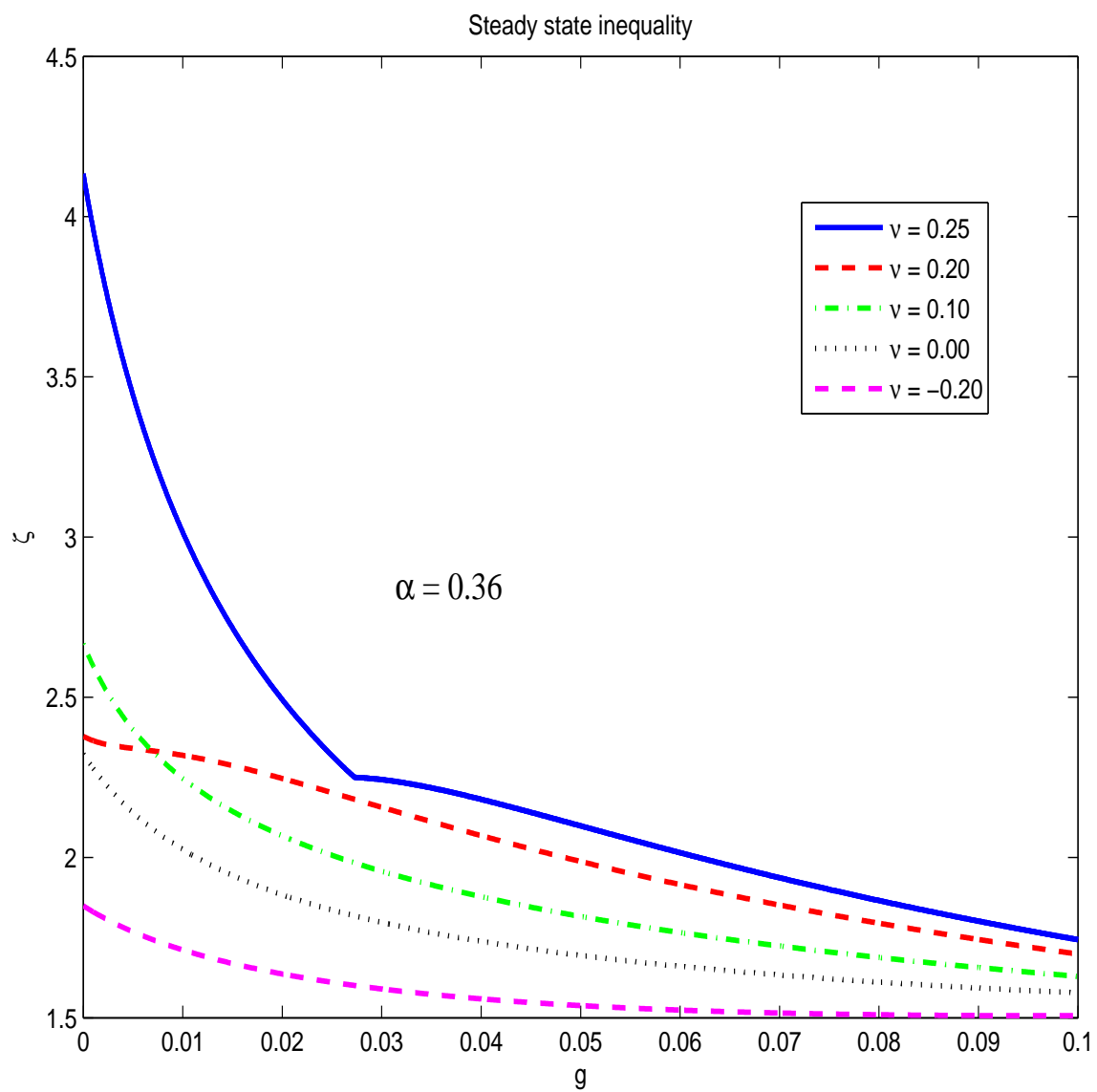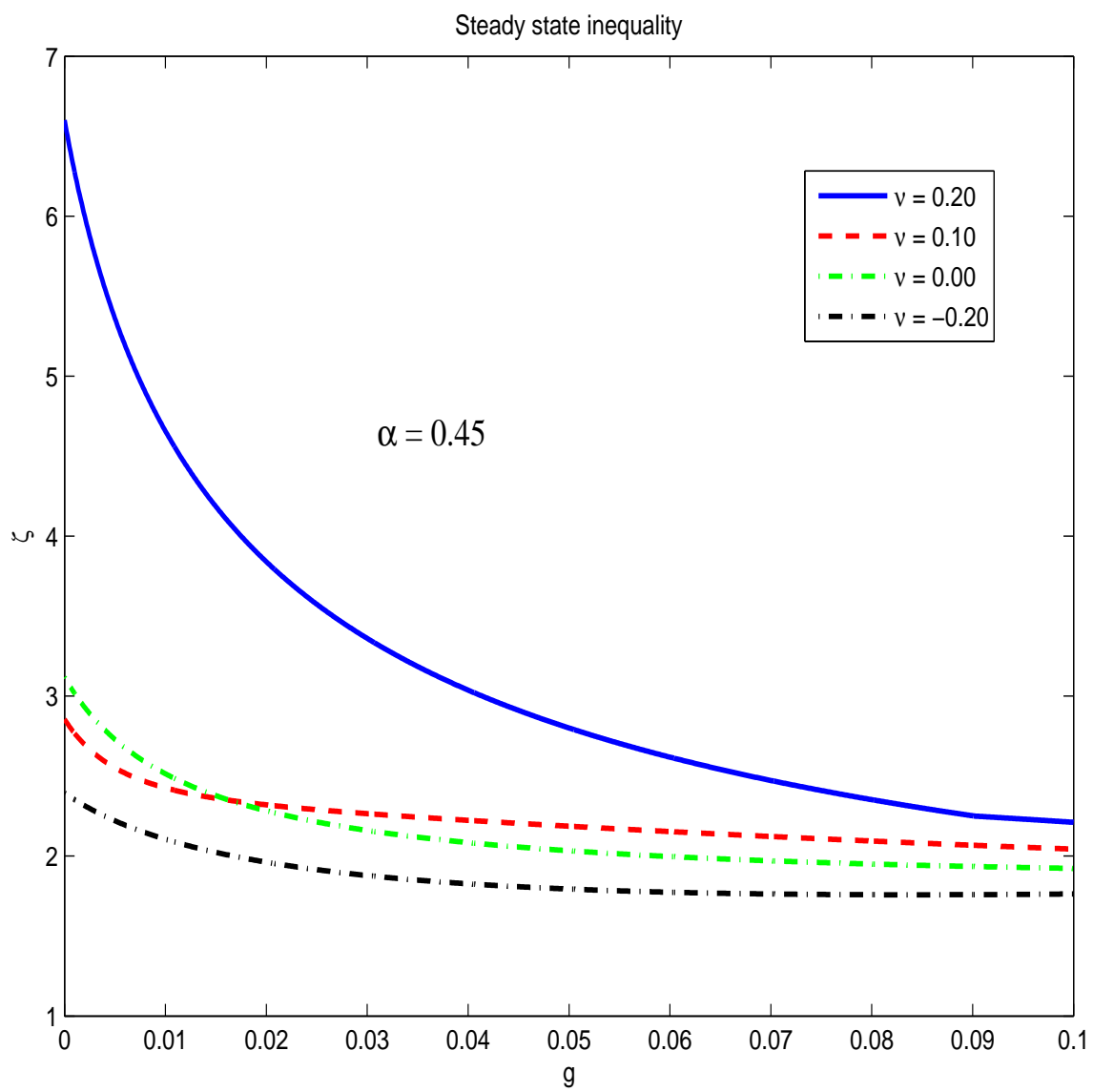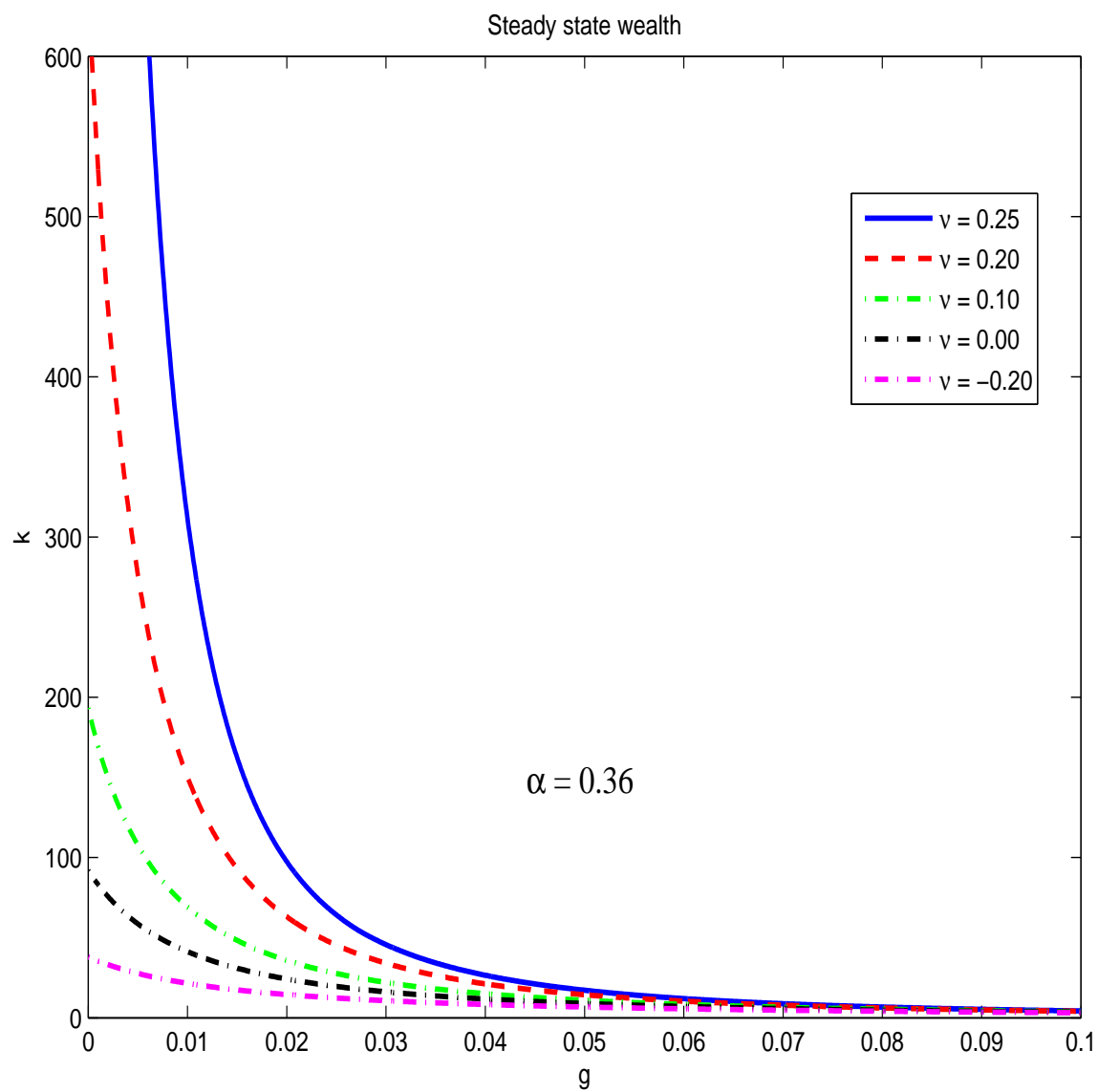
Figure 5: Steady State Wealth
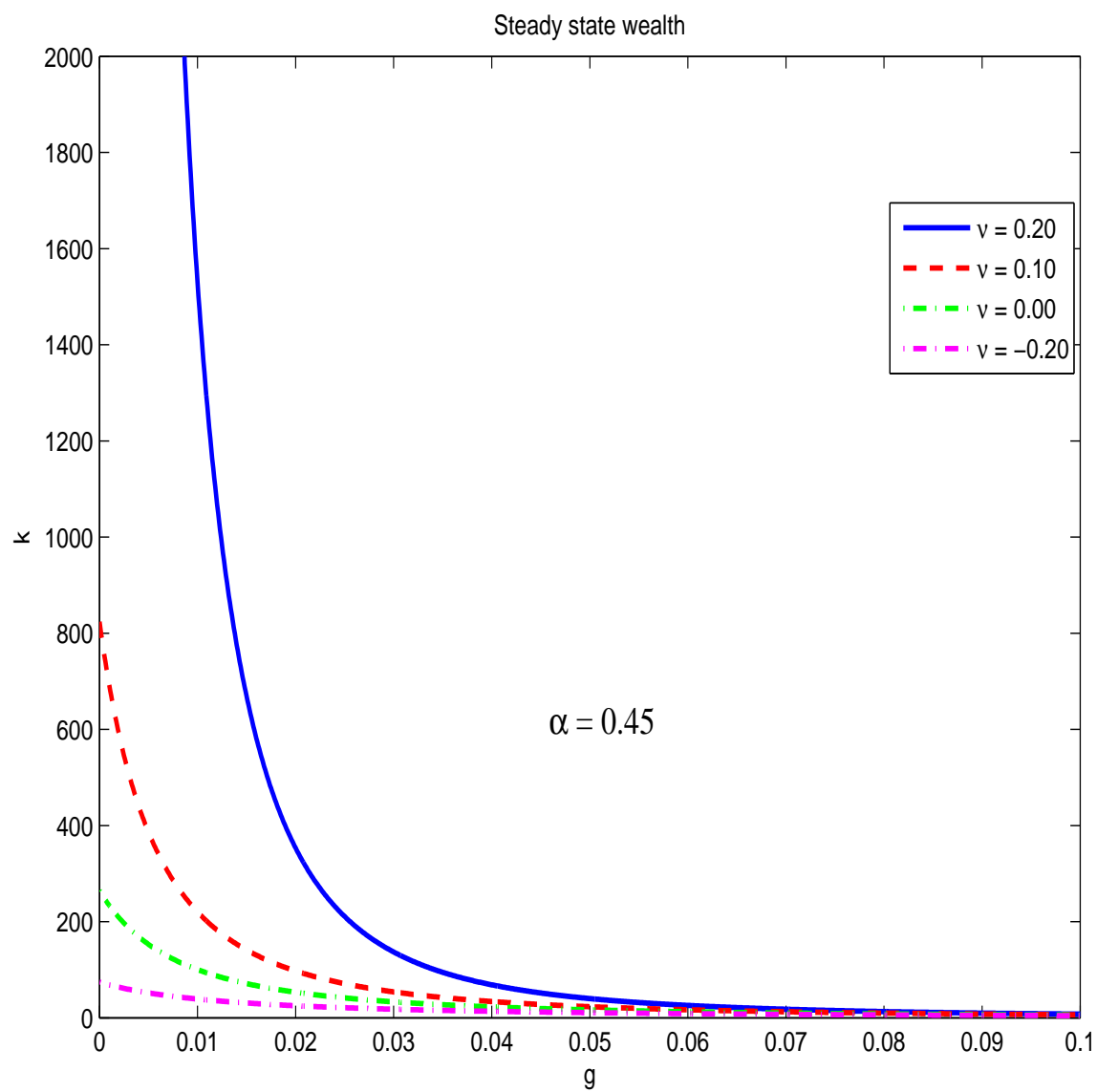
Figure 6: Capitalist Labor Supply
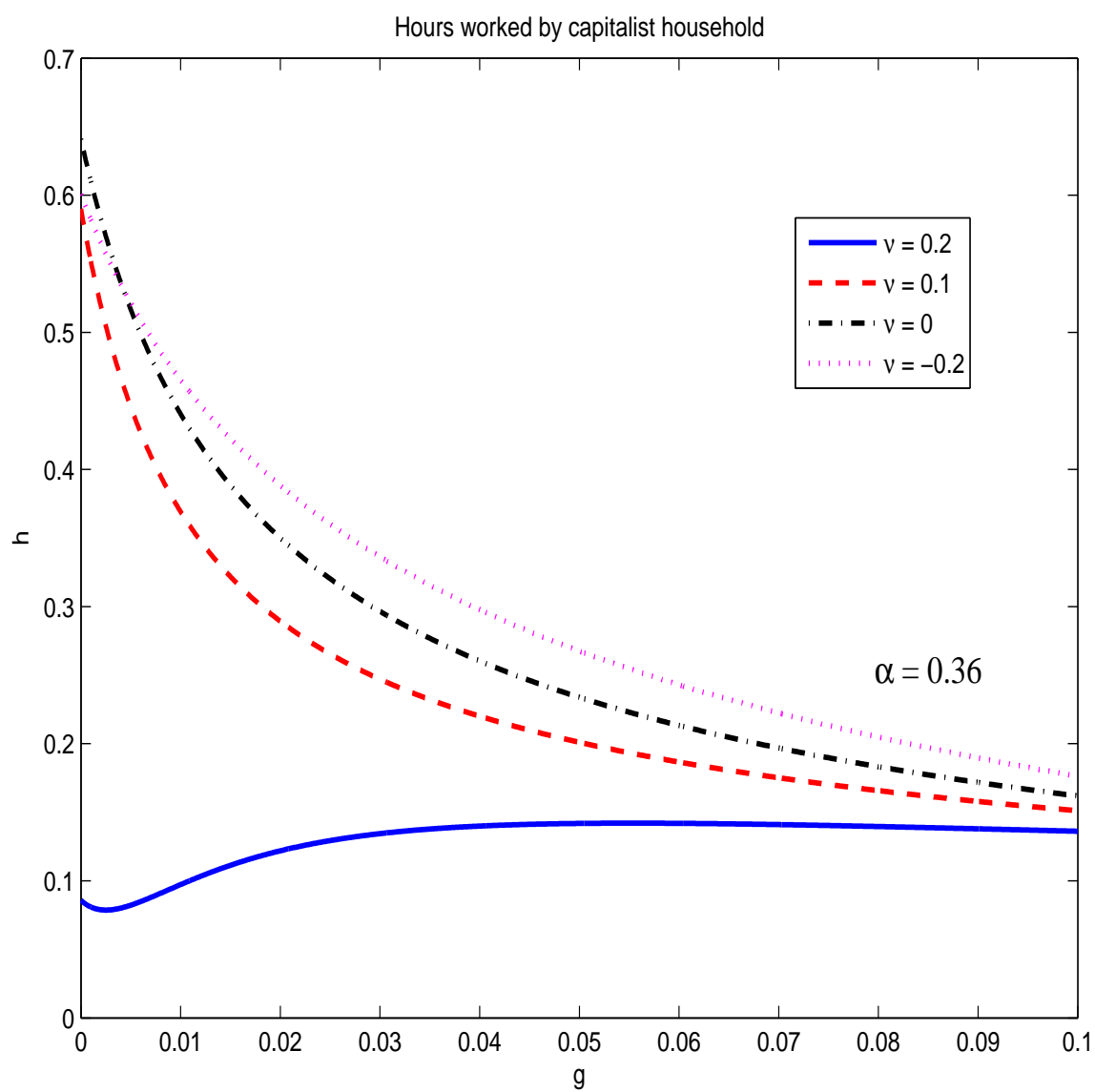
Figure 7: Capitalist Labor Supply
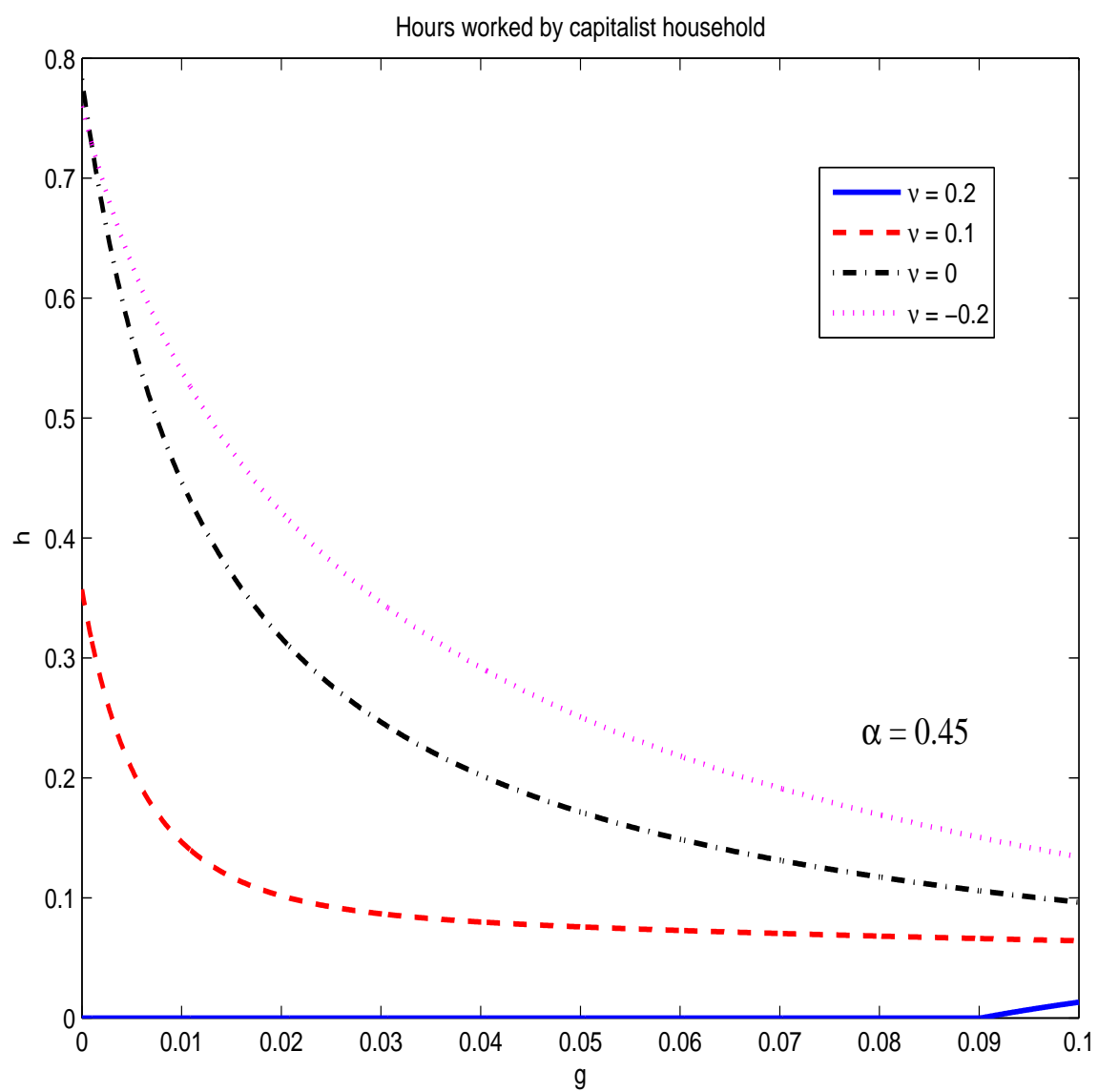
Figure 8: Decomposition of Capitalist Labor Supply

Decomposition of capitalist's hours
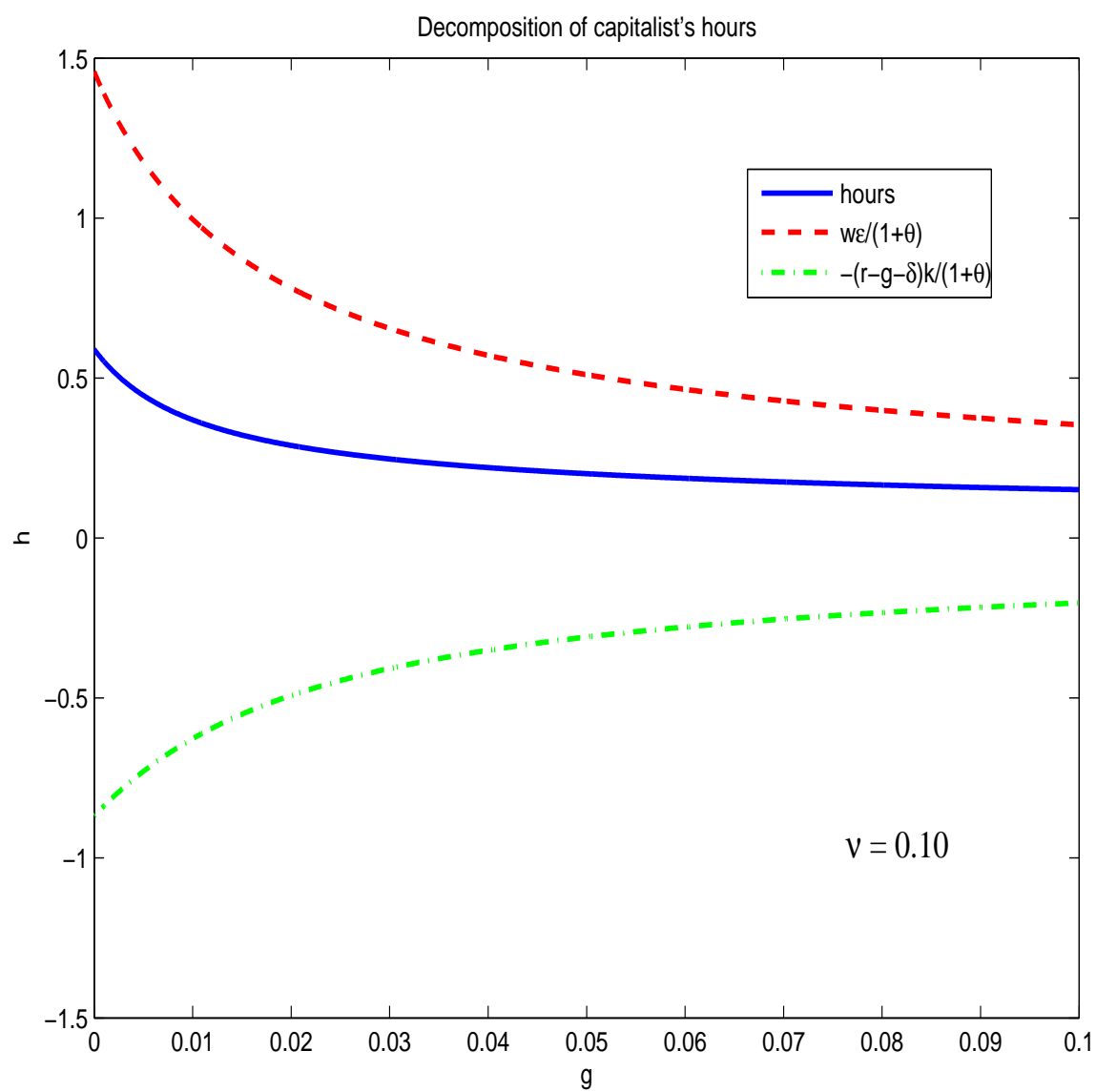
Figure 9: Decomposition of Long Run Inequality
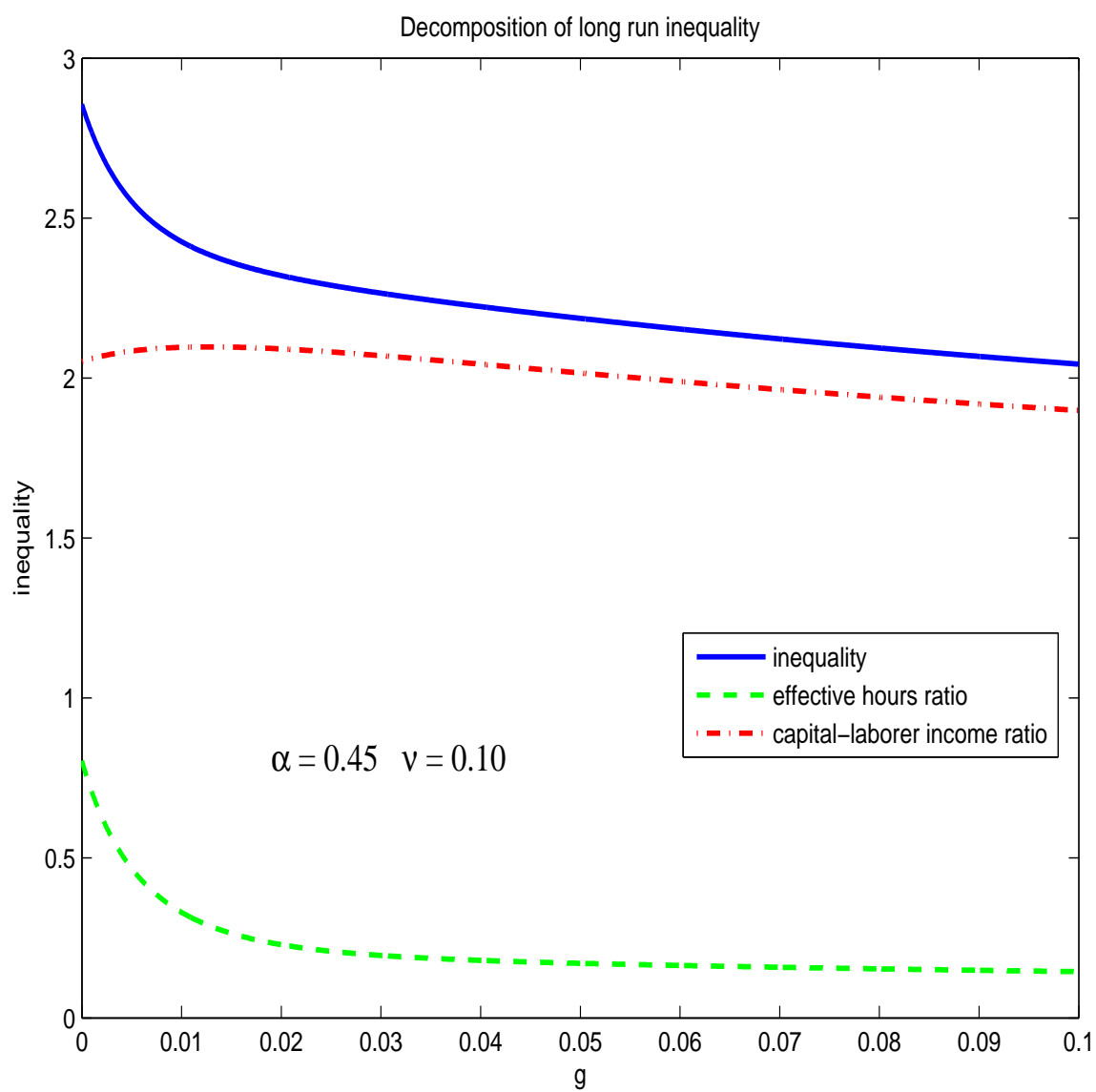

Decomposition of long run inequality

$\alpha = 0.36 \quad \nu = 0.10$
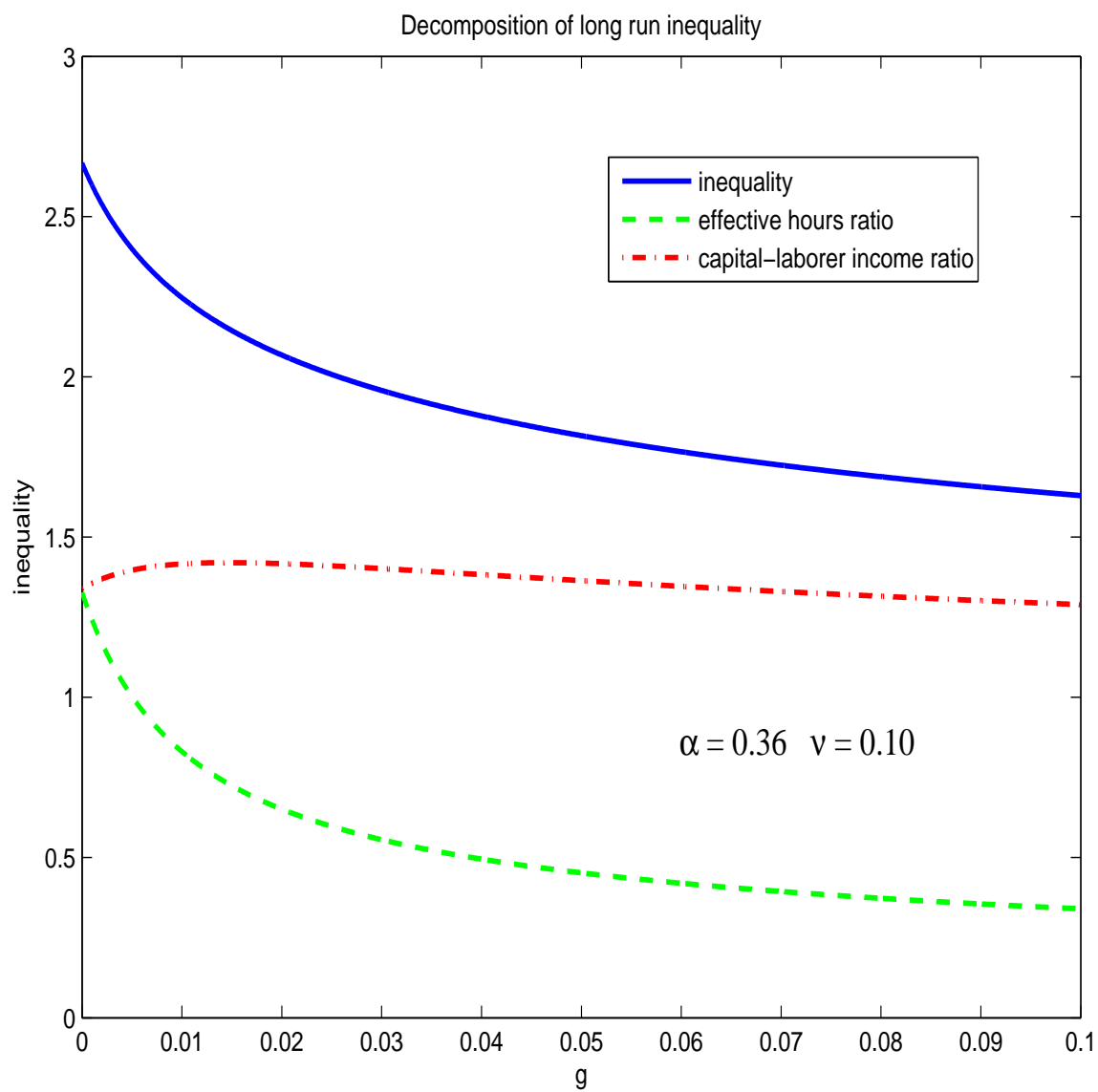
Figure 10: Decomposition of Long Run Inequality

Decomposition of long run inequality

$\alpha = 0.45 \quad \nu = 0.10$

Legend:
- inequality
- effective hours ratio
- capital-laborer income ratio

Figure 11: Decomposition of Long Run Inequality

Decomposition of long run inequality

$\alpha = 0.36 \quad \nu = 0.25$
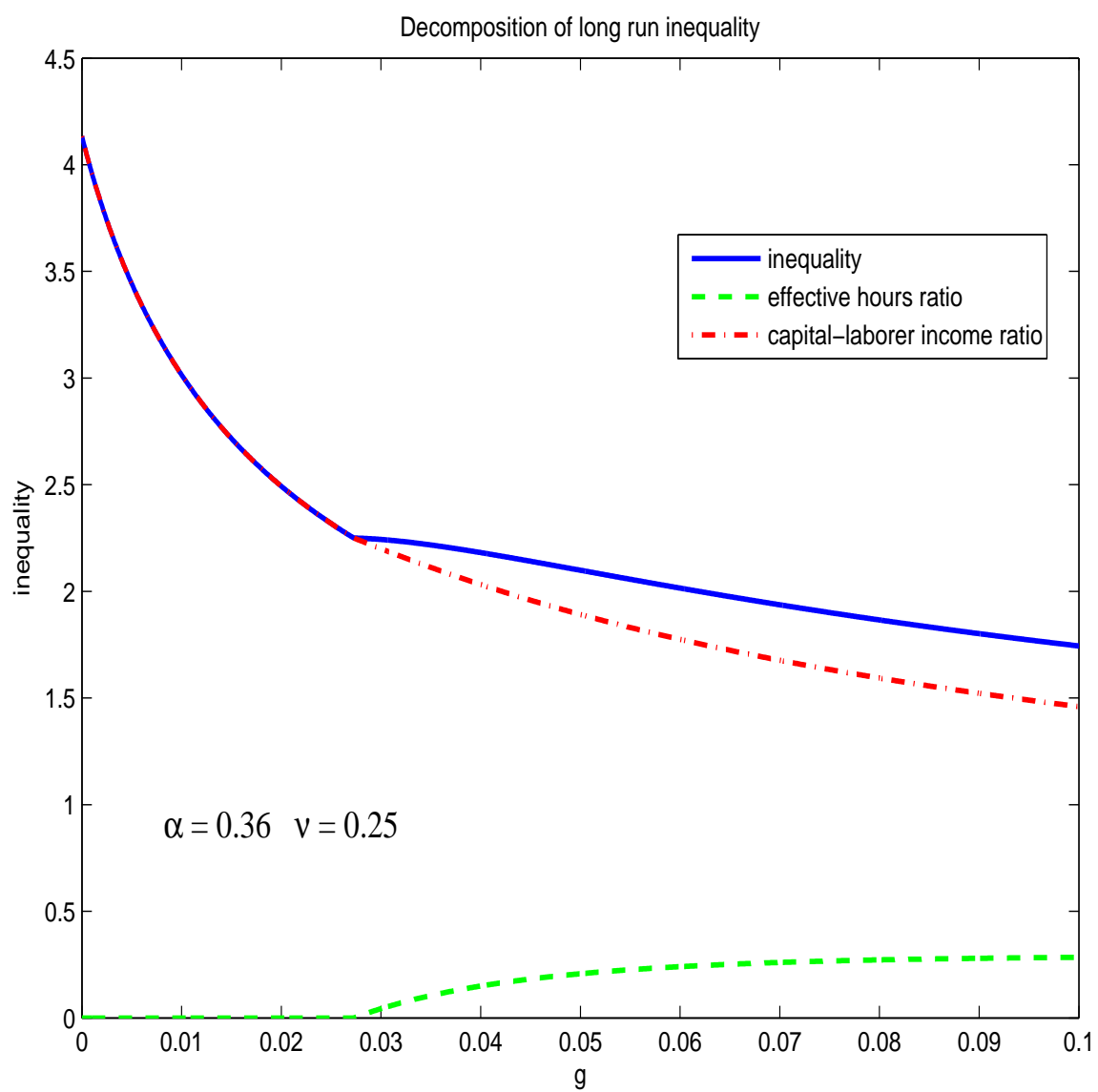
Figure 12: Decomposition of Long Run Inequality

Decomposition of long run inequality
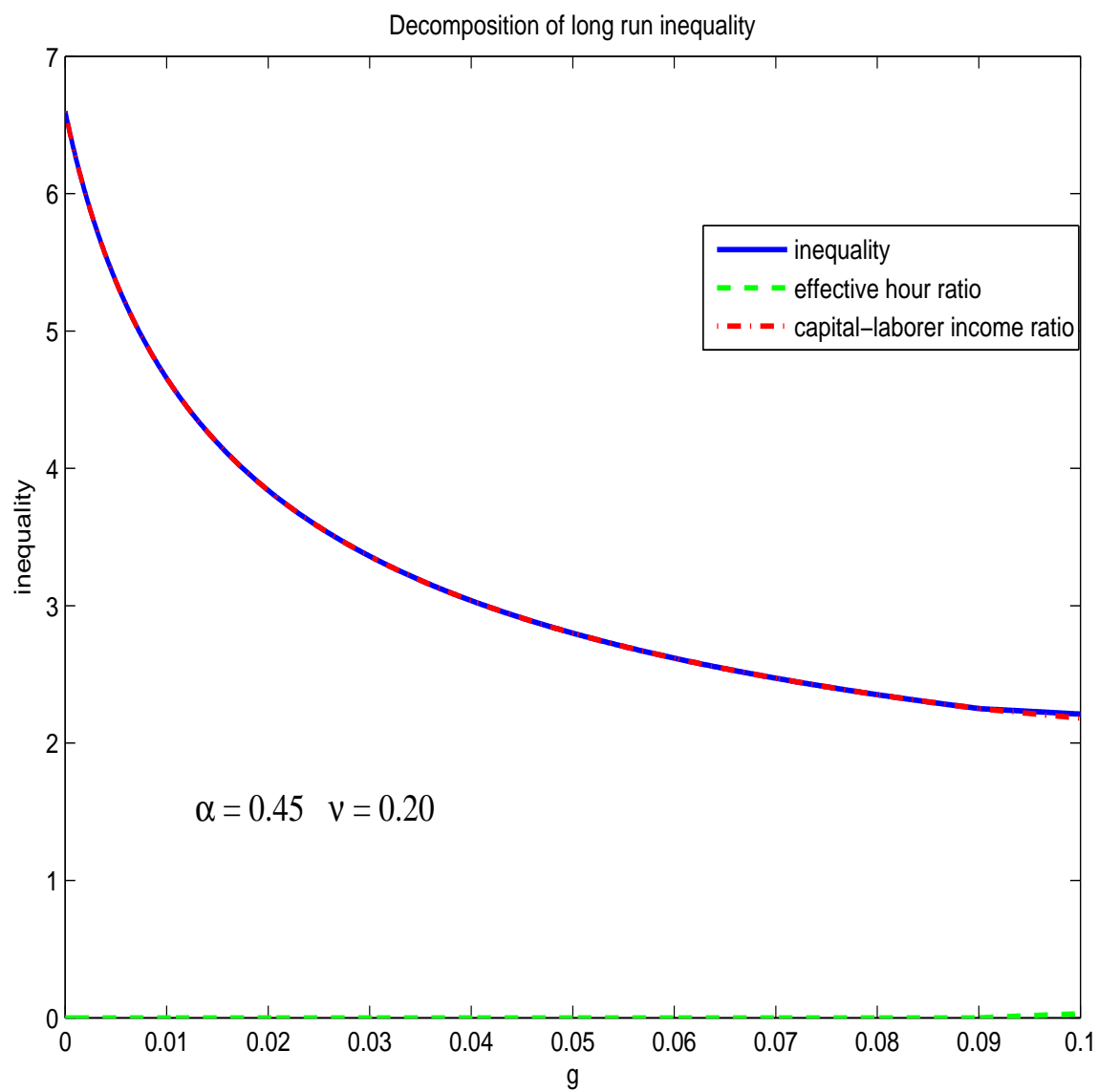
Figure 13: Steady State with Labor Productivity Risk
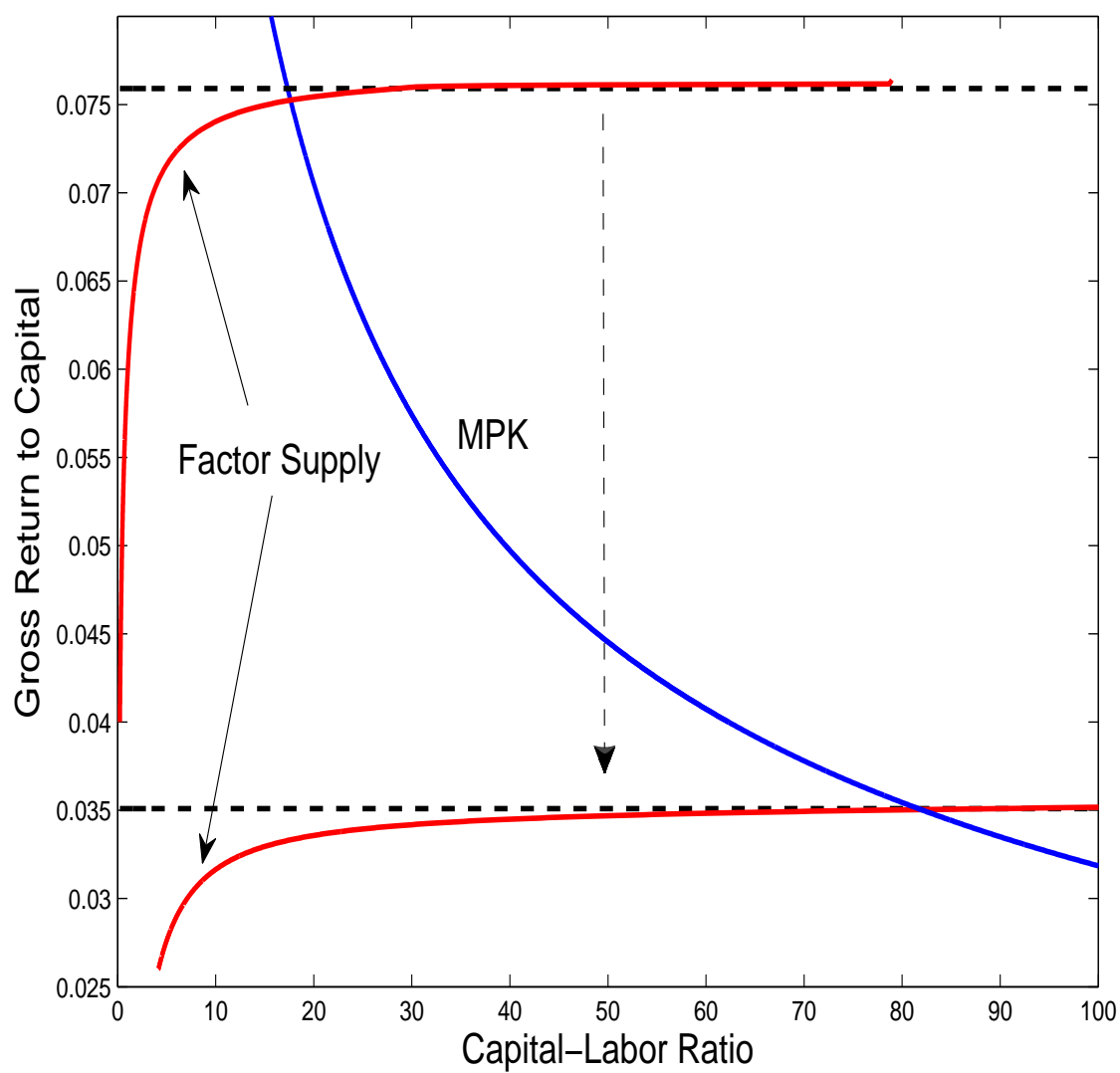
Figure 14: Saving by Capitalists
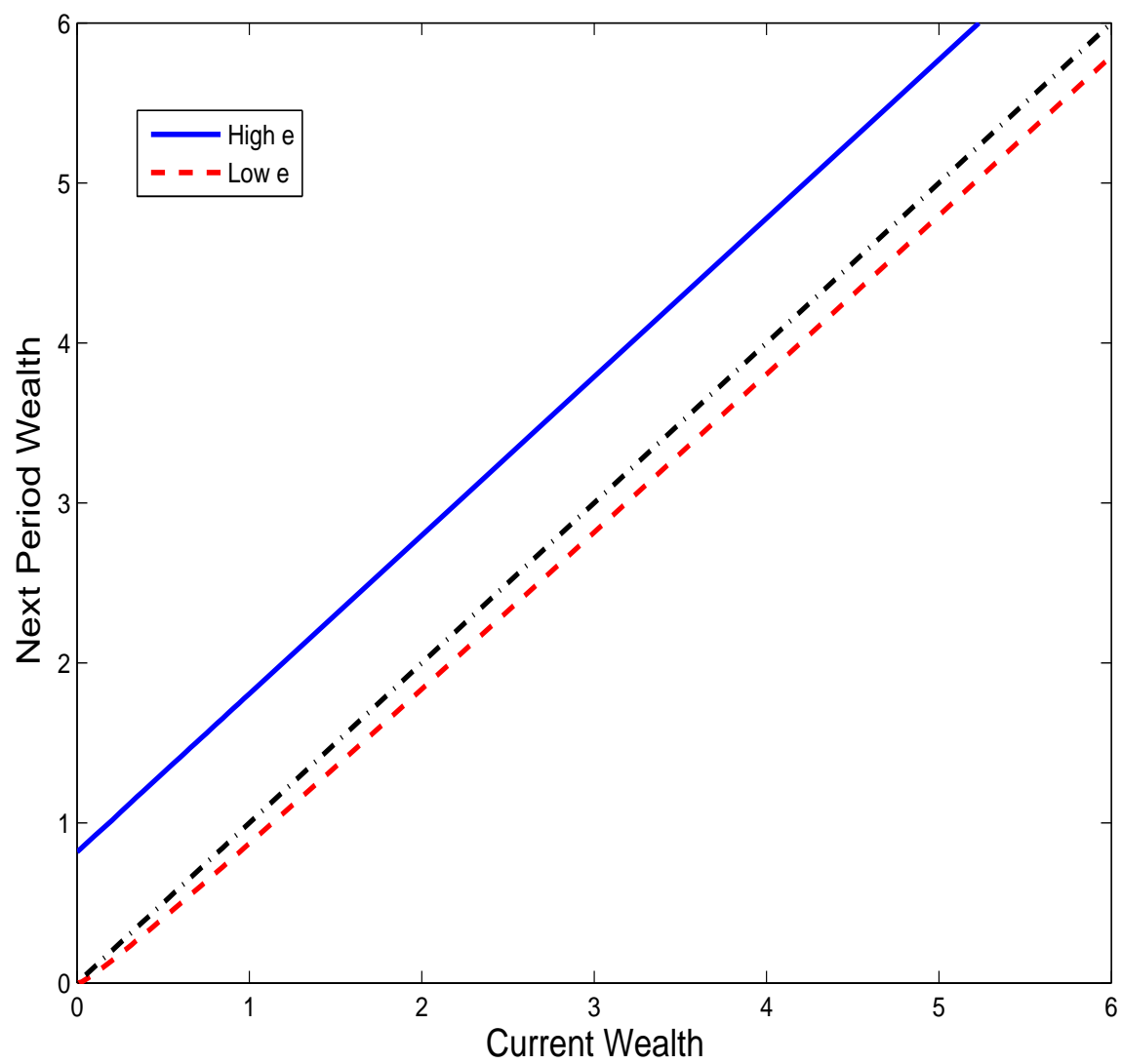
Figure 15: Saving by Workers
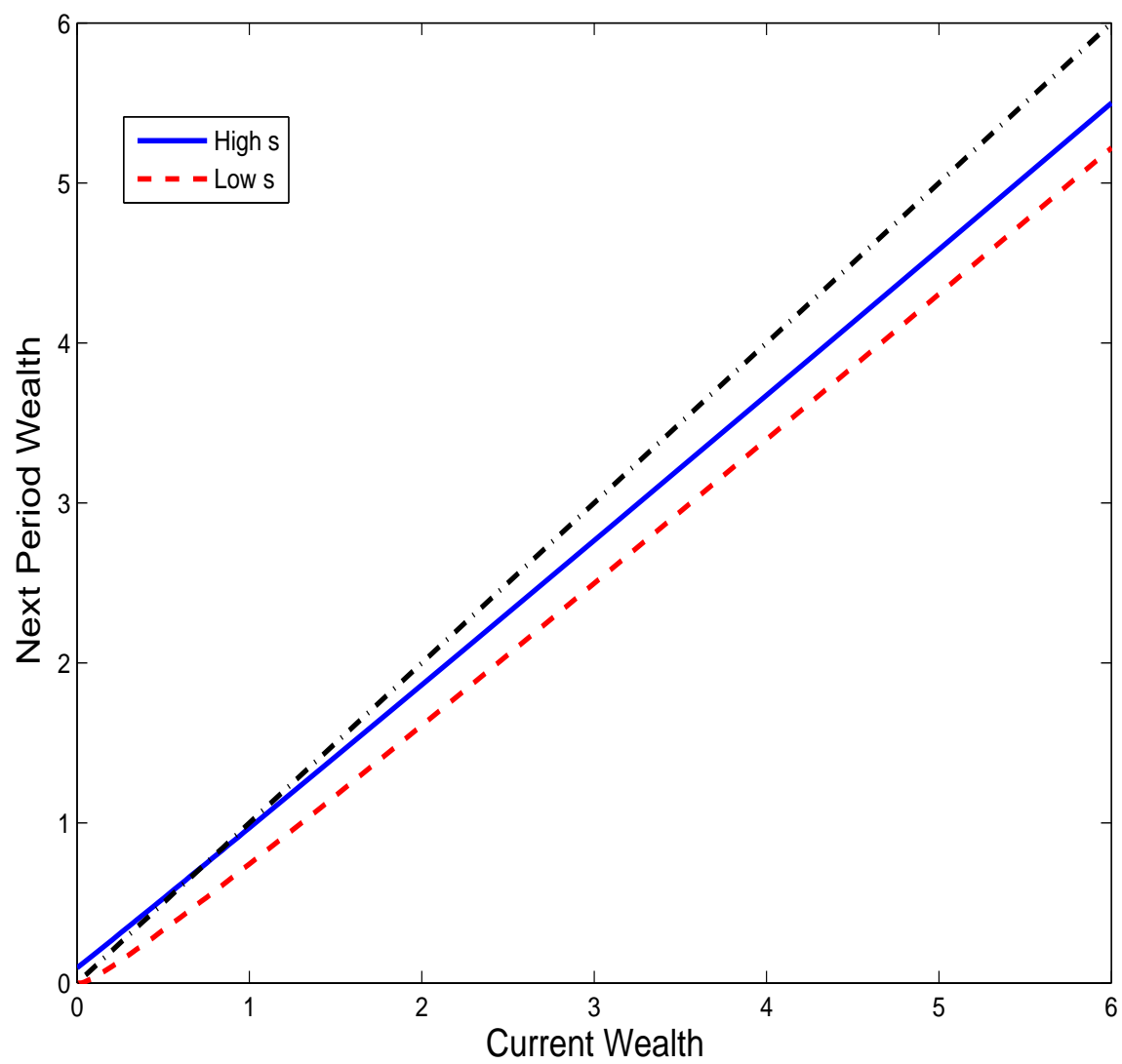
Figure 16: Lorenz Curves
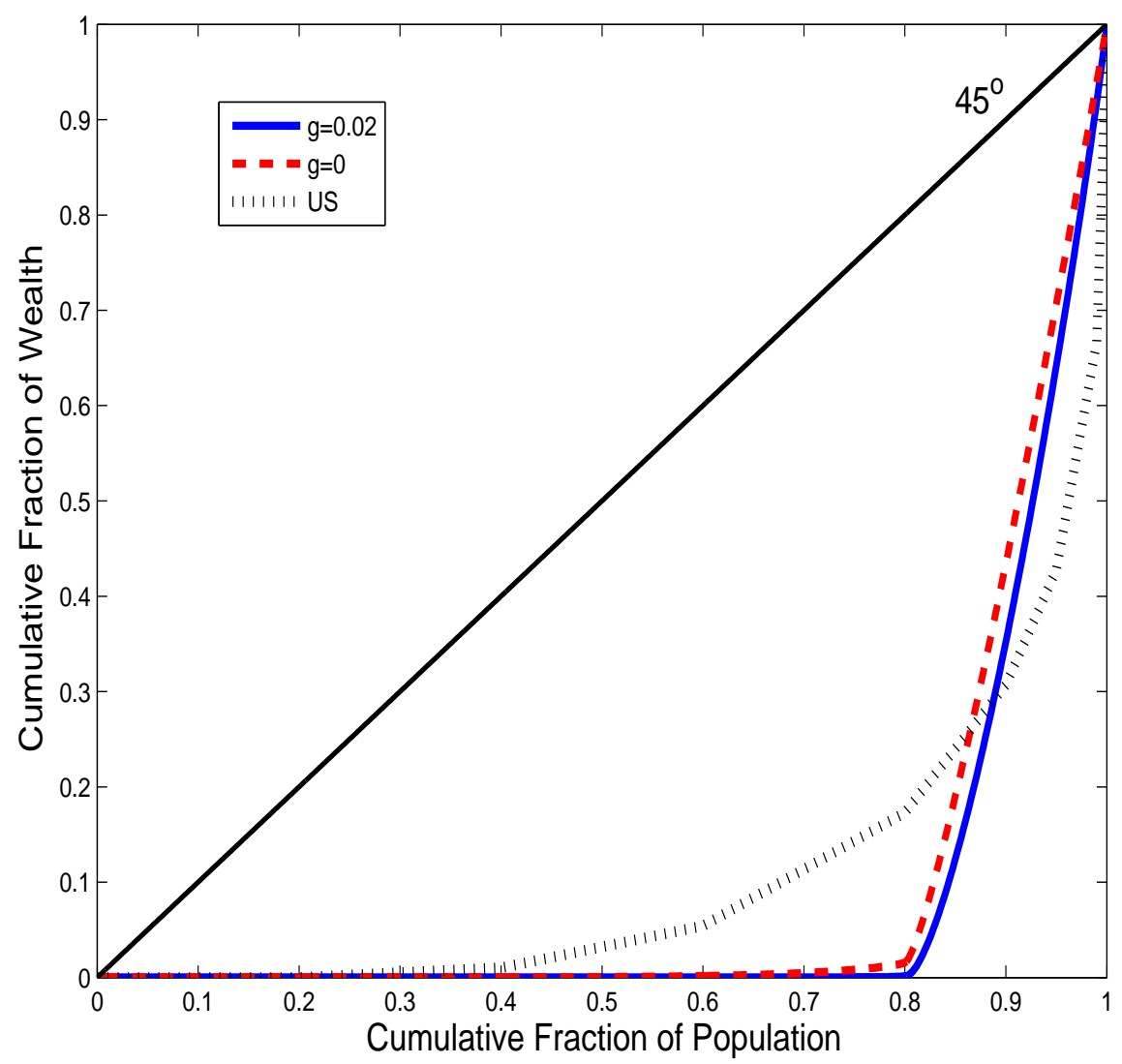
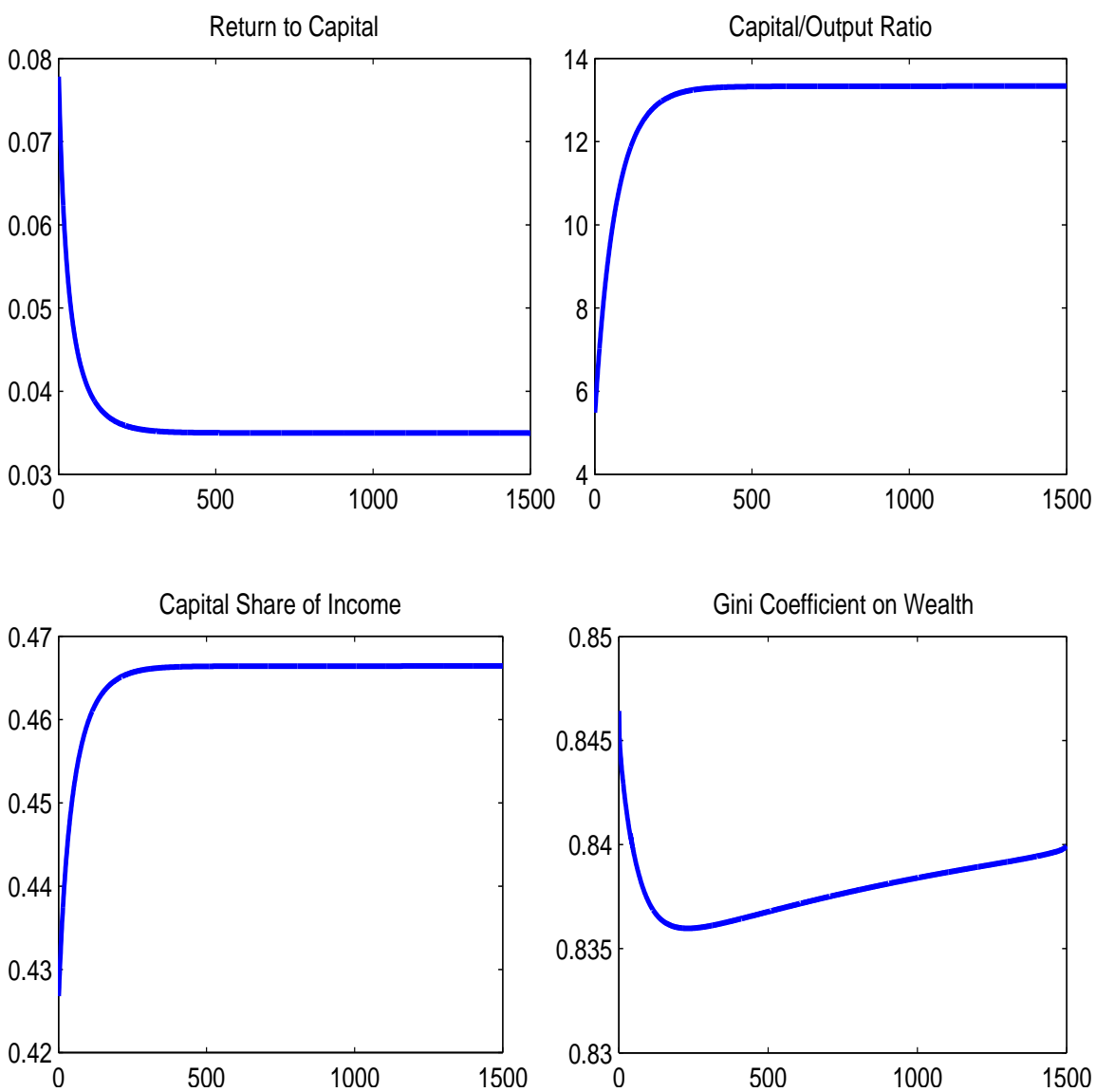Figure 17: Labor Supply by Capitalists
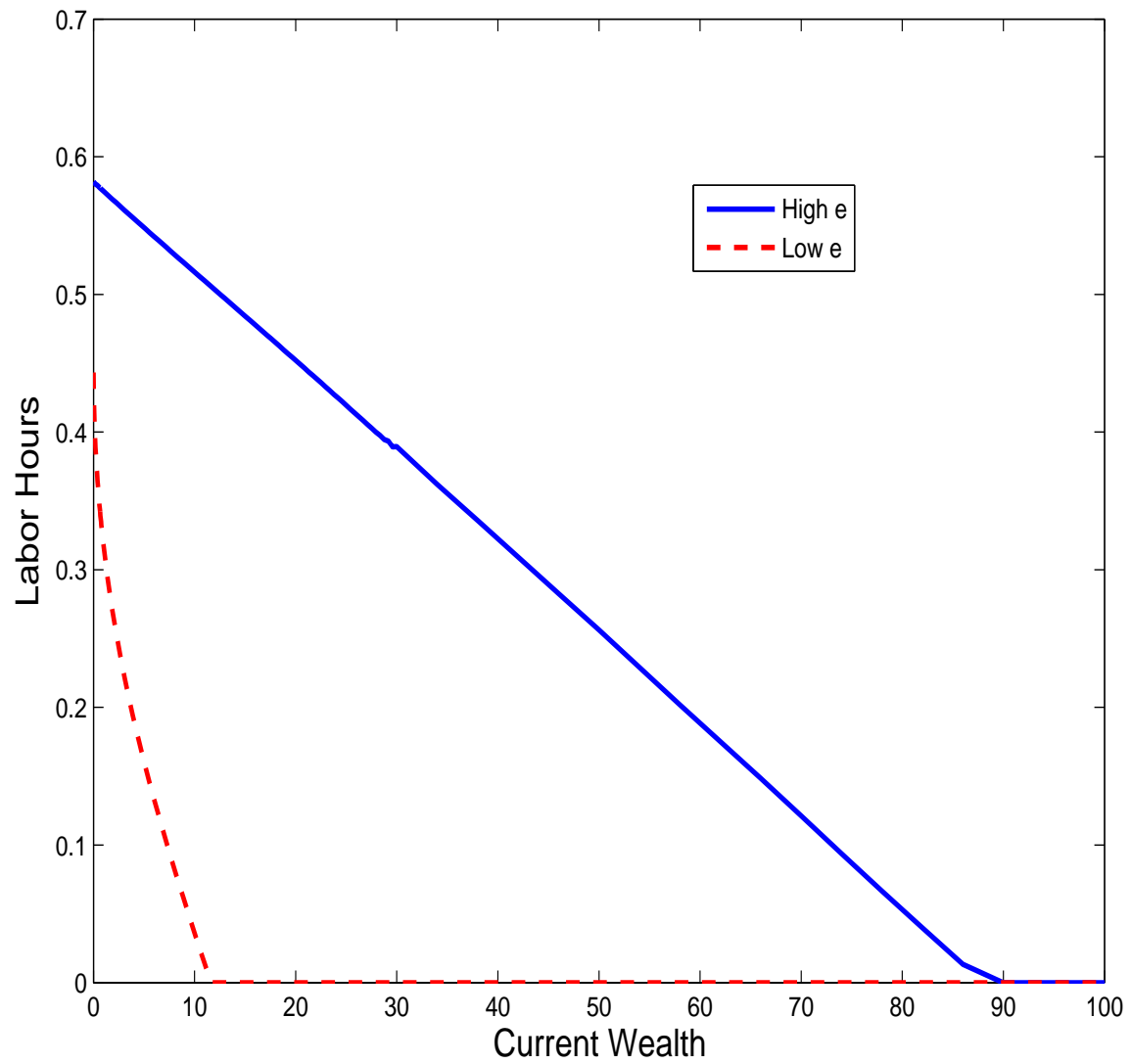
Figure 18: Transitional Dynamics

Equilibrium in the Capital Market
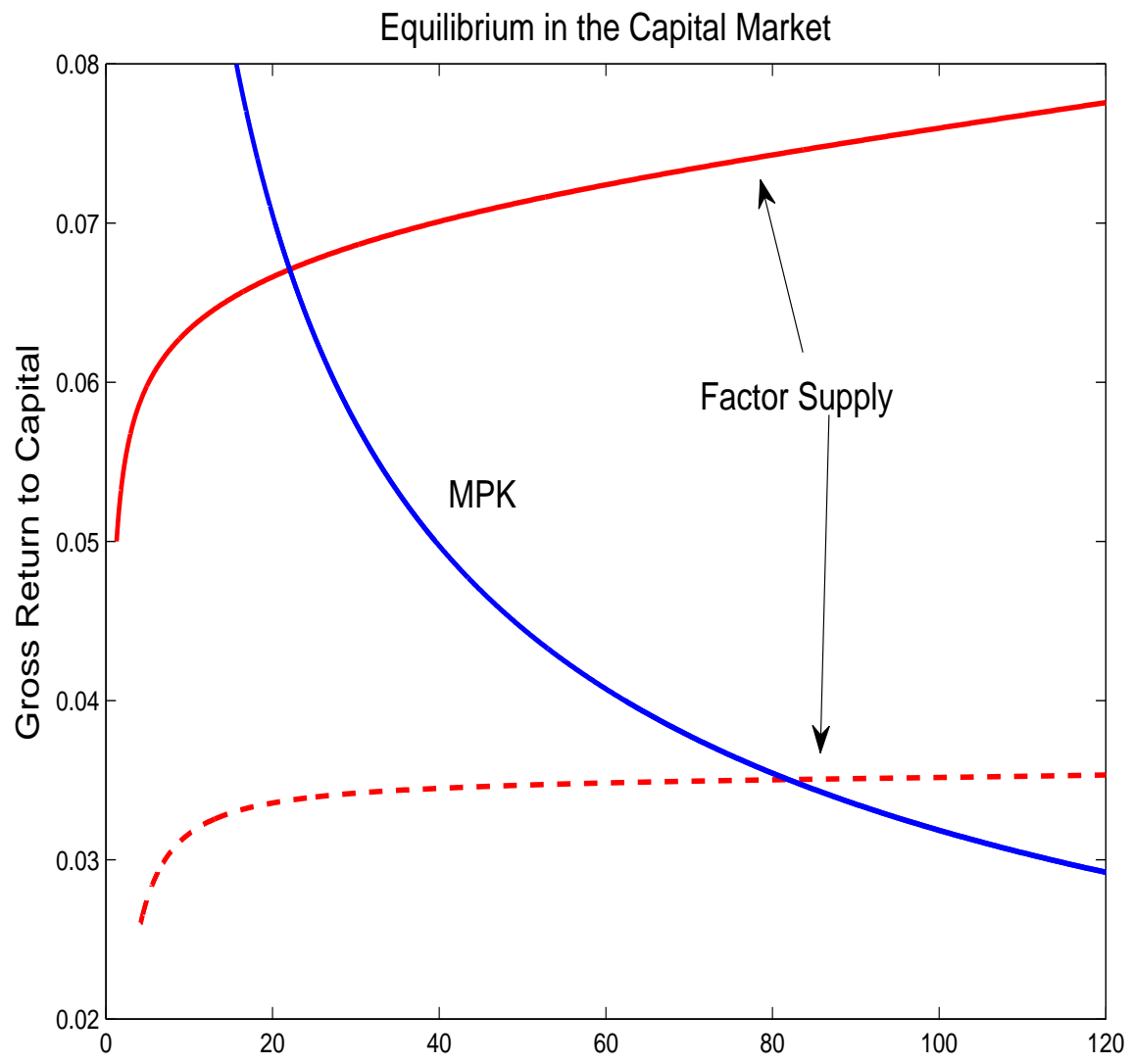
Figure 20: Lorenz Curves
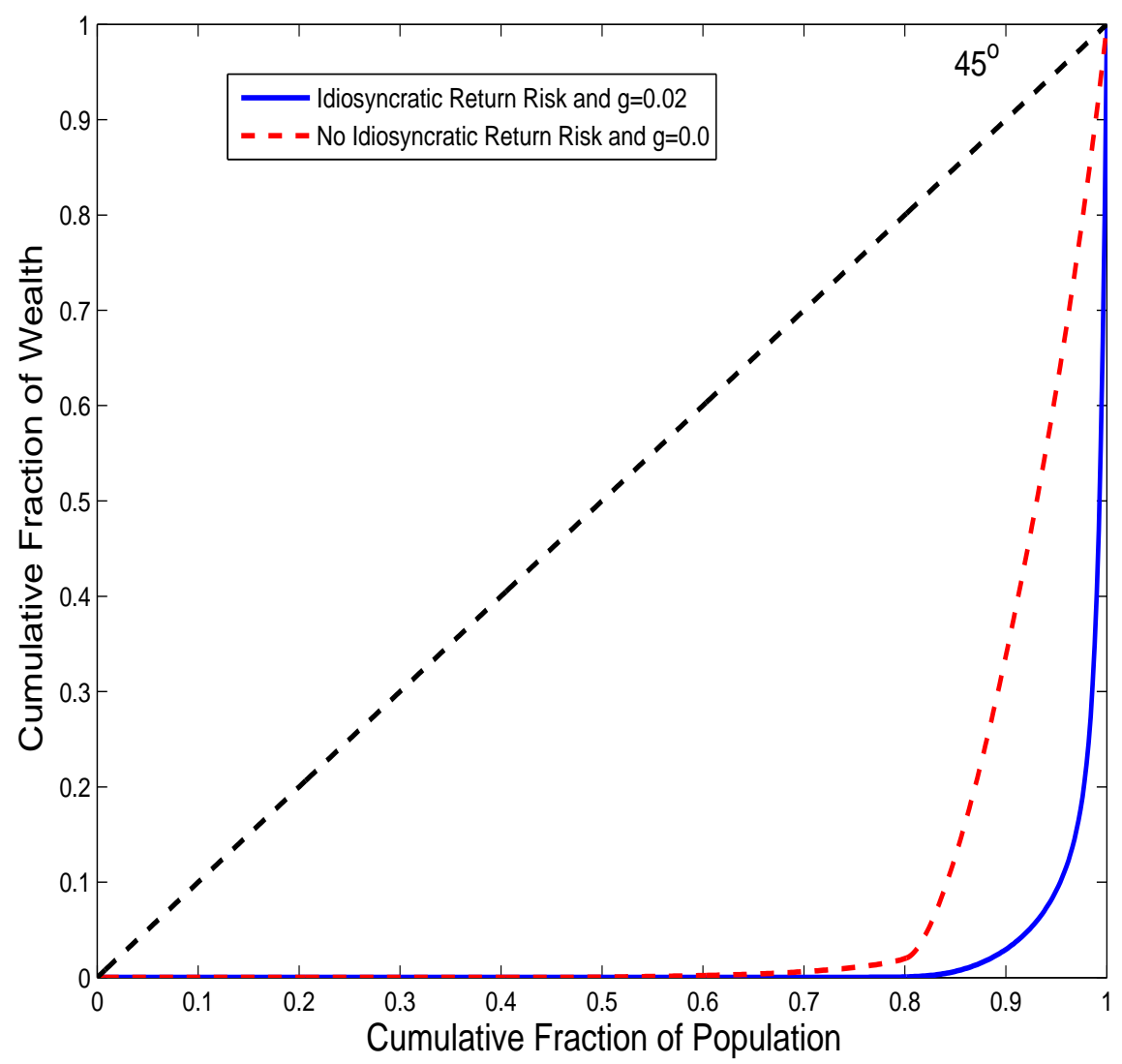
Figure 21: Transitional Dynamics

Return to Capital

Capital/Income
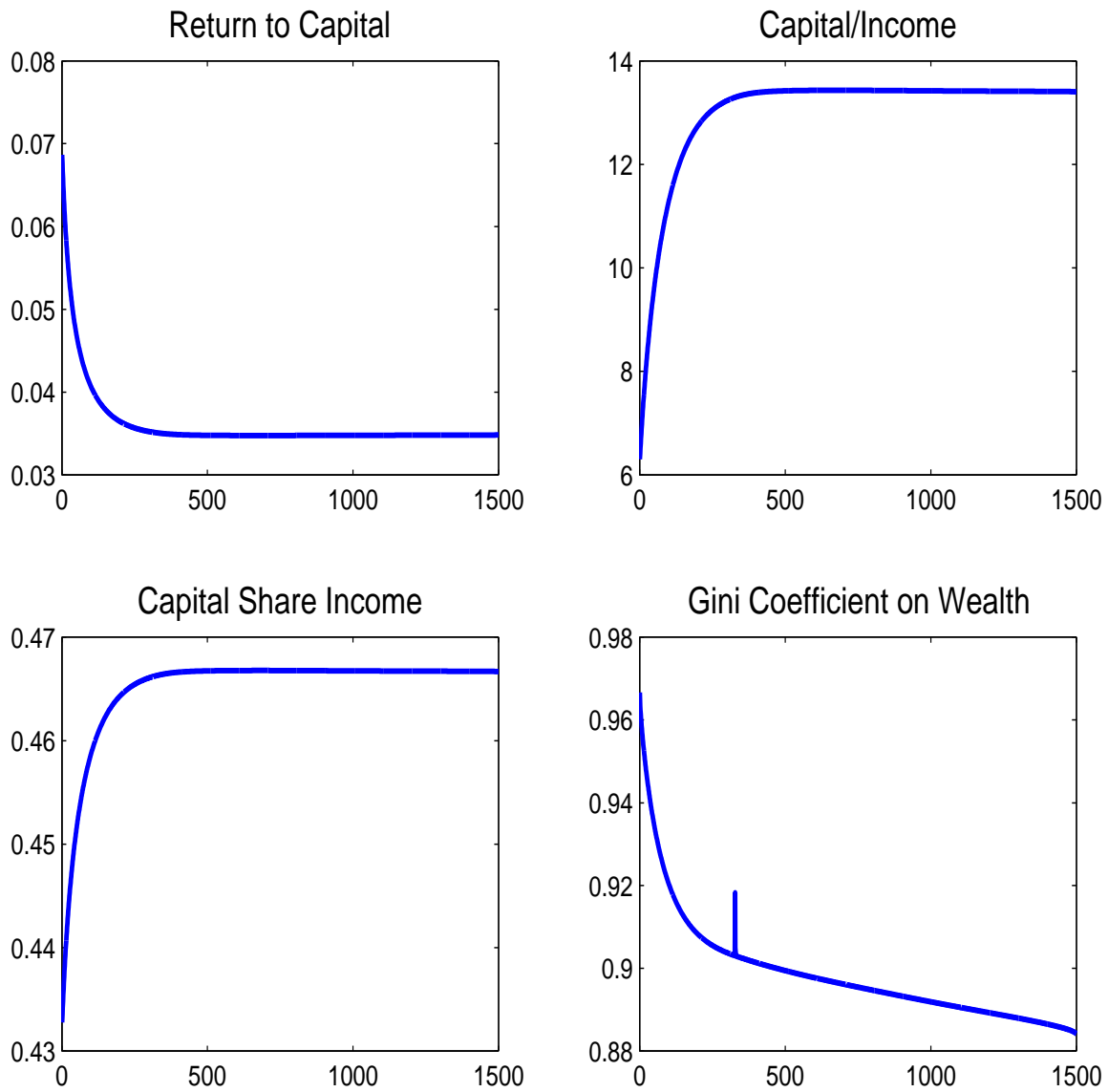
Capital Share Income

Gini Coefficient on Wealth

Figure 22: Steady State with Technological Progress