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Frequency Dependence in a Real-Time Monetary Policy Rule

Richard Ashley, Kwok Ping Tsang, and Randal J. Verbrugge

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We estimate a monetary policy rule for the US allowing for possible frequency dependence—i.e., allowing the central bank to respond differently to more persistent innovations than to more transitory innovations, in both the unemployment rate and the inflation rate. Our estimation method uses real-time data in these rates—as did the FOMC—and requires no a priori assumptions on the pattern of frequency dependence or on the nature of the processes generating either the data or the natural rate of unemployment. Unlike other approaches, our estimation method allows for possible feedback in the relationship. Our results convincingly reject linearity in the monetary policy rule, in the sense that we find strong evidence for frequency dependence in the key coefficients of the central bank's policy rule: i.e., the central bank's federal funds rate response to a fluctuation in either the unemployment or the inflation rate depended strongly on the persistence of this fluctuation in the recently observed (real-time) data. These results also provide useful insights into how the central bank's monetary policy rule has varied between the Martin-Burns-Miller and the Volcker-Greenspan time periods.

Richard Ashley is at Virginia Tech (ashleyr@vt.ed); Kwok Ping Tsang (corresponding author) is at Virginia Tech (byront@vt.edu); and Randal J. Verbrugge is at the Federal Reserve Bank of Cleveland (Randal.Verbrugge@clev.frb.org). The authors would like to thank anonymous referees for helpful comments.

1 Introduction

1.1 Literature Review

Following Taylor (1993), there has been an intense focus on Taylor-type monetary policy rules, such as:

$$i_t = \alpha + \phi_\pi \pi_t + \phi_u u_t + e_t \tag{1}$$

where i_t is the federal funds rate, π_t is the annualized inflation rate from period $t-1$ to period t, u_t is a measure of real activity (output gap or unemployment rate) in period t, and e_t is a stationary exogenous monetary shock. There are many variants of Equation (1). Theory often suggests forward-looking versions (e.g. Clarida, Gali and Gertler (2000)); real-time lags in data collection motivate the use of lagged inflation and real activity, i.e., a backward-looking monetary policy rule (e.g. McCallum (1997)); and interest rate smoothing considerations (as well as the statistical properties of i_t) motivate adding lags of i_t to the right-hand side of (1).[1] A number of studies have used variants of Equation (1) to conclude that the central bank's policy changed markedly starting with Volcker; see, e.g. Clarida, Gali and Gertler (2000) and Judd and Rudebusch (1998).[2]

Taylor originally developed this monetary policy rule as a descriptive device. More recently, a Taylor-type rule (with appropriate parameter values) is found to be optimal in a dynamic New Keynesian macroeconomic model – see Woodford (2003) or Gali (2009) for a textbook treatment. Decades ago, however, Friedman (1968) and Phelps (1968) introduced the concept of a natural rate of unemployment; and a related literature on the Phillips curve argues for the existence of a time-varying natural rate of unemployment. If there is a time-varying natural rate, it implies that not all unemployment rate movements are economically equivalent. A reasonable central

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[1]Consolo and Favero (2009) argues, in the forward-looking context, that the inertia is an artifact of a weak-instrument problem for expected inflation. Forward-looking monetary policy rules are not considered here, because doing so would require a substantial number of valid instruments for π_{t+j} and u_{t+j}, and lagged values do not provide useful instruments due to the identification problem discussed by Jondeau, Bihan and Galles (2004). Rudebusch (2002) disagrees with the interest rate smoothing interpretation; using evidence from the term structure, he shows that monetary policy inertia is more likely due to persistent shocks that the central bank faces.

[2]Using the the time-varying parameter framework, Boivin (2001), Cogley and Sargent (2005), Kim and Nelson (2006) and McCulloch (2007) come to a similar conclusion. However, Sims (2001) and Sims and Zha (2006) find that there is less evidence for significant changes in the reaction coefficients ϕ if one allows for time-varying variance in the monetary policy shock.

bank will thus respond differently to natural rate movements than to fluctuations about the natural rate. Hence, a linear policy rule such as Equation (1) – with a constant value for ϕ_u – must be seriously misspecified.

One means of addressing this issue is to use an "unemployment gap" for u_t in Equation (1), as in Ball and Tchaidze (2002) and McCulloch (2007). To calculate the unemployment gap, one needs an explicit estimate of the natural rate of unemployment. Such estimates are inherently problematic in that they typically hinge upon untested (and perhaps untestable) auxiliary assumptions about the natural rate data generating process (such as an explicit formulation of its persistence), which may well be incorrect. They are also generally based upon two-sided filters, which (as noted below) are known to distort the dynamics of time series relationships. Using an output gap instead of an unemployment gap in Equation (1) does not improve matters.

Similarly, the central bank likely responded differently to more persistent innovations in inflation than to less persistent innovations. Some analysts have attempted to address this issue by making use of "core inflation" measures in Equation (1). This expedient is valid, however, only if all movements in the core inflation measure are identically persistent, which is emphatically not the case – e.g., see Bryan and Meyer (2002) and Dolmas and Wynne (2008).

Two prominent recent studies use variants of Equation (1) to understand monetary policy behavior and its evolution over time. Orphanides (2002) uses real-time data and shows that the Federal Open Market Committee's (FOMC) forecast of inflation is biased during the Great Inflation period. He finds that the Federal Reserve actually reacts aggressively to its biased forecast; but standard analysis using *ex post* data leads to the conclusion that the FOMC only responds weakly to inflation. Ball and Tchaidze (2002) ask if the FOMC behaves differently between the 'old economy' period from 1987 to 1995, and the 'new economy' period from 1996 to 2000. Using a monetary policy rule that contains only inflation and the unemployment rate, they find that the estimates differ considerably between the two periods: during the 'old economy' period the FOMC reacts strongly to both inflation and unemployment rate fluctuations. For each one-percentage-point rise in inflation, they find that the FOMC raises the interest rate by 1.4 or 1.6 points, depending on the definition of inflation. And for each one-percentage-point rise in unemployment rate, they find that the FOMC cuts the interest rate by almost 2 points. In contrast, in the 'new economy' period they find that the FOMC's reaction to unemployment

rate fluctuations is much smaller. Ball and Tchaidze argue that these results are driven not by a change in policy but by a change in the non-accelerating-inflation rate of unemployment (NAIRU) in this later period. Allowing the estimated NAIRU to change over time and replacing unemployment with unemployment gap in the monetary policy rule, Ball and Tchaidze find that the FOMC's behavior does not change between the two periods.[3]

1.2 The Present Paper

In this paper, we estimate the central bank's monetary policy rule using a new method proposed by Ashley and Verbrugge (2009), described in Section 2. This method allows for the possibility the the policy rule reacts in different ways to fluctuations of differing persistence in the real-time data. In particular, our method allows us to estimate whether (and how) the central bank differentially responds in real time to perceived changes in either the unemployment rate or the inflation rate with persistence levels varying in steps from permanent ("zero-frequency") to completely transitory ("high-frequency"). We use a 36-month moving window in the analysis, which implies that persistence can vary in 19 steps, from completely "permanent" – referring to a fluctuation with an average reversion period of more than 36 months – to "temporary," referring to a fluctuation with an average reversion period of just two months.[4] Explicitly allowing for varying persistence in the central bank's responses to fluctuations in the unemployment rate renders explicit modeling of a time-varying NAIRU unnecessary.

Because a moving window on the data set is used, our method is gracefully compatible with the real-time data on inflation and unemployment rates which are actually available to the central bank policymakers whose behavior is being modelled. Moreover, because our Fourier decomposition of the data in the window for time t uses only the real-time data for that rate actually available at time t, our partitioning of each rate into persistence (or frequency)

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[3]This issue is re-visited in Section 5.4 below, where we find that allowing for frequency dependence in the monetary policy rule relationship eliminates any need to assume a change in the NAIRU across these two subperiods. See Tasci and Verbrugge (2014) for a current discussion of the NAIRU and NIIRU concepts.

[4]Sections 2 and 4 below describe how the real-time data on the inflation rate and the unemployment rate are decomposed into frequency components which add up to the original data series. 'Reversion period' is intuitively defined as follows: a fluctuation which tends to self-reverse on a time scale shorter than the reversion period associated with a given frequency component will have little impact on this frequency component. Section 2 exposits this decomposition in detail; Section 4 motivates this concept using an explicit example with a short (10-month) window. Table 1 summarizes the 19 frequency components and reversion periods allowed by the 36-month window actually used here.

components is, by construction, a backward-looking ("one-sided") filtering. Two-sided filters – e.g., those applied to both the explanatory and dependent variables in studies such as Cochrane (1989) – distort relationships amongst variables which are in feedback with one another, because two-sided filtering inherently mixes up future and past values of the time series. (See Ashley and Verbrugge (2009) for a more detailed exposition of this point.[5]) Thus, our analysis provides consistent estimates of the frequency dependence in monetary policy rules even where – as one might expect – there is feedback between the federal funds rate and the inflation/unemployment explanatory variates in the policy rule.[6]

While the original Taylor-type monetary policy response function is attractive in its simplicity, our frequency-dependent extension of it broadens its generality and descriptive power, yielding novel results. More explicitly, we find that the FOMC responds differently to highly persistent innovations in the unemployment rate – which one might largely identify with natural rate fluctuations – than it does to more transitory innovations. Similarly, the central bank's responses to inflation-rate innovations are also frequency-dependent. These findings – that the policy response coefficients in the FOMC's policy response function were not actually constants, but instead depended on the persistence of fluctuations in the macroeconomic variables these coefficient are multiplying – imply that a model with constant coefficients (such as Equation (1) above) is seriously mis-specified and hence yields inconsistent (and misleadingly unstable) parameter estimates. Appropriately allowing for frequency dependence in the monetary response coefficients yields a clearer picture of how the FOMC's actual policy rule has evolved over time – e.g., how it differs for the Martin-Burns-Miller (MBM) period (roughly March, 1960 to August, 1979) versus the Volcker-Greenspan-Bernanke (VGB) period, here taken to run from September, 1979 to August, 2008. In particular, estimates of Equation (1) as it stands imply that the FOMC was responsive to unemployment rate fluctuations in the MBM period but not in the VGB period; in contrast, our results in Section 5.3 show that the FOMC was significantly responsive to unemployment rate fluctuations in both periods once one allows for frequency dependence in the response coefficients. In addition, in Section 5.4 we re-visit the Ball and Tchaidze (2002) result

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[5]For the same reason, applying a two-sided filter to each of two time series likewise distorts the crosscorrelations between them, even in the absence of feedback.

[6]For this reason – and because such calculations are incompatible with the use of real-time data – two-sided cross-spectral estimates are not quoted here.

described at the end of Section 1.1 above. Allowing for frequency dependence in the monetary response coefficients, we find strong evidence that the FOMC's responses to both inflation and unemployment fluctuations were significant in both the "old economy" and the "new economy" periods, without any need to posit time variation in the NAIRU.

We do not interpret our results as implying that that the FOMC explicitly decomposed fluctuations in the inflation and unemployment rates into different frequency or persistence levels, and then mechanically followed a Taylor-type rule of the form we estimate, although it is certainly arguable that these policymakers had something like this in mind.[7] Instead, the goal of the paper is to utilize a richer statistical model to describe the behavior of the central bank, allowing the data to better inform us as to the manner in which the central bank has responded to fluctuations in these macroeconomic variables.

2 Modeling Frequency Dependence

In this section we discuss the technique used here for modeling frequency dependence in the monetary policy rule.[8] The idea of regression in the frequency domain can be traced back to Hannan (1963) and Engle (1974, 1978), and is further developed in Tan and Ashley (1999a and 1999b), who developed a real-valued reformulation of Engle's (1974) complex-valued framework.

Consider the ordinary regression model:

$$Y = X\beta + e \quad e \sim N(0, \sigma^2 I) \tag{2}$$

where Y and e are each $T \times 1$ and X is $T \times K$. Now define a $T \times T$ matrix A, whose $(s,t)^{th}$

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[7]For example, Meyer, Venkatu and Zaman (2013) at the Federal Reserve Bank of Cleveland comment that: "By specifying the inflation threshold in terms of its forecasted values, the FOMC will still be able to 'look through' transitory price changes, like they did, for example, when energy prices spiked in 2008. At that time, the year-over-year growth rate in the Consumer Price Index (CPI) jumped up above 5.0 percent but subsequently plummeted below zero a year later when the bottom fell out on energy prices. At the time, the Committee maintained the federal funds rate target at 2.0 percent, choosing not to react to the energy price spike."

[8]See Ashley and Verbrugge (2009) for details; this section (and an analogous section in Ashley and Tsang (2012) provide the most up-to-date descriptions, however. Additional descriptions are given in Ashley and Verbrugge (2007a,b)and in Ashley and Li (2014).

element is given by:

$$a_{s,t} = \begin{cases} (\frac{1}{T})^{\frac{1}{2}} & \text{for } s = 1; \\ (\frac{2}{T})^{\frac{1}{2}} \cos(\frac{\pi s(t-1)}{T}) & \text{for } s = 2, 4, 6, ..., (T-2) \text{ or } (T-1); \\ (\frac{2}{T})^{\frac{1}{2}} \sin(\frac{\pi(s-1)(t-1)}{T}) & \text{for } s = 3, 5, 7, ..., (T-1) \text{ or } T; \\ (\frac{1}{T})^{\frac{1}{2}}(-1)^{t+1} & \text{for } s = T \text{ when } T \text{ is even.} \end{cases} \tag{3}$$

It can be shown that A is an orthonormal matrix, so its transpose is its inverse and Ae is still distributed $N(0, \sigma^2 I)$. Pre-multiplying the regression model (2) by A thus yields,

$$AY = AX\beta + Ae \rightarrow Y^* = X^*\beta + e^*, e^* \sim N(0, \sigma^2 I) \tag{4}$$

where Y^* is defined as AY, X^* is defined as AX, and e^* is defined as Ae. The dimensions of the of Y^*, X^*, and e^* arrays are the same as those of Y, X, and e in Equation (1), but the T components of Y^* and e^* and the rows of X^* now correspond to frequencies instead of time periods.

To fix ideas, we initially focus on the jth component of X, i.e., column j of the X matrix, corresponding to the $j-1^{st}$ explanatory variable if there is an intercept in the model. The T frequency components are partitioned into M frequency bands, and M $T \times 1$ dimensional dummy variable vectors, $D^{*1}, ..., D^{*M}$, are defined as follows: for elements that fall into the sth frequency band, $D^{*s,j}$ equals $X^*_{\{j\}}$, and the elements are zero otherwise. The regression model can then be written as:

$$Y^* = X^*_{\{j\}}\beta_{\{j\}} + \sum_{m=1}^{M} \beta_{j,m} D^{*m,j} + e^* \tag{5}$$

where $X^*_{\{j\}}$ is the X^* matrix with its jth column deleted and $\beta_{\{j\}}$ is the β vector with its jth component deleted.

To test whether the jth component of β is frequency-dependent (i.e., to test whether the effect of the jth variable in X on Y is frequency or persistence dependent) one can then simply test the null hypothesis that $H_0 : \beta_{j,1} = \beta_{j,2} = ... = \beta_{j,M}$.

In the present application, we focus on two columns of X: the real-time inflation rate and the real-time inflation rate; these columns are denoted j and k below. By the same reasoning

used above, one can quantify (and test for) frequency dependence in the two model coefficients β_j and β_k corresponding to these two columns by re-writing the regression model (Equation 5) as:

$$Y^* = X^*_{\{j,k\}}\beta_{\{j,k\}} + \sum_{m=1}^{M}\beta_{j,m}D^{*m,j} + \sum_{m=1}^{M}\beta_{k,m}D^{*m,k} + e^*. \tag{6}$$

To make this regression equation a bit more intuitive, one can back-transform Equation (6) back into the time domain by pre-multiplying both sides of this equation with the inverse of A, which (because A is an orthonormal matrix) is just its transpose:

$$A'Y^* = A'X^*_{\{j,k\}}\beta_{\{j,k\}} + A'\sum_{m=1}^{M}\beta_{j,m}D^{*m,j} + A'\sum_{m=1}^{M}\beta_{k,m}D^{*m,k} + A'e^*. \tag{7}$$

This yields the time-domain specification:

$$Y = X_{\{j,k\}}\beta_{\{j,k\}} + \sum_{m=1}^{M}\beta_{j,m}D^{m,j} + \sum_{m=1}^{M}\beta_{k,m}D^{m,k} + e. \tag{8}$$

where $X_{\{j,k\}}$ is the original X matrix, omitting columns j and k and $\beta_{\{j,k\}}$ is the original β vector, omitting components j and k

Note that now the dependent variable is the same time series (Y) as in the original model, Equation (1). Similarly, all of the explanatory variables – except for the jth and k^{th} – are the same as in the original model. Indeed, the only difference is that these two explanatory variables have each been replaced by M new variables: i.e., the explanatory variable X_j has been replaced by $D^{1,j}...D^{M,j}$ and the the explanatory variable X_k has been replaced by $D^{1,k}...D^{M,k}$. Each of these M variables can be viewed as a bandpass-filtered version of the original data (the jth or k^{th}column of the X matrix), with the nice property that the M frequency component variables corresponding to column j of the X matrix add up precisely to the jth column of X and the M frequency component variables corresponding to column k of the X matrix add up precisely to the kth column of X.

In other words, the jth column of X – for example – is now partitioned into M parts. Reference to definition of the A matrix in Equation (3) shows that the first (low-frequency)

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component (corresponding to $m = 1$) is proportional to the sample average of the data for this explanatory variable. Similarly, the last component (corresponding to $m = M$) is essentially a sequence of changes in the data, and hence is the highest-frequency component that can be extracted from the data on this variable.[9] To test for frequency dependence in the regression coefficient on this j^{th} regressor, then, all that one need do is test the joint null hypothesis that $\beta_{j,1} = \beta_{j,2} = ... = \beta_{j,M}$. Similarly, X_k is replaced by $D^{1,k}...D^{M,k}$ and one tests the null hypothesis that $\beta_{k,1} = \beta_{k,2} = ... = \beta_{k,M}$.

However, because the A transformation mixes up past and future values (as in any Fourier-based bandpass filter), it can be shown that these M frequency components are correlated with the model error term e if there is feedback between Y and either of these two explanatory variables, leading to inconsistent estimation of the parameters $\beta_{j,1}, \beta_{j,2}, ...\beta_{j,M}$ and $\beta_{k,1}, \beta_{k,2}, ...\beta_{k,M}$ in that case. Feedback between the federal funds rate and inflation or unemployment rates is certainly likely, so this is an important issue here. To avoid this problem in general, Ashley and Verbrugge (2009) suggest modifying the procedure described above in order to obtain a one-sided filter for partitioning a variable into its frequency components. In particular, they suggest decomposing X_j, the jth explanatory variable data vector, into frequency components by applying the transformation described above within a moving three-year window, retaining only the most recent frequency component values calculable from this window. This leads to a one-sided, rather than a two-sided bandpass filter; the filtering of X_k is modified analogously.

Thus, in the results reported below, the transformation matrix A defined in Equation (3) is of dimension $T = 36 \times 36$ months and this windowing causes the lowest frequency component of each filtered series to now become a moving average, flexibly allowing for any (possibly nonlinear) trend in the original data.

This moving-window approach is used in the present paper for an additional, crucial reason: the moving window makes it possible to use real-time data for the values of X_j and X_k used in each window; in this way the analysis is consistent in each period with the data which were available to the policymakers at the time.

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[9]The average for the $m = 1$ component becomes a moving average of the data once windowing is introduced immediately below. Because the data used here are so very persistent – so as to match the specification of a typical Taylor-type monetary policy rule – an estimated linear trend is removed from the data for each window prior to the application of the filter; this would be unnecessary with the clearly I(0) data to which our procedure would ordinarily be applied.

Restricting the length of the window to $T = 36$ implies that the lowest frequency (longest period) component of X_j or X_k cannot distinguish between fluctuations with a reversion period of 36 months and fluctuations with reversion periods longer than this.[10] On the other hand, it has the nice feature that the matrix A defined in Equation (3) involves only 19 distinct frequencies, so that M is at most 19. Hence, Equation (8) involves the estimation of at most $2(19 - 1)$ additional coefficients in order to model any frequency dependence in the two relationships. This total of 19 distinct frequencies includes the 36-month moving average formed by the first row of A (corresponding to a frequency of zero) and 18 distinct positive frequencies, corresponding to the remaining rows of A, taken pairwise where the sine and cosine yield distinct rows. Section 4, below, illustrates this calculation of the number of distinct frequencies – and also supplies more of the intuition behind the frequency components – for the special case where the window length is set to ten for expositional simplicity; Table 1 tabulates the 19 frequency components possible with a the 36-month window length used here, explicitly relating each one to its reversion period and to the corresponding row (or rows) of the A matrix.

Thus, it is is feasible in this moving-window framework to estimate all of the possible distinct coefficients – i.e., $\beta_{j,1}, \beta_{j,2}, \cdots \beta_{j,19}$ and $\beta_{k,1}, \beta_{k,2}, \cdots \beta_{k,19}$ – without selecting frequency bands. This choice is not necessarily optimal, however, because the estimation of $2 \times 19 = 38$ coefficients can use up so many degrees of freedom in the dataset as to adversely, and needlessly, impact the precision with which inferences can be made. Moreover, reference to Table 1 makes it clear that, although the frequencies associated with these 19 components are equally spaced on the interval from zero to π, the corresponding reversion periods (which are proportional in each case to the reciprocal of the frequency) are quite unevenly spaced. In particular, note that the five components corresponding to $\beta_{j,13}, \cdots \beta_{j,19}$ all have reversion periods of between two and three months. While it is an empirical issue whether or not the FOMC pays any attention to i_t or u_t fluctuations this brief, we find that it is not useful to estimate six additional coefficients for each of these two variables in an attempt to quantify the degree to which the FOMC distinguishes between fluctuations with periods between two and three months in length. For this reason we routinely aggregate the seven highest frequency components for each of the two variables into

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[10]Similar empirical results to those reported below were obtained using a 48-month window as a robustness check. These results are available from the authors upon request.

a single component; this is, in effect, assuming that $\beta_{j,13}, \cdots \beta_{j,19}$ are all essentially equal and that $\beta_{k,13}, \cdots \beta_{k,19}$ are all essentially equal.

In addition, one might be interested in even more highly aggregated frequency bands so as to facilitate exposition of one's results on economic or intuitive grounds, as in Ashley and Tsang (2013) and Ashley and Li (2014), where the frequency components are aggregated into just three bands. Here we will do the same, defining a "low frequency band" corresponding to fluctuations with reversion periods in excess of 36 months, a "medium frequency" band with reversion periods between 12 and 36 months, and a "high frequency" band corresponding to reversion periods of less than 12 months. Looking ahead at Tables 1 and 4 in Section 5, the low frequency band consists of the first component; the medium frequency component is the sum of the components for reversion periods of 12, 18, and 36 months; and the high frequency component comprises the sum of the remaining 15 components, which all revert more quickly. Yes, this particular partitioning of the 19 bands into just three is, in a sense, a bit arbitrary. On the other hand, these three aggregated bands are interpretable in terms of the roughly the same calendar that the FOMC's policymakers live on.

Lastly, when decomposing X_j using a window, one must confront the problem of "edge effects" near the window endpoints. As in Dagum (1978) and Stock and Watson (1999), this problem is dealt with by augmenting the window data with projected data, here for an additional 3 months. Thus, the 36-month window incorporating the real-time data on X_j as of period t includes 33 past values of X_j – as known at time t – plus projections (forecasts) of its values for months $t+1$ to $t+3$. This window of data is then used, as described above, to compute the M components of X_j – i.e., the vectors $D^{1,j}...D^{M,j}$. The 33^d element of each of these vectors is then used as the period-t filtered value of X_j for this particular frequency. We find that the the estimated values of $\beta_{j,1}, \beta_{j,2}, ...\beta_{j,M}$ and their estimated standard errors are not sensitive to the number of projection periods (as long as at least one month of projections is used), nor to the details of how the projections (forecasts) are produced.[11] Since it is well known that the FOMC

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[11]In this paper both the unemployment and inflation series are projected by combining several distinct, backward-looking forecasting models using a model-averaging technique. This choice is not particularly consequential, however. For example, we find that using an AR(4) model with monthly dummies – or an AR(3) or AR(5) model instead – yields similar results with respect to the $\beta_{j,1}, \beta_{j,2}, ...\beta_{j,M}$ estimates. Details regarding these projection models – and RATS code implementing the frequency decomposition in this way – are available from the authors. A ready-to-use Windows-based executable file implementing the decomposition using projection based solely on seasonal dummies plus an AR(p) model is also available.

makes extensive use of forecasts in its decision-making, utilizing projections of this nature in our windowed bandpass filters seems particularly appropriate.

This is a good point at which to contrast the frequency decomposition used here with superficially similar procedures in the existing literature. For example, in contrast to trend-cycle decomposition methods (e.g., Beveridge-Nelson), our approach does not decompose an explanatory variable like X_j into just two components: an arbitrarily-persistent $I(1)$ or $I(1)$-like trend and a stationary $I(0)$ fluctuation. Our decomposition instead produces M components (adding up to X_j) which span the complete range of persistence levels permitted by the chosen window length. And it allows the data itself – via regression analysis – to quantify how the coefficients $\beta_{j,1}, ..., \beta_{j,M}$ vary across all of these persistence levels. Further, our decomposition still yields consistent parameter estimation where (as is typically the case with economic relationships) one cannot rule out feedback (or reverse-causality) – in contrast to the earlier spectral regression models cited at the outset of this section, which employ two-sided filtering. Finally, our approach is uniquely appropriate to the present analysis of the FOMC's Taylor Rule behavior, because the central bank surely bases its actual policy decisions on real-time data. In particular, the current real-time history of each of the relevant explanatory variables (the inflation and unemployment rates) corresponds exactly to the data which we use in each window for the decomposition of the current value of each variable into its frequency/persistence components.

We note, in this context, that an analogous kind of analysis based on the gain and phase of a transfer function model for the federal funds rate – as in Box and Jenkins (1976, Part III) – would be problematic because such models characteristically involve lagged values of the dependent and explanatory variables. For one thing, models containing lagged variables are inherently awkward when using real-time data because it is not clear whether the period-t datum to be used for X_j lagged, say, two periods should be the value of for that period as known currently (i.e., in period t) or at the time (i.e., in period $t-2$). In addition, transfer function gain and phase plots are substantially more challenging to interpret than our $\beta_{j,1}, ..., \beta_{j,M}$ coefficients, especially where (as here) bi-directional causality is likely. For example, Granger (1969) notes, "in many realistic economic situations, however, one suspects that feedback is occurring. In these situations the coherence and phase diagrams become difficult or impossible to interpret,

particularly the phase diagram."[12]

3 The Appeal of this Frequency-based Approach to Dis-aggregation by Persistence Level

The focus of this paper is to investigate, in a data-driven way, the degree and manner to which the FOMC has responded to persistent innovations in the unemployment rate and the inflation rate differently than it has to more transitory fluctuations in those variables. Thus the objective of partitioning two of the explanatory variable time series – X_j and X_k in Section 2 above, which are the unemployment and inflation rates in the present application – is not the bandpass filtering *per se*. Rather, we decompose the unemployment and inflation rates into frequency components so that we can separately estimate the impact of fluctuations of distinctly different persistence levels in these two variables on the federal funds rate and make inferences concerning these differential impacts.

No representation is made here that the bandpass filtering described in Section 2 above is asymptotically optimal – e.g., as in Koopmans (1974) or Christiano and Fitzgerald (2003) – although the relevance of asymptotic optimality in filtering data which here are of sample length ca. 36 is debatable.[13] On the other hand, our method of decomposing a time series into M frequency components has several very nice characteristics, which make this decomposition approach overwhelmingly well-suited to the present application:

1) The M frequency components that are generated from an explanatory variable (i.e., from a column of X) by construction partition it. That is, these M components add up precisely to the original observed data on this column of X. This makes estimation and inference with regard to frequency dependence (or its inverse, persistence dependence) in the corresponding regression

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[12]These difficulties notwithstanding, some transfer function model results with the present data are given in Appendix 1, available from the authors. (And at the end of this manuscript.)

[13]In this context we note that it feasible – albeit somewhat awkward – to iteratively employ a Christiano-Fitzgerald (2003) low-pass filter to partition the data in such a way that the frequency components still add up to the original data. This procedure involves applying the filter repeatedly, at each iteration varying the frequency threshold and applying the filter to the residuals from the previous iteration. This procedure is, of course, no longer even asymptotically optimal, but it does yield frequency components which still add up to the original data – as ours do automatically. Experiments with decompositions along these lines did not yield noticeably distinct results with regard to inferences on the regression model coefficients.

coefficient particularly straightforward: one can simply replace this explanatory variable in the regression model by a linear form in the M components and analyze the resulting M coefficient estimates.

2) Due to the moving windows used, this particular way of partitioning the data on an explanatory variable into these M frequency components by construction utilizes backward-looking (i.e., one-sided) filters. As demonstrated in Ashley and Verbrugge (2009), this feature is crucial to consistent OLS coefficient estimation where there is bi-directional Granger-causality (i.e., feedback) between the dependent variable and the explanatory variable being decomposed by frequency. The dependent variable in the present context is the federal funds rate, which is quite likely to be in a feedback relationship with the unemployment and inflation rates.

3) Finally, this way of partitioning the data on an explanatory variable into frequency/persistence components is not just mathematically valid and straightforward, it is also intuitively appealing. In particular – in contrast to many analyses in the frequency domain – our decompositions are not a 'black box.' The next section illustrates this point with a simple example.

4 An Illustrative Example with a Very Short Window

An example with a window ten periods in length illustrates the sense in which the frequency components defined above are extracting components of, say, X_j of differing levels of persistence. This window length is sufficient large as to illustrate the point, while sufficiently small as to yield an expositionally manageable example.[14] In particular, Table 2 displays the multiplication of the matrix A – whose elements are defined in Equation (3) – by the ten-component sub-vector of X_j corresponding to a window beginning in the particular period 21 and ending in period 30.

The first row of the A matrix is just a constant. The operation of this row of A on this particular ten-dimensional sub-vector of X_j is just calculating the sample mean over these ten observations. Thus, as this window progresses through the entire sample of data X_j, the first component of the vector formed by multiplying each ten-dimensional sub-vector of X_j on the left by A represents a one-sided, real-time, nonlinear trend estimate based (in this example)

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[14]As described above, the empirical implementation in this paper uses a window 36 months in length. See Table 1 for an explicit listing of the component frequencies, the corresponding reversion periods, and the corresponding A matrix rows for this 36-dimensional A matrix.

on a 10-period moving average. This is the "zero-frequency" component of the full X_j vector, corresponding to a sinusoidal reversion period unbounded in length. This component of X_j includes all of its variation at frequencies so low (i.e., reversion periods so large) that they are essentially invisible in a window which is only ten periods in length.

Higher-frequency components of X_j are, conversely, distinguishable using this window. The "Period" column in Table 1 is the number of observations over which the sine or cosine used in the corresponding row of the A matrix completes one full cycle. This is ten observations for rows two and three of this A matrix, $\frac{10}{2} = 5$ observations for rows four and five, $\frac{10}{3} = 3\frac{1}{3}$ observations for rows six and seven, $\frac{10}{4} = 2\frac{1}{2}$ observations for rows eight and nine, and $\frac{10}{5} = 2$ observations for row ten.[15] In the most common convention, the frequency is defined as $\frac{\pi}{2}$ times the inverse of cycle length (period) of the corresponding sine or cosine for that row of the A matrix, in which case the frequencies run from zero (for row one) to π for row ten.

To see intuitively why multiplication of the X_j vector by, for example, rows two and three extract only slowly-varying fluctuations in X_j, notice that these two rows are smoothly varying weights that will be applied to the ten components of X_j in forming its dot (or scalar) products with these two rows. Slowly-varying fluctuations in X_j will thus have a large impact on these two dot products, whereas rapidly-reverting variations in X_j will have little effect on the values of these two dot products. Hence, components two and three of the matrix product AX_j will 'contain' only those parts of X_j which are slowly varying.

Conversely, it is evident upon inspection of the last row of the A matrix that only high-frequency fluctuations – i.e., fluctuations which reverse in just two months or so – will contribute significantly to the tenth component of AX_j.

Thus, the first rows of the A matrix are distinguishing and extracting what are sensibly the "low-frequency" or "large period"or "highly persistent" or "relatively permanent" components of this ten-month X_j sub-vector as the window moves through the sample. Concomitantly, the last rows of the A matrix are distinguishing and extracting what are sensibly the "high-frequency" or "small period"or "low persistence" or "relatively temporary" components of this X_j sub-vector.

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[15]The number of observations in the sub-vector is an even integer – ten – in this example, implying that the sine and cosine terms are multiples of one another for what becomes a singleton last (tenth) row of the A matrix.

5 Empirical Results

5.1 Data Description and Plan of This Section

We use real-time data on the unemployment rate (u_t) and the inflation rate (π_t) from St. Louis Federal Reserve Bank ALFRED data set, so that the data we are analyzing correspond closely to those which were available to the FOMC at the time it set the federal funds rate (i_t).[16] The federal funds rate itself is not revised; thus, the end-of month observations available from the St. Louis FRED dataset are used for this variable.[17]

More specifically, we use the civilian unemployment rate for u_t and we use the inflation rate defined as the 12-month growth rate in percentage terms – i.e., $100 ln(CPI_t/CPI_{t-12})$ – where CPI_t is the non-seasonally adjusted Consumer Price Index for urban wage earners and clerical workers until February 1978 and the Consumer Price Index for all urban consumers thereafter.[18] The availability of real-time observations for use in constructing u_t and π_t constrains our sample period to begin in March 1960; our sample period ends in August 2008, just prior to the point when the sample variation in i_t becomes minimal.

Following Clarida, Gali, and Gertler (2000), we primarily consider two sub-sample periods. The first of these is March 1960 to August 1979, which roughly corresponds to the Martin-Burns-Miller period and is here denoted 'MBM'.[19] The second sub-sample runs from September 1979 to August 2008; it covers Volcker's, Greenspan's and part of Bernanke's tenures; it is hence here denoted denoted 'VGB'. Most of the VGB period is also referred to as the 'Great Moderation' – see McConnell and Perez-Quiros (2000) and Kim and Nelson (1999) – as it is characterized by low variance in most macroeconomic variables. Since the onset of the Great Recession, of course, most macroeconomic variables have become more volatile.

We also, at the end of this section, estimate and compare models utilizing two sub-samples (one running from September 1987 to December 1995 and another running from January 1996

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[16]Source: http://research.stlouisfed.org/fred2/. See Orphanides (2001) for evidence that estimated monetary policy rules are likely not robust to the vintage of the data.

[17]Source: http://alfred.stlouisfed.org/.

[18]As noted above, real-time values data are used, so the value for CPI_{t-12} in π_t is that which was available when CPI_t was released. Also, since inflation and unemployment data become available with a 1-month lag, we match the federal funds rate in each month with the data released in that month. For example, in April 2009 we use the inflation and unemployment data for March 2009.

[19]See Meltzer (2005) for a discussion of the high inflation rate during the latter part of this period.

to December 2000), so as to match up with the 'old economy' and 'new economy' periods defined in Ball and Tchaidze (2002). These results shed light on whether the FOMC's monetary policy rule in fact shifted between these two sub-periods, once we account for the frequency dependence in the policy rule.

Each observation on u_t and π_t is decomposed into frequency (persistence) components as described in Section 2. These are used to obtain the empirical results for the paper, organized as follows in the remainder of this section:

- Section 5.2 Results for an Ordinary (But Dynamic) Taylor-type Policy Rule Model Ignoring Possible Frequency Dependence

- Section 5.3 Results for the Frequency Dependent Model

    - 5.3.1 "Full Disaggregation" Results

    - 5.3.2 "Polynomial Smoothed" Results

    - 5.3.2 "Three-Band Model" Results

- Section 5.4 'New Economy' Versus 'Old Economy' Comparison

In Section 5.2 the usual Taylor-type monetary policy rule model discussed as Equation (1) in Section 1.1 is generalized to allow for dynamics, in the form of lags in i_t. The latter generalization is econometrically necessary, so as to yield model errors free from serial correlation; and it is economically interesting, in that such dynamics correspond to the FOMC acting so as to smooth the time path of the federal funds rate. In Section 5.3 the model is extended to allow for frequency dependence in the coefficients on π_t and u_t – as in Equation (8) of Section 2. There, and in Section 5.4, we obtain and discuss the central empirical results of the paper.

Finally, before focusing on regression model estimates, it is useful to consider time plots and log-spectrum plots of the frequency component time series of π_t and u_t, obtained using the one-sided real-time decomposition analysis described in Section 2. The time plots of these components should be smoother for the lower frequency components, and more noisy-looking for the higher frequency components. Concomitantly, the log-spectrum plots should indicate that the spectral power peaks up at zero frequency for the lowest frequency component and that this peak should move to higher frequencies for the higher frequency components.

Displaying these results for all nineteen frequency component time series of both π_t and u_t would require $19 \times 4 = 76$ plots, so we here present these plots for what we call the "calendar-based 3-band" decomposition below. In this decomposition the nineteen frequency components are aggregated into just three:

- "Low-Band" corresponding to reversion periods greater than 36 months

- "Mid-Band" corresponding to reversion periods between 12 and 36 months, inclusive

- "High-Band" corresponding to reversion periods less than 12 months

The 19 frequencies allowed by a 36-month window are equi-spaced; reference to Table 1, however, shows that (because they are proportional to the reciprocals of the frequencies) the corresponding reversion periods are decidedly not equi-spaced. Consequently, these three bands contain (aggregate together) unequal fractions of the 19 distinct frequency components. In particular, the low-band contains only the zero-frequency component, which is basically a nonlinear trend extracted using a 36-month moving average. The mid-band contains three frequency components, with reversion periods of 12, 18, and 36 months; and the high-band consists the remaining fifteen distinct frequency components, with reversion periods ranging from 2 to 9 months.

The time plots for these three components, partitioning π_t and u_t and displayed as Figures 1 and 2, behave just as one might expect: the low-band plots are quite smooth, the high-band plots look very noisy, and the mid-band plots are intermediate in smoothness.

Figures 3 and 4 display the corresponding log-spectrum plots. Notice how the spectral power in the log-spectrum plots for the two low-band components piles up over the greater-than-36 months abscissa, because the largest period resolvable by the windows used here is 36 months. Next note how the log-spectrum plot for the mid-band component of each of the two time series has most of its area (spectral power) to the left of the 9-month period abscissa and that it features a notable peak between 12 and 36 months. The little peaks in the log-spectrum plot for the mid-band component of π_t at reversion periods less than 6 months are likely artifacts of the fact that these spectral estimates are obtained using all of the sample data on each of these six components, whereas the bandpass filtering was done using a moving window passing through the data set. The log-spectrum plots for the high-band components of the two series are,

in contrast, relatively featureless with, again, a few minor artifactual variations. Overall, these plots reflect the fact that both π_t and u_t are quite persistent, leading to log-spectral values which are much larger for the low-band components; there is also clear evidence of some structure in the mid-band components; the high-band components, in contrast, appear to consist mostly consist of noise.

5.2 Results for an Ordinary (But Dynamic) Taylor-type Policy Rule Model Ignoring Possible Frequency Dependence

The federal funds rate (i_t) is highly persistent; we therefore re-specify the simple Taylor-type monetary rule given as Equation (1) in Section 1.1 to include enough lags in i_t as to yield serially uncorrelated fitting errors. Two lags sufficed:

$$i_t = \delta_1 i_{t-1} + \delta_2 i_{t-2} + (1 - \delta_1 - \delta_2)(\alpha + \phi_\pi \pi_t + \phi_u u_t) + e_t. \tag{9}$$

This equation is arranged so that lagged values of π_t and u_t need not be included; this is overwhelmingly appropriate in view of the use of real-time data on these two variables here, but engenders the minor complication that Equation (9) must be estimated via non-linear least squares (NLS) instead of OLS. The term $(\alpha + \phi_\pi \pi_t + \phi_u u_t)$ can be interpreted as the target interest rate, with the central bank eliminating a fraction $(1 - \delta_1 - \delta_2)$ of the gap between the current target and the current federal funds rate each month.[20]

Table 3 displays NLS estimates of δ_1, δ_2, ϕ_π, and ϕ_u separately over the MBM and VGB subperiods defined in Section 5.1. As noted above, the inclusion of two lags in i_t in this model (and in the models discussed in Sections 5.3 and 5.4 below, as well) suffices to yield serially uncorrelated model fitting errors; Eicker-White standard errors are quoted for all coefficient

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[20]Equation (9) implicitly assumes that i_t, π_t, and u_t are all stationary – i.e., $I(0)$ – time series. Empirically, it is difficult to reject the null of a unit root for these variables over the full sample, given that all of them display a good deal of persistence over time and that unit root tests have low power. These unit root null hypotheses are all strongly rejected, however, in each of the MBM and VGB subperiods when these periods are considered separately. This is in keeping with what one might expect, given the arguments in Clarida, Gali and Gertler (1999) to the effect that stationarity for these variables is implied by the theoretical models in which Taylor-type monetary policies play a role. Partitioning the sample also to some degree alleviates the problem of time-varying variance mentioned in Sims and Zha (2001, 2006), but we use Eicker-White standard error estimates throughout nevertheless.

estimates, here and below, to account, at least asymptotically, for any heteroskedasticity in ϵ_t.[21]
The coefficients δ_1 and δ_2 can be taken to quantify 'interest rate smoothing' behavior by the
FOMC, so it is noteworthy that the null hypothesis that both of these coefficients are zero can
be rejected with $P < 0.0005$ in these two models and, indeed, in all of the models estimated
here.

Ignoring – as is, of course, standard in the literature – any possible dependence of ϕ_π and
ϕ_u on the persistence of the fluctuations in the observed values of π_t and u_t, the coefficient
estimates in Table 3 indicate that the FOMC exhibited statistically significant policy responses
to fluctuations in π in both the MBM and VGB periods. The FOMC's response to inflation
rate fluctuations was notably larger in the MBM than in the VGB period, however: on average
the FOMC increased the federal funds rate by only 0.70% for every 1% increase in the inflation
rate in MGM period, whereas in the VGB period the estimated response is 1.4%. In contrast,
the FOMC's response to a 1% increase in the unemployment rate is statistically significant only
in the MBM period, and of only modest economic significance (-0.91%) even then.

Section 5.3 below presents evidence that these results are actually artifactual: allowing for
frequency dependence in the two policy reaction coefficients yields interestingly different results,
indicating that the omission of distinct frequency components of π_t and u_t in Equation (9) is
so substantially mis-specifying this Taylor-type monetary policy as to yield seriously misleading
conclusions as to the FOMC's past behavior.

5.3   Results for the Frequency Dependent Model

Here we re-specify the conventional Taylor-type monetary rule of Equation (1) to both incor-
porate dynamics – i.e., interest rate smoothing, in the form of lags in i_t – and to allow for the
possibility that the coefficients on π_t and u_t depend on the persistence levels of the fluctuations

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[21]Possible parameter estimation distortion due to three outlying observations in the fitting errors – for July
1973, May 1980, and Feb 1981 – was addressed using dummy variables to shift the intercept. The estimated
coefficients on these dummy variables were always highly significant – and (negative, negative, positive) in signs,
respectively – but their exclusion did not substantively affect the inference results reported below. Consequently,
the listing of these coefficient estimates – and the model intercept term (α) – is, for simplicity, suppressed in the
results tables here.

in these variates. This yields the model:

$$i_t = \delta_1 i_{t-1} + \delta_2 i_{t-2} + (1 - \delta_1 - \delta_2) \left(\alpha + \sum_{j=1}^{13} \phi_{\pi,j} \pi_t^j + \sum_{k=1}^{13} \phi_{u,k} u_t^k \right) + \epsilon_t. \qquad (10)$$

This regression model equation is, of course, just a special case of Equation (8), derived in Section 2 above; the 13 components of the real-time inflation rate $(\pi_t^1, ..., \pi_t^{13})$ and of the real-time unemployment rate $(u_t^1, ..., u_t^{13})$ were obtained as detailed in that section.[22]

5.3.1 "Full Disaggregation" Results

In the columns headed "Full Disaggregation" Table 4 presents our NLS estimation results for the inflation-rate policy response coefficients $(\phi_{\pi,1}, ..., \phi_{\pi,13})$ and unemployment-rate policy response coefficients $(\phi_{u,1}, ..., \phi_{u,13})$ over the two sample sub-periods, MBM and VGB.

Identifying the lowest-frequency component of the unemployment rate – i.e., (u_t^1) – in this model as the 'natural rate of unemployment,' and supposing, as is commonly done, that the FOMC responds weakly or not at all to perceived fluctuations in this natural rate, one would expect that $\phi_{u,1}$ would be negligible. This expectation is satisfied for both the MBM period coefficient (-0.247 ± 0.159) and for the VGB period coefficient (-0.070 ± 0.441).

The rows at the foot of Table 4 display the p-values at which two null hypotheses with regard to the inflation response coefficients $(\phi_{\pi,1}, ..., \phi_{\pi,13})$ and the unemployment rate response coefficients $(\phi_{u,1}, ..., \phi_{u,13})$ can be rejected.

The first set of tests address the issue of whether the FOMC's policy reaction function pays any attention at all to fluctuations the inflation and unemployment rates: if the coefficients on $\phi_{\pi,1}, ..., \phi_{\pi,13}$ are all zero, then there is no dependence in the FOMC's reactions to fluctuations in π_t. Similarly, if the coefficients on $\phi_{u,1}, ..., \phi_{u,13}$ are all zero, then there is no dependence in the FOME's reactions to fluctuations in u_t. Note that there is very strong evidence that the FOMC pays attention to fluctuations in the inflation rate in both periods. With regard to fluctuations in the unemployment rate, in contrast, there is very strong evidence that the FOMC pays attention to these fluctuations in the during the VGB period, but no evidence for

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[22]Recall that the seven highest-frequency components, with reversion periods less than or equal to three months, are aggregated together. Thus, there are only 13 components for each of these variates in Equation (10) rather than 19. See Table 1 for details.

any response at all to unemployment rate fluctuations during the MBM period.

The second set of tests address the issue of frequency dependence: if the coefficients on $\phi_{\pi,1}, ..., \phi_{\pi,13}$ are all equal, then there is no frequency (or persistence) dependence in the FOMC's reactions to fluctuations in π_t. Similarly, if the coefficients on $\phi_{u,1}, ..., \phi_{u,13}$ are all equal, then there is no frequency (or persistence) dependence in the FOMC's reactions to fluctuations in u_t. Note that there is no evidence for frequency dependence of either kind for the model estimated over the data from the MBM period, whereas there is very strong evidence for frequency dependence in the unemployment rate response coefficient using the data for the VGB period.

The interpretation of the individual estimated response coefficients for particular frequency bands – e.g. $\hat{\phi}_{u,9} = -5.736$ for the model estimated using data from the MBM – is not particularly useful. This is because actual fluctuations in the inflation rate themselves contain an array of persistence levels: one is never going to see an inflation rate fluctuation which is a pure sinusoid with a period of 4.5 months. Still, one expects to find that the individual $\hat{\phi}_{\pi,1}, ..., \hat{\phi}_{\pi,13}$ coefficients are mostly either positive or statistically insignificant and this is so for both the MBM and VGB periods. Similarly, one expects to find that the individual $\hat{\phi}_{u,1}, ..., \hat{\phi}_{u,13}$ coefficients are mostly either negative or statistically insignificant and this, too, is so for both the MBM and VGB periods.

The testing results discussed above clearly indicate that there is a non-constant pattern – i.e. frequency or persistence dependence – in the unemployment rate reaction coefficients $(\phi_{u,1}, ..., \phi_{u,13})$ during the VGB period. Discerning and interpreting this pattern, however, is hindered by the substantial amount of sampling variation in the individual coefficient estimates.

5.3.2 "Polynomial Smoothed" Results

The substantial sampling variation in the $\hat{\phi}_{\pi,1}, ..., \hat{\phi}_{\pi,13}$ and $\hat{\phi}_{u,1}, ..., \hat{\phi}_{u,13}$ coefficient estimates motivated us to smooth (and sharpen) these estimates by assuming that they vary smoothly across the 13 frequency bands. This smoothing was implemented by assuming that the frequency variation in each of these parameter groups can be captured by a kth order polynomial and instead estimating the $(k+1)$ coefficients in each polynomial. This tactic both smooths the parameter variation across the frequency bands and – presuming that $k \ll 13$ – also substantially reduces the number of parameters to be estimated. Here we found that the coefficient corresponding

to $k = 3$ was statistically insignificant in all four cases, so quadratic polynomial smoothing was used.[23] These estimates are displayed in the two "Polynomial Smoothed" columns of Table 4.

Now two distinct patterns in the response coefficients emerge for the two sample periods. During the MBM period, the FOMC basically responded positively to inflation shocks and negatively to unemployment shocks. There is also some modest statistical evidence – at the 4% level – for frequency dependence with respect to the response to inflation fluctuations during this period, in that the FOMC apparently ignored inflation rate fluctuations with reversion periods of four months or less.

During the VGB period, in contrast, the statistical evidence for frequency dependence with respect to the inflation rate is stronger, with the null hypothesis of no frequency dependence rejected at $P = .024$. And the evidence for frequency dependence with respect to fluctuations in the unemployment rate is extremely strong in this period: the null hypothesis of no frequency dependence is rejected with $P = .005$. Looking at the two (smoothed) patterns of coefficient estimates for the VGB period, the estimated coefficient pattern for the inflation responses is stark: only the very lowest frequency inflation rate fluctuations prompted a response from the FOMC. For the unemployment rate reaction coefficient variation in the VGB period, our results suggest that the FOMC ignored unemployment rate fluctuations with reversion periods larger than three years – presumably as being due to changes in the natural rate of unemployment – whereas for less-persistent fluctuations it reacted with interest rate cuts fairly robustly.

5.3.3 "Three-Band Model" Results

In this section we analyze the estimated frequency response coefficients $(\hat{\phi}_{\pi,1}, ..., \hat{\phi}_{\pi,13}$ and $\hat{\phi}_{u,1}, ..., \hat{\phi}_{u,13})$ aggregated and smoothed in a different way. This approach is more 'calendar driven' than the polynomial-smoothing approach – and thus, perhaps, a bit *ad hoc* – but it likely corresponds fairly closely to the way the FOMC itself views the real-time data with which it conducts actual monetary policy. In particular, we aggregate the full number of mathematically distinct frequency components implied by our 36-month moving window into just three bands:

- "Low Frequency Band (reversion periods: > 36 months)

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[23]This kind of smoothing was introduced by Almon (1965); see Johnston (1972, pp. 294-295) for a condensed description.

- "Medium Frequency Band (reversion periods: ≥ 12 months and ≤ 36 months)

- "High Frequency Band (reversion periods: < 12 months)

The Low Frequency band comprises what we think the FOMC takes to be 'long-term' or even 'permanent' fluctuations. The Medium Frequency band – consisting of the three components with reversion periods of 12, 18, and 36 months – comprise the fluctuations which we think the FOMC considers 'business cycle variations.' And the High Frequency band – consisting of all the components with reversion periods ranging from 2 months up to 9 months – comprise what we think the FOMC considers to be a combination of current news ('shocks') and measurement noise.

Returning to Table 4, note that the this aggregation did not noticeably impact the adjusted R^2 for the regression model, so the parameter restrictions imposed by this aggregation are not materially affecting the model's ability to model the i_t variation in either period. Consistent with this observation, the pattern of the inference results – both with regard to testing the null hypotheses that the response coefficients are zero and with regard to testing whether they vary across the frequency bands – are all quite similar to those reported for the 'polynomial smoothed' approach. In particular, there is strong evidence that observed fluctuations in the inflation rate affect the FOMC's behaviour – and in a significantly frequency dependent manner – in both the MGM and VGB periods. The same is true for observed fluctuations in the unemployment rate, but only in the VGB period.

Turning to the coefficient estimates for the individual bands, the Low Frequency band coefficient on unemployment rate fluctuations is statistically insignificant for both periods, supporting the notion that the FOMC considers these to primarily be changes in the natural rate. The statistical evidence for the FOMC responding to unemployment rate fluctuations in the Medium and High frequency bands is very weak, although at least of a sensible sign, for the MGM period, but much stronger – and especially for the High Frequency band fluctuations – in the VGB period.

With regard to the inflation rate reaction coefficients, in the MGM period the FOMC appears to respond, but strongly, only to Low Frequency band fluctuations. In the VGB period, the FOMC's response to Low Frequency band fluctuations is even larger. We note, however, that

there is some evidence here that the FOMC actually tended to lower the federal funds rate for a High Frequency band inflation rate shock in the VGB period; this result is not all that strong, however, so it could simply be sampling error in an estimate of a coefficient with a population value of zero.

5.4 Comparison with the Ball and Tchaidze (2002) Results

As mentioned at the end of Section 1.1 Ball and Tchaidze (2002) find it necessary to invoke a time-varying NAIRU in order to explain their observation that a Taylor-type policy rule based on a constant NAIRU implies that the FOMC responded to unemployment rate fluctuations in the 'old economy' (from 1987 to 1995) but does not appear to respond to unemployment rate fluctuations in the 'new economy' – which is to say, after the end of 1995.

Table 5 presents Taylor-type rule coefficient estimates which essentially reproduce these Ball and Tchaidze results in the context of a model not allowing for frequency dependence in the response coefficients. Table 6 presents inference p-value results obtained from re-estimating this model over these two sub-periods and using the methods described above to allow for "Three-band Model" frequency dependence. There, in contrast, we find very strong evidence that the FOMC responded to fluctuations of both types during both the 'old economy' and the 'new economy' time periods. Evidently, there is no need to posit – and try to separately estimate – a time-varying NAIRU once one appropriately accounts for the dependence of the response coefficients on the persistence in the inflation and unemployment rate fluctuations.

6 Conclusions

This paper presents the practical implementation of a new way of specifying an econometric regression model, allowing for flexible disaggregation of one or more of the explanatory variables – which may be real-time measures and/or in feedback with the dependent variable – into frequency (or persistence) components which add up to the original sample data. This decomposition allows us obtain richer conclusions as to how fluctuations in these explanatory variables with a distinct level of persistence will impact the dependent variable.

This new estimation technology is here applied to the estimation and analysis of Taylor-type

monetary response policy functions recently used by the U.S. central bank, first in the Martin-Burns-Miller (MBM) period and then in the Volcker-Greenspan-Bernanke (VGB) period. We find strong evidence for frequency dependence in the FOMC's inflation and unemployment rate response coefficient.

In particular, the FOMC in both periods varied the federal funds rate in response to a current movement in the real-time inflation rate if and only if this change was observed to be a low-frequency – i.e., highly persistent – fluctuation. In contrast, the FOMC essentially ignored inflation rate fluctuations with reversion periods of 36 months or less.

With regard to the FOMC's responses to real-time unemployment rate movements, we find that the FOMC's monetary policy rule was still frequency/persistence dependent, but in a distinct way during each chairmanship period. Consistent with the notion of a natural rate, the FOMC did not significantly respond to low frequency unemployment rate fluctuations – with reversion periods of 36 months or more – in either period. In contrast, in the MBM period the FOMC's responses to middle-frequency fluctuations in the unemployment rate – i.e., with reversion periods in the range of 12 to 36 months – are statistically significant and more significant than its responses to high frequency fluctuations. And in the VGB period the FOMC responded significantly to higher frequency unemployment rate fluctuations, with the overall evidence for frequency dependence in the unemployment rate reaction coefficient much more marked in this period than in the MBM period.

In contrast, a model ignoring the frequency dependence in these Taylor-type monetary response functions can only distinguish in the data that the FOMC was less responsive to unemployment rate fluctuations in the VGB period.

Thus, we reach two general conclusions in this paper:

1) *Frequency dependence is statistically and economically important in analyzing the FOMC's historical monetary policy rules. Policy has clearly distinguished between (and react differentially to) inflation and unemployment rate fluctuations of varying degrees of persistence, with the pattern of these differential reactions interestingly specific to which chairmanship period is addressed.*

2) *The empirical analysis of historical monetary policy reaction rules and – by extension – other macroeconomic relationships, without giving the data a chance to appropriately allow for fre-*

quency/persistence dependence in at least some key coefficients, is quite likely to miss out on uncovering some interesting features in the data. Distorted, and unnecessarily over-simplified inferences are thus likely consequences of ignoring frequency dependence in estimating such relationships.

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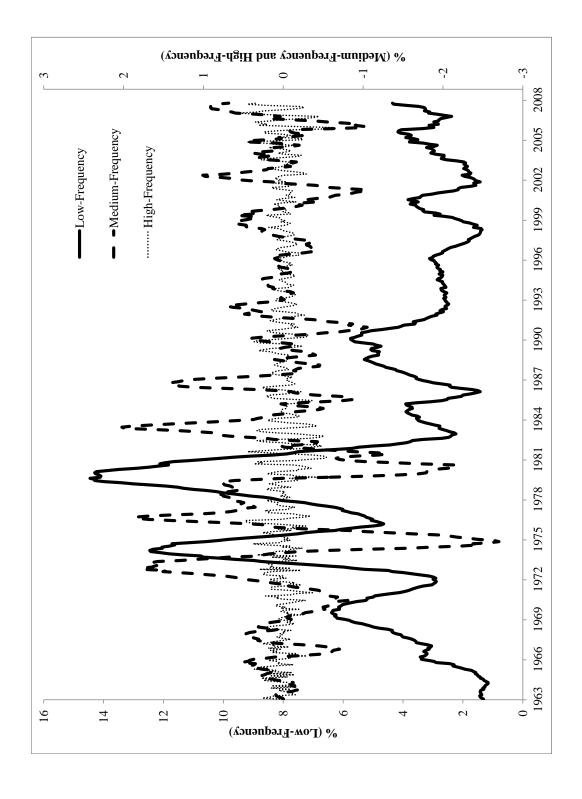Figure 1: Plots of the 3 Aggregate Frequency Components of Inflation Rate

Figure 2: Plots of the 3 Aggregate Frequency Components of Unemployment Rate

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Figure 3: Log Spectra of the Three Aggregate Frequency Components of Inflation Rate*

*A solid line is plotted for the medium frequency component and a dashed line is used for the high frequency component.
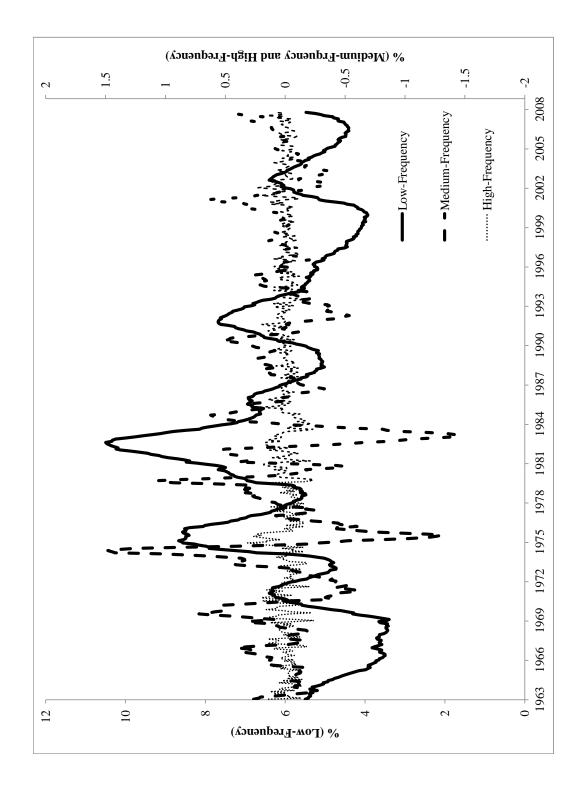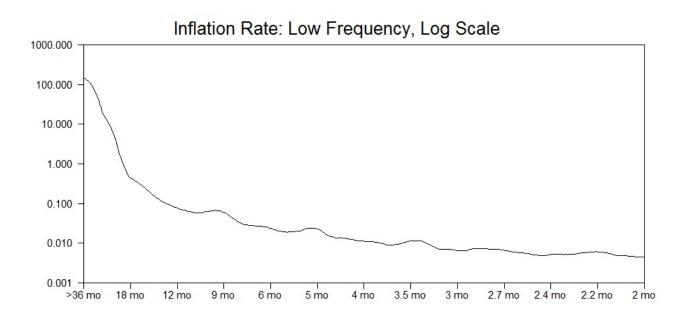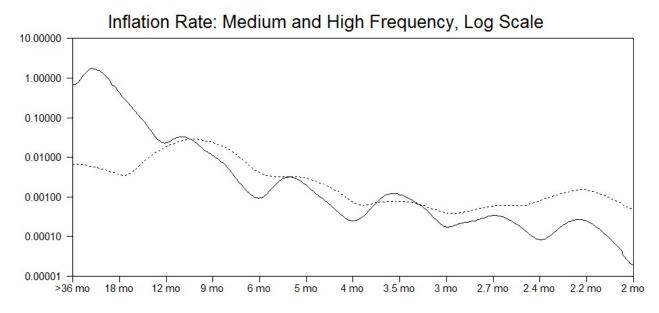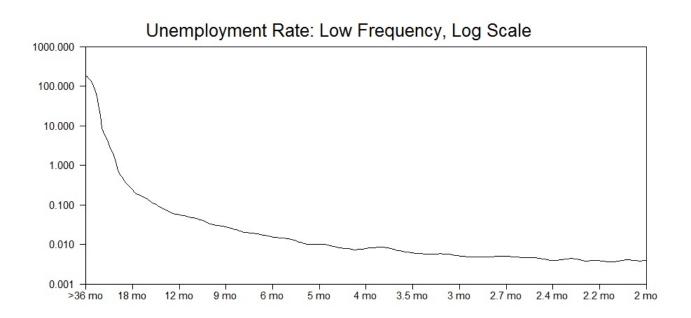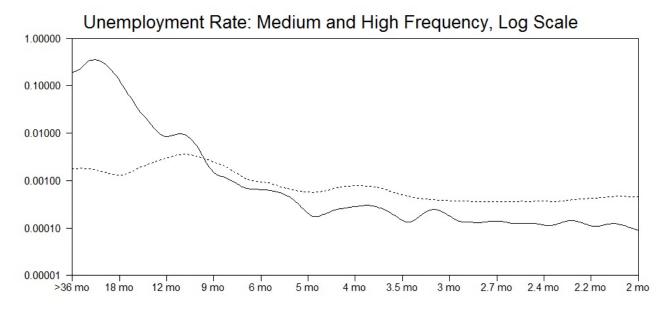
Figure 4: Log Spectra of the Three Aggregate Frequency Components of Unemployment Rate*

*A solid line is plotted for the medium frequency component and a dashed line is used for the high frequency component.

Frequency Component	Frequency	Reversion Period[a]	Row Number(s) in A[b]
1	0	>36	1
2	$\pi/18$	$36/1 = 36.00$	2,3
3	$2\pi/18$	$36/2 = 18.00$	4,5
4	$3\pi/18$	$36/3 = 12.00$	6,7
5	$4\pi/18$	$36/4 = 9.00$	8,9
6	$5\pi/18$	$36/5 = 7.20$	10,11
7	$6\pi/18$	$36/6 = 6.00$	12,13
8	$7\pi/18$	$36/7 = 5.14$	14,15
9	$8\pi/18$	$36/8 = 4.50$	16,17
10	$9\pi/18$	$36/9 = 4.00$	18,19
11	$10\pi/18$	$36/10 = 3.60$	20,21
12	$11\pi/18$	$36/11 = 3.27$	22,23
13	$12\pi/18$	$36/12 = 3.00$	24,25
14	$13\pi/18$	$36/13 = 2.77$	26,27
15	$14\pi/18$	$36/14 = 2.57$	28,29
16	$15\pi/18$	$36/15 = 2.40$	30,31
17	$16\pi/18$	$36/16 = 2.25$	32,33
18	$17\pi/18$	$36/17 = 2.12$	34,35
19	$18\pi/18$	$36/18 = 2.00$	36

Table 1: **Frequencies and Reversion Periods for a 36-Month Window** [a]In months, calculated as 2π divided by the frequency. The sinusoids comprising the elements of the row(s) of the A matrix corresponding to this reversion period complete a full cycle in this many months. Thus, the scalar product of such a row with a time-series vector whose fluctuations self-reverse substantially slower than this will be very small. [b]The A matrix is defined in Equation (3).

Period	Matrix A										Data
> 10	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	$X_j(21)$
10	0.45	0.36	0.14	-0.14	-0.36	-0.45	-0.36	-0.14	0.14	0.36	$X_j(22)$
10	0.00	0.26	0.43	0.43	0.26	0.00	-0.26	-0.43	-0.43	-0.26	$X_j(23)$
5	0.45	0.14	-0.36	-0.36	0.14	0.45	0.14	-0.36	-0.36	0.14	$X_j(24)$
5	0.00	0.43	0.26	-0.26	-0.43	0.00	0.43	0.26	-0.26	-0.43	$X_j(25)$
3.3	0.45	-0.14	-0.36	0.36	0.14	-0.45	0.14	0.36	-0.36	-0.14	$X_j(26)$
3.3	0.00	0.43	-0.26	-0.26	0.43	0.00	-0.43	0.26	0.26	-0.43	$X_j(27)$
2.5	0.45	-0.36	0.14	0.14	-0.36	0.45	-0.36	0.14	0.14	-0.36	$X_j(28)$
2.5	0.00	0.26	-0.43	0.43	-0.26	0.00	0.26	-0.43	0.43	-0.26	$X_j(29)$
2	0.32	-0.32	0.32	-0.32	0.32	-0.32	0.32	-0.32	0.32	-0.32	$X_j(30)$

Table 2: **An Example With a Window of Length Ten Periods**. The first row of A time the data vector simply yields $1/\sqrt{T}$ times the sample mean of the data in this ten-period window. As the window moves through the data set, this operation extracts any, possibly nonlinear, trend as a moving average. Rows two and three take a weighted average of the window data, using smoothly-varying weights which take a full ten periods to reverse, so any fluctuation in window data that reverses in a couple of periods yields a small value. The product of row ten and the window data is essentially calculating five changes in the data which occur during the window period. A long, smooth variation in the window data yields a small value for this frequency component.

Sample	δ_1	δ_2	ϕ_π	ϕ_u	$\bar{R}^2$
MBM Period	0.578 (0.089)	0.246 (0.081)	0.702 (0.163)	-0.913 (0.289)	0.893
VGB Period	0.611 (0.126)	0.292 (0.116)	1.420 (0.497)	0.202 (0.505)	0.912

Table 3: **Estimates for the Non-Frequency-Dependent Taylor-type Model: Equation (9) of Section 5.2.** Nonlinear least squares estimates are quoted for Equation (9), with White-Eicker standard errors in parentheses. The estimates for the intercept term and the dummy variables used to eliminate the three outliers are not quoted.

	Period (months)	Full Disaggregation		Polynomial Smoothed		Three-Band Model	
		MBM Period	VGB Period	MBM Period	VGB Period	MBM Period	VGB Period
		estimate (std.error)	*estimate* (std.error)	*estimate* (std.error)	*estimate* (std.error)	*estimate* (std.error)	*estimate* (std.error)
δ_1		1.220 (0.094)	1.297 (0.069)	1.250 (0.090)	1.299 (0.073)	1.264 (0.087)	1.328 (0.073)
δ_2		−0.309 (0.091)	−0.346 (0.070)	−0.330 (0.096)	−0.355 (0.075)	−0.349 (0.089)	−0.384 (0.076)
$p\ (H_0 : \delta_1 = \delta_2 = 0)$		0.000	0.000	0.000	0.000	0.000	0.000
$\phi_{\pi,1}$	> 36	0.751 (0.117)	1.867 (0.474)	0.826 (0.161)	1.90 (3.88)	0.796 (0.140)	1.817 (0.443)
$\phi_{\pi,2}$	36	2.281 (0.716)	1.190 (1.802)	1.732 (0.460)	−0.195 (0.753)	1.875 (0.440)	0.273 (1.390)
$\phi_{\pi,3}$	18	2.689 (1.249)	−0.095 (2.122)	2.565 (0.792)	−1.719 (1.257)		
$\phi_{\pi,4}$	12	1.734 (2.266)	−7.792 (3.508)	3.324 (1.062)	−2.742 (1.645)		
$\phi_{\pi,5}$	9.0	1.453 (2.501)	−8.777 (3.189)	4.010 (1.295)	−3.264 (1.931)	2.216 (2.038)	−5.361 (2.423)
$\phi_{\pi,6}$	7.2	−0.369 (3.221)	−3.754 (4.586)	4.622 (1.533)	−3.285 (2.169)		
$\phi_{\pi,7}$	6.0	−1.724 (6.299)	−5.331 (10.241)	5.161 (1.827)	−2.805 (2.443)		
$\phi_{\pi,8}$	5.1	3.749 (4.297)	1.506 (4.998)	5.626 (2.221)	−1.825 (2.855)		
$\phi_{\pi,9}$	4.5	7.623 (5.596)	−11.651 (11.038)	6.018 (2.747)	−0.343 (3.485)		
$\phi_{\pi,10}$	4.0	−0.785 (10.673)	9.432 (17.328)	6.336 (3.420)	1.639 (4.374)		
$\phi_{\pi,11}$	3.6	−1.879 (11.015)	−1.786 (6.428)	6.580 (4.245)	4.122 (5.530)		
$\phi_{\pi,12}$	3.3	5.569 (7.140)	5.365 (11.642)	6.752 (5.220)	7.105 (6.946)		
$\phi_{\pi,13}$	≤ 3.0	2.800 (3.443)	7.146 (7.707)	6.849 (6.345)	10.590 (8.610)		
$\phi_{u,1}$	> 36	−0.247 (0.159)	−0.070 (0.441)	−0.370 (0.182)	−0.167 (0.305)	−0.309 (1.771)	−0.172 (0.374)
$\phi_{u,2}$	36	−2.232 (1.390)	−3.513 (2.934)	−2.987 (1.415)	−4.953 (1.938)	−2.449 (1.190)	−4.254 (2.685)
$\phi_{u,3}$	18	−3.878 (1.862)	−7.500 (6.710)	−5.131 (2.513)	−8.982 (3.340)		
$\phi_{u,4}$	12	−5.814 (3.428)	−36.189 (12.242)	−6.802 (3.392)	−12.254 (4.387)		
$\phi_{u,5}$	9.0	−19.031 (8.752)	−8.396 (13.805)	−8.000 (4.053)	−14.770 (5.092)	−7.870 (4.498)	−14.187 (6.179)
$\phi_{u,6}$	7.2	−15.195 (8.752)	−52.189 (19.956)	−8.725 (4.505)	−16.528 (5.482)		
$\phi_{u,7}$	6.0	−5.002 (9.750)	−16.692 (22.387)	−8.977 (4.759)	−17.529 (5.600)		
$\phi_{u,8}$	5.1	−16.722 (9.408)	−8.752 (18.034)	−8.757 (4.836)	−17.773 (5.530)		
$\phi_{u,9}$	4.5	−5.736 (8.871)	−62.290 (23.682)	−8.064 (4.769)	−17.261 (5.416)		
$\phi_{u,10}$	4.0	−15.454 (10.521)	−38.180 (29.569)	−6.898 (4.613)	−15.991 (5.496)		
$\phi_{u,11}$	3.6	0.760 (8.800)	−21.177 (15.995)	−5.259 (4.462)	−13.964 (6.058)		
$\phi_{u,12}$	3.3	−9.167 (11.752)	−26.754 (22.039)	−3.147 (4.462)	−11.180 (7.298)		
$\phi_{u,13}$	≤ 3.0	0.540 (3.808)	−9.183 (11.215)	−0.563 (4.782)	−7.639 (9.234)		
Testing for Zero Coefficients							
$p\ (H_0 : \phi_{\pi,j} = 0\ \forall j)$		0.000	0.006	0.000	0.000	0.000	0.000
$p\ (H_0 : \phi_{u,j} = 0\ \forall j)$		0.568	0.001	0.116	0.012	0.157	0.012
Testing for Frequency Dependence							
$p\ (H_0 : \phi_{\pi,j} = \phi_{\pi,k}\ \forall j \neq k)$		0.749	0.224	0.039	0.024	0.000	0.012
$p\ (H_0 : \phi_{u,j} = \phi_{u,k}\ \forall j \neq k)$		0.787	0.000	0.136	0.005	0.150	0.004
R^2		0.981	0.988	0.981	0.987	0.981	0.987

Table 4: **OLS Estimates for the Frequency-Dependent Taylor Rule: Equation(10) of Section 5.3.** White-Eicker standard errors are quoted in parentheses. The estimate for the intercept term and the dummy variables used to eliminate the three outliers are not quoted.

Sample	δ_1	δ_2	ϕ_π	ϕ_u	R^2
Sep 1987 - Dec 1995	0.419 (0.159)	0.254 (0.119)	0.678 (0.384)	-2.058 (0.333)	0.902
Jan 1996 - Dec 2000	0.190 (0.089)	0.243 (0.093)	0.444 (0.172)	-0.121 (0.25)	0.232

Table 5: **Estimates for the Non-Frequency-Dependent Taylor-type Rule: 'Old' Versus 'New' Economy**. Nonlinear least squares estimates are quoted for Equation (9), using the Ball and Tchaidze (2002) sub-periods; see discussion in Section 5.5. White-Eicker standard errors are quoted in parentheses; the estimates for the intercept term and the dummy variables used to eliminate the three outliers are not quoted.

Sample	Unemployment Rate	Inflation Rate
Sep 1987 - Dec 1995	0.000	0.000
Jan 1996 - Dec 2000	0.001	0.000

Table 6: **Test of Taylor-type Rule Policy Unresponsiveness in the 'Old' Versus the 'New' Economy, Allowing for Frequency Dependence in the Relationship.** Rejection p-values for the null hypothesis that the coefficients on the three aggregate frequency components for inflation or unemployment rate are all zero. These are results from re-estimation of Equation (10), using the Ball and Tchaidze (2002) sub-periods; see discussion in Section 5.5.

Appendix: Gain and Phase of Transfer Function Models

The "Full Disaggregation" model of Section 5.3.1 is estimated, to obtain

$$i_t = \hat{\delta}_1 i_{t-1} + \hat{\delta}_2 i_{t-2} + (1 - \hat{\delta}_1 - \hat{\delta}_2) \left(\hat{\alpha} + \sum_{j=1}^{13} \hat{\phi}_{\pi,j} \pi_t^j + \sum_{j=1}^{13} \hat{\phi}_{u,j} u_t^j \right) + \epsilon_t. \tag{11}$$

Here, we construct the gain and phase of linear transfer functions that approximate the filters of the unemployment rate and inflation rate series implied by this estimated equation during the VGB period. In particular, we utilize our estimated values for $\hat{\phi}_{\pi,1}...\hat{\phi}_{\pi,13}$ and $\hat{\phi}_{u,1}...\hat{\phi}_{u,13}$ to construct the left-hand side variables used in estimating:

$$\sum_{j=1}^{13} \hat{\phi}_{\pi,j} \pi_t^j = \frac{A(L)}{C(L)} \pi_t + U_{1,t},$$

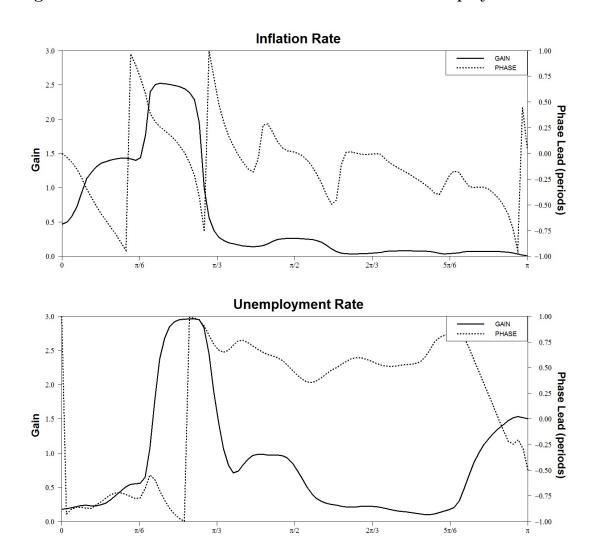$$\sum_{j=1}^{13} \hat{\phi}_{u,j} u_t^j = \frac{B(L)}{D(L)} u_t + U_{2,t}.$$

where π_t is the real-time inflation rate, u_t is the real-time unemployment rate, and where the $U_{1,t}$ and $U_{2,t}$ are error processes which may have moving average components. We then compute the gain and phase of the estimated lag polynomial quotients $\frac{A(L)}{C(L)}$ and $\frac{B(L)}{D(L)}$; these functions are plotted versus period below.[24]

Interpreting these plots, however, is somewhat of a challenge. Indeed, that is why the body of this paper concentrates solely on the estimated frequency components $\hat{\phi}_{\pi,1}...\hat{\phi}_{\pi,13}$ and $\hat{\phi}_{u,1}...\hat{\phi}_{u,13}$, as each of these bears a clear and straightforward interpretation. For example, $\hat{\phi}_{\pi,4}$ in Table 4 of Section 5 is the estimated Taylor-type policy rule coefficient for fluctuations in the inflation rate with a period of 12 months; thus, a negative value for this estimated coefficient indicates inflation accommodation with respect to inflation fluctuations which tend to self-reverse on a time-scale of 12 months.

In contrast, the gain and phase of the $\frac{A(L)}{C(L)}$ and $\frac{B(L)}{D(L)}$ filters whose calculation is described above are not so easy to interpret. For one thing – outside of elementary cases – gain and phase

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[24]Using the standard Box-Jenkins (1976) methodology, our final model for $C(L)$ consists of 14 consecutive lags, while our model of $A(L)$ consists of 19 non-consecutive lags between 0 and 24, with $U_{1,t}$ modeled with four MA terms. This model effectively removes significant partial autocorrelation in the residuals out to 24 months and yields an adjusted R^2 of 99%. Our final model for $D(L)$ consists of 9 lags, while our model of $B(L)$ consists of 18 lags between 0 and 25, with $U_{2,t}$ modeled with three MA terms. This model again effectively removes significant partial autocorrelation in the residuals out to 24 months and yields an adjusted R^2 of 93%. The moving average representations of these transfer functions were estimated out to 100 lags (in u_t) and to 200 lags in (π_t), and then Fourier-transformed. In keeping with standard practice, the Fourier transforms of the transfer functions - here, complex series of length 192 - were smoothed with a flat window prior to computing the implied gain and phase displayed in the figures.

Figure A: Gain and Phase Plots for Inflation and Unemployment Rates

Inflation Rate

Unemployment Rate

plots are well-known to be relatively opaque objects. Moreover, a feature of our study is that real-time data on π_t and u_t are used. But this implies that the operation of a lag operator – e.g., L^5 in the $\frac{A(L)}{C(L)}$ lag operator on π_t defined above – is immediately problematic: does it produce the inflation rate lagged five periods as known now or does it produce the then-currently-known value of the inflation rate from five periods ago?