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Optimal Contracts, Aggregate Risk, and the Financial Accelerator*

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This paper derives the optimal lending contract in the financial accelerator model of Bernanke, Gertler and Gilchrist (1999), hereafter BGG. The optimal contract includes indexation to the aggregate return on capital, household consumption, and the return to internal funds. This triple indexation results in a dampening of fluctuations in leverage and the risk premium. Hence, compared with the contract originally imposed by BGG, the privately optimal contract implies essentially no financial accelerator.

Key words: Agency costs, CGE models, optimal contracting.

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1. Introduction.

The financial accelerator model of Bernanke, Gertler, and Gilchrist (1999), hereafter BGG, is widely used as a convenient mechanism for integrating financial factors into an otherwise standard DSGE model. The BGG model embeds the costly state verification (CSV) model of Townsend (1979) into an environment with risk neutral entrepreneurs, risk-averse households, and aggregate risk. Appealing to insurance concerns and the apparent lack of state contingency in the world, BGG assume that the lending contract between the entrepreneur and lender is characterized by a lender return that is invariant to innovations in aggregate variables. These aggregate innovations then feed directly into entrepreneurial net worth which is crucial in the BGG model because agency costs are diminished by increases in net worth. For example, since the lender's return is fixed, a positive productivity shock shifts wealth to entrepreneurs and thus lowers agency costs. This sets in motion a contemporaneous financial accelerator, a virtuous circle in which higher net worth drives up the price of capital, which in turn increases net worth, etc. This process thus amplifies the effect of the shock. But the vital first step in this amplification is the assumption that the lender's return is pre-determined. The importance of this assumption is well known. For example, consider the following comment from Chari (2003) on BGG:

> *These authors have an economy with risk neutral agents called entrepreneurs and risk averse agents called households. They claim that an optimal contract in the presence of aggregate risk has the return paid by entrepreneurs to be a constant, independent of the current aggregate shock. I have trouble understanding this result. Surely, entrepreneurs should and would provide insurance to households against aggregate shocks. One way of providing such insurance is to provide a high return to households when their income from other sources is low and a low return when their income from other sources is high. My own guess is that if they allowed the return to households to be state contingent, then aggregate shocks would have no effects on the decisions of households and would be absorbed entirely by entrepreneurs. Before we push this intriguing financial accelerator mechanism much further, I think it would be wise to make sure that we get the microeconomics right.*

In this paper, we take up the task suggested by Chari (2003) and attempt to get "the microeconomics right". In particular, we solve for the privately optimal contract (POC) in the original BGG framework. We then contrast this POC with the contract assumed in BGG, and analyze the implications for macroeconomics fluctuations.

Our principle results include the following. First, as anticipated by Chari (2003), the financial contract imposed in the BGG model is not privately optimal. That is, lenders and borrowers would both prefer a different loan contract. Second, the POC has the loan repayment vary in response to innovations in the observed aggregate shocks, but not quite in the manner anticipated by Chari (2003). In particular, the POC has debt repayment linked to innovations in three observables: the return on capital, the level of household consumption, and the entrepreneur's valuation of net worth.[1] The consumption insurance portion of the POC was anticipated by Chari (2003). But the POC also provides a hedge to entrepreneurs who prefer greater net worth when the return to internal funds is high. Taken together, we show that the POC is a state-contingent debt contract that dampens fluctuations in leverage and the risk premium.

The mechanism is as follows. Consider a positive aggregate shock to the return on capital due to higher productivity. In BGG, this shock leads to a large increase in the entrepreneur's net worth because the lender's return is predetermined. Ceteris paribus, higher net worth lowers leverage and sets in motion the financial accelerator. Under the POC, the promised repayment and thus the lender's return on the loan are positively indexed to these innovations, such that the innovation to productivity is shared by the lender and the entrepreneur. This more modest movement in net worth largely eliminates the financial accelerator.

Quantitatively, our results are important. In our benchmark calibration the conditional welfare cost of BGG compared with the POC is a 0.023% perpetual increase in the annual flow of household consumption. In terms of amplification, in an economy with sticky prices the response of output on impact to monetary policy shocks is more than 100% larger with BGG contracts than with the POC. For TFP shocks, the difference is about 60% larger on impact. In both cases, leverage and the risk-premium are nearly constant under the POC such that the economy mimics its frictionless counterpart (apart from a

[1] To develop intuition, we focus on how the debt contract is indexed to these three distinct observables. But this is a partial equilibrium argument: In general equilibrium, all of these observables are functions of the underlying states such that one cannot so easily separate the indexation into three distinct components. (The entrepreneur's valuation of internal funds is observable as we show below that it is the discounted sum of future leverage and the risk-free rate.)

steady state distortion).

Our results on the POC are related to Krishnamurthy (2003). Krishnamurthy (2003) introduces insurance markets into a three period model where borrowing is secured by collateral as in Kiyotaki and Moore (1997). These insurance markets allow for state contingent debt that is indexed to aggregate shocks. Krishnamurthy (2003) proves that such insurance eliminates any feedback from collateral values to investment, and thus reduces collateral amplification to zero. Similarly, Di Tella (2013) shows that allowing for contracts that condition on the aggregate state completely eliminates the financial amplification resulting from productivity shocks in the infinite horizon model of Brunnermeier and Sannikov (2014 forthcoming). We have a similar result here: the POC has the level of debt repayment indexed one-for-one to the aggregate return on capital, so bankruptcy rates do not respond to innovations in the return to capital. By itself this indexation dramatically reduces the financial accelerator.

An interesting question is how risk-aversion on the part of entrepreneurs would change our results. We cannot satisfactorily answer this question since the optimality of the debt contract in the underlying CSV framework relies on the assumption that the entrepreneur's payoff is linear. Mookherjee and Png (1989) and Winton (1994) have shown that the optimal contract with a risk averse principal and risk averse agent is generally not simple debt. But if we simply assume a debt contract with risk-averse entrepreneurs, we conjecture that the POC will still have the level of debt repayment indexed one-for-one to the aggregate return on capital, but depending on the level of risk-aversion the consumption insurance that entrepreneurs provide will be mitigated (or even eliminated). In any event, we leave these questions for future work.

The paper proceeds as follows. The next section outlines the competitive equilibrium of the model. Section 3 contrasts the contract indexation to BGG. The quantitative analysis, including welfare implications, are carried out in Section 4. Section 5 provides some sensitivity analysis on the financial accelerator by examining the privately optimal contract and the BGG contract in a model with sticky prices and more exogenous shocks. Concluding comments are provided in Section 6.

2. The Model.

Households.

The typical household consumes the final good C_t and sells labor input L_t to the firm at real wage w_t. Preferences are given by

$$U(C_t, L_t) \equiv \frac{C_t^{1-\sigma}}{1-\sigma} - B\frac{L_t^{1+\eta}}{1+\eta}.$$

The household budget constraint is given by

$$C_t + D_t \leq w_t L_t + R_t^d D_{t-1} + \Pi_t$$

The household chooses the level of deposits D_t which are then used by the lender to fund the entrepreneurs (more details below). As developed below, the lender's return on its portfolio of loans is realized at time-t, and this return is passed on one-for-one to the depositors. Hence, the gross real return on time *t-1* deposits is realized at time-t (R_t^d) and is not a preset rate. We thus allow (R_t^d) to be conditional on aggregate shocks. The household owns shares in the final goods firms, capital-producing firms, and the lender. Only the capital-producing firms will generate profits (Π_t) in equilibrium. The household's optimization conditions are given by:

$$-U_L(t)/U_c(t) = w_t \tag{1}$$

$$1 = E_t M_{t+1} R_{t+1}^d \tag{2}$$

where $M_{t+1} \equiv \beta \frac{U\prime(c_{t+1})}{U\prime(c_t)}$, which is the pricing kernel.

Final goods firms.

Final goods are produced by competitive firms who hire labor and rent capital in competitive factor markets at real wage w_t and rental rate r_t. The production function is Cobb-Douglass where A_t is the random level of total factor productivity:

$$Y_t = A_t\left(K_t^f\right)^{\alpha}(L_t)^{1-\alpha} \tag{3}$$

The realization of total factor productivity is publicly observed at the beginning of time-t. The variable K_t^f denotes the amount of capital available for time-t production. This is different than the amount of capital at the end of the previous period as some is lost because of monitoring costs. The optimization conditions are:

$$mpl_t = w_t \tag{4}$$

$$mpk_t = r_t \tag{5}$$

where mpl_t and mpk_t denote the marginal products of labor and capital, respectively.

New Capital Producers.

The production of new capital is subject to adjustment costs. In particular, investment firms take $I_t\vartheta(I_t)$ consumption goods and transform them into I_t investment goods that are sold at price Q_t. Their profits are thus given by $Q_tI_t - I_t\vartheta(I_t)$, where the function ϑ is convex. We find it convenient to normalize $\vartheta(I_{ss}) = 1$, $\vartheta'(I_{ss}) = 0$ and $\vartheta''(I_{ss}) = \psi$, where I_{ss} is the steady-state level of investment. Variations in investment lead to variations in the price of capital, which is the key to the financial accelerator mechanism.

Lenders.

The representative lender accepts deposits from households and provides loans to the continuum of entrepreneurs. These loans are intertemporal, with the loans made at the end of time t being paid back in time t+1. The gross real return on these loans is denoted by R_{t+1}^L. Each individual loan is subject to idiosyncratic and aggregate risk, but since the lender holds an entire portfolio of loans only aggregate risk remains. The lender has no other source of funds, so the level of loans will equal the level of deposits. Hence, dividends are given by, $Div_{t+1} = R^l_{t+1}D_t - R^d_{t+1}D_t$. The lending market is competitive so the lender takes the rates of return as given. We assume the lender is thus a passive pass-through entity, whose primary function is to hold a diversified portfolio of loans across the many entrepreneurs. Thus in equilibrium $R^l_t = R^d_t, \forall\, t$, and the lender earns zero profits.

<u>Entrepreneurs and the Loan Contract.</u>

There is a continuum of entrepreneurs indexed by i with preferences that are linear in consumption. The entrepreneurs discount the future at rate β, and are the sole accumulators of physical capital. As in BGG we assume this capital must be liquidated and all capital repurchased each period. At the end of period t, the each entrepreneur thus sells all of his accumulated capital in a capital market populated with the continuum of entrepreneurs. BGG make this assumption to ensure that agency problems affect the entire capital stock, and not just marginal investment. The market price of capital is Q_t. The sale of capital generates net worth NW^i_t for a typical entrepreneur, where the superscript *i* indexes the individual entrepreneur. All of this capital is then re-purchased, along with any net additions to the capital stock, by the collection of entrepreneurs. The time *t* purchase of capital by entrepreneur *i* is given by K^i_{t+1}. This purchase is financed with entrepreneurial net worth (NW^i_t) and external financing from a lender. This external finance takes the form of an intertemporal loan with repayment occurring in time *t+1*.

The entrepreneur's ability to repay the loan will be dependent upon the intertemporal return to capital. This return is a product of two factors, the aggregate return to capital and the idiosyncratic return of each entrepreneur. The aggregate return to capital (R^k_{t+1}) is publicly observed and is given by:

$$R^k_{t+1} \equiv \frac{r_{t+1}+(1-\delta)Q_{t+1}}{Q_t}. \tag{6}$$

where r_{t+1} is the rental rate and δ is the capital depreciation rate. That is, a unit of capital costs Q_t at the end of time *t*, while a unit of capital generates rental rate r_{t+1} and re-sale value $(1-\delta)Q_{t+1}$ in period *t+1*. As for the idiosyncratic return, one unit of capital purchased at the end of time-t is transformed into ω_{t+1}^i units of capital in time t+1, where ω_{t+1}^i is an idiosyncratic random variable with density $\phi(\omega)$, cumulative distribution $\Phi(\omega)$, and a mean of one. We assume that ω_{t+1}^i is uncorrelated with R_{t+1}^k. The total return on the capital project is thus a product of two independent random variables, $\omega_{t+1}^i R_{t+1}^k$. In summary, after purchasing a unit of capital at price Q_t at the end of time *t*, entrepreneur *i* earns $\omega_{t+1}^i[r_{t+1}+(1-\delta)Q_{t+1}]$ in period *t+1*.

In contrast to the aggregate return to capital, the realization of ω_{t+1}^i is directly observed only by entrepreneur *i*. The lender can observe the realization only if a monitoring cost is paid, a cost that destroys part of the capital produced by the project. We assume that this monitoring cost is linear in the project outcome, $\mu\omega_{t+1}^i R_{t+1}^k K_{t+1}^i$, where K_{t+1}^i denotes the time-t capital purchase of entrepreneur *i*. The assumption of linearity allows for aggregation, but is non-standard in the optimal contracting literature.[2]

Under the assumption that payoffs are linear in the project outcome, Townsend (1979) demonstrates that the optimal contract between the entrepreneur and lender is risky debt in which monitoring only occurs if the promised payoff is not forthcoming.[3] In the present context we proceed as follows. The appendix demonstrates that if the entrepreneur's value function is linear, then the debt contract is optimal. To complete the logical circle, we will show that if risky debt is the optimal contract, then the entrepreneur's value function is linear.

[2] We follow BGG and Carlstrom and Fuerst (1997), by assuming that there is enough inter-period anonymity such that today's contract cannot be based on previous realizations of the idiosyncratic shock. As noted by these authors, this assumption vastly simplifies the analysis for otherwise the optimal contract would depend upon the entire history of each entrepreneur (see, for example Gertler (1992)).

[3] The risky debt result also assumes that the equilibrium contract must be in pure strategies, i.e., no random audits.

Under the assumption that risky debt is optimal, the loan contract is characterized by a reservation value of the idiosyncratic shock that separates repayment from default. Debt repayment does not occur (i.e., "bankruptcy") for sufficiently low values of the idiosyncratic shock, $\omega_{t+1}^{i} \le \varpi_{t+1}^{i}$. Note that ϖ_{t+1}^{i} is realized in time *t+1* and thus can be a function of R_{t+1}^{k}. Let Z_{t+1}^{i} denote the promised gross loan rate from the entrepreneur to the lender such that Z_{t+1}^{i} is defined by

$$Z_{t+1}^{i}(Q_t K_{t+1}^{i} - NW_t^{i}) \equiv \varpi_{t+1}^{i} R_{t+1}^{k} Q_t K_{t+1}^{i}. \tag{7}$$

The left-hand side of (7) denotes the promised level of repayment, with the term in parentheses being the size of the loan. The right-hand side defines the level of the idiosyncratic shock that makes the entrepreneur just able to repay. We find it convenient to express this in terms of the leverage ratio $\bar{\kappa}_t^{i} \equiv \left(\frac{Q_t K_{t+1}^{i}}{NW_t^{i}}\right)$ such that (7) becomes

$$Z_{t+1}^{i} \equiv \varpi_{t+1}^{i} R_{t+1}^{k} \frac{\bar{\kappa}_t^{i}}{\bar{\kappa}_t^{i}-1} \tag{8}$$

Let $f(\varpi_{t+1}^{i})$ and $g(\varpi_{t+1}^{i})$ denote the entrepreneur's share and lender's share of aggregate project income:

$$f(\varpi) \equiv \int_{\varpi}^{\infty} \omega\phi(\omega)d\omega - [1-\Phi(\varpi)]\varpi \tag{9}$$

$$g(\varpi) \equiv [1-\Phi(\varpi)]\varpi + (1-\mu)\int_{0}^{\varpi} \omega\phi(\omega)d\omega. \tag{10}$$

Entrepreneur i's net worth NW_t^{i} is leveraged into a project size of $Q_t K_{t+1}^{i} = \bar{\kappa}_t^{i} NW_t^{i}$. Conditional on the aggregate return on capital R_{t+1}^{k}, the expected entrepreneurial payoff and lender return are thus given by:

$$Entrepreneur\ payoff = R_{t+1}^{k} Q_t K_{t+1}^{i} f(\varpi_{t+1}^{i}) = R_{t+1}^{k} f(\varpi_{t+1}^{i}) \bar{\kappa}_t^{i} NW_t^{i} \tag{11}$$

$$Lender\ return = R_{t+1}^{l,i} = \frac{R_{t+1}^{k} g(\varpi_{t+1}^{i}) Q_t K_{t+1}^{i}}{(Q_t K_{t+1}^{i} - NW_t^{i})} = R_{t+1}^{k} g(\varpi_{t+1}^{i}) \frac{\bar{\kappa}_t^{i}}{\bar{\kappa}_t^{i}-1}. \tag{12}$$

We assume that the distribution of aggregate shocks has a bounded support. We further assume that these bounds are tight enough such that the entrepreneur's expected return to internal funds, $E_t R^k_{t+1} f\left(\varpi^i_{t+1}\right)\bar{\kappa}^i_t$, will always exceed 1/β. Consequently, each entrepreneur postpones consumption indefinitely.

To avoid self-financing in the long run, we assume that each entrepreneur faces probability (1-γ) of death each period. Entrepreneurs receive the news at the beginning of the period whether they will die at the end of the period. Dying entrepreneurs will thus choose to consume all of their net worth before exiting the economy. The dead are then replaced by an equal number of new entrepreneurs. New entrepreneurs each need a trivial amount of initial net worth to begin activity. We assume that this comes from a lump sum transfer from the existing entrepreneurs. But since this transfer can be arbitrarily small, and since only aggregate net worth matters in this setting, we neglect these transfers in what follows.

In summary, the entrepreneur sets $c^{e,i}_t = NW^i_t$, with probability (1-γ), and $NW^i_{t+1} = R^k_{t+1} f\left(\varpi^i_{t+1}\right)\bar{\kappa}^i_t NW^i_t$, with probability γ. Let V^i_t denote the value function of entrepreneur i before he receives the realization of his death shock. The Bellman equation is thus given by:

$$V^i_t = (1-\gamma)c^{e,i}_t + \beta\gamma max_{\bar{\kappa}^i_t,\varpi^i_{t+1}} E_t V^i_{t+1} \tag{13}$$

where the maximization is subject to the lender's participation constraint (equation (16) below). To prove that the value function is linear in net worth, we guess the form of the value function and then verify that this form satisfies the Bellman equation. Hence, we conjecture that the value function is given by $V^i_t \equiv V_t NW^i_t$, where V_t is constant across entrepreneurs. We will verify this conjecture below. Substituting in the consumption decision, $c^{e,i}_t = NW^i_t$, the law of motion for net worth, $NW^i_{t+1} = R^k_{t+1} f\left(\varpi^i_{t+1}\right)\bar{\kappa}^i_t NW^i_t$, and the conjectured value function, the Bellman equation is given by

$$V_t = (1-\gamma) + max_{\bar{\kappa}^i_t,\varpi^i_{t+1}} \beta\gamma\bar{\kappa}^i_t E_t V_{t+1} R^k_{t+1} f\left(\varpi^i_{t+1}\right) \tag{14}$$

We must show that $\varpi^i_{t+1} = \varpi_{t+1}$ and $\bar{\kappa}^i_t = \bar{\kappa}_t$ in order to verify our conjectured value function. On the other side of the contract we have the lender whose opportunity cost is linked to the return on deposits. We can thus write the end of time-t contracting problem as:

$$max_{\bar{\kappa}_t^i, \varpi_{t+1}^i} \bar{\kappa}_t^i \beta\gamma E_t V_{t+1} R_{t+1}^k f\left(\varpi_{t+1}^i\right) \tag{15}$$

subject to

$$E_t M_{t+1} R_{t+1}^k g\left(\varpi_{t+1}^i\right) \bar{\kappa}_t^i \geq (\bar{\kappa}_t^i - 1) \tag{16}$$

The lender's participation constraint (16) comes from combining the definition of the lender's return (12) and $R_{t+1}^l = R_{t+1}^d$. We then use (2) and take expectations. The first order conditions to this problem are given by:

$$\beta\gamma V_{t+1} f'\left(\varpi_{t+1}^i\right) + \Lambda_t^i M_{t+1} g'\left(\varpi_{t+1}^i\right) = 0 \tag{17}$$

$$\beta\gamma E_t V_{t+1} R_{t+1}^k f\left(\varpi_{t+1}^i\right) + \Lambda_t^i \left[E_t M_{t+1} R_{t+1}^k g\left(\varpi_{t+1}^i\right) - 1\right] = 0 \tag{18}$$

$$E_t R_{t+1}^k M_{t+1} g\left(\varpi_{t+1}^i\right) \frac{\bar{\kappa}_t^i}{\bar{\kappa}_t^i - 1} = 1 \tag{19}$$

where Λ_t^i denotes the multiplier on the constraint (16), and equations (17) and (18) reflect the optimal choice of ϖ_{t+1}^i and $\bar{\kappa}_t^i$, respectively. Recall that ϖ_{t+1}^i is a function of time *t+1* variables such that (17) holds state-by-state. Combining (17)-(18), we have that $\varpi_{t+1}^i = \varpi_{t+1}$, is constant across entrepreneurs. The constraint, expression (19) then implies that $\bar{\kappa}_t^i = \bar{\kappa}_t$, is also constant across entrepreneurs. Returning to the Bellman equation (14), if the optimal ϖ_{t+1}^i and $\bar{\kappa}_t^i$ are constant across entrepreneurs, then so is V_t. Hence, we have verified our conjecture that the value function is given by $V_t^i = V_t NW_t^i$, with a time-varying slope coefficient V_t that is constant across entrepreneurs.[4] Using (18) we can write the Bellman equation as

[4] In a related environment, Krishnamurthy (2003) demonstrates that although borrowers are risk-neutral in consumption, they may be risk-averse in net worth. This risk-aversion arises in Krishnamurthy (2003) because the borrower's production technology is concave, and the collateral constraint is not always binding. In the BGG model studied here, the production technology is linear and the need for external finance is a permanent feature of the CSV framework, so the entrepreneurs' payoff is linear in net worth. But as demonstrated here, the slope coefficient in the value function varies with the aggregate state.

$$V_t = (1-\gamma) + \Lambda_t \tag{20}$$

Hence, the entrepreneur's valuation of net worth is the weighted sum of the unit payoff of eating upon death, and the gain to holding on to net worth if the agent survives.

The linearity of the value function implies that we need only track aggregate net worth. That is, all entrepreneurs receive the same loan terms, with the size of the loan simply scaled to reflect each entrepreneur's net worth. Summing over all entrepreneurs we have

$$NW_{t+1} = \gamma R_{t+1}^{k} f(\varpi_{t+1})\bar{\kappa}_t NW_t. \tag{21}$$

We henceforth drop the entrepreneur index i.

Returning to (17), the **privately optimal contract (POC)** is thus described by the ϖ_{t+1} that satisfies:

$$\frac{\Lambda_t M_{t+1}}{\beta\gamma[1-\gamma+\Lambda_{t+1}]} = \frac{-f'(\varpi_{t+1})}{g'(\varpi_{t+1})} \equiv F(\varpi_{t+1}), \tag{22}$$

where the second order condition requires $F'(\varpi_{t+1}) > 0$. From the perspective of time *t*, the conditional mean behavior of ϖ_{t+1} is constrained by (18)-(19). But (22) indicates that the default cut-off is indexed to time *t+1* variables in a natural way. When C_{t+1} is low (M_{t+1} is high), the optimal ϖ_{t+1} (and thus the lender's return) increases as a form of consumption insurance to the household. Similarly, when the cost of external finance is high (Λ_{t+1} is high), the contract calls for a lower ϖ_{t+1} such that the entrepreneur holds on to more net worth.

Market Clearing and Equilibrium.

In equilibrium household deposits fund the entrepreneurs' projects, $D_t = Q_t K_{t+1} - NW_t$. Net of monitoring costs, the amount of capital available for production is given by $K_t^f = \Upsilon(\varpi_t)K_t$, where $\Upsilon(\varpi_t) \equiv f(\varpi_t) + g(\varpi_t) = 1 - \mu\int_0^{\varpi_t} x\phi(x)dx$. As noted earlier, the deposit rate is tied to the return on loans, such that $R_t^l = R_t^d$. The competitive equilibrium is defined by the variables $\{C_t, L_t, I_t, K_{t+1}, \varpi_t, \Lambda_t, \bar{\kappa}_t, \mathrm{C}_t^e, Q_t, R_t^l\}$ that satisfy

$$R_t^l = R_t^k g(\varpi_t) \frac{\bar{\kappa}_{t-1}}{(\bar{\kappa}_{t-1} - 1)} \tag{23}$$

$$-U_L(t)/U_c(t) = mpl_t \tag{24}$$

$$E_t M_{t+1} R_{t+1}^k g(\varpi_{t+1}) \frac{\bar{\kappa}_t}{(\bar{\kappa}_t - 1)} = 1 \tag{25}$$

$$\frac{\Lambda_{t-1} M_t}{\beta\gamma[1-\gamma+\Lambda_t]} = F(\varpi_t) \tag{26}$$

$$\Lambda_t = \beta\gamma E_t[(1-\gamma) + \Lambda_{t+1}] R_{t+1}^k f(\varpi_{t+1}) \bar{\kappa}_t \tag{27}$$

$$Q_t K_{t+1} = \gamma[Q_t(1-\delta) + mpk_t] f(\varpi_t) K_t \bar{\kappa}_t \tag{28}$$

$$K_{t+1} = (1-\delta)\Upsilon(\varpi_t) K_t + I_t \tag{29}$$

$$C_t + I_t \vartheta\left(\frac{I_t}{I_{ss}}\right) + \mathrm{C}_t^e \leq A_t(\Upsilon(\varpi_t) K_t)^\alpha (L_t)^{1-\alpha} \tag{30}$$

$$\mathrm{C}_t^e = (1-\gamma)[Q_t(1-\delta) + mpk_t] f(\varpi_t) K_t \bar{\kappa}_t \tag{31}$$

$$Q_t = \vartheta(I_t) + I_t \vartheta'(I_t) \tag{32}$$

where we have used $M_{t+1} \equiv \beta \frac{U\prime(c_{t+1})}{U\prime(c_t)}$, $\bar{\kappa}_t \equiv \left(\frac{Q_t K_{t+1}}{NW_t}\right)$, $F(\varpi_t) \equiv \frac{-f'(\varpi_t)}{g'(\varpi_t)}$, and $R_{t+1}^k \equiv \frac{mpk_{t+1} + (1-\delta) Q_{t+1}}{Q_t}$. The marginal products are defined as: $mpl_t \equiv (1-\alpha) Y_t / L_t$, and $mpk_t \equiv a Y_t / (\Upsilon(\varpi_t) K_t)$, with $Y_t \equiv A_t(\Upsilon(\varpi_t) K_t)^\alpha (L_t)^{1-\alpha}$.

Many of the expressions in (23)-(32) are familiar including labor choice (24), investment adjustment costs (32), and the resource constraints (29)-(30). The differences between this agency cost model and a standard RBC model can be seen in a few equations. Equations (23) and (25)-(27) characterize the solution to the contracting problem, and (28) and (31) define entrepreneurial consumption and net worth accumulation. The key agency cost distortion is given by $g(\varpi_{t+1})\frac{\bar{\kappa}_t}{(\bar{\kappa}_t-1)}$. This underlying friction is explicit in (25) as the investment choice is distorted by the term. This suggests that there may be welfare gains to minimizing fluctuations in this distortion. As we will see below, the POC will vary repayment rates in such a way as to dampen fluctuations in this distortion.

3. Comparing the POC to BGG.

Because of BGG's prominence in the literature, it is particularly useful to compare the POC contract and the contract imposed by BGG. The key assumption in the BGG model is that the lender's return is pre-determined. This is not an implication of the modeling framework, but is instead an assumption imposed on the model. As BGG write, "Since entrepreneurs are risk neutral, we *assume* that they bear all the aggregate risk associated with the contract" (BGG, page 1385, emphasis added). There are two problems with this assumption. First, the household's risk is linked to the marginal utility of consumption, not to the return on capital. Second, the entrepreneur's valuation of net worth is not constant but depends upon the aggregate state. Formally, the original BGG contracting problem is given by

$$max_{\bar{\kappa}_t,\varpi_{t+1}} E_t R^k_{t+1}\bar{\kappa}_t f(\varpi_{t+1}) \tag{33}$$

subject to

$$R^k_{t+1} g(\varpi_{t+1})\bar{\kappa}_t \geq \frac{(\bar{\kappa}_t-1)}{E_t M_{t+1}} \tag{34}$$

where the lender's return R^l_t is pre-determined and satisfies $E_t M_{t+1} R^l_t = 1$. The optimization conditions are given by:

$$f'(\varpi_{t+1}) + \Lambda_{t+1} g'(\varpi_{t+1}) = 0 \tag{35}$$

$$E_t R_{t+1}^k f(\varpi_{t+1}) + E_t \Lambda_{t+1}\left[R_{t+1}^k g(\varpi_{t+1}) - \frac{1}{E_t M_{t+1}}\right] = 0 \tag{36}$$

$$R_{t+1}^k g(\varpi_{t+1})\bar{\kappa}_t = \frac{(\bar{\kappa}_t - 1)}{E_t M_{t+1}} \tag{37}$$

As with the POC, ϖ_{t+1} is a function of time *t+1* information. Compared to POC, the key expression is (37). Under the BGG contract, the default cut-off ϖ_{t+1} will move oppositely with innovations in R_{t+1}^k to ensure that the lender's return is pre-determined. This is quite different from the behavior of ϖ_{t+1} under the POC in which the cut-off varies with innovations in the household's and entrepreneur's valuation of funds.

The differences in BGG vs. POC are particularly transparent in log-linear form, so we look at the linearized versions of POC and BGG. We begin with the POC. The appendix shows that we can express the POC in log-linear form as follows:

$$\widetilde{\omega}_t^{POC} = \left(\frac{1-\nu(\kappa-1)}{\Theta_g(\kappa-1)}\right)\kappa_{t-1} + \frac{1}{\Psi}(m_t - E_{t-1}m_t) - \frac{\beta}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \tag{38}$$

$$z_t^{POC} = E_{t-1}r_t^{l,POC} + \frac{(1-\Theta_g)[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\kappa_{t-1} + (r_t^k - E_{t-1}r_t^k) + \frac{1}{\Psi}(m_t - E_{t-1}m_t) - \frac{\beta}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \tag{39}$$

$$r_t^{l,POC} = E_{t-1}r_t^{l,POC} + \left(r_t^k - E_{t-1}r_t^k\right) + \frac{\Theta_g}{\Psi}(m_t - E_{t-1}m_t) - \frac{\beta\Theta_g}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \tag{40}$$

$$\lambda_t = E_t \sum_{j=0}^{\infty} \beta^j \left(\kappa\nu\kappa_{t+j} + r_{t+j+1}^{l,POC}\right) \tag{41}$$

$$E_t\left(r_{t+1}^{l,POC} + m_{t+1}\right) = 0, \tag{42}$$

where $\kappa \equiv Q_{ss}K_{SS}/NW_{ss}$, $\Psi \equiv \frac{\varpi_{ss}F'(\varpi_{ss})}{F(\varpi_{ss})} > 0$, $\Theta_g \equiv \frac{\varpi_{ss}g'(\varpi_{ss})}{g(\varpi_{ss})}$, $0 < \Theta_g < 1$, $\Theta_f \equiv \frac{\varpi_{ss}f'(\varpi_{ss})}{f(\varpi_{ss})} < 0$, and $\nu \equiv \left[\frac{\Psi}{(\kappa-1)\Psi - \kappa\theta_f}\right]$. The lower case letters denote log deviations of the corresponding endogenous variables, and κ_t and $\widetilde{\omega}_t$ denotes the log deviations of $\bar{\kappa}_t$ and ϖ_t, respectively.

The POC has three key characteristics that are transparent from a partial equilibrium perspective. We will thus discuss separately the three innovations in (39)-(40), although in general equilibrium all of these observables are functions of the underlying state. First, the promised repayment and lender's return are scaled one-for-one by innovations in r_t^k such that the default cut-off is sterilized from these innovations. That is, the coefficient on r_t^k innovations in (39)-(40) is unity, while the r_t^k innovation is absent from (38). This is quite natural. There are two sources of uncertainty within the underlying CSV problem: unobserved idiosyncratic shocks, and the observed aggregate return on capital. Bankruptcy and costly monitoring are part of the optimal debt contract as the mechanism to ensure truthful revelation of the idiosyncratic shock. But there is no need for such a deterrent for observed aggregate shocks. A second key feature of the POC is that it provides consumption insurance to the household in that the lender's return (40) is increasing when the marginal utility of consumption is unexpectedly high (m_t is high). The higher lender return is then passed on to the household via increases in the deposit rate. Third, the POC provides for a hedge to the entrepreneur in that when the return to internal funds is high (λ_t is high), the repayment to the lender (39) and (40) declines such that the entrepreneur can build up net worth.

The appendix also demonstrates how to express the BGG contract in log-linear form:

$$\widetilde{\omega}_t^{BGG} = \frac{[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\kappa_{t-1} - \frac{1}{\Theta_g}\left(r_t^k - E_{t-1}r_t^k\right) \tag{43}$$

$$z_t^{BGG} = r_{t-1}^{l,BGG} + \frac{(1-\Theta_g)[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\kappa_{t-1} + \left(\frac{\Theta_g-1}{\Theta_g}\right)\left(r_t^k - E_{t-1}r_t^k\right) \tag{44}$$

$$r_{t-1}^{l,BGG} = E_{t-1}r_t^k - \nu\kappa_{t-1} \tag{45}$$

$$r_t^{l,BGG} + E_t(m_{t+1}) = 0, \tag{46}$$

Unlike the POC, the BGG default cut-off (43) will depend directly upon innovations in r_t^k. For the calibration used below, $\Theta_g \approx 1$, such that the BGG default rates fall sharply with innovations in r_t^k. This then implies that the promised repayment (44) declines modestly with innovations in r_t^k. But the key expression is (45): the lender's return is pre-determined. This implies that innovations in r_t^k are entirely absorbed by entrepreneurial net worth, and that the contract is missing the household and entrepreneurial hedging motives of the POC.

Although BGG and POC differ only by innovations, the inertial dynamics of net worth imply that these differences will have persistent consequences. Linearizing the behavior of aggregate net worth and using the two contracts we have

$$nw_t^{POC} = nw_{t-1}^{POC} + E_{t-1}r_t^{l,POC} + \kappa\nu\kappa_{t-1} + \left(r_t^k - E_{t-1}r_t^k\right) + \frac{\Theta_f}{\Psi}(m_t - \mathrm{E}_{t-1}m_t) - \frac{\beta\Theta_f}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \quad (47)$$

$$nw_t^{BGG} = nw_{t-1}^{BGG} + r_{t-1}^{l,BGG} + \kappa\nu\kappa_{t-1} + \kappa\left(r_t^k - E_{t-1}r_t^k\right). \quad (48)$$

As with the lender's return, the differences in the two contracts differ by the response of net worth to innovations in three aggregate variables: the return on capital, the household's marginal utility of consumption, and the entrepreneur's valuation of net worth. Here we will focus on the magnitude of these effects. Under our baseline calibration, $\kappa = 2$, such that the BGG contract responds to r_t^k by twice as much as POC. As for the other innovations, the POC provides insurance both to the household (consumption insurance) and the entrepreneur (financing insurance) in a symmetric fashion. These insurance effects are non-trivial: our baseline calibration implies $\frac{\Theta_f}{\Psi} = -2.17$.

The difference between the POC and BGG are most clearly evident for the case of a net worth shock. That is, suppose we append (47)-(48) with an innovation in entrepreneurial net worth that is realized at the beginning of the period (actualized as a lump sum transfer from household income to entrepreneurs). There are three fundamental effects coming from such a shock. First, an innovation in net worth leads to an increase in the price of capital (because entrepreneurs are the purchasers of capital) and thus an innovation in r_t^k. Second, the increase in net worth ameliorates the agency distortion such that investment spending increases and consumption declines, i.e., a positive innovation in m_t. Third, the higher level of net worth lowers the marginal valuation of net worth, i.e., a negative innovation in λ_t. Under the POC, the latter two effects work against the r_t^k innovation such that there is essentially no movement in net worth (we will see this quantitatively below). But under BGG, the latter two effects are entirely absent, and the r_t^k effect is magnified because of leverage. In summary, under POC an innovation in net worth is largely sterilized by movements in the required repayment rate, while under BGG, the net worth innovation sets in motion a contemporaneous financial accelerator that magnifies this initial shock.

The POC thus contains two insurance or hedging motives. Which of these two POC insurance effects dominates? The two insurance terms in (39)-(40) can be combined, by noting that in general equilibrium innovations in the household's pricing kernel are linked to innovations in the deposit rate. The household's pricing kernel implies

$$(m_t - E_{t-1}m_t) = \sigma(c_t - E_{t-1}c_t) = -\left(E_t \sum_{j=0}^{\infty} r_{t+j+1}^{l,POC} - E_{t-1} \sum_{j=0}^{\infty} r_{t+j+1}^{l,POC}\right) \qquad (49)$$

Similarly, (41) implies that

$$(\lambda_t - E_{t-1}\lambda_t) = E_t \sum_{j=0}^{\infty} \beta^j\left(\kappa\nu\kappa_{t+j} + r_{t+j+1}^{l,POC}\right) - E_{t-1} \sum_{j=0}^{\infty} \beta^j\left(\kappa\nu\kappa_{t+j} + r_{t+j+1}^{l,POC}\right) \qquad (50)$$

Assuming β = 1, we can substitute (49)-(50) into (39)-(40) to yield:

$$z_t^{POC} = E_{t-1}r_t^{l,POC} + \frac{(1-\Theta_g)[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\kappa_{t-1} + (r_t^k - E_{t-1}r_t^k) - \frac{\kappa\nu}{\Psi}\left(E_t \sum_{j=0}^{\infty} \kappa_{t+j} - E_{t-1} \sum_{j=0}^{\infty} \kappa_{t+j}\right) \qquad (51)$$

$$r_t^{l,POC} = E_{t-1}r_t^{l,POC} + \left(r_t^k - E_{t-1}r_t^k\right) - \frac{\kappa\nu\Theta_g}{\Psi}\left(E_t \sum_{j=0}^{\infty} \kappa_{t+j} - E_{t-1} \sum_{j=0}^{\infty} \kappa_{t+j}\right) \qquad (52)$$

Thus, in general equilibrium the combined effect of these two insurance motives results in one strategy: dampen fluctuations in leverage. That is, when leverage is unexpectedly high, the repayment levels decline such that net worth rises and leverage is pulled back down.

Recall that the distortion in capital accumulation in (25) is given by $g(\varpi_{t+1})\frac{\bar{\kappa}_t}{(\bar{\kappa}_t - 1)}$. Up to a log-linear approximation this spread between the return on capital and the lending rate is proportional to leverage:

$$E_t\left(r_{t+1}^k - r_{t+1}^l\right) = \nu\kappa_t. \tag{53}$$

(See the appendix for details.) Similarly, the *ex ante* risk premium in the model is given by $E_t(z_{t+1}^{POC} - r_{t+1}^l)$ which from (51) is also proportional to leverage. Hence, by dampening movements in leverage, the POC dampens movements in the investment distortion and in the risk premium. In sharp contrast, the BGG lender repayment is pre-determined such that movements in leverage, the investment distortion, and the risk premium are all magnified.

4. Quantitative Analysis.

We investigate the competitive equilibrium under POC and BGG. Our benchmark calibration largely follows BGG (1999). The discount factor β is set to 0.99. Utility is assumed to be logarithmic in consumption ($\sigma = 1$), and the elasticity of labor is assumed to be 3 ($\eta = 1/3$). We choose the constant B to normalize steady-state labor to unity. The production function parameters include $\alpha = 0.35$, investment adjustment costs $\psi = 0.50$, and quarterly depreciation is $\delta = .025$. We parameterize the adjustment cost such that in the steady state the price of capital is equal to unity. As for the credit-related parameters, we calibrate the model to be consistent with: (i) a steady state spread between Z and R^d of 200 bp (annualized), (ii) a quarterly bankruptcy rate of .75%, and (iii) a leverage ratio of $\kappa = 2$. These values imply a survival rate of $\gamma = 0.94$, a standard deviation of the idiosyncratic productivity shock of 0.28, and a monitoring cost of μ = 0.63. In the linearized model (see appendix), this then implies an agency cost elasticity of ν = 0.188.[5]

Our baseline analysis assumes that total factor productivity follows an AR(1) process with ρ^A = 0.95. But first to develop intuition, Figure 1 presents impulse response functions for the case of a 1% iid TFP shock, ρ^A = 0. Note that the POC responds to this iid shock in something of an iid fashion. That is, there is very little persistence. In particular, leverage (and thus the risk premium) is essentially unchanged. This arises because of the POC's indexed repayment. The TFP shock leads to a positive innovation in r_t^k, a negative innovation in m_t (as consumption rises), and a negative innovation in λ_t (because the movement in net worth is larger than the movement in desired capital). Quantitatively, these latter two insurance motives largely cancel as leverage hardly moves under the POC, and the lender's return responds one-to-one to innovations in r_t^k. Matters could not be more different with BGG. Because the lender's rate is pre-determined, the TFP shock leads to a sharp increase in net worth and a symmetric decline in leverage. This net worth expansion leads to an amplification (relative to the POC) of output and investment. These effects diminish only slowly as entrepreneurial net worth returns to normal levels.

Figure 2 looks at the case of an auto-correlated TFP shock. Compared to the POC, BGG shifts net worth and thus consumption towards the entrepreneur. The persistent movement in net worth leads to a persistent decline in leverage and the risk premium and hence an amplification of investment and output. But under the POC, the lender repayment moves on impact to largely eliminate all movements in leverage and the risk premium. Hence, the POC dynamics will be roughly identical to a frictionless model (in which leverage and the risk premium are unchanged by the shock).

Table 1 provides a welfare analysis of the POC and BGG under the baseline calibration and sensitivity analysis over the persistence in the shock process and the degree of investment adjustment costs. The standard deviation of TFP is set to 0.01. Under the POC, this generates a standard deviation

[5] BGG (1999) calibrated $(R^k - R^d)$ to 200 bp. But the model's risk premium is $(Z - R^d)$, so we calibrate this spread to 200 bp. Our calibration leads to a larger BGG financial accelerator in that the BGG calibration implies ν = 0.04, while our calibration has ν = 0.188.

of household consumption equal to 0.037 (and a SD of 0.037 for aggregate consumption). This standard deviation of household consumption increases to 0.038 under BGG (and a SD of 0.041 for aggregate consumption).

The welfare measures we report are computed based on a second-order approximation to the nonlinear equilibrium conditions of the model and the two contracts. Our preferred welfare measure is the household and entrepreneur's value function where the state variables are evaluated at the deterministic steady state (which is the same across contracts). Thus, we employ a conditional welfare measure. We follow closely the computational strategy outlined in Schmitt-Grohe and Uribe (2005).

For the baseline calibration the welfare gain is fairly small, a perpetual annual flow of 0.027% of aggregate consumption. But as point of comparison, Lucas (1987) estimates that the welfare cost of consumption variability is a consumption flow of 0.068% (assuming consumption variability of 0.037). Hence, the welfare cost of the BGG contract (compared to POC) is of the same order of magnitude as the welfare cost of business cycle variability. The problem with the BGG contract is that employment and investment over-respond to TFP shocks because of the financial accelerator. This implies negative welfare consequences for both households and entrepreneurs. That is, the POC is a Pareto improvement over the BGG contract.

The welfare losses of the BGG contract double for the case of iid shocks. The efficient response to an iid shock is a temporary increase in employment and investment. But under BGG the initial boom in net worth leads to a protracted increase in output. An increase in investment adjustment costs implies an increase in the financial accelerator that is operative under the BGG contract. Hence, the welfare cost of BGG is also larger for the higher level of adjustment costs.

5. Sensitivity of the financial accelerator to other shocks.

As a form of sensitivity analysis on the positive aspects of the model, we investigate adding sticky prices and other exogenous shocks to the analysis. The focus is on the financial accelerator and how it is affected by the two alternative loan contracts, BGG and POC. We integrate sticky prices via the familiar Dynamic New Keynesian (DNK) methodology. Note that there is no nominal stickiness between the lender and entrepreneur such that the POC is unchanged. The DNK model is standard so we dispense with the formal derivation, see, for example, Woodford (2003) for details. Imperfect competition distorts factor prices such that the marginal productivity of capital and labor in (4) and (5) are pre-multiplied by marginal cost. Marginal cost in turn is affected by the path of inflation such that in log deviations we have a relationship between inflation (π_t) and marginal cost (ζ_t):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \zeta_t \tag{54}$$

The model is then closed with the familiar Fisher equation linking real and nominal interest rates:

$$i_t = E_t \pi_{t+1} + \sigma(E_t c_{t+1} - c_t) \tag{55}$$

and an interest rate policy for the central bank. In log deviations the policy rule is given by:

$$i_t = \phi_\pi \pi_t + \varepsilon_t^m. \tag{56}$$

where ε_t^m is an exogenous policy movement with autocorrelation $\rho^m = 0.50$. We also consider a shock to the financial market. In particular, we augment the evolution of net worth with an exogenous and persistent change in net worth that takes the form of a lump sum transfer from households to entrepreneurs that occurs at the beginning of time-t. By altering the path of net worth, this shock will alter leverage and risk premia. We assume that this net worth shock is persistent with $\rho^{NW} = 0.70$. All three of the aggregate shocks (TFP, monetary policy, and net worth) are observed at the beginning of the period. The DNK parameter calibration is standard with $\phi_\pi = 1.5$ and $\kappa = 0.025$.

Figures 5-7 report the impulse response functions to a 1% TFP shock ($\rho^A = 0.95$), a 25 bp (quarterly) monetary policy shock, and a 1% net worth shock. The key to understanding all three experiments is to focus on the behavior of net worth. The indexation under the POC leads to sharp changes in the lender's return such that the change in net worth is quite modest when compared to BGG. And as noted earlier, the persistence in net worth implies that the initial change in net worth drives all subsequent dynamics. As suggested earlier, the poster boy for this mechanism is the net worth shock. In the BGG model, a 1% exogenous shock to net worth leads to a 10% movement in net worth (recall that leverage = 2) because the financial accelerator kicks in: higher net worth boosts the price of capital, the higher price of capital boosts net worth, etc. This implies a sharp decline in leverage and the risk premium. But these effects are entirely absent in POC. In fact, net worth actually declines slightly on impact such that leverage (and the risk premium) increases. This initial decline occurs because of the large endogenous response of the repayment rate to the exogenous movement in net worth. Since the exogenous net worth transfer is persistent, but the contract only responds to innovations, the POC has the repayment rate over-compensate for the net worth transfer so that the average leverage deviations are minimized. The POC thus involves over-shooting of leverage and the risk premium as it transitions back to the steady state. But in comparison to BGG, these leverage movements are trivial.

In summary, the BGG model delivers sharp amplification of all three shocks, but only because the non-optimality of the BGG contract leads to sharp movements in net worth, leverage and the risk premium. This amplification is even manifested in the comparison of the TFP response in the flexible price model (figure 2) vs. the sticky price model (figure 5). With the BGG contract and sticky prices, the innovation in net worth leads to such a stimulus to investment that the TFP "supply" shock is augmented with "demand" characteristics such that marginal cost rises on impact.[6] This endogenous decline in mark-ups in the DNK model leads to a larger increase in the rental rate and thus the return on capital. Hence, net worth moves by more with sticky prices than it does with flexible prices. This net worth movement leads to a further amplification of real activity in the sticky price model. In contrast, under the POC, the net worth movement in both flexible and sticky prices is quite comparable such that the real

[6] The behavior of marginal cost can be inferred from the time path of inflation and the Phillips curve (54).

response to a TFP shock is modestly dampened by the addition of sticky prices as marginal cost falls and mark-ups rise. This is the typical effect of sticky prices in DNK models.

6. Conclusion.

Two basic functions of financial markets are to intermediate between borrowers and lenders, and to provide a mechanism to hedge risk. Both of these motivations are present here. The risky debt contract is the optimal method of intermediation as it mitigates the informational asymmetries arising from the CSV problem. An important feature of the optimal contract between lenders and borrowers is indexation to observed aggregate shocks. The optimal level of indexation provides for both consumption insurance for households and a net worth hedge for entrepreneurs. The original analysis of BGG ignored both of these effects, which implied a sub-optimal amplification of aggregate shocks that is not part of competitive behavior. In contrast, the POC model derived here incorporates these optimal indexation effects.[7] The financial accelerator is thus sharply muted when compared to BGG.

It is an open empirical question to assess the degree of contract indexation in the data. At face value, it may appear that indexation is scarce.[8] But there are two cautions to this assertion. First, less explicit features of lending relationships may serve a similar purpose. Second, the myriad layers of hedging devices in financial markets may result in macro behavior that is more akin to the indexation under the POC. Carlstrom, Fuerst, Ortiz and Paustian (2014) use familiar Bayesian methods to estimate a BGG-style macro model in which the degree of repayment indexation is a parameter to estimate.

[7] In a companion paper, Carlstrom, Fuerst, and Paustian (2013) explore the Pareto efficiency implications of the POC.
[8] Halonen-Akatwijuka and Hart (2014) try to explain why optimal contracts may not condition on all verifiable aggregate states and hence remain imperfectly indexed.

Although the specific results vary depending on observables, they estimate a significant level of debt indexation in the model. Further the estimated level of indexation implies a trivial financial accelerator.

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APPENDIX.

1. Linearized Model.

$$E_t(r_{t+1}^k - r_{t+1}^l) = \nu\kappa_t \tag{A1}$$

$$nw_t^{POC} = nw_{t-1}^{POC} + E_{t-1}r_t^d + \kappa\nu\kappa_{t-1} + (r_t^k - E_{t-1}r_t^k) + \frac{\Theta_f}{\Psi}(m_t - \mathrm{E}_{t-1}m_t) - \frac{\beta\Theta_f}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \tag{A2}$$

$$r_t^{l,POC} = E_{t-1}r_t^d + \left(r_t^k - E_{t-1}r_t^k\right) + \frac{\Theta_g}{\Psi}(m_t - E_{t-1}m_t) - \frac{\beta\Theta_g}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \tag{A3}$$

$$z_t^{POC} = E_{t-1}r_t^d + \frac{(1-\Theta_g)[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\kappa_{t-1} + (r_t^k - E_{t-1}r_t^k) + \frac{1}{\Psi}(m_t - E_{t-1}m_t) - \frac{\beta}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \tag{A4}$$

$$\varpi_t^{POC} = \left(\frac{1-\nu(\kappa-1)}{\Theta_g(\kappa-1)}\right)\kappa_{t-1} + \frac{1}{\Psi}(m_t - E_{t-1}m_t) - \frac{\beta}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \tag{A5}$$

$$\lambda_t = E_t \sum_{j=0}^{\infty} \beta^j \left(\kappa\nu\kappa_{t+j} + r_{t+j}^d\right) \tag{A6}$$

$$r_t^k = \epsilon q_t + (1-\epsilon)mpk_t - q_{t-1} \tag{A7}$$

$$\kappa_{t-1} = (q_{t-1} + k_t - nw_{t-1}) \tag{A8}$$

$$\sigma c_t + \eta l_t = \alpha k_t + a_t - \alpha l_t \tag{A9}$$

$$E_t r_{t+1}^d = \sigma(E_t c_{t+1} - c_t) \tag{A10}$$

$$q_t = \psi i_t \tag{A11}$$

$$k_{t+1} = \delta i_t + (1-\delta)k_t \tag{A12}$$

$$c_{ss}c_t + c_{ss}^e nw_t + i_{ss}i_t = a_t + \alpha k_t + (1-\alpha)l_t \tag{A13}$$

where $\epsilon \equiv \frac{1-\delta}{mpk_{ss}+(1-\delta)}$. Also we have $\kappa \equiv K_{SS}/NW_{SS}$, Q_{SS}= 1, $R_{SS}^s = 1/\beta$. Finally, in the linear model we set $\mu\int_0^{\varpi_{SS}} \omega\phi(\omega)d\omega \approx 0$ such that monitoring costs do not appear above.

2. POC and BGG contracts log-linearized.

The POC contract.

Using (20), we can rewrite (17)-(19) as

$$\beta\gamma(1-\gamma+\Lambda_{t+1})f'(\varpi_{t+1})+\Lambda_t M_{t+1}g'(\varpi_{t+1})=0 \tag{A14}$$

$$\beta\gamma E_t(1-\gamma+\Lambda_{t+1})R^k_{t+1}f(\varpi_{t+1})+\Lambda_t\left[E_t M_{t+1}R^k_{t+1}g(\varpi_{t+1})-1\right]=0 \tag{A15}$$

$$E_t R^k_{t+1}M_{t+1}g(\varpi_{t+1})=\frac{\bar{\kappa}_t-1}{\bar{\kappa}_t} \tag{A16}$$

In the steady-state, these are given by:

$$\gamma(1-\gamma+\Lambda_{ss})f'(\varpi_{ss})=-\Lambda_{ss}g'(\varpi_{ss}) \tag{A18}$$

$$\beta\gamma(1-\gamma+\Lambda_{ss})R^k_{ss}f(\varpi_{ss})=\Lambda_{ss}[1-\beta R^k_{ss}g(\varpi_{ss})] \tag{A19}$$

$$[1-\beta R^k_{ss}g(\varpi_{ss})]\kappa=1 \tag{A20}$$

$$1=\gamma R^k_{ss}f(\varpi_{ss})\kappa \tag{A21}$$

where (A21) is the steady-state version of (21). We can solve for the steady state values as implicit functions of the underlying parameters:

$$F(\varpi_{ss})\equiv-\frac{f'(\varpi_{ss})}{g'(\varpi_{ss})}=\frac{\beta}{\gamma} \tag{A22}$$

$$R^k_{ss}[\gamma f(\varpi_{ss})+\beta g(\varpi_{ss})]=1 \tag{A23}$$

$$\kappa-1=\frac{\beta g(\varpi_{ss})}{\gamma f(\varpi_{ss})} \tag{A24}$$

$$\Lambda_{ss}=\frac{\beta(1-\gamma)}{(1-\beta)} \tag{A25}$$

Using these steady-state expressions, we can log-linearize equations (A14)-(A16) as:

$$\Psi\widetilde{\omega}_t = m_t + \lambda_{t-1} - \beta\lambda_t \tag{A26}$$

$$\kappa_t + E_t\left(r_{t+1}^k + \Theta_f\widetilde{\omega}_{t+1}\right) = \lambda_t - \beta E_t\lambda_{t+1} \tag{A27}$$

$$E_t\left(r_{t+1}^k + m_{t+1} + \Theta_{\mathrm{g}}\widetilde{\omega}_{t+1}\right) = \left(\frac{1}{\kappa-1}\right)\kappa_t \tag{A28}$$

The lower case letters denote log deviations of the corresponding endogenous variables, and κ_t and $\widetilde{\omega}_t$ denotes the log deviations of $\bar{\kappa}_t$ and ϖ_t, respectively. The constants are given by $\Psi \equiv \frac{\varpi_{ss}F'(\varpi_{ss})}{F(\varpi_{ss})} > 0$, $\Theta_{\mathrm{g}} \equiv \frac{\varpi_{ss}g\prime(\varpi_{ss})}{g(\varpi_{ss})}$, $0 < \Theta_{\mathrm{g}} < 1$, and $\Theta_{\mathrm{f}} \equiv \frac{\varpi_{ss}f\prime(\varpi_{ss})}{f(\varpi_{ss})} < 0$. In the steady state, (A22) and (A24) imply that $\frac{-\Theta_{\mathrm{f}}}{\Theta_{\mathrm{g}}} = (\kappa - 1)$. Scrolling (A26) forward and substituting in (A27), we can solve for $E_t\widetilde{\omega}_{t+1}$:

$$(\Psi - \Theta_{\mathrm{f}})E_t\widetilde{\omega}_{t+1} = \kappa_t + E_t\left(r_{t+1}^k + m_{t+1}\right) \tag{A29}$$

Using this in A28 yields:

$$E_t\left(r_{t+1}^k - r_{t+1}^l\right) = \left[\frac{\Psi}{(\kappa-1)\Psi - \kappa\theta_f}\right]\kappa_t \equiv \nu\kappa_t \tag{A30}$$

where we have used

$$E_t\left(r_{t+1}^l + m_{t+1}\right) = 0. \tag{A31}$$

From (A29), this then implies that

$$E_{t-1}\widetilde{\omega}_t = \left(\frac{1+\nu}{\Psi - \Theta_{\mathrm{f}}}\right)\kappa_{t-1} = \left(\frac{1-\nu(\kappa-1)}{\Theta_{\mathrm{g}}(\kappa-1)}\right)\kappa_{t-1} \tag{A32}$$

where the latter equality uses (A30) to eliminate Ψ. Using this expression in (A26) yields

$$\widetilde{\omega}_t^{POC} = \left(\frac{1-\nu(\kappa-1)}{\Theta_{\mathrm{g}}(\kappa-1)}\right)\kappa_{t-1} + \frac{1}{\Psi}(m_t - E_{t-1}m_t) - \frac{\beta}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \tag{A33}$$

From (8) and (12), the log-linearized promised payment and lender's return are given by:

$$z_t = \widetilde{\omega}_t + r_t^k - \frac{1}{\kappa-1}\kappa_{t-1} \tag{A34}$$

$$r_t^l = -\frac{1}{(\kappa-1)}\kappa_{t-1} + \Theta_g\widetilde{\omega}_t + r_t^k \tag{A35}$$

Using the expression for $\widetilde{\omega}_t^{POC}$ we can write these as:

$$z_t^{POC} = E_{t-1}r_t^l + \frac{(1-\Theta_g)[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\kappa_{t-1} + \left(r_t^k - E_{t-1}r_t^k\right) + \frac{1}{\Psi}(m_t - E_{t-1}m_t) - \frac{\beta}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \tag{A36}$$

$$r_t^{l,POC} = E_{t-1}r_t^l + \left(r_t^k - E_{t-1}r_t^k\right) + \frac{\Theta_g}{\Psi}(m_t - E_{t-1}m_t) - \frac{\beta\Theta_g}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \tag{A37}$$

Recall that the expected return on lending is linked to the households pricing kernel via (A31). Finally, solving (A27) forward we have

$$\lambda_t = E_t\sum_{j=0}^{\infty}\beta^j\left(\kappa\nu\kappa_{t+j} + r_{t+j+1}^l\right) \tag{A38}$$

Expressions (A33), and (A36)-(A38) are those used in the text.

<u>BGG Contract:</u>

The optimization conditions for the BGG contract are given by:

$$f'(\varpi_{t+1}) + \Lambda_{t+1}g'(\varpi_{t+1}) = 0 \tag{A39}$$

$$E_tR_{t+1}^kf(\varpi_{t+1}) + E_t\Lambda_{t+1}\left[R_{t+1}^kg(\varpi_{t+1}) - \frac{1}{E_tM_{t+1}}\right] = 0 \tag{A40}$$

$$R_{t+1}^kg(\varpi_{t+1})\bar{\kappa}_t = \frac{(\bar{\kappa}_t-1)}{E_tM_{t+1}} \tag{A41}$$

Importantly, expression (A41) holds state-by-state, and not in expected value. The log-linear version of these conditions are given by:

$$\Psi\widetilde{\omega}_{t+1} = -\lambda_{t+1} \tag{A42}$$

$$\kappa_t + E_t\left(r_{t+1}^k + m_{t+1} + \Theta_f \widetilde{\omega}_{t+1}\right) = -E_t \lambda_{t+1} \tag{A43}$$

$$r_{t+1}^k + \Theta_g \widetilde{\omega}_{t+1} + E_t m_{t+1} = \left(\frac{1}{\kappa - 1}\right)\kappa_t \tag{A44}$$

Expressions (A29) and (A30) also hold for the BGG contract as they are a result of taking the expected value of the optimization conditions. Using (A29)-(A30), we can solve (A44) for the time-t default cut-off:

$$\widetilde{\omega}_t^{BGG} = \frac{[1 - \nu(\kappa - 1)]}{\Theta_g(\kappa - 1)}\kappa_{t-1} - \frac{1}{\Theta_g}\left(r_t^k - E_{t-1} r_t^k\right) \tag{A45}$$

The expressions (A34)-(A35) then imply expressions for the lender's return and the promised repayment:

$$r_{t-1}^{l,BGG} = E_{t-1} r_t^k - \nu \kappa_{t-1} \tag{A46}$$

$$z_t^{BGG} = r_{t-1}^l + \frac{(1 - \Theta_g)[1 - \nu(\kappa - 1)]}{\Theta_g(\kappa - 1)}\kappa_{t-1} + \left(\frac{\Theta_g - 1}{\Theta_g}\right)\left(r_t^k - E_{t-1} r_t^k\right) \tag{A47}$$

The lender's pre-determined return is again linked to the pricing kernel as

$$r_t^l + E_t(m_{t+1}) = 0. \tag{A48}$$

Expressions (A45)-(A48) are those used in the text.

3. <u>The optimality of the debt contract.</u>

In this appendix we show that if the entrepreneur's value function is linear in net worth, then the optimal contract is risky debt. Suppose that the entrepreneur's value function is given by $V_t NW_t^i$. The linear return on the entrepreneur's project is $\omega_{t+1}^i R_{t+1}^k$, where ω has a unit mean and is iid. A critical assumption is that the idiosyncratic productivity ω_{t+1}^i is independent of the aggregate return to capital R_{t+1}^k. The entrepreneur reports ω_{t+1}^i to the lender, and R_{t+1}^k is observed by all. Because of this linearity and observability, we can without loss of generality normalize the repayment function as $R_{t+1}^k P(\omega_{t+1}^i; R_{t+1}^k)$, for some function $P(\cdot)$. Incentive compatibility implies that $P(\cdot)$ is independent of ω_{t+1}^i on the no-monitoring set. Following Townsend (1979) and Williamson (1986), the monitoring interval is given by $A_{t+1} = [0, \varpi_{t+1}]$, where $P(\omega_{t+1}^i; R_{t+1}^k) = P_{t+1}$, for $\omega_{t+1}^i \geq \varpi_{t+1}$. The time t+1 payoffs are then given by:

$$E_t ent_{t+1} = E_t V_{t+1} \bar{\kappa}_t NW_t^i R_{t+1}^k \left[E_t\left(\omega_{t+1}^i\right) - P_{t+1}(1 - \Phi_{t+1}) - X_{t+1}\right] \tag{A47}$$

$$E_t len_{t+1} = E_t M_{t+1} \bar{\kappa}_t NW_t^i R_{t+1}^k [P_{t+1}(1 - \Phi_{t+1}) + X_{t+1} - \mu \Phi_{t+1}] \tag{A48}$$

where $\Phi_{t+1}$is the CDF evaluated at ϖ_{t+1}, and we define

$$X_{t+1} \equiv \int_0^{\varpi_{t+1}} P(\omega)\phi(\omega) d\omega \tag{A49}$$

The linearity of the entrepreneur's value function is exploited in (A47). That is, we can pull the expectations operator into the expression and use $E_t\left(\omega_{t+1}^i\right) = 1$. This would not be possible if the entrepreneur's payoff were nonlinear or if V_{t+1} or R_{t+1}^k were correlated with ω_{t+1}^i. The optimal contract maximizes $E_t ent_{t+1}$, subject to the lender's participation constraint:

$$E_t len_{t+1} \geq (\bar{\kappa}_t - 1) NW_t^i$$

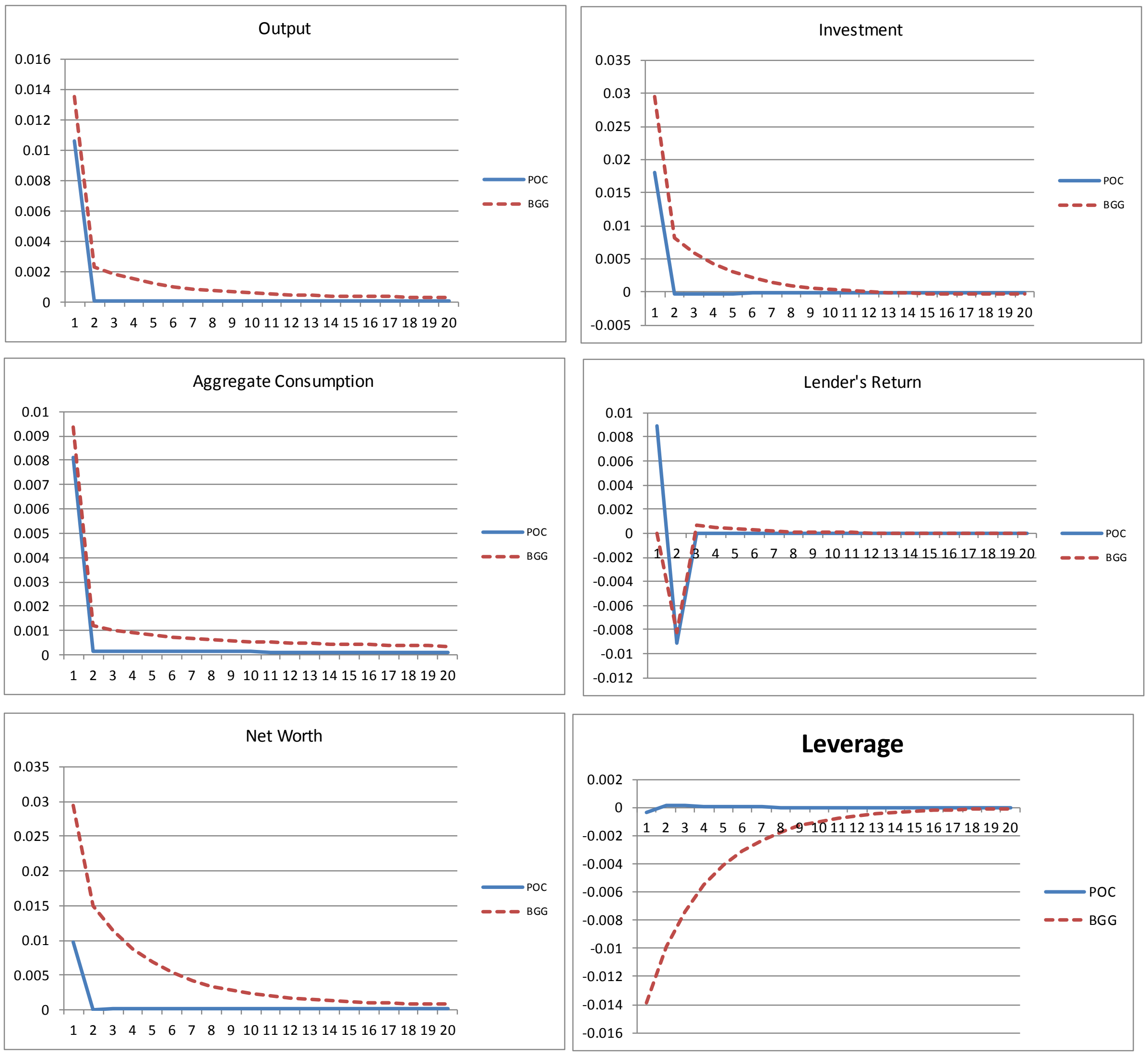
We have already noted that incentive compatibility implies that the repayment function must be constant over the no-monitoring set. The final step is to demonstrate that when monitoring does occur, the lender seizes the entire project, that is $P\left(\omega_{t+1}^i\right) = \omega_{t+1}^i$ for all $\omega_{t+1}^i \in A_{t+1}$. But this result is standard given the linear expressions (A47) and (A48). See, for example, Townsend (1979) or Williamson (1986).

Table 1: Welfare Comparison* (POC-BGG).

	Baseline (ρ=0.95, $\psi = 0.5$)	**iid Shocks (ρ=0)**	**More persistence (ρ=0.99)**	**Lower adjustment costs ($\psi = 0.25$)**	**Higher adjustment costs ($\psi = 2$)**
Household welfare gain as % of household consumption	0.022	0.047	0.010	0.010	0.044
Entrepreneur welfare gain as % of entrepreneurial consumption	0.048	0.070	0.064	0.058	0.010
Total welfare gain as % of aggregate consumption	0.027	0.051	0.021	0.020	0.037

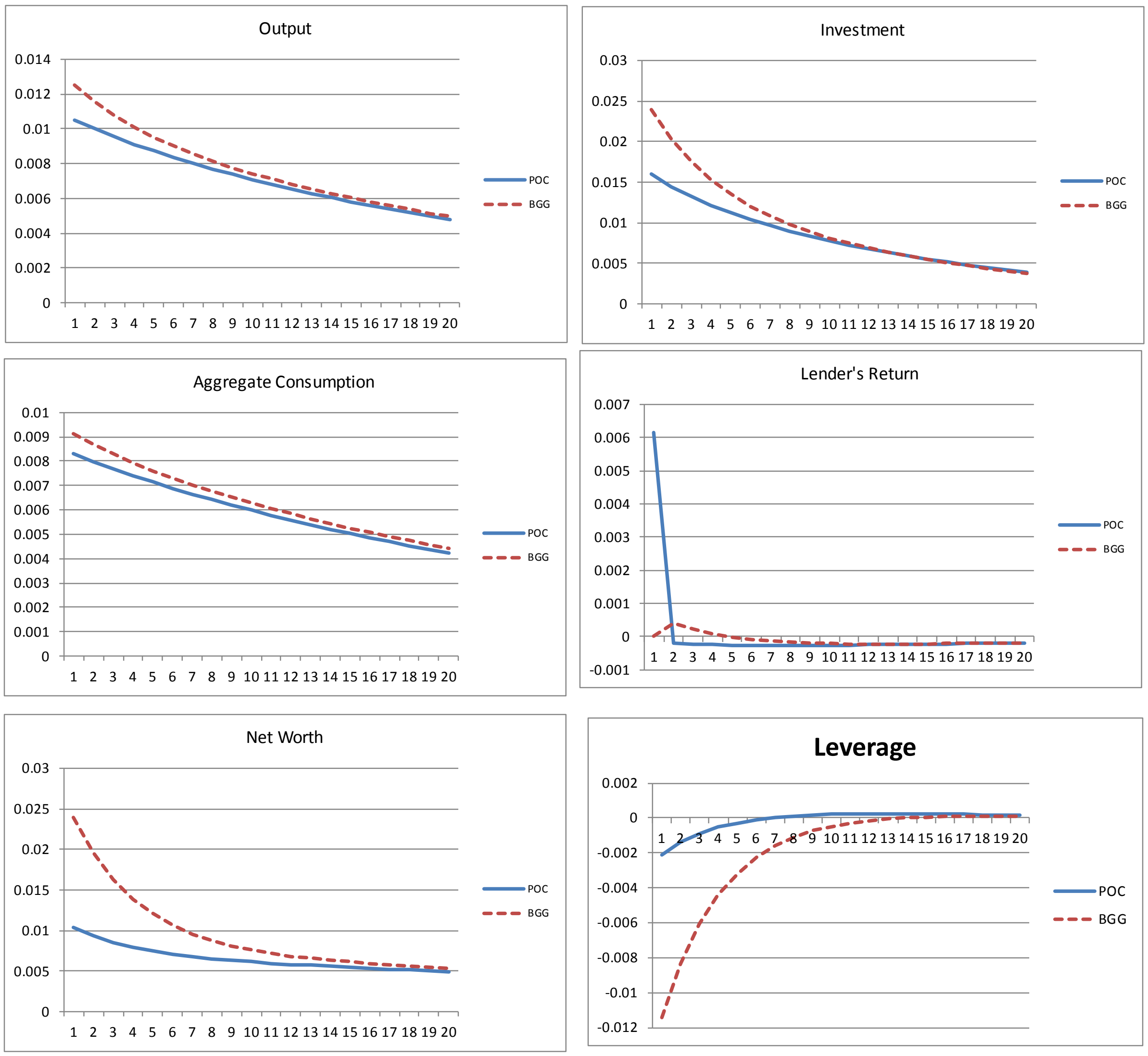
*The entries represent the difference in the household and entrepreneurial value function under the two contracts (POC-BGG), evaluated at the steady state. Since the entrepreneur's value function is linear, we consider a representative entrepreneur that holds the entire stock of net worth. To turn these utility flows into consumption equivalents, we divide this utility difference by the steady state marginal utility of consumption. The total welfare gain is the sum of the household and entrepreneur's welfare gain, expressed in consumption units and then taken as a fraction of aggregate consumption. The calibration is discussed in the text. The total welfare gain thus represents the perpetual flow increase in consumption that equates lifetime utility under the two contracts. For example, 0.027 is a perpetual increase in annual consumption of 0.027% (or a one-time increase of 2.7%).

Figure 1: iid TFP shock with flexible prices.



The response of output, investment, aggregate consumption, the lender's return, net worth, and leverage, to a 1% innovation in the TFP process.

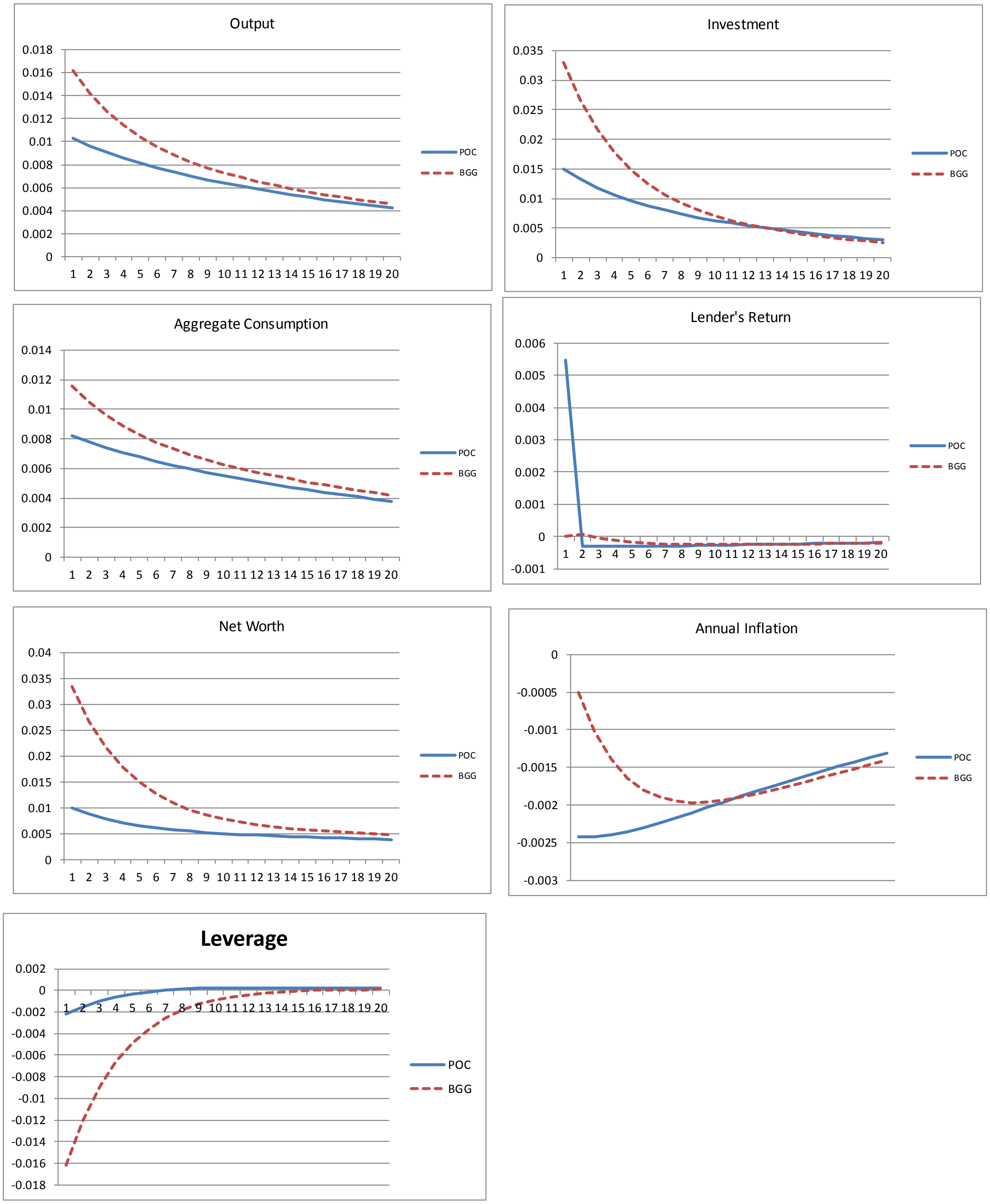
Figure 2: Persistent TFP shock with flexible prices.



The response of output, investment, aggregate consumption, the lender's return, net worth, and leverage, to a 1% innovation into TFP process with AR(1) coefficient of 0.95.

<u>Figure 3: TFP shock with sticky prices.</u>

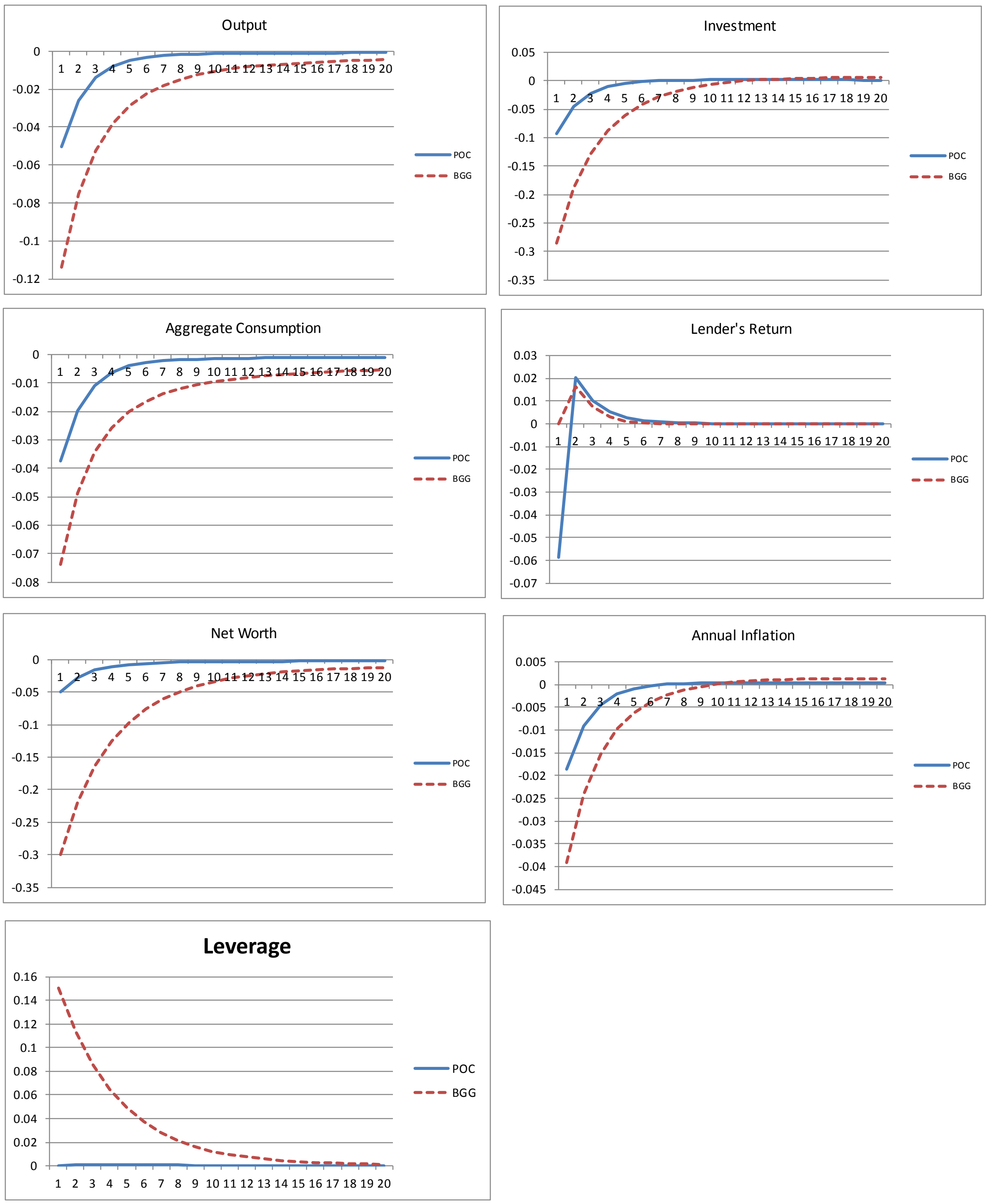
(Impulse response to a 1% TFP shock.)



The response of output, investment, aggregate consumption, the lender's return, net worth, inflation (annualized), and leverage, a 1% innovation into TFP process with AR(1) coefficient of 0.95.

Figure 4: Monetary shock with sticky prices.

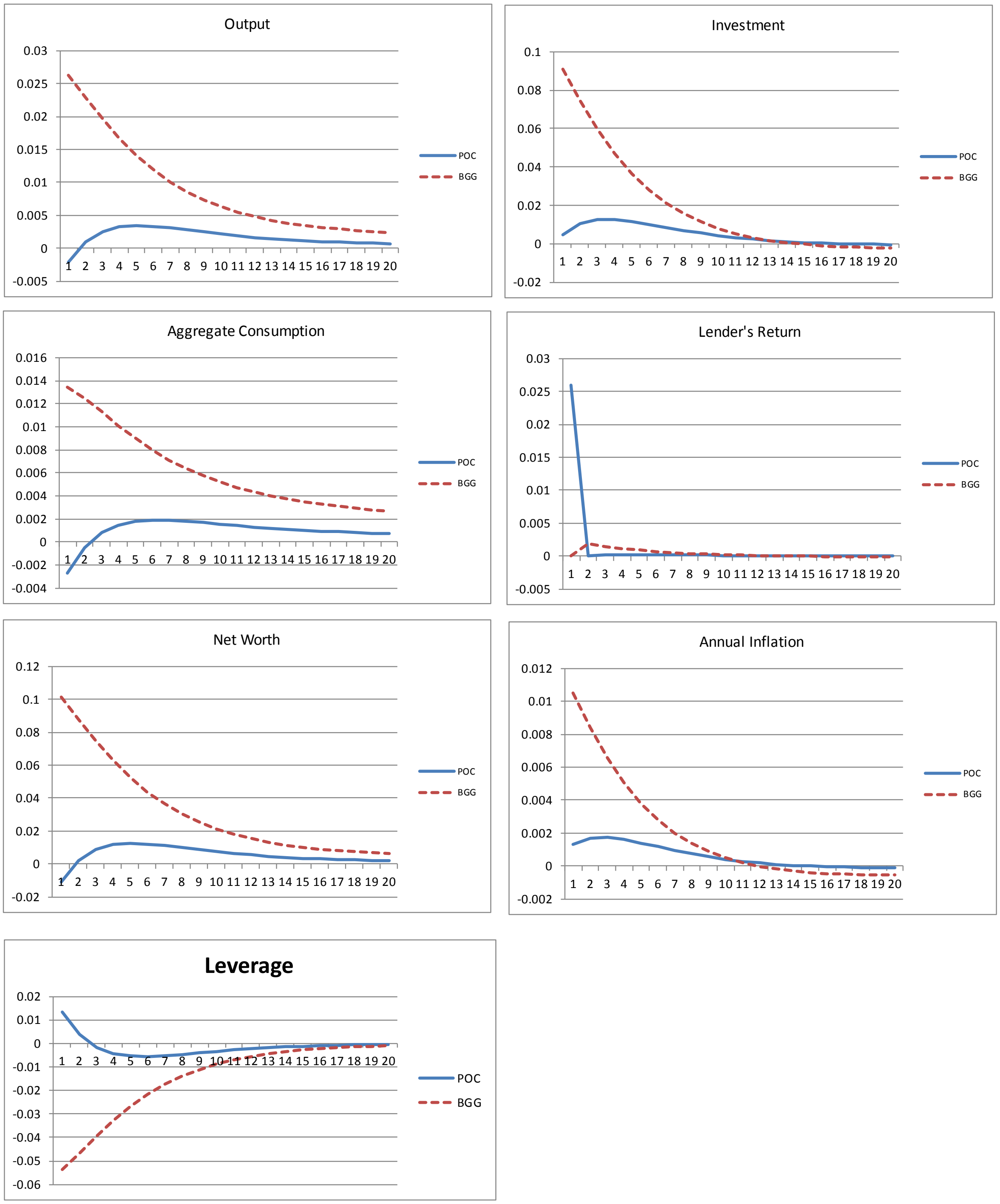
(Impulse response to a 25 bp quarterly policy shock.)



The response of output, investment, aggregate consumption, the lender's return, net worth, inflation (annualized), and leverage, to an innovation of 25 b.p. (quarterly) monetary policy shock with serial correlation coefficient of 0.5.

Figure 5: Net worth shock with sticky prices.

(Impulse response to a 1% shock to net worth.)



The response of output, investment, aggregate consumption, the lender's return, net worth, inflation (annualized), and leverage, to a 1% innovation in entrepreneur's net worth with AR(1) coefficient of 0.9.