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**Targeting Long Rates in a Model with
Segmented Markets**

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Matthias Paustian



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Targeting Long Rates in a Model with Segmented Markets

Charles T. Carlstrom, Timothy S. Fuerst, and Matthias Paustian

This paper develops a model of segmented financial markets in which the net worth of financial institutions limits the degree of arbitrage across the term structure. The model is embedded into the canonical Dynamic New Keynesian (DNK) framework. We estimate the model using data on the term premium. Our principal results include the following. First, the estimated segmentation coefficient implies a nontrivial effect of central bank asset purchases on yields and real activity. Second, there are welfare gains to having the central bank respond to the term premium, eg., including the term premium in the Taylor rule. Third, a policy that directly targets the term premium sterilizes the real economy from shocks originating in the financial sector. A term premium peg can have significant welfare effects.

Key words: Agency costs, CGE models, optimal contracting.

JEL codes: C68, E44, E61.

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To restate my main point, I believe that measures of bond market risk premiums--for example, estimates of the expected excess returns on long-term Treasury securities relative to Treasury bills and on credit-risky bonds relative to Treasury securities--may turn out to be useful inputs into the monetary policy framework.

FRB Governor Jeremy Stein, March 21, 2014, "Incorporating Financial Stability Considerations into a Monetary Policy Framework".

So what then might the Fed do if its target interest rate, the overnight federal funds rate, fell to zero?...A more direct method, which I personally prefer, would be for the Fed to begin announcing explicit ceilings for yields on longer-maturity Treasury debt. The Fed could enforce these interest-rate ceilings by committing to make unlimited purchases of securities....If this program were successful, not only would yields on medium-term Treasury securities fall, but yields on longer-term public and private debt would likely fall as well.

FRB Governor Ben S. Bernanke, November 21, 2002, "Deflation: Making Sure 'It' Doesn't Happen Here".

1. Introduction.

In the aftermath of the 2008 financial crisis many central banks have adopted unconventional policies, including outright purchases of long term government debt. These bond purchases raise a number of research questions for macro theory. Under what conditions can such purchases have aggregate effects? If they have aggregate consequences, how do term premia movements affect inflation and economic activity? What are appropriate policy rules for such interventions? To answer such questions, this paper develops a model of the term premium in which central bank purchases can affect the yield structure independently of the anticipated path of short term interest rates. The model is embedded into an otherwise canonical medium-scale DNK model where long-term bonds are necessary to finance investment purchases. This implies that both new and old policy questions can be examined in a unified framework.

The key features of the model include the following. First, the short term bond market is segmented from the long term bond market in that only financial intermediaries can purchase long term debt. Households can access the long-term debt instruments indirectly by providing deposits to intermediaries. Second, the ability of intermediaries to arbitrage the yield gap between the short term deposit rate and long term lending rate is limited by net worth. That is, a simple hold-up problem constrains the amount of deposits that can be supported by a given level of intermediary net worth. Third, the intermediary faces adjustment costs in rapidly varying the size of its portfolio in the wake of shocks. These assumptions imply that central bank purchases of long-term bonds will have a significant effect on long yields. Finally these long-term yields affect real economic activity because

of our final assumption: capital investment is financed by the issuance of long term bonds which sell in the same market that absorbs long term Treasuries. Taken together, these assumptions imply that central bank purchases of long-term bonds will have a significant and persistent effect on long yields and real activity.

We use the model to consider the efficacy of alternative policies linked to the term premium. This is a natural policy in the context of the model as the distortion arising from market segmentation is, to a linear approximation, equal to the term premium. Hence, we show that there are significant welfare gains to including the term premium in a traditional Taylor rule operating on the short term rate. We also consider policies that utilize a Taylor-type rule over the long rate. Such a long rate policy sterilizes the rest of the model economy from shocks originating in the financial system. This sterilization is directly analogous to the classic Poole (1970) result that a FFR targets sterilize the economy from money demand shocks.

The papers closest in spirit to the current work are Gertler and Karadi (2011, 2013) and Chen, Curdia, and Ferrero (2013). There are two crucial similarities between these papers and the present work. First, there is some friction that limits the ability to arbitrage across the short-term and long-term bond markets. This implies that the long rate is not the expected average of short rates, i.e., there is a term premium. Second, the market segmentation has real effects because some portion of real activity is financed in the segmented market. Gertler and Karadi (2013) assume that the entire capital stock is re-financed each period by the purchase of equity claims in this market by intermediaries. Chen et al. (2013) assume that a small subset of consumers finance their consumption in the segmented market. In contrast, the current paper assumes that new investment is financed in the segmented market with the issuance of long term debt. Both of these assumptions will magnify the effects of segmentation because investment is the most interest-sensitive component of aggregate expenditure, and the long term debt assumption implies that persistent interest rate movements have larger effects. Hence, a central bank bond purchase policy will have a much larger effect in the present paper than in the models of Gertler and Karadi (2013) and Chen et al. (2011).

A complementary approach to modeling the term premium in a DSGE model has been pursued by Rudebusch and Swanson (2008, 2012). These authors study an otherwise standard DNK model with no market segmentation effects. To capture a time-varying term premium, they solve the model using third-order

approximation. Rudebusch and Swanson (2008) use a specification for household preferences commonly used in the business cycle literature. In this case they find essentially no steady state term premium and no time-variation in the premium. In contrast, Rudebusch and Swanson (2012) use Epstein-Zin (EZ) preferences. If the EZ risk aversion coefficient is set high enough, they are able to match a significant steady state term premium and time variation in the premium. Importantly, in both of these papers the higher order approximation and the EZ preferences have a trivial effect on the decision rules for real variables such as output and investment. That is, the model with EZ preferences is able to generate plausible movements in the term premium although the behavior of the real variables is unaltered by this time variation. Although Rudebusch and Swanson do not pursue policy issues, their results suggest that time variation in the term premium should have little consequence for monetary policy.

The paper proceeds as follows. The next section develops the theoretical model. Section 3 presents our quantitative results including our estimation of the key model parameters. Section 3 also focuses on how the segmentation affects the IRFs to shocks, and the efficacy of central bank policies that directly or indirectly target the term premium. Section 4 concludes.

2. The Model.

The economy consists of households, financial intermediaries (FI's), and firms. We discuss each in turn.

Households.

Each household maximizes the utility function

$$E_0 \sum_{s=0}^{\infty} \beta^s e^{rn_{t+s}} \left\{ \ln(C_{t+s} - hC_{t+s-1}) - B \frac{H_{t+s}^{1+\eta}(j)}{1+\eta} \right\} \quad (1)$$

where C_t is consumption, h is the degree of habit formation, $H_t(j)$, is the labor input of household j , and e^{rn_t} is a shock to the discount factor. This intertemporal preference shock follows the stochastic process

$$rn_t = \rho_{rn} rn_{t-1} + \varepsilon_{rn,t}, \quad (2)$$

with $\varepsilon_{rn,t} \sim i.i.d. N(0, \sigma_{rn}^2)$. Each household is a monopolistic supplier of specialized labor, $H_t(j)$, as in Erceg et al. (2000). A large number of competitive employment agencies combine this specialized labor into a homogenous labor input sold to intermediate firms, according to

$$H_t = \left[\int_0^1 H_t(j)^{1/(1+\lambda_{w,t})} dj \right]^{1+\lambda_{w,t}} \quad (3)$$

The desired markup of wages over the household's marginal rate of substitution, $\lambda_{w,t}$, follows the exogenous stochastic process

$$\log \lambda_{w,t} = (1 - \rho_w) \log \lambda_w + \rho_w \log \lambda_{w,t-1} + \varepsilon_{w,t} - \theta_w \varepsilon_{w,t-1}, \quad (4)$$

with $\varepsilon_{w,t} \sim i.i.d. N(0, \sigma_w^2)$. This is the wage markup shock. Profit maximization by the perfectly competitive employment agencies implies that the real wage (W_t) paid by intermediate firms for their homogenous labor input is

$$W_t = \left[\int_0^1 W_t(j)^{-1/\lambda_{w,t}} dj \right]^{-\lambda_{w,t}} \quad (5)$$

Every period a fraction θ_w of households cannot freely alter their nominal wage, so their real wage follows the indexation rule

$$W_t(j) = \frac{\Pi_{t-1}^w}{\Pi_t} W_{t-1}(j) \quad (6)$$

The remaining fraction of households chooses instead an optimal real wage $W_t(j)$ by maximizing

$$E_t \left\{ \sum_{s=0}^{\infty} \theta_w^s \beta^s \left[-e^{rn_{t+s}} B \frac{H_{t+s}(j)^{1+\psi}}{1+\psi} + \Lambda_{t+s} W_t(j) H_{t+s}(j) \right] \right\} \quad (7)$$

subject to the labor demand function coming from the employment agencies, and where Λ_{t+s} is the household's marginal utility of consumption. The existence of state contingent securities ensures that household consumption (and thus Λ_{t+s}) is the same across all households. The household also earns income by renting capital to the intermediate goods firm.

The household has two means of intertemporal smoothing: short term deposits (D_t) in the financial intermediaries (FI), and accumulation of physical capital (K_t). Households also have access to the market in short term government bonds (“T-bills”). But since T-bills are perfect substitutes with deposits, and the supply of T-bills moves endogenously to hit the central bank’s short-term interest rate target, we treat D_t as the household’s net resource flow into the FI’s. To introduce a need for intermediation, we assume that all investment purchases must be financed by issuing new “investment bonds” that are ultimately purchased by the FI. We find it convenient to use the perpetual bonds suggested by Woodford (2001). In particular, these bonds are perpetuities with cash flows of 1, κ , κ^2 , etc. Let Q_t denote the time-t price of a new issue. Given the time pattern of the perpetuity payment, the new issue price Q_t summarizes the prices at all maturities, eg., κQ_t is the time-t price of the perpetuity issued in period t-1. The duration and (gross) yield to maturity on these bonds are defined as: duration = $(1 - \kappa)^{-1}$, gross yield to maturity = $Q_t^{-1} + \kappa$. Let CI_t denote the number of new perpetuities issued in time-t to finance investment. In time-t, the household’s nominal liability on past issues is given by:

$$F_{t-1} = CI_{t-1} + \kappa CI_{t-2} + \kappa^2 CI_{t-3} + \dots \quad (8)$$

We can use this recursion to write the new issue as

$$CI_t = (F_t - \kappa F_{t-1}) \quad (9)$$

The representative’s household constraints are thus given by:

$$C_t + \frac{D_t}{P_t} + P_t^k I_t + \frac{F_{t-1}}{P_t} \leq \frac{W_t}{P_t} H_t + R_t^k K_t - T_t + \frac{D_{t-1}}{P_t} R_{t-1} + \frac{Q_t(F_t - \kappa F_{t-1})}{P_t} + div_t \quad (10)$$

$$K_{t+1} \leq (1 - \delta)K_t + I_t \quad (11)$$

$$P_t^k I_t \leq \frac{Q_t(F_t - \kappa F_{t-1})}{P_t} = \frac{Q_t CI_t}{P_t} \quad (12)$$

where P_t is the price level, P_t^k is the real price of capital, R_{t-1} is the gross nominal interest rate on deposits, R_t^k is the real rental rate, T_t are lump-sum taxes, and div_t denotes the dividend flow from the FI’s. The household also receives a profit flow from the intermediate goods producers and the new capital producers, but this is entirely standard so we dispense from this added notation for simplicity. The “loan-in-advance” constraint (12) will increase the private cost of purchasing investment goods. Although for simplicity we place capital accumulation

within the household problem, this model formulation is isomorphic to an environment in which household-owned firms accumulate capital subject to the loan constraint. The first order conditions to the household problem include:

$$\Lambda_t = E_t \beta \Lambda_{t+1} \frac{R_t}{\Pi_{t+1}} \quad (13)$$

$$\Lambda_t P_t^k M_t = E_t \beta \Lambda_{t+1} [R_t^k + (1 - \delta) P_{t+1}^k M_{t+1}] \quad (14)$$

$$\Lambda_t Q_t^l M_t = E_t \beta \Lambda_{t+1} \frac{[1 + \kappa Q_{t+1} M_{t+1}]}{\Pi_{t+1}} \quad (15)$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is gross inflation. Expression (13) is the familiar Fisher equation. The capital accumulation expression (14) is distorted relative to the familiar by the time-varying distortion M_t , where $M_t \equiv 1 + \frac{\vartheta_t}{\Lambda_t}$, and ϑ_t is the multiplier on the loan-in-advance constraint (12). The endogenous behavior of this distortion is fundamental to the real effects arising from market segmentation.

Financial Intermediaries.

The FI's in the model are a stand-in for the entire financial nexus that uses accumulated net worth (N_t) and short term liabilities (D_t) to finance investment bonds (F_t) and the long-term government bonds (B_t). The FIs are the sole buyers of the investment bonds and long term government bonds. We again assume that government debt takes the form of Woodford-type perpetuities that provide payments of 1, κ , κ^2 , etc. Let Q_t denote the price of a new-debt issue at time- t . The time- t asset value of the current and past issues of investment bonds is:

$$Q_t C I_t + \kappa Q_t [C I_{t-1} + \kappa C I_{t-2} + \kappa^2 C I_{t-3} + \dots] = Q_t F_t \quad (16)$$

The FIs balance sheet is thus given by:

$$\frac{B_t}{P_t} Q_t + \frac{F_t}{P_t} Q_t = \frac{D_t}{P_t} + N_t = L_t N_t \quad (17)$$

where L_t denotes leverage. Note that on the asset side, investment lending and long term bond purchases are perfect substitutes to the FI. Let $R_{t+1}^L \equiv \left(\frac{1 + \kappa Q_{t+1}}{Q_t} \right)$. The FI's time- t profits are then given by:

$$prof_t \equiv \frac{P_{t-1}}{P_t} [(R_t^L - R_{t-1}^d)L_{t-1} + R_{t-1}]N_{t-1} \quad (18)$$

The FI will pay out some of these profits as dividends (div_t) to the household, and retain the rest as net worth for subsequent activity. In making this choice the FI discounts dividend flows using the household's pricing kernel augmented with additional impatience. The FI accumulates net worth because it is subject to a financial constraint: the FI's ability to attract deposits will be limited by its net worth. We will use a simple hold-up problem to generate this leverage constraint, but a wide variety of informational restrictions will generate the same constraint. The FI chooses dividends, net worth, and leverage to solve:

$$V_t \equiv \max_{N_t, div_t, L_t} E_t \sum_{j=0}^{\infty} (\beta\zeta)^j \Lambda_{t+j} div_{t+j} \quad (19)$$

subject to the financing constraint developed below and the following budget constraint:

$$div_t + N_t[1 + f(N_t)] \leq \frac{P_{t-1}}{P_t} [(R_t^L - R_{t-1}^d)L_{t-1} + R_{t-1}]N_{t-1} \quad (20)$$

The function $f(N_t) \equiv \frac{\psi_n}{2} \left(\frac{N_t - N_{ss}}{N_{ss}} \right)^2$, denotes an adjustment cost function that dampens the ability of the FI to adjust the size of its portfolio in response to shocks.

The hold-up problem works as follows (see the appendix for further details). At the beginning of period $t+1$, but before aggregate shocks are realized, the FI can choose to default on its planned repayment to depositors. In this event, depositors can seize at most fraction $(1 - \mu_t)$ of the FI's assets, where μ_t is a function of net worth and the other state variables. If the FI defaults, the FI is left with $\mu_t R_{t+1}^L L_t N_t$, which it pays out to households and exits the world. To ensure that the FI will always re-pay the depositor, the time- t incentive compatibility constraint is thus given by:

$$E_t V_{t+1} \geq \mu_t L_t N_t E_t \Lambda_{t+1} \frac{P_t}{P_{t+1}} R_{t+1}^L \quad (21)$$

We will calibrate the model so that this constraint is binding in the steady state (and thus binding for small shocks around the steady state). If we assumed no adjustment costs ($\psi_n = 0$) and that the net worth solution is interior, the FI's value function is linear and given by:

$$V_t = \frac{P_{t-1}}{P_t} \Lambda_t [(R_t^L - R_{t-1}^d) L_{t-1} + R_{t-1}^d] N_{t-1} \equiv X_t N_{t-1}. \quad (22)$$

In this case, net worth naturally cancels out of (21) so that the model aggregates. But because of the convex adjustment cost in net worth accumulation, the FI's value function will include a time-varying additive term:

$$V_t = X_t N_{t-1} - g_t$$

where $g_t \geq 0$, and $g_{ss} = 0$. The term g_t is a function of aggregate variables that are exogenous to the FI. This implies that in general we cannot aggregate net worth and consider a representative entrepreneur as the incentive constraint (21) will vary with net worth (for a given $g_t > 0$, it will be less severe as net worth increases). To avoid this problem, we assume that μ_t is symmetric with the convexity in the adjustment cost function. In particular, we assume that the fraction of assets that the FI can keep in case of default is increasing in net worth and defined by:

$$\mu_t \equiv \Phi_t \left[1 - \frac{1}{N_t} \left(\frac{E_t g_{t+1}}{E_t X_{t+1}} \right) \right] \quad (23)$$

where Φ_t is an exogenous stochastic process that represents exogenous changes in the financial friction. Note that (23) implies that higher net worth makes it easier for the FI to shield assets and thus makes the hold-up problem more severe. This increased severity is chosen to match the decreased severity coming from the value function. In any event, assumption (23) implies that the binding incentive constraint (22) is given by:

$$E_t \frac{P_t}{P_{t+1}} \Lambda_{t+1} \left[\left(\frac{R_{t+1}^L}{R_t^d} - 1 \right) L_t + 1 \right] = \Phi_t L_t E_t \Lambda_{t+1} \frac{P_t}{P_{t+1}} \frac{R_{t+1}^L}{R_t^d} \quad (24)$$

As anticipated, leverage is a function of aggregate variables but is independent of each FI's net worth, i.e., there is now a representative entrepreneur. Log-linearizing this expression we have:

$$(E_t r_{t+1}^L - r_t) = \nu l_t + \left[\frac{1 + L_{ss}(s-1)}{L_{ss}-1} \right] \phi_t \quad (25)$$

where $\nu \equiv (L_{ss} - 1)^{-1}$, is the elasticity of the interest rate spread to leverage, s denotes the gross steady-state term premium, and the financial shock $\phi_t \equiv \ln(\Phi_t)$, follows an AR(1) process:

$$\phi_t = (1 - \rho_\phi) \phi_{ss} + \rho_\phi \phi_{t-1} + \varepsilon_{\phi,t}, \quad (26)$$

Increases in ϕ_t will exacerbate the hold-up problem, and thus are “credit shocks” which will increase the spread and lower real activity.

Qualitatively the log-linearized expression (25) for leverage is identical to the corresponding relationship in the more complex costly-state- verification (CSV) environment of, for example, Bernanke, Gertler, and Gilchrist (1999). In a CSV model, the primitives include: (i) idiosyncratic risk, (ii) death rate, and (iii) monitoring cost. One typically chooses these to match values for (i) leverage, (ii) interest rate spread, and (iii) default rate. The hold-up model has only two primitives: (i) the impatience rate ζ , and (ii) the fraction of assets that can be seized Φ . In comparison to the hold-up model, the extra primitive in the CSV framework thus allows it to match one more moment of the financial data (default rates). One important quantitative difference is that interest rate spreads are more responsive to leverage in our framework than in the CSV model calibrated to the same steady state leverage. For example, suppose we calibrated a CSV model to a leverage of 6.0, a risk premium of 100 bp, and a quarterly default rate of 0.205% (the default rate in the hold-up model is 0%). This would imply $\nu = 0.097$. In the hold-up model analyzed here, a leverage of 6.0 implies $\nu = 0.20$, about twice as large as the CSV counterpart.

Since the incentive constraint (24) is now independent of net worth, the FI takes leverage as given. The FI’s optimal accumulation decision is then given by:

$$\Lambda_t[1 + N_t f'(N_t) + f(N_t)] = E_t \beta \zeta \Lambda_{t+1} \frac{P_t}{P_{t+1}} [(R_{t+1}^L - R_t^d)L_t + R_t^d] \quad (27)$$

Equations (24) and (27) are fundamental to the model as they summarize the limits to arbitrage between the return on long term bonds and the rate paid on short term deposits. The leverage constraint (24) limits the FI’s ability to attract deposits and eliminate the arbitrage opportunity between the deposit and lending rate. Hence, the expectations theory of the term structure holds within the long bond market, but not between the short and long debt market. In essence, the segmentation decouples the short rate from the rest of the term structure. Increases in net worth allow for greater arbitrage and thus can eliminate this market segmentation. Equation (27) limits this arbitrage in the steady-state ($\zeta < 1$) and dynamically ($\psi_n > 0$). Since the FI is the sole means of investment

finance, this market segmentation means that central bank purchases that alter the supply of long term debt will have repercussions for investment loans because net worth and deposits cannot quickly sterilize the purchases.

Final good producers.

Perfectly competitive firms produce the final consumption good Y_t combining a continuum of intermediate goods according to the CES technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{1/(1+\epsilon_p)} di \right]^{1+\epsilon_p} \tag{28}$$

Profit maximization and the zero profit condition imply that the price of the final good, P_t , is the familiar CES aggregate of the prices of the intermediate goods.

Intermediate goods producers.

A monopolist produces the intermediate good i according to the production function

$$Y_t(i) = A_t K_t(i)^\alpha H_t(i)^{1-\alpha} \tag{29}$$

where $K_t(i)$ and $H_t(i)$ denote the amounts of capital and labor employed by firm i . The variable $\ln A_t$ is the exogenous level of TFP and evolves according to:

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_{a,t}, \tag{30}$$

Every period a fraction θ_p of intermediate firms cannot choose its price optimally, but resets it according to the indexation rule

$$P_t(i) = P_{t-1}(i) \Pi_{t-1}^{\theta_p}, \tag{31}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is gross inflation. The remaining fraction of firms chooses its price $P_t(i)$ optimally, by maximizing the present discounted value of future profits

$$E_t \left\{ \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s} / P_{t+s}}{\Lambda_t / P_t} [P_t(i) (\prod_{k=1}^s \Pi_{t+k-1}^{l_p}) Y_{t+s}(i) - W_{t+s} H_{t+s}(i) - P_{t+s} \rho_{t+s} K_{t+s}(i)] \right\} \quad (32)$$

where the demand function comes from the final goods producers.

New Capital Producers.

New capital is produced according to the production technology that takes I_t investment goods and transforms them into $\mu_t \left[1 - S \left(\frac{I_t}{I_{t-1}} \right) \right] I_t$ new capital goods. The time- t profit flow is thus given by

$$P_t^k \mu_t \left[1 - S \left(\frac{I_t}{I_{t-1}} \right) \right] I_t - I_t \quad (33)$$

where the function S captures the presence of adjustment costs in investment, as in Christiano et al. (2005), and is given by $S \left(\frac{I_t}{I_{t-1}} \right) \equiv \frac{\psi_i}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2$. These firms are owned by households and discount future cash flows with Λ_t .

The investment shock follows the stochastic process

$$\log \mu_t = \rho_\mu \log \mu_{t-1} + \varepsilon_{\mu,t}, \quad (34)$$

where $\varepsilon_{\mu,t}$ is *i.i.d.* $N(0, \sigma_\mu^2)$.

Central Bank Policy.

We assume that the central bank follows a familiar Taylor rule over the short rate (T-bills and deposits):

$$\ln(R_t) = (1 - \rho) \ln(R_{ss}) + \rho \ln(R_{t-1}) + (1 - \rho)(\tau_\pi \pi_t + \tau_y y_t^{gap}) + \epsilon_t^r \quad (35)$$

where $y_t^{gap} \equiv (Y_t - Y_t^f)/Y_t^f$, denotes the deviation of output from its flexible price counterpart, and ϵ_t^r is an exogenous and auto-correlated policy shock with AR(1) coefficient ρ_r . We will think of this as the Federal Funds Rate (FFR). Below we will also investigate the efficacy of putting the term-premium into the Taylor rule. The supply of T-bills is endogenous, varying as needed to support the FFR target. As for the long term policy, the central bank will choose between: (i) an exogenous path for the quantity of long term debt available to FIs, or (ii) a policy rule for the long term bond yield. We will return to this below.

Fiscal policy is entirely passive. Government expenditures are set to zero. Lump sum taxes move endogenously to support the interest payments on the short and long debt.

Loglinearized Model.

To gain further intuition and to derive the term premium, we first log-linearize the model. Let $b_t \equiv \ln\left(\frac{\bar{B}_t}{\bar{B}_{SS}}\right)$, and $f_t \equiv \ln\left(\frac{\bar{F}_t}{\bar{F}_{SS}}\right)$, where $\bar{B}_t \equiv Q_t \frac{B_t}{P_t}$, and $\bar{F}_t \equiv Q_t \frac{F_t}{P_t}$, denote the real market value of the bonds available to FIs. We will focus on bonds of 10-year maturities, so R_t^{10} will denote their gross yield. L_{SS} denotes steady state leverage. Using lower case letters to denote log deviations, the log-linearized model is given by the following:

$$\lambda_t = \frac{1}{(1-\beta h)(1-h)} E_t [\beta h c_{t+1} - (1 + \beta h^2) c_t + h c_{t-1}] + \frac{1}{1-\beta h} (r n_t - \beta h E_t r n_{t+1}) \quad (36)$$

$$r n_t + \eta h_t - \lambda_t = m r s_t \quad (37)$$

$$\pi_t^w - \iota_w \pi_{t-1} = \kappa_w (m r s_t - w_t) + \beta (\pi_{t+1}^w - \iota_w \pi_t) + \epsilon_t^w \quad (38)$$

$$w_t = w_{t-1} + \pi_t^w - \pi_t \quad (39)$$

$$\lambda_t = E_t \lambda_{t+1} + r_t - E_t \pi_{t+1} \quad (40)$$

$$\lambda_t + p_t^k + m_t = E_t \{ \lambda_{t+1} + [1 - \beta(1 - \delta)] r_{t+1}^k + \beta(1 - \delta) (p_{t+1}^k + m_{t+1}) \} \quad (41)$$

$$\lambda_t + q_t + m_t = E_t \lambda_{t+1} - E_t \pi_{t+1} + \beta \kappa E_t (q_{t+1} + m_{t+1}) \quad (42)$$

$$(1 - \kappa_I) (p_t^k + i_t) = f_t - \kappa (f_{t-1} + q_t - q_{t-1} - \pi_t) \quad (43)$$

$$r_{t+1}^L = \frac{\kappa q_{t+1}}{R_{SS}^L} - q_t \quad (44)$$

$$r_t^{10} = -\left(\frac{R_{SS}^L - \kappa}{R_{SS}^L}\right) q_t \quad (45)$$

$$E_t(r_{t+1}^L - r_t) = \left(\frac{1}{L_{SS}-1}\right) l_t + \left[\frac{1+(s-1)L_{SS}}{L_{SS}-1}\right] \phi_t \quad (46)$$

$$\psi n_t = \left[\frac{sL_{SS}}{1+L_{SS}(s-1)}\right] E_t(r_{t+1}^L - r_t) + \left[\frac{(s-1)L_{SS}}{1+L_{SS}(s-1)}\right] l_t \quad (47)$$

$$\frac{\bar{B}_{SS}}{L_{SS}N_{SS}} b_t + \left(1 - \frac{\bar{B}_{SS}}{L_{SS}N_{SS}}\right) f_t = n_t + l_t \quad (48)$$

$$w_t = mc_t + mpl_t \quad (49)$$

$$r_t^k = mc_t + mpk_t \quad (50)$$

$$\pi_t = \frac{\kappa\pi}{1+\beta l_p} mc_t + \frac{\beta}{1+\beta l_p} E_t \pi_{t+1} + \frac{l}{1+\beta l_p} \pi_{t-1} + \epsilon_t^p \quad (51)$$

$$p_t^k = \psi_i[(i_t - i_{t-1}) - \beta E_t(i_{t+1} - i_t)] - \mu_t \quad (52)$$

$$\left(1 - \frac{I_{SS}}{Y_{SS}}\right) c_t + \frac{I_{SS}}{Y_{SS}} i_t = a_t + \alpha k_t + (1 - \alpha) h_t \quad (53)$$

$$k_{t+1} = (1 - \delta)k_t + \delta(\mu_t + i_t) \quad (54)$$

$$r_t = \rho r_{t-1} + (1 - \rho)(\tau_\pi \pi_t + \tau_y y_t^{gap}) + \epsilon_t^r \quad (55)$$

To close the model, we need one more equation outlining the policy rule for the long term debt market. Before a discussion of these policy options, several comments are in order.

First, equation (41) highlights the economic distortion, m_t , arising from the segmented markets. Solving this forward we have:

$$p_t^k + m_t = E_t \sum_{j=0}^{\infty} [\beta(1 - \delta)]^j \{ [1 - \beta(1 - \delta)] r_{t+j}^k - (r_{t+j} - \pi_{t+j+1}) \} \quad (56)$$

As is clear from (56), the segmentation distortion, m_t , acts like a mark-up or excise tax on the price of new capital goods. What is this distortion? Using (42) and (44) we have

$$m_t = E_t \sum_{j=0}^{\infty} (\beta\kappa)^j \Xi_{t+j}, \quad (57)$$

where

$$\Xi_{t+j} \equiv \beta\kappa q_{t+j+1}^i - q_{t+j}^i - r_{t+j} \approx r_{t+j}^L - r_{t+j} \quad (58)$$

The distortion is thus the discounted sum of the future one-period loan to deposit spreads. As discussed above, this spread exists because of the assumed market segmentation.

Second, the market segmentation distortion is joined by the two familiar distortions in this familiar DNK model. The nominal price rigidity induces a time-varying wedge between rental rates and the marginal product of capital (50). The price and wage rigidity create a time-varying “labor wedge” between the marginal product of labor and the marginal rate of substitution (see (38) and (49)). To anticipate our empirical estimates, the nominal wage rigidity is significantly more sticky than the nominal price rigidity, so that the time-varying labor wedge is of fundamental importance. Because of this our discussion below will focus on the interaction between the segmentation distortion and the labor wedge.

Third, the *term premium* can be defined as the difference between the observed yield on a 10-year bond (see (45)) and the corresponding yield implied by applying the expectation hypothesis (EH) of the term structure to the series of short rates. The price of this hypothetical EH bond satisfies

$$r_t = E_t \frac{\kappa q_{t+1}^{EH}}{R_{SS}} - q_t^{EH} \quad (59)$$

while its yield is given by

$$r_t^{EH,10} = \left(\frac{R_{SS} - \kappa}{R_{SS}} \right) q_t^{EH}. \quad (60)$$

Using these definitions, the term premium can be expressed as

$$term\ premium \equiv tp_t \equiv (r_t^{10} - r_t^{EH,10}) = - \left(\frac{R_{SS}^L - \kappa}{R_{SS}^L} \right) q_t + \left(\frac{R_{SS} - \kappa}{R_{SS}} \right) q_t^{EH} \quad (61)$$

Solving the bond prices in terms of the future short rates, we have

$$\begin{aligned} tp_t &= \left(\frac{R_{SS}^L - \kappa}{R_{SS}^L} \right) \sum_{j=0}^{\infty} \left(\frac{\kappa}{R_{SS}^L} \right)^j E_t r_{t+j}^L - \left(\frac{R_{SS} - \kappa}{R_{SS}} \right) \sum_{j=0}^{\infty} \left(\frac{\kappa}{R_{SS}} \right)^j E_t r_{t+j} \\ &\approx (1 - \beta\kappa) \sum_{j=0}^{\infty} (\beta\kappa)^j E_t (r_{t+j}^L - r_{t+j}) \end{aligned} \quad (62)$$

Comparing (57) and (62), the distortion m_t is closely proxied by the term premium. Hence, a policy that eliminates fluctuations in the term premium will largely eliminate fluctuations in the market segmentation distortion.

Fourth, the loan-deposit spread arises because of the segmentation effects summarized in (46)-(47). Equation (46) expresses the endogenous response of leverage to higher expected returns on intermediation, while equation (47) summarizes the FIs desire to accumulate more net worth in response to the profit opportunity of the

spread. The model's dynamics collapse to the familiar DNK model if we set $\psi_n = 0$, so that net worth can move instantaneously to eliminate all arbitrage opportunities. But if $\psi_n > 0$, then the segmentation acts like an endogenous adjustment cost to investment. That is, increases in investment necessitate an increase in investment bonds (48), but this drives up the one-period spread (46) and thus m_t . The net worth adjustment cost (47) implies that this effect cannot be entirely undone by movements in net worth.

Fifth, the previous suggests that a policy that stabilizes the term premium will likely be welfare improving (unless the interaction with the sticky price distortion is significant). This suggests the efficacy of a central bank including the term premium in a Taylor type rule. But we can take this argument one step further. Under a policy that directly targets the term premium the supply of long debt held by FIs will be endogenous. In particular, (48) separates out from the rest of the model, and defines the behavior of long bonds that move endogenously to support the long rate target. This implies that "credit shocks", those proxied by ϕ_t , will have no effect on real activity or inflation. That is, a long rate policy sterilizes the real economy from financial shocks. This is analogous to the classic result of Poole (1970) in which an interest rate target sterilizes the real economy from shocks to money demand.

Sixth and finally, the assumption that the long bonds are nominal implies that monetary policy shocks will have real effects even in a flexible price model. This is seen most clearly in (43). Innovations in inflation will erode the existing real value of investment debt thus making increased issuance less costly. This effect disappears if the debt is only one period ($\kappa = 0$), or if the debt is indexed to inflation (κ is a real payment).

Debt market policies.

To close the model, we need one more restriction that will pin down the behavior in the long debt market. We will consider two different policy regimes for this market: (i) exogenous debt, and (ii) endogenous debt. We will discuss each in turn.

Exogenous debt. The variable b_t denotes the real value of long term government debt on the balance sheet of FI's. There are two distinct reasons why this variable could fluctuate. First, the central bank could engage in long bond purchases ("quantitative easing," or QE). Second, the fiscal authority could alter the mix of short debt to long debt in its maturity structure. We will model both of these scenarios as exogenous movements in long debt. Under either scenario, the long yield r_t^{10} will be endogenous. Our benchmark experiments will hold this variable fixed at steady state, $b_t = 0$. We will also explore QE policies. To model a persistent and hump-shaped QE policy shock we will use an AR(2):

$$b_t = \rho_1^b b_{t-1} + \rho_2^b b_{t-2} + \epsilon_t^b \quad (63)$$

Within such an exogenous debt regime, we will also consider policies in which the Taylor rule for the short rate responds to some measure of the term premium:

$$r_t = \rho r_{t-1} + (1 - \rho)(\tau_\pi \pi_t + \tau_y y_t^{gap} + \tau_{tp} tp_t) \quad (64)$$

where the term premium (tp_t) is defined as in (61). As noted earlier, there are reasons to think that such a policy may be welfare-improving.

Endogenous debt. The polar opposite scenario is a policy under which the central bank pegs the term-premium $tp_t = 0$. Under this policy regime the level of long debt b_t will be endogenous. Under the term premium peg, the balance sheet of the intermediary and the leverage constraint, FI net worth and household deposits will be constant. Hence, any increase of FI holdings of investment debt is achieved via the central bank purchasing government bonds. The proceeds from this sale effectively finances loans for investment.

3. Quantitative Results.

a. **Estimation and Calibration.**

We calibrate several parameters to match long run features of US data, features that are not well identified by the high frequency business cycle data. We interpret a model period as a quarter, and set $\beta = 0.99$. The production parameters are given by $\alpha = 0.33$, and $\delta = 0.02$. The elasticity parameters are set at $\epsilon_p = \epsilon_w = 5$, implying a 20% mark-up in both price and wage-setting. We use evidence on interest rate spreads and leverage to pin down two primitive parameters. The steady-state loan-deposit spread and leverage ratio are given by:

$$\zeta = \left(\frac{R_{SS}^{10}}{R_{SS}^{10,EH}} \right)^{-1}$$

$$L_{SS} = \left[1 + (\Phi_{SS} - 1) \left(\frac{R_{SS}^{10}}{R_{SS}^{10,EH}} \right) \right]^{-1}$$

We will choose the parameters ζ and Φ_{SS} , to match a term premium of 100 annual bp, and a leverage level of $L_{SS} = 6$. This is the same calibration as in Gertler and Karadi (2013). The government and investment bonds will both be calibrated to a duration of 40 quarters, $(1 - \kappa)^{-1} = 40$. We also need to calibrate the balance sheet proportion, $\frac{\bar{B}_{SS}}{N_{SS}}$. This is proportional to the fraction of FI assets held as long term debt: $\frac{\bar{B}_{SS}}{N_{SS}} = \frac{\bar{B}_{SS}}{\bar{F}_{SS} + \bar{B}_{SS}} * L_{SS}$. Given our interpretation of the FIs as the financial sector, we set the ratio of government securities to total FI assets to $\frac{\bar{B}_{SS}}{\bar{F}_{SS} + \bar{B}_{SS}} = 40\%$, comparable to data on outstanding government debt and investment spending.

The remaining parameters are estimated using familiar Bayesian techniques. The model includes seven exogenous shocks so that we need at least seven observables. We treat as observables the growth rates of real GDP, real gross private domestic investment, real wages, and the PCE index. Employment and real wages are measured as in Carlstrom, Fuerst, Ortiz and Paustian (2014). The two interest rate measures are the effective federal funds rate, and the series on the term premium on a 10-year Treasury estimated by Adrian, Crump, and Moench (2013). The term premium series is available starting in 1962:1. We end our estimation before the zero bound and the financial crisis, so our estimation period is 1962:1-2008:4. All data is de-meanned.

Distribution functions and priors on the estimated coefficients are outlined in Table 1. These largely follow the literature (see Carlstrom et al. (2014)). The key parameter of interest is the portfolio adjustment cost parameter ψ_n . For this we assume a diffuse uniform prior between 0 and 10. Recall that the model collapses to the standard DNK model if $\psi_n = 0$. The net worth elasticity is given by $1/\psi_n$, so that our priors imply an elasticity between 0.1 and infinity.

The coefficient estimates are also reported in Table 1. A few comments are in order. First, the estimates include modest levels of wage and price indexation. But the estimates imply a much higher degree of wage stickiness compared to price stickiness. Although nominal wages are indexed to overall price inflation ($\iota_w = 0.51$), only 4% of nominal wages are freely re-set each period compared to 25% of nominal prices ($\theta_w = 0.96$, $\theta_p = 0.75$). As noted above, this suggests that the labor wedge will be of central importance.

Second, the four main drivers of output are shocks to TFP, the investment technology, credit shocks, and natural rate shocks. The fraction of output variance explained by these three shocks is 21%, 36%, 28%, and 13%, respectively. Hence, we will concentrate on these four shocks below.

Third, and finally, the point estimate of the adjustment cost parameter is $\psi_n = 0.79$, or a net worth elasticity of 1.27. The 90% confidence window on this parameter is wide, with estimated elasticities between 0.81 and 2.95. We will conduct sensitivity analysis on this parameter below.

b. A QE Shock.

Although not our primary focus, it is natural to first consider a QE-type shock in the estimated model. Figure 1 graphs the change in the Fed's bond portfolio relative to the government debt in the hands of the domestic public. The QE policies are quite apparent. We will consider a QE shock that decreases b_t by 6.5%, comparable to the magnitude in Figure 1 (roughly \$300 billion).¹ To match the persistent nature of this expansion we set $\rho_1^b = 1.8$, and $\rho_2^b = -0.81$. Empirical estimates of the response of the 10 year yield to these QE shocks vary from no effect to over 45 bp (eg., the evidence discussed in Chen et al. (2013)). Using the

¹ Recall that b_t is the amount of government debt held by the FI's, so that a QE shock is a decrease in b_t .

estimated model parameters, the impulse response to the QE shock is exhibited in Figure 2. The policy shock has a modest (19 bp at its peak) but persistent effect on the 10 year yield, with most of this movement being driven by changes in the term premium. This term premium effect dissipates as net worth responds and segmentation returns to steady state levels, so that the long rate is eventually driven by the path of the short rate. The policy has a persistent and significant effect on investment and output, while consumption at first declines modestly before rising subsequently. The increased output leads to a policy-induced increase in the funds rate. The funds rate eventually overshoots its long run level, thus leading to a persistent decline in the long rate.

Sensitivity analysis on the QE experiment is reported in Table 1. The first observation is that the quantitative results are only modestly affected by the size of the adjustment costs on net worth, ψ_n . The peak investment response only varies from 3.1% to 5.2% as we vary the adjustment cost parameter from one end of the 90% confidence interval to the other end. A key parameter is the calibrated duration of the investment bonds. Longer maturities for the investment bonds lead to much larger effects on both real activity and the long bond term premium. For example, as we move from 1 year bonds to 20 year bonds, the peak investment response increases from 1.6% to 5.7%. As the maturity of the investment bonds decreases, more of the QE purchase is absorbed by movements in the yield and not movements in real activity. Recall that without a borrowing constraint for investment, changes in the term premium would have no real effects.²

c. **Other shocks under exogenous and endogenous debt policies.**

Figures 3-9 look at the effect of the estimated shocks. Each figure considers two different policy scenarios: (i) the “exogenous debt” case in which the long bond portfolio is held fixed and the short rate is conducted according to a familiar Taylor rule, and (ii) the “endogenous debt” case in which the bond portfolio responds endogenously to hold the term premium fixed. The former is the assumption used in estimating the

² If we vary the duration of government debt, κ , with an exogenous debt policy there will be no real impact. Government bond duration separates out from the rest of the system (see equations 39 and 40) so κ only affects bond pricing and the term premium.

model. But as noted earlier, the term premium peg is of interest because it largely stabilizes the market segmentation distortion so that the IRFs to these shocks will mirror their DNK counterparts.

As noted earlier, the shocks driving output variability include shocks to investment, TFP, credit markets, and the natural rate, respectively. We will review the shocks in this order. For the TFP and investment shocks in Figures 3-4, the increase in the term premium dampens investment and thus output behavior. The term premium increases because the shocks increase the demand for investment (a demand-side channel) and increase the real value of existing investment debt (a supply-side channel) thus making it more costly to absorb additional debt. By altering investment behavior, the increase in the term premium also changes the composition of output, shifting it more towards consumption.

The credit shock is charted in Figure 5. In this linear world, these shocks can be interpreted in two ways. First, and as modeled above, they can be thought of as shocks to the hold-up problem. Second, they can instead be interpreted as shocks to the FI's impatience parameter. In both cases, they imply exogenous increase in the term premium. As with the previous shocks, the increase in the term premium increases the relative cost of investment, leading to an increase in consumption but a decline in overall output. As discussed above, these shocks are entirely sterilized under a term-premium peg. The central bank passively engages in QE purchases of an order of magnitude comparable to the QE experiment in Figure 2.

Finally, the shock to the natural rate is something of a pure demand shock in that there is an increase in the desire for current consumption (see Figure 6). This naturally crowds out investment, but leads to a sustained increase in output. The decline in the term premium comes from a decline in desired investment, and a decline in the real value of existing investment debt. As with the credit shock, the term premium is countercyclical for natural rate shocks. In contrast, the term premium is procyclical for the TFP and investment shocks. For the estimated model as a whole, the implied term premium is mildly procyclical (the correlation with output is 0.12).

The remaining three shocks (monetary policy, price mark-ups, and wage mark-ups) are displayed in Figures 7-9. All three have only modest effects on the term premium so that the behavior follows from the familiar DNK model.

d. **Welfare consequences of a Taylor rule including the term premium.**

In this section we consider the effect of a central bank including the term premium in its FFR Taylor rule. In particular, suppose that the Taylor Rule is given by:

$$r_t = \rho r_{t-1} + (1 - \rho)(\tau_\pi \pi_t + \tau_y y_t^{gap} + \tau_{tp} tp_t) \quad (65)$$

where the term premium (tp_t) is defined above. As an initial experiment, we set the remainder of the Taylor rule at the estimated parameter values in Table 1, and consider the welfare consequences of alternative values for τ_{tp} .

The first step in the analysis is to ensure equilibrium determinacy. Figure 10 looks at equilibrium determinacy for the Taylor rule that includes the term premium. For determinacy under a term premium rule, the response of the short rate to the term premium cannot be too large. At the baseline calibration for the Taylor rule, this restriction is $\tau_{tp} < 0.74$. To understand the intuition for this upper bound, consider the long run response of the policy rate to a permanent shift in inflation:

$$\frac{dr}{d\pi} = \frac{1}{(1+\tau_{tp})} \left(\tau_\pi + \tau_y \frac{dy^{gap}}{d\pi} + \tau_{tp} \frac{dr^L}{d\pi} \right) \quad (66)$$

where we have used the fact that the long run level of the term premium is just the loan-deposit spread. The Taylor Principle is that this policy rate response must exceed unity. How does the loan rate respond to an innovation in inflation? An innovation to inflation leads to an erosion of the real value of the existing debt, which lowers the hold-up problem and thus the long rate, i.e., $\frac{dr^L}{d\pi} < 0$. Hence, if the central bank responds positively to movements in the term premium then it is effectively lowering its response to an innovation in inflation. By this logic, a negative response to the term premium is consistent with determinacy. Further, Figure 10 implies that if this response is not too negative, then the response to inflation can be significantly below unity.

Figures 11 look at the welfare consequence of alternative τ_{tp} for the estimated parameter values. The preferred response is negative, so we only plot the welfare function for $\tau_{tp} < 0$. The optimal response occurs at $\tau_{tp} = -1.20$, implying that a 50 bp increase in the term premium should lead the central bank to lower its funds rate target by 60 bp. The welfare gain, relative to not responding to the term premium, is significant: 1.7% of consumption in perpetuity.

Finally, Table 3 considers two stark policies. In both cases, the central bank uses the baseline Taylor rule (without a response to the term premium). In terms of long debt policy, we consider two extremes: (i) the level of long debt in circulation is held fixed (so that the term premium is endogenous), vs. (ii) the term premium is pegged (so that the level of long debt is endogenous). Note that the term premium peg does not completely stabilize the market segmentation distortion m_t , but its variability is lowered by an order of magnitude. The overall welfare gain is quite large, 2.35% of aggregate consumption in perpetuity. Almost all of this gain comes from the credit and natural rate shocks.

Why are credit and natural rate shocks so important here? There are three underlying distortions in the model: (i) the mark-up of prices over marginal cost creates a wedge between the marginal product of capital and the rental rate, (ii) the combined price and wage mark-up creates a labor wedge between the marginal product of labor and the marginal rate of substitution, and (iii) the market segmentation distortion that creates a wedge between the price of investment goods and the production cost ($M_t > 1$). Because the estimated frequency of price adjustment is significantly larger than wage adjustment, the latter two distortions are key. Natural rate and credit shocks are “demand” shocks in that output and inflation move together. An increase in inflation leads to a decline in the two key distortions as real wages fall, and inflation erodes the value of existing investment debt and thus leads to a decline in the term premium. Hence, the labor wedge and segmentation distortion positively co-move with credit shocks and shocks to the natural rate. But this is disastrous from a welfare perspective. Under a term premium peg, this co-movement goes trivially to zero.

4. Conclusion.

This paper has built a model to analyze the Quantitative Easing policy used by the Fed during the recent ZLB environment. At the core of any such model is an assumption about market segmentation. In the present model we assume the short term money market is segmented from the long term bond market. Households buy long-term debt instruments indirectly by providing deposits to intermediaries. But intermediaries are limited in their ability to arbitrage the return differentials because the amount of deposits is constrained by an intermediary's net worth. Risk neutral intermediaries would immediately increase net worth to eliminate these movements but the intermediary faces adjustment costs in varying the size of its portfolio. Finally these long-term yields affect real economic activity because of a loan-in-advance constraint for capital investment.

We show that the real impact of this segmentation is meaningful. These real effects arise because the assumed segmentation introduces a time-varying wedge or distortion on the cost of investment goods. But any wedge needs a remedy. We emphasize two results. First, a monetary policy that includes the term premium in a Taylor rule can dampen movements in the market segmentation distortion. In particular, welfare is improved modestly when the short-term rate responds negatively to the term premium. Second, a policy that makes the balance sheet endogenous by directly targeting the term premium will sterilize credit shocks. The advantage of this sterilization depends quite naturally on the importance of credit shocks in the business cycle.

We have assumed that government and private sector bonds are perfect substitutes. Hence, when government bonds are purchased from intermediaries, they respond by replacing public with private debt one for one. In practice, this link is less strong because of imperfect substitutability. Hence, our model is likely to give an upper bound on the impact of asset purchases. It would be useful to extend the model to include imperfect substitutability. We leave this to future work.

APPENDIX.

A. Nonlinear equilibrium conditions:

$$\Lambda_t = \frac{b_t}{C_t - hC_{t-1}} - E_t \frac{\beta h b_{t+1}}{C_{t+1} - hC_t} \quad (\text{A1})$$

$$\Lambda_t = E_t \beta \Lambda_{t+1} \frac{R_t}{\Pi_{t+1}} \quad (\text{A2})$$

$$(W_t^*)^{(1+\epsilon_w \eta)} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{G_{1t}}{G_{2t}} \quad (\text{A3})$$

$$G_{1t} = W_t^{\epsilon_w(1+\eta)} b_t B H_t^{1+\eta} + \beta \theta_w \left(\frac{\Pi_{t+1}}{\Pi_t^w} \right)^{\epsilon_w(1+\eta)} G_{1t+1} \quad (\text{A4})$$

$$G_{2t} = \Lambda_t W_t^{\epsilon_w} H_t + \beta \theta_w \left(\frac{\Pi_{t+1}}{\Pi_t^w} \right)^{(\epsilon_w - 1)} G_{2t+1} \quad (\text{A5})$$

$$W_t^{1-\epsilon_w} = (1 - \theta_w)(W_t^*)^{1-\epsilon_w} + \theta_w \left(\frac{\Pi_{t-1}^w}{\Pi_t} \right)^{1-\epsilon_w} W_{t-1}^{1-\epsilon_w} \quad (\text{A6})$$

$$\Lambda_t P_t^k M_t = E_t \beta \Lambda_{t+1} [R_{t+1}^k + (1 - \delta) P_{t+1}^k M_{t+1}] \quad (\text{A7})$$

$$\Lambda_t Q_t^l M_t = E_t \beta \Lambda_{t+1} \frac{[1 + \kappa_l Q_{t+1}^l M_{t+1}]}{\Pi_{t+1}} \quad (\text{A8})$$

$$V_t^h = b_t \left\{ \ln(C_t - hC_{t-1}) - d_t^w B \frac{H_t^{1+\eta}}{1+\eta} \right\} + \beta E_t V_{t+1}^h \quad (\text{A9})$$

$$R_t^k = MC_t MPK_t \quad (\text{A10})$$

$$W_t = MC_t MPL_t \quad (\text{A11})$$

$$\Pi_t^* = \frac{\epsilon_p}{\epsilon_p - 1} \frac{X_{1t}}{X_{2t}} \Pi_t \quad (\text{A12})$$

$$X_{1t} = MC_t \Lambda_t Y_t + \beta \theta_p \Pi_t^{-\epsilon_p} \Pi_{t+1}^{\epsilon_p} X_{1t+1} \quad (\text{A13})$$

$$X_{2t} = \Lambda_t Y_t + \beta \theta_p \Pi_t^{\epsilon_p(1-\epsilon_p)} \Pi_{t+1}^{\epsilon_p - 1} X_{2t+1} \quad (\text{A14})$$

$$\Pi_t^{1-\epsilon_p} = (1 - \theta_p)(\Pi_t^*)^{1-\epsilon_p} + \theta_p \Pi_{t-1}^{\epsilon_p(1-\epsilon_p)} \quad (\text{A15})$$

$$d_t = \Pi_t^{\epsilon_p} [(1 - \theta_p)(\Pi_t^*)^{-\epsilon_p} + \theta_p \Pi_{t-1}^{-\epsilon_p} d_{t-1}] \quad (\text{A16})$$

$$d_t^w = (1 - \theta_w) \left(\frac{W_t^*}{W_t} \right)^{-\epsilon_w(1+\eta)} + \theta_w \left(\frac{W_{t-1}}{W_t} \right)^{-\epsilon_w(1+\eta)} \left(\frac{\Pi_t}{\Pi_{t-1}^w} \right)^{\epsilon_w(1+\eta)} d_{t-1}^w \quad (\text{A17})$$

$$C_t + I_t = Y_t \quad (\text{A18})$$

$$Y_t = \frac{A_t K_t^\alpha L_t^{1-\alpha}}{d_t} \quad (\text{A19})$$

$$K_t = (1 - \delta)K_{t-1} + \mu_t \left[1 - S \left(\frac{I_t}{I_{t-1}} \right) \right] I_t \quad (\text{A20})$$

$$P_t^k \mu_t \left[1 - S \left(\frac{I_t}{I_{t-1}} \right) - \frac{I_t}{I_{t-1}} S' \left(\frac{I_t}{I_{t-1}} \right) \right] = 1 - \frac{\beta \Lambda_{t+1}}{\Lambda_t} P_{t+1}^k \mu_{t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 S' \left(\frac{I_{t+1}}{I_t} \right) \quad (\text{A21})$$

$$\bar{B}_t + \bar{F}_t \leq N_t L_t \quad (\text{A22})$$

$$L_t = \frac{E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}}}{\left[E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}} + (\Phi_t - 1) E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}} \left(\frac{R_{t+1}^L}{R_t^d} \right) \right]} \quad (\text{A23})$$

$$P_t^k I_t \leq \bar{F}_t - \kappa \frac{\bar{F}_{t-1}}{\Pi_t} \frac{Q_t}{Q_{t-1}} \quad (\text{A24})$$

$$\Lambda_t [1 + N_t f'(N_t) + f(N_t)] = E_t \beta \zeta \Lambda_{t+1} \frac{P_t}{P_{t+1}} [(R_{t+1}^L - R_t^d) L_t + R_t^d] \quad (\text{A25})$$

$$R_{t+1}^L = \frac{1 + \kappa Q_{t+1}}{Q_t} \quad (\text{A26})$$

$$R_t^{10} = Q_t^{-1} + \kappa \quad (\text{A27})$$

where

$$f(N_t) \equiv \frac{\psi_n}{2} \left(\frac{N_t - N_{ss}}{N_{ss}} \right)^2$$

$$S \left(\frac{I_t}{I_{t-1}} \right) \equiv \frac{\psi_i}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2$$

$$\bar{B}_t \equiv Q_t \frac{B_t}{P_t}$$

$$\bar{F}_t \equiv Q_t^l \frac{F_t}{P_t}$$

B. Steady State:

We choose B so that $H_{SS} = 1$. We also normalize $\mu_{SS} = A_{SS} = 1$.

$$\Lambda_{SS} = \frac{(1 - \beta h)}{(1 - h)C_{SS}}$$

$$1 = \beta R_{SS}$$

$$B = W_{SS}\Lambda_{SS}$$

$$R_{SS}^k = \frac{M_{SS}[1 - \beta(1 - \delta)]}{\beta}$$

$$M_{SS} = \frac{\beta}{(1 - \beta\kappa)Q_{SS}^I}$$

$$R_{SS}^k = MC_{SS}MPK_{SS}$$

$$W_{SS} = MC_{SS}MPL_{SS}$$

$$1 = \frac{\epsilon_p}{\epsilon_p - 1} \frac{X_{1SS}}{X_{2SS}}$$

$$X_{1SS} = \frac{MC_{SS}\Lambda_{SS}Y_{SS}}{1 - \beta\theta_p}$$

$$X_{2SS} = \frac{\Lambda_{SS}Y_{SS}}{1 - \beta\theta_p}$$

$$d_{SS} = 1$$

$$G_{1SS} = \frac{W_{SS}^{\epsilon_w(1+\eta)} B H_{SS}^{1+\eta}}{1 - \beta\theta_w}$$

$$G_{2SS} = \frac{\Lambda_{SS} W_{SS}^{\epsilon_w} H_{SS}}{1 - \beta\theta_w}$$

$$d_{SS}^w = 1$$

$$W_{SS} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{B H_{SS}^\eta}{\Lambda_{SS}}$$

$$C_{SS} + I_{SS} = Y_{SS}$$

$$Y_{SS} = K_{SS}^\alpha$$

$$\delta K_{SS} = I_{SS}$$

$$P_{SS}^k = 1$$

$$\bar{B}_{SS} + \bar{F}_{SS} = N_{SS} \left[1 + (\Phi_{SS} - 1) \left(\frac{R_{SS}^L}{R_{SS}} \right) \right]^{-1}$$

$$I_{SS} = \bar{F}_{SS}(1 - \kappa)$$

$$1 = \beta \zeta R_{SS}^L$$

$$Q = (R_{SS}^L - \kappa)^{-1}$$

$$R_{SS}^{10} = R_{SS}^L$$

Some simplifications:

$$M_{SS} = \frac{(\beta R_{SS}^L - \beta \kappa)}{(1 - \beta \kappa)} > 1$$

$$MC_{SS} = \frac{\epsilon_p - 1}{\epsilon_p} < 1$$

$$\frac{K_{SS}}{Y_{SS}} = \frac{\beta \alpha MC_{SS}}{M_{SS} [1 - \beta(1 - \delta)]}$$

$$K_{SS} = \left(\frac{K_{SS}}{Y_{SS}} \right)^{1/(1-\alpha)}$$

$$C_{SS} = Y_{SS} - \delta K_{SS}$$

C. Calibration:

$$\beta = 0.99, \beta R_{SS}^L = 1.0025, L_{SS} = 6.$$

$$(1 - \Phi_{SS})(1.0025) = \frac{L_{SS} - 1}{L_{SS}}$$

$$\text{Duration} = 40 = (1 - \kappa)^{-1}$$

$$\frac{\bar{B}_{SS}}{\bar{B}_{SS} + \bar{F}_{SS}} = 40\%$$

D. Details of the FI's value function.

The FI's problem is given by:

$$V_t \equiv \max_{N_t, div_t} E_t \sum_{j=0}^{\infty} (\beta\zeta)^j \Lambda_{t+j} div_{t+j} \quad (A1)$$

$$div_t + N_t[1 + f(N_t)] \leq X_t N_{t-1} \quad (A2)$$

where

$$X_t \equiv \frac{P_{t-1}}{P_t} [(R_t^L - R_{t-1}^d)L_{t-1} + R_{t-1}]. \quad (A3)$$

The function $f(N_t) \equiv \frac{\psi_n}{2} \left(\frac{N_t - N_{ss}}{N_{ss}} \right)^2$, denotes an adjustment cost function that dampens the ability of the FI to adjust the size of its portfolio in response to shocks. We assume that leverage is given exogenously to the FI. We will return to this below. Assuming an interior solution, the FI's accumulation choice is given by:

$$[N_t f'(N_t) + f(N_t)] = \frac{E_t \beta \zeta \Lambda_{t+1} X_{t+1} - \Lambda_t}{\Lambda_t} \equiv z_t \quad (A4)$$

This implies that N_t is a function only of the forecasted market spread:

$$N_t = h(z_t) \quad (A5)$$

We now conjecture the form of the value function:

$$V_t = \Lambda_t X_t N_{t-1} - g_t. \quad (A6)$$

We check this conjecture by putting it into the Bellman equation. The Bellman equation is given by:

$$V_t = \Lambda_t X_t N_{t-1} - \Lambda_t N_t [1 + f(N_t)] + \beta \zeta E_t V_{t+1} \quad (A7)$$

Substituting in (A6) we have:

$$-g_t = -\Lambda_t N_t [1 + f(N_t)] + \beta \zeta E_t (\Lambda_{t+1} X_{t+1} N_t - g_{t+1}) \quad (A8)$$

Using (A4) we have:

$$g_t = \Lambda_t N_t f(N_t) + \beta \zeta E_t g_{t+1} - N_t \Lambda_t [N_t f'(N_t) + f(N_t)] \quad (A9)$$

Using (A5) we have:

$$g_t = \Lambda_t [h(z_t) f(h(z_t)) - h(z_t) z_t] + \beta \zeta E_t g_{t+1} \quad (A10)$$

Or:

$$g_t = E_t \sum_{j=0}^{\infty} (\beta \zeta)^j \Lambda_{t+j} [h(z_{t+j}) f(h(z_{t+j})) - h(z_{t+j}) z_{t+j}] \quad (\text{A11})$$

Hence g_t is a function of the current and forecasted market spreads z_t . In particular, g_t is independent of net worth. Note that as net worth rises the relative importance of this affine term declines so that the elasticity of the value function with respect to net worth ($\frac{d \log V_t}{d \log N_{t-1}} \geq 1$) is decreasing in the level of net worth.

Let us now turn to leverage. We assumed that leverage was exogenous to the FI. This means that the FI cannot alter his leverage level by acquiring more net worth (the only FI-specific variable). The hold-up constraint is given by:

$$E_t V_{t+1}^i \geq \mu_t^i L_t N_t^i E_t \Lambda_{t+1} \frac{P_t}{P_{t+1}} R_{t+1}^L \quad (\text{A12})$$

Note that we have introduced a FI-specific index i so that we can demonstrate that leverage is independent of i . We will calibrate the model so (A12) is binding in the steady state (and thus binding for small shocks around the steady state). We need to show that this constraint does not depend upon individual net worth. Since the value function includes a linear term, we need to reverse-engineer the hold-up function μ_t^i . This is straightforward. Let us assume that the hold-up parameter is given by:

$$\mu_t^i \equiv \Phi_t \left[1 - \frac{1}{N_t^i} \left(\frac{E_t g_{t+1}}{E_t \Lambda_{t+1} X_{t+1}} \right) \right] = \Phi_t \frac{E_t \Lambda_{t+1} X_{t+1} N_t^i - E_t g_{t+1}}{E_t \Lambda_{t+1} X_{t+1} N_t^i} \quad (\text{A13})$$

Note that (A13) implies that higher net worth makes it easier for the FI to shield assets and thus makes the hold-up problem more severe. This increased severity is chosen to match the changing net worth elasticity of the value function (A6). Assumption (A13) implies that the binding incentive constraint (A12) is given by:

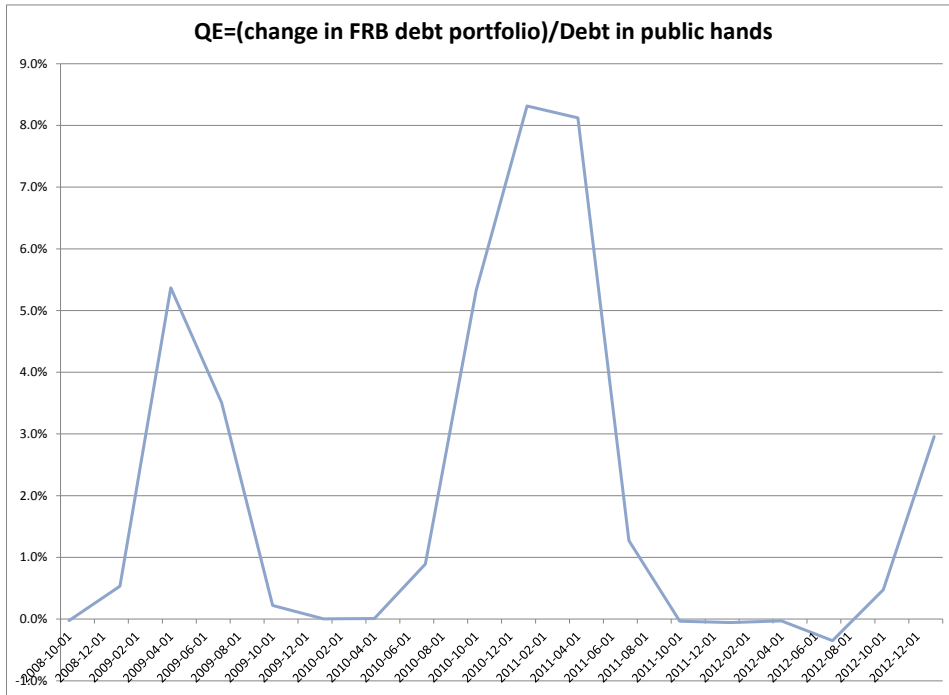
$$E_t \Lambda_{t+1} \frac{P_t}{P_{t+1}} \left[\left(\frac{R_{t+1}^L}{R_t^d} - 1 \right) L_t + 1 \right] = \Phi_t L_t E_t \Lambda_{t+1} \frac{P_t}{P_{t+1}} \frac{R_{t+1}^L}{R_t^d} \quad (\text{A14})$$

As planned, leverage is now a function only of interest rates, etc., outside the control of the FI. That is, leverage is independent of individual net worth. Expression (A14) is the one used in the paper.

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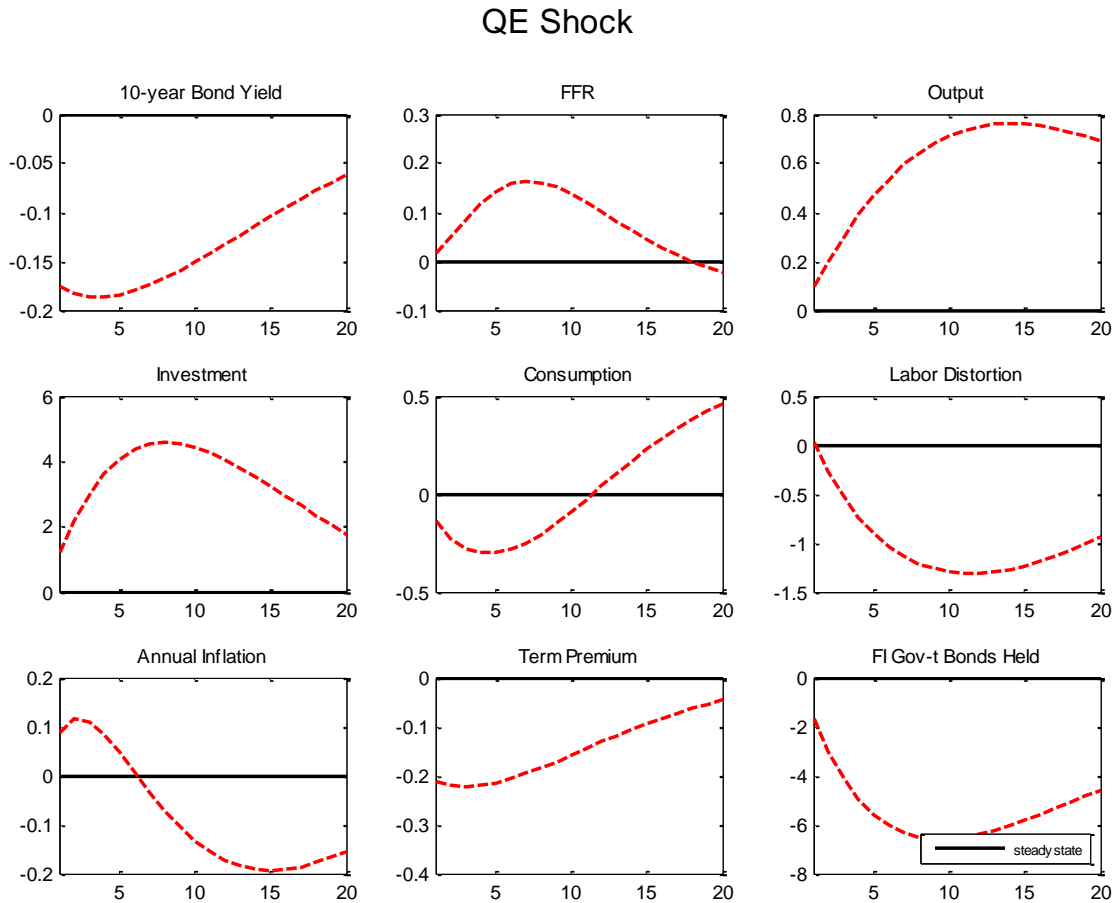
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Figure 1:



Source: FRB St. Louis.

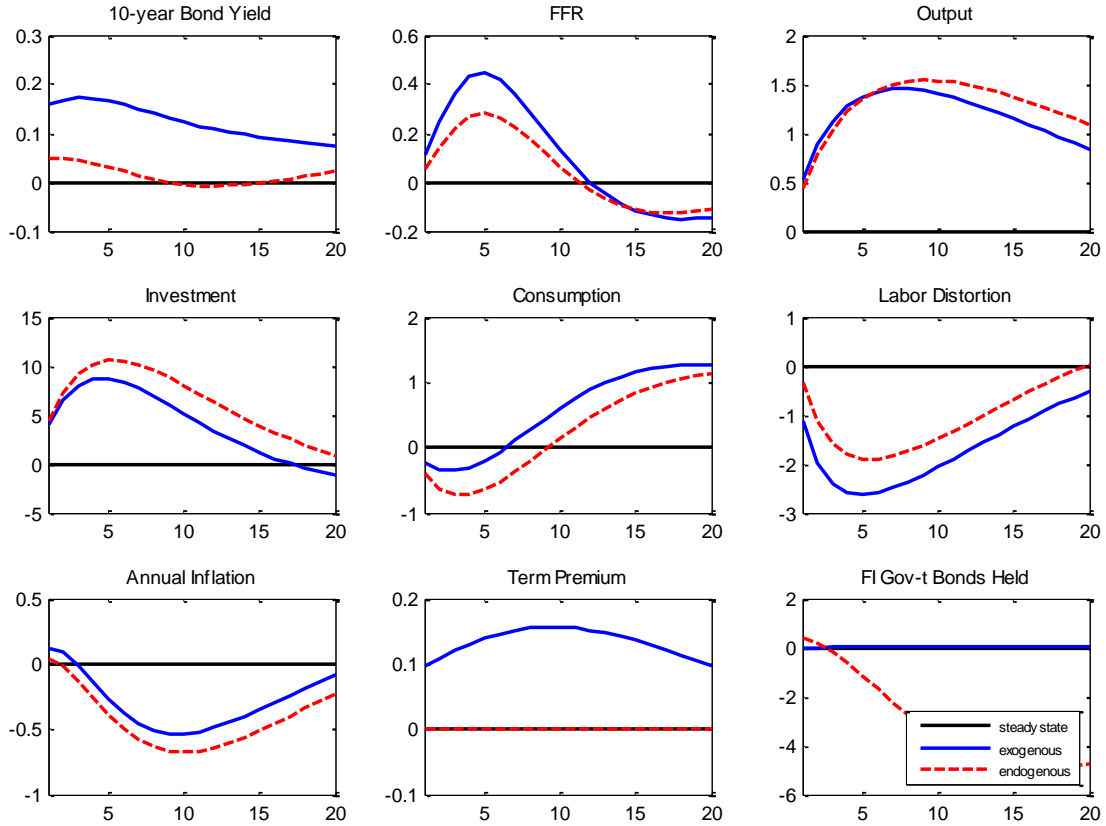
Figure 2: QE experiment, baseline parameter values.



Legend: All variables are in percentage points and all rates are annualized. The variable “Labor Distortion” is the ratio of the marginal product of labor to the marginal rate of substitution.

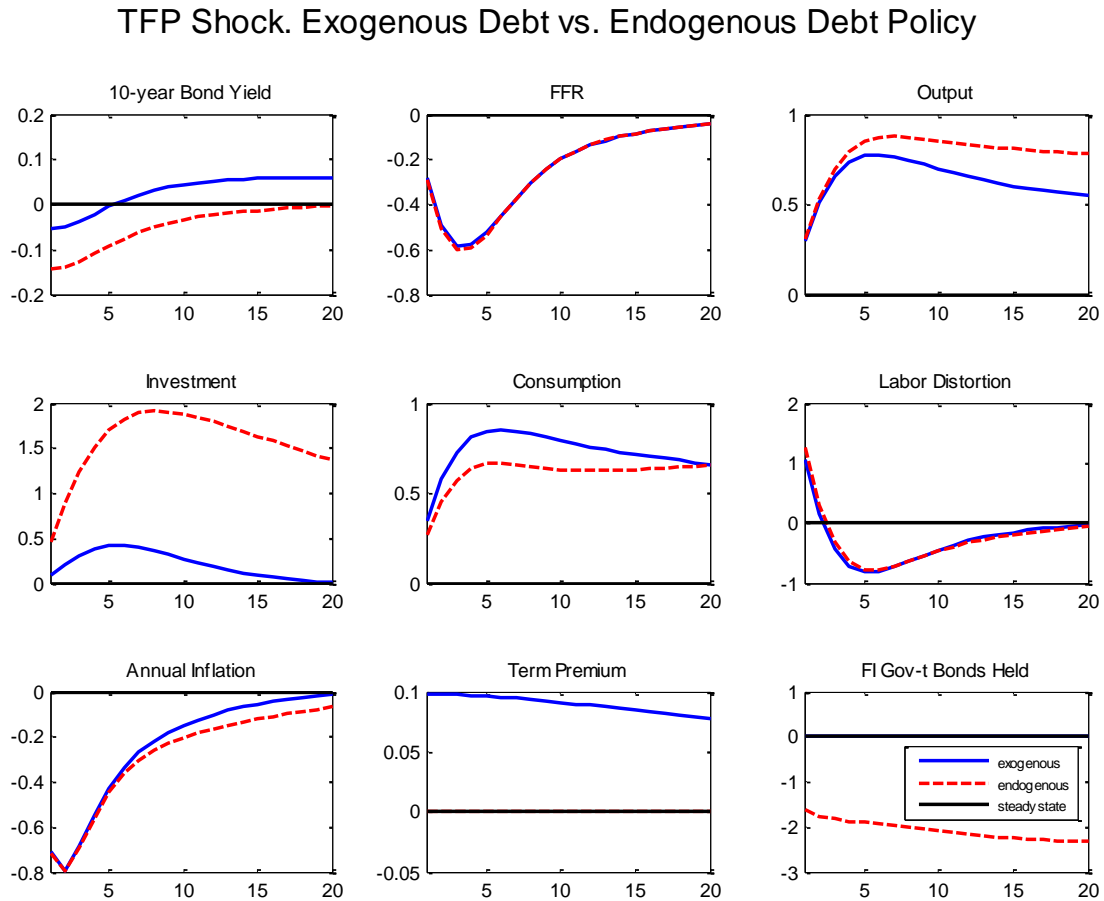
Figure 3: One SD investment shock under exogenous and endogenous debt policies.

Investment Shock. Exogenous Debt vs. Endogenous Debt Policy



Legend: All variables are in percentage points and all rates are annualized. The variable “Labor Distortion” is the ratio of the marginal product of labor to the marginal rate of substitution.

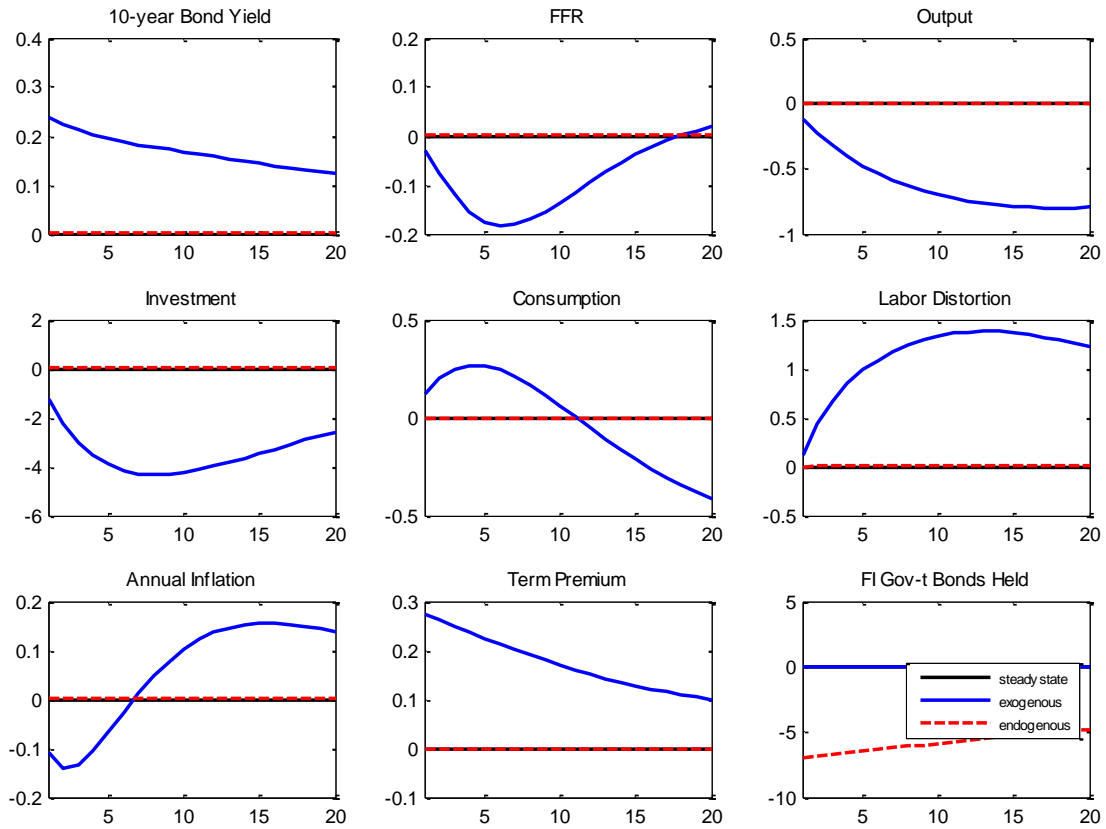
Figure 4: One SD TFP shock under exogenous and endogenous debt policies.



Legend: All variables are in percentage points and all rates are annualized. The variable “Labor Distortion” is the ratio of the marginal product of labor to the marginal rate of substitution.

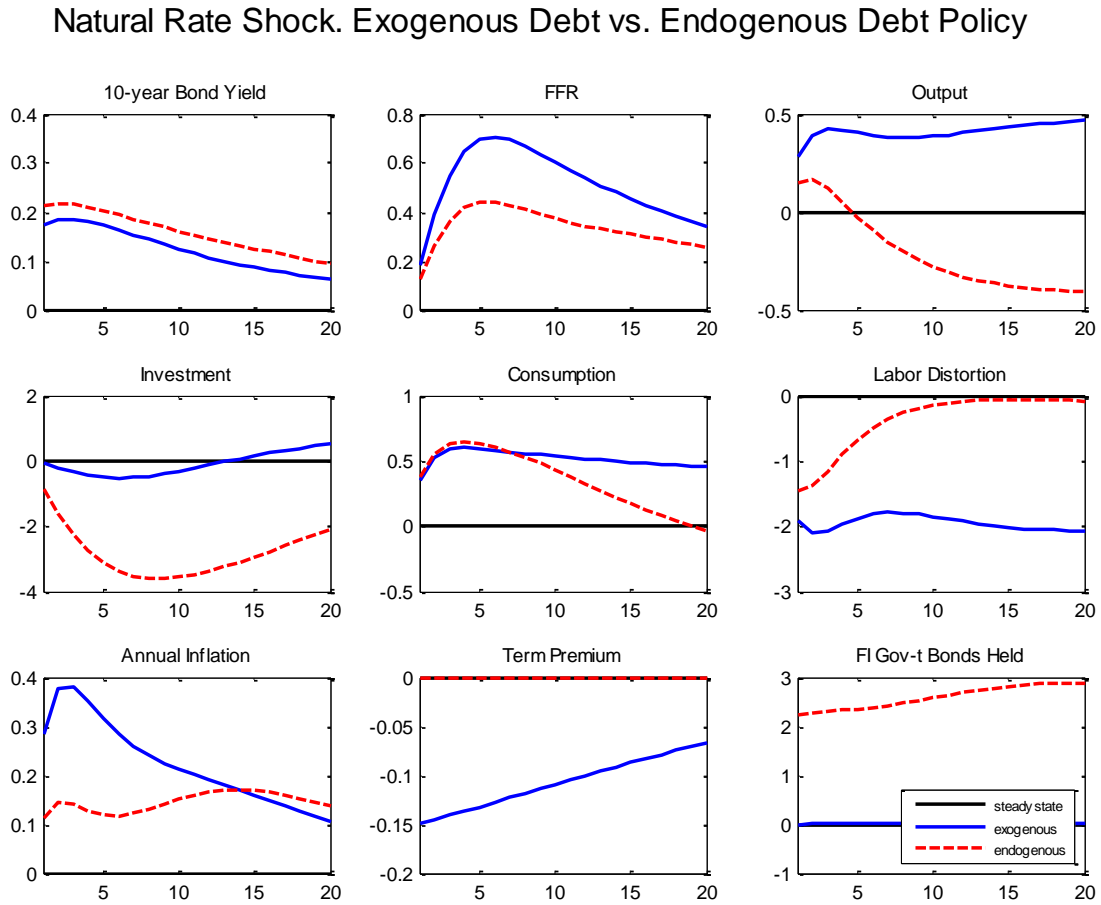
Figure 5: One SD credit shock under exogenous and endogenous debt policies.

Credit Shock. Exogenous Debt vs. Endogenous Debt Policy



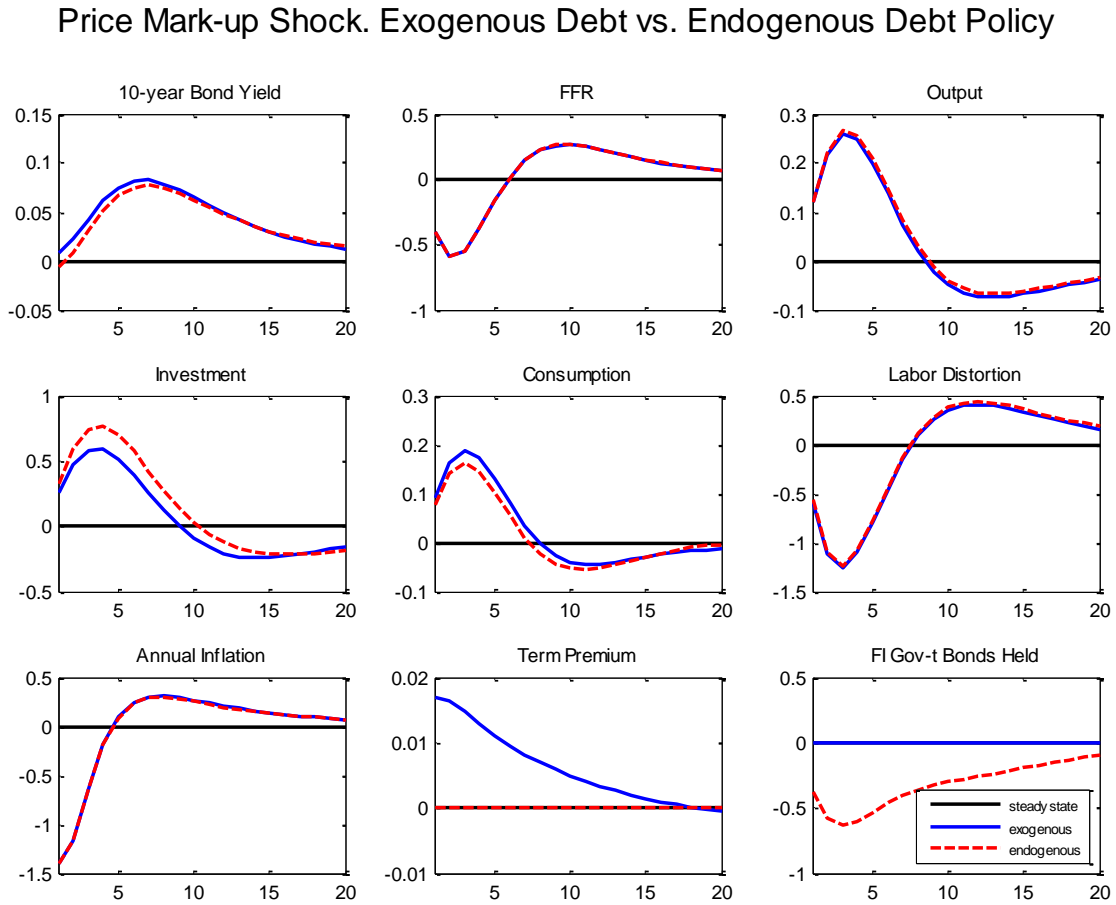
Legend: All variables are in percentage points and all rates are annualized. The variable “Labor Distortion” is the ratio of the marginal product of labor to the marginal rate of substitution.

Figure 6: One SD natural rate shock under exogenous and endogenous debt policies.



Legend: All variables are in percentage points and all rates are annualized. The variable “Labor Distortion” is the ratio of the marginal product of labor to the marginal rate of substitution.

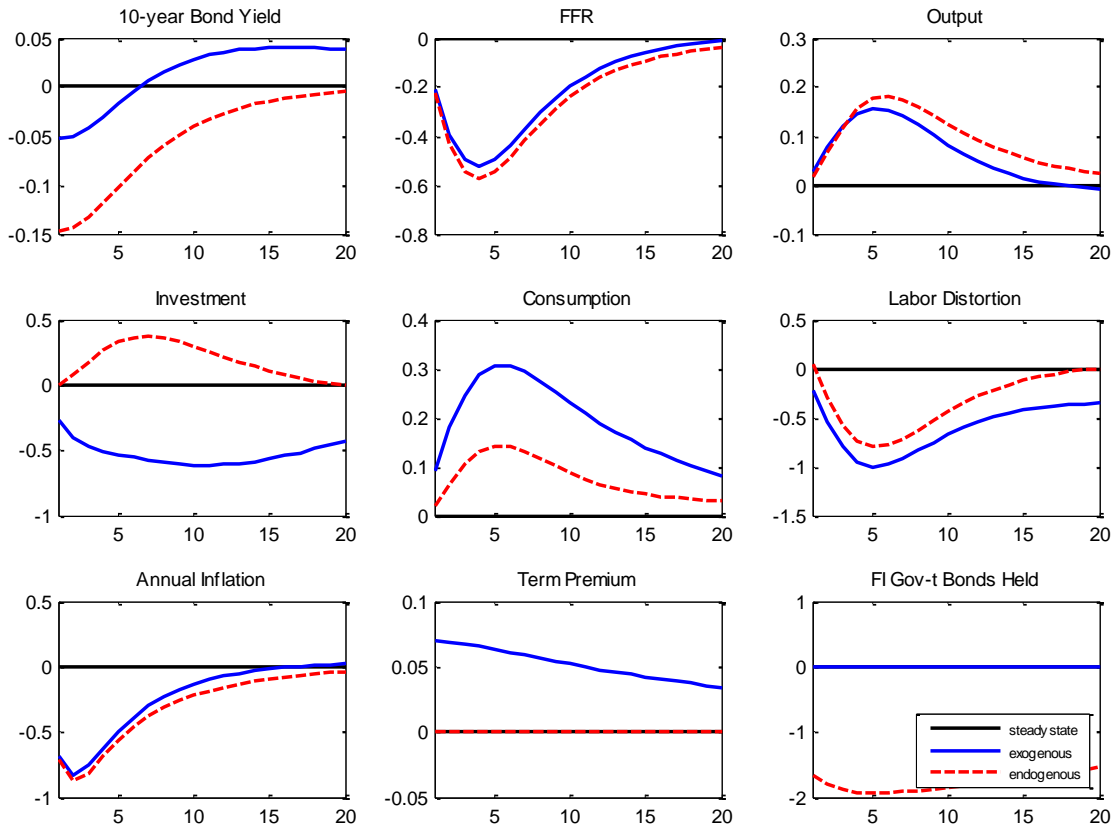
Figure 7: One SD price mark-up shock under exogenous and endogenous debt policies.



Legend: All variables are in percentage points and all rates are annualized. The variable “Labor Distortion” is the ratio of the marginal product of labor to the marginal rate of substitution.

Figure 8: One SD wage mark-up shock under exogenous and endogenous debt policies.

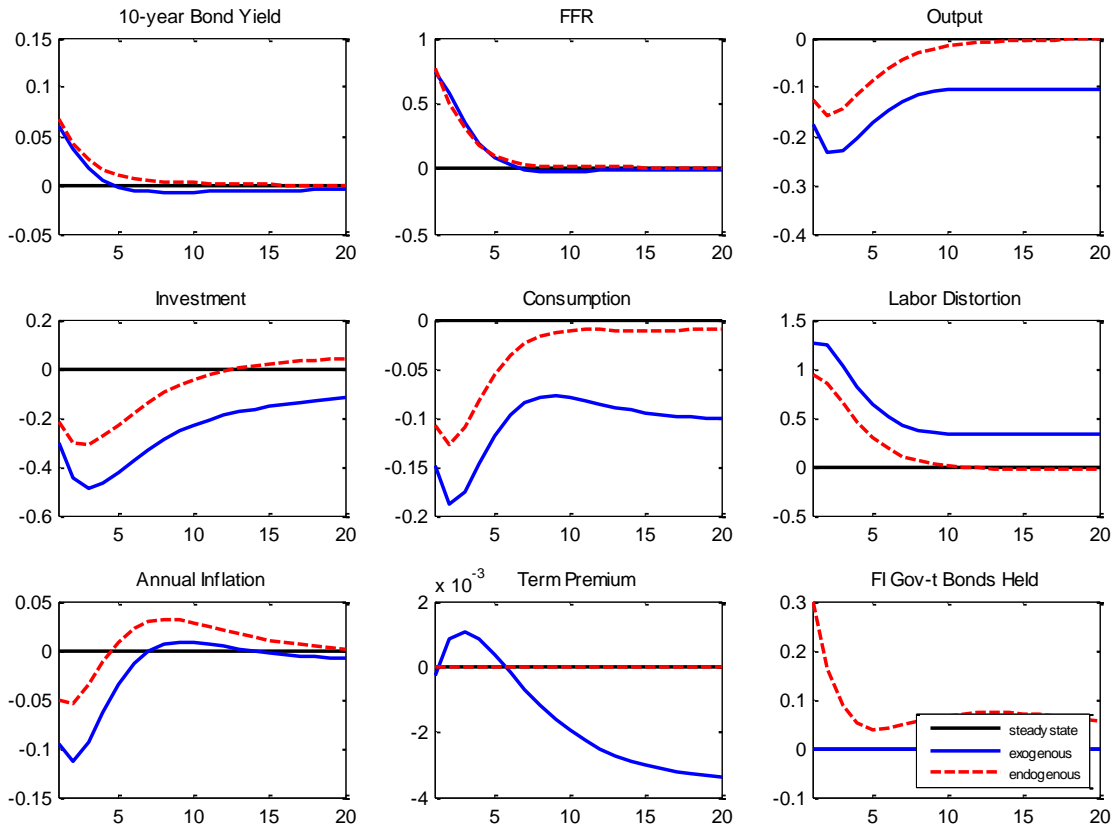
Wage Mark-up Shock. Exogenous Debt vs. Endogenous Debt Policy



Legend: All variables are in percentage points and all rates are annualized. The variable “Labor Distortion” is the ratio of the marginal product of labor to the marginal rate of substitution.

Figure 9: One SD monetary shock under exogenous and endogenous debt policies.

Monetary Policy Shock. Exogenous Debt vs. Endogenous Debt Policy



Legend: All variables are in percentage points and all rates are annualized. The variable “Labor Distortion” is the ratio of the marginal product of labor to the marginal rate of substitution.

Figure 10: Equilibrium determinacy under a term premium Taylor Rule.

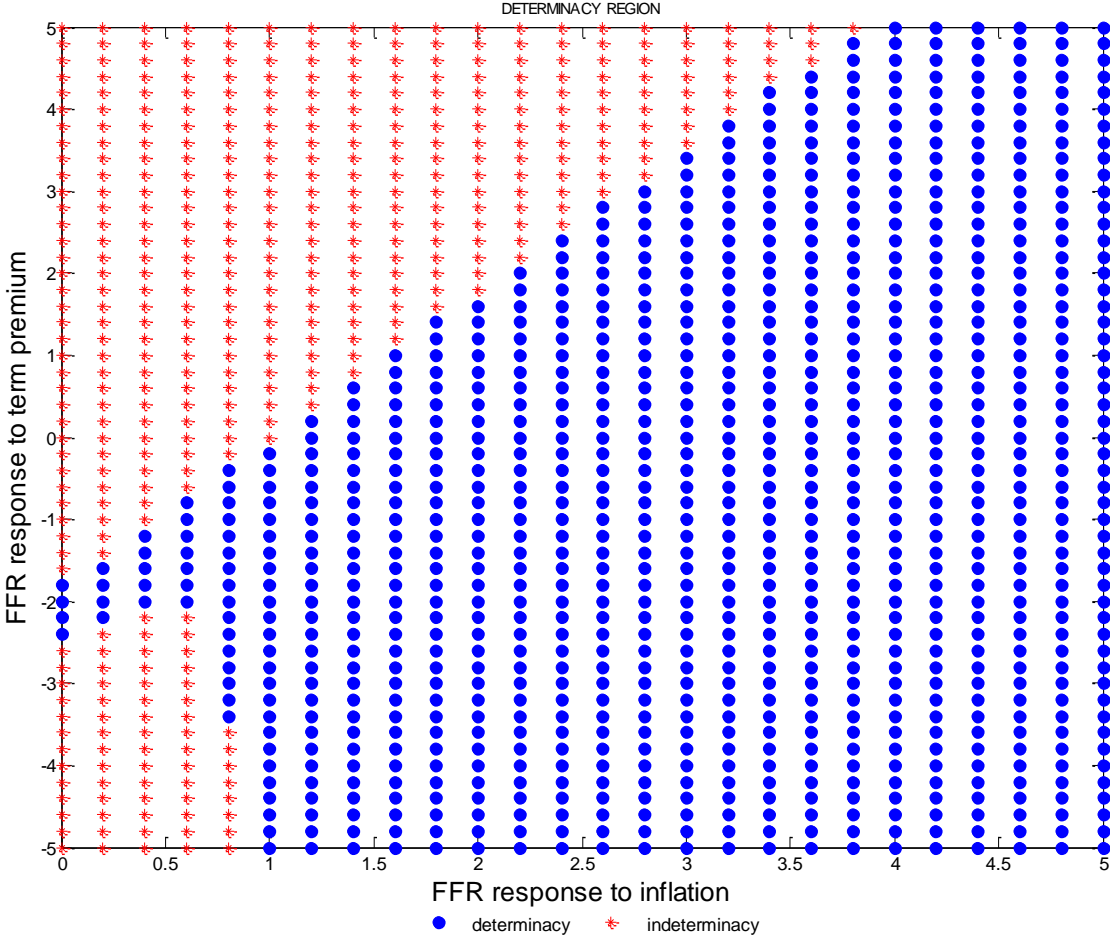
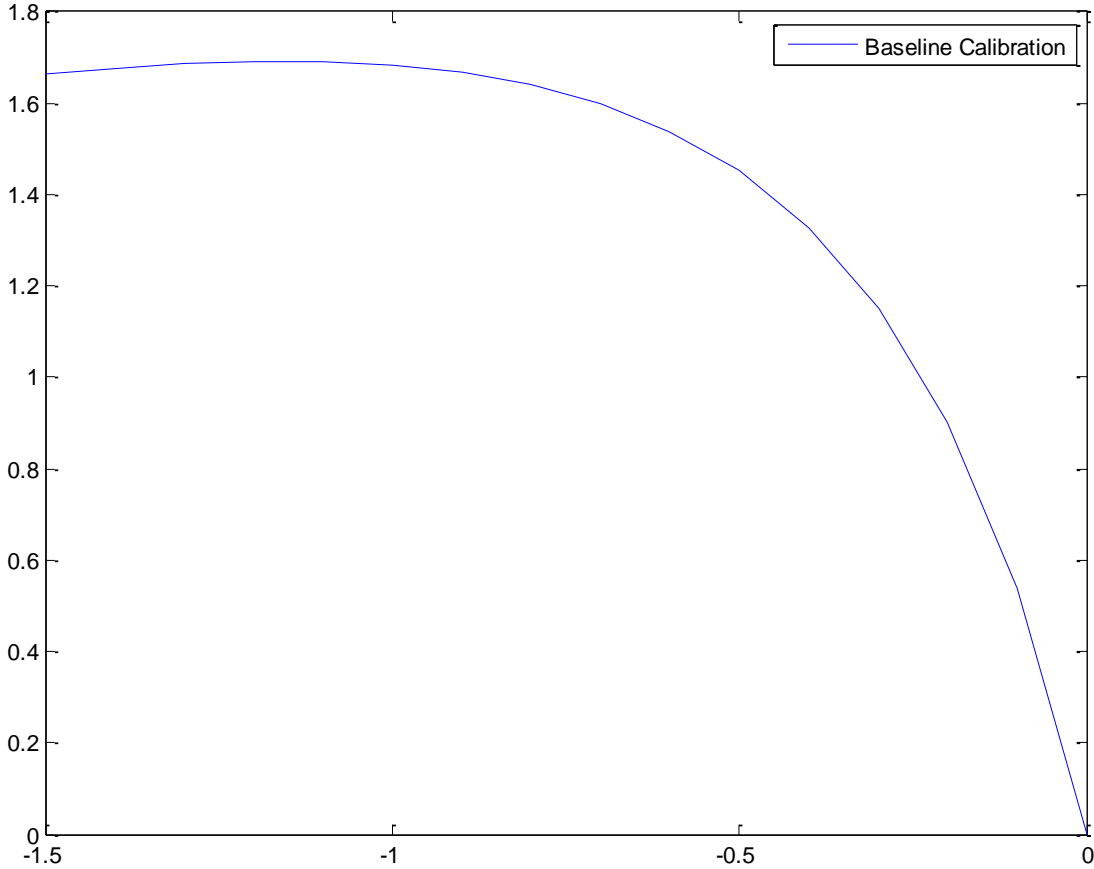


Figure 11: Welfare Consequences of Taylor Rule with term premium response.



The units are in consumption perpetuities, i.e., 0.5 means a perpetual increase in consumption equal to 0.5% of steady-state consumption, or a one-time increase of 50%. The welfare change is on the vertical axis, and the term premium coefficient in the Taylor Rule is on the horizontal axis. The peak welfare gain occurs at $\tau_{prem} = -1.2$.

Table 1: Model Estimation

Coefficient	Prior			Posterior		
	Prior density	Prior mean	pstdev	Post. Mean	5%	95%
η	G	2.00	0.75	2.0259	1.2673	2.7526
h	B	0.60	0.10	0.6225	0.5760	0.6687
ψ_n	U	5.00	2.89	0.7850	0.3389	1.2394
ϕ	G	3.00	1.00	3.2821	2.1857	4.3639
τ_π	N	1.50	0.10	1.4202	1.2828	1.5493
τ_ν	N	0.50	0.10	0.4906	0.3566	0.6292
ρ_i	B	0.80	0.10	0.7712	0.7309	0.8109
ι_p	B	0.60	0.10	0.4175	0.2752	0.5610
ι_w	B	0.60	0.10	0.5110	0.4085	0.6205
κ_{pc}	B	0.20	0.10	0.0860	0.0104	0.1544
κ_w	B	0.20	0.10	0.0002	0.0001	0.0004
ρ_a	B	0.60	0.20	0.9921	0.9841	0.9997
ρ_μ	B	0.60	0.20	0.8695	0.8281	0.9122
ρ_ϕ	B	0.60	0.20	0.9821	0.9682	0.9963
ρ_{mk}	B	0.60	0.20	0.6650	0.4945	0.8405
ρ_{mkw}	B	0.60	0.20	0.2059	0.1036	0.3027
ρ_m	B	0.60	0.20	0.1564	0.0646	0.2515
ρ_{rn}	B	0.60	0.20	0.9483	0.9212	0.9751
σ_a	I	0.50	1.00	0.6481	0.5936	0.7030
σ_i	I	0.50	1.00	7.3454	5.5735	9.2124
σ_{mp}	I	0.10	1.00	0.2151	0.1935	0.2368
σ_{mk}	I	0.10	1.00	0.2442	0.1830	0.3049
σ_{mkw}	I	0.10	1.00	0.4840	0.4103	0.5569
σ_{rn}	I	0.10	1.00	0.1588	0.1179	0.2000
σ_ψ	I	0.50	1.00	2.7196	1.9449	3.4826

N stands for Normal, B-Beta, G-Gamma, U-Uniform, I-Inverted-Gamma distribution. Posterior percentiles are from 2 chains of 100,000 draws generated using a Random Walk Metropolis algorithm. We discard the initial 50,000 and retain one every 5 subsequent draws.

Table 2: Sensitivity Analysis of QE Shock

Parameter Value*	Peak Investment Response	Peak Ten-Year Yield Response	Peak Inflation Response
$\psi_n = 0$	0	0	0
$\psi_n = 0.34$	3.14	-0.14	0.06
$\psi_n = 0.78$ (baseline)	4.57	-0.19	0.12
$\psi_n = 1.24$	5.23	-0.21	0.15
Duration = 4 q	1.57	-0.31	0.01
Duration = 20 q	3.56	-0.21	0.06
Duration = 80 q	5.71	-0.18	0.18

*All parameter values are held at their estimated or calibrated values unless otherwise noted.
Duration is duration of the investment bond.

Table 3: Comparing two stark policies.*

Here we consider two extreme policy choices: holding the balance sheet fixed ($b_t = 0$), vs. a term premium peg, ($tp_t = 0$).

	Credit shocks only	Investment shocks only	TFP shocks only	Discount shocks only	All four shocks
Welfare gain of term premium peg	0.47	0.12	0.13	1.63	2.35

All four shocks.	$b_t = 0$	$tp_t = 0$	$b_t = 0$ (with subsidies)	$tp_t = 0$ (with subsidies)	$b_t = 0$ (flexible wages)	$tp_t = 0$ (flexible wages)
Welfare gain of term premium peg	--	2.35	--	1.89	--	1.80
Mean of segmentation wedge	1.08	1.06	1.02	1	1.14	1.05
Mean of labor wedge	1.74	1.56	1.15	0.99	1.57	1.57

*The welfare units are in consumption perpetuities, i.e., 2.35 means a perpetual increase in consumption equal to 1.82% of steady-state consumption, or a one-time increase of 235%. In the non-stochastic steady state without subsidies, the segmentation wedge is $M_{ss} = 1.07$. The labor wedge is the ratio of the marginal product of labor to the marginal rate of substitution. In the non-stochastic steady state without subsidies, the labor wedge is equal to 1.56. In the case of subsidies, we have $M_{ss} = 1$, and the labor wedge is equal to 1.