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Privately Optimal Contracts and Sub-Optimal Outcomes in a Model of Agency Costs

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This paper derives the privately optimal lending contract in the celebrated financial accelerator model of Bernanke, Gertler and Gilchrist (1999). The privately optimal contract includes indexation to the aggregate return on capital, household consumption, and the return to internal funds. Although privately optimal, this contract is not welfare maximizing as it leads to a sub-optimally high price of capital. The welfare cost of the privately optimal contract (when compared to the planner outcome) is significant. A menu of time-varying taxes and subsidies can decentralize the planner's allocations.

Key words: Agency costs, CGE models, optimal contracting.

JEL codes: C68, E44, E61.

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1. Introduction.

The financial accelerator model of Bernanke, Gertler, and Gilchrist (1999), hereafter BGG, is widely used as a convenient mechanism for integrating financial factors into an otherwise standard DSGE model. The BGG model embeds the costly state verification (CSV) model of Townsend (1979) into an environment with risk neutral entrepreneurs, risk averse households, and aggregate risk. Appealing to insurance concerns, BGG assume that the lending contract between the entrepreneur and lender is characterized by a lender return that is invariant to innovations in aggregate variables. Instead, these aggregate innovations feed directly into entrepreneurial net worth. The behavior of net worth is crucial in the BGG model because the agency costs are diminished by increases in net worth. For example, since the lender's return is fixed, a positive productivity shock shifts wealth to entrepreneurs and thus lowers agency costs. This sets in motion a financial accelerator, a virtuous circle in which higher net worth drives up the price of capital, which in turn increases net worth, etc. This process thus amplifies the effect of the shock. But the vital first step in this amplification is the assumption that the lender's return is pre-determined. Hence, BGG's insurance assumption is fundamental to the financial accelerator in their model. The importance of this insurance assumption is well known. For example, consider the following comment of Chari (2003):

> *A final misgiving [is] about a central ingredient of this model. This comment really applies to Bernanke, Gertler and Gilchrist, upon which this paper is based. These authors have an economy with risk neutral agents called entrepreneurs and risk averse agents called households. They claim that an optimal contract in the presence of aggregate risk has the return paid by entrepreneurs to be a constant, independent of the current aggregate shock. I have trouble understanding this result. Surely, entrepreneurs should and would provide insurance to households against aggregate shocks. One way of providing such insurance is to provide a high return to households when their income from other sources is low and a low return when their income from other sources is high. My own guess is that if they allowed the return to households to be state contingent, then aggregate shocks would have no effects on the decisions of households and would be absorbed entirely by entrepreneurs. Before we push this intriguing financial accelerator mechanism much further, I think it would be wise to make sure that we get the microeconomics right*.

This paper revisits BGG's key assumption and confirms the insurance intuition of Chari. But there are two other effects not anticipated by Chari (2003). First, since entrepreneurs value net worth more when the return to internal funds is high, there is a hedging motive on the part of entrepreneurs. Second, since aggregate shocks are observed by all parties, the optimal contract will include indexation to these shocks.

In this paper, we solve for the privately optimal contract (POC) that incorporates all of these characteristics. We then contrast this POC with the contract assumed in BGG, and analyze the implications for macroeconomics fluctuations. Finally, we solve the problem of an informationally constrained planner. Our principle results include the following. First, the financial contract imposed in the BGG model is not privately optimal. That is, lenders and borrowers would both prefer a different loan contract. Second, the privately optimal contract (POC) has the loan repayment varying in response to innovations in the observed aggregate shocks. Third, the POC is not socially optimal because it leads to a steady-state price of capital that is too high compared to the social optimum. Fourth, the social welfare costs of the privately optimal contract are significant. In our benchmark calibration the conditional welfare cost of the POC is equal to a 0.35% increase in the annual flow of household consumption. Under an alternative calibration, this cost rises to 0.49% in an annual flow. Finally, we demonstrate that the planner outcome represents a Pareto improvement over the POC. We show that the planner outcome can be implemented with a set of state-dependent taxes on household income and monitoring costs.

Our results on the privately optimal contract are related to Krishnamurthy (2003). Krishnamurthy (2003) introduces insurance markets into a three period model where borrowing is secured by collateral as in Kiyotaki and Moore (1997). These insurance markets allow for state contingent debt that is indexed to aggregate shocks. Krishnamurthy (2003) shows that such insurance eliminates any feedback from collateral values to investment, and thus reduces collateral amplification to zero. We have a similar result here: the privately optimal contract has the level of debt repayment indexed one-for-one to the aggregate return on capital, so that bankruptcy rates do not respond to innovations in the return to capital. By itself this indexation dramatically reduces the financial accelerator. But there are two novel twists in the current framework. First since households are risk averse and entrepreneurs are risk-neutral, the POC features a loan repayment rate linked to aggregate consumption. Second, since the entrepreneurs prefer high net worth when the return to internal funds is high, the POC features a repayment linked to the return to internal funds. Taken together, these indexation effects leads to a modest but suboptimal magnification of aggregate shocks when compared to the planner.

We conjecture that our conclusions would be qualitatively unchanged if entrepreneurs were risk averse, but with lower levels of risk aversion than households. But there is a problem with following this approach. The optimality of the debt contract in the underlying CSV framework relies on the assumption that the entrepreneur's payoff is linear. This means that moving away from risk-neutrality also means moving away from the debt contract. The prominence of the original BGG framework suggests that we first pursue these issues in their original setting before exploring contracting with risk-averse entrepreneurs.

Two other notable precedents for the current paper are Lorenzoni (2008) and Jeanne and Korinek (2010). Although the modeling details differ across the papers, both examine situations in which borrowing is constrained either by limited commitment (Lorenzoni (2008)) or asset value (Jeanne and Korinek (2010)). The common conclusion of the two papers is that the competitive equilibrium is inefficient because of a pecuniary externality. Similar pecuniary externalities are present in this paper. In particular, when negotiating the loan contract, the lender and entrepreneur take as given the aggregate price of capital. But their choice of the repayment rate affects bankruptcy probabilities and thus the resource cost of bankruptcy. In the model these costs are in terms of lost physical capital, so that higher bankruptcy rates imply higher replacement investment rates and thus a higher price of capital. Within the model a higher price of capital sub-optimally shifts consumption towards entrepreneurs. These effects are not internalized with general equilibrium effects because entrepreneurs are the sole holders of physical capital, and their savings decisions are passive. Hence, in the competitive equilibrium the private agents choose a repayment rate and default rate that are too high when compared to the planner.

The paper proceeds as follows. The next section outlines the competitive equilibrium of the model. Section 3 contrasts the contract indexation to BGG. Section 4 contrasts the competitive equilibrium with the constrained social planner's allocation. The quantitative analysis including welfare implications are carried out in Section 5. Section 6 provides some sensitivity analysis on the financial accelerator by examining the privately optimal contract and the BGG contract in a model with sticky prices and more exogenous shocks. Concluding comments are provided in Section 7.

2. The Model.

Households.

The typical household consumes the final good (C_t) and sells labor input (L_t) to the firm at real wage w_t. Preferences are given by

$$U(C_t, L_t) \equiv \frac{C_t^{1-\sigma}}{1-\sigma} - B\frac{L_t^{1+\eta}}{1+\eta}.$$

The household budget constraint is given by

$$C_t + D_t + Q_t^L S_t \leq w_t L_t + R_{t-1}^D D_{t-1} + (Q_t^L + Div_t) S_{t-1}$$

The household chooses the level of deposits (D_t) which are then used by the lender to fund the entrepreneurs (more details below). The (gross) real rate R_t^d on these deposits is known at time-t. The household owns shares in the final goods firms, capital-producing firms, and in the lender. The former two are standard, so we simply focus on the shares of the lender. This share price is denoted by Q_t^L with Div_t denoting lender dividends, and S_t the number of shares held by the representative household (in equilibrium S_t = 1). The optimization conditions include:

$$-U_L(t)/U_c(t) = w_t \tag{1}$$

$$U_c(t) = E_t \beta U_c(t+1) R_t^d \tag{2}$$

Final goods firms.

Final goods are produced by competitive firms who hire labor and rent capital in competitive factor markets at real wage w_t and rental rate r_t. The production function is Cobb-Douglass where A_t is the random level of total factor productivity:

$$Y_t = A_t\left(K_t^f\right)^{\alpha}(L_t)^{1-\alpha} \quad (3)$$

The variable K_t^f denotes the amount of capital available for time-t production. This is different than the amount of capital at the end of the previous period as some is lost because of monitoring costs. The optimization conditions include:

$$mpl_t = w_t \quad (4)$$

$$mpk_t = r_t \quad (5)$$

<u>New Capital Producers.</u>

The production of new capital is subject to adjustment costs. In particular, investment firms take $I_t\vartheta(I_t)$ consumption goods and transform them into I_t investment goods that are sold at price Q_t. Their profits are thus given by $Q_tI_t - I_t\vartheta(I_t)$, where the function ϑ is convex. We find it convenient to normalize $\vartheta(I_{ss}) = 1,\ \vartheta'(I_{ss}) = 0$ and $\vartheta''(I_{ss}) = \psi$, where I_{ss} is the steady-state level of investment. Variations in investment lead to variations in the price of capital, which are key to the financial accelerator mechanism.

<u>Lenders.</u>

The representative lender accepts deposits from households (promising sure return R_t^d) and provides loans to the continuum of entrepreneurs. These loans are intertemporal, with the loans made at the end of time t being paid back in time t+1. The gross real return on these loans is denoted by R_{t+1}^L. Each individual loan is subject to idiosyncratic and aggregate risk, but since the lender holds an entire portfolio of loans, only aggregate risk remains. The lender has no other source of funds, so the level of loans will equal the level of deposits. Hence, dividends are given by, $Div_{t+1} = R_{t+1}^L D_t - R_t^d D_t$. The lending market is competitive so that the lender takes as given the rates of return. The intermediary seeks to maximize its equity value by choice of the deposits it accepts:

$$Q_t^L = max_{D_t} E_t \sum_{j=1}^{\infty} \beta^j \frac{U_c(t+j)}{U_c(t)} Div_{t+j} \tag{6}$$

The FOC of the lender's problem is:

$$E_t M_{t+1} \left[R_{t+1}^L - R_t^d \right] = 0 \tag{7}$$

where $M_{t+1} \equiv \beta \frac{U\prime(c_{t+1})}{U\prime(c_t)}$. Expression (7) shows that in expectation the lender makes zero profits but ex-post profits and losses can occur. We assume that losses are covered by households as negative dividends. This is similar to the standard assumption in the Dynamic New Keynesian (DNK) model, eg.,Woodford (2003). That is, the sticky price firms are owned by the household and pay out profits to the household. These profits are typically always positive (for small shocks) because of the steady state mark-up over marginal cost. Similarly, one could introduce a steady-state wedge (eg., monopolistic competition among lenders) in the lender's problem so that dividends are always positive. But this assumption would have a trivial effect on the model's dynamics so we dispense from it for simplicity.

Confirming Chari's (2003) intuition, the expression for the equity value of the bank (6) implies that the household prefers a lender that delivers a dividend stream that co-varies negatively with household consumption. The lender is providing loans to the entrepreneurs. Hence, the household prefers a loan contract that requires the entrepreneur to pay back more in periods of low consumption, and vice versa. This is in sharp contrast to BGG who assume that the lender makes zero profits state by state, ie., $R_{t+1}^L = R_t^d$. Under the BGG assumption the dividend is fixed at zero and thus cannot provide a consumption hedge to the household.

<u>Entrepreneurs and the Loan Contract.</u>

There is a continuum of entrepreneurs with preferences that are linear in consumption. As in BGG and Carlstrom and Fuerst (1997), there is sufficient linearity in the model so that we can consider a representative entrepreneur. The entrepreneurs discount the future at rate β, and are the sole accumulators of physical capital. The time t+1 rental rate and capital price are denoted by r_{t+1} and Q_{t+1}, respectively, implying that the gross return to holding capital from time-t to time t+1 is given by:

$$R_{t+1}^{k} \equiv \frac{r_{t+1}+(1-\delta)Q_{t+1}}{Q_t}. \tag{8}$$

At the end of period t, the entrepreneurs sell all of their accumulated capital, and then re-purchase it along with any net additions to the capital stock. This purchase is financed with entrepreneurial net worth (NW_t) and external financing from a lender. External financing is subject to a one-period CSV problem.[1] In particular, one unit of capital purchased at the end of time-t is transformed into ω_{t+1} units of capital in time t+1, where ω_{t+1} is an idiosyncratic random variable with density $\phi(\omega)$ and cumulative distribution $\Phi(\omega)$ and a mean of one. The realization of $\omega_{t+1}$is directly observed by the entrepreneur, but the lender can observe the realization only if a monitoring cost is paid, a cost that is linear in the project size, $\mu\omega_{t+1}Q_tK_{t+1}$. As pointed out by Carlstrom and Fuerst (1997), the assumption of linearity allows for aggregation, but is non-standard in the optimal contracting literature. Under the assumption that the entrepreneurs' and lenders' contract payoffs are linear in the project outcome, Townsend (1979) demonstrates that the optimal contract between the entrepreneur and intermediary is risky debt in which monitoring only occurs if the promised payoff is not forthcoming.[2] The lender's valuation of loan repayment is linear (see (6)). We find it convenient to first assume that the entrepreneur's value function is linear. The appendix demonstrates that in this case the debt contract is optimal. We will then use the debt contract to show that the value function is in fact linear.[3] This will thus validate our initial linearity assumption.

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[1] We follow BGG and Carlstrom and Fuerst (1997), by assuming that there is enough inter-period anonymity so that today's contract cannot be based on previous realizations of the idiosyncratic shock. As noted by these authors, this assumption vastly simplifies the analysis for otherwise the optimal contract would depend upon the entire history of each entrepreneur (see, for example Gertler (1992)).

[2] The risky debt result also assumes that the equilibrium must be in pure strategies, i.e., no random audits.

[3] In a related environment, Krishnamurthy (2003) demonstrates that although borrowers are risk-neutral in consumption, they may be risk-averse in net worth. This risk-aversion arises in Krishnamurthy (2003) because the borrower's production technology is concave, and the collateral constraint is not always binding. In the BGG model studied here, the production technology is linear and the need for external finance is a permanent feature of the CSV

The debt contract is characterized by a reservation value of the idiosyncratic shock that separates repayment from default. Debt repayment does not occur for sufficiently low values of the idiosyncratic shock, $\omega_{t+1} < \varpi_{t+1}$. Let Z_{t+1} denote the promised gross rate-of-return so that Z_{t+1} is defined by

$$Z_{t+1}(Q_t K_{t+1} - NW_t) \equiv \varpi_{t+1} R_{t+1}^{k} Q_t K_{t+1} \,. \tag{9}$$

We find it convenient to express this in terms of the leverage ratio $\bar{\kappa}_t \equiv \left(\frac{Q_t K_{t+1}}{NW_t}\right)$ so that (9) becomes

$$Z_{t+1} \equiv \varpi_{t+1} R_{t+1}^{k} \frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} \tag{10}$$

Let $f(\varpi_{t+1})$ and $g(\varpi_{t+1})$ denote the entrepreneur's share and lender's share of the project outcome:

$$f(\varpi) \equiv \int_{\varpi}^{\infty} \omega\phi(\omega)d\omega - [1 - \Phi(\varpi)]\varpi \tag{11}$$

$$g(\varpi) \equiv [1 - \Phi(\varpi)]\varpi + (1 - \mu)\int_{0}^{\varpi} \omega\phi(\omega)d\omega \,. \tag{12}$$

The entrepreneur's net worth NW_t is leveraged into a project size of $Q_t K_{t+1}$, so that the entrepreneur and lender returns are given by:

$$Entrepreneur\ payoff = R_{t+1}^{k} Q_t K_{t+1} f(\varpi_{t+1}) = R_{t+1}^{k} f(\varpi_{t+1})\bar{\kappa}_t NW_t \tag{13}$$

$$Lender\ return = R_{t+1}^{L} = \frac{R_{t+1}^{k} g(\varpi_{t+1}) Q_t K_{t+1}}{(Q_t K_{t+1} - NW_t)} = R_{t+1}^{k} g(\varpi_{t+1}) \frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} \tag{14}$$

In general equilibrium, the entrepreneur's expected return to internal funds $(E_t R_{t+1}^{k} f(\varpi_{t+1})\bar{\kappa}_t)$ will always exceed 1/β, so that the entrepreneur will postpone consumption indefinitely. To avoid self-financing in the long run by accumulating sufficient internal funds, we assume that fraction (1-γ) of the entrepreneurs die each period, where dying means eating their accumulated net worth and exiting the economy. They are then replaced by an equal number of new entrepreneurs. Let NW_t denote the net worth of the representative entrepreneur at the beginning of time-t, after the loans from the previous

framework, so the entrepreneurs' payoff is linear in net worth. Although not risk averse in net worth, we will show that entrepreneurs care about the covariance of the debt contract with aggregate shocks.

period have been settled, but before the death realization has occurred. The representative entrepreneur sets $c_t^e = NW_t$ with probability (1-γ), and $NW_{t+1} = R_{t+1}^k f(\varpi_{t+1})\bar{\kappa}_t NW_t$ with probability γ. These linear decision rules imply that the value function will be linear in net worth. The Bellman equation for the representative entrepreneur is thus given by:

$$V_t NW_t = (1-\gamma)c_t^e + \beta\gamma max_{\bar{\kappa}_t,\varpi_{t+1}} E_t V_{t+1} NW_{t+1} \tag{15}$$

where V_t is the time-varying slope coefficient, and the maximization is over the terms of the subsequent debt contract. The representative entrepreneur sets $c_t^e = NW_t$, and $NW_{t+1} = R_{t+1}^k f(\varpi_{t+1})\bar{\kappa}_t NW_t$. Substituting in the trivial consumption decision we have

$$V_t = (1-\gamma) + \beta\gamma max_{\bar{\kappa}_t,\varpi_{t+1}} \bar{\kappa}_t E_t V_{t+1} R_{t+1}^k f(\varpi_{t+1}) \tag{16}$$

Hence, we have confirmed that the value function is linear in net worth. Further, note that: (i) the value function V_t is the discounted sum of future leverage and returns to capital, and (ii) the entrepreneur would prefer a contract in which NW_{t+1} is positively correlated with V_{t+1}. As for aggregate net worth, integrating over all entrepreneurs we have $NW_{t+1} = \gamma R_{t+1}^k f(\varpi_{t+1})\bar{\kappa}_t NW_t$.

On the other side of the contract we have the lender whose return is linked to the return on deposits via (2) and (7):

$$E_t R_{t+1}^L M_{t+1} = 1 \tag{17}$$

We can thus write the end of time-t contracting problem as:

$$max_{\bar{\kappa}_t,\varpi_{t+1}} \beta\gamma\bar{\kappa}_t E_t V_{t+1} R_{t+1}^k f(\varpi_{t+1}) \tag{18}$$

subject to

$$E_t R_{t+1}^k \bar{\kappa}_t M_{t+1} g(\varpi_{t+1}) \geq (\bar{\kappa}_t - 1) \tag{19}$$

Note that the payoffs of both the entrepreneur and the lender are linear in R^k_{t+1}. After some re-arrangement, the optimization conditions include:

$$\beta\gamma V_{t+1} f'(\varpi_{t+1}) + \Lambda_t M_{t+1} g'(\varpi_{t+1}) = 0 \tag{20}$$

$$\beta\gamma E_t V_{t+1} R^k_{t+1} f(\varpi_{t+1}) \bar{\kappa}_t = \Lambda_t \tag{21}$$

$$E_t R^k_{t+1} M_{t+1} g(\varpi_{t+1}) \bar{\kappa}_t = (\bar{\kappa}_t - 1) \tag{22}$$

where Λ_t denotes the multiplier on the constraint (19). The default cut-off ϖ_{t+1} is state-contingent so that (20) holds state-by-state, ie., ϖ_{t+1} will be a function of V_{t+1} and M_{t+1}. Using (21) we can write the Bellman equation as

$$V_t = (1-\gamma) + \Lambda_t \tag{23}$$

so that the entrepreneur's valuation of net worth is (not surprisingly) linked to the shadow value of the contract constraint. The **privately optimal contract (POC)** is thus described by the ϖ_{t+1} that satisfies:

$$\frac{\Lambda_t M_{t+1}}{\beta\gamma[1-\gamma+\Lambda_{t+1}]} = \frac{-f'(\varpi_{t+1})}{g'(\varpi_{t+1})} \equiv F(\varpi_{t+1}) \tag{24}$$

where satisfaction of the second order condition implies $F'(\varpi_{t+1}) > 0$. Expression (24) implies that the default cut-off is indexed to aggregate variables in a natural way. When household consumption is low (M_{t+1} is high), the optimal ϖ_{t+1} (and thus the lender's return) increases as a form of consumption insurance to the household. Similarly, when the cost of external finance is high (Λ_{t+1} is high), the contract calls for a lower ϖ_{t+1} so that the entrepreneur may hold on to more net worth (NW_{t+1} positively covaries with V_{t+1}).[4]

[4] Although the value function is linear, the entrepreneur's marginal valuation of net worth varies with the aggregate state. Despite this the debt contract still includes 100% confiscation of net worth in default states. Why does the optimal contract not provide for less than complete confiscation in high valuation states (and thus leave the entrepreneur with more net worth)? The answer is familiar in the CSV setting. The more efficient way of boosting net worth in high valuations states is to reduce the default cut-off (ϖ_{t+1}) instead of confiscating less than 100% in

<u>Market Clearing and Equilibrium.</u>

In equilibrium the household holds the shares of the lender, $S_t = 1$, and the lender funds the entrepreneurs' projects, $D_t = Q_t K_{t+1} - NW_t$. Net of monitoring costs, the amount of capital available for production is given by $K_t^f = \Upsilon(\varpi_t) K_t$. The competitive equilibrium is defined by the variables $\{C_t, L_t, I_t, K_{t+1}, \varpi_t, \Lambda_t, \bar{\kappa}_t, \mathrm{C}_t^{\mathrm{e}}, Q_t, R_t^d\}$ that satisfy

$$U_c(t) = R_t^d E_t \beta U_c(t+1) \tag{25}$$

$$-U_L(t)/U_c(t) = mpl_t \tag{26}$$

$$\frac{\Lambda_{t-1} M_t}{\beta\gamma[1-\gamma+\Lambda_t]} = \frac{-f'(\varpi_t)}{g'(\varpi_t)} \equiv F(\varpi_t) \tag{27}$$

$$\Lambda_t = \beta\gamma E_t[(1-\gamma) + \Lambda_{t+1}] R_{t+1}^k f(\varpi_{t+1}) \bar{\kappa}_t \tag{28}$$

$$E_t R_{t+1}^k M_{t+1} g(\varpi_{t+1}) \bar{\kappa}_t = (\bar{\kappa}_t - 1) \tag{29}$$

$$Q_t K_{t+1} = \gamma[Q_t(1-\delta) + mpk_t] f(\varpi_t) K_t \bar{\kappa}_t \tag{30}$$

$$K_{t+1} \leq (1-\delta)\Upsilon(\varpi_t) K_t + I_t \tag{31}$$

$$C_t + I_t \vartheta\left(\frac{I_t}{I_{ss}}\right) + \mathrm{C}_t^{\mathrm{e}} \leq A_t (\Upsilon(\varpi_t) K_t)^\alpha (L_t)^{1-\alpha} \tag{32}$$

$$\mathrm{C}_t^{\mathrm{e}} = (1-\gamma)[Q_t(1-\delta) + mpk_t] f(\varpi_t) K_t \tag{33}$$

$$Q_t = \vartheta(I_t) + (I_t)\vartheta'(I_t) \tag{34}$$

where we have used $\bar{\kappa}_t \equiv \left(\frac{Q_t K_{t+1}}{NW_t}\right)$, $R_{t+1}^k \equiv \frac{mpk_{t+1} + (1-\delta) Q_{t+1}}{Q_t}$, and $\Upsilon(\varpi_t) \equiv f(\varpi_t) + g(\varpi_t) = 1 - \mu \int_0^{\varpi_t} x\phi(x) dx$. Note that $\Upsilon'(\varpi_t) = -\mu \varpi_t \phi(\varpi_t) < 0$. The marginal products are defined as: $mpl_t \equiv (1-\alpha) Y_t / L_t$, and $mpk_t \equiv a Y_t / (\Upsilon(\varpi_t) K_t)$, where $Y_t \equiv A_t (\Upsilon(\varpi_t) K_t)^\alpha (L_t)^{1-\alpha}$. Before proceeding to a quantitative analysis of the POC, we first contrast the POC with the contract imposed by BGG.

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default states. The reason is obvious: default has a resource cost associated with it, and complete confiscation lowers the probability of default. See the appendix for details.

3. Comparing the POC to BGG.

BGG assume that the lender's return is equal to the deposit rate state-by-state, i.e., lender profits are zero state-by-state. This is not an implication of the modeling framework, but is instead an assumption imposed on the model. As BGG write, "Since entrepreneurs are risk neutral, we *assume* that they bear all the aggregate risk associated with the contract" (BGG, page 1385, emphasis added). There are two problems with this assumption. First, the household's risk is linked to consumption, not to the return on capital. Second, the entrepreneur cares about the covariance between net worth and the return on capital. The POC includes both of these motivations in (27), while the contract assumed by BGG does not. The behavior of bankruptcy rates in BGG is given implicitly by

$$g(\varpi_t) = \frac{R^d_{t-1}(\overline{\kappa}_{t-1}-1)}{R^k_t\overline{\kappa}_{t-1}}. \tag{35}$$

It is useful to compare the BGG and POC contracts, especially (27) and (35) as they imply sharply different behavior for ϖ_t and lender returns (see (14)). This is particularly transparent in log-linear form, so we proceed to look at the linearized versions of POC and BGG. In log-linear form (lower case letters), equations (21)-(24) for the POC are given by:

$$\Psi\varpi_{t+1} = m_{t+1} + \lambda_t - \gamma F_{ss}\lambda_{t+1} \tag{36}$$

$$\kappa_t + E_t\left(r^k_{t+1} + \Theta_f\varpi_{t+1}\right) = \lambda_t - \gamma F_{ss}E_t\lambda_{t+1} \tag{37}$$

$$E_t\left(r^k_{t+1} + m_{t+1} + \Theta_g E_t\varpi_{t+1}\right) = \left(\frac{1}{\kappa-1}\right)\kappa_t \tag{38}$$

where $\Psi \equiv \frac{\varpi_{ss}F'(\varpi_{ss})}{F(\varpi_{ss})}$, with $\Psi > 0$ by the second order condition, $\Theta_g \equiv \frac{\varpi_{ss}g\prime(\varpi_{ss})}{g(\varpi_{ss})}$, $0 < \Theta_g < 1$, and $\Theta_f \equiv \frac{\varpi_{ss}f\prime(\varpi_{ss})}{f(\varpi_{ss})} < 0$. Taking expectations in (36) and combining with (37)-(38) we have a convenient expression for the spread between the return on capital and the lender's return:

$$E_t\left(r^k_{t+1} - r^L_{t+1}\right) = \left[\frac{(\Psi-\theta_f+\theta_g)-\kappa\theta_g}{(\kappa-1)(\Psi-\theta_f+\theta_g)}\right]\kappa_t \equiv \nu\kappa_t \tag{39}$$

Note that increases in leverage are associated with increases in the spread. From (10) and (14), the promised payment and lender's return are given by:

$$z_t = \varpi_t + r_t^k - \frac{1}{\kappa-1}\kappa_{t-1} \tag{40}$$

$$r_t^l = -\frac{1}{(\kappa-1)}\kappa_{t-1} + \Theta_g\varpi_t + r_t^k \tag{41}$$

Combining the previous, we can express the POC in log-linear form:

$$\varpi_t^{POC} = \left(\frac{1-\nu(\kappa-1)}{\Theta_g(\kappa-1)}\right)\kappa_{t-1} - \frac{\sigma}{\psi}(c_t - E_{t-1}c_t) - \frac{\beta}{\psi}(\lambda_t - E_{t-1}\lambda_t) \tag{42}$$

$$z_t^{POC} = r_{t-1}^d + \frac{(1-\Theta_g)[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\kappa_{t-1} + (r_t^k - E_{t-1}r_t^k) - \frac{\sigma}{\Psi}(c_t - E_{t-1}c_t) - \frac{\beta}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \tag{43}$$

$$r_t^{l,POC} = r_{t-1}^d + \left(r_t^k - E_{t-1}r_t^k\right) - \frac{\sigma\Theta_g}{\Psi}(c_t - E_{t-1}c_t) - \frac{\beta\Theta_g}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \tag{44}$$

$$\lambda_t = E_t\sum_{j=0}^{\infty}\beta^j\left(\Xi\kappa_{t+j} + r_{t+j}^d\right) \tag{45}$$

The constant Ξ is given by:

$$\Xi \equiv \left[1 + \nu + \Theta_f\left(\frac{1-\nu(\kappa-1)}{\Theta_g(\kappa-1)}\right)\right] = \left[1 + \nu + F_{ss}\left(\nu - \frac{1}{(\kappa-1)}\right)\right]. \tag{46}$$

Recall that the multiplier (45) is linked to the value function in (23). Hence, expression (45) links the entrepreneur's valuation of internal funds to the present discounted value of future leverage and thus, via (39), the future spread between the return on capital and the risk-free rate.

In contrast, BGG impose a contract in which ϖ_t behaves according to (35).[5] Proceeding as before, we can express the BGG contract in log-linear form:

[5]The full set of BGG contract conditions are given by (35) and

$$\beta\gamma f'(\varpi_{t+1}) + \frac{\Lambda_{t+1}}{R_t^d}g'(\varpi_{t+1}) = 0$$

$$\beta\gamma E_t R_{t+1}^k f(\varpi_{t+1})\bar{\kappa}_t = E_t\Lambda_{t+1}$$

$$\varpi_t^{BGG} = \frac{[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\kappa_{t-1} - \frac{1}{\Theta_g}(r_t^k - E_{t-1}r_t^k) \tag{47}$$

$$z_t^{BGG} = r_{t-1}^d + \frac{(1-\Theta_g)[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\kappa_{t-1} + \left(\frac{\Theta_g-1}{\Theta_g}\right)\left(r_t^k - E_{t-1}r_t^k\right) \tag{48}$$

$$r_t^{l,BGG} = r_{t-1}^d \tag{49}$$

The key difference between the POC and BGG is the response of the default threshold ϖ_t and the promised repayment z_t to innovations. Both of these effects are summarized in the lender's return, expressions (44) and (49). In comparison to BGG, the POC has three distinct characteristics. First, other things equal, the innovation in r_t^k has no effect on bankruptcy rates. Second, the POC provides consumption insurance to the household with the lender's return increasing when consumption is unexpectedly low, an effect that is increasing in household risk aversion. Third, the POC provides financing insurance to the entrepreneur by lowering repayment rates when the shadow cost of external finance is unexpectedly high (λ_t is high). All three of these effects, which are a feature of the optimal contract, are absent in BGG.

Although the lender returns between POC and BGG differ only by innovations, the inertial dynamics of net worth implies that these differences will have long-lived consequences. Linearizing the behavior of aggregate net worth and using the two contracts we have

$$nw_{t+1}^{POC} = nw_t^{POC} + r_t^d + \left\{\frac{\Theta_f[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)} + 1 + \nu\right\}\kappa_t + (r_{t+1}^k - E_t r_{t+1}^k) - \frac{\sigma\Theta_f}{\Psi}(c_{t+1} - \mathrm{E}_t c_{t+1}) - \frac{\beta\Theta_f}{\Psi}(\lambda_t - E_{t-1}\lambda_t) \tag{50}$$

$$nw_{t+1}^{BGG} = nw_t^{BGG} + r_t^d + \left\{\frac{\Theta_f[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)} + 1 + \nu\right\}\kappa_t + \left(1 - \frac{\Theta_f}{\Theta_g}\right)(r_{t+1}^k - E_t r_{t+1}^k) \tag{51}$$

As with the lender's return, the differences in the two contracts differ by the response of net worth to innovations in three aggregate variables: the return on capital, household consumption, and the entrepreneur's valuation of net worth. Here we will focus on the magnitude of these effects. Under our baseline calibration, the term $\left(1-\frac{\Theta_f}{\Theta_g}\right)$ is approximately equal to 2, so that the BGG contract responds to r_{t+1}^k by twice as much as POC. This difference between BGG and POC is magnified with higher levels of leverage. As for the other innovations, the POC provides insurance both to the household (consumption insurance) and the entrepreneur (financing insurance) in a symmetric fashion. These insurance effects are non-trivial: our baseline calibration implies $\frac{\Theta_f}{\Psi}=-2.17$. For the aggregate shocks considered below, these latter two forms of indexation tend to cancel out. That is, innovations in aggregate consumption are tightly correlated with innovations in the entrepreneur's valuation of net worth. Hence, the most important quantitative difference between BGG and POC is that the latter has default rates perfectly indexed to innovations in r_{t+1}^k so that net worth moves one-for-one with these shocks. In any event, as is clear from (50)-(51), these different responses to innovations will have a long-lived effect on the path of net worth, and thus the future path of leverage and the risk premium.

4. The Planner's Problem.

In this section we consider a planner who maximizes a weighted sum of the lifetime utility flow of the representative household and entrepreneur. We will subsequently use the planner's behavior as a useful comparison to POC and BGG. The linearity in the model implies that we can aggregate entrepreneurial consumption. With a utility weight of ϵ on the continuum of entrepreneurs, the planner maximizes:

$$E_t \sum_{j=0}^{\infty} \beta^j \left[U\left(C_{t+j}, L_{t+j}\right)+\epsilon C_{t+j}^e\right] \tag{52}$$

subject to the resource constraints and private optimality. We assume that the planner is constrained by the competitive behavior of the agents. However, the planner can affect this behavior by levying distortionary taxes and subsidies. The private informational barrier on observing entrepreneurial payoffs implies that the planner cannot directly levy taxes on the entrepreneurs. Hence, we must focus on the other agents. Fundamentally, the planner cares about three margins: the household's labor choice, the household's savings choice, and the distribution of consumption across agents. The first two can be influenced by wedges on the relevant margins: a tax of τ_t^L on labor income, and a tax of τ_t^d on interest income from deposits.

But the third margin, the allocation of consumption across agents, is more difficult as the planner cannot directly tax entrepreneurs because their income is unobserved by the planner. From (31), the aggregate consumption of entrepreneurs is linked directly to the lending contract via the bankruptcy threshold ϖ_t. The most direct tax to influence this threshold is a wedge on monitoring costs. Hence, we assume that the planner has access to a time-varying tax on monitoring costs so that the private cost of monitoring becomes $(1+\tau_t^\mu)\mu$. With this wedge, the lender's payoff is given by:

$$h(\varpi;\tau^\mu) \equiv g(\varpi) - \mu\tau^\mu \int_0^\varpi \omega\phi(\omega)d\omega \tag{53}$$

Movements in this tax will directly feed into movements in bankruptcy rates.[6] As with the household income taxes, these tax proceeds are rebated to the household in a lump-sum manner.

To summarize, the planner maximizes (52) with the assistance of three wedges: τ_t^d, τ_t^L, and τ_t^μ. These wedges alter the following equilibrium conditions:

$$U_c(t) = R_t^d E_t \beta U_c(t+1)(1-\tau_t^d) \tag{25a}$$

$$-U_L(t)/U_c(t) = mpl_t(1-\tau_t^L) \tag{26a}$$

$$\beta\gamma(1-\gamma+\Lambda_{t+1})f'(\varpi_{t+1}) + \Lambda_t M_{t+1} h'(\varpi_{t+1}) = 0 \tag{27a}$$

$$\beta\gamma E_t V_{t+1} R_{t+1}^k f(\varpi_{t+1})\bar{\kappa}_t = \Lambda_t \tag{28a}$$

$$E_t R_{t+1}^k M_{t+1} h(\varpi_{t+1})\bar{\kappa}_t = (\bar{\kappa}_t - 1) \tag{29a}$$

The planner thus maximizes (52) subject to (25a)-(29a) and (30)-(34), by choosing allocations, prices, and taxes.

[6] Alternatively, the planner could levy taxes/subsidies on the promised repayment rate Z_{t+1}.With this wedge, the lender's payoff is given by

$$h(\varpi;\tau^z) \equiv g(\varpi)(1-\tau^z) + \tau^z(1-\mu)\int_0^\varpi \omega\phi(\omega)d\omega$$

where τ^z is the tax/subsidy on the repayment rate.

This optimization problem can be greatly simplified by noting that many of the constraints are not restrictions per se, but instead can be thought of as defining or "backing out" the needed market price or tax that supports the planner's allocation. This simplification will leave us with the planner choosing allocations, while the market prices and taxes are chosen to support this outcome. In particular, equations (25a)-(29a), and (30), can be used to solve for R_t^d, Λ_t, $\bar{\kappa}_t$, and the three supporting taxes, as functions of the allocations. We are thus left with:

$$K_{t+1} \leq (1-\delta)\Upsilon(\varpi_t)K_t + I_t \tag{54}$$

$$C_t + I_t\vartheta(I_t) + C_t^e \leq A_t(\Upsilon(\varpi_t)K_t)^{\alpha}(L_t)^{1-\alpha} \tag{55}$$

$$C_t^e = (1-\gamma)[Q_t(1-\delta) + mpk_t]f(\varpi_t)K_t \tag{56}$$

with the price of capital given implicitly by $Q_t = \vartheta(I_t) + (I_t)\vartheta'(I_t)$. The planner thus chooses the allocations $\{\varpi_t, K_{t+1}, I_t, C_t, C_t^e, L_t\}$ to maximize (50) subject to (54)-(56).

Constraints (54)-(55) are familiar resource constraints and need no elaboration. But the novelty here is constraint (56). Along with choosing work effort and capital accumulation, the planner seeks to efficiently allocate consumption between the agents. But this allocation is complicated by monitoring costs. The planner can redistribute consumption from entrepreneurs to households by varying the cut-off value ϖ_t. Higher values of ϖ_t lower entrepreneurial consumption and increase household consumption, but this reallocation comes at the expense of a lower capital stock via $\Upsilon(\varpi_t)$ in (54)-(55). Hence, the planner's problem is ultimately one of consumption-sharing across agents, where the level of sharing is constrained because of monitoring costs. The planner can decentralize the optimal ϖ_t behavior with the use of the monitoring tax in (53).

An unusual feature of (56) is the presence of a market price in the planner's constraint set. If the planner could choose the price of capital directly, she could achieve an efficient consumption allocation ($U_c(t) = \epsilon$) with no resource cost, ie., $\varpi_t = 0$, and the CSV problem would disappear from the model. It is for this reason that we assume the planner *cannot* levy a per unit tax/subsidy on the sale of new capital for this would allow the planner to choose the price of capital independently of the level of investment.[7] We ignore this possibility because it makes things too simple for the planner. That is, since entrepreneurs are inelastic savers and are the only holders of capital in the BGG model, a capital tax effectively gives the planner a lump sum tax. In a model such as Carlstrom and Fuerst (1997), where both households and entrepreneurs hold capital, the capital tax could not be used so effectively. More generally, in more elaborate models it is surely not the case that a capital tax is isomorphic to a lump sum tax.

This discussion makes clear the pecuniary externality present in the competitive equilibrium of the model. When negotiating the loan contract, the entrepreneur and lender take as given the price of capital. But a higher ϖ_t implies a larger destruction of capital via monitoring activity. This then implies a higher level of investment, and via the convexity in the investment production function, a higher price of new capital, Q_t. General equilibrium forces do not fully internalize this effect because only the entrepreneurs accumulate capital, and they do so passively until they die. Hence, under the POC, ϖ_t and the market price of capital will be too high when compared to the planner.[8]

Let Λ_{1t}, Λ_{2t}, and Λ_{3t}, denote the multipliers on (54)-(56), respectively. We find it convenient to treat Q_t parametrically as defined by $Q_t = \vartheta(I_t) + (I_t)\vartheta'(I_t)$, so that $Q_I(t)$ denotes the derivative of Q with respect to investment. The following are the FONC to the planner's problem:

$$\Lambda_{1t} - \Lambda_{3t} x_t Q_I(t)(1-\delta)\Upsilon(\varpi_t)K_t = U_c(t)Q_t \tag{57}$$

[7] Suppose the new-capital producer maximized:

$$Q_t(1-\tau_t^q)I_t - I_t\phi(I_t).$$

where τ_t^q is a tax on new capital levied on the seller. This implies the following first-order condition:

$$Q_t = \frac{\phi(I_t)+I_t\phi'(I_t)}{(1-\tau_t^q)}.$$

Hence, by varying τ_t^q, the planner could achieve any capital price that is desired, and thus, via (56), any desired level of entrepreneurial consumption. This means that (56) will no longer be a constraint, and ϖ_t = 0. As noted, this is the (uninteresting!) case of perfect consumption sharing, and the CSV problem drops from the model. A similar result holds in Jeanne and Korinek (2010): a time-varying subsidy on asset purchases can eliminate the borrowing constraint and achieve the frictionless allocation.

[8] This pecuniary externality would thus be significantly diminished if the investment production technology implied a steady state price of capital that was independent of steady state investment. For example, if the investment technology was in terms of changes in the flow of investment, $\phi(I_t/I_{t-1})$, then there would be no pecuniary externality in the steady state, and a trivial welfare cost of the POC. In such an environment, the welfare cost would come only from suboptimal variations in the price of capital in response to shocks, but since the steady state did not have this distortion, these variations would have a modest welfare cost.

$$\Lambda_{3t} = U_c(t) - \epsilon \tag{58}$$

$$-U_L(t) = U_c(t)mpl_t - \Lambda_{3t}\alpha mpl_t x_t \tag{59}$$

$$\Lambda_{1t} = \beta E_t \Upsilon(\varpi_{t+1}) \begin{Bmatrix} \Lambda_{1t+1}(1-\delta) + U_c(t+1)mpk_{t+1} \\ -\Lambda_{3t+1}x_{t+1}[\alpha mpk_{t+1} + (1-\delta)Q_{t+1}] \end{Bmatrix} \tag{60}$$

$$\frac{\Upsilon'(\varpi_t)}{f'(\varpi_t)} = \frac{\Lambda_{3t}(1-\gamma)[Q_t(1-\delta)+mpk_t]}{[\Lambda_{1t}(1-\delta)+U_c(t)mpk_t+\Lambda_{3t}x_t(1-\alpha)mpk_t]} \tag{61}$$

where we define

$$x_t \equiv (1-\gamma)\frac{f(\varpi_t)}{\Upsilon(\varpi_t)}, \tag{62}$$

and we have used $U_c(t) = \Lambda_{2t}$.

It is instructive to compare the planner's behavior (57)-(61) to the competitive equilibrium. The competitive equilibrium includes the marginal conditions

$$-U_L(t) = U_c(t)mpl_t \tag{63}$$

$$E_t \beta U_c(t+1) R_{t+1}^k \frac{\bar{\kappa}_t}{\bar{\kappa}_{t-1}} g(\varpi_{t+1}) = U_c(t) \tag{64}$$

The competitive equilibrium has employment (63) satisfying the traditional RBC margin, but the investment decisions (64) is distorted relative to familiar RBC behavior. Comparing (63)-(64) to the complementary (59)-(60) it is quite clear that the planner's allocations will differ sharply from the competitive equilibrium.

There are two notable differences. First, leverage ratios and net worth do not constrain the planner because the tax on interest income can be varied to motivate any desired level of savings from the households. Second, the multiplier Λ_{3t} alters both of the planner's conditions (59)-(60) considerably from the competitive equilibrium. From (58), the multiplier Λ_{3t} denotes the difference in the marginal utilities between the entrepreneur and the household. The planner wants to equate these two (and thus set Λ_{3t} = 0) by transferring consumption units. But (56) constrains the planner: entrepreneurial consumption can be altered only by altering variables in (56). It is this constraint that colors all the planner's choices. Consider first the planner's choice of ϖ_t. Since $f'(\varpi_t)$ and $\Upsilon'(\varpi_t)$ are both negative, (61) implies that

Λ_{3t} is positive. That is, the planner sets $\varpi_t > 0$ and tolerates the associated costs of positive bankruptcy rates only because on the margin he desires to transfer consumption units from the entrepreneur back to the household. But the positive monitoring costs imply that the planner is ultimately frustrated and does not achieve equal marginal utilities ($U_c(t) > \epsilon$).

This incomplete redistribution illuminates the remaining differences between the planner and the competitive equilibrium. Because marginal output units do not flow entirely to the higher-marginal-utility household, the planner prefers a lower level of work effort as implied by (59), and a lower level of physical capital as implied by (60). Further, since reductions in the price of capital lead directly to a redistribution from the entrepreneur to the household, the planner prefers a lower price of capital as implied by (57). As suggested above, this last effect is the most important quantitatively.

5. Quantitative Analysis.

Our benchmark calibration largely follows BGG. The discount factor β is set 0.99. Utility is assumed to be logarithmic in consumption ($\sigma = 1$), and the elasticity of labor is assumed to be 3 ($\eta = 1/3$). The production function parameters include $\alpha = 0.35$, investment adjustment costs $\psi = 0.50$, and quarterly depreciation is $\delta = .025$. As for the credit-related parameters, we calibrate the model to be consistent with: (i) a steady state spread between Z and R^d of 200 bp (annualized), (ii) a quarterly bankruptcy rate of .75%, and (iii) a leverage ratio of $\kappa = 2$. These values imply a death rate of $\gamma = 0.98$, a standard deviation of the idiosyncratic productivity shock of 0.28, and a monitoring cost of $\mu = 0.63$. In the linearized model (see appendix), this then implies $\nu = 0.188$. We assume that total factor productivity follows an AR(1) process with $\rho^A = 0.95$. The financial accelerator is driven by fluctuations in the price of capital. The size of these movements is driven by the convexity in the investment production function. As noted, we parameterize the adjustment cost so that in the steady state of the competitive equilibrium the price of capital is equal to unity. Steady state investment is equal to

$$I_{ss} = K_{ss}[1 - (1 - \delta)\Upsilon(\varpi_{ss})] \tag{65}$$

Hence, the level of steady-state investment is affected by ϖ_{ss}, and this will differ between the planner and competitive equilibrium. Let I_{ss}^{POC} denote the steady state level of investment in the POC competitive equilibrium (BGG has the same steady-state). We thus center the investment technology at this point: $\vartheta(I_{ss}^{POC}) = 1$, $\vartheta'(I_{ss}^{POC}) = 0$ and $\vartheta''(I_{ss}^{POC}) = \psi$.

We investigate three allocations: (i) the planner, (ii) competitive equilibrium under POC, and (iii) BGG. To reiterate, under a laissez faire assumption only the POC is a competitive equilibrium as it is the optimal contract. The planner and BGG allocations would be supported under a competitive equilibrium only if there are time-varying governmental interventions. For the planner's behavior we need to assume a welfare weight for entrepreneurial consumption (ϵ). We find it convenient to choose the baseline value of this weight such that the steady state level of capital is identical for the planner and the POC.

To develop intuition, Figure 1 presents impulse response functions for the case of a 1% iid TFP shock, $\rho^A = 0$ (we plot aggregate consumption which is the sum of household and entrepreneurial consumption). Note that the planner responds to this iid shock in something of an iid fashion. That is, there is very little persistence in the planner's behavior because net worth is not a state variable, and physical capital has modest effects on persistence. The POC behavior is similar to the planner, but with modest amplification in output and investment. Matters are much different with BGG. Because the lender's rate is pre-determined, the TFP shock leads to a sharp increase in net worth. This net worth expansion leads to an amplification (relative to the planner) of output and investment. These effects diminish only slowly as entrepreneurial net worth returns to normal levels.

Figure 2 looks at the case of an auto-correlated TFP shock. In comparison to the planner, both POC and BGG again over-respond to the shock. Note in particular that bankruptcy rates decline very modestly under the planner so that entrepreneurial consumption (which is proportional to net worth) rises only modestly. It is noteworthy that the planner tolerates very little movement in the monitoring threshold ϖ_t. This is quite intuitive. The underlying informational friction is static, so that the optimal level of monitoring is roughly constant. But under both POC and especially BGG, the financial contract shifts net worth and thus consumption towards the entrepreneur. The persistent movement in net worth leads to a decline in the risk premium and hence a sub-optimal amplification of investment and output.

The dynamic version of the pecuniary externality is manifested in figures 1-2. Comparing the planner to the POC, the price of capital moves more under the POC than under the planner. This leads to a sharper movement of net worth and thus entrepreneurial consumption under POC than under the planner. This is socially suboptimal as $U_c(t) > \epsilon$. The planner avoids this sharp increase in entrepreneurial consumption but at the cost of lower investment.

Table 1 provides a welfare analysis of the three allocations (POC, BGG, and planner) under the baseline calibration and a calibration with a higher leverage rate. For the higher leverage case, we hold fixed the bankruptcy rate and risk premium, and vary the other agency cost parameters to achieve a leverage rate of 4. To see if there are Pareto improvements, data is also presented for household and entrepreneurial welfare. In all cases the results are reported as numerical differences from the planner's welfare levels. The welfare measures we report are computed based on a second-order approximation to the nonlinear equilibrium conditions of each model. Our preferred welfare measure is the conditional expectation of a weighted average of household and entrepreneurial discounted lifetime utility. The conditional welfare measure is chosen since agents in the model solve an explicitly conditional optimization problem. As noted, we choose the baseline weight on entrepreneurial utility (ϵ) such that the capital stock in the steady state is the same for the planner problem as for the BGG model and the POC model.

For the baseline calibration the welfare gain is large, a perpetual annual flow of 0.35% of household consumption for POC, and 0.38% for BGG (we are using an annual discount rate of 4%, and exploiting the fact that the calibration is log utility). As point of comparison, Lucas (1987) estimates that the welfare cost of US business cycles is on the order of a consumption flow of 0.05%. The welfare costs of the POC and BGG contracts are an order of magnitude larger than these Lucas estimates. The reason is that the POC and BGG are distorted steady states and thus have level effects when compared to the planner. Lucas's (1987) analysis abstracts from these first-order effects by holding fixed mean behavior. This steady-state distortion manifests itself in two related ways: (i) the cut-off value for monitoring is 6.57% lower under the planner (0.455) than under the POC (0.487), and (ii) the price of capital is 1.9% lower under the planner. Essentially, the planner finds a more cost-effective way of redistributing consumption from the entrepreneur to the household: a lower Q_{ss} and a lower ϖ_{ss}. Interestingly, most of the welfare cost under POC comes from these steady-state effects, although this share is much smaller for BGG. Evidently this is a result of the mild amplification of TFP shocks under POC versus the pronounced amplification under BGG. That is, both BGG and POC feature distorted steady states, but BGG compounds the problem by introducing sharp fluctuations from these steady states.

For the baseline experiment the planner outcome in Table 1 is not a Pareto improvement over POC and BGG. But the loss to the household is quite small compared to the gain to the entrepreneur. This suggests that POC and BGG must be within the planner's frontier. Figure 3 confirms this. Figure 3 traces out the planner's welfare frontier in the neighborhood of the baseline ϵ = 1.01 (that equalized steady state capitals). As we vary ϵ, the planner's steady state capital stock changes. Hence, we consider modest movements in ϵ, so that the corresponding movements in the planner's steady state capital stock are small, and our quadratic approximation retains a reasonable level of accuracy. For example, in Figure 3, we vary ϵ from 0.91 to 1.11, and the planner's steady state capital stock varies from 3.50 to 3.67, a 2.5% deviation from the POC steady state of 3.58. We use the calculated quadratic welfare functions to approximate planner welfare levels conditioned at the same steady state capital stock as BGG and POC. This approach means that we do not trace out the entire welfare frontier of the planner, but only the frontier in the neighborhood of BGG and POC conditional welfare. Figure 3 demonstrates that both BGG and POC are significantly inside the planner welfare frontier. Figure 4 presents the complementary welfare frontier but for a POC and BGG calibration with a steady state leverage of 4. In this case, BGG and POC are both well inside the planner welfare frontier. Using the centered ϵ = 0.84 (again to equalize steady state capital stocks), the conditional welfare cost of BGG and POC is quite large: an annual consumption flow increase of 0.49% for POC, and 0.62% for BGG.

6. Sensitivity of the financial accelerator to other shocks.

As a form of sensitivity analysis on the positive aspects of the model, we investigate adding sticky prices and other exogenous shocks to the analysis. The focus is on the financial accelerator and how it is affected by the two alternative loan contracts, BGG and POC. We integrate sticky prices via the familiar Dynamic New Keynesian (DNK) methodology. Note that there is no nominal stickiness between the lender and entrepreneur so that the POC is unchanged. The DNK model is standard so we dispense with the formal derivation, see, for example, Woodford (2003) for details. Imperfect competition distorts factor prices so that the marginal productivity of capital and labor in (4) and (5) are pre-multiplied by marginal cost. Marginal cost in turn is affected by the path of inflation so that in log deviations we have a relationship between inflation (π_t) and marginal cost (ζ_t):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \zeta_t \qquad (66)$$

The model is then closed with the familiar Fisher equation linking real and nominal interest rates:

$$i_t = E_t \pi_{t+1} + r_t^d \qquad (67)$$

and an interest rate policy for the central bank. In log deviations this policy rule is given by:

$$i_t = \phi_\pi \pi_t + \varepsilon_t^m. \qquad (68)$$

where ε_t^m is an exogenous policy movement with autocorrelation $\rho^m = 0.50$. We also consider a shock to the financial market. In particular, we augment the evolution of net worth with an exogenous and persistent change in net worth that can be viewed as re-allocating wealth from the household sector to the entrepreneurs. By altering net worth, this shock will alter leverage and risk premia. We assume that this wealth shock is persistent with $\rho^{NW} = 0.70$. The DNK parameter calibration is standard with $\phi_\pi = 1.5$ and $\kappa = 0.025$.

Figures 5-7 report the impulse response functions to a 1% TFP shock ($\rho^A = 0.95$), a 25 bp (quarterly) monetary policy shock, and a 1% net worth shock. The key to understanding all three experiments is to focus on the behavior of net worth. The indexation under the POC leads to sharp changes in the lender's return so that the change in net worth is quite modest when compared to BGG. And as noted earlier, the persistence in net worth implies that the initial change in net worth drives all subsequent dynamics. The poster boy for this mechanism is the net worth shock. In the BGG model, a 1% exogenous shock to net worth leads to a 10% movement in net worth (recall that leverage = 2) because the financial accelerator kicks in: higher net worth boosts the price of capital, the higher price of capital boosts net worth, etc. But this is entirely absent in POC. In fact, net worth actually declines on impact because the entrepreneur prefers to pay out the exogenous net worth windfall as higher lender returns.

In summary, the BGG model delivers sharp amplification of all three shocks, but only because the non-optimality of the BGG contract leads to sharp movements in net worth. This amplification is even manifested in the comparison of the TFP response in the flexible price model (figure 2) vs. the sticky price model (figure 5). Because of the hump-shape behavior in inflation, marginal cost rises on impact, so that the increase in the rental rate and thus the return on capital is amplified in the sticky price framework. Hence, net worth moves by more with sticky prices than it does with flexible prices. This net worth movement leads to a further amplification of real activity in the sticky price model. In contrast, under the POC, the net worth movement in both flexible and sticky prices is quite comparable so that the real response to a TFP shock is modestly dampened by the addition of sticky prices as marginal cost falls and mark-ups rise. This is the typical effect of sticky prices in DNK models.

7. Conclusion.

Two basic functions of financial markets are to intermediate between borrowers and lenders, and to provide a mechanism to hedge risk. Both of these motivations are present here. The risky debt contract is the optimal method of intermediation as it mitigates the informational asymmetries arising from the CSV problem. This then allows for funds to flow from the household-lenders to the entrepreneurial-borrowers. An important feature of the optimal contract between lenders and borrowers is indexation to observed aggregate shocks. The original analysis of BGG ignored these hedging effects, which implied an amplification of aggregate shocks that is not part of competitive behavior. In contrast, the POC model derived here incorporates these optimal indexation effects. The financial accelerator is thus sharply muted when compared to BGG. But the POC is not socially optimal. As in Lorenzoni (2008) and Jeanne and Korinek (2010), in environments with credit constraints, financial markets can go awry. This is the case here. Competing interests in the loan contract result in a price of capital that is sub-optimally high.

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APPENDIX.

1. Linearized Model (POC).

$$E_t(r_{t+1}^k - r_{t+1}^l) = \nu\kappa_t \tag{A1}$$

$$nw_t = \kappa\frac{\gamma}{\beta}\left(r_t^k - r_t^l\right) + \frac{\gamma}{\beta}(r_t^l + nw_{t-1}) + \gamma\kappa\frac{rp}{\beta}(k_t + q_t + r_t^k) \tag{A2}$$

$$r_t^{l,POC} = r_{t-1}^d + \left(r_t^k - E_{t-1}r_t^k\right) - \frac{\sigma\Theta_g}{\Psi}(c_t - \mathrm{E}_{t-1}c_t) - \frac{\beta\Theta_g}{\Psi}(\tilde{\Lambda}_t - E_{t-1}\tilde{\Lambda}_t) \tag{A3}$$

$$z_t^{POC} = r_{t-1}^d + \frac{(1-\Theta_g)[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\kappa_{t-1} + (r_t^k - E_{t-1}r_t^k) - \frac{\sigma}{\Psi}(c_t - E_{t-1}c_t) - \frac{\beta}{\Psi}(\tilde{\Lambda}_t - E_{t-1}\tilde{\Lambda}_t) \tag{A4}$$

$$\varpi_t^{POC} = \left(\frac{1-\nu(\kappa-1)}{\Theta_g(\kappa-1)}\right)\kappa_{t-1} - \frac{\sigma}{\Psi}(c_t - E_{t-1}c_t) - \frac{\beta}{\Psi}(\tilde{\Lambda}_t - E_{t-1}\tilde{\Lambda}_t) \tag{A5}$$

$$\tilde{\Lambda}_t = E_t\sum_{j=0}^{\infty}\beta^j\left(\Xi\kappa_{t+j} + r_{t+j}^d\right) \tag{A6}$$

$$r_t^k = \epsilon q_t + (1-\epsilon)mpk_t - q_{t-1} \tag{A7}$$

$$\kappa_{t-1} = (q_{t-1} + k_t - nw_{t-1}) \tag{A8}$$

$$\sigma c_t + \eta l_t = \alpha k_t + a_t - \alpha l_t \tag{A9}$$

$$r_t^d = \sigma(E_t c_{t+1} - c_t) \tag{A10}$$

$$q_t = \psi i_t \tag{A11}$$

$$k_{t+1} = \delta i_t + (1-\delta)k_t \tag{A12}$$

$$c_{ss}c_t + c_{ss}^e nw_t + i_{ss}i_t = a_t + \alpha k_t + (1-\alpha)l_t \tag{A13}$$

where $\epsilon \equiv \frac{1-\delta}{mpk_{ss}+(1-\delta)}$. Also we have $\kappa \equiv K_{SS}/NW_{SS}$, Q_{SS}= 1, $R_{SS}^s = 1/\beta$. Finally, we set $\mu\int_0^{\varpi_{ss}}\omega\phi(\omega)d\omega \approx 0$ so that monitoring costs do not appear in (A12).

2. The optimality of the debt contract.

The representative entrepreneur begins the period with net worth NW_t^i. We assume that the entrepreneur's value function is linear in net worth, given by $V_t NW_t^i$. Under this assumption, we will show that the optimal contract is risky debt. Using risky debt, the text demonstrates that his value function is linear, thus validating the linearity assumption and completing the circle. The entrepreneur has linear preferences in consumption and discounts the future at rate β. In equilibrium, the return on internal funds will always be high enough that he will postpone consumption until death. With probability (1-γ) he dies and eats $c_t^e = NW_t^i$. With probability γ, he lives and uses all of his net worth to fund a project, with leverage rate $\bar{\kappa}_t$. The per unit return on the entrepreneur's project is $\omega_{t+1}^i R_{t+1}^k$, where ω has a unit mean and is iid. The entrepreneur reports ω_{t+1}^i to the mechanism, and R_{t+1}^k is observed by all. Because of this observability, we can normalize the repayment function as $R_{t+1}^k R(\omega_{t+1}^i)$. Incentive compatibility implies that $R\left(\omega_{t+1}^i\right)$ is constant on the no-monitoring set, $R\left(\omega_{t+1}^i\right) = R_{t+1}$. The monitoring interval is given without loss of generality by $A_{t+1} = [0, \varpi_{t+1}]$, where $R(\varpi_{t+1}) = R_{t+1}$. The expected returns of the entrepreneur and lender are given by:

$$E_t ent_{t+1} = E_t \bar{\kappa}_t NW_t^i R_{t+1}^k V_{t+1}[1 - R_{t+1}(1 - \Phi_{t+1}) - X_{t+1}]$$

$$E_t len_{t+1} = E_t \bar{\kappa}_t NW_t^i R_{t+1}^k M_{t+1}[R_{t+1}(1 - \Phi_{t+1}) + X_{t+1} - \mu\Phi_{t+1}]$$

where $\Phi_{t+1}$is the CDF evaluated at ϖ_{t+1}, and we define

$$X_{t+1} \equiv \int_0^{\varpi_{t+1}} R(\omega)\phi(\omega)d\omega$$

The optimal contract maximizes $E_t ent_{t+1}$, subject to the lender's participation constraint:

$$E_t len_{t+1} \geq (\bar{\kappa}_t - 1)NW_t^i$$

Lemma: In the event of monitoring, all of the entrepreneur's assets are seized, ie., $R(\omega) = \omega$. Hence, if the entrepreneur's value function is linear, then the optimal contract is risky debt.

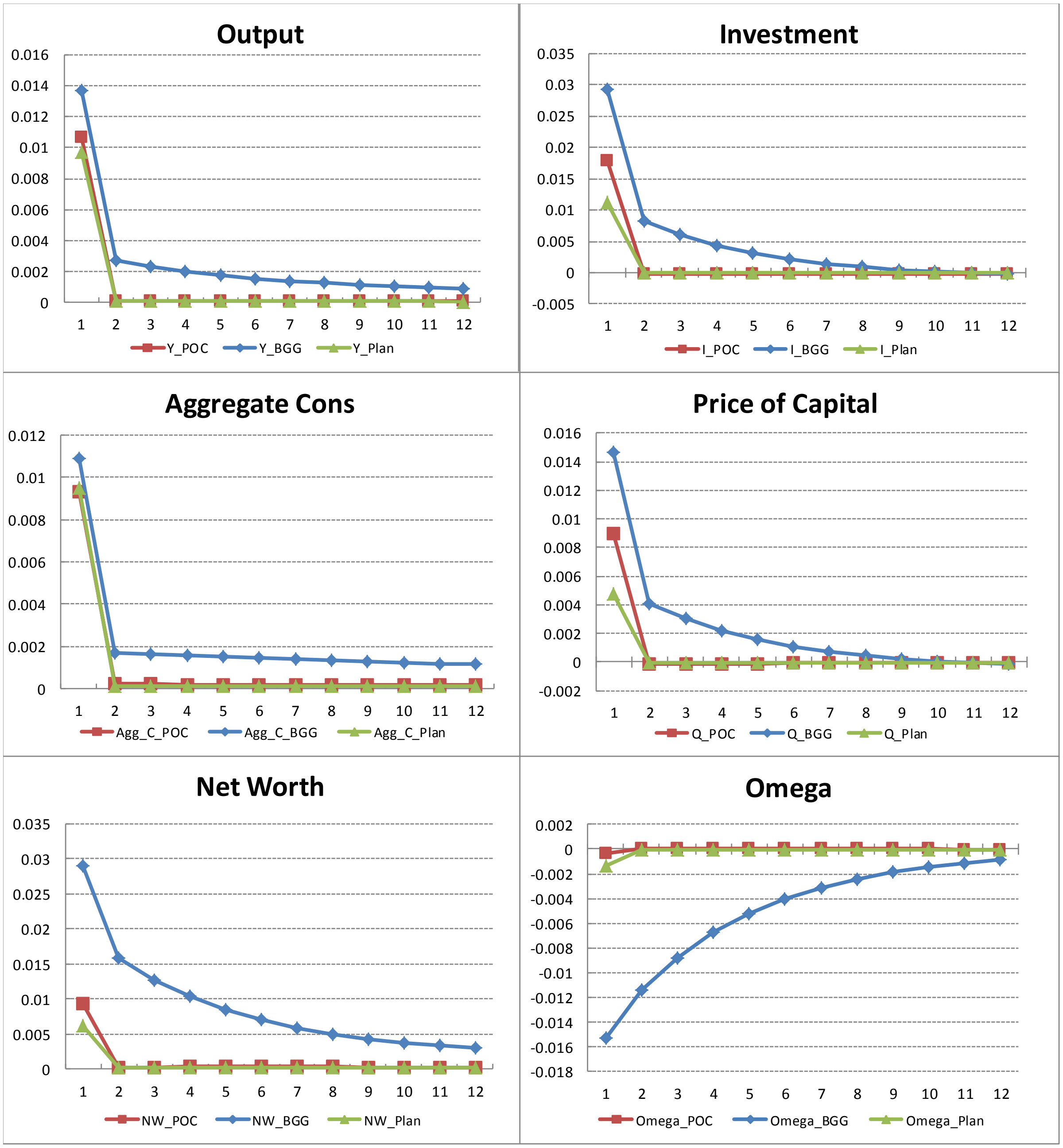
Proof: The proof is by contradiction. The repayment to the lender in default states is given by X_{t+1}. Suppose that $R(\omega) < \omega$ in some states. But this means we can keep X_{t+1} fixed, but lower ϖ_{t+1} by equating $R(\omega) = \omega$. This is welfare-improving as it relaxes the lender constraint (since Φ_{t+1} is lower) by lowering monitoring costs. In essence, the most efficient way of altering X_{t+1} is to vary ϖ_{t+1}. ■

Table 1: Welfare Comparison.

Welfare comparison*	**Baseline Calibration (ϵ = 1.01, K_{ss} = 3.58)**		**Higher leverage (ϵ = 0.84, K_{ss} = 3.03)**	
	Planner-POC	**Planner-BGG**	**Planner-POC**	**Planner-BGG**
Conditional welfare cost	0.348	0.376	0.492	0.619
Household conditional welfare	-0.119	-0.096	0.064	0.209
Entrepreneur conditional welfare	0.462	0.467	0.509	0.488
Steady state welfare cost	0.347	0.347	0.490	0.490
$\%\boldsymbol{\Delta\varpi_{ss}}$	-6.57%	-6.57%	-2.73%	-2.73%
$\%\boldsymbol{\Delta Q_{ss}}$	-1.9%	-1.9%	-1.5%	-1.5%

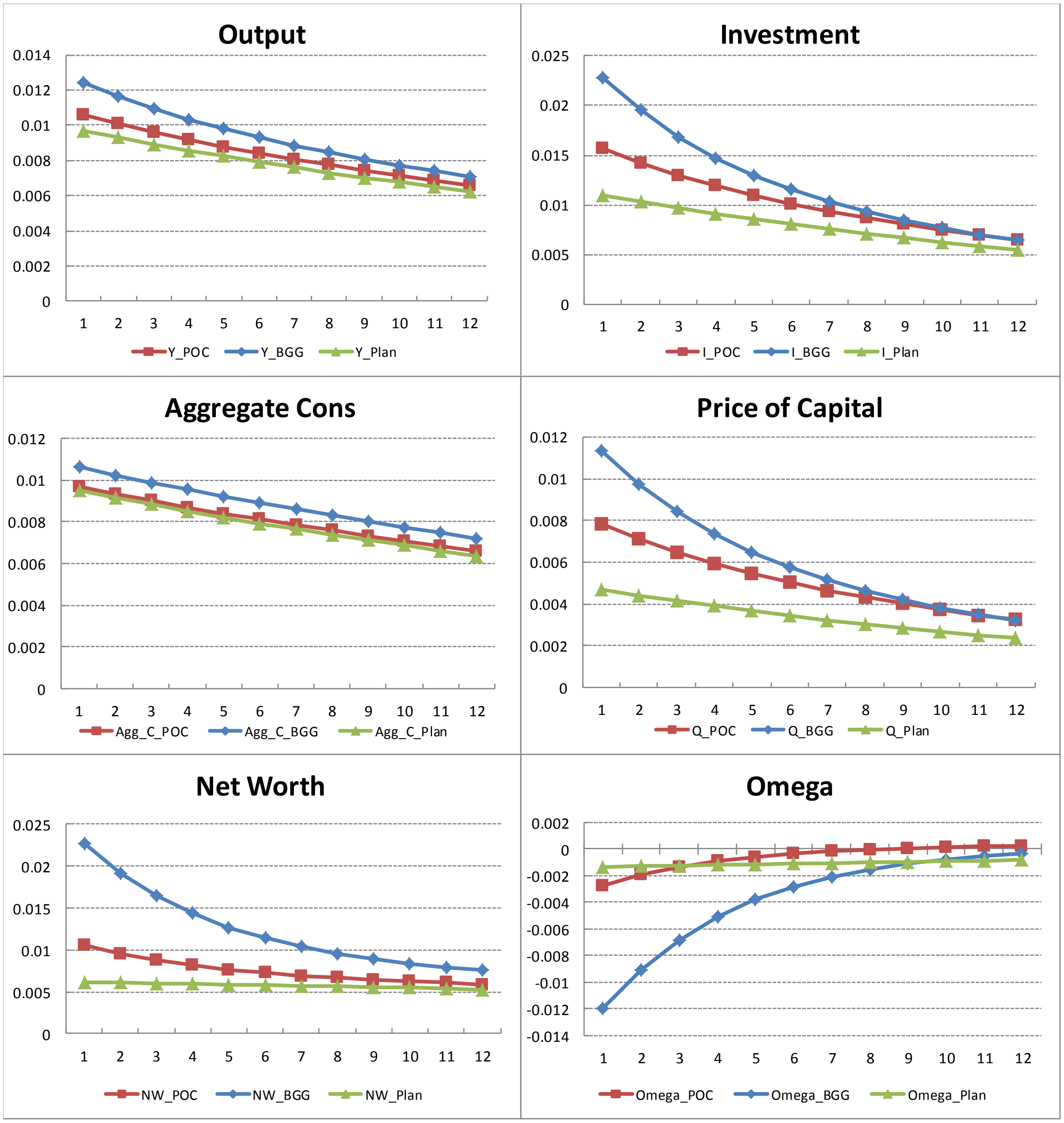
*Entries represent expected lifetime welfare conditional on being at the steady state capital stock. The calibration is discussed in the text. The planner's weight on the entrepreneurs is chosen to equate the planner's steady-state capital stock to the POC. Since the calibration is log utility, these numbers represent the perpetual flow increase in household consumption, eg., 0.35 is a perpetual increase in annual household consumption of 0.35%.

Figure 1: iid TFP shock.



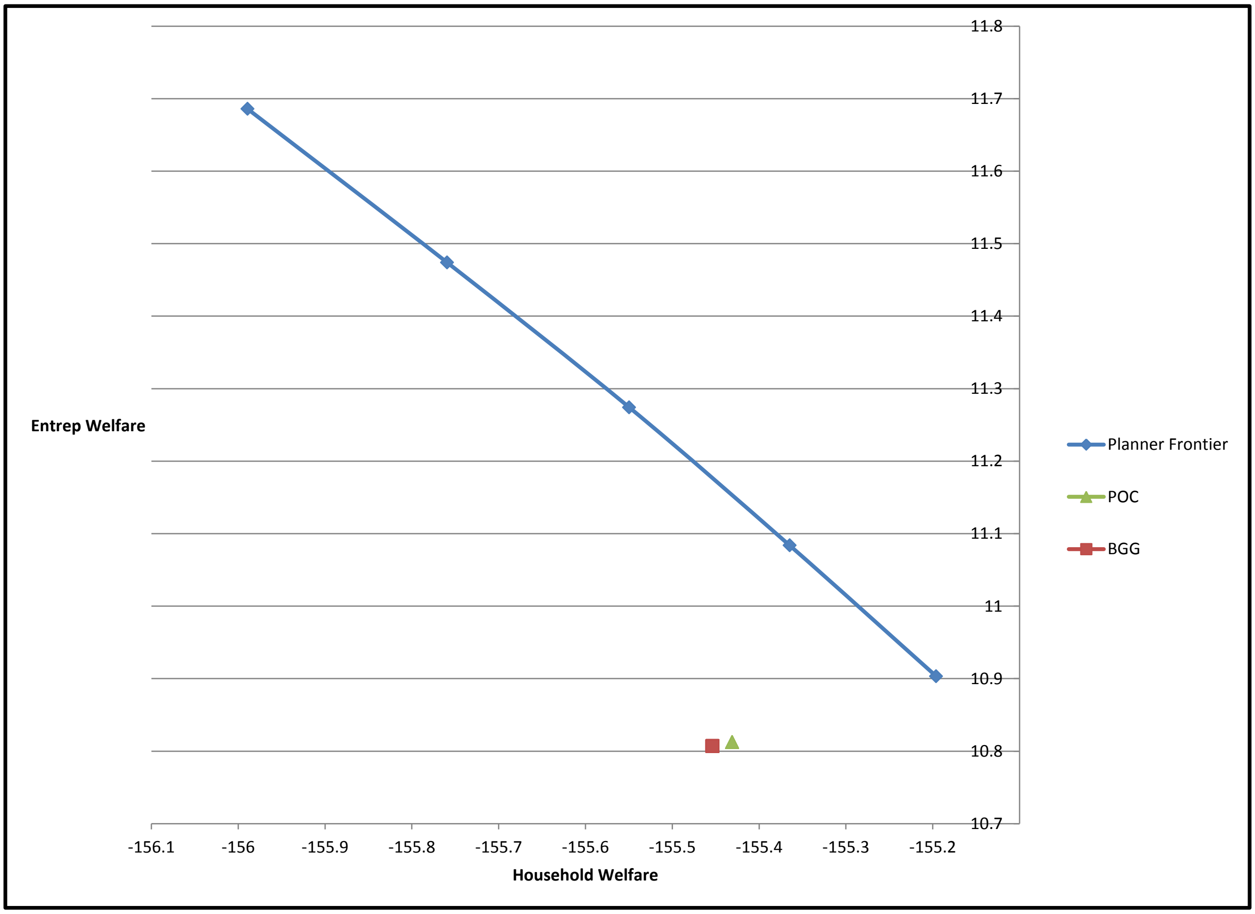
The response of output, investment, aggregate consumption, the price of capital, net worth, and the bankruptcy threshold, to a 1% iid TFP shock.

Figure 2: auto-correlated TFP shock.



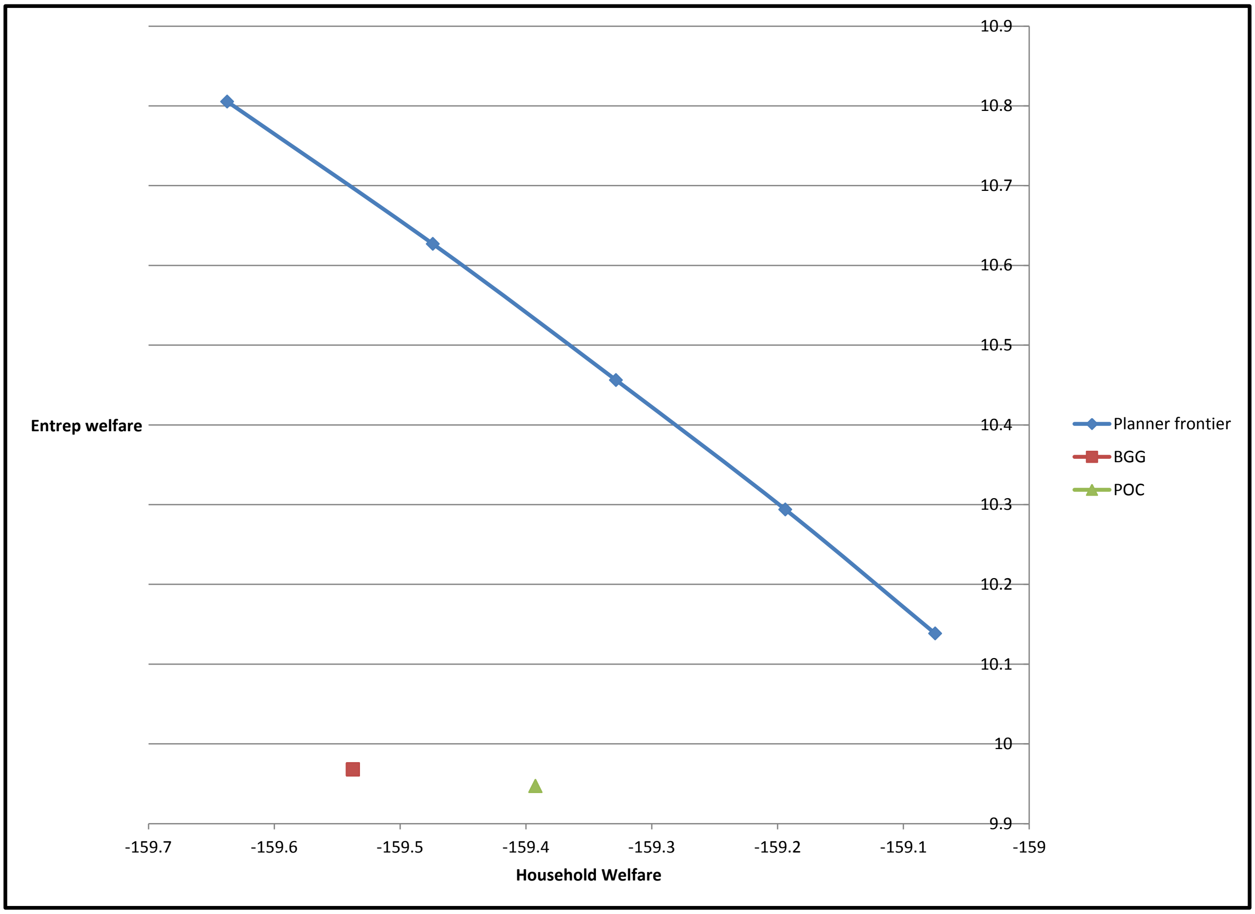
The response of output, investment, aggregate consumption, the price of capital, net worth, and the bankruptcy threshold, to a 1% autocorrelated TFP shock.

<u>Figure 3: Planner welfare frontier.</u>



***Baseline calibration including leverage of 2.**

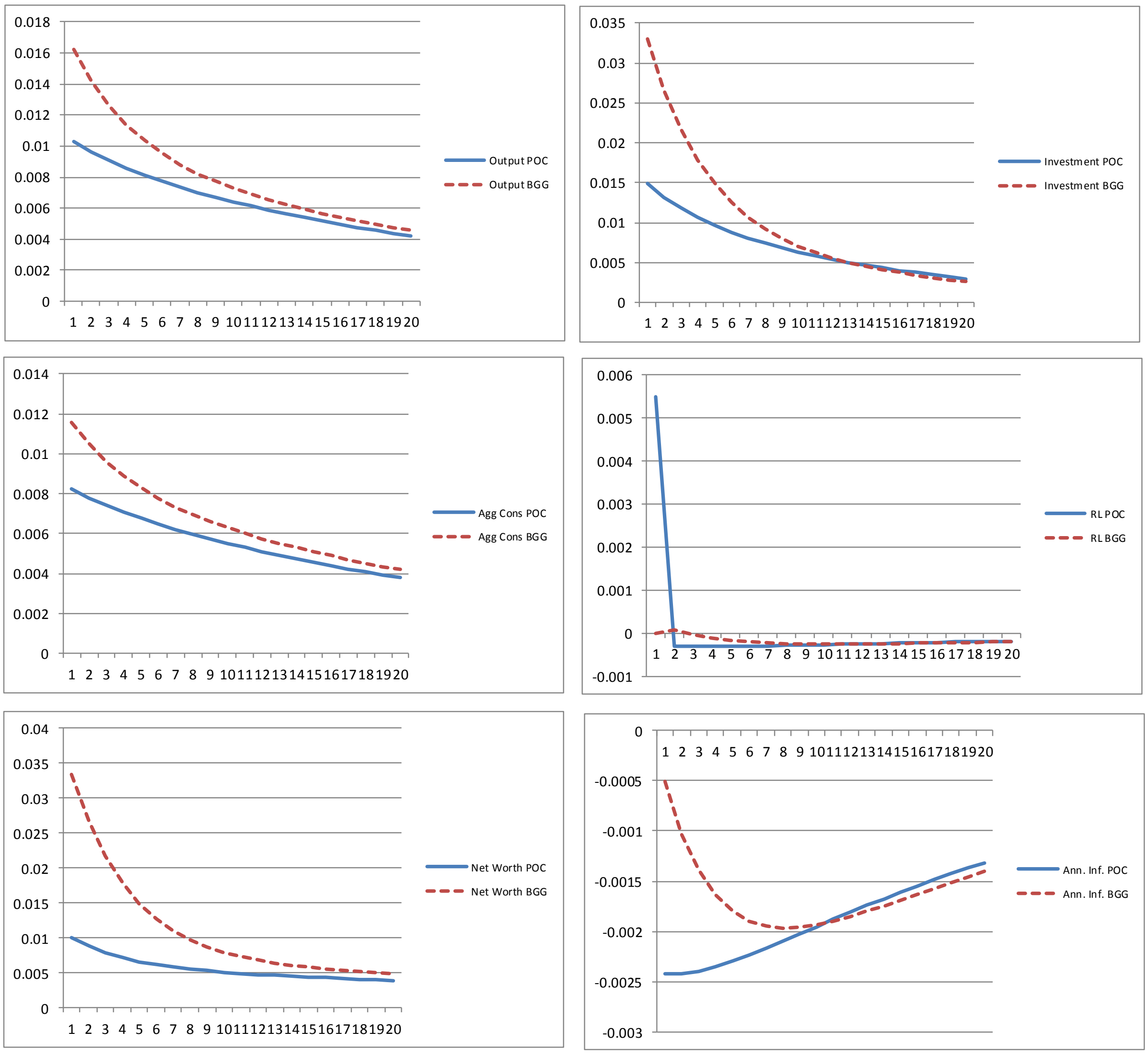
Figure 4: Planner welfare frontier.



***Calibration with leverage = 4.**

<u>Figure 5: TFP shock with sticky prices.</u>

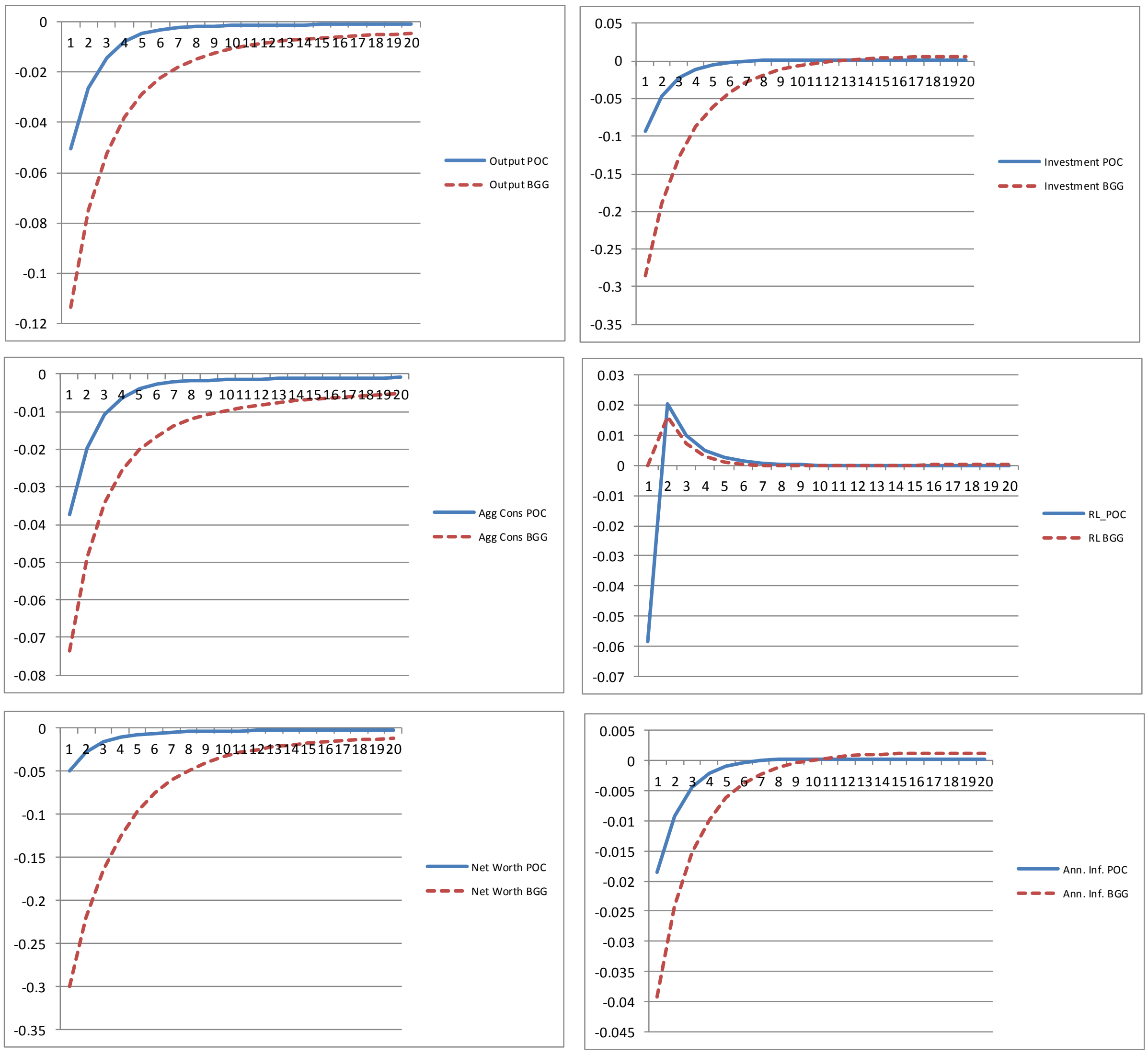
(Impulse response to a 1% TFP shock.)



The response of output, investment, aggregate consumption, the lender's return, net worth, and inflation (annualized) to a 1% autocorrelated TFP shock.

Figure 6: Monetary shock with sticky prices.

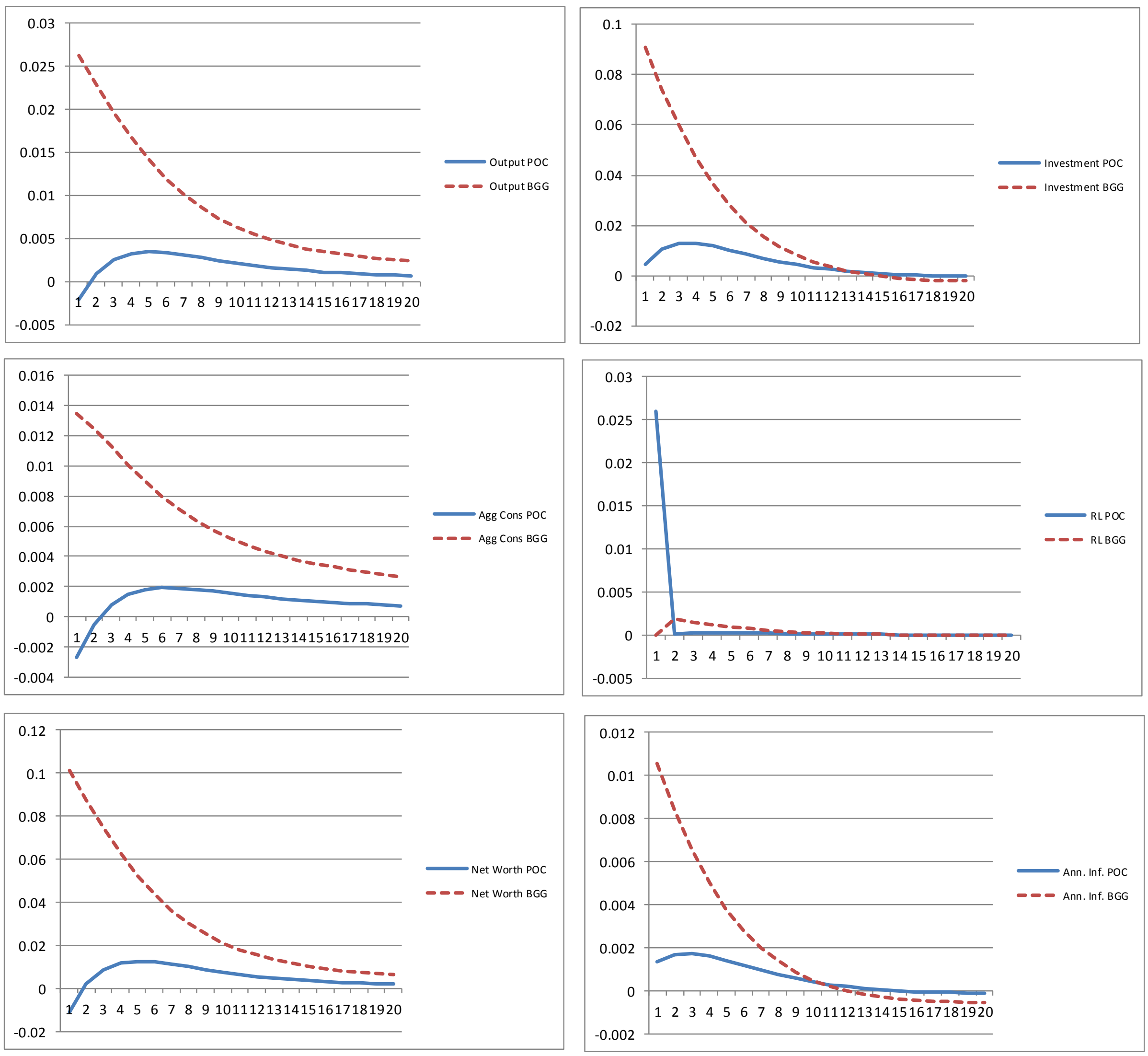
(Impulse response to a 25 bp quarterly policy shock.)



The response of output, investment, aggregate consumption, the lender's return, net worth, and inflation (annualized) to a 25 b.p. (quarterly) autocorrelated monetary policy shock.

Figure 7: Net worth shock with sticky prices.

(Impulse response to a 1% shock to net worth.)



The response of output, investment, aggregate consumption, the lender's return, net worth, and inflation (annualized) to a 1% autocorrelated increase in entrepreneur's net worth.