Estimating Contract Indexation in a Financial Accelerator Model

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This paper addresses the positive implications of indexing risky debt to observable aggregate conditions. These issues are pursued within the context of the celebrated financial accelerator model of Bernanke, Gertler and Gilchrist (1999). The principal conclusions include: (1) the estimated level of indexation is significant, (2) the business cycle properties of the model are significantly affected by this degree of indexation, (3) the importance of investment shocks in the business cycle depends upon the estimated level of indexation, and (4) although the data prefers the financial model with indexation over the frictionless model, they have remarkably similar business cycle properties for non-financial exogenous shocks.

Key words: Agency costs; financial accelerator; business cycles.

JEL Codes: E32, E44.

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1. Introduction.

The fundamental function of credit markets is to channel funds from savers to entrepreneurs who have some valuable capital investment project. These efforts are hindered by agency costs arising from asymmetric information. A standard result in a subset of this literature, the costly state verification (CSV) framework, is that risky debt is the optimal contract between risk-neutral lenders and entrepreneurs. The modifier risky simply means that there is a non-zero chance of default. In the CSV model external parties can observe the realization of the entrepreneur's idiosyncratic production technology only by expending a monitoring cost. Townsend (1979) demonstrates that risky debt is optimal in this environment because it minimizes the need for verification of project outcomes. This verification is costly but necessary to align the incentives of the firm with the bank.

Aggregate conditions will also affect the ability of the borrower to repay the loan. But since aggregate variables are observed by both parties, it may be advantageous to have the loan contract indexed to the behavior of aggregate variables. Therefore, even when loan contracts cannot be designed based on private information, we can exploit common information to make these financial contracts more state-contingent. That is, why should the loan contract call for costly monitoring when the event that leads to a poor return is observable by all parties?[2] Carlstrom, Fuerst, and Paustian (2013) examine questions of this type within the financial accelerator of Bernanke, Gertler, and Gilchrist (1999), hereafter BGG. Carlstrom et al. (2013) demonstrate that the privately optimal contract in the BGG model includes indexation to: (i) the aggregate return to capital (which we will call R^k-indexation), (ii) the marginal utility of wealth (which we will call λ-indexation), and, (iii) the shadow cost of external financing.

In this paper we explore the business cycle implications of indexing the BGG loan contract to the aggregate return to capital and to the marginal utility of wealth. There are at least two reasons why this is an interesting exercise. First, as noted above, Carlstrom et al. (2013) demonstrate that the privately optimal contract in the BGG framework includes indexation of this very type. Second, indexation of this

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[2]This is the logic behind Shiller and Weiss's (1999) suggestion of indexing home mortgages to movements in aggregate house prices.

type is not so far removed from some financial contracts we do observe. For example, indexing repayment to innovations in the marginal utility of wealth is a close approximation to indexation to movements in the risk free rate of interest. There are many debt instruments that are directly linked to market interest rate of this type, e.g., adjustable rate mortgages. More generally, since we are assuming that the CSV framework proxies for agency cost effects in the entire US financial system, it seems reasonable to include some form of indexation to mimic the myriad ex post returns on external financing. For example, in contrast to the model assumption where entrepreneurs get zero in the event of bankruptcy, this is clearly not the implication of Chapter 11 bankruptcy. In any event, we use familiar Bayesian methods to estimate the degree of contract indexation to the return to capital and the marginal utility of wealth.

To avoid misspecification problems in the estimation we need a complete model of the business cycle. We use the recent contribution of Justiniano, Primiceri, and Tambalotti (2011), hereafter JPT, as our benchmark. A novelty of the JPT model is that it includes two shocks to the capital accumulation technology. The first shock is a non-stationary shock to the relative cost of producing investment goods, the "investment specific technology shock" (IST). The second is a stationary shock to the transformation of investment goods into installed capital, the "marginal efficiency of investment shock" (MEI). For business cycle variability, JPT find that the IST shocks are irrelevant, while the MEI shocks account for a substantial portion of business cycle fluctuations.

Our principle results include the following. First, the estimated level of R^k-indexation significantly exceeds unity, much higher than the assumed BGG indexation of approximately zero. A model with R^k-indexation fits the data significantly better when compared to BGG. This is because the BGG model's prediction for the risk premium in the wake of a MEI shock is counterfactual. A MEI shock lowers the price of capital and thus leads to a sharp decline in entrepreneurial net worth in the BGG model. But under R^k-indexation, the required repayment falls also so that net worth moves by significantly less.

Second, with R^k-indexation, this financial model and JPT have remarkably similar business cycle

properties for *non-financial* exogenous shocks. For example, for the case of MEI shocks, the estimated level of indexation leads to net worth movements in the financial model that accommodate real behavior quite similar to the response of JPT to an MEI shock. We also nest financial shocks into the JPT model by treating fluctuations in these two financial variables as serially correlated measurement error. This model horserace results in the R^k-indexation model dominating BGG, which in turn significantly dominates JPT. The financial models are improvements over JPT in two ways. The financial models make predictions for the risk premium and leverage on which JPT is silent, and the financial models introduce other exogenous shocks, e.g., shocks to net worth or idiosyncratic variance, that are irrelevant in JPT.

Third, we find that whether financial shocks or MEI shocks are more important drivers of the business cycle depends upon the level of indexation. Under BGG, financial shocks account for a significant part of the variance of investment spending. But under the estimated level of R^k-indexation, financial shocks become much less important and the MEI shocks are again of paramount importance.

Two prominent papers closely related to the current work are Christiano, Motto, and Rostagno (2010), and DeGraeve (2008). They each use Bayesian methods to estimate versions of the BGG framework in medium-scale macro models. Both papers conclude that the model with financial frictions provides a better fit to the data when compared to its frictionless counterpart. The chief novelty of the current paper is to introduce contract indexation into the BGG framework, and demonstrate that it is empirically relevant, altering the business cycle properties of the model. Neither of the previous papers considered indexation of this type.

The paper proceeds as follows. Section 2 presents a simple example that illustrates the importance of contract indexation to the financial accelerator. Section 3 develops the DSGE model. Section 4 presents the estimation results. Section 5 concludes.

2. Why does indexation matter? A simple example.

This section presents a simple intuitive example that demonstrates the importance of indexation in determining the size of the financial accelerator. Consider a world with agency costs in which the portion of net worth owned by entrepreneurs (nw_t) has a positive effect on the value of capital (q_t):

$$q_t = p * nw_t + \epsilon_t^d \tag{1}$$

where the expression is in log deviations and ϵ_t^d is an exogenous shock to capital prices, e.g., a shock to MEI in the general equilibrium model below. Equation (1) is a manifestation of agency costs in that the distribution of net worth across lenders and borrowers affects asset prices. The idea is that higher net worth in the hands of entrepreneurs makes it easier for them to access a loan with which to buy capital, so that higher levels of net worth act like a demand channel on asset prices. In the general equilibrium model below, the value of p is a function of the agency cost and (installed) capital adjustment cost parameters.

The entrepreneur accumulates net worth to mitigate the agency problems involved in direct lending. The agency problem arises from a CSV problem in the entrepreneur's production technology. The entrepreneur takes one unit of input and creates ω_t units of capital, where the unit-mean random variable ω_t is privately observed by the entrepreneur but can be verified by the lender only by paying a cost. This CSV problem makes equity finance problematic, so that the optimal contract is given by a risky debt contract with a promised repayment of r_t^p. The repayment r_t^p cannot be indexed to ω_t because it is privately observed. But it can be indexed to the aggregate price of capital:

$$r_t^p = \chi q_t. \tag{2}$$

This form of indexation is similar to indexing to the rate of return on capital in the general equilibrium model developed below.

Entrepreneurial net worth accumulates with the profit flow from the investment project, but decays via consumption of entrepreneurs (which is a constant fraction of net worth). Log-linearized this evolution is given by:

$$nw_t = \kappa\left(q_t - r_t^p\right) + nw_{t-1} + r_t^p + \epsilon_t^n \qquad (3)$$

where $\kappa > 1$ denotes leverage (the ratio of project size to net worth) and ϵ_t^n is an exogenous shock to net worth. Using the indexation assumption (2), we can express (3) as

$$nw_t = q_t[\kappa - \chi(\kappa - 1)] + nw_{t-1} + \epsilon_t^n \qquad (4)$$

Note that since $\kappa > 1$, the slope of the net worth equation is decreasing in the level of indexation.

Equations (1) and (4) are a simultaneous system in net worth and the price of capital. We can solve for the two endogenous variables as a function of the pre-determined and exogenous variables:

$$nw_t = \frac{nw_{t-1} + \epsilon_t^n + \epsilon_t^d[\kappa - \chi(\kappa - 1)]}{\{1 - p[\kappa - \chi(\kappa - 1)]\}} \qquad (5)$$

$$q_t = \frac{p(nw_{t-1} + \epsilon_t^n) + \epsilon_t^d}{\{1 - p[\kappa - \chi(\kappa - 1)]\}} \qquad (6)$$

The inverse of the denominator in (5)-(6) is the familiar "multiplier" arising from two endogenous variables with positive feedback. This then implies that exogenous shocks are "multiplied" or "financially accelerated", and that the degree of this multiplication depends upon the level of indexation. The effect of indexation on the financial multiplier is highly nonlinear. Figure 1 plots the multiplier for $\kappa = 2$, and $p = 0.45$, both of these values roughly correspond to the general equilibrium analysis below. Note that moving from $\chi = 0$ to $\chi = 1$, has an enormous effect on the multiplier. But there are sharp diminishing returns so the multiplier is little changed as we move from $\chi = 1$ to $\chi = 2$. This suggests, and we confirm below, that the data can distinguish $\chi = 0$ from, say, $\chi = 1$, but that indexation values in excess of unity will have similar business cycle characteristics and thus be difficult to identify.

Consider three special cases of indexation: $\chi = 0$, $\chi = 1$, and $\chi = {}^{\kappa}/_{(\kappa - 1)}$. The first is the implicit assumption in BGG; the second implies complete indexation; the third eliminates the financial accelerator altogether. In these cases, net worth and asset prices are given by:

Indexation	Net worth	Capital price	Multiplier (p=0.45, κ =2)
$\chi = 0$	$\dfrac{nw_{t-1} + \epsilon_t^n + \kappa\epsilon_t^d}{1 - p\kappa}$	$\dfrac{p(nw_{t-1} + \epsilon_t^n) + \epsilon_t^d}{1 - p\kappa}$	10
$\chi = 1$	$\dfrac{nw_{t-1} + \epsilon_t^n + \epsilon_t^d}{1 - p}$	$\dfrac{p(nw_{t-1} + \epsilon_t^n) + \epsilon_t^d}{1 - p}$	1.82
$\chi = {}^{\kappa}\!/_{(\kappa - 1)}.$	$nw_{t-1} + \epsilon_t^n$	$p(nw_{t-1} + \epsilon_t^n) + \epsilon_t^d$	1

For both $\chi = 0$ and $\chi = 1$, exogenous shocks to asset prices and net worth have multiple effects on the equilibrium levels of net worth and capital prices. Since $\kappa > 1$, this effect is much larger under BGG's assumption of no indexation ($\frac{1}{1-p\kappa} \gg \frac{1}{1-p}$). Further, under the BGG assumption, exogenous shocks to asset prices (ϵ_t^d) have an added effect as they are weighted by leverage. But for all levels of indexation, there are always agency cost effects in that the price of capital is affected by the level of entrepreneurial net worth. The financial multiplier effects are traced out in Figure 2: an exogenous shock to asset prices has a much larger effect on both net worth and asset prices in the BGG framework. Finally, since $\kappa \approx 2$ the financial accelerator largely disappears when $\chi = 2$.

Before proceeding, it is helpful to emphasize the two parameters that are crucial in our simple example as they will be manifested below in the richer general equilibrium environment. Our reduced form parameter p in equation (1) is the agency cost parameter. In a Modigliani-Miller world we would have p = 0, as the distribution of net worth would have no effect on asset prices or real activity. Second, the indexation parameter χ determines the size of the financial accelerator, i.e., how do unexpected movements in asset prices feed in to net worth? These are two related but logically distinct ideas. That is, one can imagine a world with agency costs ($p > 0$), but with very modest accelerator effects ($\chi = {}^{\kappa}\!/_{(\kappa - 1)}.$). To anticipate our empirical results, this is the parameter set that wins the model horse race. That is, the data is consistent with an agency cost model but with trivial accelerator effects. In such an

environment, financial shocks (e.g., shocks to net worth) will affect real activity, but other real shocks (e.g., MEI shocks) will not be accelerated.

3. The Model.

The benchmark model follows the JPT framework closely. The model of agency costs comes from BGG with the addition of exogenous contract indexation. The BGG loan contract is between lenders and entrepreneurs, so we focus on these two agents first before turning to the familiar framework of JPT.

Lenders.

The representative lender accepts deposits from households (promising a sure real return R_t^d) and provides loans to the continuum of entrepreneurs. These loans are intertemporal, with the loans made at the end of time *t* being paid back in time *t+1*. The realized gross real return on these loans is denoted by R_{t+1}^L. Each individual loan is subject to idiosyncratic and aggregate risk, but since the lender holds an entire portfolio of loans, only the aggregate risk remains. The lender has no other source of funds, so the level of loans will equal the level of deposits. Hence, real dividends are given by $Div_{t+1} = (R_{t+1}^L D_t - R_t^d D_t)$. The intermediary seeks to maximize its equity value which is given by:

$$Q_t^L = E_t \sum_{j=1}^{\infty} \frac{\beta^j \Lambda_{t+j}}{\Lambda_t} Div_{t+j} \tag{9}$$

where Λ_t is the marginal utility of real income for the representative household that owns the lender. The FOC of the lender's problem is:

$$E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[R_{t+1}^L - R_t^d \right] = 0 \tag{10}$$

The first-order condition shows that in expectation, the lender makes zero profits, but ex-post profits and losses can occur.[3] We assume that losses are covered by households as negative dividends. This is similar to the standard assumption in the Dynamic New Keynesian (DNK) model, e.g., Woodford (2003). That is,

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[3] In contrast, BGG assume that bank profits are always zero ex post so that the lender's return in pre-determined. This is not a feature of the optimal contract. See Carlstrom et al. (2013) for details.

the sticky price firms are owned by the household and pay out profits to the household. These profits are typically always positive (for small shocks) because of the steady state mark-up over marginal cost. Similarly, one could introduce a steady-state wedge (e.g., monopolistic competition among lenders) in the lender's problem so that dividends are always positive. But this assumption would have no effect on the model's dynamics so we dispense from it for simplicity.

Entrepreneurs and the Loan Contract.

Entrepreneurs are the sole accumulators of physical capital. At the beginning of period t, the entrepreneurs sell all of their accumulated capital to "capital agencies" at beginning-of-period capital price Q_t^{beg}. At the end of the period, the entrepreneurs purchase the entire capital stock $\bar{K}_t$, including any net additions to the stock, at end-of-period price Q_t. This re-purchase of capital is financed with entrepreneurial net worth (NW_t) and external financing from a lender. The external finance takes the form of a one period loan contract. The gross return to holding capital from time-t to time $t+1$ is given by:

$$R_{t+1}^k \equiv \frac{Q_{t+1}^{beg}}{Q_t}. \tag{11}$$

Below we show that $Q_{t+1}^{beg} = Q_{t+1}(1-\delta) + [\rho_{t+1}u_{t+1} - a(u_{t+1})] \approx Q_{t+1}(1-\delta) + \rho_{t+1}$, the latter term coinciding with the expression in BGG. Variations in R_{t+1}^k are the source of aggregate risk in the loan contract. The external financing is subject to a costly-state-verification (CSV) problem because of idiosyncratic risk. In particular, one unit of capital purchased at time-t is transformed into ω_{t+1} units of capital in time $t+1$, where ω_{t+1} is a idiosyncratic random variable with density $\phi(\omega)$ and cumulative distribution $\Phi(\omega)$. The realization of ω_{t+1} is directly observed by the entrepreneur, but the lender can observe the realization only if a monitoring cost is paid, a cost that is fraction μ_{mc} of the project outcome. Assuming that the entrepreneur and lender are risk-neutral, Townsend (1979) demonstrates that the optimal contract between entrepreneur and intermediary is risky debt in which monitoring only occurs if

the promised payoff is not forthcoming. Payoff does not occur for sufficiently low values of the idiosyncratic shock, $\omega_{t+1} < \varpi_{t+1}$. Let R^p_{t+1} denote the promised gross rate-of-return so that R^p_{t+1} is defined by

$$R^p_{t+1}(Q_t\bar{K}_t - NW_t) \equiv \varpi_{t+1}R^k_{t+1}Q_t\bar{K}_t .\tag{12}$$

We find it convenient to express this in terms of the leverage ratio $\bar{\kappa}_t \equiv \left(\frac{Q_t\bar{K}_t}{NW_t}\right)$ so that (4) becomes

$$R^p_{t+1} \equiv \varpi_{t+1}R^k_{t+1}\frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1}\tag{13}$$

With $f(\varpi_{t+1})$ and $g(\varpi_{t+1})$ denoting the entrepreneur's share and lender's share of the project outcome, respectively, the lender's ex post realized $t+1$ return on the loan contract is defined as:

$$R^L_{t+1} \equiv \frac{R^k_{t+1}g(\varpi_{t+1})Q_t\bar{K}_t}{Q_t\bar{K}_t - NW_t} = R^k_{t+1}g(\varpi_{t+1})\frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1}\tag{14}$$

where

$$f(\varpi) \equiv \int_{\varpi}^{\infty} \omega\phi(\omega)d\omega - [1 - \Phi(\varpi)]\varpi\tag{15}$$

$$g(\varpi) \equiv [1 - \Phi(\varpi)]\varpi + (1 - \mu_{mc})\int_0^{\varpi} \omega\phi(\omega)d\omega\tag{16}$$

Recall that the lender's stochastic discount factor comes from the household, and the lender's return is linked to the return on deposits via (10):

$$E_t R^L_{t+1}\Lambda_{t+1} = R^d_t E_t\Lambda_{t+1}\tag{17}$$

As for the entrepreneur, Carlstrom, Fuerst and Paustian (2013) show that the entrepreneur's value function is linear with a time-varying coefficient we denote by V_t, where V_t satisfies:

$$V_t = (1 - \gamma) + \beta\gamma\bar{\kappa}_t E_t V_{t+1}R^k_{t+1}f(\varpi_{t+1})\tag{18}$$

Using this valuation and expression (17), the end-of-time-t contracting problem is thus given by:

$$max_{\bar{\kappa}_t, \varpi_{t+1}}\{E_t V_{t+1}R^k_{t+1}\bar{\kappa}_t f(\varpi_{t+1})\}\tag{19}$$

subject to

$$E_t R^k_{t+1}\frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1}\Lambda_{t+1}g(\varpi_{t+1}) \geq R^d_t E_t\Lambda_{t+1}\tag{20}$$

An important observation is that the choice of ϖ_{t+1} can be made contingent on public information available in time $t+1$. Indexation of this type is optimal. After some re-arrangement, the contract optimization conditions include:

$$V_{t+1}f'(\varpi_{t+1}) = \left[\frac{E_t V_{t+1}f'(\varpi_{t+1})}{E_t \Lambda_{t+1}g'(\varpi_{t+1})}\right]\Lambda_{t+1}g'(\varpi_{t+1}) \tag{21}$$

$$(\bar{\kappa}_t - 1)E_t V_{t+1}R_{t+1}^k f(\varpi_{t+1}) = \left[\frac{-E_t V_{t+1}f'(\varpi_{t+1})}{E_t \Lambda_{t+1}g'(\varpi_{t+1})}\right]E_t \Lambda_{t+1}R_{t+1}^k g(\varpi_{t+1}) \tag{22}$$

$$E_t \Lambda_{t+1}R_{t+1}^k \frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1}g(\varpi_{t+1}) = R_t^d E_t \Lambda_{t+1} \tag{23}$$

A key result is given by (21). The optimal monitoring cut-off ϖ_{t+1} is independent of innovations in R_{t+1}^k. The second-order condition implies that the ratio $\frac{-f'(\varpi_{t+1})}{g'(\varpi_{t+1})}$, is increasing in ϖ_{t+1}. Hence, another implication of the privately optimal contract is that ϖ_{t+1} (and thus the optimal repayment rate R_{t+1}^p) is an increasing function of (innovations in) the marginal utility of wealth Λ_{t+1}, and a decreasing function of (innovations in) the entrepreneur's valuation V_{t+1}.

The monitoring cut-off implies the behavior of the repayment rate (see (13)). In log deviations, Carlstrom, Fuerst, and Paustian (2013) show that the repayment rate under the optimal contract is given by:

$$\hat{r}_t^p = r_{t-1}^d + \frac{(1-\Theta_g)[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\kappa_{t-1} + (\hat{r}_t^k - E_{t-1}\hat{r}_t^k) + \frac{1}{\Psi}(\hat{\lambda}_t - E_{t-1}\hat{\lambda}_t) - \frac{1}{\Psi}(\hat{v}_t - E_{t-1}\hat{v}_t) \tag{24}$$

$$\hat{v}_t = E_t \sum_{j=0}^{\infty} \beta^{j+1}\left(\Xi\hat{\kappa}_{t+j} + \hat{r}_{t+j+1}^k\right) \tag{25}$$

where the hatted lower case letters denote log deviations, and the positive constants Ξ and Ψ are defined in the appendix. Innovations in $\hat{v}_t$ will be driven by innovations in $\hat{r}_t^k$. Hence, for parsimony we will estimate an indexation rule of the form:

$$\hat{r}_t^p = \Omega_{t-1} + \chi_k(\hat{r}_t^k - E_{t-1}\hat{r}_t^k) + \chi_\lambda(\lambda_t - E_{t-1}\lambda_t) \tag{26}$$

where Ω_{t-1} are the pre-determined variables that affect the repayment rate, e.g., $\bar{\kappa}_{t-1}$. As noted in the example sketched in Section 2, different indexation values will have dramatic effects on the financial accelerator. The original BGG model assumes that the lender's return was pre-determined. From (13)-

(14), this implies that $\chi_\lambda = 0$ and $\chi_k \approx 0$, where χ_k is modestly negative ($\chi_k = -0.01$, in our

benchmark calibration).

Entrepreneurs have linear preferences and discount the future at rate β. Given the high return to

internal funds, they will postpone consumption indefinitely. To limit net worth accumulation and ensure

that there is a need for external finance in the long run, we assume that fraction (1-γ) of the entrepreneurs

die each period. Their accumulated assets are sold and the proceeds transferred to households as

consumption. Given the exogenous death rate, aggregate net worth accumulation is described by

$$NW_t = \gamma NW_{t-1} \bar{\kappa}_{t-1} R_t^k f(\varpi_t) \eta_{nw,t} \tag{27}$$

where $\eta_{nw,t}$ is an exogenous disturbance to the distribution of net worth. We assume it follows the

stochastic process

$$log\, \eta_{nw,t} = \rho_{nw} log\, \eta_{nw,t-1} + \varepsilon_{nw,t}, \tag{28}$$

with $\varepsilon_{nw,t}$ i.i.d. N(0,σ_{nw}^2). Equation (27) implies that NW_t is determined by the realization of R_t^k and the

response of ϖ_t to these realizations. NW_t then enters the contracting problem in time t so that the

realization of R_t^k is propagated forward.

As in Christiano, Motto, and Rostagno (2010), and Gilchrist, Ortiz and Zakrajšek (2009), we also

consider time variation in the variance of the idiosyncratic shock ω_t. The variance of ω_t is denoted by σ_t

and follows the exogenous stochastic process given by

$$log\, \sigma_t = \rho_\sigma log \sigma_{t-1} + \varepsilon_{\sigma,t},$$

Shocks to this variance will alter the risk premium in the model.

Final good producers.

Perfectly competitive firms produce the final consumption good Y_t combining a continuum of

intermediate goods according to the CES technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{1/(1+\lambda_{p,t})}\, di \right]^{1+\lambda_{p,t}} \tag{29}$$

The elasticity $\lambda_{p,t}$ follows the exogenous stochastic process

$$log\ \lambda_{p,t} = (1 - \rho_p)\ log\ \lambda_p + \rho_p\ log\ \lambda_{p,t-1} + \varepsilon_{p,t} - \theta_p \varepsilon_{p,t-1}, \tag{30}$$

where $\varepsilon_{p,t}$ is *i.i.d.* $N(0,\sigma_p^2)$. Fluctuations in this elasticity are price markup shocks. Profit maximization

and the zero profit condition imply that the price of the final good, P_t, is the familiar CES aggregate of the

prices of the intermediate goods.

Intermediate goods producers.

A monopolist produces the intermediate good *i* according to the production function

$$Y_t(i) = max\{A_t^{1-\alpha}K_t(i)^\alpha L_t(i)^{1-\alpha} - A_t Y_t^{\frac{\alpha}{1-\alpha}}F; 0\}, \tag{31}$$

where $K_t(i)$ and $L_t(i)$ denote the amounts of capital and labor employed by firm *i*. *F* is a fixed cost of

production, chosen so that profits are zero in steady state. The variable A_t is the exogenous non-

stationary level of TFP progress. Its growth rate $(z_t \equiv \Delta ln A_t)$ is given by

$$z_t = (1 - \rho_z)\gamma_z + \rho_z z_{t-1} + \varepsilon_{z,t}, \tag{32}$$

with $\varepsilon_{z,t}$ *i.i.d.*$N(0,\sigma_z^2)$. The other non-stationary process Y_t is linked to the investment sector and is

discussed below.

Every period a fraction ξ_p of intermediate firms cannot choose its price optimally, but resets it

according to the indexation rule

$$P_t(i) = P_{t-1}(i)\pi_{t-1}^{\iota_p}\pi^{1-\iota_p}, \tag{33}$$

where $\pi_t \equiv P_t/P_{t-1}$ is gross inflation and π is its steady state. The remaining fraction of firms chooses its

price $P_t(i)$ optimally, by maximizing the present discounted value of future profits

$$E_t\left\{\sum_{s=0}^\infty \xi_p^s \frac{\beta^s \Lambda_{t+s}/P_{t+s}}{\Lambda_t/P_t}\left[P_t(i)\left(\prod_{k=1}^s \pi_{t+k-1}^{\iota_p}\pi^{1-\iota_p}\right)Y_{t+s}(i) - W_{t+s}L_{t+s}(i) - P_{t+s}\rho_{t+s}K_{t+s}(i)\right]\right\} \tag{34}$$

where the demand function comes from the final goods producers, Λ_t/P_t is the marginal utility of

nominal income for the representative household, and W_t is the nominal wage.

Employment agencies

Firms are owned by a continuum of households, indexed by $j \in [0,1]$. Each household is a monopolistic

supplier of specialized labor, $L_t(j)$, as in Erceg et al. (2000). A large number of competitive employment

agencies combine this specialized labor into a homogenous labor input sold to intermediate firms,

according to

$$L_t = \left[\int_0^1 L_t(j)^{1/(1+\lambda_{w,t})} \, dj \right]^{1+\lambda_{w,t}} \tag{35}$$

As in the case of the final good, the desired markup of wages over the household's marginal rate of

substitution, $\lambda_{w,t}$, follows the exogenous stochastic process

$$\log \lambda_{w,t} = (1 - \rho_w)\log \lambda_w + \rho_w \log \lambda_{w,t-1} + \varepsilon_{w,t} - \theta_w \varepsilon_{w,t-1}, \tag{36}$$

with $\varepsilon_{w,t}$ *i.i.d.* N $(0, \sigma_w^2)$. This is the wage markup shock. Profit maximization by the perfectly

competitive employment agencies implies that the wage paid by intermediate firms for their homogenous

labor input is

$$W_t = \left[\int_0^1 W_t(j)^{-1/\lambda_{w,t}} dj \right]^{-\lambda_{w,t}} \tag{37}$$

Capital agencies.

The capital stock is managed by a collection of perfectly competitive capital agencies. These firms are

owned by households and discount cash flows with Λ_t, the marginal utility of real income for the

representative household. At the beginning of period t, these agencies purchase the capital stock $\overline{K}_{t-1}$

from the entrepreneurs at beginning-of-period price Q_t^{beg}. The agencies produce capital services by

varying the utilization rate u_t which transforms physical capital into effective capital according to

$$K_t = u_t \overline{K}_{t-1}. \tag{38}$$

Effective capital is then rented to firms at the real rental rate ρ_t. The cost of capital utilization is $a(u_t)$

per unit of physical capital. The capital agency then re-sells the capital to entrepreneurs at the end of the

period at price Q_t. The profit flow is thus given by:

$$Q_t(1 - \delta)\,\overline{K}_{t-1} + [\rho_t u_t - a(u_t)]\,\overline{K}_{t-1} - Q_t^{beg}\,\overline{K}_{t-1} \tag{39}$$

Profit maximization implies

$$Q_t^{beg} = Q_t(1 - \delta) + [\rho_t u_t - a(u_t)] \tag{40}$$

$$\rho_t = a'(u_t) \tag{41}$$

In steady state, u = 1, a(1) = 0 and $\vartheta \equiv$ a''(1)/a'(1). Hence, in the neighbourhood of the steady state

$$Q_t^{beg} \approx Q_t(1 - \delta) + \rho_t \tag{42}$$

which is consistent with BGG's definition of the intertemporal return to holding capital $R_t^k \equiv \frac{\rho_t + (1-\delta)Q_t}{Q_{t-1}}$.

New Capital Producers.

New capital is produced according to the production technology that takes I_t investment goods and transforms them into $\mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$ new capital goods. The time-t profit flow is thus given by

$$Q_t \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t - P_t^I I_t \tag{43}$$

where P_t^I is the relative price of the investment good. The function S captures the presence of adjustment costs in investment, as in Christiano et al. (2005). The function has the following convenient steady state properties: *S = S' = 0 and S'' >0.* These firms are owned by households and discount future cash flows with Λ_t, the marginal utility of real income for the representative household. JPT refer to the investment shock μ_t as a shock to the marginal efficiency of investment (MEI) as it alters the transformation between investment and installed capital. JPT conclude that this shock is the primary driver of output and investment at business cycle frequencies. The investment shock follows the stochastic process

$$log\mu_t = \rho_\mu log\mu_{t-1} + \varepsilon_{\mu,t}, \tag{44}$$

where $\varepsilon_{\mu,t}$ is *i.i.d.*N $\left(0, \sigma_\mu^2\right)$.

Investment Producers.

A competitive sector of firms produce investment goods using a linear technology that transforms one consumption good into Υ_t investment goods. The exogenous level of productivity Υ_t is non-stationary with a growth rate ($\upsilon_t \equiv \Delta log\Upsilon_t$) given by

$$v_t = (1 - \rho_v)\gamma_v + \rho_v v_{t-1} + \varepsilon_{v,t}.$$

The constant returns production function implies that the price of investment goods (in consumption units) is equal to $\frac{1}{\Upsilon_t}$.

Households.

Each household maximizes the utility function

$$E_t \left\{ \sum_{s=0}^{\infty} \beta^s \, b_{t+s} \left[\ln(C_{t+s} - hC_{t+s-1}) - \varphi \, \frac{L_{t+s}(j)^{1+\psi}}{1+\psi} \right] \right\}, \tag{45}$$

where C_t is consumption, h is the degree of habit formation and b_t is a shock to the discount factor. This intertemporal preference shock follows the stochastic process

$$log b_t = \rho_b log b_{t-1} + \varepsilon_{b,t}, \tag{46}$$

with $\varepsilon_{b,t} \sim i.i.d.N$ $(0, \sigma_b^2)$. Since technological progress is nonstationary, utility is logarithmic to ensure the existence of a balanced growth path. The existence of state contingent securities ensures that household consumption is the same across all households. The household's flow budget constraint is

$$C_t + T_t + D_t + \frac{B_t}{P_t} \leq \frac{R_{t-1}B_{t-1}}{P_t} + \frac{W_t(j)}{P_t} L_t(j) + D_{t-1}R_{t-1}^d + profits_t, \tag{47}$$

where D_t denotes real deposit at the lender, T_t is lump-sum taxes, and B_t is holdings of nominal government bonds that pay gross nominal rate R_t. The term $profits_t$ denotes the combined profit flow of all the firms owned by the representative agent including lenders, intermediate goods producers, capital agencies, and new capital producers. Every period a fraction ξ_w of households cannot freely set its wage, but follows the indexation rule

$$W_t(j) = W_{t-1}(j)(\pi_{t-1}e^{z_{t-1} + \frac{\alpha}{1-\alpha}v_t})^{\iota_w}(\pi e^{\gamma_z + \frac{\alpha}{1-\alpha}\gamma_v})^{1-\iota_w}, \tag{48}$$

The remaining fraction of households chooses instead an optimal wage $W_t(j)$ by maximizing

$$E_t \left\{ \sum_{s=0}^{\infty} \xi_w^s \beta^s \left[-b_{t+s}\varphi \, \frac{L_{t+s}(j)^{1+\psi}}{1+\psi} + \frac{\Lambda_{t+s}}{P_{t+s}} W_t(j)L_{t+s}(j) \right] \right\} \tag{49}$$

subject to the labor demand function coming from the firm.

The government

A monetary policy authority sets the nominal interest rate following a feedback rule of the form

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\emptyset_\pi} \left(\frac{X_t}{X_t^*}\right)^{\phi_x}\right]^{1-\rho_R} \left[\frac{X_t/X_{t-1}}{X_t^*/X_{t-1}^*}\right]^{\phi_{dx}} \eta_{mp,t},$$ (50)

where R is the steady state of the gross nominal interest rate. The interest rates responds to deviations of inflation from its steady state, as well as to the level and the growth rate of the GDP gap (X_t/X_t^*). The monetary policy rule is also perturbed by a monetary policy shock, $\eta_{mp,t}$, which evolves according to

$$\log \eta_{mp,t} = \rho_{mp} \log \eta_{mp,t-1} + \varepsilon_{mp,t},$$ (51)

where $\varepsilon_{mp,t}$ is $i.i.d.$N $(0, \sigma_{mp}^2)$. Public spending is determined exogenously as a time-varying fraction of output.

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t,$$ (52)

where the government spending shock g_t follows the stochastic process

$$\log g_t = \left(1 - \rho_g\right) \log g + \rho_g \log g_{t-1} + \varepsilon_{g,t},$$ (53)

with $\varepsilon_{g,t} \sim i.i.d.N(0, \sigma_g^2)$. The spending is financed with lump sum taxes.

Market clearing.

The aggregate resource constraints are given by:

$$C_t + \frac{I_t}{Y_t} + G_t + a(u_t) \overline{K}_{t-1} = Y_t$$ (54)

$$\overline{K}_t = (1 - \delta)\left(1 - \mu_{mc} \int_0^{\varpi_t} \omega \phi(\omega) d\omega\right) \overline{K}_{t-1} + \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t,$$ (55)

This completes the description of the model. We now turn to the estimation of the linearized model.

4. Estimation.

The linearized version of the model equations are collected in the appendix. The three fundamental agency cost parameters are the steady state idiosyncratic variance (σ_{ss}), the entrepreneurial survival rate (γ), and the monitoring cost fraction (μ_{mc}). In contrast to DeGraeve (2008), we follow Christiano et al. (2010), and calibrate these parameters to be consistent with long run aspects of US

financial data. We follow this calibration approach because these parameters are pinned down by long run or steady state properties of the model, not the business cycle dynamics that the Bayesian estimation is trying to match. In any event, these three parameters are calibrated to match the steady state levels of the risk premium ($R^p - R^d$), leverage ratio ($\bar{\kappa}$), and default rate ($\Phi(\varpi_{ss})$). In particular, they are chosen to deliver a 200 bp annual risk premium (BAA-Treasury spread), a leverage ratio of $\bar{\kappa} = 1.95$, and a quarterly default rate of 0.03/4. These imply an entrepreneurial survival rate of $\gamma = 0.98$, a standard deviation of $\sigma_{ss} = 0.28$, and a monitoring cost of $\mu_{mc} = 12\%$. A key expression in the log-linearized model is the reduced-form relationship between the risk premium and leverage:

$$E_t \hat{r}^k_{t+1} - \hat{r}^d_t = \nu(\hat{q}_t + \hat{k}_t - \hat{n}_t) + \hat{\sigma}_t \qquad (56)$$

(See the appendix for details.) The value of ν implied by the previous calibration is $\nu = 0.041$. This is thus imposed in the estimation of the financial models.[4] For JPT we have $\nu = 0$. Steady state relationships also imply that we calibrate $\delta = 0.025$, and $(1-1/g) = 0.22$. The remaining parameters are estimated using familiar Bayesian techniques as in JPT. For the non-financial parameters of the model we use the same priors as in JPT.

We treat as observables the growth rates of real GDP, consumption, investment, the real wage, and the relative price of investment. The other observables include employment, inflation, the nominal rate, leverage, and the risk premium. Employment is measured as the log of per capita hours. Inflation is the consumption deflator, and the nominal rate is the federal funds rate. The series for leverage comes from Gilchrist, Ortiz and Zakrajsek (2009). The risk premium is the spread between the BAA and ten year Treasury. The time period for the estimation is 1954:3-2009:1. We choose the end of the sample period to avoid the observed zero bound on the nominal rate.

We estimate four versions of the model. Along with all the exogenous shocks outlined in the paper, we also include autocorrelated measurement error between the model's risk premium and the

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[4] As a form of sensitivity analysis, we also estimated ν in the financial models. We found that the estimation is quite sensitive to priors, again suggesting that it is not well identified by business cycle dynamics.

observed risk premium. Autocorrelated measurement error is also included for leverage. The first model we label JPT as it corresponds to the model without agency costs ($v = 0$). Note that to match the observed financial variables, the JPT model will assign all risk premium and leverage variation to autocorrelated measurement error. The remaining three models have operative agency costs ($v = 0.041$). Recall that the optimal contract has the form given in (26). Our three estimates consider variations on this basic form. In the model labeled BGG we impose the level of indexation implicitly assumed by BGG: $\chi_k = -0.01$, $\chi_\lambda = 0$. For the model labeled R^k-indexation, we set $\chi_\lambda = 0$ and estimate the value of χ_k. For the model labeled R^k & λ-indexation, we estimate both indexation parameters. We use diffuse priors on the indexation parameters with a uniform distribution centered at 0 and with a standard deviation of 2.

The agency cost models also include two financial shocks: (i) time-varying movements in idiosyncratic risk, and (ii) exogenous redistributions of net worth. Both of these shocks are irrelevant in the JPT model in which lending is not subject to the CSV problem. We posit priors for the standard deviation and autocorrelation of these financial shocks in a manner symmetric with the non-financial exogenous processes in JPT.

The estimation results are summarized in Table 1. The BGG, R^k-indexation, and R^k & λ-indexation agency cost models dominate the JPT model as the JPT model cannot capture the forecastability of leverage and the risk premium that is in the data. Comparing BGG and R^k-indexation, the data rejects the BGG level of indexation preferring a level of contract indexation that is economically significant: $\chi_k = 1.70$ with a 90% confidence interval between 1.36 and 2.05. Results are on a similar range in the case of the R^k & λ-indexation estimation with $\chi_k = 2.23$ with a 90% confidence interval between 1.52 and 2.99. As suggested by the example in section 2, this level of indexation will imply significantly different responses to shocks compared to the BGG assumption. We will see this manifested in the IRF below. The estimated level of indexation to the marginal utility of wealth is $\chi_\lambda = 0.67$ with a 90% confidence interval between 0 and 1.38. The combination of the two indexation parameters under the R^k & λ-indexation specification generates dynamics that are similar to those of the R^k-indexation specification alone.

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Two other differences in parameter estimates are worth some comment. First, the BGG model estimates a significantly smaller size for investment adjustment costs (S'') in the table: $S'' = 1.68$ for BGG, but 2.99 for R^k-indexation, 2.50 for R^k & λ-indexation, and 2.92 for JPT. The level of adjustment costs has two contrasting effects. First, lower adjustment costs will increase the response of investment to aggregate shocks. Second, lower adjustment costs imply smaller movements in the price of installed capital (Q_t) and thus smaller financial accelerator effects in the BGG model.

A second important difference in parameter estimates is in the standard deviation of the shocks. Compared to JPT, the BGG model estimates a significantly smaller volatility in the MEI shocks, and instead shifts this variance on to net worth shocks. Recall that the principle conclusion of JPT is the importance of the MEI shocks in the business cycle. But we once again end up with the JPT conclusion with regards to the importance of MEI shocks in the R^k-indexation and R^k & λ-indexation models. An interesting question we take up below is why the BGG model downplays these shocks so significantly.

Table 2 reports the variance decomposition of three key variables: GDP, investment, and the risk premium. The JPT results are replicated here: the MEI shocks account for a substantial amount of business cycle variability in GDP (60% at the 8-quarter horizon) and investment (77% at the 8-quarter horizon). This conclusion is largely unchanged with R^k-indexation and R^k & λ-indexation. Evidently the estimated level of indexation results in real behaviour similar to a model without agency costs. This is particularly clear in the IRFs presented in Figure 3 that we discuss below.

In contrast to the R^k-indexation and R^k & λ-indexation models, BGG places much less weight on the MEI shocks and instead shifts this variance to the financial shocks (the idiosyncratic variance and net worth shocks) and the monetary policy shock. For the case of investment at the 8-quarter horizon, the BGG model places 15% of the variance on the MEI shocks (compared to 68% for the R^k-indexation and R^k & λ-indexation models, and 77% for JPT). The importance of the two financial shocks increases from 13% under R^k & λ-indexation and 14% under R^k-indexation, to 44% for BGG. The estimated level of financial shocks depends critically upon the estimated level of indexation.

The advantage of the financial models is showcased in the variance decomposition of the risk premium. By assumption, JPT assigns 100% of this variation to measurement error. In contrast, the financial models explain large portions of the risk premia movement by forces within the model. For example, at the 8-quarter horizon, the R^k indexation model assigns less than 50% to measurement error, and the R^k & λ-indexation assigns less than 10% to measurement error. This predictability of the risk premium is echoed by DeGraeve (2008).

Why does the BGG model downplay the MEI shocks and thus shift variance to the other shocks? The answer is quite apparent from Figure 3a. The figure sets all parameter values to those estimated in the R^k-indexation model, except for the levels of indexation ($\chi_k \approx 0$ for BGG and $\chi_k = 2.23$ and $\chi_\lambda = 0.67$ for the R^k & λ-indexation model), and the level of agency cost effects ($\nu = 0$ for JPT).[5] A positive innovation in MEI leads to a fall in the price of capital. Since the BGG contract is not indexed to the return to capital, the shock leads to a sharp decline in entrepreneurial net worth, and thus a sharp increase in the risk premium. This procyclical movement in the risk premium is in sharp contrast to the data. Hence, the Bayesian estimation in the BGG model estimates only a small amount of variability coming from the MEI shocks. Notice that in the R^k-indexation and R^k & λ-indexation models net worth is almost unchanged in response to an MEI shock, so that the impact effect on the risk premium is countercyclical. The main difference among models is the behavior of the repayment that under credit contract indexation is lowered in response to the drop in the return on capital, while in BGG it remains unchanged. The R^k-indexation and R^k & λ-indexation models are thus consistent with MEI shocks driving the cycle, and the risk premium being countercyclical. The similarity of the R^k-indexation and JPT model is also apparent: the two IRFs to an MEI shock largely lie on top of one another.

Since the BGG model downplays the importance of MEI shocks, and shifts this variance to other shocks. Figures 3b-3c plot the IRFs to the two financial shocks. The good news with the two financial shocks is that the spread is now countercyclical. But the difficulty with the financial shocks is that they

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[5]Alternatively we could have considered the IRFs for each model at each model's parameter estimates. These IRFs are similar to those reported here, but we find Figure 3 more intuitive as it is holding all other parameters fixed except for the degree of indexation and the presence of agency costs.

result in countercyclical consumption. This is the familiar co-movement puzzle that arises when a positive shock in one sector (e.g., higher net worth mitigates agency costs in capital accumulation) leads to a downward production movement in the other sector. However, this co-movement problem does not arise with risk premium shocks in the R^k-indexation and R^k & λ-indexation models. As an aside, note that a shock to net worth has a larger effect on net worth and capital prices in the BGG model. This is just a manifestation of the multiplier intuition outlined in section 2.

Figure 3d plots the IRF to a monetary shock. In the case of BGG, the IRFs exhibit plausible co-movement and countercyclical spreads. The BGG estimation does not put more weight on these policy shocks because the funds rate is an observable, and thus limits possible interest rate variability. In contrast, it is quite clear why the Indexation model puts so little weight on monetary policy shocks. In the case of Indexation, the spread is procyclical, a clear counterfactual prediction.

As a form of sensitivity analysis, Table 3 presents the estimation results for i.i.d. measurement error in the financial variables. The R^k & λ-indexation now wins the model horse race, with JPT coming in significantly worse than the three financial models. The degree of R^k indexation is much larger than the case with autocorrelated measurement error. Further the level of indexation to the marginal utility of wealth is estimated to be significantly positive, in line with the theory outlined above.

5. Conclusion.

This paper began as an empirical investigation of the importance of agency costs and contract indexation in the business cycle. To reiterate, our principle results include the following. First, the financial models appear to be an improvement over the financial-frictionless JPT. Second, R^k-indexation appears to be an important characteristic of the data. Third, the importance of financial shocks (net worth and idiosyncratic variance) in explaining the business cycle is significantly affected by the estimated degree of R^k-indexation. In short, we find evidence for the importance of financial shocks in the business cycle. But the evidence also suggests that the effect of non-financial shocks on real activity is unaffected

by the inclusion of financial forces in the model. That is, the results suggest the importance of financial shocks, but not the existence of a financial accelerator. This analysis thus implies that Bayesian estimation of financial models should include estimates of contract indexation. Empirical analyses that impose zero contract indexation likely distort both the source of business cycle shocks and their transmission mechanism.

APPENDIX.

1. Linearized System of Equations:

$$\hat{y}_t = \frac{y+F}{y}\left[\alpha \hat{k}_t + (1-\alpha)\hat{L}_t\right] \tag{A1}$$

$$\hat{\rho}_t = \hat{w}_t + \hat{L}_t - \hat{k}_t \tag{A2}$$

$$\hat{s}_t = \alpha\hat{\rho}_t + (1-\alpha)\hat{w}_t \tag{A3}$$

$$\hat{\pi}_t = \frac{\beta}{1+\beta\iota_p}E_t\hat{\pi}_{t+1} + \frac{\iota_p}{1+\beta\iota_p}\hat{\pi}_{t-1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta\iota_p)\xi_p}\hat{s}_t + \hat{\lambda}_{p,t} \tag{A4}$$

$$\hat{\lambda}_t = \frac{h\beta e^{\gamma_z}}{(e^{\gamma_z}-h\beta)(e^{\gamma_z}-h)}E_t\hat{c}_{t+1} - \frac{e^{2\gamma_z}+h^2\beta}{(e^{\gamma_z}-h\beta)(e^{\gamma_z}-h)}\hat{c}_t + \frac{he^{\gamma_z}}{(e^{\gamma_z}-h\beta)(e^{\gamma_z}-h)}\hat{c}_{t-1} + \frac{h\beta e^{\gamma_z}\rho_z - he^{\gamma_z}}{(e^{\gamma_z}-h\beta)(e^{\gamma_z}-h)}\hat{z}_t + \frac{e^{\gamma_z}-h\beta\rho_b}{(e^{\gamma_z}-h\beta)}\hat{b}_t +$$

$$\left[\frac{h\beta e^{\gamma_z}\rho_v - he^{\gamma_z}}{(e^{\gamma_z}-h\beta)(e^{\gamma_z}-h)}\right]\left(\frac{\alpha}{1-\alpha}\right)\hat{v}_t \tag{A5}$$

$$\hat{\lambda}_t = \hat{R}_t + E_t(\hat{\lambda}_{t+1} - \hat{z}_{t+1} - \hat{\pi}_{t+1} - \left(\frac{\alpha}{1-\alpha}\right)\hat{v}_{t+1}) \tag{A6}$$

$$\hat{\rho}_t = \vartheta\hat{u}_t \tag{A7}$$

$$E_t\hat{r}_{t+1}^k = \hat{\lambda}_t - E_t\hat{\lambda}_{t+1} + E_t\hat{z}_{t+1} + \left(\frac{\alpha}{1-\alpha}\right)E_t\hat{v}_{t+1} \tag{A8}$$

$$\hat{q}_t = -\hat{\mu}_t + e^{2(\gamma_z+\gamma_v)}S''\left(\hat{\imath}_t - \hat{\imath}_{t-1} + \hat{z}_t + \left(\frac{1}{1-\alpha}\right)\hat{v}_t\right) - \beta e^{2(\gamma_z+\gamma_v)}S''E_t\left(\hat{\imath}_{t+1} - \hat{\imath}_t + \hat{z}_{t+1} + \left(\frac{1}{1-\alpha}\right)\hat{v}_{t+1}\right) \tag{A9}$$

$$\hat{k}_t = \hat{u}_t + \hat{\bar{k}}_{t-1} - \hat{z}_t - \left(\frac{1}{1-\alpha}\right)\hat{v}_t \tag{A10}$$

$$\hat{\bar{k}}_t = (1-\delta)e^{-(\gamma_z+\gamma_v)}\left(\hat{\bar{k}}_{t-1} - \hat{z}_t - \left(\frac{1}{1-\alpha}\right)\hat{v}_t\right) + \left[1-(1-\delta)e^{-(\gamma_z+\gamma_v)}\right](\hat{\mu}_t + \hat{\imath}_t) \tag{A11}$$

$$\hat{w}_t = \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{w}_{t+1} - \frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\beta\iota_w)\xi_w}\hat{g}_{w,t} + \frac{\iota_w}{1+\beta}\hat{\pi}_{t-1} - \frac{1+\beta\iota_w}{1+\beta}\hat{\pi}_t + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} + \frac{\iota_w}{1+\beta}\left(\hat{z}_{t-1} +\right.$$

$$\left.\left(\frac{\alpha}{1-\alpha}\right)\hat{v}_{t-1}\right) - \frac{1+\beta\iota_w - \beta\rho_z}{1+\beta}\hat{z}_t - \frac{1+\beta\iota_w - \beta\rho_v}{1+\beta}\left(\frac{\alpha}{1-\alpha}\right)\hat{v}_t + \hat{\lambda}_{w,t} \tag{A12}$$

$$\hat{g}_{w,t} = \hat{w}_t - (\psi\hat{L}_t + \hat{b}_t - \hat{\lambda}_t) \tag{A13}$$

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)[\phi_\pi\hat{\pi}_t + \phi_x(\hat{x}_t - \hat{x}_t^*)] + \phi_{dx}[(\hat{x}_t - \hat{x}_{t-1}) - (\hat{x}_t^* - \hat{x}_{t-1}^*)] + \hat{\eta}_{mp,t} \tag{A14}$$

$$\hat{x}_t = \hat{y}_t - \frac{\rho k}{y}\hat{u}_t \tag{A15}$$

$$\frac{1}{g}\hat{y}_t = \frac{1}{g}\hat{g}_t + \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{\imath}_t + \frac{\rho k}{y}\hat{u}_t \tag{A16}$$

$$\hat{r}_t^d = \hat{R}_t - E_t\hat{\pi}_{t+1} \tag{A17}$$

$$\hat{r}_t^k = \beta e^{-(\gamma_z+\gamma_v)}(1-\delta)\hat{q}_t + \left[1-\beta e^{-(\gamma_z+\gamma_v)}(1-\delta)\right]\hat{\rho}_t - \hat{q}_{t-1} \tag{A18}$$

For the **agency cost model**, we replace (A8) with

$$E_t\hat{r}_{t+1}^k = \hat{\lambda}_t - E_t\hat{\lambda}_{t+1} + E_t\hat{z}_{t+1} + \left(\frac{\alpha}{1-\alpha}\right)E_t\hat{v}_{t+1} + \nu(\hat{q}_t + \hat{k}_t - \hat{n}_t) + \hat{\sigma}_t \tag{A8'}$$

And add the following equations:

$$\hat{n}_t = \kappa\frac{\gamma}{\beta}(\hat{r}_t^k - \hat{r}_t^l) + \frac{\gamma}{\beta}(\hat{r}_t^l + \hat{n}_{t-1}) + \gamma\kappa\frac{rp}{\beta}(\hat{k}_{t-1} + \hat{q}_{t-1} + \hat{r}_t^k) - \hat{z}_t - \left(\frac{1}{1-\alpha}\right)\hat{v}_t + \hat{\eta}_{nw,t} \tag{A19}$$

$$\hat{r}_t^l = \hat{r}_{t-1}^d + [1 + \theta_g(\chi_k - 1)](\hat{r}_t^k - E_{t-1}\hat{r}_t^k) + \theta_g\chi_\lambda(\lambda_t - E_{t-1}\lambda_t) \tag{A20}$$

2. The Derivation of A8' and A20.

The optimal contract (20)-(22) can be expressed as

$$V_{t+1}f'(\varpi_{t+1}) = \left[\frac{E_t V_{t+1} f'(\varpi_{t+1})}{E_t \Lambda_{t+1} g'(\varpi_{t+1})}\right] \Lambda_{t+1} g'(\varpi_{t+1}) \tag{A21}$$

$$\bar{\kappa}_t E_t R_{t+1}^k V_{t+1} f(\varpi_{t+1}) = \frac{-E_t V_{t+1} f'(\varpi_{t+1})}{E_t \Lambda_{t+1} g'(\varpi_{t+1})} R_t^d E_t \Lambda_{t+1} \tag{A22}$$

$$E_t \Lambda_{t+1} R_{t+1}^k \frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} g(\varpi_{t+1}) = R_t^d E_t \Lambda_{t+1} \tag{A23}$$

It is convenient to define $F(\varpi_{t+1}) \equiv \frac{-f'(\varpi_{t+1})}{g'(\varpi_{t+1})}$, where $\Psi \equiv \frac{\varpi_{ss} F'(\varpi_{ss})}{F(\varpi_{ss})} > 0$, by the second order condition. Linearizing (A21)-(A23) we have

$$\Psi(\varpi_{t+1} - E_t \varpi_{t+1}) = (\lambda_{t+1} - E_t \lambda_{t+1}) + (v_{t+1} - E_t v_{t+1}) \tag{A24}$$

$$E_t(\hat{r}_{t+1}^k - \hat{r}_t^d) + \kappa_t = (\Psi - \theta_f) E_t \varpi_{t+1} \tag{A25}$$

$$E_t(\hat{r}_{t+1}^k - \hat{r}_t^d) = \left(\frac{1}{\kappa - 1}\right)\kappa_t - \theta_g E_t \varpi_{t+1} \tag{A26}$$

where $\theta_g \equiv \frac{\varpi_{ss} g'(\varpi_{ss})}{g(\varpi_{ss})}$, with $0 < \theta_g < 1$, and $\theta_f \equiv \frac{\varpi_{ss} f'(\varpi_{ss})}{f(\varpi_{ss})} < 0$. Solving (A25)-(A26) we have:

$$E_t \varpi_{t+1} = \frac{\kappa}{\kappa - 1} \frac{1}{(\Psi - \theta_f + \theta_g)} \kappa_t \tag{A27}$$

$$E_t(\hat{r}_{t+1}^k - \hat{r}_t^d) = \left[\frac{(\Psi - \theta_f + \theta_g) - \kappa \theta_g}{(\kappa - 1)(\Psi - \theta_f + \theta_g)}\right]\kappa_t \equiv \nu \kappa_t \tag{A28}$$

Using the definition of leverage and the deposit rate, (A28) is the same as (A8'). The linearized lender return and promised payment are given by:

$$\hat{r}_{t+1}^l = \hat{r}_{t+1}^k + \theta_g \varpi_{t+1} - \left(\frac{1}{\kappa - 1}\right)\kappa_t \tag{A29}$$

$$\hat{r}_{t+1}^p = \hat{r}_{t+1}^k + \varpi_{t+1} - \left(\frac{1}{\kappa - 1}\right)\kappa_t \tag{A30}$$

Combining (A24) and (A30) we have:

$$\hat{r}_t^p = r_{t-1}^d + \frac{(1 - \Theta_g)[1 - \nu(\kappa - 1)]}{\Theta_g(\kappa - 1)}\kappa_{t-1} + (\hat{r}_t^k - E_{t-1}\hat{r}_t^k) + \frac{1}{\Psi}(\hat{\lambda}_t - E_{t-1}\hat{\lambda}_t) - \frac{1}{\Psi}(\hat{v}_t - E_{t-1}\hat{v}_t) \tag{A31}$$

where from (18) we have

$$\hat{v}_t = E_t \sum_{j=0}^{\infty} \beta^{j+1}\left(\Xi \hat{\kappa}_{t+j} + \hat{r}_{t+j+1}^k\right) \tag{A32}$$

where $\Xi \equiv \left[1 + \frac{\beta}{\gamma}\left(\nu - \frac{1}{(\kappa - 1)}\right)\right]$. Innovations in $\hat{v}_t$ are dominated by innovations in $\hat{r}_t^k$, so for parsimony we will estimate a promised payment of the form

$$\hat{r}_t^p = E_{t-1}\hat{r}_t^p + \chi_k(\hat{r}_t^k - E_{t-1}\hat{r}_t^k) + \chi_\lambda(\lambda_t - E_{t-1}\lambda_t) \tag{A33}$$

Combining this with (A29)-(A30) we have:

$$\hat{r}_t^l = \hat{r}_{t-1}^d + [1 + \theta_g(\chi_k - 1)](\hat{r}_t^k - E_{t-1}\hat{r}_t^k) + \theta_g \chi_\lambda(\lambda_t - E_{t-1}\lambda_t) \tag{A34}$$

This is just (A20).

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Figure 1: the multiplier as a function of indexation.
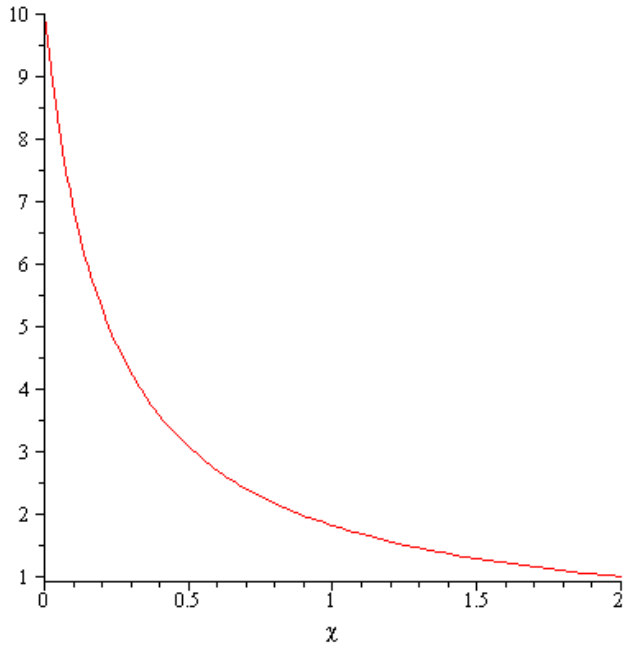
Figure 2: a shock to asset demand.
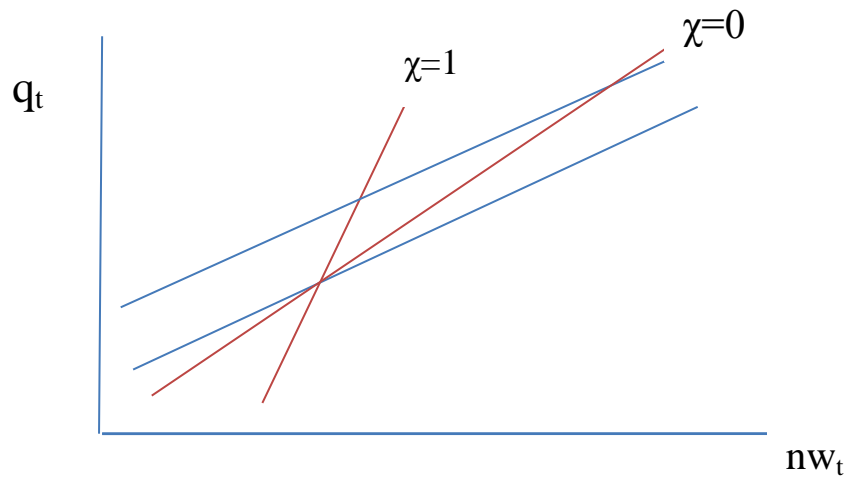
(Asset price is blue line. Net worth evolution is red line.) Demand shock shifts up asset price. The new equilibrium in (n,q) space depends upon the level of indexation.   Lower levels of indexation amplify these effects.

Figure 3.a. Impulse Response Functions to a One Standard Deviation Marginal Efficiency of Investment Shock
Keeping parameters constant to the R^k indexation model except R^k and lambda indexation parameters (χ_{k} and $\chi\lambda$)
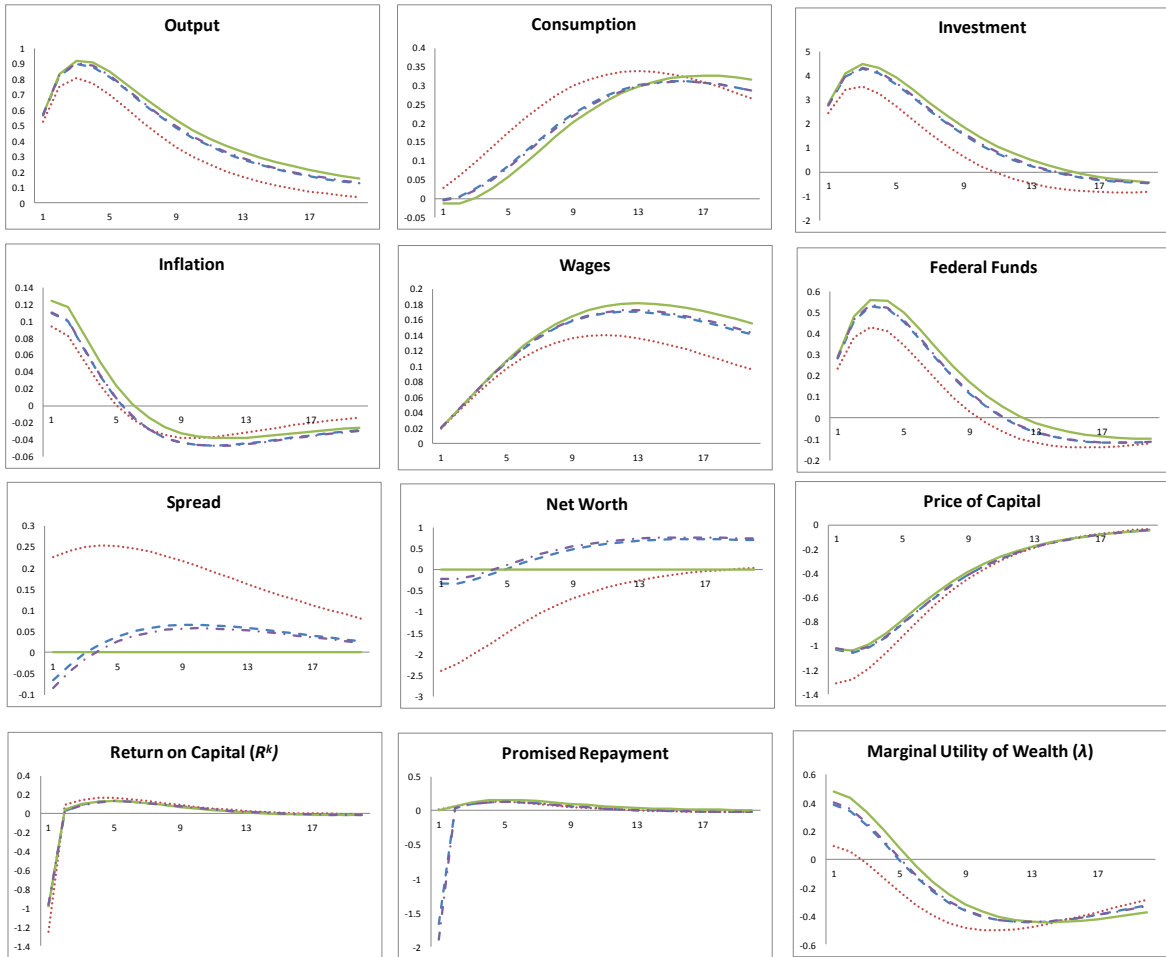
JPT   BGG   Indexation to R^k Indexation to R^k and λ

Figure 3.b. Impulse Response Functions to a One Standard Deviation Net Worth Shock
Keeping parameters constant to the R^k indexation model except R^k and lambda indexation parameters (χ_k and $\chi\lambda$)

Figure 3.c. Impulse Response Functions to a One Standard Deviation Idiosyncratic Variance Shock
Keeping parameters constant to the R^k indexation model except R^k and lambda indexation parameters (χ_{R_k} and $\chi\lambda$)

Figure 3.d. Impulse Response Functions to a One Standard Deviation Monetary Policy Shock
Keeping parameters constant to the R^k indexation model except R^k and lambda indexation parameters (χ_k and $\chi\lambda$)

Table 1: Models Estimations and Models Comparisons with BAA - T10 credit spread and leverage data.
All models have autocorrelated measurement errors in the credit spread and leverage series.

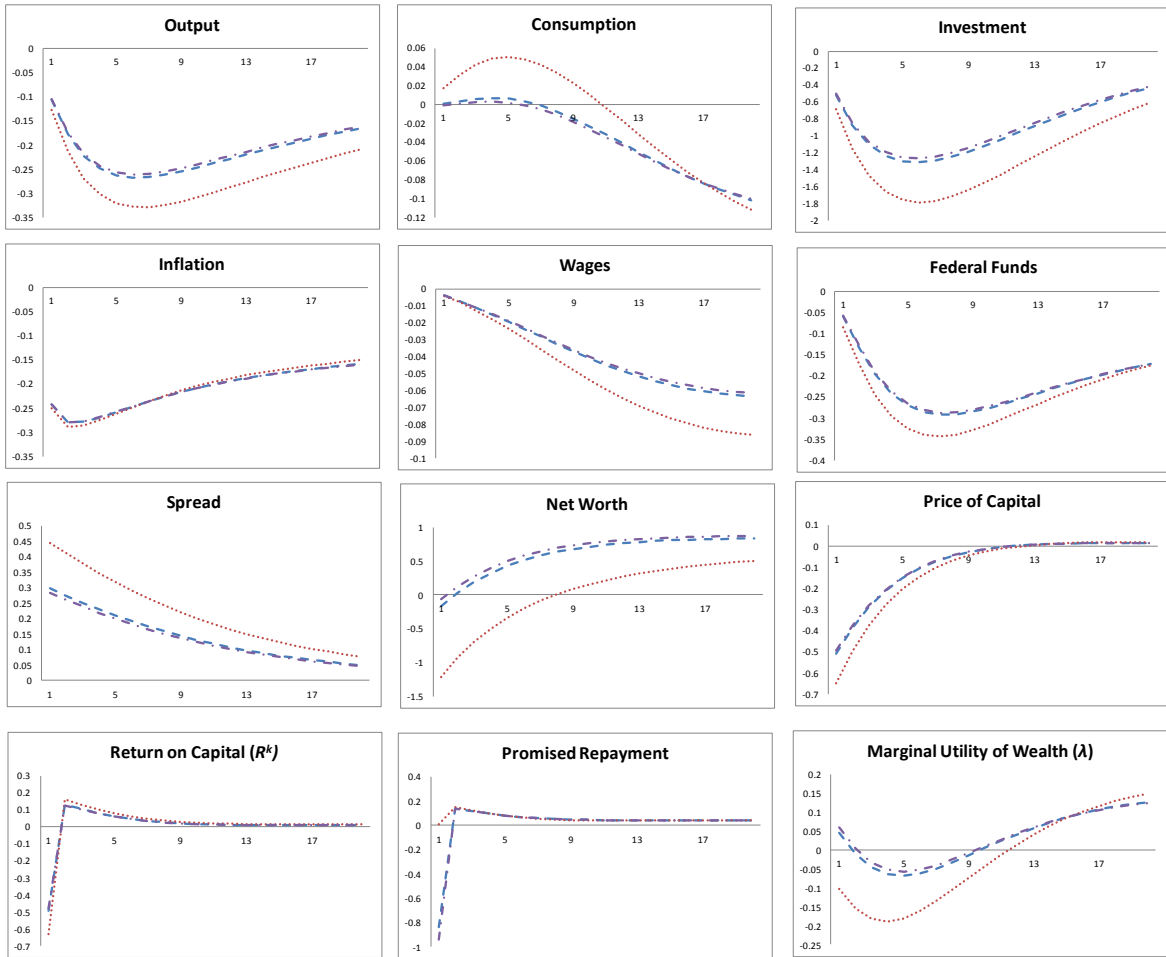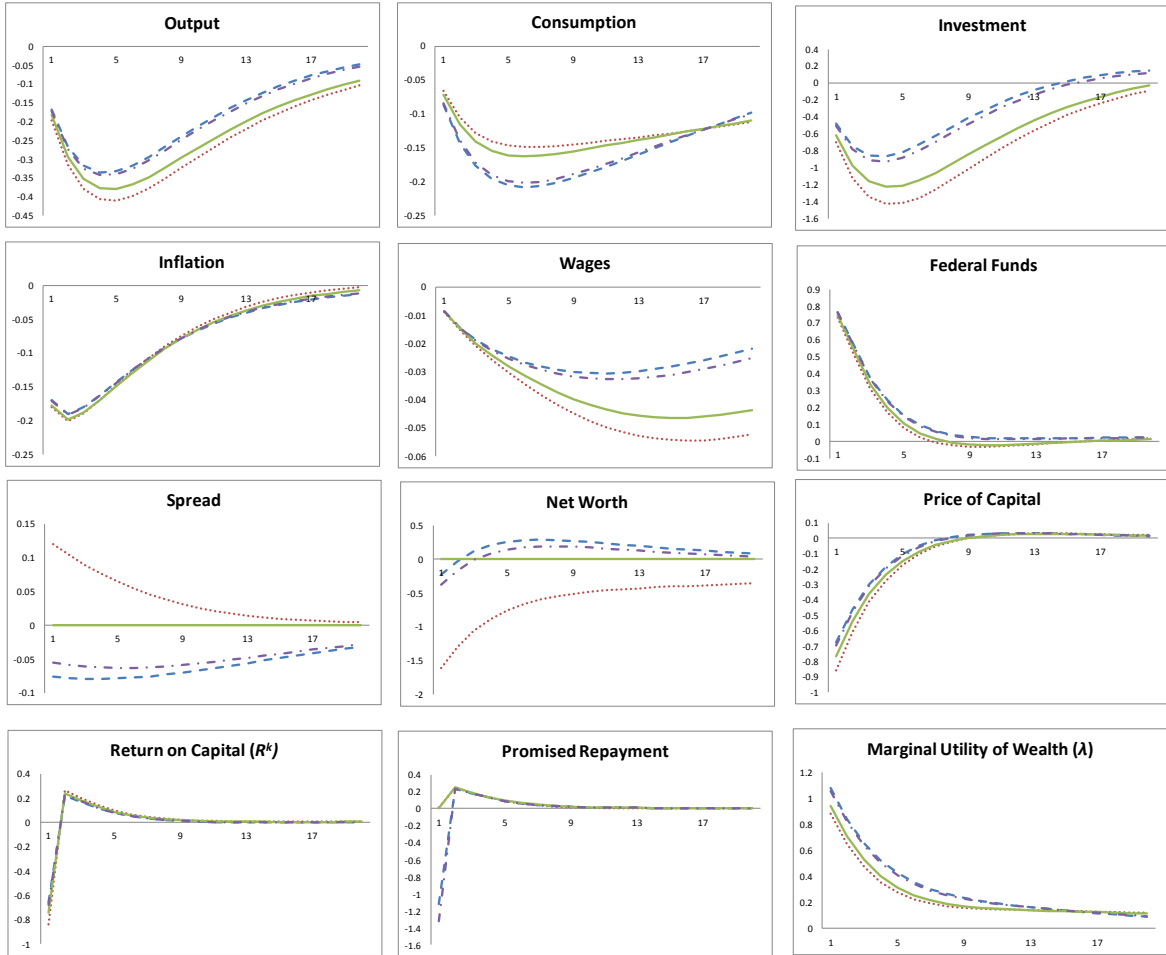| Coefficient	Description	Prior density [e]	prior mean	pstdev	JPT Model [a] post. mean	5%	95%	BGG Model [b] post. mean	5%	95%	Indexation to R^k Model [c] post. mean	5%	95%	Indexation to R^k and λ Model [d] post. mean	5%	95%
Log data density					-1590.5			-1555.8			-1511.1			-1524.3		
Posterior Model Probability					0%			0%			100%			0%		
α	Capital share	N	0.30	0.05	0.17	0.15	0.18	0.17	0.15	0.19	0.17	0.16	0.19	0.17	0.16	0.18
ι_p	Price indexation	B	0.50	0.15	0.15	0.05	0.25	0.14	0.08	0.24	0.17	0.08	0.27	0.14	0.05	0.23
ι_w	Wage indexation	B	0.50	0.15	0.19	0.10	0.27	0.12	0.06	0.19	0.16	0.09	0.24	0.15	0.07	0.22
γ_z	SS technology growth rate	N	0.50	0.03	0.49	0.46	0.53	0.45	0.42	0.47	0.49	0.46	0.52	0.45	0.41	0.49
γ_u	SS IST growth rate	N	0.50	0.03	0.52	0.44	0.59	0.46	0.41	0.49	0.50	0.47	0.52	0.48	0.44	0.50
h	Consumption habit	B	0.50	0.10	0.85	0.81	0.90	0.83	0.69	0.89	0.84	0.80	0.89	0.86	0.82	0.89
λ_p	SS mark-up goods prices	N	0.15	0.05	0.19	0.15	0.23	0.17	0.11	0.20	0.20	0.17	0.23	0.17	0.14	0.20
λ_w	SS mark-up wages	N	0.15	0.05	0.11	0.03	0.20	0.12	0.06	0.15	0.12	0.08	0.15	0.09	0.05	0.14
$\log L^{ss}$	SS hours	N	0.00	0.50	0.04	-0.75	0.71	0.38	-0.12	1.05	-0.03	-0.43	0.51	0.74	0.14	1.32
$100(\pi - 1)$	SS quarterly inflation	N	0.50	0.10	0.62	0.47	0.76	0.69	0.54	0.84	0.60	0.51	0.69	0.64	0.53	0.74
$100(\beta^{-1} - 1)$	Discount factor	G	0.25	0.10	0.17	0.09	0.26	0.22	0.17	0.30	0.26	0.16	0.35	0.21	0.12	0.31
ψ	Inverse frisch elasticity	G	2.00	0.75	3.10	2.10	3.91	4.39	2.87	5.78	4.05	2.92	5.30	4.17	3.00	5.14
ξ_p	Calvo prices	B	0.66	0.10	0.76	0.71	0.81	0.73	0.69	0.78	0.77	0.72	0.82	0.75	0.70	0.80
ξ_w	Calvo wages	B	0.66	0.10	0.78	0.72	0.85	0.76	0.71	0.85	0.72	0.59	0.85	0.74	0.68	0.80
ϑ	Elasticity capital utilization costs	G	5.00	1.00	4.90	3.76	6.09	4.54	3.76	5.05	4.59	3.98	5.40	5.62	3.81	7.19
S''	Investment adjustment costs	G	4.00	1.00	2.92	2.31	3.50	1.68	1.30	1.94	2.99	2.62	3.37	2.50	1.97	3.07
ϕ_p	Taylor rule inflation	N	1.70	0.30	1.50	1.15	1.78	1.19	0.96	1.46	1.72	1.49	1.94	1.75	1.36	2.07
ϕ_y	Taylor rule output	N	0.13	0.05	0.05	0.01	0.07	0.02	0.00	0.04	0.10	0.06	0.14	0.09	0.05	0.13
ϕ_{dy}	Taylor rule output growth	N	0.13	0.05	0.27	0.22	0.31	0.23	0.20	0.25	0.25	0.21	0.30	0.26	0.21	0.31
ρ_R	Taylor rule smoothing	B	0.60	0.20	0.83	0.78	0.87	0.78	0.73	0.82	0.85	0.82	0.89	0.84	0.81	0.87
ρ_{mp}	Monetary policy	B	0.40	0.20	0.10	0.02	0.18	0.08	0.02	0.15	0.08	0.01	0.14	0.08	0.01	0.14
ρ_z	Neutral technology growth	B	0.60	0.20	0.32	0.22	0.42	0.30	0.15	0.40	0.32	0.23	0.41	0.29	0.20	0.38
ρ_g	Government spending	B	0.60	0.20	1.00	0.99	1.00	0.99	0.96	1.00	0.99	0.99	1.00	0.99	0.99	1.00
ρ_u	IST growth	B	0.60	0.20	0.47	0.20	0.78	0.38	0.07	0.54	0.41	0.29	0.53	0.35	0.23	0.45
ρ_p	Price mark-up	B	0.60	0.20	0.94	0.90	0.98	0.93	0.87	0.97	0.91	0.85	0.96	0.94	0.90	0.98
ρ_w	Wage mark-up	B	0.60	0.20	0.97	0.96	0.99	0.94	0.89	0.97	0.97	0.94	0.99	0.97	0.95	0.98
ρ_b	Intertemporal preference	B	0.60	0.20	0.60	0.49	0.71	0.51	0.45	0.62	0.61	0.49	0.73	0.58	0.48	0.68
θ_p	Price mark-up MA	B	0.50	0.20	0.65	0.50	0.80	0.58	0.48	0.71	0.59	0.43	0.76	0.68	0.57	0.79
θ_w	Wage mark-up MA	B	0.50	0.20	0.99	0.98	1.00	0.98	0.93	0.99	0.96	0.92	1.00	0.99	0.98	1.00
ρ_σ	Idiosyncratic variance	B	0.60	0.20	-	-	-	0.94	0.82	1.00	0.98	0.97	1.00	0.98	0.95	1.00
ρ_{nw}	Net worth	B	0.60	0.20	-	-	-	0.77	0.69	0.88	0.82	0.69	0.97	0.66	0.43	0.89
ρ_μ	Marginal efficiency of investment	B	0.60	0.20	0.75	0.69	0.82	0.42	0.30	0.52	0.67	0.54	0.79	0.77	0.71	0.82
ρ_{rpme}	Risk premium measurement error	B	0.60	0.20	0.65	0.38	0.92	0.93	0.85	0.99	0.94	0.83	1.00	0.42	0.13	0.69
ρ_{levme}	Leverage measurement error	B	0.60	0.20	0.94	0.90	0.98	0.91	0.83	0.98	0.92	0.87	0.97	0.91	0.86	0.97
ν	Elasticity risk premium	N	0.05	0.02	**0**	-	-	0.041	-	-	0.041	-	-	0.041	-	-
χ_k	Indexation to R^k	U	0.00	2.00	**0**	-	-	**BGG**	-	-	1.70	1.36	2.05	2.23	1.52	2.99
χ_λ	Indexation to Marginal Utility	U	0.00	2.00	**0**	-	-	**0**	-	-	**0**	-	-	0.67	0.00	1.38

standard deviation of shocks

	Description	Prior density	prior mean	pstdev	post. mean	5%	95%	post. mean	5%	95%	post. mean	5%	95%	post. mean	5%	95%
σ_{mp}	Monetary policy	I	0.20	1.00	0.25	0.23	0.28	0.28	0.24	0.33	0.25	0.22	0.27	0.25	0.22	0.27
σ_z	Neutral technology growth	I	0.50	1.00	0.89	0.80	0.98	0.87	0.76	1.01	0.90	0.81	0.98	0.87	0.79	0.96
σ_g	Government spending	I	0.50	1.00	0.36	0.33	0.40	0.37	0.33	0.40	0.36	0.33	0.40	0.36	0.33	0.40
σ_u	IST growth	I	0.50	1.00	0.59	0.53	0.66	0.58	0.51	0.64	0.59	0.53	0.64	0.58	0.52	0.64
σ_p	Price mark-up	I	0.10	1.00	0.22	0.18	0.27	0.24	0.19	0.37	0.22	0.18	0.26	0.24	0.20	0.27
σ_w	Wage mark-up	I	0.10	1.00	0.34	0.30	0.38	0.38	0.31	0.58	0.31	0.25	0.36	0.34	0.30	0.38
σ_b	Intertemporal preference	I	0.10	1.00	0.03	0.02	0.04	0.04	0.03	0.07	0.03	0.02	0.04	0.04	0.02	0.05
σ_σ	Idiosyncratic variance	I	0.50	1.00	-	-	-	0.09	0.07	0.15	0.09	0.07	0.10	0.09	0.07	0.11
σ_{nw}	Net worth	I	0.50	1.00	-	-	-	0.86	0.59	1.36	0.47	0.16	0.80	1.17	0.42	1.83
σ_μ	Marginal efficiency of investment	I	0.50	1.00	5.26	4.50	6.08	3.58	3.01	4.00	5.73	5.09	6.38	4.51	3.81	5.21
σ_{rpme}	Risk premium measurement error	I	0.50	1.00	0.19	0.17	0.20	0.08	0.07	0.09	0.08	0.07	0.09	0.07	0.06	0.08
σ_{levme}	Leverage measurement error	I	0.50	1.00	8.03	7.22	8.78	6.74	5.14	7.52	7.86	7.08	8.59	7.24	6.72	7.77

Note: calibrated coefficients: δ = 0.025, g implies a SS government share of 0.22.
 For the agency cost models (BGG and Indexation) the following parameters are also calibrated: entrepreneurial survival rate γ = 0.98, a SS risk premium rp = 0.02/4, and a SS leverage ratio κ = 1.95.
[a] In JPT model there are not financial (risk premium and net worth) shocks. The elasticity of risk premium, ν, is set to 0 and the indexation parameters, χ_k and χ_λ, are irrelevant and set to 0.
[b] In BGG model there are financial shocks and the elasticity of risk premium, ν, is calibrated to 0.041, while the indexation to the return of capital parameter, χ_k, is set to the implied in BGG, $\chi_k = (\Theta_g - 1)/\Theta_g$ where Θ_g = 0.985, and the indexation to the marginal utility of wealth parameter, χ_λ, is set to 0.
[c] In the Indexation to R^k model there are financial shocks and the elasticity of risk premium, ν, is calibrated to 0.041, while the indexation to the return of capital parameter, χ_k, is estimated, and the indexation to the marginal utility of wealth parameter, χ_λ, is set to 0.
[d] In the Indexation to R^k and λ model there are financial shocks an the elasticity of risk premium, ν, is calibrated to 0.041, while the indexation to the return of capital parameter, χ_k, and the indexation to the marginal utility of wealth parameter, χ_λ, are both estimated.
[e] N stands for Norman, B-Beta, G-Gamma, U-Uniform, I-Inverted-Gamma distribution.
[f] Posterior percentiles are from 2 chains of 500,000 draws generated using a Random Walk Metropolis algorithm. We discard the initial 250,000 and retain one every 5 subsequent draws.

Table 2: Variance Decomposition at Different Horizons in the JPT, BGG, Indexation to R^k and Indexation to R^k & λ Models

Output

	Monetary policy	Neutral technology	Government	Investment specific technology	Price mark-up	Wage mark-up	Intertemporal preference	Marginal efficiency of investment	Net Worth	Idiosyncratic variance	Measurement error of risk premium	Measurement error of leverage
4 quarters												
JPT	6.1	14.2	4.3	2.2	5.8	2.8	5.3	59.4	-	-	0.0	
BGG	16.8	18.6	6.1	2.1	9.3	0.6	7.2	24.0	5.4	9.7	0.0	0.0
R^k Indexation	5.5	16.1	4.3	2.4	6.1	2.9	6.1	52.5	0.4	3.8	0.0	0.0
R^k & λ Indexation	7.9	13.5	3.3	2.8	4.3	0.2	4.8	57.8	1.6	3.9	0.0	0.0
8 quarters												
JPT	6.4	7.4	3.0	1.7	9.4	8.5	3.6	59.9	-	-	0.0	0.0
BGG	20.2	10.3	4.4	1.5	16.6	0.5	5.7	16.1	10.3	14.4	0.0	0.0
R^k Indexation	5.6	8.9	3.0	2.0	10.0	9.4	4.5	50.8	0.9	5.0	0.0	0.0
R^k & λ Indexation	9.0	7.3	2.1	2.4	7.5	0.2	3.4	60.3	2.5	5.4	0.0	0.0
16 quarters												
JPT	5.4	5.6	2.6	1.3	10.9	22.2	2.4	49.7	-	-	0.0	0.0
BGG	17.8	7.6	3.3	1.0	18.2	5.9	3.8	10.1	15.1	17.2	0.0	0.0
R^k Indexation	4.4	6.1	2.5	1.8	10.9	22.9	3.2	40.7	1.7	5.8	0.0	0.0
R^k & λ Indexation	8.6	5.9	1.6	2.2	9.8	4.3	2.4	54.6	3.6	7.0	0.0	0.0
1000 quarters												
JPT	2.0	2.1	16.5	0.5	4.5	54.8	0.9	18.7	-	-	0.0	0.0
BGG	9.8	4.1	4.9	0.5	10.3	35.3	2.0	5.2	11.7	16.1	0.0	0.0
R^k Indexation	2.6	3.8	6.6	1.3	7.0	43.1	1.9	24.9	2.0	6.9	0.0	0.0
R^k & λ Indexation	5.2	3.5	4.7	1.5	6.6	32.9	1.4	33.1	3.0	8.1	0.0	0.0

Investment

	Monetary policy	Neutral technology	Government	Investment specific technology	Price mark-up	Wage mark-up	Intertemporal preference	Marginal efficiency of investment	Net Worth	Idiosyncratic variance	Measurement error of risk premium	Measurement error of leverage
4 quarters												
JPT	4.1	2.7	0.0	0.1	4.9	0.3	2.0	85.9	-	-	0.0	0.0
BGG	15.7	5.1	0.1	0.2	9.1	2.7	3.4	30.1	16.3	17.2	0.0	0.0
R^k Indexation	3.3	1.5	0.0	0.3	5.1	0.4	1.6	78.5	2.2	7.2	0.0	0.0
R^k & λ Indexation	5.2	2.1	0.4	0.5	3.4	0.8	2.3	75.6	4.2	5.4	0.0	0.0
8 quarters												
JPT	4.0	7.4	0.0	0.4	7.4	1.3	2.4	77.1	-	-	0.0	0.0
BGG	13.8	9.9	0.1	0.7	12.4	2.2	2.8	14.5	23.4	20.1	0.0	0.0
R^k Indexation	2.7	4.2	0.0	0.2	7.7	1.7	1.8	67.6	4.2	9.8	0.0	0.0
R^k & λ Indexation	4.9	5.1	0.4	0.3	5.4	0.7	2.5	67.5	6.1	7.1	0.0	0.0
16 quarters												
JPT	3.6	12.2	0.0	1.6	9.4	4.4	2.4	66.5	-	-	0.0	0.0
BGG	10.7	10.9	0.1	1.5	12.4	1.9	2.0	9.6	29.1	21.9	0.0	0.0
R^k Indexation	2.2	5.7	0.0	0.5	8.8	4.6	1.6	56.3	7.5	12.8	0.0	0.0
R^k & λ Indexation	4.5	7.4	0.4	0.3	7.1	1.0	2.3	59.0	8.6	9.3	0.0	0.0
1000 quarters												
JPT	3.1	11.5	1.5	2.4	8.3	11.6	2.1	59.5	-	-	0.0	0.0
BGG	9.3	9.7	0.1	1.9	11.0	6.7	1.8	8.9	28.2	22.5	0.0	0.0
R^k Indexation	2.2	5.2	0.2	0.6	8.0	7.0	1.5	52.2	8.2	15.0	0.0	0.0
R^k & λ Indexation	4.1	7.0	0.5	0.4	6.7	5.4	2.1	54.6	8.6	10.4	0.0	0.0

Observed Risk Premium

	Monetary policy	Neutral technology	Government	Investment specific technology	Price mark-up	Wage mark-up	Intertemporal preference	Marginal efficiency of investment	Net Worth	Idiosyncratic variance	Measurement error of risk premium	Measurement error of leverage
4 quarters												
JPT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-	100.0	0.0
BGG	1.9	0.1	0.0	0.3	0.0	0.0	0.2	4.2	30.2	35.9	27.2	0.0
R^k Indexation	2.9	1.1	0.0	0.2	0.3	0.2	0.6	0.9	5.4	38.8	49.6	0.0
R^k & λ Indexation	0.0	0.2	1.9	1.9	0.0	0.6	0.5	5.0	52.4	27.6	9.9	0.0
8 quarters												
JPT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-	100.0	0.0
BGG	1.2	0.2	0.0	0.2	0.0	0.0	0.1	4.3	38.3	29.2	26.4	0.0
R^k Indexation	3.2	1.9	0.0	0.2	0.5	0.1	1.0	1.4	10.5	32.7	48.5	0.0
R^k & λ Indexation	0.2	0.3	1.7	2.0	0.2	0.9	0.3	3.4	57.2	27.3	6.5	0.0
16 quarters												
JPT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-	100.0	0.0
BGG	0.8	0.7	0.0	0.1	0.2	0.1	0.1	4.1	39.8	24.7	29.5	0.0
R^k Indexation	3.0	3.1	0.1	0.3	0.8	0.2	1.4	2.5	14.2	26.0	48.4	0.0
R^k & λ Indexation	0.5	1.3	1.5	2.2	0.6	1.2	0.4	3.7	57.0	26.7	5.0	0.0
1000 quarters												
JPT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-	100.0	0.0
BGG	0.6	1.2	0.0	0.2	0.2	0.4	0.1	3.3	33.7	21.4	38.8	0.0
R^k Indexation	2.3	3.5	0.1	0.5	0.7	0.7	1.2	2.2	13.2	22.9	52.7	0.0
R^k & λ Indexation	0.6	2.6	1.4	2.8	0.8	1.7	0.4	4.0	53.7	27.5	4.4	0.0

Table 3: Models Estimations and Models Comparisons with BAA - T10 credit spread and leverage data.
All models have **i.i.d.** measurement errors in these series.

		Prior			JPT Model [a]			BGG Model [b]			Indexation to R^k Model [c]			Indexation to R^k and λ Model [d]		
Log data density					-1849.6			-1677.5			-1617.8			-1601.5		
Posterior Model Probability					0%			0%			0%			100%		
Coefficient	Description	Prior density [e]	prior mean	pstdev	Posteriors [f] post. mean	5%	95%	Posteriors post. mean	5%	95%	Posteriors post. mean	5%	95%	Posteriors post. mean	5%	95%
α	Capital share	N	0.30	0.05	0.16	0.15	0.18	0.16	0.15	0.16	0.16	0.14	0.17	0.16	0.14	0.18
ι_p	Price indexation	B	0.50	0.15	0.18	0.10	0.26	0.29	0.21	0.39	0.22	0.16	0.30	0.26	0.12	0.40
ι_w	Wage indexation	B	0.50	0.15	0.22	0.12	0.32	0.12	0.09	0.18	0.18	0.12	0.24	0.15	0.07	0.23
γ_z	SS technology growth rate	N	0.50	0.03	0.48	0.45	0.52	0.49	0.46	0.50	0.47	0.43	0.52	0.42	0.37	0.48
γ_u	SS IST growth rate	N	0.50	0.03	0.51	0.49	0.53	0.46	0.44	0.48	0.52	0.48	0.55	0.47	0.44	0.50
h	Consumption habit	B	0.50	0.10	0.83	0.78	0.88	0.88	0.85	0.90	0.89	0.86	0.92	0.86	0.82	0.90
λ_p	SS mark-up goods prices	N	0.15	0.05	0.21	0.13	0.28	0.20	0.17	0.23	0.26	0.21	0.31	0.22	0.18	0.27
λ_w	SS mark-up wages	N	0.15	0.05	0.17	0.14	0.21	0.10	0.07	0.17	0.21	0.15	0.26	0.19	0.11	0.24
$\log L^{ss}$	SS hours	N	0.00	0.50	0.39	-0.42	1.27	-0.05	-0.35	0.19	0.29	-0.18	0.67	0.48	0.16	0.85
$100(\pi-1)$	SS quarterly inflation	N	0.50	0.10	0.58	0.45	0.69	0.60	0.53	0.68	0.83	0.70	0.94	0.68	0.59	0.77
$100(\beta^{-1}-1)$	Discount factor	G	0.25	0.10	0.16	0.08	0.25	0.13	0.08	0.17	0.15	0.08	0.22	0.14	0.08	0.20
ψ	Inverse frisch elasticity	G	2.00	0.75	3.59	2.62	4.79	2.97	2.42	3.34	2.43	2.00	2.91	3.17	2.28	4.23
ξ_p	Calvo prices	B	0.66	0.10	0.80	0.76	0.84	0.75	0.72	0.76	0.75	0.71	0.79	0.75	0.69	0.81
ξ_w	Calvo wages	B	0.66	0.10	0.70	0.66	0.75	0.86	0.74	0.93	0.56	0.47	0.62	0.62	0.53	0.71
ϑ	Elasticity capital utilization costs	G	5.00	1.00	1.49	0.98	2.14	4.34	3.82	5.34	4.51	3.59	5.63	5.15	3.72	6.35
S''	Investment adjustment costs	G	4.00	1.00	2.20	1.75	2.68	2.97	2.61	3.18	3.72	3.16	4.24	4.21	2.58	6.03
ϕ_p	Taylor rule inflation	N	1.70	0.30	1.39	1.25	1.52	1.09	0.96	1.34	2.25	1.88	2.72	1.97	1.72	2.24
ϕ_y	Taylor rule output	N	0.13	0.05	0.05	0.03	0.07	0.02	0.01	0.04	0.08	0.04	0.12	0.09	0.06	0.11
ϕ_{dy}	Taylor rule output growth	N	0.13	0.05	0.27	0.22	0.32	0.27	0.21	0.30	0.14	0.10	0.17	0.17	0.12	0.23
ρ_R	Taylor rule smoothing	B	0.60	0.20	0.79	0.74	0.83	0.77	0.75	0.81	0.83	0.80	0.87	0.82	0.80	0.85
ρ_{mp}	Monetary policy	B	0.40	0.20	0.14	0.04	0.23	0.12	0.06	0.18	0.07	0.01	0.12	0.09	0.01	0.16
ρ_z	Neutral technology growth	B	0.60	0.20	0.36	0.27	0.46	0.17	0.11	0.28	0.38	0.30	0.46	0.34	0.23	0.45
ρ_g	Government spending	B	0.60	0.20	1.00	0.99	1.00	0.99	0.99	1.00	1.00	0.99	1.00	1.00	1.00	1.00
ρ_u	IST growth	B	0.60	0.20	0.33	0.23	0.45	0.48	0.34	0.59	0.37	0.23	0.50	0.33	0.20	0.43
ρ_p	Price mark-up	B	0.60	0.20	0.89	0.83	0.95	0.96	0.90	0.99	0.96	0.92	1.00	0.94	0.90	0.99
ρ_w	Wage mark-up	B	0.60	0.20	1.00	0.99	1.00	0.96	0.94	0.98	0.98	0.97	0.99	0.98	0.97	1.00
ρ_b	Intertemporal preference	B	0.60	0.20	0.61	0.52	0.72	0.51	0.43	0.61	0.63	0.55	0.69	0.66	0.56	0.76
θ_p	Price mark-up MA	B	0.50	0.20	0.65	0.52	0.78	0.64	0.58	0.74	0.69	0.59	0.80	0.66	0.52	0.82
θ_w	Wage mark-up MA	B	0.50	0.20	0.98	0.97	0.99	0.99	0.98	1.00	0.85	0.78	0.92	0.92	0.86	0.97
ρ_σ	Idiosyncratic variance	B	0.60	0.20	-	-	-	0.93	0.91	0.95	0.98	0.97	1.00	0.99	0.97	1.00
ρ_{nw}	Net worth	B	0.60	0.20	-	-	-	0.75	0.71	0.82	0.52	0.41	0.64	0.67	0.57	0.79
ρ_μ	Marginal efficiency of investment	B	0.60	0.20	0.70	0.62	0.78	0.59	0.52	0.66	0.75	0.70	0.80	0.75	0.67	0.82
ρ_{rpme}	Risk premium measurement error	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
ρ_{levme}	Leverage measurement error	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
ν	Elasticity risk premium	N	0.05	0.02	**0**	-	-	**0.041**	-	-	**0.041**	-	-	**0.041**	-	-
χ_k	Indexation to R^k	U	0.00	2.00	**0**	-	-	**BGG**	-	-	2.97	2.54	3.46	3.21	2.91	3.46
χ_λ	Indexation to Marginal Utility	U	0.00	6.00	**0**	-	-	**0**	-	-	**0**	-	-	1.04	0.39	1.68

standard deviation of shocks

		Prior density	prior mean	pstdev	post. mean	5%	95%	post. mean	5%	95%	post. mean	5%	95%	post. mean	5%	95%
σ_{mp}	Monetary policy	I	0.20	1.00	0.26	0.23	0.29	0.26	0.25	0.28	0.25	0.23	0.28	0.25	0.22	0.27
σ_z	Neutral technology growth	I	0.50	1.00	0.90	0.81	0.99	0.95	0.83	1.00	0.91	0.82	1.00	0.89	0.80	0.98
σ_g	Government spending	I	0.50	1.00	0.36	0.33	0.40	0.39	0.34	0.40	0.36	0.33	0.40	0.36	0.33	0.40
σ_u	IST growth	I	0.50	1.00	0.58	0.53	0.63	0.56	0.53	0.60	0.58	0.53	0.64	0.58	0.52	0.64
σ_p	Price mark-up	I	0.10	1.00	0.23	0.19	0.27	0.20	0.17	0.23	0.24	0.20	0.27	0.23	0.19	0.27
σ_w	Wage mark-up	I	0.10	1.00	0.31	0.27	0.35	0.33	0.31	0.35	0.28	0.24	0.32	0.28	0.25	0.32
σ_b	Intertemporal preference	I	0.10	1.00	0.03	0.02	0.04	0.04	0.03	0.04	0.03	0.02	0.04	0.03	0.02	0.04
σ_σ	Idiosyncratic variance	I	0.50	1.00	-	-	-	0.16	0.12	0.19	0.14	0.10	0.18	0.16	0.13	0.18
σ_{nw}	Net worth	I	0.50	1.00	-	-	-	1.92	1.49	2.27	3.21	2.57	3.86	2.21	1.68	2.74
σ_μ	Marginal efficiency of investment	I	0.50	1.00	4.19	3.52	4.81	4.36	3.37	6.03	6.03	5.25	6.84	6.89	4.49	9.35
σ_{rpme}	Risk premium measurement error	I	0.50	1.00	0.19	0.17	0.20	0.08	0.07	0.11	0.08	0.07	0.09	0.07	0.06	0.08
σ_{levme}	Leverage measurement error	I	0.50	1.00	9.97	9.94	10.00	6.85	6.00	8.12	5.80	4.85	7.00	7.07	6.32	7.80

Note: calibrated coefficients: $\delta = 0.025$, g implies a SS government share of 0.22.

For the agency cost models (BGG and Indexation) the following parameters are also calibrated: entrepreneurial survival rate $\gamma = 0.98$, a SS risk premium $rp = 0.02/4$, and a SS leverage ratio $\kappa = 1.95$.

[a] In JPT model there are not financial (risk premium and net worth) shocks. The elasticity of risk premium, ν, is set to 0 and the indexation parameters, χ_k and χ_λ, are irrelevant and set to 0.

[b] In BGG model there are financial shocks and the elasticity of risk premium, ν, is calibrated to 0.041, while the indexation to the return of capital parameter, χ_k, is set to the implied in BGG, $\chi_k = (\Theta_g - 1)/\Theta_g$ where $\Theta_g = 0.985$, and the indexation to the marginal utility of wealth parameter, χ_λ, is set to 0.

[c] In the Indexation to R^k model there are financial shocks and the elasticity of risk premium, ν, is calibrated to 0.041, while the indexation to the return of capital parameter, χ_k, is estimated, and the indexation to the marginal utility of wealth parameter, χ_λ, is set to 0.

[d] In the Indexation to R^k and λ model there are financial shocks an the elasticity of risk premium, ν, is calibrated to 0.041, while the indexation to the return of capital parameter, χ_k, and the indexation to the marginal utility of wealth parameter, χ_λ, are both estimated.

[e] N stands for Norman, B-Beta, G-Gamma, U-Uniform, I-Inverted-Gamma distribution.

[f] Posterior percentiles are from 2 chains of 500,000 draws generated using a Random Walk Metropolis algorithm. We discard the initial 250,000 and retain one every 5 subsequent draws.