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Privately Optimal Contracts and Suboptimal Outcomes in a Model of Agency Costs

Charles T. Carlstrom, Timothy S. Fuerst, and Matthias Paustian

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This paper derives the privately optimal lending contract in the celebrated financial accelerator model of Bernanke, Gertler, and Gilchrist (1999). The privately optimal contract includes indexation to the aggregate return on capital and household consumption. Although privately optimal, this contract is not welfare maximizing, as it exacerbates fluctuations in real activity. The household's desire to hedge business cycle risk, leads, via the financial contract, to greater business cycle risk. The welfare cost of the privately optimal contract (when compared to the planner's outcome) is significant. A menu of time-varying taxes and subsidies on household income and monitoring costs can decentralize the planner's allocations. But just one wedge, a time-varying tax on monitoring costs, can come close to achieving the planner's allocation. This can also be decentralized with a time-varying subsidy on loan repayment rates.

Key words: Agency costs, CGE models, optimal contracting.

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Charles T. Carlstrom is at the Federal Reserve Bank of Cleveland (charles.t.carlstrom@clev.frb.org). Timothy S. Fuerst is at the University of Notre Dame and the Federal Reserve Bank of Cleveland (tfuerst@nd.edu). Matthias Paustian is at the Bank of England (matthias.paustian@bankofengland.co.uk). The views expressed in this paper are those of the authors and not necessarily those of the Bank of England. The authors acknowledge helpful comments from Enrique Martinez-Garcia, two anonymous referees, and the editor.

1. Introduction.

The financial accelerator model of Bernanke, Gertler, and Gilchrist (1999), hereafter BGG, is widely used as a convenient mechanism for integrating financial factors into an otherwise standard DSGE model. The BGG model embeds the costly state verification (CSV) model of Townsend (1979) into an environment with risk neutral entrepreneurs, risk averse households, and aggregate risk. Appealing to insurance concerns, BGG assume that the lending contract between the entrepreneur and lender is characterized by a lender return that is invariant to innovations in aggregate variables. Instead, these aggregate innovations feed directly into entrepreneurial net worth. The behavior of net worth is crucial in the BGG model because the agency costs are diminished by increases in net worth. For example, a positive productivity shock shifts wealth to entrepreneurs, lowers agency costs, and thus amplifies the effect of the shock. Hence, BGG's insurance assumption is key to the financial accelerator in their model. The importance of this insurance assumption is well known. For example, consider the following comment of Chari (2003):

> *A final misgiving [is] about a central ingredient of this model. This comment really applies to Bernanke, Gertler and Gilchrist, upon which this paper is based. These authors have an economy with risk neutral agents called entrepreneurs and risk averse agents called households. They claim that an optimal contract in the presence of aggregate risk has the return paid by entrepreneurs to be a constant, independent of the current aggregate shock. I have trouble understanding this result. Surely, entrepreneurs should and would provide insurance to households against aggregate shocks. One way of providing such insurance is to provide a high return to households when their income from other sources is low and a low return when their income from other sources is high. My own guess is that if they allowed the return to households to be state contingent, then aggregate shocks would have no effects on the decisions of households and would be absorbed entirely by entrepreneurs. Before we push this intriguing financial accelerator mechanism much further, I think it would be wise to make sure that we get the microeconomics right*.

This paper revisits BGG's key assumption and confirms the intuition of Chari. Surprisingly, however, we find that that this insurance amplifies the financial accelerator of BGG.

Our principle results include the following. First, the financial contract imposed in the BGG model is not privately optimal. That is, lenders would increase their equity value by offering a loan contract different than the one imposed by BGG. Second, the privately optimal loan contract has the loan repayment varying in response to innovations in the return on capital and innovations in consumption growth. That is, the privately optimal loan contract is indexed to the realization of aggregate shocks. Third, the privately optimal contract is not socially optimal because it leads to large fluctuations in leverage and bankruptcy rates. The social welfare costs of the privately optimal contract are significant. In our benchmark calibration the conditional welfare cost of the privately optimal contract is equal to a 0.26% increase in the annual flow of household consumption (or a one-off payment of 26% of household consumption). Under an alternative calibration, this cost rises to 0.56% in an annual flow. Finally, we demonstrate that the planner outcome represents a Pareto improvement over the competitive equilibrium. In essence, a household's desire to hedge business cycle risk, leads in the aggregate via the financial contract, to greater business cycle risk. We show that the planner outcome can be implemented with a set of state-dependent taxes on household income and monitoring costs.

Our results on the privately optimal contract are related to Krishnamurthy (2003). Krishnamurthy (2003) introduces insurance markets into a three period model where borrowing is secured by collateral as in Kiyotaki and Moore (1997). These insurance markets allow for state contingent debt that is indexed to aggregate shocks as in our framework. Krishnamurthy (2003) shows that such insurance eliminates any feedback from collateral values to investment, and thus reduces collateral amplification to zero. We have a similar result here: the privately optimal contract has the level of debt repayment indexed one-for-one to the aggregate return on capital, so that bankruptcy does not respond to innovations in the return to capital. By itself this indexation dramatically reduces the financial accelerator. But there is a novel twist here that arises in the BGG framework because households are risk averse and entrepreneurs are risk-neutral: households prefer a loan contract that has repayment linked to aggregate consumption. This mechanism is absent in Krishnamurthy as all agents are risk-neutral in his model. Hence, our conclusion is quite different from Krishnamurthy. In the BGG modeling environment, the household's desire to hedge consumption risk does not eliminate but actually amplifies the financial accelerator. That is, the original insurance assumption of BGG actually understates the financial accelerator present in the model.

The previous discussion highlights the importance of the risk neutrality assumption in BGG. Our conclusions would be qualitatively unchanged if entrepreneurs were risk averse, but with lower levels of risk aversion than households. But there is a problem with following this approach. The optimality of the debt contract in the underlying CSV framework relies on the assumption that the entrepreneur is risk neutral. This means that moving away from risk-neutrality also means moving away from the debt contract. The prominence of the original BGG framework suggests that we first pursue these issues in their original setting before exploring contracting with risk-averse entrepreneurs.

Two other notable precedents for the current paper are Lorenzoni (2008) and Jeanne and Korinek (2010). Although the modeling details differ across the papers, both examine situations in which borrowing is constrained either by limited commitment (Lorenzoni (2008)) or asset value (Jeanne and Korinek (2010)). The common conclusion of the two papers is that the competitive equilibrium is inefficient because of a pecuniary externality.

Similar pecuniary externalities are present in this paper. In particular, the lender does not internalize the effect of its contemporaneous hedging activities on the subsequent evolution of entrepreneurial net worth, and thus aggregate consumption and the stochastic discount factor. There are two related effects, one deterministic, the other stochastic. First, the lender desires to charge a higher repayment rate today, but this implies lower entrepreneurial net worth and higher risk premium tomorrow. Since the CSV contract is one-period, the lender does not internalize the effect of today's repayment rate on future net worth and credit conditions. Second, there is a corresponding stochastic effect. The atomistic household desires to own shares in a lender that provides countercyclical dividends. Responding to this incentive, the lender enters into a lending contract with risk-neutral entrepreneurs who willingly provide this desired insurance by paying countercyclical lending rates. In the aggregate, this results in greater subsequent volatility of aggregate net worth, the risk premium, and consumption. These subsequent effects are not internalized by the lender who takes as given the future path of net worth in this one-period relationship. But the lending contract the lender pursues leads to greater consumption variability and thus to a change in the stochastic discount factor. These effects are reinforcing in that the higher variability leads to an even greater desire for consumption hedging in the loan contract.

The paper proceeds as follows. The next section outlines the competitive equilibrium of the model. Section 3 contrasts the contract indexation to BGG. Section 4 contrasts the competitive equilibrium with the constrained social planner's allocation. The quantitative analysis including welfare implications are carried out in Section 5. Section 6 provides some sensitivity analysis on the financial accelerator by examining the privately optimal contract and the BGG contract in a model with sticky prices and more exogenous shocks. Concluding comments are provided in Section 7.

2. The Model.

Households.

The typical household consumes the final good (C_t) and sells labor input (L_t) to the firm at real wage w_t. Preferences are given by

$$U(C_t, L_t) \equiv \frac{C_t^{1-\sigma}}{1-\sigma} - B\frac{L_t^{1+\eta}}{1+\eta}.$$

The household budget constraint is given by

$$C_t + D_t + Q_t^L S_t \leq w_t L_t + R_{t-1}^D D_{t-1} + (Q_t^L + Div_t) S_{t-1}$$

The household chooses the level of deposits (D_t) which are then used by the lender to fund the entrepreneurs (more details below). The (gross) real rate R_t^d on these deposits is known at time-t. The household owns shares in the final goods firms, capital-producing firms, and in the lender. The former two are standard, so we simply focus on the shares of the lender. This share price is denoted by Q_t^L with Div_t denoting lender dividends, and S_t the number of shares held by the representative household (in equilibrium S_t = 1). The optimization conditions include:

$$-U_L(t)/U_c(t) = w_t \tag{1}$$

$$U_c(t) = E_t \beta U_c(t+1) R_t^d \tag{2}$$

Final goods firms.

Final goods are produced by competitive firms who hire labor and rent capital in competitive factor markets at real wage w_t and rental rate r_t. The production function is Cobb-Douglass where A_t is the random level of total factor productivity:

$$Y_t = A_t\left(K_t^f\right)^{\alpha}(L_t)^{1-\alpha} \tag{3}$$

The variable K_t^f denotes the amount of capital available for time-t production. This is different than the amount of capital at the end of the previous period as some is lost because of monitoring costs. The optimization conditions include:

$$mpl_t = w_t \tag{4}$$

$$mpk_t = r_t \tag{5}$$

New Capital Producers.

The production of new capital is subject to adjustment costs. In particular, investment firms take $I_t\vartheta(\frac{I_t}{I_{ss}})$ consumption goods and transform them into I_t investment goods that are sold at price Q_t. Their profits are thus given by $Q_tI_t - I_t\vartheta(\frac{I_t}{I_{ss}})$, where the function ϑ is convex with $\vartheta(1) = 1,\ \vartheta'(1) = 0$ and $\vartheta''(1) = \psi$. Variations in investment lead to variations in the price of capital, which is key to the financial accelerator mechanism.

Lenders.

The representative lender accepts deposits from households (promising sure return R_t^d) and provides loans to the continuum of entrepreneurs. These loans are intertemporal, with the loans made at the end of time t being paid back in time t+1. The gross real return on these loans is denoted by R_{t+1}^L. Each individual loan is subject to idiosyncratic and aggregate risk, but since the lender holds an entire portfolio of loans, only aggregate risk remains. The lender has no other source of funds, so the level of loans will equal the level of deposits. Hence, dividends are given by, $Div_{t+1} = R^L_{t+1}D_t - R^d_t D_t$. The intermediary seeks to maximize its equity value which is given by:

$$Q^L_t = E_t \sum_{j=1}^{\infty} \beta^j \frac{U_c(t+j)}{U_c(t)} Div_{t+j} \tag{6}$$

The FOC of the lender's problem is:

$$E_t \frac{\beta U_c(t+1)}{U_c(t)} \left[R^L_{t+1} - R^d_t\right] = 0 \tag{7}$$

The first-order condition shows that in expectation, the lender makes zero profits, but ex-post profits and losses can occur. As we discuss later, this is in sharp contrast to BGG who assume that the lender makes zero profits state by state, $R^L_{t+1} = R^d_t$.

We assume that losses are covered by households as negative dividends. This is similar to the standard assumption in the Dynamic New Keynesian (DNK) model, eg.,Woodford (2003). That is, the sticky price firms are owned by the household and pay out profits to the household. These profits are typically always positive (for small shocks) because of the steady state mark-up over marginal cost. Similarly, one could introduce a steady-state wedge (eg., monopolistic competition among lenders) in the lender's problem so that dividends are always positive. But this assumption would have no effect on the model's dynamics so we dispense from it for simplicity.

Confirming Chari's (2003) intuition, the expression for the equity value of the bank (6) implies that the household prefers a lender that delivers a dividend stream that co-varies negatively with household consumption. The lender is providing loans to the entrepreneurs. Hence, the household prefers a loan contract that requires the entrepreneur to pay back more in periods of low consumption, and vice versa. As we will see below, such a lending contract is privately optimal but socially costly as it exacerbates fluctuations in aggregate activity by making leverage ratios and risk premia countercyclical.

Household’s desire to hedge consumption risk actually amplifies the financial accelerator. That is, the original insurance assumption of BGG actually understates the financial accelerator present in the model.

Entrepreneurs and the Loan Contract.

Entrepreneurs are the sole accumulators of physical capital. The time t+1 rental rate and capital price are given by r_{t+1} and Q_{t+1}, respectively, implying that the gross return to holding capital from time-t to time t+1 is given by:

$$R_{t+1}^{k} \equiv \frac{r_{t+1}+(1-\delta)Q_{t+1}}{Q_t}. \tag{8}$$

At the end of period t, the entrepreneurs sell all of their accumulated capital, and then re-purchase it along with any net additions to the capital stock. This purchase is financed with entrepreneurial net worth (NW_t) and external financing from a lender. The external financing is subject to a CSV problem. In particular, one unit of capital purchased at the end of time-t is transformed into ω_{t+1} units of capital in time t+1, where ω_{t+1} is a idiosyncratic random variable with density $\phi(\omega)$ and cumulative distribution $\Phi(\omega)$ and a mean of one. The realization of $\omega_{t+1}$is directly observed by the entrepreneur, but the lender can observe the realization only if a monitoring cost is paid. We follow BGG and Carlstrom and Fuerst (1997), by assuming that there is enough inter-period anonymity so that today’s contract cannot be based on previous realizations of the idiosyncratic shock. As noted by these authors, this assumption vastly simplifies the analysis for otherwise the optimal contract would depend upon the entire history of each entrepreneur (see, for example Gertler (1992)). Assuming that the entrepreneur and lender are risk-neutral, Townsend (1979) demonstrates that the optimal contract between entrepreneur and intermediary is risky debt in which monitoring only occurs if the promised payoff is not forthcoming. Payoff does not occur for sufficiently low values of the idiosyncratic shock, $\omega_{t+1} < \varpi_{t+1}$. Let Z_{t+1} denote the promised gross rate-of-return so that Z_{t+1} is defined by

$$Z_{t+1}(Q_t K_{t+1} - NW_t) \equiv \varpi_{t+1} R_{t+1}^{k} Q_t K_{t+1}\ . \tag{9}$$

We find it convenient to express this in terms of the leverage ratio $\bar{\kappa}_t \equiv \left(\frac{Q_t K_{t+1}}{NW_t}\right)$ so that (9) becomes

$$Z_{t+1} \equiv \varpi_{t+1} R^k_{t+1} \frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} \quad (10)$$

The CSV problem takes as exogenous the return on capital (R^k_{t+1}) and the opportunity cost of the lender. With $f(\varpi_{t+1})$ and $g(\varpi_{t+1})$ denoting the entrepreneur's share and lender's share of the project outcome, respectively, the lender's ex post realized t+1 return on the loan contract is defined as:

$$R^L_{t+1} \equiv \frac{R^k_{t+1} g(\varpi_{t+1}) Q_t K_{t+1}}{(Q_t K_{t+1} - NW_t)} \equiv R^k_{t+1} g(\varpi_{t+1}) \frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} \quad (11)$$

where

$$f(\varpi) \equiv \int_{\varpi}^{\infty} \omega \phi(\omega) d\omega - [1 - \Phi(\varpi)]\varpi \quad (12)$$

$$g(\varpi) \equiv [1 - \Phi(\varpi)]\varpi + (1 - \mu) \int_0^{\varpi} \omega \phi(\omega) d\omega \quad (13)$$

Recall that the lender's return is linked to the return on deposits via (7):

$$E_t R^L_{t+1} U_c(t+1) = R^d_t E_t U_c(t+1) \quad (14)$$

The contracting problem takes as given the deposit rate R^d_t and the random variables $U_c(t+1)$ and R^k_{t+1}. The end-of-time-t contracting problem is thus given by:

$$max_{K_{t+1}, \varpi_{t+1}} E_t R^k_{t+1} Q_t K_{t+1} f(\varpi_{t+1}) \quad (15)$$

subject to

$$E_t R^k_{t+1} Q_t K_{t+1} U_c(t+1) g(\varpi_{t+1}) \geq R^d_t E_t U_c(t+1)[Q_t K_{t+1} - NW_t] \quad (16)$$

For a given level of net worth, the choice of K_{t+1} determines the size of the loan, and ϖ_{t+1} implicitly determines the state-contingent interest rate on the loan. After some re-arrangement, the optimization conditions include:

$$f'(\varpi_{t+1}) + \Lambda_t U_c(t+1) g'(\varpi_{t+1}) = 0 \tag{17}$$

$$(\bar{\kappa}_t - 1) E_t R^k_{t+1} f(\varpi_{t+1}) = \Lambda_t E_t U_c(t+1) R^k_{t+1} g(\varpi_{t+1}) \tag{18}$$

$$E_t U_c(t+1) R^k_{t+1} \frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} g(\varpi_{t+1}) = R^d_t E_t U_c(t+1) \tag{19}$$

where Λ_t denotes the multiplier on the constraint (16). Note that ϖ_{t+1} is state-contingent so that (17) holds state-by-state. Expression (19) states that the return to the lender (adjusted for covariances) is equal to the certain return R^d_t in expected value. This follows directly from the assumption that the lender maximizes its equity value. In contrast, BGG impose that (19) must hold state-by-state, i.e., the lender's return is pre-determined and exactly equal to the deposit rate. Under the POC, the cut-off value for ϖ_{t+1} varies state-by-state and is given implicitly by:

$$\Lambda_t U_c(t+1) = \frac{-f'(\varpi_{t+1})}{g'(\varpi_{t+1})} \equiv F(\varpi_{t+1}) \tag{20}$$

The **privately optimal contract (POC)** is thus defined by the default cut-off ϖ_{t+1} and leverage ratio $\frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1}$ that satisfy (19)-(20), with Λ_t given by (18). Note that under the POC, ϖ_{t+1} is a function of innovations in household consumption, but does not respond to innovations in R^k_{t+1}. Below we will compare the POC to the contract imposed by BGG.

Entrepreneurs have linear preferences and discount the future at rate β. Given the high return to internal funds, they will postpone consumption indefinitely. To limit net worth accumulation and ensure that there is a need for external finance in the long run, we assume that fraction (1-γ) of the entrepreneurs die each period. These dying entrepreneurs consume their accumulated net worth and exit the economy. Given the exogenous death rate, aggregate net worth accumulation is described by

$$\mathrm{NW_t} = \gamma NW_{t-1} \bar{\kappa}_{t-1} R^k_t f(\varpi_t) \tag{21}$$

The behavior of net worth thus depends upon the response of ϖ_t to innovations in aggregate behavior.

Because the POC is closely related to the results of Krishnamurthy (2003), we pause to explore these relationships here. A novel insight of Krishnamurthy (2003) is that although borrowers are risk-neutral in consumption, they may be risk-averse in net worth. This arises in Krishnamurthy (2003) because the production technology is concave, and the collateral constraint is not always binding. In the BGG model, the production technology is linear and the need for external finance is a permanent feature of the CSV framework, so the entrepreneurs' payoff is linear in net worth. Although not risk averse in net worth, entrepreneurs do care about the covariance of the debt contract with aggregate shocks. The entrepreneur's t+1 payoff is given by

$$R^k_{t+1} Q_t K_{t+1} f(\varpi_{t+1}) = \left[R^k_{t+1} f(\varpi_{t+1}) \bar{\kappa}_t\right] NW_t \tag{22}$$

Note that the entrepreneur's payoff is linear in net worth, but the slope coefficient in brackets is time-varying and stochastic. Since f is decreasing, the entrepreneur would prefer a loan contract in which R^k_{t+1} and ϖ_{t+1}covary negatively (recall that the contract is negotiated in time t). But since g is increasing, the lender prefers just the opposite. Because R^k_{t+1} enters both the entrepreneur's and lender's payoff linearly, the POC splits the difference: for a given level of household consumption, the POC has ϖ_{t+1} unresponsive to R^k_{t+1} innovations. From (10), this means that debt repayment is indexed one-to-one to R^k_{t+1} innovations. This is directly analogous to Krishnamurthy (2003) who shows that the optimal hedging contract eliminates the wealth redistribution arising from fluctuations in asset prices by linking repayment levels to asset prices.

Market Clearing and Equilibrium.

In equilibrium the household holds the shares of the lender, $S_t = 1$, and the lender funds the entrepreneurs' projects, $D_t = Q_t K_{t+1} - NW_t$. Net of monitoring costs, the amount of capital available for production is given by $K^f_t = m(\varpi_t) K_t$. The competitive equilibrium is defined by the variables $\{C_t, L_t, I_t, K_{t+1}, \varpi_t, \Lambda_t, \bar{\kappa}_t, \mathrm{C}^{\mathrm{e}}_{\mathrm{t}}, Q_t, R^d_t\}$ that satisfy

$$U_c(t) = R_t^d E_t \beta U_c(t+1) \tag{23}$$

$$-U_L(t)/U_c(t) = mpl_t \tag{24}$$

$$f'(\varpi_{t+1}) + \Lambda_t U_c(t+1) g'(\varpi_{t+1}) = 0 \tag{25}$$

$$(\bar{\kappa}_t - 1) E_t R_{t+1}^k f(\varpi_{t+1}) = \Lambda_t E_t U_c(t+1) R_{t+1}^k g(\varpi_{t+1}) \tag{26}$$

$$E_t U_c(t+1) R_{t+1}^k \frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} g(\varpi_{t+1}) = R_t^d E_t U_c(t+1) \tag{27}$$

$$Q_t K_{t+1} = \gamma [Q_t(1-\delta) + mpk_t] f(\varpi_t) K_t \bar{\kappa}_t \tag{28}$$

$$K_{t+1} \leq (1-\delta) m(\varpi_t) K_t + I_t \tag{29}$$

$$C_t + I_t \vartheta\left(\frac{I_t}{I_{ss}}\right) + \mathrm{C_t^e} \leq A_t (m(\varpi_t) K_t)^{\alpha} (L_t)^{1-\alpha} \tag{30}$$

$$\mathrm{C_t^e} = (1-\gamma)[Q_t(1-\delta) + mpk_t] f(\varpi_t) K_t \tag{31}$$

$$Q_t = \vartheta\left(\frac{I_t}{I_{ss}}\right) + \left(\frac{I_t}{I_{ss}}\right) \vartheta'\left(\frac{I_t}{I_{ss}}\right) \tag{32}$$

where we have used $\bar{\kappa}_t \equiv \left(\frac{Q_t K_{t+1}}{NW_t}\right)$, $R_{t+1}^k \equiv \frac{mpk_{t+1} + (1-\delta) Q_{t+1}}{Q_t}$, and $m(\varpi_t) \equiv f(\varpi_t) + g(\varpi_t) = 1 - \mu \int_0^{\varpi_t} x\phi(x)dx$. Note that $m'(\varpi_t) = -\mu \varpi_t \phi(\varpi_t)$. The marginal products are defined as: $mpl_t \equiv (1-\alpha) Y_t / L_t$, and $mpk_t \equiv a Y_t / (m(\varpi_t) K_t)$, where $Y_t \equiv A_t (m(\varpi_t) K_t)^{\alpha} (L_t)^{1-\alpha}$.

The model is closed by specifying a rule for ϖ_t. Under the POC contract, we scroll (25) back to period t and collect an additional state variable, Λ_{t-1}. Alternatively, the BGG contract scrolls back (27) and assumes that it holds state-by-state, and not just in expectation. Hence, lagged leverage $\bar{\kappa}_{t-1}$ is an additional state variable with the BGG contract. This suggests that a key issue in the model is the nature of the contracting problem defined by ϖ_t. We will now contrast the POC competitive equilibrium with the BGG contract and the solution to the constrained planner's problem.

3. Comparing the POC to BGG.

In contrast to the POC given by (20), BGG assume that the lender's return is equal to the deposit rate state-by-state, i.e., lender profits are zero state-by-state. This is not an implication of the modeling framework, but is instead an assumption. As BGG write, "Since entrepreneurs are risk neutral, we *assume* that they bear all the aggregate risk associated with the contract" (BGG, page 1385, emphasis added). The problem with this assumption is that household risk is linked to consumption, not to the return on capital. The POC provides this consumption insurance; the contract assumed by BGG does not. The behavior of bankruptcy rates in BGG is given implicitly by

$$g(\varpi_{t+1}) = \frac{R_t^d(\bar{\kappa}_t - 1)}{R_{t+1}^k \bar{\kappa}_t} \tag{33}$$

It is useful to compare (20) and (33). In BGG bankruptcy rates depend negatively on the return to capital, but under POC bankruptcy rates do not respond to the return on capital. This necessarily implies that in the POC the promised repayment Z_{t+1} is indexed one-for-one to innovations in the return on capital. The POC has bankruptcy rates rise when consumption falls, while bankruptcy does not depend on consumption in the BGG model. This response to consumption comes about because under the POC the risk-neutral entrepreneur is willing to offer consumption insurance to the household.

We can log-linearize both models to gain further insight. In log-linear form (lower case), the equations (17)-(19) for the POC are given by:

$$\Psi\varpi_{t+1} = (\lambda_t - \sigma c_{t+1}) \tag{34}$$

$$(\lambda_t - \sigma E_t c_{t+1}) = \frac{\kappa}{\kappa - 1}\kappa_t + (\Theta_f - \Theta_g)E_t\varpi_{t+1} \tag{35}$$

$$E_t(r_{t+1}^k - r_{t+1}^L) = \left(\frac{1}{\kappa - 1}\right)\kappa_t - \Theta_g E_t\varpi_{t+1} \tag{36}$$

where $\Psi \equiv \frac{\varpi_{ss}F'(\varpi_{ss})}{F(\varpi_{ss})}$, with $\Psi > 0$ by the second order condition, $\Theta_{\mathrm{g}} \equiv \frac{\varpi_{ss}g\prime(\varpi_{ss})}{g(\varpi_{ss})}$, $0 < \Theta_{\mathrm{g}} < 1$, and $\Theta_{\mathrm{f}} \equiv \frac{\varpi_{ss}f\prime(\varpi_{ss})}{f(\varpi_{ss})} < 0$. Taking expectations in (34) and combining with (35)-(36) we have a convenient expression for the risk spread in terms of leverage:

$$E_t\left(r_{t+1}^k - r_{t+1}^L\right) = \left[\frac{(\Psi-\theta_f+\theta_g)-\kappa\theta_g}{(\kappa-1)(\Psi-\theta_f+\theta_g)}\right]\kappa_t \equiv \nu\kappa_t \tag{37}$$

Note that increases in leverage are associated with increases in the risk premium. Ceteris paribus, deterioration in borrower net worth increases this premium. Using (36)-(37) to solve for ϖ_{t+1} in (34) we have that the POC default threshold is given by:

$$\varpi_{t+1}^{POC} \equiv \frac{[1-\nu(\kappa-1)]}{\Theta_{\mathrm{g}}(\kappa-1)}\kappa_t - \frac{\sigma}{\Psi}(c_{t+1} - \mathrm{E}_{\mathrm{t}}c_{t+1}) \tag{38}$$

From (9) and (11), the promised payment and lender's return is given by:

$$z_{t+1} = \varpi_{t+1} + r_{t+1}^k - \frac{1}{\kappa-1}\kappa_t \tag{39}$$

$$r_{t+1}^l \equiv -\frac{1}{(\kappa-1)}\kappa_t + \Theta_{\mathrm{g}}\varpi_{t+1} + r_{t+1}^k \tag{40}$$

Substituting (32) into these expression we have that under the POC these are given by

$$z_{t+1}^{POC} = r_t^d + \frac{(1-\Theta_{\mathrm{g}})[1-\nu(\kappa-1)]}{\Theta_{\mathrm{g}}(\kappa-1)}\kappa_t + \left(r_{t+1}^k - E_t r_{t+1}^k\right) - \frac{\sigma}{\Psi}(c_{t+1} - \mathrm{E}_{\mathrm{t}}c_{t+1}) \tag{41}$$

$$r_{t+1}^{l,POC} = r_t^d + \left(r_{t+1}^k - E_t r_{t+1}^k\right) - \frac{\sigma\Theta_{\mathrm{g}}}{\Psi}(c_{t+1} - \mathrm{E}_{\mathrm{t}}c_{t+1}) \tag{42}$$

The POC is thus defined by (38) and (41)-(42).

The POC contract is quite different than the one imposed by BGG. As mentioned above, BGG assume that (19) holds state-by-state (the lender return equals pre-determined deposit rate state-by-state). The BGG contract is thus given by (34)-(35) and

$$r_{t+1}^k - r_t^d = \left(\frac{1}{\kappa-1}\right)\kappa_t - \Theta_{\mathrm{g}}\varpi_{t+1} \tag{43}$$

Using this expression we have that the BGG contract is given by

$$\varpi_{t+1}^{BGG} = \frac{[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\kappa_t - \frac{1}{\Theta_g}(r_{t+1}^k - E_t r_{t+1}^k) \tag{44}$$

$$z_{t+1}^{BGG} = r_t^d + \frac{(1-\Theta_g)[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\kappa_t + \left(\frac{\Theta_g - 1}{\Theta_g}\right)(r_{t+1}^k - E_t r_{t+1}^k) \tag{45}$$

$$r_{t+1}^{l,BGG} = r_t^d \tag{46}$$

The key difference between the POC and BGG is the response of the default threshold ϖ_{t+1} and the promised repayment z_{t+1} to innovations in consumption and the return to capital. Under the POC, the household receives consumption insurance from the entrepreneur. For example, when aggregate consumption falls unexpectedly the POC has the entrepreneur increase the promised repayment to the lender. That is, when the marginal utility of consumption is high, the POC has the lender's dividend stream being high. This positive covariance is preferred by households and increases the equity value of the lender. This consumption insurance is totally missing from BGG, so that a lender that offered the BGG contract would have a lower equity value. This suggests that the BGG contract would not arise in a competitive financial market. If all lenders are offering the BGG contract, an individual lender can increase its equity value by offering the POC contract.

The second difference between the two contracts has nothing to do with risk aversion as it arises even with σ = 0. Under the POC the promised repayment moves one-for-one with innovations in the return on capital, thus implying that the bankruptcy rate is unaffected by these innovations. With BGG everything is reversed. Since $\Theta_g \approx 1$, z_{t+1} is nearly constant and ϖ_{t+1} responds nearly one for one with innovations in r_{t+1}^k. Thus under BGG, bankruptcy costs fluctuate with these observed aggregate shocks.

Although the POC is privately optimal, we will see below that it does not maximize social welfare because it exacerbates fluctuations in agency costs by exacerbating fluctuations in net worth and the risk premium (see (37)). In log deviations, the evolution of net worth (21) is given by:

$$nw_{t+1} = nw_t + \kappa_t + r^k_{t+1} + \Theta_f \varpi_{t+1} \tag{47}$$

Using the alternative expressions for the two contracts (POC and BGG) we have

$$nw^{POC}_{t+1} = nw^{POC}_t + r^d_t + \left\{\frac{\Theta_f[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)} + 1 + \nu\right\}\kappa_t + (r^k_{t+1} - E_t r^k_{t+1}) - \frac{\sigma\Theta_f}{\Psi}(c_{t+1} - \mathrm{E_t} c_{t+1}) \tag{48}$$

$$nw^{BGG}_{t+1} = nw^{BGG}_t + r^d_t + \left\{\frac{\Theta_f[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)} + 1 + \nu\right\}\kappa_t + \left(1 - \frac{\Theta_f}{\Theta_g}\right)(r^k_{t+1} - E_t r^k_{t+1}) \tag{49}$$

The two lending contracts differ by the response of net worth to innovations in aggregate variables. Under the POC, innovations in the return on capital are shared equally by the lender and the entrepreneur. But under BGG, net worth responds to productivity innovations by twice as much than under the POC (for the calibration used below, $\frac{\Theta_f}{\Theta_g} \approx -0.95$).

The more important difference in net worth behavior comes from consumption innovations. Under the BGG contract, net worth is entirely unresponsive to consumption shocks. But under the POC, net worth responds *sharply* to consumption innovations: for the calibration used below, $-\frac{\sigma\Theta_f}{\Psi} \approx 11.9$ (!). atomistic households privately prefer owning shares in a bank that offers the POC lending contract because it provides a hedge against consumption risk (and the entrepreneur is indifferent since he is risk neutral). Consequently, the lender seeking to maximize its equity value will offer such a contract. However, this privately optimal behavior is socially costly as it results in sharp movements in net worth and the risk premium, which individual households do not internalize. For example, suppose that a negative TFP shock leads to a decline in consumption. Under the POC, net worth declines sharply as a result. This decline in net worth is highly persistent and implies a persistent increase in the risk premium.

4. The Planner's Problem.

In this section we consider a social planner who maximizes a weighted sum of the lifetime utility flow of the representative household and entrepreneur. The linearity in the model implies that we can aggregate entrepreneurial consumption. With a utility weight of ϵ on the entrepreneurs, the planner maximizes:

$$E_t \sum_{j=0}^{\infty} \beta^j \left[U\left(C_{t+j}, L_{t+j}\right) + \epsilon C_{t+j}^e\right] \tag{50}$$

subject to the resource constraints and private optimality. We assume that the planner is constrained by the competitive behavior of the agents. However, the planner can affect this behavior by levying distortionary taxes and subsidies. The private informational barrier on observing entrepreneurial payoffs implies that the planner cannot directly levy taxes on the entrepreneurs. Hence, we must focus on the other agents. Fundamentally, the planner cares about three margins: the household's labor choice, the household's savings choice, and the distribution of consumption across agents. The first two can be influenced by wedges on the relevant margins: a tax of τ_t^L on labor income, and a tax of τ_t^d on interest income from deposits.

But the third margin, the allocation of consumption across agents, is more difficult as the planner cannot directly tax entrepreneurs because their income is unobserved. From (31), the aggregate consumption of entrepreneurs is linked directly to the lending contract via the bankruptcy threshold ϖ_t. The most direct tax to influence this threshold is a wedge on monitoring costs. Hence, we assume that the planner has access to a time-varying tax on monitoring costs so that the private cost of monitoring becomes $(1+\tau_t^\mu)\mu$. With this wedge, the lender's payoff is given by:

$$h(\varpi; \tau^\mu) \equiv g(\varpi) - \mu\tau^\mu \int_0^{\varpi} \omega\phi(\omega)d\omega \tag{51}$$

Movements in this tax will directly feed into movements in bankruptcy rates.[1] As with the household income taxes, these tax proceeds are rebated to the household in a lump-sum manner.

To summarize, the planner maximizes (50) with the assistance of three wedges: τ_t^d, τ_t^L, and τ_t^μ. These wedges alter the following equilibrium conditions:

$$U_c(t) = R_t^d E_t \beta U_c(t+1)(1-\tau_t^d) \tag{23a}$$

[1] Alternatively, the planner could levy taxes/subsidies on the promised repayment rate Z_{t+1}.With this wedge, the lender's payoff is given by

$$h(\varpi; \tau^z) \equiv g(\varpi)(1-\tau^z) + \tau^z(1-\mu) \int_0^{\varpi} \omega\phi(\omega)d\omega$$

where τ^z is the tax/subsidy on the repayment rate.

$$-U_L(t)/U_c(t) = mpl_t(1-\tau_t^L) \tag{24a}$$

$$f'(\varpi_{t+1}) + \Lambda_t U_c(t+1)h'\left(\varpi_{t+1};\tau_{t+1}^{\mu}\right) = 0 \tag{25a}$$

$$(\bar{\kappa}_t - 1)E_t R_{t+1}^k f(\varpi_{t+1}) = \Lambda_t E_t U_c(t+1)R_{t+1}^k h\left(\varpi_{t+1};\tau_{t+1}^{\mu}\right) \tag{26a}$$

$$E_t U_c(t+1)R_{t+1}^k \frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} h\left(\varpi_{t+1};\tau_{t+1}^{\mu}\right) = R_t^d E_t U_c(t+1) \tag{27a}$$

The planner thus maximizes (50) subject to (23a)-(27a) and (28)-(32), by choosing allocations, prices, and taxes.

This optimization problem can be greatly simplified by noting that many of the constraints are not restrictions per se, but instead can be thought of as defining or "backing out" the needed market price or tax that supports the planner's allocation. This simplification will leave us with the planner choosing allocations, while the market prices and taxes are chosen to support this outcome. In particular, equations (23a)-(27a), and (28), can be used to solve for R_t^d, Λ_t, $\bar{\kappa}_t$, and the three supporting taxes, as functions of the allocations. The three taxes are given implicitly by:

$$g'(\varpi_{t+1}) - \mu\tau_{t+1}^{\mu}\varpi_{t+1}\phi(\varpi_{t+1}) = \frac{-f'(\varpi_{t+1})}{\Lambda_t U_c(t+1)} \tag{52}$$

$$(1-\tau_t^d) = \left[\frac{U_c(t)}{E_t\beta U_c(t+1)}\right]\left[\frac{E_t U_c(t+1)}{E_t U_c(t+1)R_{t+1}^k \frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} h\left(\varpi_{t+1};\tau_{t+1}^{\mu}\right)}\right] \tag{53}$$

$$(1-\tau_t^L) = \frac{-U_L(t)/U_c(t)}{mpl_t} \tag{54}$$

where the multiplier is given by

$$\Lambda_t = \frac{(\bar{\kappa}_t - 1)E_t R_{t+1}^k f(\varpi_{t+1})}{E_t U_c(t+1)R_{t+1}^k h\left(\varpi_{t+1};\tau_{t+1}^{\mu}\right)} \tag{55}$$

We are thus left with:

$$K_{t+1} \leq (1-\delta)m(\varpi_t)K_t + I_t \tag{56}$$

$$C_t + I_t \vartheta\left(\frac{I_t}{I_{ss}}\right) + C_t^e \leq A_t (m(\varpi_t)K_t)^\alpha (L_t)^{1-\alpha} \tag{57}$$

$$C_t^e = (1-\gamma)[Q_t(1-\delta) + mpk_t] f(\varpi_t) K_t \tag{58}$$

with the price of capital given implicitly by $Q_t = \vartheta\left(\frac{I_t}{I_{ss}}\right) + \left(\frac{I_t}{I_{ss}}\right)\vartheta'(\frac{I_t}{I_{ss}})$. The planner thus chooses the allocations $\{\varpi_t, K_{t+1}, I_t, C_t, C_t^e, L_t\}$ to maximize (50) subject to (56)-(58).

Constraints (56)-(57) are familiar resource constraints and need no elaboration. But the novelty here is constraint (58). Along with choosing work effort and capital accumulation, the planner seeks to shift consumption risk to the risk neutral entrepreneur. But this desire to efficiently share risk is complicated by monitoring costs. The planner can redistribute consumption from entrepreneurs to households by varying the cut-off value ϖ_t. Higher values of ϖ_t lower entrepreneurial consumption and increase household consumption, but this reallocation comes at the expense of lower output via $m(\varpi_t)$ in (24). Hence, the planner's problem is ultimately one of risk-sharing across agents, where the level of sharing is constrained because of monitoring costs. The planner can decentralize this ϖ_t behavior with the use of the monitoring tax in (52).

Let Λ_{1t}, Λ_{2t}, and Λ_{3t}, denote the multipliers on (56)-(58), respectively. We find it convenient to treat Q_t parametrically as defined by (32) so that $Q_I(t)$ denotes the derivative of (32) with respect to investment. The following are the FONC to the planner's problem:

$$\Lambda_{1t} - \Lambda_{3t} x_t Q_I(t)(1-\delta) m(\varpi_t) K_t = U_c(t) Q_t \tag{59}$$

$$\Lambda_{3t} = U_c(t) - \epsilon \tag{60}$$

$$-U_L(t) = U_c(t) mpl_t - \Lambda_{3t} \alpha mpl_t x_t \tag{61}$$

$$\Lambda_{1t} = \beta E_t m(\varpi_{t+1}) \begin{Bmatrix} \Lambda_{1t+1}(1-\delta) + U_c(t+1) mpk_{t+1} \\ -\Lambda_{3t+1} x_{t+1}[\alpha mpk_{t+1} + (1-\delta) Q_{t+1}] \end{Bmatrix} \tag{62}$$

$$\frac{m'(\varpi_t)}{f'(\varpi_t)} = \frac{\Lambda_{3t}(1-\gamma)[Q_t(1-\delta) + mpk_t]}{[\Lambda_{1t}(1-\delta) + U_c(t) mpk_t + \Lambda_{3t} x_t (1-\alpha) mpk_t]} \tag{63}$$

where we define

$$x_t \equiv (1-\gamma) \frac{f(\varpi_t)}{m(\varpi_t)}, \tag{64}$$

and we have used $U_c(t) = \Lambda_{2t}$.

It is instructive to compare the planner's behavior (59)-(63) to the competitive equilibrium. The competitive equilibrium includes the marginal conditions

$$-U_L(t) = U_c(t)mpl_t \tag{65}$$

$$E_t\beta U_c(t+1)R^k_{t+1}\frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1}g(\varpi_{t+1}) = U_c(t) \tag{66}$$

The competitive equilibrium has employment (65) satisfying the traditional RBC margin, but the investment decisions (66) is distorted relative to familiar RBC behavior. Comparing (65)-(66) to the complementary (61)-(62) it is quite clear that the planner's allocations will differ sharply from the competitive equilibrium. There are two notable differences. First, leverage ratios and net worth do not constrain the planner because the tax on interest income can be varied to motivate any desired level of savings from the households. Second, the multiplier Λ_{3t} alters both of the planner's conditions (61)-(62) considerably from the competitive equilibrium. From (60), the multiplier Λ_{3t} denotes the difference in the marginal utilities between the entrepreneur and the household. The planner wants to equate these two (and thus set Λ_{3t} = 0) by transferring consumption units. But (58) constrains the planner: entrepreneurial consumption can be altered only by altering variables in (58). It is this constraint that colors all the planner's choices. Consider first the planner's choice of ϖ_t. Since $f'(\varpi_t)$ and $m'(\varpi_t)$ are both negative, (63) implies that Λ_{3t} is positive. That is, the planner sets $\varpi_t > 0$ and tolerates the associated costs of positive bankruptcy rates only because on the margin he desires to transfer consumption units from the entrepreneur back to the household. But the positive monitoring costs imply that the planner is ultimately frustrated and does not achieve equal marginal utilities ($U_c(t) > \epsilon$). This incomplete redistribution illuminates the remaining differences between the planner and the competitive equilibrium. Because marginal output units do not flow entirely to the higher-marginal-utility household, the planner prefers a lower level of work effort as implied by (61), and a lower level of physical capital as implied by

(62). Further, since reductions in the price of capital lead directly to a redistribution from the entrepreneur to the household, the planner prefers a lower price of capital as implied by (59).

One can see this distribution motive clearly by considering a special case. Suppose we ignored the private information barrier and assumed that the planner had access to a lump sum transfer on entrepreneurs that could be used to transfer consumption across agents. In this case (58) would no longer be a constraint and Λ_{3t} would be identically zero. The planner would set ϖ_t identically to zero, and choose labor and investment behavior identical to a two-agent RBC model in which one agent is risk-neutral agent and provides perfect consumption insurance to households. That is, the employment and investment margins would be given by:

$$-U_L(t) = U_c(t)mpl_t \tag{67}$$

$$U_c(t) = \beta E_t U_c(t+1) R_{t+1}^k \tag{68}$$

Hence, if the planner was not constrained in his ability to redistribute income, he would choose the traditional RBC behavior. (The consumption insurance provided by entrepreneurs implies that this two-agent RBC economy will respond sharply to TFP shocks compared to a one-agent RBC model.)

Before closing, it is helpful to comment on a tax that we assume the planner *cannot* levy. If the planner could levy a per unit tax/subsidy on the sale of new capital, then the planner has the ability to choose the price of capital independently of the level of investment. That is, suppose the new-capital producer maximizes:

$$Q_t(1-\tau_t^q)I_t - I_t\phi(\tfrac{I_t}{I_{ss}}).$$

where τ_t^q is a tax on new capital levied on the seller. This implies the following first-order condition:

$$Q_t = \frac{\phi\left(\frac{I_t}{I_{ss}}\right)+\left(\frac{I_t}{I_{ss}}\right)\phi'\left(\frac{I_t}{I_{ss}}\right)}{(1-\tau_t^q)}.$$

Hence, by varying τ_t^q, the planner can achieve any capital price that is desired, and thus, via (58), any desired level of entrepreneurial consumption. This means that (58) will no longer be a constraint, Λ_{3t} = 0, and ϖ_t = 0. This is again the case of perfect consumption insurance, and the CSV problem drops from the model.[2] We ignore this possibility because it makes things too simple for the planner. That is, since entrepreneurs are inelastic savers and are the only holders of capital in the BGG model, a capital tax effectively gives the planner a lump sum tax. In a model such as Carlstrom and Fuerst (1997), where both households and entrepreneurs hold capital, the capital tax could not be used so effectively. More generally, in more elaborate models it is surely not the case that a capital tax is isomorphic to a lump sum tax.

5. Quantitative Analysis.

Our benchmark calibration will largely follow BGG. The discount factor β is set 0.99. Utility is assumed to be logarithmic in consumption (σ = 1), and the elasticity of labor is assumed to be 3 (η = 1/3). The production function parameters include α = 0.35, investment adjustment costs ψ = 0.25, and quarterly deprecation is δ = .025. As for the credit-related parameters, we calibrate the model to be consistent with: (i) a steady state spread between $R^k$and R^d of 200 bp (annualized), (ii) monitoring costs μ = 0.12, and (iii) a leverage ratio of κ = K/NW = 1.95. These values imply a death rate of γ = 0.98, a standard deviation of the idiosyncratic productivity shock of 0.28, and a quarterly bankruptcy rate of .75% (ϖ_{ss} = 0.486). In the linearized model (see appendix), this then implies ν = 0.041. We assume that total factor productivity follows an AR(1) process with ρ^A = 0.95. The financial accelerator is driven by fluctuations in the price of capital. The size of these movements is driven by the capital adjustment cost ψ. Hence we perform sensitivity analysis over this parameter.

[2] A similar result holds in Jeanne and Korinek (2010): a time-varying subsidy on asset purchases can eliminate the borrowing constraint and achieve the first-best allocation.

We investigate three allocations: (i) the planner, (ii) competitive equilibrium under POC, and (iii) BGG. To reiterate, under a laissez faire assumption only the POC is a competitive equilibrium as it maximizes equity value. The planner and BGG allocations would be supported under a competitive equilibrium only if there are time-varying governmental interventions. For the planner's behavior we need to assume a welfare weight for entrepreneurial consumption (ϵ). We find it convenient to choose the baseline value of this weight such that the steady state level of capital is identical for the planner and the POC. We can then vary ϵ and trace out the planner's welfare frontier. Because movements in ϵ result in changes in the planner's steady-state capital stock, we consider only modest variations in ϵ so that our approximation methods retain a reasonable level of accuracy. This means that we do not trace out the entire welfare frontier of the planner, but only the frontier in the neighborhood of the BGG and POC steady state.

To develop intuition, Figure 1 presents impulse response functions for the case of a 1% iid TFP shock, $\rho^A = 0$ (we plot aggregate consumption which is the sum of household and entrepreneurial consumption). Note that the planner responds to this iid shock in something of an iid fashion. That is, there is very little persistence in the planner's behavior because net worth is not a state variable, and physical capital has modest effects on persistence. Matters are much different with BGG and POC. Because of the financial accelerator, both BGG and POC over-respond to the TFP shock (in comparison to the planner). This amplification is particularly strong under POC. The consumption insurance under the POC implies that the increase in consumption leads to a sharp fall in the bankruptcy cut-off (ϖ_t) and thus a significant increase in entrepreneurial net worth. This surge in net worth leads to a sharp increase in investment. These effects diminish only slowly as entrepreneurial net worth returns to normal levels.

Figure 2 looks at the case of an auto-correlated TFP shock. In comparison to the planner, both POC and BGG over-respond to the shock. Again, the planner's response to the shock is less persistent than BGG and POC. Note in particular that bankruptcy rates decline very modestly under the planner so that entrepreneurial consumption (which is proportional to net worth) rises only modestly. But under both BGG and especially the POC, the financial contract shifts net worth and thus consumption sharply towards the entrepreneur. The persistent movement in net worth leads to a decline in the risk premium, and hence a sub-optimal amplification of investment and output.

Table 1 reports the steady state and standard deviation of some key variables in the model for our baseline calibration and a calibration with higher adjustment costs. As noted earlier, the baseline welfare weight on entrepreneurs is chosen so that the steady state capital stocks are the same under the planner and the competitive equilibrium. This table does not present the BGG steady states because the steady state of the POC and BGG are identical. There are two key comments regarding Table 1. First, the POC has a significantly higher threshold value of ϖ_t, ie., a higher rate of monitoring, than does the planner. This effect is accentuated for the case of higher adjustment costs. Second, the consumption-hedging in the POC contract leads to volatile net worth behavior, and thus significantly higher output variability in POC compared to the planner.

Before proceeding we return to the nature of the pecuniary externality present in the model. As noted above, the planner tolerates costly bankruptcy because on the margin consumption is more valued by the household. But as Table 1 demonstrates, the planner has a lower monitoring level than does the POC.[3] This is because under the POC, the lender does not internalize the effect of today's repayment rate (Z_t) and monitoring rate (ϖ_t) on the future evolution of net worth and the risk premium. That is, a high repayment rate today is beneficial to the lender, but comes at the cost of lower entrepreneurial net worth and thus higher bankruptcy rates tomorrow. Since the loan contracts are one-period, this latter effect is not internalized. As noted in the introduction, this externality has a stochastic counterpart in that the consumption hedging that occurs under the POC has adverse consequences for the subsequent evolution of net worth and thus credit conditions. The appendix develops this intuition more carefully by comparing the household budget constraint to the planner's constraint set.

[3] This is for our baseline choice of ϵ. While it holds for a wide range of ϵ, for extremely low levels of ϵ the planner will choose a higher monitoring level than does the POC.

Table 2 provides a welfare analysis of the three allocations (POC, BGG, and planner). The table presents both unconditional welfare, and welfare conditional on the steady state level of capital. To see if there are Pareto improvements, data is also presented for household and entrepreneurial welfare. In all cases the results are reported as numerical differences from the planner's welfare levels. The welfare measures we report are computed based on a second-order approximation to the nonlinear equilibrium conditions of each model. Our preferred welfare measure is the conditional expectation of a weighted average of household and entrepreneurial discounted lifetime utility. The conditional welfare measure is chosen since agents in the model solve an explicitly conditional optimization problem. As noted, we choose the baseline weight on entrepreneurial utility (ϵ) such that the capital stock in the steady state is the same for the planner problem as for the BGG model and the POC model.

Let us first focus on comparing the planner to the POC. For the baseline calibration we see that the planner's allocation is a Pareto improvement. The conditional welfare gain is large, an annual flow of 0.26% of household consumption. Since the calibration is log utility, this is equivalent to a one-off payment equal to 26% of steady state household consumption (using a discount rate of 4%). These effects are magnified for higher adjustment costs and higher levels of autocorrelation. With $\psi = 0.75$, the conditional welfare loss is a consumption flow of 0.56% of household consumption.

The welfare losses of the POC are significant. As point of comparison, Lucas (1987) estimates that the welfare cost of US business cycles is on the order of a consumption flow of 0.05%. The welfare costs of the POC are an order of magnitude larger than these Lucas estimates. The reason is that the POC is a distorted steady state, eg., the cut-off value for monitoring is significantly higher under the POC than under the planner. This creates a steady-state welfare loss. This first-order cost is then amplified by the fluctuations in net worth induced by the POC consumption-hedging. Lucas's (1987) analysis abstracts from these first-order effects by holding fixed the mean path of consumption.

Comparing BGG to the planner, it is curious that for the benchmark calibration, the welfare costs of the BGG contract are quite modest, 0.03% in conditional welfare. For high adjustment costs and high autocorrelation, these welfare effects become larger, 0.11% in conditional welfare. But in all cases the planner allocation is not a Pareto improvement. This is primarily because we set the baseline ϵ to match steady state capital stocks.

Figure 3 traces out the planner's welfare frontier in the neighborhood of the baseline ϵ = 1.15 (that equalized steady state capitals). As we vary ϵ, the planner's steady state capital stock changes. Hence, we consider small movements in ϵ, so that the corresponding movements in the steady state capital stock are also modest. We can then use the calculated quadratic welfare functions to approximate planner welfare levels conditioned at the same steady state capital stock as BGG and POC. Figure 3 demonstrates that both BGG and POC are inside the planner welfare frontier. As anticipated by Table 2, BGG is only modestly inside, while POC is significantly below the frontier. Recall that under BGG, the loan repayment rate is not indexed to innovations in R_t^k, so that the net worth consequences of TFP shocks is increasing in the steady state level of leverage. Figure 4 presents the complementary welfare frontier but for a POC and BGG calibration with a steady state leverage of 4 (this implies a higher entrepreneurial death rate). In this case, BGG and POC are both significantly inside the planner welfare frontier. Using the baseline ϵ = 0.38 (again to equalize steady state capital stocks), the conditional welfare cost of BGG and POC is an annual consumption flow increase of 0.36%.

It is noteworthy that the planner tolerates very little movement in the monitoring threshold ϖ_t. This is quite intuitive. The underlying informational friction is static, so that the optimal level of monitoring is roughly constant. This suggests that the planner allocation can be closely approximated in the competitive equilibrium with only the time-varying tax on monitoring costs that leads the private agents to keep bankruptcy rates constant. The POC is affected by this tax so that ϖ_{t+1} is defined by

$$g'(\varpi_{t+1}) - \mu\tau_{t+1}^{\mu}\varpi_{t+1}\phi(\varpi_{t+1}) = \frac{-f'(\varpi_{t+1})}{\Lambda_t U_c(t+1)} \tag{69}$$

To keep ϖ_{t+1} constant, the tax rate τ_{t+1}^{μ} must be countercyclical. The logic is straightforward. Under the POC, the lender will offer a contract that provides consumption insurance by having repayment rates and bankruptcy rates that are countercyclical. But by raising the private cost of monitoring in low-consumption states, the state-contingent tax τ_{t+1}^{μ} undoes the lender's desire to offer such a state-contingent loan contract. This single tax policy comes close to achieving the planner level of welfare. For example, with such a tax policy, the conditional welfare cost of the POC in the baseline scenario is only 0.023, essentially the steady-state welfare cost.

6. Sensitivity of the financial accelerator to other shocks.

As a form of sensitivity analysis on the positive aspects of the model, we investigate adding sticky prices and other exogenous shocks to the analysis. The focus is on the financial accelerator and how it is affected by the two alternative loan contracts, BGG and POC. We integrate sticky prices via the familiar Dynamic New Keynesian (DNK) methodology. This is standard so we dispense with the formal derivation, see, for example, Woodford (2003) for details. Imperfect competition distorts factor prices so that the marginal productivity of capital and labor in (4) and (5) are pre-multiplied by marginal cost. Marginal cost in turn is affected by the path of inflation so that in log deviations we have a relationship between inflation (π_t) and marginal cost (mc_t):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa mc_t \tag{70}$$

The model is then closed with the familiar Fisher equation linking real and nominal interest rates:

$$i_t = E_t \pi_{t+1} + r_t^d \tag{71}$$

and an interest rate policy for the central bank. In log deviations this policy rule is given by:

$$i_t = \phi_\pi \pi_t + \varepsilon_t^m. \tag{72}$$

where ε_t^m is an exogenous policy movement with autocorrelation $\rho^m = 0.50$. We also consider an exogenous shock to the household discount rate (ε_t^d) so that

$$r_t^d = \sigma(E_t c_{t+1} - c_t) + \varepsilon_t^d \tag{73}$$

where ε_t^d is auto-correlated with coefficient $\rho^d = 0.56$ (as estimated in Carlstrom, Fuerst, Ortiz, and Paustian (2012)). The parameter calibration is standard with $\phi_\pi = 1.5$ and $\kappa = 0.025$.

Figures 5-7 report the impulse response functions to a 1% TFP shock ($\rho^A = 0.95$), a 25 bp (quarterly) monetary policy shock, and a 25 bp (quarterly) discount rate shock. We will discuss each in turn. Because it directly affects aggregate supply, the TFP shock tends to lower inflation. This is what we see in the BGG model. But there is an offsetting effect under POC. Because the shock leads to an innovation in consumption, the insurance mechanism in the POC contract reduces the repayment rate and sharply boosts net worth. This increase in net worth lowers agency costs and thus increases investment and acts like a demand shock. Thus, inflation under the POC actually *increases* with a positive TFP shock. Because of this increase in inflation, the TFP shock under sticky prices is significantly amplified compared to the case with flexible prices (Figure 2) for the case of the POC contract. All of these effects take time to dissipate because of the persistence in net worth.

Figure 6 presents the results for an exogenous 25 bp (quarterly) increase in the nominal interest rate. Once again, there is significant magnification with the POC compared to BGG. This magnification comes from movements in net worth and hence investment.

Finally figure 7 presents the results for a 25 bp (quarterly) discount rate shock. The shock increases the household's desire for current consumption. Under BGG, the modest movement in output comes almost entirely from the movement in household consumption. But under POC, the increase in household consumption leads to a sharp increase in net worth. This then feeds into a sharp increase in investment and thus aggregate output. The significant difference between BGG and POC suggests that any shock that primarily effects household consumption will have a much larger effect under POC compared to BGG.

7. Conclusion.

Two basic functions of financial markets are to intermediate between borrowers and lenders, and to provide a mechanism to hedge risk. Both of these motivations are present here. The risky debt contract is an efficient way of mitigating the informational asymmetries arising from the CSV problem. This then allows for funds to flow from the household-lenders to the entrepreneurial-borrowers. Since the entrepreneurs are risk neutral, households prefer contracts that index the loan repayment to innovations in aggregate consumption. This provides the household with a hedge against business cycle risk, and intermediaries that offer such loan contracts have a higher equity value than others. But as in Lorenzoni (2008) and Jeanne and Korinek (2010), in environments with credit constraints, financial markets can go awry. This is the case here. In this model with CSV-inspired credit constraints, the household's desire to hedge consumption risk results, paradoxically, in greater business cycle risk.

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APPENDIX

1. Linearized Model (POC).

$$E_t(r^k_{t+1} - r^l_{t+1}) = \nu\kappa_t \tag{A1}$$

$$nw_t = \kappa\frac{\gamma}{\beta}\left(r^k_t - r^l_t\right) + \frac{\gamma}{\beta}(r^l_t + nw_{t-1}) + \gamma\kappa\frac{rp}{\beta}(k_t + q_t + r^k_t) \tag{A2}$$

$$r^{l,POC}_{t+1} = r^d_t + \left(r^k_{t+1} - E_t r^k_{t+1}\right) - \frac{\sigma\Theta_{\mathrm{g}}}{\Psi}(c_{t+1} - \mathrm{E_t}c_{t+1}) \tag{A3}$$

$$z^{POC}_{t+1} = r^d_t + \frac{(1-\Theta_{\mathrm{g}})[1-\nu(\kappa-1)]}{\Theta_{\mathrm{g}}(\kappa-1)}\kappa_t + \left(r^k_{t+1} - E_t r^k_{t+1}\right) - \frac{\sigma}{\Psi}(c_{t+1} - \mathrm{E_t}c_{t+1}) \tag{A4}$$

$$\varpi^{POC}_t \equiv \frac{[1-\nu(\kappa-1)]}{\Theta_{\mathrm{g}}(\kappa-1)}\kappa_{t-1} - \frac{\sigma}{\Psi}(c_t - \mathrm{E_{t-1}}c_t) \tag{A5}$$

$$r^k_t = \epsilon q_t + (1-\epsilon)mpk_t - q_{t-1} \tag{A6}$$

$$\kappa_{t-1} = (q_{t-1} + k_t - nw_{t-1}) \tag{A7}$$

$$\sigma c_t + \eta l_t = \alpha k_t + a_t - \alpha l_t \tag{A8}$$

$$r^d_t = \sigma(E_t c_{t+1} - c_t) \tag{A9}$$

$$q_t = \psi i_t \tag{A10}$$

$$k_{t+1} = \delta i_t + (1-\delta)k_t \tag{A11}$$

$$c_{ss}c_t + c^e_{ss}nw_t + i_{ss}i_t \;\; = a_t + \alpha k_t + (1-\alpha)l_t \tag{A12}$$

where $\epsilon \equiv \frac{1-\delta}{mpk_{ss}+(1-\delta)}$. Also we have $\kappa \equiv K_{SS}/NW_{SS}$, Q_{SS}= 1, $R^s_{SS} = 1/\beta$. Finally, we set $\mu\int_0^{\varpi_{ss}}\omega\phi(\omega)d\omega \approx 0$ so that monitoring costs do not appear in (A12).

2. The Derivation of the spread.

$$f'(\varpi_{t+1}) + \Lambda_t U_c(t+1)g'(\varpi_{t+1}) = 0 \tag{A13}$$

$$(\bar{\kappa}_t - 1)E_t R_{t+1}^k f(\varpi_{t+1}) = \Lambda_t E_t U_c(t+1) R_{t+1}^k g(\varpi_{t+1}) \tag{A14}$$

$$E_t U_c(t+1) R_{t+1}^k \frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} g(\varpi_{t+1}) = R_t^d E_t U_c(t+1) \tag{A15}$$

Linearized we have:

$$\Psi \varpi_{t+1} = (\lambda_t - \sigma c_{t+1}) \tag{A16}$$

$$(\lambda_t - \sigma c_{t+1}) = \frac{\kappa}{\kappa - 1} \kappa_t + (\theta_f - \theta_g) E_t \varpi_{t+1} \tag{A17}$$

$$E_t(r_{t+1}^k - r_{t+1}^L) = \left(\frac{1}{\kappa - 1}\right) \kappa_t - \Theta_{\mathrm{g}} E_t \varpi_{t+1} \tag{A18}$$

Substitute (A16) into (A17):

$$E_t \varpi_{t+1} = \frac{\kappa}{\kappa - 1} \frac{1}{(\Psi - \theta_f + \theta_g)} \kappa_t$$

Then into (A18):

$$E_t(r_{t+1}^k - r_{t+1}^L) = \left[\frac{(\Psi - \theta_f + \theta_g) - \kappa\theta_g}{(\kappa - 1)(\Psi - \theta_f + \theta_g)}\right] \kappa_t \equiv \nu \kappa_t$$

where $\Psi \equiv \frac{\varpi_{ss} F'(\varpi_{ss})}{F(\varpi_{ss})} > 0$, by the second order condition, and $\Theta_{\mathrm{g}} \equiv \frac{\varpi_{ss} g'(\varpi_{ss})}{g(\varpi_{ss})}$, where $0 < \Theta_{\mathrm{g}} < 1$, and $\Theta_{\mathrm{f}} \equiv \frac{\varpi_{ss} f'(\varpi_{ss})}{f(\varpi_{ss})} < 0$.

3. <u>Going from the household budget constraint to the planner.</u>

Why does the household behave differently than the planner? Evidently there are some effects that the household takes as exogenous but that are internalized by the planner. To gain some insight into this, let us begin with the budget constraint of the household:

$$C_t + D_t + Q_t^L S_t \leq w_t N_t + R_{t-1}^D D_{t-1} + (Q_t^L + Div_t) S_{t-1} + P_t \tag{A19}$$

where P_t denotes the profit flow of the capital-producing firm. For future reference, $P_t \equiv Q_t I_t - I_t \vartheta(\frac{I_t}{\delta K_{ss}})$. Suppose the household internalized all factor prices and dividend flows, that is, the household internalized the following equilibrium conditions:

$Div_t = R_t^L D_{t-1} - R_{t-1}^d D_{t-1}$
$S_t = 1$
$D_t = Q_t K_{t+1} - NW_t$
$w_t = MPN_t$
$r_t = MPK_t$

$$R_t^k \equiv \frac{r_t + (1-\delta)Q_t}{Q_{t-1}}$$

Further let us define the risk premium as

$$R_t^L \equiv R_t^k g(\varpi_t)\frac{\bar{\kappa}_{t-1}}{\bar{\kappa}_{t-1}-1} \equiv R_t^k - rp_t$$

Substituting these expressions into the household's budget constraint (A19) we have

$$C_t + Q_t[K_{t+1} - (1-\delta)K_t] \leq K_t^\alpha (A_t L_t)^{1-\alpha} - \{\boldsymbol{rp_t Q_{t-1}}\}K_t + \{\boldsymbol{NW_t - R_t^L NW_{t-1}}\} + P_t \qquad \text{(A20)}$$

Suppose that the household maximized utility subject to (A20), taking as exogenous the two bold terms in braces. That is, suppose the household internalized all factor prices except for the behavior of the risk premium and net worth dynamics. It is straightforward to show that the competitive equilibrium of this framework is identical to the original competitive equilibrium. But if we substitute out for these terms in braces, we are lead to the planner's constraints (56)-(58). To see this, substitute in for rp_t and re-arrange terms:

$$C_t + Q_t[K_{t+1} - (1-\delta)K_t] \leq r_t K_t + w_t L_t + NW_t - R_t^k Q_{t-1}K_t + R_t^L(Q_{t-1}K_t - NW_{t-1}) + P_t$$

Using the definition of R_t^k:

$$C_t + Q_t K_{t+1} \leq w_t L_t + NW_t + R_t^L(Q_{t-1}K_t - NW_{t-1}) + P_t \qquad \text{(A21)}$$

We also know that:

$$\mathrm{NW_t} = \gamma[Q_t(1-\delta) + mpk_t]f(\varpi_t)K_t = \gamma R_t^k f(\varpi_t)\bar{\kappa}_{t-1}\mathrm{NW_{t-1}}$$

$$\mathrm{C_t^e} = (1-\gamma)[Q_t(1-\delta) + mpk_t]f(\varpi_t)K_t = (1-\gamma)R_t^k f(\varpi_t)\bar{\kappa}_{t-1}\mathrm{NW_{t-1}}$$

$$R_t^L \equiv R_t^k g(\varpi_t)\frac{\bar{\kappa}_{t-1}}{\bar{\kappa}_{t-1}-1}$$

Hence, we can write constraint (A21) as:

$$C_t + Q_t K_{t+1} - NW_t \leq w_t N_t + R_t^k Q_{t-1}K_t m(\varpi_t) - R_t^k Q_{t-1}K_t f(\varpi_t) + P_t$$

$$C_t + Q_t K_{t+1} + C_t^e \leq w_t N_t + R_t^k Q_{t-1}K_t m(\varpi_t) + P_t$$

$$C_t + Q_t K_{t+1} + C_t^e \leq w_t N_t + [Q_t(1-\delta) + mpk_t]K_t m(\varpi_t) + P_t$$

Using the definition of P_t, we have the planner constraints:

$$K_{t+1} \leq (1-\delta)m(\varpi_t)K_t + I_t$$

$$C_t + I_t\phi(\frac{I_t}{\delta K_{ss}}) + \mathrm{C_t^e} \leq Y_t$$

$$\mathrm{C_t^e} \leq (1-\gamma)[Q_t(1-\delta) + mpk_t]f(\varpi_t)K_t$$

To summarize, if the household maximized with respect to A20, taking as given the terms in braces, the household FONC are identical and the competitive equilibrium is the same as in the text. But if we substitute out for these two terms in braces, we are led to the planner's constraints. This then implies that the competitive equilibrium differs from the planner because the household does not internalize these terms in braces: the behavior of net worth and the risk premium. In particular, when choosing labor and consumption, the household does not internalize the effect of these choices on net worth and the risk premium. This then spills on to the CSV contracting problem. The lender does not internalize the effect of ϖ_t on the household's budget set via movements in the risk premium and net worth.

Table 1: Model Comparison.

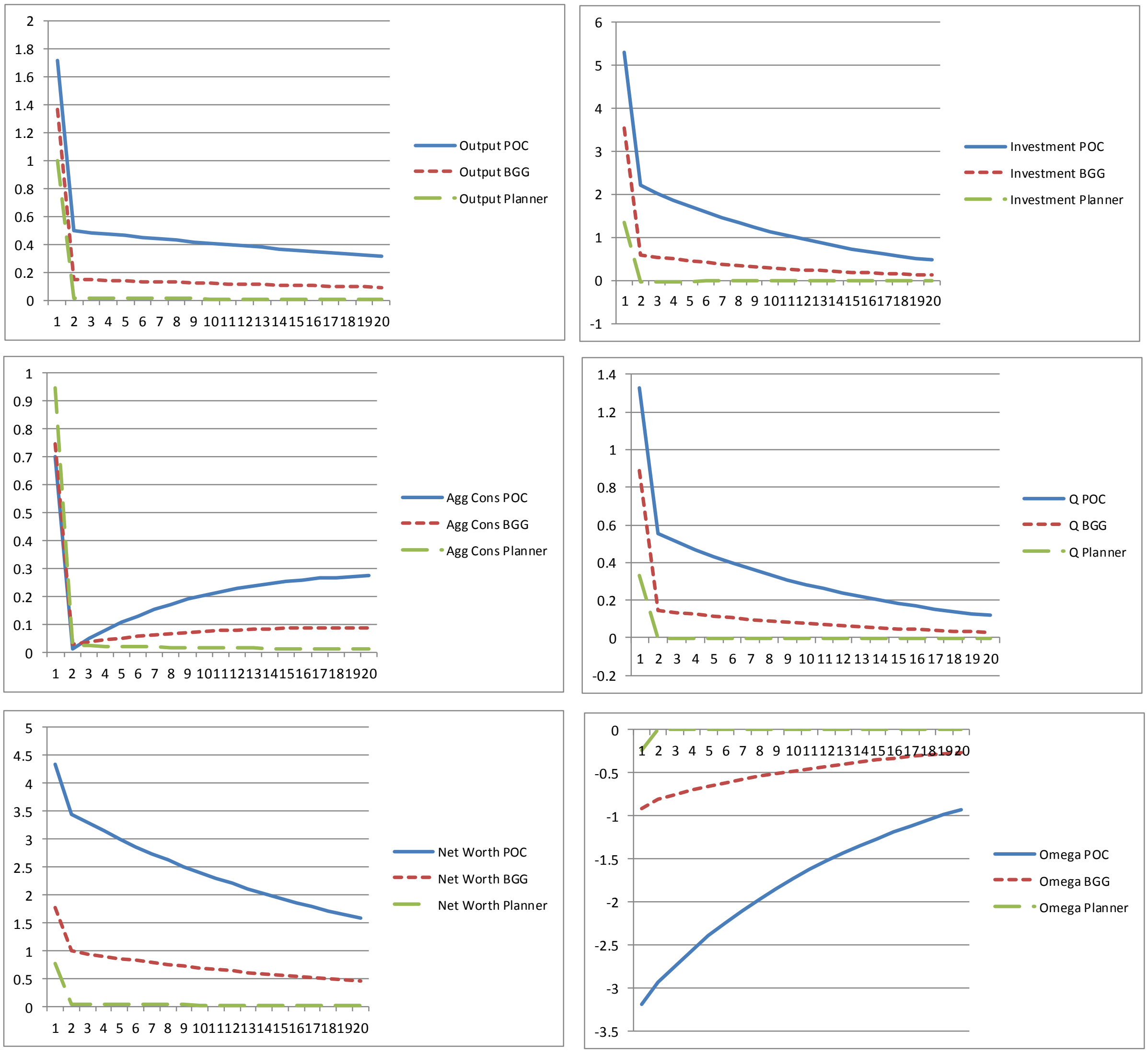
Steady states	**Baseline (ψ = 0.25, ϵ = 1.15)**		**Higher adjustment costs (ψ = 0.75, ϵ = 1.35)**	
	Planner	**POC**	**Planner**	**POC**
Household consumption	0.552	0.553	0.551	0.553
Household labor	0.249	0.249	0.249	0.249
Entrepreneurial consumption	0.070	0.068	0.072	0.068
Output	0.798	0.797	0.799	0.797
Capital stock	6.95	6.95	6.95	6.95
$\boldsymbol{\varpi_{ss}}$	0.471	0.487	0.452	0.487
Std. dev. of household consumption	0.0926	0.0987	0.0899	0.0864
Std. dev. of output	0.1513	0.2321	0.1263	0.2144

Table 2: Welfare Comparison.

Welfare comparison*	Baseline (ψ = 0.25, ϵ = 1.15)		Higher adjustment costs (ψ = 0.75, ϵ = 1.35)	
	Planner-POC	Planner-BGG	Planner-POC	Planner-BGG
Conditional welfare	0.2608	0.0255	0.5655	0.1093
Unconditional welfare	0.4659	0.0552	0.8580	0.1432
Household conditional	0.1347	-0.2114	0.2717	-0.4793
Household unconditional	0.3489	-0.1866	0.6222	-0.4487
Entrepreneur conditional	0.1101	0.2067	0.2174	0.4355
Entrepreneur unconditional	0.1021	0.2110	0.1744	0.4379
Steady state welfare	0.0217	0.0217	0.0964	0.0964

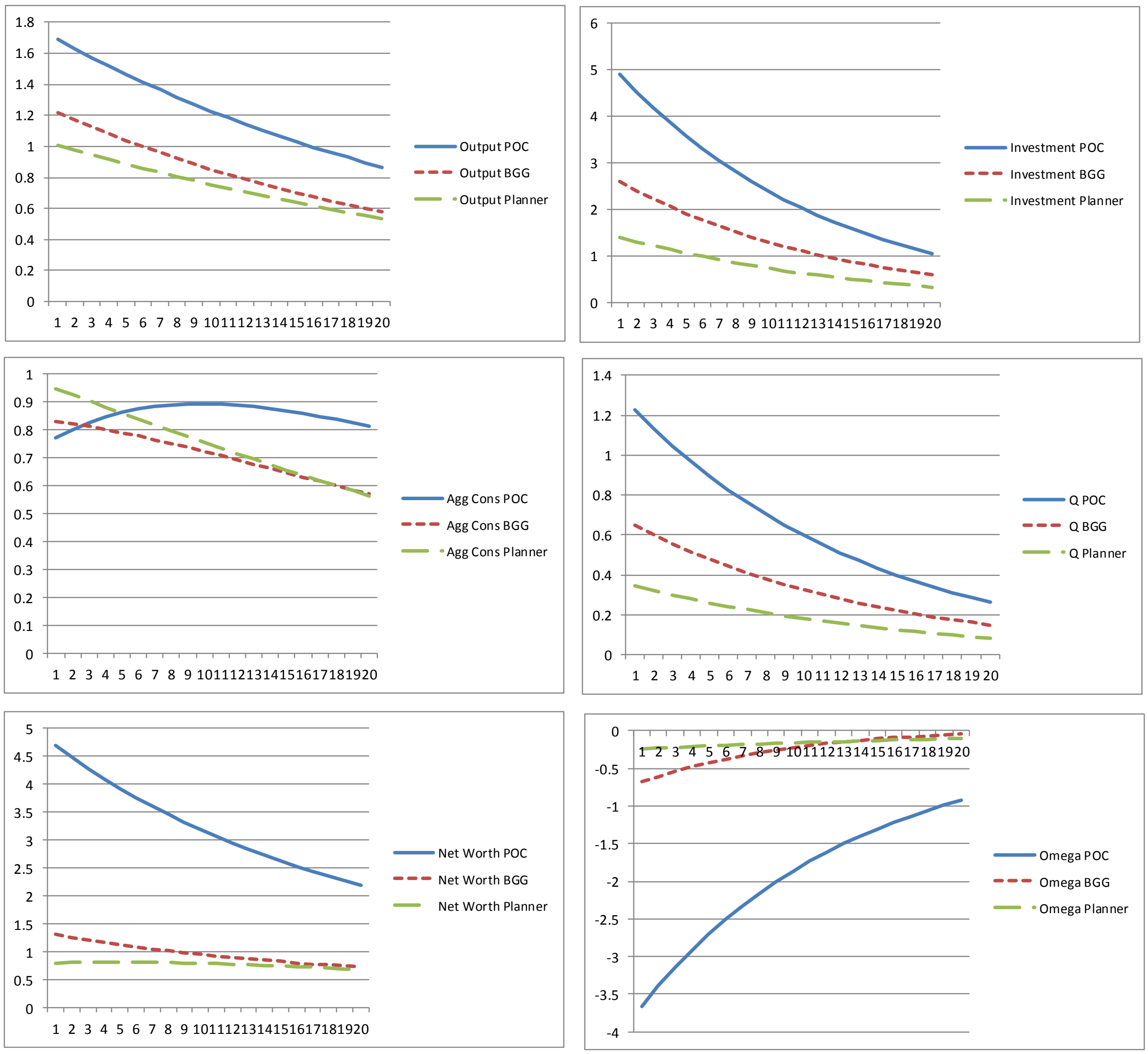
*Entries represent expected lifetime welfare conditional on beginning at the steady state capital stock. Since the calibration is log utility, these numbers represent the perpetual flow increase in household consumption, eg., 0.26 is a perpetual increase in annual household consumption of 0.26%.

Figure 1: iid TFP shock.



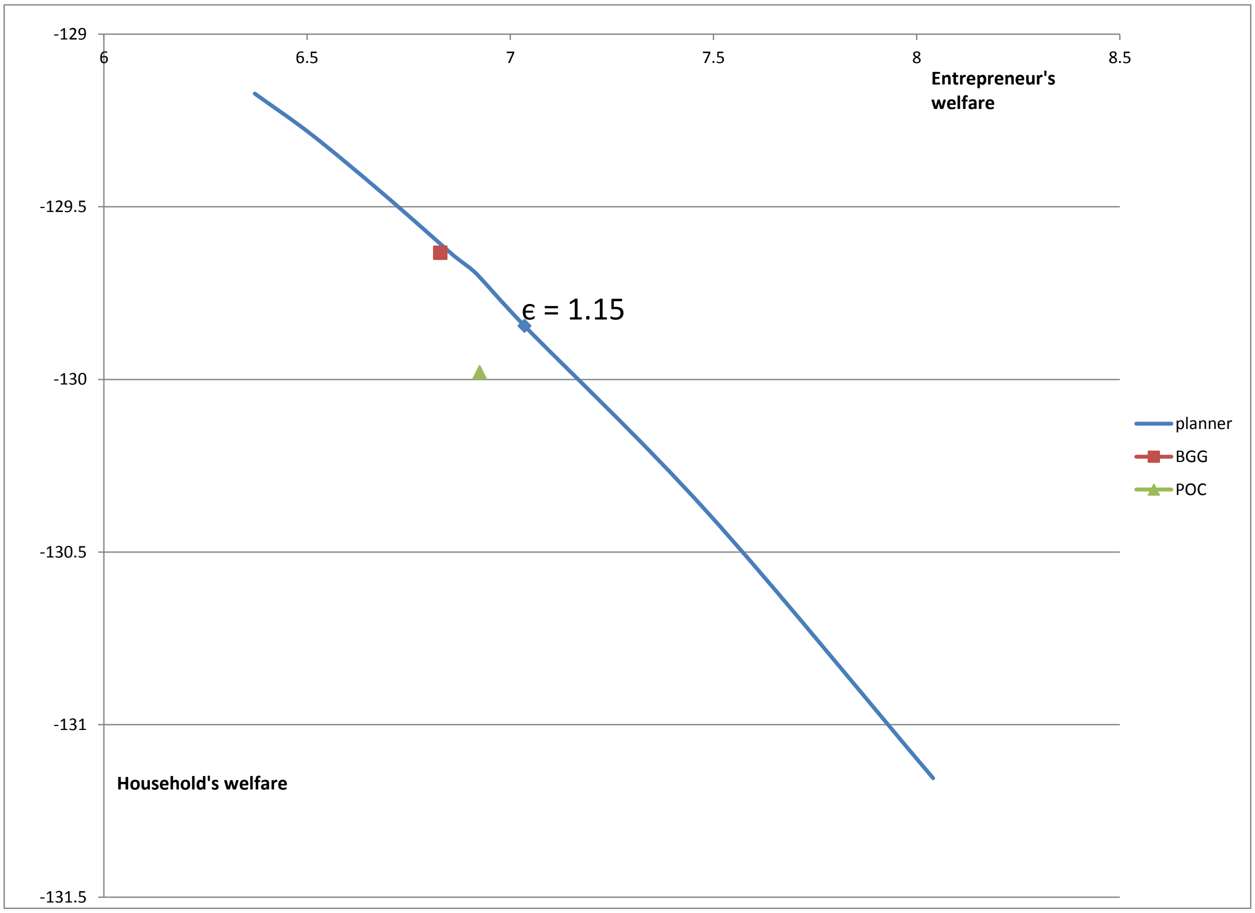
The response of output, investment, aggregate consumption, the price of capital, net worth, and the bankruptcy threshold, to a 1% TFP shock.

Figure 2: auto-correlated TFP shock.



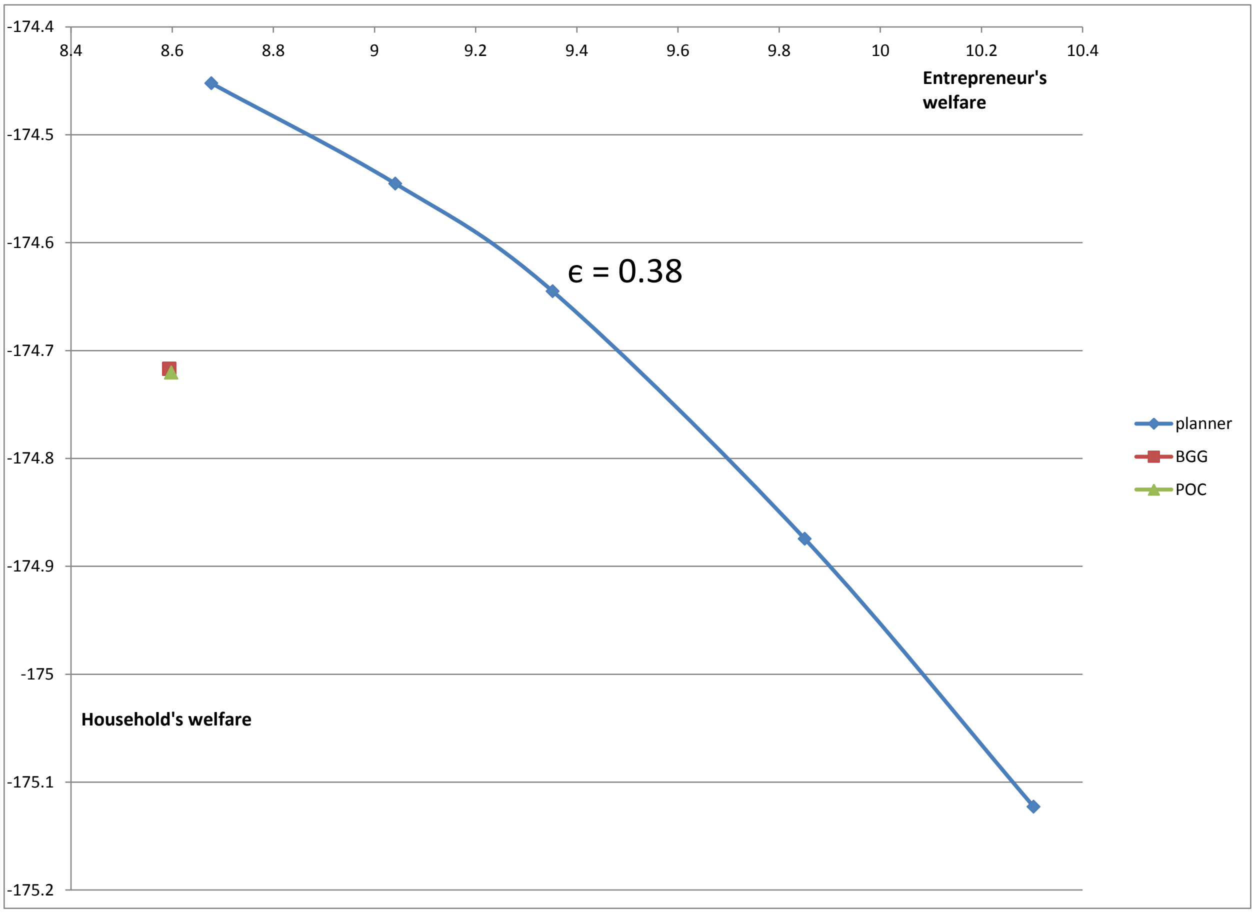
The response of output, investment, aggregate consumption, the price of capital, net worth, and the bankruptcy threshold, to a 1% TFP shock.

Figure 3: Planner welfare frontier.



***Baseline calibration including leverage of 1.954.**

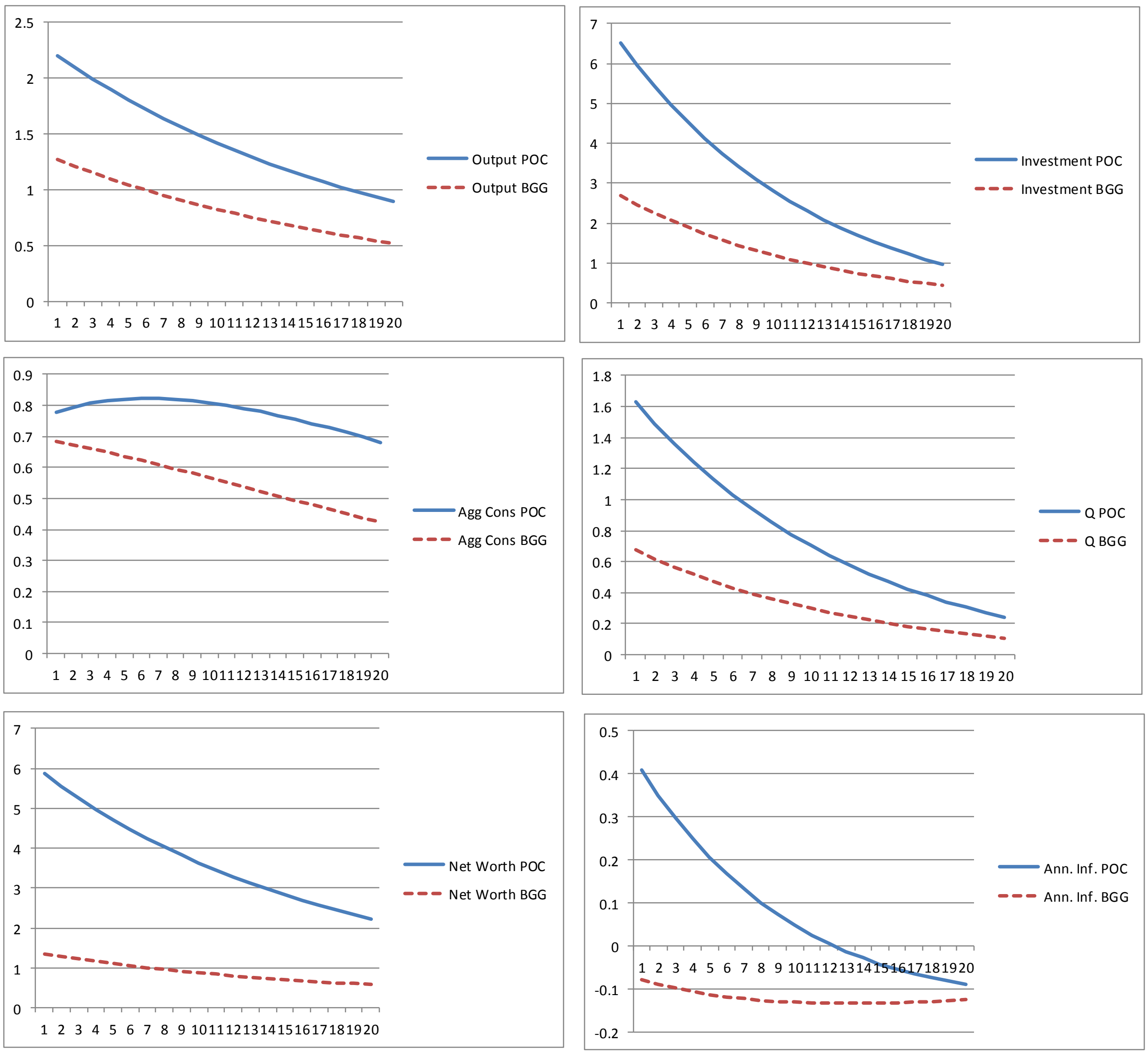
Figure 4: Planner welfare frontier.



***Baseline calibration except entrepreneurial death rate chosen to imply steady state leverage level of 4.**

Figure 5: TFP shock with sticky prices.

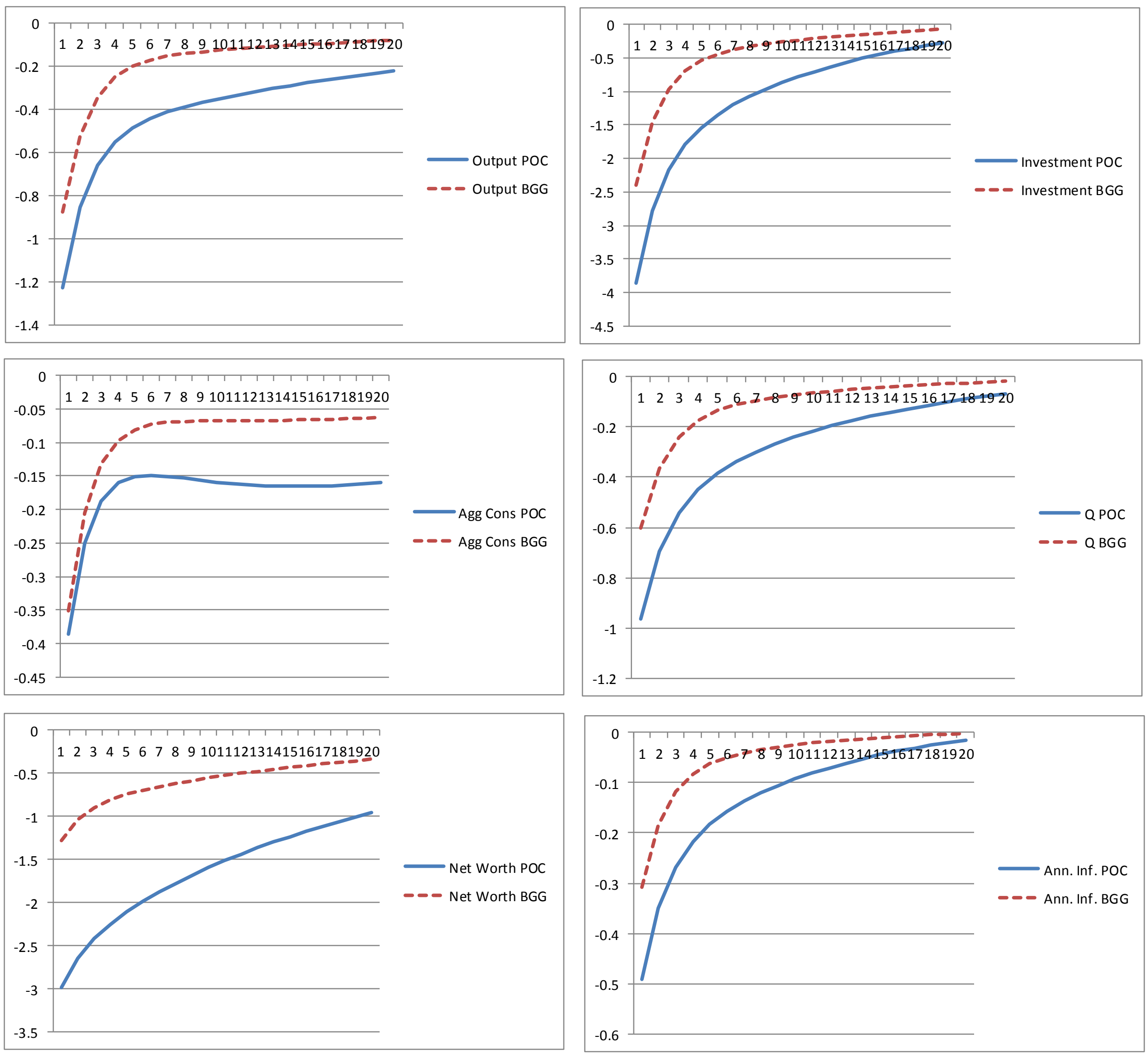
(Impulse response to a 1% TFP shock.)



The response of output, investment, aggregate consumption, the price of capital, net worth, and inflation (annualized) to a 1% TFP shock.

Figure 6: Monetary shock with sticky prices.

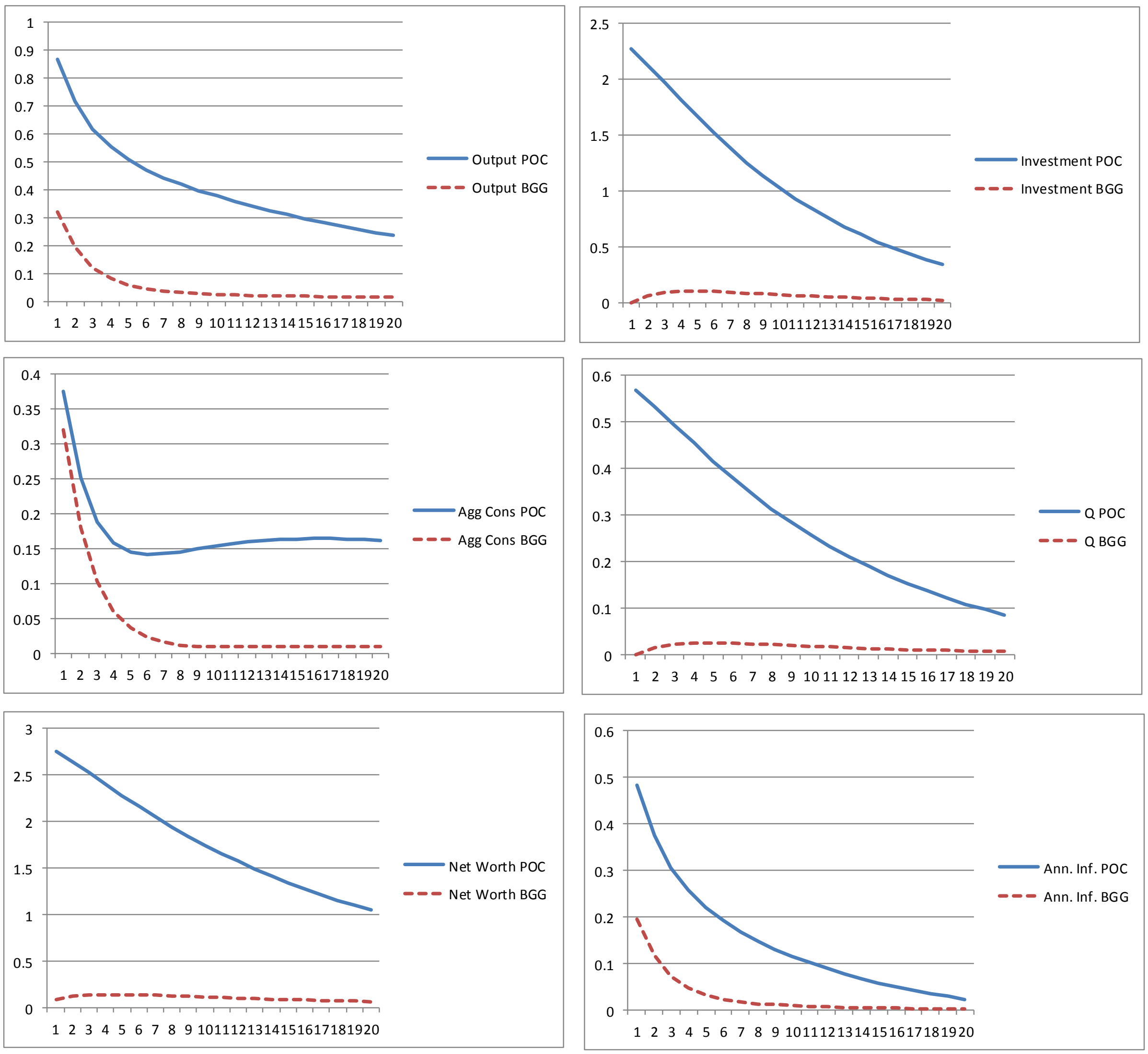
(Impulse response to a 25 bp quarterly policy shock.)



The response of output, investment, aggregate consumption, the price of capital, net worth, and inflation (annualized) to a 25 b.p. (quarterly) monetary policy shock.

Figure 7: Discount rate shock with sticky prices.

(Impulse response to a 25 bp quarterly household discount rate shock.)



The response of output, investment, aggregate consumption, the price of capital, net worth, and inflation (annualized) to a 25 b.p. (quarterly) household discount rate shock.