working paper

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Working Paper 12-38 December 2012

Leverage, Investment, and Optimal Monetary Policy

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We study optimal monetary policy in an economy where firms' debt overhangs lead to under-investment and under-production. The magnitude of this debt-induced distortion varies over the business cycle, rising significantly during recessions. When debt is contracted in nominal terms, this distortion gives rise to a balance sheet channel for monetary policy. In the presence of real and financial shocks, the monetary authority faces a trade-off between inflation and output gap stabilization. The optimal monetary policy rule prescribes that the anticipated component of inflation should be set equal to a target level, while the unanticipated component should rise in response to adverse shocks, smoothing the debt overhang distortion and the output gap.

Keywords: Debt overhang; Balance sheet channel of monetary policy; Fisher debt-deflation.

JEL Classification Number: E32, E52

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1 Introduction

The recent financial crisis in the U.S. and the earlier experience of Japan in the 1990s have highlighted the importance of balance sheet conditions of households, firms, and financial intermediaries for the macroeconomy. In particular, when debt financing and leverage are elevated, the economy becomes exposed to balance sheet shocks and more vulnerable to real shocks. Moreover, once private sector balance sheets have deteriorated, their repair may result in an exceptionally long period of subpar growth, as argued for example by Koo (2003, 2008). All this has renewed the interest among macroeconomists and policy makers in how policy can affect the economy by strengthening the balance sheets of private agents, and how policy should respond to real and financial shocks depending on balance sheet conditions.

Leverage and balance sheets, however, do not play any role in most traditional macroeconomic models. In the standard New Keynesian model, a workhorse for monetary policy analysis, neither the optimal policy response to shocks depends on the conditions of private sector balance sheets, nor monetary policy works through any balance sheet channel. The key transmission mechanism is an interest rate channel: Because of price rigidities, lowering the nominal interest rate temporarily decreases the real rate and stimulates aggregate demand, as the representative agent brings consumption and investment forward.

The recent macroeconomic literature has tried to amend this limitation of traditional models. A common approach, developed by Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Kiyotaky and Moore (1997) and Bernanke, Gertler and Gilchrist (1999), introduces credit constraints that depend on aggregate economic conditions and pro-cyclically affect the entrepreneurs' ability to borrow and invest. This type of friction, however, has proven to have limited amplification power under standard calibration and linearized solution methods, as pointed out by Kocherlakota (2000) and Córdoba and Ripoll (2004), and minor implications for the monetary policy conduct, as shown in Bernanke, Gertler and Gilchrist (1999) and Iacoviello (2005). More recently, the literature has introduced credit constraints faced by the financial sector, as in Gertler and Karadi (2009), Meh and

Moran (2010) and Gertler and Kiyotaki (2010, 2012), shifting its focus on shocks affecting the credit supply of financial intermediaries to explain credit contractions.

Our paper complements the recent literature, using the *debt overhang* framework, as developed by Myers (1977), Lamont (1995) and Occhino and Pescatori (2010), to study the balance sheet channel of monetary policy and its implications for the optimal response to shocks. Our approach does not rely on the presence of borrowing constraints or other forms of credit restrictions. Instead, it studies the implications of the presence of pre-existing debt and leverage for investment and output. With debt overhang, the probability that a firm will default on its debt obligation acts like a tax that discourages its new investment—because, in the event of default, the marginal benefit of that investment will accrue to the creditors, not to the equity holders. This leads to sub-optimal levels of investment and production. Since default risk is strongly counter-cyclical the size of the debt overhang distortion rises significantly during recessions.

Several recent studies, including Hennessy (2004), Moyen (2007) and Chen and Manso (2011), have documented the quantitative importance of this distortion. Hennessy, Levy and Whited (2007), in particular, show that the magnitude of the debt overhang drag on investment is substantial, especially for distressed firms at high risk of default. They find that debt overhang decreases the level of investment by approximately 1 to 2 percent for each percentage point increase in the leverage ratio of long-term debt to assets. Furthermore, as documented by Schularick and Taylor (2012), various measures of credit in advanced economies trended upward relative to GDP in the last 60 years, with a particularly marked increase in the 2000s. While most of this rise was due to financial deepening and deregulation, the more recent surge reflected the large accumulation of debt contracted to finance investment in the boom years. The overhang of this accumulated debt has likely constrained investment and credit in the subsequent bust years.

When pre-existing debt weighs on firms' balance sheets, adverse real and financial shocks raise the debt overhang distortion and widen the output gap. An adverse technology shock, for instance, raises firms' default probability and its associated distortion. This discourages investment and production more than it would be efficient, widening the output gap.

Monetary policy, however, can offset these distortive effects via a balance sheet channel. Suppose that debt is contracted in nominal terms, while assets are real. Then, by generating surprise inflation, the monetary authority can lower the real value of firms' liabilities, redistribute from creditors to debtors and strengthen the firms' balance sheets. This reduces the debt-overhang distortion and limits the widening of the output gap. In our context, we found this mechanism quantitatively important: For a plausible parametrization, an increase of 1 percent in unanticipated inflation narrows the output gap by one third of a percentage point.

More importantly, monetary policy *should* partially offset the distortive effects of adverse shocks. If monetary policy aims at minimizing the gap between efficient and actual output as well as at stabilizing inflation around a target level, the optimal monetary policy rule has two main features. First, the anticipated component of inflation should be set equal to the target. Raising further the overall level of inflation would not have any effect on the real level of debt and on the debt overhang distortion, and would only worsen the inflation part of the objective function. Second, the unanticipated component should systematically rise in response to adverse shocks, both real and financial, to partially offset their distortive effects and smooth the debt overhang distortion and the output gap across different states. Symmetrically, unanticipated inflation should decline when the economy is hit by favorable shocks. Without debt overhang, inflation would be perfectly stabilized. In the presence of debt overhang, however, the monetary authority should adjust the inflation rate to smooth the default probability, which is the right indicator for the size of the debt overhang distortion.

In the balance of the paper, Section 2 describes the debt overhang model, and studies the effect of shocks on the distortion and the balance sheet monetary channel; Section 3 characterizes the prominent features of an optimal monetary policy rule in a debt overhang distorted economy; Section 4 concludes.

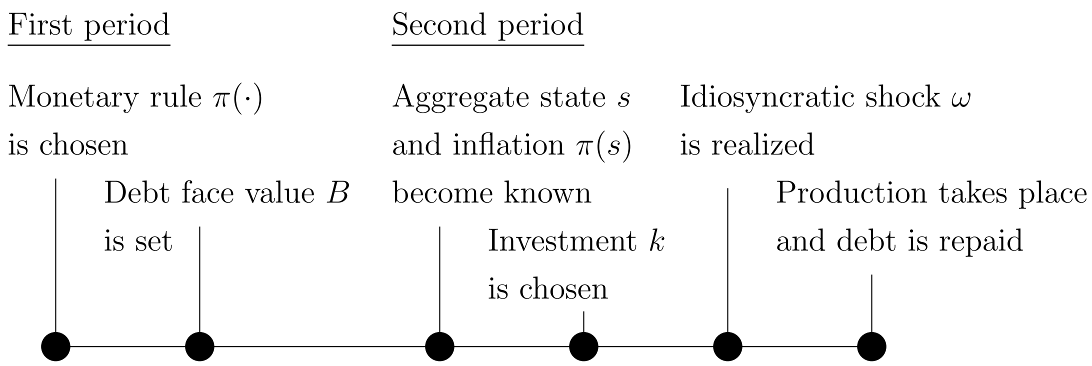

Figure 1: Timing of the model.

2 Model

We illustrate the interaction between firms' debt overhangs and monetary policy with a two-period model, outlined in Figure 1, featuring a monetary authority and a continuum of firms. The first-period price level P_0 is pre-determined. Let s be the aggregate state, which is realized at the beginning of the second period.

At the beginning of the first period, the monetary authority chooses a monetary policy rule that sets the second-period price level P contingent on the realization of the second-period aggregate state s. Equivalently, after letting $\pi \equiv \ln(P/P_0)$ be the inflation rate between the first and second period, the rule sets the inflation rate π as a function of s.[1]

After the monetary rule has been announced, firms issue nominal debt with face value B, payable at the end of the second period. Let N be the amount of funds borrowed in the first period, and let $i \equiv \ln(B/N)$ be the debt nominal yield. We assume that the demand for real funds by firms, N/P_0, is inversely related to

[1]We assume that the monetary authority controls the inflation rate directly, disregarding the issue of what is the monetary policy instrument and how the equilibrium is implemented. However, it is possible to provide examples where the monetary authority controls the money growth rate, while the inflation rate is determined endogenously, and the same equilibrium arises. We study one such example at the end of Section 3.

the real yield $r \equiv i - E_0\pi$:

$$N/P_0 = \bar{b}e^{-r},$$

where E_0 is the expectation conditional on information available in the first period and $\bar{b} > 0$ is a positive constant.[2] Substituting the expressions for r and i, we obtain:

$$\begin{aligned} N/P_0 &= \bar{b}e^{-(i-E_0\pi)} \\ N/P_0 &= \bar{b}e^{-\ln(B/N)+E_0\pi} \\ B &= \bar{b}P_0e^{E_0\pi} \\ B &= \bar{b}e^{E_0\ln(P)} \end{aligned}$$

so the nominal debt face value, B, fully incorporates anticipated inflation, increasing one-to-one with the expectations of the second-period price level and inflation rate. Stated differently, the expected log-real face value of debt is equal to a constant independent of monetary policy: $E_0 \ln(B/P) = \ln(\bar{b})$.

At the beginning of the second period, the aggregate state s is revealed. The state $s \equiv (\theta, \sigma)$ includes a standard technology shock θ and a shock to the volatility σ of the firms' idiosyncratic productivity. This latter shock is analogous to the risk shock of Christiano, Motto and Rostagno (2007), and can be interpreted as a financial shock since its direct initial impact is on firms' probability of default. The price level $P(s)$ and the inflation rate $\pi(s)$ become known as well, as prescribed by the monetary rule. Notice that, since the debt nominal face value B is given in the second period, the debt real face value

$$b \equiv B/P = \bar{b}e^{-(\pi-E_0\pi)} \tag{1}$$

[2]The corporate finance literature has identified several reasons why firms borrow, including to exploit the interest tax deductibility and to minimize several agency distortions associated with asymmetric information and moral hazard (see Myers 2001 for a review). With asymmetric information and costly state verification, for instance, risky debt has been shown to be the constrained-optimal contract (see Townsend 1979 and Gale and Hellwig 1985). More relevantly for our paper, in a moral hazard setting with hidden managerial effort, risky debt is the constrained-optimal contract that minimizes the debt overhang distortion, as shown by Innes (1990).

decreases with the realizations of the price level P and of the inflation rate π.

After the aggregate state has been revealed, firms choose their investment level k. Then, a shock ω to the firms' idiosyncratic productivity is realized and firms produce $y \equiv \omega\theta f(k)$, where $f(k) \equiv Ak^{\alpha}$ is the production function, with $A > 0$ and $\alpha \in (0, 1)$. After that, the debt becomes payable. However, firms may default on their debt. We assume that a firm's liability is limited by the nominal value of its second-period production, Py, so the nominal value of what it repays is $\min\{Py, B\}$, and the real value is $\min\{Py, B\}/P = \min\{y, b\}$.

To summarize, the crucial events in the model occur in this sequence: in the first period, the monetary authority chooses a rule for the inflation rate $\pi(\cdot)$ and, then, firms issue risky debt with nominal face value B; in the second period, the aggregate state s and the corresponding inflation rate $\pi(s)$ become known, and, then, firms make their investment decision, k, and produce output, y.

The debt overhang distortion

To study the model, we proceed backward starting with the firm's investment decision in the second period. At this stage, the debt nominal face value B, the aggregate state $s \equiv (\theta, \sigma)$, the price level $P(s)$ and the inflation rate $\pi(s)$ are all known. Each firm chooses investment k to maximize the expected firm's value, which is equal to the expected profits minus the expected debt payoff, all in real terms:

$$\max_{k}\{E\{y - \min\{y, b\}\} - k\}$$

where $y \equiv \omega\theta f(k)$, and E is the expectation given information available in the second period, after the aggregate state s is revealed but before the realization of the idiosyncratic shock ω.

The firm's first-order condition is[3]

$$\frac{\partial E\{y\}}{\partial k} - \frac{\partial E\{\min\{y, b\}\}}{\partial k} = 1.$$

The second term on the left hand side represents the distortion induced by debt financing. It is the sensitivity of the expected debt payoff $E\{\min\{y, b\}\}$ to the investment level k. The greater this term, the lower the firm's investment choice k. If debt is risk-free, the expected debt payoff is constant and, thus, there is no distortion. If there is a risk of default, however, the expected debt payoff $E\{\min\{y, b\}\}$ is positively affected by the firm's investment, inducing a conflict with the creditors and leading the firm to under-invest. For example, consider the firm's marginal decision to invest one extra-unit of resources. This unit is expected to increase the revenue by the marginal expected product $\partial E\{y\}/\partial k$. However, this unit will also increase the expected debt repayments by the firm, since the benefit of this investment will be reaped by the creditors in the case of default.

In what follows we will assume that ω is log-normal$(\mu - \sigma^2/2, \sigma)$, such that $\ln(E\{\omega\}) = \mu$. In this case, the following well-known analytical result holding for log-normal variables becomes helpful:

$$\begin{aligned} \frac{\partial E\{\min\{y, b\}\}}{\partial k} &= \frac{\partial E\{y\}}{\partial k}[1 - \Phi(\nu)], \\ \nu &= \frac{\ln(E\{y\}) - \ln(b)}{\sigma} + \frac{1}{2}\sigma \end{aligned} \tag{2}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal random variable. It helps the intuition to interpret ν as the normalized distance between $E\{y\}$ and b, i.e. the distance to default; $\Phi(\nu)$ as the adjusted probability of full debt repayment, $y \geq b$; and $1 - \Phi(\nu)$ as the probability that the firm defaults on its debt.

[3] The following second-order condition must hold as well:

$$\frac{\partial^2 E\{y\}}{\partial k^2} - \frac{\partial^2 E\{\min\{y, b\}\}}{\partial k^2} < 0$$

Then, the firm's first-order condition becomes[4]

$$\frac{\partial E\{y\}}{\partial k}\Phi(\nu) = 1$$
$$E\{\omega\}\theta A\alpha k^{\alpha-1}\Phi(\nu) = 1$$
$$\ln(k) = \frac{\ln(E\{\omega\}\theta A\alpha\Phi(\nu))}{1-\alpha} \tag{3}$$

Notice that the default probability, $1 - \Phi(\nu)$, acts like a tax on investment in the first-order condition, and is the right indicator for the size of the debt overhang distortion.

Let k^* and y^* be the efficient levels of investment and output, implicitly defined by the previous first-order condition with $\Phi(\nu)$ set equal to 1. We can easily derive the investment gap:

$$\ln(k) - \ln(k^*) = \frac{1}{1-\alpha}\ln(\Phi(\nu))$$

and the output gap $x \equiv \ln(y) - \ln(y^*)$:

$$x = \alpha(\ln(k) - \ln(k^*)) = \frac{\alpha}{1-\alpha}\ln(\Phi(\nu)). \tag{4}$$

Notice that both gaps are negative, and they decrease as the default probability, $1 - \Phi(\nu)$, increases. The debt overhang distortion, measured by the default probability, depresses actual investment and output below their efficient levels, and leads to negative investment and output gaps.

[4] The second-order condition becomes

$$E\{\omega\}\theta f''(k)\Phi(\nu) + E\{\omega\}\theta f'(k)\Phi'(\nu)\frac{1}{\sigma}\frac{f'(k)}{f(k)} < 0$$
$$(\alpha - 1) + \frac{\Phi'(\nu)}{\Phi(\nu)}\alpha\frac{1}{\sigma} < 0$$
$$\frac{1}{\sigma}\frac{\alpha}{1-\alpha}\frac{\Phi'(\nu)}{\Phi(\nu)} < 1$$

This condition implicitly constrains the repayment probability Φ. For instance, under our preferred parametrization, i.e. $\alpha = 1/3$ and $\sigma = 0.1$, the previous inequality holds for $\Phi > 89.7\%$, i.e. for default risks up to 10.3%. Higher values of σ relax the constraint: for instance, if $\alpha = 1/3$ and $\sigma = 1$, then the previous inequality holds for $\Phi > 5.8\%$.

The balance sheet channel of monetary policy

We study the mechanics of the model by first looking at the effects of aggregate variables on the distance to default ν. The equilibrium of the model is described by the system of equations (1) through (4). As shown in Appendix A, the system can be reduced to the following equation in the unknown ν:

$$\nu = \frac{1}{\sigma}\left\{\frac{\ln(E\{\omega\}\theta A)}{1-\alpha} + \alpha\frac{\ln(\alpha\Phi(\nu))}{1-\alpha} - \ln(\bar{b}) + (\pi - E_0\pi)\right\} + \frac{1}{2}\sigma. \tag{5}$$

Equations (4) and (5) together fully characterize the equilibrium of the economy. By linearizing these two equations, we can derive the sensitivity of the output gap, x, to changes of technology, θ, inflation surprise, $\pi - E_0\pi$, and risk, σ:

$$\hat{x} = \frac{\bar{\chi}}{1-\bar{\chi}}\left\{\frac{1}{1-\alpha}\tilde{\theta} + (\pi - E_0\pi) - (\bar{\nu} - \bar{\sigma})\hat{\sigma}\right\} \tag{6}$$

where $\bar{\chi} \equiv \frac{1}{\bar{\sigma}}\frac{\alpha}{1-\alpha}\frac{\Phi'(\bar{\nu})}{\Phi(\bar{\nu})} \in (0,1)$; bars over variables denote steady state values derived in the absence of aggregate shocks; while hats and tildes over variables denote, respectively, deviations and log-deviations from steady state values. (See Appendix A for details of the derivation.)

Holding the inflation rate constant, an adverse technology shock widens the output gap. This occurs because the shock lowers expected output, which raises the default probability and, thus, the debt overhang distortion—the shock lowers actual output by more than it would be efficient. Holding inflation constant, then, technology shocks move output and the output gap in the same direction, unlike in the standard New Keynesian model where actual output and efficient output decrease by the same amount, and the output gap remains unchanged. An adverse risk shock, i.e. a shock that raises σ, widens the output gap as well, by directly increasing the default probability.

As to the balance sheet channel of the monetary policy, the first thing to notice is that anticipated inflation does not have any real effects. Inflation that is anticipated at the time when debt is issued is fully incorporated in the debt contract and does not have any effect on the debt overhang distortion and investment. If the scale of the price level doubles, the nominal face value of debt doubles and the debt overhang distortion is unaffected. Hence, price level policies that differ by a scale factor implement the same real allocation. Equivalently, all policies for inflation that differ by a constant implement the same real allocation.

Deviations of inflation from its anticipated value, however, do have real effects. Inflation that is not anticipated when debt is issued, but is known before the investment decision, lowers the real face value of firms' liabilities, strengthens the firms' balance sheets, decreases the default probability and the debt overhang distortion, and encourages investment and output. Since there is no effect on efficient output, the output gap narrows. The debt overhang distortion, then, generates an expectations-augmented aggregate supply curve, equation (6), with a trade-off between unanticipated inflation and the output gap.[5] This mechanism can be easily incorporated into a business cycle model with debt overhang, like the one of Occhino and Pescatori (2010), similarly to the way Christiano, Motto and Rostagno (2007) introduce the "Fisher debt-deflation channel" into a financial accelerator model.

Rearranging equation (6), we obtain an expectations-augmented Phillips curve:

$$\pi = E_0\pi + \kappa\hat{x} + u, \tag{7}$$

where $\kappa \equiv 1/\bar{\chi} - 1$ and $u \equiv -\tilde{\theta}/(1-\alpha) + (\bar{\nu} - \bar{\sigma})\hat{\sigma}$ is a term that collects exogenous disturbances. Notice that both technology and risk shocks shift this curve, similarly to traditional cost-push shocks, presenting the monetary authority with a trade-off between inflation stabilization and output gap stabilization: In response to an adverse shock, the monetary authority can either perfectly stabilize the inflation rate or the output gap, but not both. This contrasts with the standard version of the New Keynesian model, exemplified by Clarida, Galí and Gertler (1999). In that framework, technology shocks and other shocks (with the exception of cost push shocks) do not shift the Phillips curve and do not force a trade-off, so stabilizing inflation is equivalent to stabilizing the output gap. In particular, in response

[5] Incidentally, notice that any inflation that is not anticipated even at the time when investment is chosen does not have any effect on the investment decision and on the real allocation. To have real effects, inflation must be unanticipated when debt is issued but anticipated when investment is chosen.

to technology shocks, the monetary authority can keep the output gap perfectly closed by fully stabilizing inflation. As pointed out by Blanchard and Galí (2007), this property of the standard New Keynesian framework, which they label *divine coincidence*, contrasts with the widespread consensus that monetary authorities face a trade-off between stabilizing inflation and the output gap and that perfect inflation stabilization is not optimal. These more realistic positive and normative implications arise naturally in our debt overhang framework.

For a quantitative evaluation, we adopt the following parametrization: $\alpha = 1/3$, $\bar{\sigma} = 0.1$ and $\Phi(\bar{\nu}) = 0.98$ in the steady state without aggregate shocks. The value for the power coefficient of capital in the production function, α, is standard. With this value, a 1 percent increase of the default probability corresponds, approximately, to a 0.5 percent decline in the output gap, as implied by equation (4). The value for $\bar{\sigma}$ implies that the volatility of idiosyncratic productivity is 10 percent, well within the range of plausible values. The value for $\Phi(\bar{\nu})$ implies a 2 percent default probability, which matches the average annual default rate for All Corporates from Moody's.

The previous calibrated values imply $\bar{\nu} = 2.05$, $\Phi'(\bar{\nu}) = 0.05$, $\bar{\chi} = 0.25$, and $\frac{\bar{\chi}}{1-\bar{\chi}} = 0.33$. Then, equation (6) states that: a 1 percent increase in surprise inflation decreases the distortion, and narrows the output gap by 0.33 percentage points; a 1 percent decrease in the technology shock increases the distortion and widens the output gap by 0.5 percentage points; and an increase in the risk shock of 1 percentage point increases the distortion and widens the output gap by 0.65 percentage points.

The top 4 panels of Figure 2 show the effects of shocks in the non-linear model. The dashed lines in the top 4 panels plot the response of the default probability and the output gap to the technology shock and the risk shock, for constant inflation rate. The figure points to an important asymmetry: The effects of adverse shocks are larger than those of favorable shocks, so the mechanisms that we describe tend to be stronger during recessions. Approximately, the size of the effect of an adverse shock is 50 percent larger than the size of the effect of a favorable shock.

This asymmetry becomes more evident as ampler shocks are considered.

3 Optimal monetary policy rule

In this section, we characterize the main features of an optimal monetary policy rule. We assume that the monetary authority aims at minimizing the gap between efficient and actual output as well as at stabilizing inflation around a target level, say $\overline{\pi}$. More specifically, it aims at minimizing the expected weighted average of the squared output gap, x^2, which is strictly decreasing in x since x is negative, and of the squared deviation of the inflation rate from target:

$$\frac{1}{2}E_0\{\gamma x^2 + (1-\gamma)(\pi - \overline{\pi})^2\}, \tag{8}$$

where $\gamma \in [0, 1]$.

The monetary authority chooses a monetary policy rule, i.e. a function that expresses the price level P, or equivalently the inflation rate π, as a function of s. The equilibrium of the economy is fully characterized by equations (4) and (5). The optimal monetary policy problem, then, is

$$\min_{\pi(s),x} \text{(8) subject to: (4) and (5).} \tag{9}$$

Appendix B shows that the solution is:

$$\gamma \left\{ x\frac{\chi}{1-\chi} - E_0\left\{ x\frac{\chi}{1-\chi} \right\} \right\} + (1-\gamma)(\pi - \overline{\pi}) = 0. \tag{10}$$

where $\chi \equiv \frac{1}{\sigma}\frac{\alpha}{1-\alpha}\frac{\Phi'(\nu)}{\Phi(\nu)}$. For interpretation, it helps recalling that the output gap, x, is a decreasing function of the default probability, $1 - \Phi(\nu)$.

First, notice that, as long as inflation stabilization appears in the objective function, i.e. $\gamma < 1$, the anticipated inflation component should be set equal to the target level: $E_0\pi = \overline{\pi}$. This can be easily derived taking the expectations of both sides of equation (10). When setting the monetary policy rule, the monetary authority recognizes that the face value of debt B depends on the inflation rate that agents anticipate at the time the debt is issued, so it depends on the monetary policy rule itself. Raising the anticipated component of inflation would not have any effect on the real allocation, and would only worsen the inflation part of the objective function, so optimal policy calls for setting average inflation equal to target.[6]

As to the unanticipated component of inflation, it is helpful to begin with the two extreme cases, $\gamma = 0$ and $\gamma = 1$, respectively.

When the monetary authority focuses on inflation stabilization only ($\gamma = 0$), optimal policy obviously calls for perfectly stabilizing of the inflation rate at the target inflation rate: $\pi = \overline{\pi}$ for all states. This case is depicted with the dashed lines in the top 6 panels of Figure 2.

When the monetary authority focuses on output gap minimization only ($\gamma = 1$), optimal policy calls for smoothing the debt overhang distortion and the output gap across states.[7] If σ is constant, so there are no risk shocks, the distance to default, ν, the default probability, $1 - \Phi(\nu)$, and the output gap, x, should be constant across states. Interestingly, this can be achieved with a policy aimed at fully stabilizing nominal income, $Y \equiv PE\{y\}$. It is easy to check that this implies a constant distance to default across states, and so constant default probabilities and constant output gap. The optimality of a nominal income targeting policy follows intuitively from the fact that firms' debt is set in nominal terms while firms' assets depend on the nominal value of their output. If σ is not constant, the default probability and the output gap do vary with σ, but their variability is small. This case is depicted with the solid lines in the top 6 panels of Figure 2.

[6]As in other models with commitment, the monetary authority has an incentive to deviate and to deny its own commitment after the agents have made their expectations-based choice. After the debt is issued and the face value B is set, the monetary authority has an incentive to raise the price level P in order to decrease the debt overhang distortion and to encourage investment. In an environment without commitment, this would induce an inflationary bias. Here, however, we focus on the case of commitment.

[7]Notice that, when $\gamma = 1$ and inflation does not appear in the monetary authority's objective function, the optimal policy for inflation is determined only up to a constant, since, as we saw in Section 2, all policies for inflation that differ by a constant implement the same real allocation. When $\gamma < 1$, however, the monetary authority aims at inflation stabilization as well, and the optimal policy for inflation is unique.

In the intermediate case, $\gamma \in (0, 1)$, optimal policy strikes a balance between inflation stabilization and smoothing the debt overhang distortion and the output gap. The specific state-contingent rule depends on the specific value for γ in the objective function of the monetary authority and its relative aversion toward inflation variability and output gap. The optimal response to shocks is intermediate between the two extreme cases considered above, $\gamma = 0$ and $\gamma = 1$, as can be seen from the dotted lines in the top 6 panels of Figure 2, which refer to the case $\gamma = 0.8$.

The unanticipated component of inflation should respond to shocks, both real and financial, partially offsetting their effects on the output gap. Unanticipated inflation should rise in response to adverse shocks that raise the default probability and widen the output gap, and should decline in response to favorable shocks. These prescriptions, which are qualitatively similar to the ones obtained by De Fiore, Teles and Tristani (2011) in a model with financial frictions, contrast with the ones of the standard New Keynesian model that prescribes inflation stabilization in response to technology shocks and is silent about the response to financial shocks. Also, adverse real and financial shocks raise the default probability while lowering output. Hence, in equilibrium, the inflation rate, the default probability and the debt overhang distortion are counter-cyclical, rising during recessions, while the output gap is pro-cyclical.

In the two panels of Figure 3, we show the trade-off that the monetary authority faces when choosing its rule. The left panel shows the volatility of inflation and output gap when the economy is subject to technology shocks only and the inflation rate responds optimally, for all possible different values of γ. The right panel does the same for the case when the economy is subject to risk shocks only. The figure shows that, approximately, the output gap volatility can be reduced by 1 percent by accepting a larger inflation volatility by 3 percent.

Money growth rate

In our analysis, we have assumed that the monetary authority controls the inflation rate directly. Here, we show that the same equilibrium arises in an example where the monetary authority controls the money growth rate, while the inflation rate is determined endogenously. Suppose that the money demand is proportional to nominal output. Then, the money growth rate is equal to the inflation rate plus the output growth rate. Rather than setting a rule in terms of the inflation rate, the monetary authority can set a rule for the corresponding money growth rate. One equilibrium associated with the rule expressed in terms of the money growth rate is the same as the one associated with the rule expressed in terms of the inflation rate.

The bottom two panels of Figure 2 show the response of the money growth rate, after setting the output growth rate equal to zero in the absence of shocks. In the case of technology shocks only, a constant money growth rate rule implies that the inflation rate rises in response to adverse shocks and declines in response to favorable shocks in such a way that the nominal income Y, the distance to default ν, the default probability $1 - \Phi(\nu)$ and the output gap x are all constant across states. This rule is clearly optimal if the monetary authority is concerned of output gap minimization only. If the monetary authority aims at inflation stabilization as well, it should lower the money growth rate in response to an adverse shock, to limit the rise of the inflation rate. In the case of risk shocks only, however, a constant money growth rate rule implies that the inflation rate rises and the output gap declines in response to an adverse shock. The monetary authority, then, faces a trade-off: it should raise the money growth rate in order to smooth the output gap, but it should lower it in order to stabilize inflation. The optimal policy depends on the weight on these two objectives.

4 Conclusion

In this paper, we have studied how monetary policy should respond to real and financial shocks in an environment where leverage distorts firms' investment decisions. This debt-induced distortion is counter-cyclical, rising in response to adverse shocks, and asymmetric, with much larger effects during recessions. It gives rise to a balance sheet channel for monetary policy, working through inflation that is unanticipated when debt is issued, while anticipated when investment is chosen.

In the presence of real and financial shocks, the monetary authority faces a trade-off between inflation stabilization and output gap stabilization. The optimal monetary policy rule prescribes that the anticipated component of inflation should be set equal to the target level, while the unanticipated component should be contingent on the shocks, rising in response of adverse real and financial shocks, with the aim of smoothing the probability of default, the debt overhang distortion and the output gap across different states. A policy targeting nominal income comes close to perfectly stabilizing the default probability and the output gap. The main features of our monetary policy rule, especially monetary easing in response to adverse real and financial shocks that raise the probability of default, are likely to be valid more generally in models where leverage and balance sheet conditions are important for the macroeconomic outcome.

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A Derivation of equations (5) and (6)

This appendix derives equations (5) and (6) from the system of equations (1) through (4), which describes the equilibrium of the model.

Using the definition $y \equiv \omega\theta A k^{\alpha}$, and the expressions for k and b from equations (1) and (3), equation (2) becomes:

$$\nu = \frac{\ln(E\{\omega\}\theta A k^{\alpha}) - \ln(b)}{\sigma} + \frac{1}{2}\sigma$$
$$\nu = \frac{1}{\sigma}\left\{\ln(E\{\omega\}\theta A) + \alpha\frac{\ln(E\{\omega\}\theta A\alpha\Phi(\nu))}{1-\alpha} - \ln\left(\bar{b}e^{-(\pi - E_0\pi)}\right)\right\} + \frac{1}{2}\sigma$$
$$\nu = \frac{1}{\sigma}\left\{\frac{\ln(E\{\omega\}\theta A)}{1-\alpha} + \alpha\frac{\ln(\alpha\Phi(\nu))}{1-\alpha} - \ln(\bar{b}) + (\pi - E_0\pi)\right\} + \frac{1}{2}\sigma$$

which is equation (5).

Taking a first-order Taylor expansion of equation (5) with respect to ν, $\ln(\theta)$, $(\pi - E_0\pi)$ and σ, we obtain:

$$\hat{\nu} = \frac{1}{\bar{\sigma}}\left\{\frac{1}{1-\alpha}\tilde{\theta} + \frac{\alpha}{1-\alpha}\frac{\Phi'(\bar{\nu})}{\Phi(\bar{\nu})}\hat{\nu} + (\pi - E_0\pi) - (\bar{\nu} - \bar{\sigma})\tilde{\sigma}\right\}$$
$$\hat{\nu} = \frac{1}{1-\bar{\chi}}\frac{1}{\bar{\sigma}}\left\{\frac{1}{1-\alpha}\tilde{\theta} + (\pi - E_0\pi) - (\bar{\nu} - \bar{\sigma})\hat{\sigma}\right\}$$

where $\bar{\chi} \equiv \frac{1}{\bar{\sigma}}\frac{\alpha}{1-\alpha}\frac{\Phi'(\bar{\nu})}{\Phi(\bar{\nu})} \in (0,1)$; bars over variables denote steady state values derived in the absence of aggregate shocks; while hats and tildes over variables denote, respectively, deviations and log-deviations from steady state values.[8]

Finally, taking a first-order Taylor expansion of equation (4), and using the previous result, we obtain:

$$\hat{x} = \frac{\alpha}{1-\alpha}\frac{\Phi'(\bar{\nu})}{\Phi(\bar{\nu})}\hat{\nu}$$
$$\hat{x} = \frac{\alpha}{1-\alpha}\frac{\Phi'(\bar{\nu})}{\Phi(\bar{\nu})}\frac{1}{1-\bar{\chi}}\frac{1}{\bar{\sigma}}\left\{\frac{1}{1-\alpha}\tilde{\theta} + (\pi - E_0\pi) - (\bar{\nu} - \bar{\sigma})\hat{\sigma}\right\}$$
$$\hat{x} = \frac{\bar{\chi}}{1-\bar{\chi}}\left\{\frac{1}{1-\alpha}\tilde{\theta} + (\pi - E_0\pi) - (\bar{\nu} - \bar{\sigma})\hat{\sigma}\right\}$$

which is equation (6).

[8]Notice that $\bar{\chi} < 1$ follows from the second-order condition in footnote 4.

B Solution of the optimal policy problem (9)

This appendix solves the optimal monetary policy problem (9).

Notice that monetary policy rules that differ by a scale factor ($P' \equiv \kappa P$, or equivalently $\pi' \equiv \ln(\kappa) + \pi$) induce the same real allocation. If you double up the price level, the face value of debt doubles and there is no change in the real allocation. However, the second term in the objective function, equal to $E_0(\pi - \overline{\pi})^2 = E_0(\pi - E_0\pi)^2 + (E_0\pi - \overline{\pi})^2$, is minimized by the rule such that $E_0\pi = \overline{\pi}$. Then, without loss of generality, we can restrict attention to rules where the expected value of inflation is the target inflation rate, adding the constraint

$$E_0\pi = \overline{\pi} \tag{11}$$

to the optimal problem (9). Other rules have the same output gap but a larger inflation gap, so they are not optimal.

The optimal problem (9), then, becomes:

$$\min_{\pi(s)} (8) \text{ subject to: } (11), (4), \text{ and } (5).$$

After substituting the expression $E_0\pi = \overline{\pi}$ in the constraint (5),

$$\min_{\pi(s)} \frac{1}{2} E_0 \left\{ \gamma x^2 + (1-\gamma)(\pi - \overline{\pi})^2 \right\}$$

$$\text{subject to: } E_0\pi = \overline{\pi},$$

$$x = \frac{\alpha}{1-\alpha} \ln(\Phi(\nu)),$$

$$\nu = \frac{1}{\sigma} \left\{ \frac{\ln(E\{\omega\}\theta A)}{1-\alpha} + \alpha \frac{\ln(\alpha\Phi(\nu))}{1-\alpha} - \ln(\overline{b}) + (\pi - \overline{\pi}) \right\} + \frac{1}{2}\sigma.$$

Equivalently,

$$\min_{\pi(s)} \frac{1}{2} E_0 \left\{ \gamma x^2 + (1-\gamma)(\pi - \overline{\pi})^2 \right\}$$

$$\text{subject to: } E_0\pi = \overline{\pi},$$

$$\text{where: } x \equiv \frac{\alpha}{1-\alpha} \ln(\Phi(\nu)),$$

$$\nu \equiv \frac{1}{\sigma} \left\{ \frac{\ln(E\{\omega\}\theta A)}{1-\alpha} + \alpha \frac{\ln(\alpha\Phi(\nu))}{1-\alpha} - \ln(\overline{b}) + (\pi - \overline{\pi}) \right\} + \frac{1}{2}\sigma.$$

After defining $\hat{\pi} \equiv \pi - \overline{\pi}$,

$$\min_{\hat{\pi}(s)} \frac{1}{2} E_0 \left\{ \gamma x^2 + (1-\gamma)\hat{\pi}^2 \right\}$$

$$\text{subject to: } E_0 \hat{\pi} = 0,$$

$$\text{where: } x \equiv \frac{\alpha}{1-\alpha} \ln(\Phi(\nu)),$$

$$\nu \equiv \frac{1}{\sigma} \left\{ \frac{\ln(E\{\omega\}\theta A)}{1-\alpha} + \alpha \frac{\ln(\alpha\Phi(\nu))}{1-\alpha} - \ln(\bar{b}) + \hat{\pi} \right\} + \frac{1}{2}\sigma.$$

Letting $q(s)$ be the probability of the aggregate state given information available in the first period,

$$\min_{\hat{\pi}(s)} \frac{1}{2} \sum_s \left\{ \gamma x^2 + (1-\gamma)\hat{\pi}^2 \right\} q(s)$$

$$\text{subject to: } \sum_s \hat{\pi} q(s) = 0,$$

$$\text{where: } x \equiv \frac{\alpha}{1-\alpha} \ln(\Phi(\nu)),$$

$$\nu \equiv \frac{1}{\sigma} \left\{ \frac{\ln(E\{\omega\}\theta A)}{1-\alpha} + \alpha \frac{\ln(\alpha\Phi(\nu))}{1-\alpha} - \ln(\bar{b}) + \hat{\pi} \right\} + \frac{1}{2}\sigma.$$

Letting λ be the Lagrange multiplier (constant across states), the Lagrangian is

$$\mathcal{L} = \sum_s \left\{ \frac{1}{2}\gamma x^2 + \frac{1}{2}(1-\gamma)\hat{\pi}^2 + \lambda\hat{\pi} \right\} q(s)$$

$$\text{where: } x \equiv \frac{\alpha}{1-\alpha} \ln(\Phi(\nu)),$$

$$\nu \equiv \frac{1}{\sigma} \left\{ \frac{\ln(E\{\omega\}\theta A)}{1-\alpha} + \alpha \frac{\ln(\alpha\Phi(\nu))}{1-\alpha} - \ln(\bar{b}) + \hat{\pi} \right\} + \frac{1}{2}\sigma.$$

Taking the first-order conditions with respect to $\hat{\pi}(s)$, for all states s,

$$\gamma x \frac{\partial x}{\partial \hat{\pi}} + (1-\gamma)\hat{\pi} + \lambda = 0$$

$$\text{where: } \frac{\partial x}{\partial \hat{\pi}} = \frac{\alpha}{1-\alpha} \frac{\Phi'(\nu)}{\Phi(\nu)} \frac{\partial \nu}{\partial \hat{\pi}},$$

$$\frac{\partial \nu}{\partial \hat{\pi}} = \frac{1}{\sigma} \left\{ \frac{\alpha}{1-\alpha} \frac{\Phi'(\nu)}{\Phi(\nu)} \frac{\partial \nu}{\partial \hat{\pi}} + 1 \right\}.$$

Letting $\chi \equiv \frac{1}{\sigma} \frac{\alpha}{1-\alpha} \frac{\Phi'(\nu)}{\Phi(\nu)}$, we obtain

$$\frac{\partial \nu}{\partial \hat{\pi}} \equiv \chi \frac{\partial \nu}{\partial \hat{\pi}} + \frac{1}{\sigma}$$

$$\frac{\partial \nu}{\partial \hat{\pi}} \equiv \frac{1}{\sigma} \frac{1}{1-\chi}$$

and then

$$\frac{\partial x}{\partial \hat{\pi}} = \frac{\alpha}{1-\alpha}\frac{\Phi'(\nu)}{\Phi(\nu)}\frac{1}{\sigma}\frac{1}{1-\chi}$$
$$\frac{\partial x}{\partial \hat{\pi}} = \frac{\chi}{1-\chi}$$

so the previous first-order conditions become:

$$\gamma x \frac{\chi}{1-\chi} + (1-\gamma)\hat{\pi} + \lambda = 0.$$

To obtain an expression for λ, we take the expectation of both sides and use $E_0\hat{\pi} = 0$:

$$E_0\left\{\gamma x \frac{\chi}{1-\chi} + (1-\gamma)\hat{\pi} + \lambda\right\} = 0$$
$$\lambda = -\gamma E_0\left\{x\frac{\chi}{1-\chi}\right\}.$$

Substituting this expression for λ into the previous first-order conditions, and using $\hat{\pi} \equiv \pi - \overline{\pi}$, we obtain

$$\gamma\left\{x\frac{\chi}{1-\chi} - E_0\left\{x\frac{\chi}{1-\chi}\right\}\right\} + (1-\gamma)(\pi - \overline{\pi}) = 0$$

where $\chi \equiv \frac{1}{\sigma}\frac{\alpha}{1-\alpha}\frac{\Phi'(\nu)}{\Phi(\nu)}$. This is solution (10).

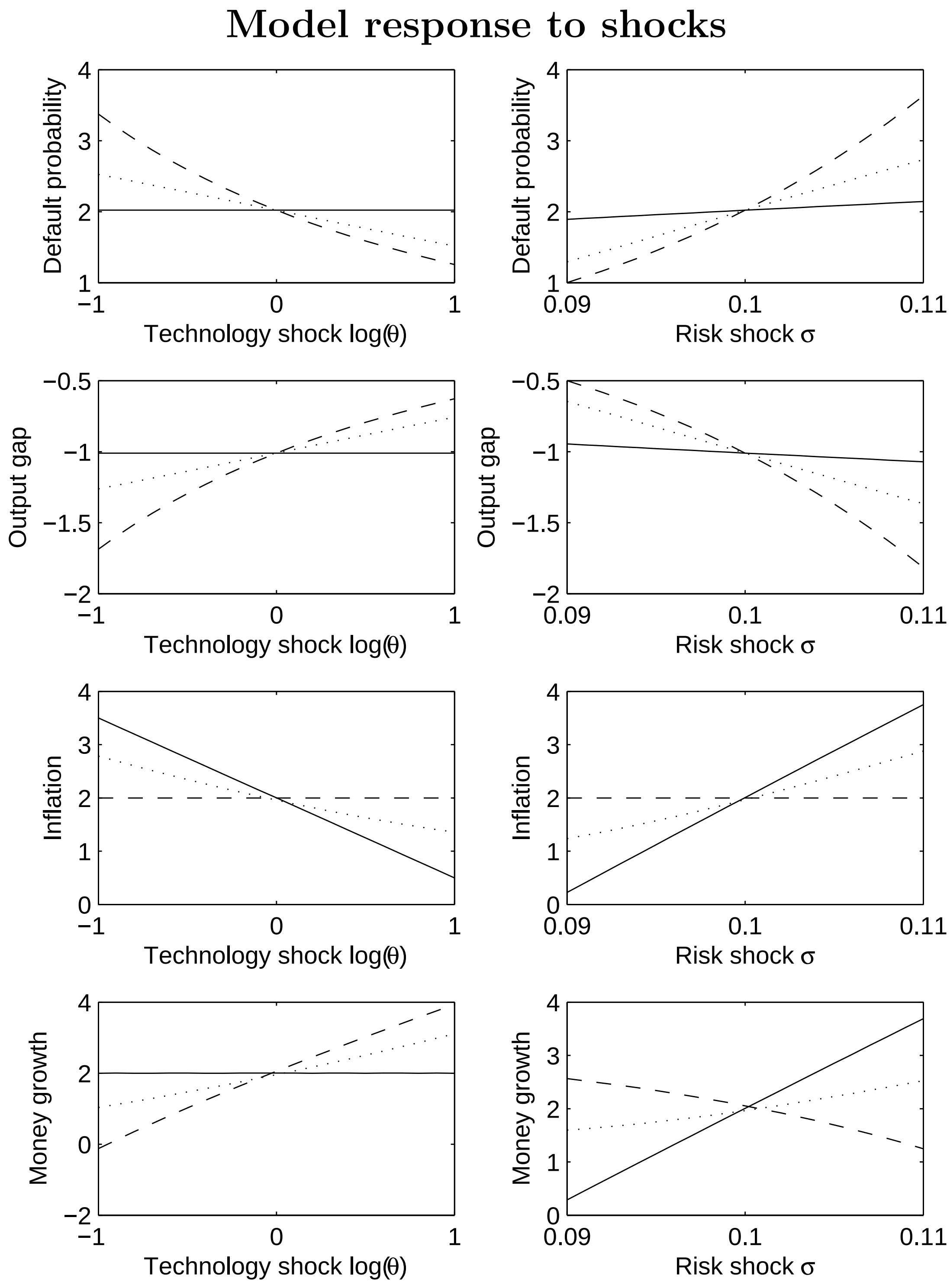

Figure 2: Model response to shocks depending on the monetary authority's objective function. The solid, dashed and dotted lines refer, respectively, to the cases where: the monetary authority aims at minimizing the output gap; the monetary authority aims at stabilizing inflation around a target level; the monetary authority aims at both objectives. All variables are multiplied by 100.

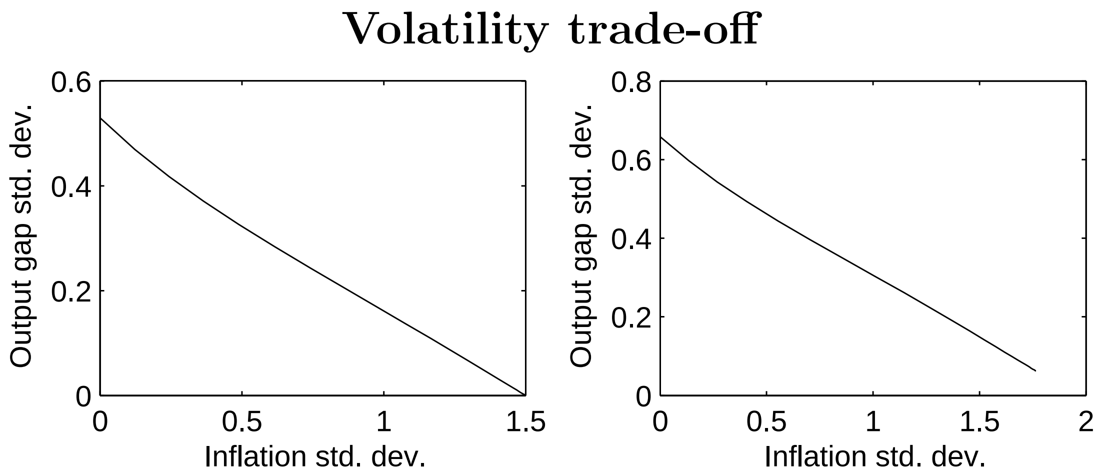

Figure 3: Trade-off between volatility of the output gap and volatility of the inflation rate that the monetary authority faces when choosing its rule. The frontier is traced for different values of the relative aversion of the monetary authority toward inflation variability and output gap. All variables are multiplied by 100.