

w o r k i n g
p a p e r

12 35

**Fiscal Multipliers under an
Interest Rate Peg of
Deterministic vs. Stochastic Duration**

Charles T. Carlstrom, Timothy S. Fuerst, and
Matthias Paustian



FEDERAL RESERVE BANK OF CLEVELAND

Working papers of the Federal Reserve Bank of Cleveland are preliminary materials circulated to stimulate discussion and critical comment on research in progress. They may not have been subject to the formal editorial review accorded official Federal Reserve Bank of Cleveland publications. The views stated herein are those of the authors and are not necessarily those of the Federal Reserve Bank of Cleveland or of the Board of Governors of the Federal Reserve System.

Working papers are available at:

www.clevelandfed.org/research.

**Fiscal Multipliers under an Interest Rate Peg of
Deterministic vs. Stochastic Duration**

Charles T. Carlstrom, Timothy S. Fuerst, Matthias Paustian

This paper revisits the size of the fiscal multiplier. The experiment is a fiscal expansion under the assumption of a pegged nominal rate of interest. We demonstrate that a quantitatively important issue is the articulation of the exit from the policy experiment. If the monetary-fiscal expansion is stochastic with a mean duration of T periods, the fiscal multiplier can be unboundedly large. However, if the monetary-fiscal expansion is for a fixed T periods, the multiplier is much smaller. Our explanation rests on a Jensen's inequality type argument: the deterministic multiplier is convex in duration, and the stochastic multiplier is a weighted average of the deterministic multipliers. The quantitative difference in the two multipliers also arises in a model with capital, and in the baseline nonlinear model. However, the differences between the two is less pronounced in the nonlinear models.

Keywords: fiscal multiplier, fixed interest rates, new Keynesian model, zero lower bound.

JEL Classification: E32.

Charles T. Carlstrom is at the Federal Reserve Bank of Cleveland (charles.t.carlstrom@clev.frb.org). Timothy S. Fuerst is at the University of Notre Dame and the Federal Reserve Bank of Cleveland (tfuerst@nd.edu). Matthias Paustian is at the Bank of England (matthias.paustian@bankofengland.co.uk). The authors thank Larry Christiano, Martin Eichenbaum, Sergio Rebelo, Jesper Linde, and Simon Price for comments on an earlier draft. The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England.

1 Introduction

A timely policy question is the size of the government spending multiplier under the case in which the central bank pegs the nominal rate of interest. The policy experiment is an increase in government spending, financed with lump sum taxes, during a period in which the nominal interest rate is held fixed. The analysis is typically motivated by a ‘liquidity trap’ scenario, but the key assumption is that the nominal rate during this period is pegged, not necessarily zero. For example, there are many historical periods in which the central bank accommodated a fiscal expansion by moderating interest rate movements.

The literature has typically considered two versions of the exit from the policy expansion. First, a stochastic exit: in each period the continuation probability of the fiscal expansion is constant so that there is an uncertain final date but a constant mean duration of T periods. Second, a deterministic exit: the expansion continues for a known duration of T periods. In both experiments the expected value of the fiscal expenditure is the same (from the vantage point of the initial period). Further the analysis is conducted in a linear model. A priori one would thus anticipate that the impact multiplier under a deterministic peg would be similar to that under a stochastic peg with the same expected duration. For example, Christiano, Eichenbaum, and Rebelo (2011) transition without comment from stochastic to deterministic exit as they move from a labor-only model to a model that includes physical capital.

In contrast, the contribution of this paper is to demonstrate it matters quantitatively how the exit from the joint monetary fiscal expansion is modeled. For example in a labor-only model and an expansion of duration $T = 6$, the stochastic exit multiplier is 4.90, while the deterministic exit multiplier is 1.09. As we increase T , the stochastic exit multiplier approaches an asymptote and becomes unboundedly positive, but then reverses to become unboundedly negative. In contrast, the deterministic exit multiplier is a smooth and modestly increasing function of T . Similar results arise in a model with physical capital. Using the parameterization in Christiano, Eichenbaum, and Rebelo (2011) and with $T = 4$, the deterministic exit multiplier is 2.3, while the stochastic exit multiplier is 66.9 (!). Evidently, the manner of exit from the expansion is of critical importance.

The explanation for our key finding is related to Jensen’s inequality. The deterministic exit fiscal multiplier is a convex function of the duration of the stimulus at constant interest rates. Intuitively, the fiscal multiplier under a stochastic exit averages the deterministic multipliers across all possible durations. It then follows from the strong convexity that this mean multiplier is much larger than the multiplier

evaluated at the mean.¹ Essentially, the stochastic exit multiplier can be strongly influenced by the low probability event of the monetary-fiscal expansion lasting for a very long time. In that event, the fiscal multiplier in the linearized model is extremely large. But the approximation error from a linearized solution is also large in this event. In line with findings by Braun and Waki (2010), we document that the linear stochastic exit multiplier can be very inaccurate when compared to the true nonlinear multiplier, while the corresponding linear deterministic exit multiplier in our model environment remains accurate.

We explore the distinction between the two exit strategies by analyzing a hybrid exit in which there is a stochastic interest rate peg up until a maximum time period after which exit is immediate. Surprisingly for even large values of this truncation date, the hybrid multiplier is much closer to the corresponding deterministic multiplier than the stochastic multiplier. This suggests that the deterministic multiplier is a preferred way of thinking about fiscal multipliers under an interest rate peg.

There is a rapidly growing literature on the fiscal multiplier at the zero bound. The extant literature documents the sensitivity of the multiplier to parameter values as in Christiano, Eichenbaum, and Rebelo (2011), lump-sum vs. distortionary taxes as in Drauztenburg and Uhlig (2011), timing of spending vis-a-vis the exit from the zero lower bound as emphasized by Woodford (2011) and Christiano, Eichenbaum, and Rebelo (2011), linear vs. nonlinear solution technique as highlighted by Braun and Waki (2010), and an open versus closed economy setting as in Fujiwara and Ueda (2012) etc. Our results suggest that there is another important issue for the size of the multiplier: how to model the exit from the fiscal-monetary expansion.

The paper begins with the results under a stochastic peg in a labor-only model. One novelty is to demonstrate the link between the size of the multiplier and the parameter border between equilibrium determinacy and indeterminacy. Because this border is an asymptote, we also show that the fiscal multiplier is extremely sensitive to small changes in either the mean duration or various parameter values. Section 3 considers the complementary policy experiment in which the expansion is for a deterministic period of time. The key conclusion is that the size of the multiplier is dramatically affected by this alternative statement of how the policy expansion is expected to end. Section 4 investigates the hybrid peg. Section 5 provides some intuition for our main results. Section 6 provides some sensitivity analysis by examining a model with physical capital, and the nonlinear baseline model. Section 7 concludes.

¹We thank Jesper Linde for suggesting to frame our explanation in terms of this convexity argument.

2 The model with a stochastic interest rate peg

We examine the familiar DNK model with Calvo (1983) pricing and a linear labor-only production technology. Our baseline preferences are given by

$$U(C_t, N_t) \equiv \frac{C_t^{1-\sigma}}{1-\sigma} - \eta \frac{N_t^{1+\nu}}{1+\nu} \quad (1)$$

The linearized model is given by:

$$i_t - E_t \pi_{t+1} = -\sigma(c_t - E_t \bar{c}_{t+1}) \quad (2)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa m c_t \quad (3)$$

$$m c_t = \sigma c_t + \nu y_t \quad (4)$$

$$y_t = (1-s)c_t + s g_t \quad (5)$$

where π_t , y_t , c_t , g_t , $m c_t$, and i_t , denote inflation, output, consumption, government spending, marginal cost, and the nominal rate, respectively, all measured as deviations from the steady-state. The constant $s = \frac{G}{Y_{ss}}$ is the share of government spending in the steady state. Substituting (4)-(5) into (3) we have:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa[\sigma + \nu(1-s)]c_t + \kappa \nu s g_t. \quad (6)$$

The simple DNK model is thus given by (2), (6), and a description of monetary and fiscal policy. We consider a coordinated policy experiment in which (i) government spending is set above steady state $g_t = \bar{g} > 0$, and simultaneously (ii) the central bank announces an interest rate peg $i_t = 0$. Each period there is probability p that this policy will continue so that the expected duration of the expansion is $T = \frac{1}{1-p}$. With probability $(1-p)$ the expansion ends, at which point fiscal policy returns to steady state, $g_t = 0$, and monetary policy reverts to a typical Taylor rule:

$$i_t = \phi_\pi \pi_t + \phi_y y_t. \quad (7)$$

Under standard assumptions on ϕ_π and ϕ_y , there is a unique equilibrium after the period of the peg. Since there are no state variables nor exogenous shocks during these subsequent periods, the unique equilibrium after the policy experiment is given by $\pi_t = y_t = 0$.

The equilibrium is one in which all endogenous variables take on one value during the experiment and then revert to steady-state when the economy exits the experi-

ment. This equilibrium is given by:

$$\pi_t = \left[\frac{\kappa \nu s \sigma (1-p)}{\Delta} \right] g_t \quad (8)$$

$$c_t = \left(\frac{p \kappa \nu s}{\Delta} \right) g_t \quad (9)$$

$$y_t = \left[\frac{s \sigma [(1-p)(1-\beta p) - \kappa p]}{\Delta} \right] g_t \quad (10)$$

where

$$\Delta \equiv \sigma (1-p) (1-\beta p) - \kappa [\sigma + \nu (1-s)] p. \quad (11)$$

But there are potentially other equilibria in this model. The general solution to inflation is given by

$$\pi_t = \left[\frac{\kappa \nu s \sigma (1-p)}{\Delta} \right] g_t + c_1 e_1^t + c_2 e_2^t \quad (12)$$

where e_1 and e_2 are the roots of:

$$h(q) \equiv \beta \sigma p^2 q^2 - \{ \kappa [\sigma + \nu (1-s)] + \sigma (1+\beta) \} p q + \sigma = 0 \quad (13)$$

The constants c_1 and c_2 are potentially free variables as there is no terminal condition. The equilibrium is unique ($c_1 = c_2 = 0$) if and only if both roots of h are outside the unit circle. The function h is convex, positive at zero, decreasing at zero, and the product of the roots is $1/(\beta p^2)$. This implies that both roots are outside the unit circle if and only if $h(1) = \Delta > 0$. Hence, under the assumption of $\Delta > 0$ maintained in Christiano, Eichenbaum, and Rebelo (2011), there is a unique stationary equilibrium. But if $\Delta < 0$, there are multiple stationary equilibria and thus sunspot equilibria. A similar point has been made before by Eggertsson and Pugsley (2006), see their assumption 1. In this case, increases in government spending may actually decrease output. The case of $\Delta > 0$, is the “normal” case so that increases in government spending lead to increases in output.

The fiscal multiplier during the interest rate peg is given by:

$$\frac{dY}{dG} \equiv \left(\frac{1}{s} \right) \frac{dy_t}{dg_t} = \left[\frac{\sigma [(1-p)(1-\beta p) - \kappa p]}{\Delta} \right] \quad (14)$$

Note that as $\kappa \rightarrow \infty$, the model approaches flexible prices and the multiplier converges to

$$\frac{dY}{dG} = \left(\frac{1}{s} \right) \frac{dy_t}{dg_t} = \frac{\sigma}{\sigma + \nu(1-s)} \leq 1. \quad (15)$$

Hence, in the model with flexible prices, the multiplier cannot exceed unity. A fiscal expansion leads to a fall in consumption, an increase in the real rate, and a complementary decrease in expected inflation. But as emphasized by Christiano, Eichenbaum, and Rebelo (2011), with sticky prices and a nominal rate peg, the multiplier can be significantly greater than unity. Assuming $\Delta > 0$, the fiscal expansion leads to an increase in consumption, and given the interest rate peg a decline in the real rate, and a complementary increase in expected inflation.² Given the presence of sticky prices, the higher inflation rate is associated with higher levels of output. The potentially large multipliers with sticky prices and an interest rate peg has nothing to do with government spending, per se. All shocks are potentially magnified in this environment.

Figure 1 graphs this sticky price multiplier as function of the expected duration of the coordinated policy shock, $T = \frac{1}{1-p}$. The Figure uses the following baseline parameter values: $\beta = 0.99$, $\kappa = 0.028$, $\nu = 0.5$, $\sigma = 2$, $s = 0.2$. As a sensitivity analysis, we use non-separable preferences given by

$$U(C_t, N_t) \equiv \frac{[C_t^\gamma (1 - N_t)^{1-\gamma}]^{1-\sigma} - 1}{1 - \sigma} \quad (16)$$

In this case our calibration again follows Christiano, Eichenbaum, and Rebelo (2011) and we set $\kappa = 0.03$ and $\gamma = 0.29$. As Figure (1) shows, the fiscal multiplier increases sharply as a function of the expected duration for both preference specifications.

The multiplier can easily exceed one, and approaches infinity as T increases and Δ approaches zero. Hence, the borderline case of $\Delta = 0$, coincides with the case of an asymptotic multiplier and the existence of stationary sunspot equilibria. This borderline case occurs with a relatively short policy experiment: $T = 6.12$ quarters.

Because of the asymptote, the multiplier is extremely sensitive to changes in T , and κ . Considering the baseline case of separable preferences, with the above calibration, if $T = 6$, the multiplier is 4.90. But with $T = 5$, the multiplier is only 1.3. With $T = 7$, we are in the indeterminacy region and the multiplier is only 0.33. Holding T fixed at $T = 6$, very small changes in κ can make a huge difference (κ is typically estimated between 0.005 and 0.03). For example, if $\kappa = 0.025$ (versus 0.028 in the baseline) the multiplier drops from 4.89 to 2; for $\kappa = 0.029$, the multiplier is 30. With $\kappa = 0.03$ you are already in indeterminacy region and the multiplier is -5.

²The notable exception is the special case when the labor supply is infinitely elastic (i.e., linear labor, $\nu = 0$). In this case, inflation, consumption, and marginal cost are constant and the multiplier is always identically equal to one.

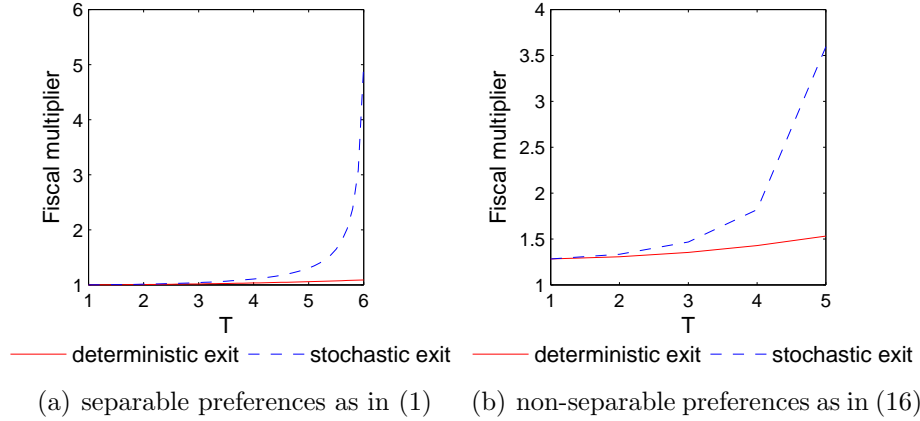


Figure 1: Fiscal multipliers in the model without physical capital accumulation.

3 A deterministic interest rate peg

In this section we consider a modest change in the policy environment. The policy experiment is no longer stochastic, continuing with probability p , but is instead deterministic, ending for sure after T periods. The endogenous variables will of course not be constant during this pegged experiment, but there is a unique equilibrium because the end of the experiment coincides with a unique equilibrium, and thus provides a terminal condition to the model.

It is convenient to express the equilibrium in terms of the path for inflation. The remaining endogenous variables can then be inferred from (3)-(5). The fundamental difference equation during the interest rate peg is given by:

$$\beta\sigma\pi_{t+2} - \{\kappa[\sigma + v(1-s)] + \sigma(1+\beta)\}\pi_{t+1} + \sigma\pi_t + \sigma\kappa v s(g_{t+1} - g_t) = 0 \quad (17)$$

The equilibrium behavior of inflation is given by

$$\pi_t = m_1 e_1^t + m_2 e_2^t \quad (18)$$

where the e 's are the roots to:

$$f(q) \equiv \beta\sigma q^2 - \{\kappa[\sigma + v(1-s)] + \sigma(1+\beta)\}q + \sigma = 0 \quad (19)$$

Note that f is convex, $f(0) > 0$, $f(1) < 0$, so that we have one root e_1 in $(0,1)$, and one root $e_2 > 1$. The constants m_1 and m_2 are uniquely determined by the following two terminal conditions

$$\pi_T = \kappa v s g_T \quad (20)$$

$$\pi_{T-1} = \left\{ \kappa \left[1 + \frac{v}{\sigma} (1 - s) \right] + (1 + \beta) \right\} \pi_T \quad (21)$$

With time-varying inflation and thus output, it is not clear how to define the fiscal multiplier in this case. But it is easy to show that the largest output effect occurs in the very first period. Let $y_1(T)$ denote the level of output in the first period with a peg that lasts T periods, so that the initial multiplier is defined as:

$$\frac{dY}{dG} \equiv \left(\frac{1}{s} \right) \frac{dy_1(T)}{dg_t}. \quad (22)$$

Figure 1 also graphs this multiplier as a function of T . Remarkably, the size of the multiplier is quite different. For example, with $T = 6$, the stochastic exit multiplier is 4.90, while the deterministic exit multiplier is 1.09. Although it is not a priori obvious which policy experiment (stochastic peg vs. deterministic peg) is a better description of reality, the explosive behavior under the stochastic peg seems quite implausible.³ Further, the extreme sensitivity to parameter values for the stochastic peg is not evident for the deterministic peg. For example, as we increase κ from $\kappa = 0.028$, to $\kappa = 0.029$, the deterministic multiplier increases only slightly from 1.0895 to 1.0930.

4 A hybrid interest rate peg

As a form of sensitivity analysis, we consider in this section a stochastic peg as in section 2, but with a certain maximum truncation date as in section 3, i.e., there is a constant probability of exit up until time T , after which date exit is certain. As with the deterministic peg, the certain maximum exit date implies that there is equilibrium determinacy for all values of T . We set the continuation probability to $p = 5/6$. When the truncation date is small, this implies an anticipated duration of somewhat less than 6 periods. The fundamental difference equation during the interest rate peg is given by:

$$p^2 \beta \sigma \pi_{t+2} - p \left\{ \kappa \left[\sigma + v (1 - s) \right] + \sigma (1 + \beta) \right\} \pi_{t+1} + \sigma \pi_t + \sigma \kappa \nu s (p g_{t+1} - g_t) = 0 \quad (23)$$

The two terminal conditions are given by (20) and

$$\pi_{T-1} = p \left\{ \kappa \left[1 + \frac{v}{\sigma} (1 - s) \right] + (1 + \beta) \right\} \pi_T + \kappa \nu s (1 - p) g_t \quad (24)$$

³A deterministic peg can deliver large multipliers if there are endogenous states such as lagged inflation. In this case there can again be asymptotes and large multipliers for modest values of T .

T period truncation	Fiscal multiplier	
	separable as in (1) (p=5/6)	non-separable as in (16) (p=4/5)
6	1.05	1.33
50	1.64	2.83
100	2.22	3.36
500	4.33	3.59
1,000	4.82	3.59
10,000	4.90	3.59
Infinity (stochastic exit)	4.90	3.59

Table 1: Fiscal multiplier in the hybrid model

For our benchmark parameter values, Table 1 reports the value of the multiplier for this hybrid peg as a function of T .

As a point of comparison, the deterministic peg with $T = 6$ has a multiplier of 1.09, while the stochastic peg with duration of $T = 6$, has a multiplier of 4.90. Table 1 suggests the peculiarity of the original stochastic peg. If the peg is stochastic up to period 50 (over 12 years!), after which it is truncated, the fiscal multiplier is only 1.64, quite close to the deterministic peg. Even for a truncation of 100 periods, the multiplier is 2.22, less than half of the pure stochastic counterpart.

5 Explaining the differences

What is the intuition for these striking differences between stochastic, deterministic exit and hybrid exit multipliers of equal mean duration? Essentially, our explanation rests on a Jensen inequality argument that we develop below. First we argue that the stochastic exit multiplier averages the deterministic exit multipliers across all possible durations. Second, we show that this deterministic exit multiplier is a convex function of the duration. It then follows from convexity and Jensen's inequality that this mean multiplier is much larger than the multiplier evaluated at the mean.

Initial output under a deterministic peg of exactly $j > 1$ periods, $y_0^d(j)$, can be written from the Euler equation as

$$y_0^d(j) = sg_0 + \frac{1-s}{\sigma} \sum_{k=1}^{j-1} \pi_k(j) \quad (25)$$

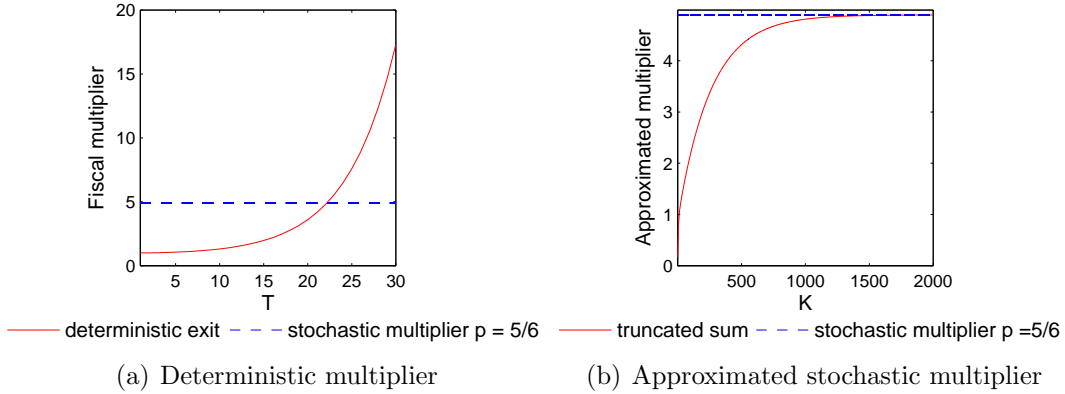


Figure 2: Deterministic and approximated stochastic exit multipliers

Thus, the growth of the fiscal multiplier as we extend the duration is proportional to the initial expected inflation rate.

$$y_0^d(j) - y_0^d(j-1) = \frac{1-s}{\sigma} \pi_1(j) \quad (26)$$

One can show that $\pi_1(j)$ increases exponentially in the duration j so that the fiscal multiplier is a convex function of the duration. Under a stochastic peg, households attach probability $(1-p)p^{j-1}$ to the event that the peg lasts exactly j periods. Hence initial output under a stochastic peg, y_0^s , is given by

$$y_0^s = \sum_{j=1}^{\infty} (1-p)p^{j-1} y_0^d(j) \quad (27)$$

Figure 2(a) plots the multiplier in the benchmark model with separable preferences for durations $d = 1, 2, \dots, 30$, which clearly shows the convexity. Convexity has been pointed out before by Erceg and Linde (2010), but without connecting it to the difference between stochastic and deterministic exit. Figure 2(b) plots an average of the deterministic exit multipliers, each weighted by $(1-p)p^{j-1}$. The infinite sum in (27) is truncated at K for the numerical implementation. Clearly, for high enough truncation point K , the weighted sum of deterministic exit multipliers converges to the stochastic exit multiplier.

Why does the stochastic exit multiplier become unboundedly large as the expected duration reaches some finite threshold value? Our intuition builds again on the insight that the stochastic exit multiplier is a weighed average of the deterministic exit multipliers. There are two forces at work. As the duration increases, the

deterministic exit fiscal multiplier grows exponentially. But the weights attached to each finite duration multiplier decay exponentially. As we outline below, there always exists a critical value $\bar{p} < 1$ such that the decay of the weights becomes arbitrarily close to the growth of the finite duration multiplier. Hence, the stochastic exit multiplier asymptotes. Importantly, the stochastic exit model has a determinate equilibrium as p approaches \bar{p} from below.

To see this, we can run time backwards from the end of the fiscal expansion by inverting expression (18), and thus express the initial inflation rate as a function of T . This initial inflation rate is exploding in T since $1/e_1 > 1$. For large T , we can ignore other terms involving the stable root so that this initial inflation rate is roughly proportional to $(1/e_1)^T$. The stochastic multiplier with mean duration $1/(1-p)$ periods is then roughly proportional to Θ , defined as

$$\Theta = (1-p) \sum_{k=1}^{\infty} \left(\frac{p}{e_1}\right)^k = (1-p) \frac{p}{e_1 - p} \quad (28)$$

The root e_1 is the stable root of (19) and thus independent of p . One can show that the stochastic model has a unique stationary equilibrium if and only if $p < e_1$. There are two implications as p approaches e_1 from below. First, the stochastic-exit multiplier becomes arbitrarily large. Second, the hybrid multiplier converges to the stochastic multiplier very slowly as the truncation point is increased.

6 Sensitivity

We document sensitivity with respect to two aspects: Adding capital and solving a nonlinear model.

6.1 Capital

Capital accumulation is an important aspect for the fiscal multiplier. If fiscal policy raises inflation expectations and thus lowers real interest rates due to the peg for nominal rates, this now crowds in not only private consumption but also private investment. Since investment is typically more interest sensitive than consumption, this increases the constant rate fiscal multiplier. As Christiano, Eichenbaum, and Rebelo (2011) and others have shown, the multiplier can be significantly larger than unity with a deterministic peg in a model with capital. We revisit the model with capital accumulation and contrast this with a stochastic peg.

We follow Christiano, Eichenbaum, and Rebelo (2011), and use the preference specification given by (16). The model is then given by the following linearized equilibrium conditions.

$$\pi_t = \kappa m c_t + \beta \pi_{t+1} \quad (29)$$

$$c_t + v n_t = m c_t + \alpha (k_{t-1} - n_t) \quad (30)$$

$$i_t - \pi_{t+1} = [\gamma(1 - \sigma) - 1](c_t - c_{t+1}) - (1 - \gamma)(1 - \sigma)v(n_t - n_{t+1}) \quad (31)$$

$$i_t - \pi_{t+1} = (1 - \epsilon)((1 - \alpha)(n_{t+1} - k_t) + m c_{t+1}) \\ + \epsilon \sigma_I (k_{t+1} - (2 - \delta)k_t) - \sigma_I (k_t - (2 - \delta)k_{t-1}) \quad (32)$$

$$\alpha k_{t-1} + (1 - \alpha)n_t = \frac{C}{Y} c_t + \frac{I}{Y\delta} (k_t - (1 - \delta)k_{t-1}) + \frac{G}{Y} g_t \quad (33)$$

Here, $v \equiv \frac{\bar{N}}{1 - \bar{N}}$. Physical capital is denoted by k_t with depreciation rate δ . The constant ϵ is given by $\beta(1 - \delta)$. We use a capital adjustment cost specification used in Christiano, Eichenbaum, and Rebelo (2011), see their equation (40) with coefficient σ_I . The calibration is the same as before, except $\delta = 0.02$, $\gamma = 0.29$, $\sigma_I = 17$, and $\alpha = 0.345$.

Because of the presence of an endogenous state variable we cannot obtain an analytical solution for the stochastic exit fiscal multiplier as in section 2. Instead, we employ a two stage procedure. In matrix form, the system in (29)-(33) can be expressed as

$$0 = AE_t Y_{t+1} + BY_t + CY_{t-1} + D. \quad (34)$$

Here, the matrix D captures terms involving the constant value of government spending prior to the exit. In the first stage, we close the above model with a Taylor rule that is in place after the exit and compute its solution via standard methods. The post exit decision rules for the vector of endogenous variables Y_t can be expressed in matrix form as $Y_t = FY_{t-1}$. In a second step, we consider the model prior to the exit that we close with $i_t = 0$. That system can be written as

$$0 = pAY_{t+1} + [(1 - p)AF + B]Y_t + CY_{t-1} + D, \quad (35)$$

and solved via standard methods.

Figure (3) compares the multipliers under a deterministic peg with that from a stochastic peg with equal mean duration. Note that the stochastic peg in the baseline calibration results in indeterminacy when the continuation probability exceeds $p = 0.75$ (corresponding to mean duration of $T = 4$).

Under the baseline calibration with $\gamma = 0.29$, the multiplier under a stochastic exit is substantially larger than its deterministic exit counterpart. For $T = 4$

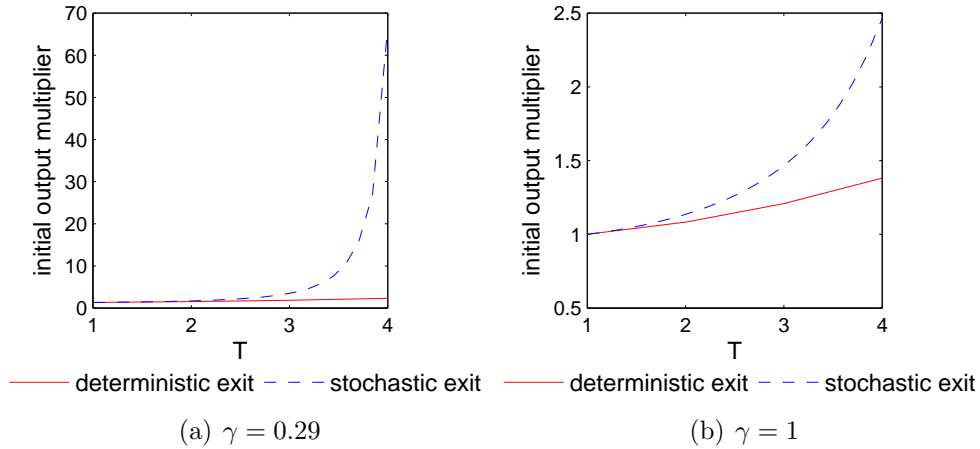


Figure 3: Fiscal multipliers in the model with physical capital accumulation

the deterministic exit multiplier is 2.3 vs. 66.87 for the stochastic exit. The large stochastic-exit multiplier arises because we are on the indeterminacy boundary. We can once again consider the hybrid model and increase the truncation point. As expected, when the stochastic exit multiplier is substantially larger than its deterministic exit counterpart, it is driven considerably by stimulus in the very distant future and the hybrid multiplier converges to the stochastic counterpart very slowly. In this regard, capital accumulation does not affect our main result.

T period truncation	Fiscal multiplier	
	$\gamma = 0.29$	$\gamma = 1$
5	2.06	1.2
50	14.43	2.41
100	22.37	2.46
500	52.83	2.47
1,000	63.55	2.47
5,000	66.87	2.47
Infinity (stochastic exit)	66.87	2.47

Table 2: Fiscal multiplier in the hybrid model with $p = 3/4$

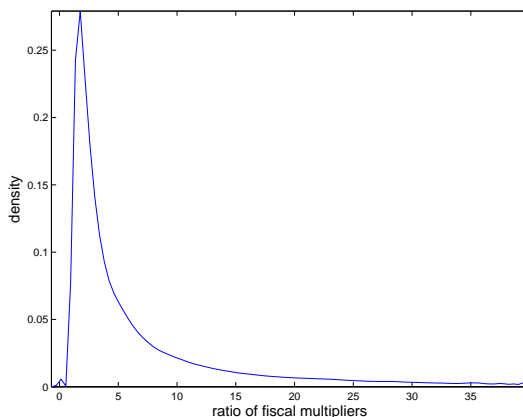


Figure 4: Density of relative fiscal multipliers (stochastic vis a vis deterministic exit)

The baseline calibration used in Figure 3(a) suggests that the fiscal multiplier under a stochastic exit can be an order of magnitude bigger than deterministic exit multiplier for modest durations of around 4 quarters. However, this finding may be sensitive to the particular parameterization. We therefore conduct sensitivity analysis. Conditioning on a mean duration of the monetary-fiscal expansion of 4 quarters, we draw parameters from a narrow range of a priori reasonable values of the models structural parameters.⁴ For each draw, we compute the ratio of the fiscal multiplier under a stochastic exit to the multiplier under a deterministic exit. The median ratio is 3.3 with a standard deviation of 7.3. After removing outliers in the right tail, Figure 4 shows the plot of the kernel density of this ratio.

Clearly, the extremely large discrepancy between the stochastic and deterministic exit multiplier evident in Figure 3(a) for $T = 4$ is an outlier. Nevertheless, across a wide range of reasonable parameters, the stochastic exit multiplier is substantially bigger than its deterministic exit counterpart.

We have also explored fiscal multipliers in the medium-scale model of Smets and Wouters (2007) using the same computational approach as outlined for the capital model above. Our key result also holds in this model, stochastic exit multipliers are larger than the corresponding deterministic exit multipliers. An additional twist is brought about by the presence of strong endogenous propagation mechanisms via habit, price and wage indexation and investment adjustment costs specified in

⁴ $\kappa \in [0.01, 0.05]$, $\alpha \in [0.3, 0.4]$, $\sigma \in [1, 3]$, $\gamma \in [0.1, 0.9]$, $\sigma_I \in [10, 20]$, $N \in [0.3, 0.5]$, and $\tau \in [1.5, 2]$. Parameters are uniformly distributed in these ranges. We draw until we have 10,000 parameters sets that deliver a determinate equilibrium under a stochastic exit with $p = 3/4$.

growth rates. As is discussed in detail in a companion paper, such endogenous propagation mechanisms can result in extremely large multipliers in response to any stimulus at constant interest rates even under a deterministic exit assumption, see Carlstrom, Fuerst, and Paustian (2012). In particular, multipliers appear to "asymptote" once the duration approaches a critical value T^d under deterministic exit. However, multipliers in the stochastic exit model also asymptote and do so at smaller critical mean duration $T^s < T^d$. Furthermore, the stochastic exit model becomes indeterminate for $T > T^s$. Confining the comparison to the determinacy region in the stochastic exit world, we have found stochastic exit multipliers to be larger than the deterministic exit multipliers in the Smets and Wouters model.

6.2 Nonlinear solution

As emphasized by Braun and Waki (2010), the large inflation responses associated with large fiscal multipliers suggest that the linear approximation may be inaccurate. As a form of sensitivity analysis, we thus consider the underlying nonlinear system. We consider Rotemberg quadratic costs of price adjustment with adjustment cost parameter ϕ . For the case of a stochastic exit, the model is given by the following equations representing market clearing for the final good, the Phillips curve and the Euler equation for bond holdings:

$$C + G = Y \left(1 - \frac{\phi}{2}(\Pi - 1)^2 \right) \quad (36)$$

$$(\epsilon - 1)\tau = +\epsilon Y^\varphi C^\sigma - \phi(\Pi - 1)\Pi + p\beta D\phi(\Pi - 1)\Pi \quad (37)$$

$$C^{-\sigma} = p\beta DC^{-\sigma} \frac{R^L}{\Pi} + (1 - p)\beta D\bar{C}^{-\sigma} \frac{R^L}{\bar{\Pi}} \quad (38)$$

Here $D > 1$ is an exogenous discount factor shock that persists with probability p and $R^L = 1$ is the value of the nominal interest rate at the lower bound. The discount shock is often employed as the rationale for the interest rate peg, ie., the discount rate shock pushes the nominal rate to zero. In the linearized models considered above, the discount shock has no effect on the fiscal multiplier. But this may not be the case in the nonlinear model as the discount shock pushes the economy far from the steady state before the fiscal expansion is considered. Upon exit, the economy reverts to the zero net-inflation steady state, whose values are denoted with a bar. We assume a sales-subsidy τ that makes the steady state efficient. The parameter ϵ is the price elasticity of demand and φ is the curvature of labor supply. Equilibrium is given by values for $\{Y, C, \Pi\}$ that solve (36) - (38) for given steady state values $\{\bar{Y}, \bar{C}, \bar{\Pi}\}$ and exogenous values for G . We set $\varphi = 0.5$, $\epsilon = 11$ and set ϕ such that

linearizing this model about a zero inflation steady state gives the same slope of the Phillips curve as in our previous analysis.

This stochastic-exit model can have multiple solutions, but as in Christiano and Eichenbaum (2012), some are not E-stable and we discard these. We also solve the deterministic exit model nonlinearly. Backing up from the last period, the deterministic model has two solutions with identical private consumption values, but low or high inflation. Conditional on choosing the low inflation equilibrium in the last period, the previous period problem again has two solutions of the same type. Repeatedly conditioning on the low inflation equilibrium, we compute the initial period fiscal multiplier.⁵

	<u>stochastic exit</u>		<u>deterministic exit</u>	
	linear	nonlinear	linear	nonlinear
model without discount factor shock				
T = 3	1.04	1.04	1.02	1.02
T = 4	1.11	1.11	1.03	1.03
T = 5	1.30	1.31	1.06	1.06
T = 6	4.90	3.75*	1.09	1.09
model with discount factor shock				
T = 3	1.04	1.04	1.02	1.02
T = 4	1.11	1.10	1.03	1.03
T = 5	1.30	1.18	1.06	1.06
T = 6	4.90	1.24	1.09	1.08

Table 3: Fiscal multipliers: linear vs. nonlinear solution. Asterisks denotes that for the non-linear stochastic exit model without discount factor shock we consider $T = 5.85$, because for higher T there is no solution.

Table 3 summarizes our results. Since the size of the discount factor shock affects the fiscal multiplier in the nonlinear model (unlike in the linear world), we need to take a stand on the size of the shock. We consider two cases. For $D = 1$ there is no discount factor shock and the interest rate is held constant at steady state exogenously. When $D = 1.02$, the economy reaches the zero lower bound and the

⁵Conditional on the high inflation equilibrium in the last period, the previous period problem has no solution, so this conditioning is natural.

interest rate is held fixed at that bound endogenously. As noted earlier, the fiscal multiplier in the linear economy is unaffected by the value for D . We consider an increase in government spending of 1% about a steady state share in GDP of 0.2. The government spending multiplier reported below is defined in terms of changes in consumption plus government spending, i.e. changes in output purely due to changes in costs of price adjustment costs are ignored.⁶

Turning to the case without a discount factor shock, we find that the stochastic exit multiplier is larger than the deterministic exit multiplier of equal mean duration also in the nonlinear model. Furthermore, the linear approximation gives very accurate fiscal multipliers for all durations considered in the table. However, we are unable to find a solution for the nonlinear stochastic exit model for $T > 5.85$, while the linear approximation results in very large multipliers in the range between $T = 5.85$ and the slightly larger indeterminacy border.

When a discount factor shock is present we again find that the stochastic exit multiplier is larger than its deterministic exit counterpart. However, the difference between the two is now much less pronounced. The underlying reason is that the system is displaced from the point around which the model is linearly approximated. We would therefore expect the local linear approximation to perform worse. This approximation error kicks in much more strongly for the stochastic exit model at modest expected durations. The reason is closely related to our intuition for why the stochastic exit multiplier is so big in the first place. It is strongly influenced by the low probability event of the expansion lasting a very long time. In this event, the linear multiplier is extremely large and the linear approximation is particularly bad. Our intuition highlighted that the linear deterministic exit multiplier is a convex function of the duration and the linear stochastic exit multiplier averages across these durations. However, the deterministic exit multiplier in the nonlinear model is no longer convex over all durations. For instance, in the baseline model with a discount factor shock, this multiplier rises to 1.66 for a duration of $T = 23$, but then falls for longer durations. At $T = 50$ the nonlinear multiplier is only 1.23. For comparison, the linear multipliers at these two durations are 5.56 and 585!

⁶The presence of a discount factor shock generally results in a fall inflation which generates adjustment costs that are part of aggregate demand. To the extent that government spending mitigates this fall in inflation, it reduces price adjustment costs which in itself reduces output. From a welfare perspective, there is nothing detrimental per-se about reducing price adjustment costs and our definition of the fiscal multiplier reflects this.

7 Conclusion

This paper has shown that the size of the fiscal multiplier under an interest rate peg depends crucially on the probability distribution of the exit date, holding constant the mean duration of the peg. Larger multipliers occur when the distribution of possible exit dates has wide support as in a stochastic exit model. This implies that the simple modeling choice of endowing agents with a known exit date from the fiscal monetary regime can result in much smaller multipliers than stochastic exit assumptions of same durations. This paper has also shown that the stochastic exit multiplier computed via a local linear approximation around the zero inflation steady state is much more prone to approximation error at modest mean durations. Approximation errors for the corresponding deterministic exit model are much smaller.

Which modeling approach should be employed? When studying the effects of fiscal policy in certain historical episodes, the interest rate is typically fixed because particular shocks push it to the zero lower bound. It is hard to assess what people expect about the probability distribution of these shocks. But we think it is a priori unreasonable to attach a non-zero probability to the central bank keeping the interest rate pegged for say, 25 years, after a one-time exogenous event. Our results on the hybrid multiplier then imply that the deterministic multiplier is a much more reasonable estimate of the true multiplier than the pure stochastic multiplier. Hence we conclude that for policy analyses of this type the deterministic approach should be preferred. In line with this view, Christiano, Eichenbaum, and Rebelo (2011) employ the deterministic exit assumption when aiming to quantify the fiscal multiplier in a medium scale macro model designed to fit macro data.

References

- BRAUN, R. A., AND Y. WAKI (2010): “On the Size of the Fiscal Multiplier When the Nominal Interest Rate is Zero,” unpublished, University of Tokyo.
- CARLSTROM, C. T., T. S. FUERST, AND M. PAUSTIAN (2012): “Inflation and output in New Keynesian models with a transient interest rate peg,” Bank of England Working Paper No 459.
- CHRISTIANO, L., AND M. EICHENBAUM (2012): “Notes on Linear Approximation, Equilibrium Multiplicity and E-Learnability in the Analysis of the Zero Lower Bound,” unpublished manuscript, Northwestern University.
- CHRISTIANO, L. J., M. EICHENBAUM, AND S. REBELO (2011): “When is the

- government spending multiplier large?," *Journal of Political Economy*, 119, 78–121.
- CORSETTI, G., A. MEIER, AND G. MUELLER (2012): "Fiscal stimulus with spending reversals," *Review of Economic Studies*, forthcoming.
- DRAUTZENBURG, T., AND H. UHLIG (2011): "Fiscal Stimulus and Distortionary Taxation," NBER Working Paper No 17111.
- EGGERTSSON, G., AND B. PUGSLEY (2006): "The Mistake of 1937: A General Equilibrium Analysis," *Monetary and Economic Studies*, 24, 1–58.
- ERCEG, C. J., AND J. LINDE (2010): "Is there a Fiscal Free Lunch in a Liquidity Trap?," Board of Governors of the Federal Reserve System, International Finance Discussion Papers No 1003.
- FUJIWARA, I., AND K. UEDA (2012): "The Fiscal Multiplier and Spillover in a Global Liquidity Trap," CAMA Working Paper 17/2012.
- SMETS, F., AND R. WOUTERS (2007): "Shocks and Frictions in US Business Cycles: a Bayesian DSGE Approach," *American Economic Review*, 97, 586–606.
- WOODFORD, M. (2011): "Simple Analytics of the Government Expenditure Multiplier," *American Economic Journal: Macroeconomics*, 3, 1–35.