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A Model of Money and Credit, With Application to the Credit Card Debt Puzzle

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Many individuals simultaneously have significant credit card debt and money in the bank. The credit card debt puzzle is: given high interest rates on credit cards and low rates on bank accounts, why not pay down debt? While some economists go to elaborate lengths to explain this, we argue it is a special case of the rate of return dominance puzzle from monetary economics. We extend standard monetary theory to incorporate consumer debt, which is interesting in its own right since developing models where money and credit coexist is a long-standing challenge. Our model is quite tractable – e.g., it readily yields nice existence and characterization results – and helps puts into context recent discussions of consumer debt.

Key words: credit card debt puzzle, search model, model with coexisting money and credit, consumer debt, liquidity.

JEL codes: C78, D11, E41

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1 Introduction

A large number of households simultaneously have a significant amount of credit card debt and a significant amount of low-interest liquid assets, such as money in their checking accounts. There are many ways to measure this, and we discuss some of the empirical issues in more detail in Section 4.2, but for now we offer these simple summary statistics from Telyukova (2006): 27% of U.S. households in 2001 had credit card debt and liquid assets both in excess of \$500; and the median household in this group revolved around \$3,800 on their credit cards even though they had \$3,000 in the bank. The so-called *credit card debt puzzle* is this: given 14% interest rates on credit cards, and 1 or 2% on bank accounts, why not pay down debt? According to Gross and Souleles (2001), "Such behavior is puzzling, apparently inconsistent with no-arbitrage and thus inconsistent with any conventional model."

Some economists have gone to elaborate lengths recently to explain such phenomena. For example, some assume that consumers cannot control themselves (Laibson et al. 2000); others assume that they cannot control their spouses (Bertaut and Haliassos 2002; Haliassos and Reiter 2003); and others hypothesize that such households are typically on the verge of bankruptcy (Lehnert and Maki 2001). These ideas are certainly interesting, may contain elements of truth, and will be discussed further below, but from the outset we want to point out that the credit card debt puzzle is actually not a new observation. Rather, it is another manifestation of the venerable *rate of return dominance puzzle* from monetary economics. Hence, insights may be gained by using models and ideas from monetary theory, and in particular, by taking seriously the notion of liquidity.[1]

[1] The idea that agents may hold assets with low rates of return because they are relatively liquid – i.e. because they have an advantage as a medium of exchange – is formalized among other places in Kiyotaki and Wright (1989), but goes back much further in the informal literature. None of this is to say that the credit card debt puzzle is *easy* to explain by recasting it in terms of rate of return dominance, which is an old and difficult issue; what we are saying is that it is *useful* to think about consumer debt through the lens of modern monetary economics.

Our hypothesis is simply that households need to have money readily available – whether this means cash on hand, or in the bank, or generally in the form of some relatively liquid assets – for contingencies where it may be difficult or costly to use credit. In addition to the usual examples like taxis and cigarettes, it is important to note that there are many big-ticket items for which this is the case. For instance, rent or mortgage payments cannot usually be made by credit card. Also, for whatever reason, many unanticipated household expenses including automotive or home repairs (heating and air conditioning, plumbing, etc.) cannot be covered using credit. It can also be difficult to pay for some medical emergencies with a credit card. Clearly, getting caught short in such events can be very costly, and so even if these contingencies are not all that frequent, agents may want to keep some money easily accessible, even if they are revolving relatively high-interest credit card debt.

The rate of return dominance question and the idea that some notion of liquidity ought to be part of the solution go back a long time. Hicks (1935) challenged monetary economists to "look frictions in the face" when framing "the central issue in the pure theory of money" as the need for an explanation of the fact that people hold money when interest rates are positive. The better-known version of his challenge is to explain "the decision to hold assets in the form of barren money, rather than of interest- or profit-yielding securities." But the same issue arises in reverse: "So long as interest rates are positive, the decision to hold money rather than lend it, or *use it to pay off old debts*, is apparently an unprofitable one" (Hicks 1935, emphasis added). This looks a lot like the credit card debt puzzle.

This paper is about using recent developments in monetary economics to address the issue. Unfortunately, for our purposes, there is no off-the-shelf model with money and credit, and it is not trivial to build one. Modern theory makes money essential by imposing some form of anonymity, which makes credit infeasible – indeed, making credit difficult is precisely what makes money essential. Here we assume agents are anonymous in some situations but not others. While this seems natural, one has to specify the environment in such a way that when agents are not anonymous credit is actually useful, which is *not* the case e.g. in Lagos and Wright (2005) because of quasi-linear utility in the centralized market of that model. Moreover, if one does specify assumptions that make credit useful when agents are not anonymous, without care, the analysis quickly becomes intractable, as is the case e.g. if one abandons quasi-linear utility in the Lagos-Wright model. In our framework, money is essential and credit is useful, but the analysis is still simple and yields strong existence and characterization results. These results help us to understand the coexistence of consumer debt and low-interest liquid assets.[2]

2 The Basic Model

Here we describe the physical environment, focusing for now on a somewhat special case to make the main point, then generalizing the results on several dimensions in Section 4 below. To begin, there is a $[0, 1]$ continuum of agents that live forever in discrete time. There is one nonstorable consumption good at each date that individuals produce using labor. Following Lagos and Wright (2005), hereafter LW, we assume agents periodically visit both centralized markets and decentralized markets. Having some decentralized trade, with certain frictions discussed below, makes money essential. Having some centralized trade is interesting for its own sake, and makes the analysis much more tractable than what one sees in models on the microfoundations of money without this feature.[3]

[2]While this is a theory paper, Telyukova (2006) analyzes in detail how a version of the approach presented here can account for salient aspects of the data *quantitatively*. We discuss this and some other empirical issues, such as discriminating between alternative hypotheses, in Section 4.2.

[3]See Molico (2006), Green and Zhou (1998,2002), Camera and Corbae (1999), Zhou (1999) or Zhu (2003,2005) for models where all trade is decentralized. Earlier monetary models like Shi (1995) or Trejos and Wright (1995) are tractable, but only because of the assumption that money is indivisible and agents can carry at most 1 unit. Shi (1996) is a previous search model with money and credit, but he also makes strong assumptions about how much money and credit agents can carry (although, to his credit, he enforces repayment via collateral, while we simply assume repayment).

Money in this economy is a perfectly divisible and storable object that is intrinsically worthless but potentially could have value as a medium of exchange. The money supply is fixed for now at M, but later we allow it to vary over time. Although we use the word money, we do not necessarily literally mean cash. It is not hard to recast the model with agents depositing their cash in bank accounts and paying for goods and services using checks or debit cards, as in He, Huang and Wright (2005, 2006). This is relevant because what we have in mind is relatively liquid assets generally: the money need not be in your pocket, and could be in your bank, but it does need to be easily accessible. Given realistic interest rates on demand deposits, money in the bank is about the same as money in your pocket (except possibly for safety considerations), so we ignore the distinction and assume money here is any perfectly liquid asset with 0 interest.[4]

In LW, each period is divided into two subperiods. In one, there is a centralized Walrasian market, and in the other, there is a decentralized market where agents meet according to a random bilateral-matching process. With the additional assumption that agents are anonymous in the decentralized market, a medium of exchange becomes essential, as is well known; see Kocherlakota (1998), Wallace (2001), Corbae et al. (2003), and Aliprantis et al. (2006) for formal discussions. After each meeting of the decentralized market agents go to a centralized market, where they engage in various activities, including working and rebalancing their money holdings. If utility is linear in some variable, such as hours worked, all agents take the same amount of money out of the centralized market, which keeps things nice and easy.

There is no role for credit in LW, and it is important to understand why. First, credit is not possible in the decentralized market because of the assumption that agents are anonymous, which we cannot relax since this is what makes money essential. Second, credit is not necessary in the centralized market because of the assumption that all agents can work and have utility

[4]Later, in Section 4.1, we introduce an additional real asset that can always be liquidated but only at a fixed cost, and determine its return endogenously.

that is linear in hours, which we do not want to relax since this is what makes the analysis tractable. How to proceed? Our idea is to introduce a third subperiod – generalized later to many subperiods – where some agents want to consume but cannot produce, which makes credit useful, and where we do not assume anonymity, which makes credit feasible. We determine whether agents use cash or credit in this market endogenously, while maintaining an essential role for money plus analytic tractability due to the other two markets.[5]

All agents want to consume in subperiod (market) 1, and $u_1(x_1)$ is their common utility function, which is strictly increasing and weakly concave. A random subset want to consume in $s = 2, 3$, and conditional on this, $u_s(x_s)$ is their utility function, which is strictly increasing and concave. All agents are able to produce in $s = 1$, and the disutility of working h_1 hours is linear, $c_1(h_1) = h_1$. A random subset are able to produce in $s = 2, 3$, and conditional on this, the disutility of working is $c_s(h_s)$, which is strictly increasing and weakly convex. When they can produce, agents transform labor one-for-one into goods, $x_s = h_s$ (it is easy to replace this assumption by a standard labor market with firms having general technologies). At any $s = 2, 3$ a random set of agents chosen in an i.i.d. manner want to consume but cannot produce, and vice-versa, while no one can do both (it is easy to generalize this). Let x_s^* solve $u_s'(x_s^*) = c_s'(x_s^*)$, and let β_s be the discount factor between s and the next subperiod, with $\beta_1\beta_2\beta_3 < 1$.

An individual's state is (m_{st}, b_{st}), denoting money and debt in subperiod s of period t. We drop the t when there is no risk of confusion, writing $m_{st} = m_s$, $m_{s,t+1} = m_{s,+1}$, etc. Let $W_s(m_s, b_s)$ be the value function. At $s = 1, 2$, the market value of money is ϕ_s, so $1/\phi_s$ is the nominal price level; there is no ϕ_3 since there is no centralized market at $s = 3$, although prices will implicitly be defined by whatever trades happen to occur. Similarly, the real interest rate

[5] Berentsen et al. (2005) and Chiu and Meh (2006) also introduce a subperiod where agents are not anonymous into LW, so that credit is feasible. We share this approach of making all agents anonymous in some markets but not others – as opposed to, say, Cavalcanti and Wallace (1999a,b) who assume some agents are anonymous in all markets and some are not anonymous in any market.

in the centralized market at $s = 1, 2$ is r_s, but there is no r_3. Our convention for notation is as follows: if you bring debt b_s into subperiod s you owe $(1 + r_s)b_s$. We assume away all enforcement problems with credit – repayment is assumed exogenously. It may be interesting to make enforcement endogenous, but this would seem to be a distraction for our purposes. Still, it is important to emphasize that this exogenous enforcement does not render money inessential: the assumption is that we can enforce agreements when the agents are known (in centralized markets) but not when they are anonymous (in decentralized markets).

We now consider behavior each subperiod (or, each market) in turn, after which we shall put things together and talk about equilibrium.

2.1 Market 1

At $s = 1$, there is a centralized market where agents solve[6]

$$
\begin{aligned}
W_1(m_1, b_1) &= \max_{x_1, h_1, m_2, b_2} \{u_1(x_1) - h_1 + \beta_1 W_2(m_2, b_2)\} \qquad (1)\\
\text{s.t. } x_1 &= h_1 + \phi_1(m_1 - m_2) - (1 + r_1)b_1 + b_2.
\end{aligned}
$$

Substituting h_1 from the budget constraint, the first-order conditions are

$$
\begin{aligned}
1 &= u_1'(x_1) \qquad (2)\\
\phi_1 &= \beta_1 W_{2m}(m_2, b_2) \qquad (3)\\
-1 &= \beta_1 W_{2b}(m_2, b_2). \qquad (4)
\end{aligned}
$$

where e.g. W_{sm} is the partial of W_s with respect to m. The envelope conditions are

$$
\begin{aligned}
W_{1m}(m_1, b_1) &= \phi_1 \qquad (5)\\
W_{1b}(m_1, b_1) &= -(1 + r_1). \qquad (6)
\end{aligned}
$$

[6] To rule out Ponzi schemes, one normally imposes a credit limit $b_j \le \bar{B}$, either explicitly or implicitly. We impose that agents pay off past debts at $s = 1$ each period, without loss in generality (given they have to pay off debt at some point, they are quite happy to pay it off in market $s = 1$ at any date t since then they have quasi-linear utility). Also, we always assume an interior solution for h; see LW for conditions to guarantee this is valid in these types of models.

Notice (2) implies $x_1 = x_1^*$ for all agents, while (3)-(4) imply (m_2, b_2) is independent of x_1 and (m_1, b_1). Also, as long as W_2 is strictly concave, there will be a unique solution for (m_1, b_1). It is simple to check that the same conditions that guarantee strict concavity in m from LW also apply here, and so $m_2 = M$ for all agents. However, we will see that W_2 is actually linear in b_2, which means we cannot pin down b_2 for any individual. This is no surprise: with a competitive market and quasi-linear utility, in equilibrium, agents are indifferent between the allocation they have and an alternative where they work a little more now, save the proceeds, and work a little less later. Although this is true for any individual, it cannot be true in the aggregate, since average labor input is pinned down by feasibility at $\bar{h}_1 = x_1^*$.

Given this, one can resolve the indeterminacy for individuals in two ways. First, one can focus on symmetric equilibria where all agents choose the same solution when they have the same set of solutions to a maximization problem, which seems innocuous since other equilibria are payoff equivalent and observationally equivalent at the aggregate level; this pins down $b_2 = \bar{b}_2$ for all agents. Alternatively, we could impose an arbitrarily small transaction cost on revolving debt in subperiod 1, which would break agents' indifference and refine away any other equilibria. In what follows we take the former route and focus on symmetric equilibria. Then, aggregating budget equations across agents, we have

$$\bar{x}_1 = \bar{h}_1 + \phi_1(\bar{m}_1 - \bar{m}_2) - (1 + r_1)\bar{b}_1 + \bar{b}_2. \tag{7}$$

In equilibrium, $\bar{h}_1 = \bar{x}_1 = x_1^*$, $\bar{m}_1 = \bar{m}_2 = M$, and $\bar{b}_1 = 0$. Hence, (7) implies $b_2 = 0$, and no agents carry debt out of market 1 into market 2; the interesting issues here instead concern debt acquired in market 2.

2.2 Market 2

At $s = 2$, a measure π of agents want to consume but cannot produce and a measure π can produce but do not want to consume; the remaining $1 - \pi$ do neither. In equilibrium, $x_2^C = h_2^P$, where x_2^C is the consumption of consumers and h_2^P the production of producers. The expected value of entering market 2 is therefore

$$W_2(m_2, b_2) = \pi W_2^C(m_2, b_2) + \pi W_2^P(m_2, b_2) + (1 - 2\pi) W_2^N(m_2, b_2), \tag{8}$$

where W_2^C, W_2^P and W_2^N are the value functions for a consumer, a producer and a nontrader. It is tedious but useful to study their problems one at a time.

For a nontrader,

$$\begin{aligned} W_2^N(m_2, b_2) &= \max_{m_3, b_3} \beta_2 W_3(m_3, b_3) \\ \text{s.t. } 0 &= \phi_2(m_2 - m_3) - (1 + r_2) b_2 + b_3. \end{aligned}$$

Although nontraders neither consume nor produce, they can adjust their portfolios, but we will see below that in equilibrium they choose not to. The solution (m_3^N, b_3^N) satisfies

$$W_{3m}(m_3^N, b_3^N) = -\phi_2 W_{3b}(m_3^N, b_3^N), \tag{9}$$

plus the budget equation. The envelope conditions are

$$W_{2m}^N(m_2, b_2) = \beta_2 W_{3m}(m_3^N, b_3^N) \tag{10}$$

$$W_{2b}^N(m_2, b_2) = \beta_2 (1 + r_2) W_{3b}(m_3^N, b_3^N). \tag{11}$$

For a consumer,

$$\begin{aligned} W_2^C(m_2, b_2) &= \max_{x_2, m_3, b_3} \{u_2(x_2) + \beta_2 W_3(m_3, b_3)\} \\ \text{s.t. } x_2 &= \phi_2(m_2 - m_3) - (1 + r_2) b_2 + b_3. \end{aligned}$$

The solution (x_2^C, m_3^C, b_3^C) satisfies

$$\phi_2 u_2'(x_2^C) = \beta_2 W_{3m}(m_3^C, b_3^C) \tag{12}$$

$$-u_2'(x_2^C) = \beta_2 W_{3b}(m_3^C, b_3^C) \tag{13}$$

plus the budget equation. The envelope conditions are

$$W^C_{2m}(m_2, b_2) = \phi_2 u_2'(x_2^C) = \beta_2 W_{3m}(m_3^C, b_3^C) \tag{14}$$

$$W^C_{2b}(m_2, b_2) = -(1+r_2)u_2'(x_2^C) = (1+r_2)\beta_2 W_{3b}(m_3^C, b_3^C). \tag{15}$$

For a producer,

$$\begin{aligned} W_2^P(m_2, b_2) &= \max_{h_2, m_3, b_3} \{-c_2(h_2) + \beta_2 W_3(m_3, b_3)\} \\ \text{s.t.}\, 0 &= h_2 + \phi_2(m_2 - m_3) - (1+r_2)b_2 + b_3. \end{aligned}$$

The solution (h_2^P, m_3^P, b_3^P) satisfies

$$\phi_2 c_2'(h_2^P) = \beta_2 W_{3m}(m_3^P, b_3^P) \tag{16}$$

$$-c_2'(h_2^P) = \beta_2 W_{3b}(m_3^P, b_3^P) \tag{17}$$

plus the budget equation. The envelope conditions are

$$W^P_{2m}(m_2, b_2) = \phi_2 c_2'(h_2^P) = \beta_2 W_{3m}(m_3^P, b_3^P) \tag{18}$$

$$W^P_{2b}(m_2, b_2) = -(1+r_2)c_2'(h_2^P) = (1+r_2)\beta_2 W_{3b}(m_3^P, b_3^P). \tag{19}$$

We cannot conclude that (m_3, b_3) is independent of (m_2, b_2), the way we could conclude that (m_2, b_2) is independent of (m_1, b_1) in the previous subperiod, since in this market we generally do not assume u_2 and c_2 are linear. But in any case,

$$W_{2m}(m_2, b_2) = \beta_2[\pi W_{3m}(m_3^C, b_3^C) + \pi W_{3m}(m_3^P, b_3^P) + (1-2\pi)\, W_{3m}(m_3^N, b_3^N)] \tag{20}$$

$$W_{2b}(m_2, b_2) = \beta_2(1+r_2)[\pi W_{3b}(m_3^C, b_3^C) + \pi W_{3b}(m_3^P, b_3^P) + (1-2\pi)\, W_{3b}(m_3^N, b_3^N)] \tag{21}$$

2.3 Market 3

In market 3 we assume trade occurs via anonymous bilateral meetings. Also, we use generalized Nash bargaining, although this is *not* crucial – e.g. Aruoba, Rocheteau and Waller (2007) analyze similar models with other bargaining solutions, Rocheteau and Wright (2005) consider price taking and price posting, while Kircher and Galenianos (2006) and Dutu, Julien and King (2007) consider versions with auctions, and the key results go through with these alternative pricing mechanisms. In any case, in market 3, you cannot use credit, due to anonymity: I will not take your IOU because I know you could renege without fear of punishment.[7] There is still an issue as to why some institution that is not anonymous does not issue interest-bearing claims – i.e. private money – that circulate in market 3. One answer is to say that such claims can be counterfeited (the government has a monopoly on the production of non-counterfeitable notes), which is a strong assumption but one that is logically consistent and provides a role for both some form of money and credit.[8]

Consider a meeting where one agent wants to consume and the other can produce. Call the former agent the *buyer* and the latter the *seller*. They bargain over the amount of consumption for the buyer x_3 and labor by the seller h_3, and also a dollar payment d from to the former to the latter. Since feasibility implies $x_3 = h_3$, we denote their common value by q. If (m_3, b_3) is the state of a buyer and $(\tilde{m}_3, \tilde{b}_3)$ the state of a seller, the outcome satisfies the generalized Nash bargaining solution,

$$(q, d) \in \arg\max \ S(m_3, b_3)^{\theta} \tilde{S}(\tilde{m}_3, \tilde{b}_3)^{1-\theta} \text{ s.t. } d \leq m_3, \tag{22}$$

where the constraint says an agent cannot give more money than he has, θ is the bargaining

[7]Anonymity is a logically coherent assumption making money essential, although one need not take it literally as a description of all monetary trade; as we discuss later, in principle alternative motives for using cash (e.g. tax avoidance) could work for our purposes.

[8]As we said above, there are related models where agents deposit money in institutions like banks and pay with instruments like checks or debit cards in the decentralized market, assuming these are also not too easy to counterfeit. This works even if individuals are anonymous because the instruments are claims on the bank and not on the consumer – consider e.g. travellers' checks. Interest on this private (inside) money is endogenous, but generally will be less than the market rate on consumer credit for several reasons, including bank operating costs, legal restrictions, or the need for banks to hold some low-interest assets as reserves. All we need is that rates on liquid assets like demand deposits are below those on consumer credit.

power of the buyer, and the surpluses are given by

$$\begin{aligned} S(m_3, b_3) &= u_3(q) + \beta_3 W_{1,+1}(m_3 - d, b_3) - \beta_3 W_{1,+1}(m_3, b_3) \\ \tilde{S}(\tilde{m}_3, \tilde{b}_3) &= -c_3(q) + \beta_3 W_{1,+1}(\tilde{m}_3 + d, \tilde{b}_3) - \beta_3 W_{1,+1}(\tilde{m}_3, \tilde{b}_3). \end{aligned}$$

Using (5) and (6), we have the neat simplification

$$\begin{aligned} S(m_3, b_3) &= u_3(q) - \beta_3 \phi_{1,+1} d \\ \tilde{S}(\tilde{m}_3, \tilde{b}_3) &= -c_3(q) + \beta_3 \phi_{1,+1} d. \end{aligned}$$

This yields the following generalization of LW (a proof is in the Appendix).

Lemma 1. *$\forall (m_3, b_3)$ and $(\tilde{m}_3, \tilde{b}_3)$, the solution to the bargaining problem is*

$$q = \begin{cases} g^{-1}(\beta_3 m_3 \phi_{1,+1}) & \text{if } m_3 < m_3^* \\ q^* & \text{if } m_3 \geq m_3^* \end{cases} \quad \text{and } d = \begin{cases} m_3 & \text{if } m_3 < m_3^* \\ m_3^* & \text{if } m_3 \geq m_3^* \end{cases} \tag{23}$$

where q^* solves $u_3'(q^*) = c_3'(q^*)$, the function $g(\cdot)$ is given by

$$g(q) \equiv \frac{\theta u_3'(q) c_3(q) + (1-\theta) u_3(q) c_3'(q)}{\theta u_3'(q) + (1-\theta) c_3'(q)}, \tag{24}$$

and $m_3^* = g(q^*)/\beta_3 \phi_{1,+1}$.

Observe that the bargaining solution (q, d) depends on the buyer's money holdings m_3 and not on any other element of (m_3, b_3) or $(\tilde{m}_3, \tilde{b}_3)$; hence we write $q = q(m_3)$ and $d = d(m_3)$ from now on. We also show in the Appendix, as in LW, that $m_3 < m_3^*$ in any equilibrium. Hence, buyers always spend all their money in market 3, $d(m_3) = m_3$, and receive $q = g^{-1}(\beta_3 m_3 \phi_{1,+1}) < q^*$ in return, which yields $\partial q / \partial m_3 = \beta_3 \phi_{1,+1} / g'(q) > 0$.

Let σ be the probability of a meeting between a buyer and seller in market 3. Then

$$\begin{aligned} W_3(m_3, b_3) = \sigma \left\{ u_3[q(m_3)] + \beta_3 W_1[m_3 - d(m_3), b_3] \right\} \\ + \sigma \mathbb{E} \left\{ -c_3[q(\tilde{m}_3)] + \beta_3 W_1[m_3 + d(\tilde{m}_3), b_3] \right\} + (1 - 2\sigma) \beta_3 W_1[m_3, b_3], \end{aligned} \tag{25}$$

where $\mathbb{E}$ is the expectation of $\tilde{m}_3$ (the money holdings of a random agent one meets, which in equilibrium is actually degenerate at $\tilde{m}_3 = M$). Differentiating (25) using (5) and (6),

$$W_{3m}(m_3, b_3) = \beta_3 \phi_{1,+1} \{\sigma e [q(m_3)] + 1 - \sigma\} \tag{26}$$

$$W_{3b}(m_3, b_3) = -\beta_3 (1 + r_{1,+1}), \tag{27}$$

where the function $e(\cdot)$ in (26) is given by

$$e(q) \equiv u_3'(q)/g'(q), \tag{28}$$

with $g(q)$ defined in (24). We assume $e'(q) < 0$; sufficient conditions for this can be found in LW.[9]

3 Equilibrium

Our definition of equilibrium is relatively standard, except there is no market-clearing condition for market 3, since with bilateral trade it clears automatically. Also, to reduce notation, we describe every agent's problem at $s = 1, 2$ in terms of choosing $(x_s, h_s, m_{s+1}, b_{s+1})$, which are implicitly functions of the state, and it is understood that for producers $x_2^P = 0$, for consumers $h_2^C = 0$, and for nontraders $x_2^N = h_2^N = 0$.[10]

Definition 1. *An* equilibrium *is a set of (possibly time-dependent) value functions* $\{W_s\}$, $s = 1, 2, 3$, *decision rules* $\{x_s, h_s, m_{s+1}, b_{s+1}\}$, $s = 1, 2$, *bargaining outcomes* $\{q, d\}$, *and prices* $\{r_s, \phi_s\}$, $s = 1, 2$, *such that:*

[9] A simple assumption that works for any preferences is $\theta \approx 1$, since $\theta = 1$ implies $g(q) = c(q)$. But even if we do not have $e'(q) < 0$ for all q, it is easy to check that $e'(q) < 0$ for any q that satisfies the second order conditions to the maximization problem in market 2, so this assumption is really not much of a restriction.

[10] We do not include the distribution of the state variable in the definition of equilibrium, but it is implicit: given an initial distribution $F_1(m, b)$ at the start of subperiod 1, the decision rules generate $F_2(m, b)$; then the decision rules at $s = 2$ generate $F_3(m, b)$; and the bargaining outcome at $s = 3$ generates $F_{1,+1}(m, b)$. Also, as we said above, we only consider equilibria where we have an interior solution for h.

1. *Optimization: In every period, for every agent,* $\{W_s\}$*,* $s = 1, 2, 3$*, solve the Bellman equations (1), (8) and (25);* $\{x_s, h_s, m_{s+1}, b_{s+1}\}$*,* $s = 1, 2$*, solve the relevant maximization problems; and* $\{q, d\}$ *solve the bargaining problem.*

2. *Market clearing: In every period,*

$$\bar{x}_s = \bar{h}_s,\ \bar{m}_{s+1} = M,\ \bar{b}_{s+1} = 0,\ s = 1, 2$$

where for any variable y*,* $\bar{y} = \int y^i di$ *denotes the aggregate.*

Definition 2. *A steady state equilibrium is an equilibrium where the endogenous variables are constant across periods, although generally not across subperiods within a period.*

We are mainly interested in equilibria where money is valued, which means it must be valued in all subperiods in every period.

Definition 3. *A monetary equilibrium is an equilibrium where, in every period,* $\phi_s > 0$*,* $s = 1, 2$*, and* $q > 0$*.*

Theorem 1. *There exists a steady state monetary equilibrium, and it is characterized by:*

1. *At* $s = 1$*, all agents choose the same consumption* $x_1 = x_1^*$*, portfolio* $(m_2, b_2) = (M, 0)$*, and as a function of their individual states* (m_1, b_1) *hours*

$$h_1 = h_1(m_1, b_1) = x_1^* - \phi_1(m_1 - M) + (1 + r_1)b_1,$$

which implies that on average h_1 *is* $\bar{h}_1 = x_1^*$*.*

2. *At* $s = 2$*,*

consumers choose $x_2 = x_2^*$*,* $m_3 = M$ *and* $b_3 = x_2^*$*;*

producers choose $h_2 = x_2^*$*,* $m_3 = M$ *and* $b_3 = -x_2^*$*;*

nontraders choose $m_3 = M$ *and* $b_3 = 0$*.*

3. At $s = 3$, in every trade $d = M$ and q solves

$$1 + \frac{\rho}{\sigma} = e(q), \tag{29}$$

where $e(q)$ is given by (28) and ρ is defined by $\frac{1}{1+\rho} = \beta_1\beta_2\beta_3$, which implies $q < q^*$.

4. Prices are given by:

$$\begin{aligned} r_1 &= \frac{u_2'(x_2^*) - \beta_2\beta_3}{\beta_2\beta_3},\ r_2 = \frac{\rho - r_1}{1+r_1}, \\ \phi_1 &= \frac{g(q)}{\beta_3 M},\ \textit{and}\ \phi_2 = \frac{\phi_1\left[\sigma e(q) + 1 - \sigma\right]}{1+r_1}. \end{aligned}$$

Proof: To begin, insert the envelope condition for W_{3b} from (27) into (13) and (17) to get

$$u_2'(x_2^C) = \beta_2\beta_3(1 + r_{1,+1}) \tag{30}$$

$$c_2'(h_2^P) = \beta_2\beta_3(1 + r_{1,+1}). \tag{31}$$

This implies $u_2'(x_2^C) = c_2'(h_2^P)$, and hence $x_2^C = h_2^P = x_2^*$. Similarly, insert the envelope condition for W_{3m} from (26) into the first order conditions (12) and (16) to get

$$\phi_2 u_2'(x_2^C) = \beta_2\beta_3\phi_{1,+1}\left\{\sigma e\left[q(m_3^C)\right] + 1 - \sigma\right\} \tag{32}$$

$$\phi_2 c_2'(h_2^P) = \beta_2\beta_3\phi_{1,+1}\left\{\sigma e\left[q(m_3^P)\right] + 1 - \sigma\right\}. \tag{33}$$

Given $e'(q) < 0$ and $q'(m) > 0$ for all $m < m_3^*$, plus $x_2^C = h_2^P = x_2^*$, we conclude $m_3^C = m_3^P$.

Similarly, inserting (27) and (26) into the first order condition for a nontrader,

$$\phi_{1,+1}\left\{\sigma e\left[q(m_3^N)\right] + 1 - \sigma\right\} = \phi_2(1 + r_{1,+1}). \tag{34}$$

Exactly the same condition results from combining (30) and (32) for a consumer, or (31) and (33) for a producer. Hence, we conclude $m_3^N = m_3^C = m_3^P = M$. From the budget equations,

$$\begin{aligned} b_3^C &= x_2^* + (1 + r_2)b_2 \\ b_3^P &= -x_2^* + (1 + r_2)b_2 \\ b_3^N &= (1 + r_2)b_2. \end{aligned}$$

This completes the description of market 2. Moving back to market 1, clearly (2) implies $x_1 = x_1^*$. Inserting the envelope conditions for W_2 and W_3 into (3) and (4), we have

$$\phi_1 = \beta_1\beta_2\beta_3\phi_{1,+1}\{\sigma e[q(M)] + 1 - \sigma\} \tag{35}$$

$$1 = \beta_1\beta_2\beta_3(1 + r_2)(1 + r_{1,+1}), \tag{36}$$

where we use in the first case the result that W_{3m} depends on m_3 but not b_3, and $m_3 = M$. Notice (36) is an arbitrage condition between r_2 and $r_{1,+1}$: if it does not hold there is no solution to the agents' problem at $s = 1$; and if it does hold then any choice of b_2 is consistent with optimization. Hence we can set $b_2 = 0$. On the other hand, (35) implies

$$(1 + \rho)\frac{\phi_1}{\phi_{1,+1}} = \sigma e[q(M)] + 1 - \sigma. \tag{37}$$

In steady state this implies (29). It is now standard (see LW) to show $q < q^*$.

This pins down the allocation. Now consider prices. We get r_1 from (30) with $x_2 = x_2^*$, and then set r_2 in terms of r_1 to satisfy the arbitrage condition (36). Given q, Lemma 1 tells us ϕ_1, and (34) tells us ϕ_2, as described in the statement of the Theorem. This is all we need for the definition of equilibrium (plus the value functions, but these are obvious). By construction, this constitutes a steady state monetary equilibrium. ■

Theorem 2. *(Rate of Return Dominance) In any steady state monetary equilibrium,*

$$\frac{\phi_{1,+1}}{\phi_2} < 1 + r_{1,+1}.$$

Proof: By (34),

$$\frac{\phi_{1,+1}}{\phi_2} = \frac{1 + r_{1,+1}}{1 - \sigma + \sigma e(q)}.$$

The result follows if $e(q) > 1$. By (29), in steady state $e(q) = 1 + \rho/\sigma > 1$. ■

4 Discussion

Here we make several remarks concerning the above Theorems and argue that they are robust to a variety of extensions. We also discuss their economic content and empirical implications. To begin, the central result of Theorem 1 is that, at $s = 2$, consumers use credit ($b_3 = x_2^*$) even though they have cash on hand ($m_2 = m_3 > 0$). The reason is that they know they may need the money at $s = 3$, when credit is not available. Notice also that the proof not only characterizes equilibrium and establishes existence, basically by construction, it also establishes that the steady state is unique, which is important for the following reason: we not only can claim that there is *some* steady state equilibrium where consumers with cash choose to use credit, we can also claim that this *must* happen, since this equilibrium is unique.[11] Theorem 2 goes on to say that agents with cash choose to use credit despite rate of return dominance – i.e. despite credit being costly in terms of interest payments.

To see this more clearly, note that (34) equates the value of a dollar's worth of cash and a dollar's worth of credit/debt coming out of market 2. The left side is a weighted average of the marginal gain if the dollar is spent in market 3, $u'(q)q'(m_3) = \beta_3\phi_{1,+1}e(q)$, and the return if it is not spent but carried forward to the next period, $\beta_3\phi_{1,+1}$. The right side is the real return (the interest saved) from using the same dollar to pay down debt, $\beta_3\phi_2(1+r_{1,+1})$. A key observation is that the return on money includes a liquidity premium: since $e(q) > 1$ in equilibrium, the value of spending a dollar at $s = 3$ is higher than the value of carrying it to next period. If one ignores this premium, and simply considers the return on carrying money across periods, then it looks like – indeed *it is* – rate of return dominance. Of course, there is a liquidity premium in

[11] Recall is the above-mentioned caveat concerning uniqueness: we impose in market 1 that all agents choose the same solution for b_2 when they have multiple solutions. As we said, other equilibria are payoff equivalent and observationally equivalent in the aggregate, so this is not much of a restriction. In any other equilibrium, prices and consumption are actually exactly as stated in Theorem 1, but individuals may choose to roll over debt between periods, which affects the timing of their labor supply but does not not any of the results concerning money and credit.

other monetary models, including LW; the big difference here is that agents make real choices to use money or credit, and both are essential in the sense that the economy is worse off if we arbitrarily shut down either one.[12]

4.1 Theoretical Extensions

Here we consider several technical extensions, and show the economic content is robust. First note that in *any* equilibrium, and not just in steady state, essentially everything in Theorems 1 and 2 holds, except that (37) does not reduce to (29). Instead, we can insert $g(q) = \beta_3 m_3 \phi_{1,+1}$ from Lemma 1 to get

$$(1+\rho)\frac{g(q)}{g(q_{+1})} = \sigma e(q_{+1}) + 1 - \sigma. \tag{38}$$

Monetary equilibria are given by (positive) bounded paths $\{q_t\}$ solving (38), along with values for the other objects satisfying the conditions above. As in most monetary models, there are multiple equilibrium paths $\{q_t\}$, including exotic dynamic and sunspot equilibria (Lagos and Wright 2003), but in all of these equilibria x_s, b_s, and r_s are exactly as in Theorem 1, and although ϕ_s will change over time, Theorem 2 holds as stated.

Another generalization is to allow the money supply to vary over time. Suppose e.g. $M_{+1} = (1+\gamma)M$, with money being injected or withdrawn in market 1 via lump sum transfers or taxes. Consider equilibria where all real variables, including q and ϕM, are stationary, which means $\phi_1/\phi_{1,+1} = 1+\gamma$ and (37) becomes $(1+\rho)(1+\gamma) = \sigma e[q(M)] + 1 - \sigma$. From the Fisher equation,

[12] We mention some technical details here. First, one can always price in the model a nominal bond traded in $s = 2$ at t and redeemed in $s = 1$ at $t+1$ using the Fisher equation, which yields the nominal interest rate $1 + i_{1,+1} = (1 + r_{1,+1})\,\phi_2/\phi_{1,+1}$ (for the sake of this discussion, assume these bonds are illiquid – they cannot be used as a medium of exchange in market 3, either because they are not tangible objects but merely book-keeping entries, say, or because they are counterfeitable in that market). Then Theorem 2 can be equivalently stated as $i_{1,+1} > 0$. The nominal rate is the opportunity cost of carrying cash, which agents are willing to pay, for the benefit of liquidity. Also, the results above are framed in terms of rates of return between $s = 2$ at t and $s = 1$ at $t+1$, because it is at $s = 2$ that the decision is made to pay with cash or credit, but we can alternatively measure returns over the entire period. From $s = 1$ at t to $s = 1$ at $t+1$, the gross return on money in steady state is 1, while the return on credit (paying down debt) is $(1 + r_2)(1 + r_{1,+1})$. We readily get $(1 + r_2)(1 + r_{1,+1}) > 1$ from (36), so rate of return dominance also holds across the entire period.

the left side is $1+i$, so we get

$$1+\frac{i}{\sigma}=e(q). \tag{39}$$

Thus, q is decreasing in i, but this does not affect the real allocation in markets 1 and 2. As is standard, the Friedman rule $i=0$ is achieved by a policy setting $\gamma=\beta_1\beta_2\beta_3-1$, and provides a lower bound on γ. At the Friedman rule, the returns on money and credit are the same; for any other feasible policy, we still get rate of return dominance.[13]

Next, note that although the baseline 3-subperiod model has some agents carrying debt and money simultaneously, they never need to roll over this debt for more than a period – they *could* roll it over, but as long as they have to pay it off sometime, given quasi-linear utility at $s=1$ this is as good a time as any. Of course, this does not mean the model speaks only to high-frequency observations, since we can make a period as long as we like (indeed, in an overlapping generations version, we could make it a lifetime). However, if one wants debt rollover to be more complicated, we can extend this version of the model to have n subperiods. For this exercise we will let the centralized and decentralized markets be open simultaneously each subperiod, and have agents transit between them as follows: those in the centralized market at s move to the decentralized market at $s+1$ with probability δ_s; and those in the decentralized market at s move to the centralized market at $s+1$ with probability 1.[14]

As long as $\delta_s>0$, agents are willing to pay an opportunity cost of carrying money at s, since they might need it at $s+1$. For convenience, set $\delta_n=0$, so everyone is in the centralized market at $s=1$, and let them all produce and have quasi-utility linear at $s=1$, so they settle all their debts at $s=1$, as in the benchmark model. In each $s\in\{2,...,n\}$, the centralized markets are

[13]Since this is a paper on positive and not normative economics, we take the policy i as exogenous and do not ask why it is what it is. It is of course optimal here to set $i=0$, for all the usual reasons, plus one: we simply assume the enforcement of credit is feasible in centralized markets, but in reality, as a referee pointed out, even if it is feasible it need not be free. At the Friedman rule $i=0$ agents are happy to use money for all trades, and we can save any costs associated with credit.

[14]The convenient aspect of this specification, adapted from Williamson (2005), is that one is never in a decentralized market two periods in a row, as in our baseline model.

like market 2 in the benchmark, except now we can more generally let productivity ω_s differ across agents and subperiods in any i.i.d. manner ($\omega_s = 0$ is the case where you cannot produce at all). Also, let agents in the centralized markets at $s > 1$ now have general utility functions $U_s(x_s, h_s)$. In the decentralized markets, agents trade bilaterally, exactly as in market 3 in the benchmark model.

In the working paper (Telyukova and Wright 2006) we establish the following generalizations of Theorems 1 and 2 for the model with n subperiods:

Theorem 3. *There exists a steady state monetary equilibrium and it implies:*

1. *For all s, every agent leaves the centralized market with the same $m_{s+1} = M$.*
2. *For all s, $U_{sx}(x_s, h_s) = k_s$ and $U_{sh}(x_s, h_s) = -k_s\omega_s$ where k_s is constant across agents in the centralized market.*
3. *If two agents have different (m_s, b_s) and the same productivity ω_s, their h_s, x_s and m_{s+1} are the same, so they have different b_{s+1}; if two agents have the same (m_s, b_s) and different ω_s, their h_s will differ and they typically have different b_{s+1}.*
4. *Agents may roll over or run up debt between $s = 2$ and $s = n$ while maintaining their holdings of m_s.*

Theorem 4. *In monetary equilibrium, for all $s \neq n$,*

$$\frac{\phi_{1,+1}}{\phi_s} < (1 + r_{s+1})(1 + r_{s+2})...(1 + r_n)(1 + r_{1,+1}). \tag{40}$$

Hence the general framework easily accommodates many trading rounds between times when agents settle, and they sometimes roll over and even run up debt for several rounds, at positive interest rates, while holding money. Given this is clear, for simplicity we revert to the baseline model with 3 markets from now on.

A referee suggested the next generalization: in market 3, in addition to having a probability σ of an anonymous meeting where you cannot use credit, there is a probability $\tilde{\sigma}$ of a nonanonymous meeting where you can. One can interpret this in terms of preference shocks: sometimes you want a good produced by someone who knows you, so you do not need cash; other times you want a good produced by someone who does not, so you need cash. It is easy to prove that in nonanonymous meetings, where credit is feasible, agents trade the efficient quantity $q = q^*$, and are indifferent between payments in any combination of cash and credit with the same value (although they must use *some* credit since m_3 is never enough in equilibrium to afford q^*). The value of q in anonymous meetings satisfies (29), as before, which depends on σ but not on $\tilde{\sigma}$. When $\sigma \to 0$, which must happen when $\tilde{\sigma} \to 1$, money becomes worthless; but for any $\tilde{\sigma} < 1$, as long as we have $\sigma > 0$ all our results go through.

The next theoretical extension was inspired by another referee, who was concerned that although the baseline model may account for the observation that consumers choose not to pay down debt when they have cash on hand, it could not account for the observation that they also hold other assets, that are less liquid than cash and have rates of return that are positive but lower than those on debt. To address this, we now suppose there is another asset a, in addition to m and b.[15] For concreteness, suppose it is a standard "Lucas tree" in fixed supply A that pays a real dividend δ each period at $s = 1$, and can be traded at price ψ_s at $s = 1, 2$. It can also be used as a means of payment at $s = 3$, but only if one pays a fixed cost p in terms of utility (it would be easy to also make it in terms of money). This is meant to capture the "penalty for early withdrawal" on some time deposits, the fixed cost involved in taking out a second mortgage or home equity loan, and so on. Thus, a is less liquid than m, but it can be accessed if necessary.

[15] This is related to recent work by Lagos (2006), Lagos and Rocheteau (2006), and Geromichalos, Licari and Suárez-Lledó (2006) on search models with multiple assets, but the important additional assumption, for our purposes, is that we introduce a fixed cost to liquidating one of them.

The market 1 problem is now

$$\begin{aligned} W_1(m_1, b_1, a_1) &= \max_{x_1, h_1, m_2, b_2, a_2} \{u_1(x_1) - h_1 + \beta_1 W_2(m_2, b_2, a_2)\} \\ \text{s.t. } \ x_1 &= h_1 + \ \phi_1(m_1 - m_2) - b_1(1 + r_1) + b_2 + a_1(\psi_1 + \delta) - \psi_1 a_2 \end{aligned}$$

We get the same first-order and envelope conditions (2)-(6) as above, plus obvious new conditions concerning a_3. The results go through about everyone choosing the same portfolio $(m_2, b_2, a_2) = (M, 0, A)$, and W being linear. At $s = 2$, we have the first-order and envelope conditions from the baseline model plus the obvious conditions concerning a_3. At $s = 3$, agents meet and bargain over q and payments d_m and d_a in terms of m and a, taking into account that the buyer has to pay liquidation cost $p(d_a)$, with $p(d_a) = p > 0$ if $d_a > 0$ and $p(0) = 0$.

For simplicity, at $s = 3$ we assume $\theta = 1$ (take-it-or-leave-it offers by buyers) and $c(q) = q$. The key assumption is that a buyer's preferences are random: with probability σ he has a *regular* meeting with utility $u_3(q)$; with probability $\hat{\sigma}$ he has an *emergency* meeting with $\hat{u}_3(\hat{q})$, where $\hat{u}_3'(q) > u_3'(q)$ for all q. With $\theta = 1$, in a regular meeting the buyer chooses (q, d_m, d_a) to solve

$$\begin{gathered} \max_{q, d_m, d_a} \{u_3(q) - \beta_3[\phi_{1,+1} d_m + (\psi_{1,+1} + \delta) d_a] - p(d_a)\} \\ \text{s.t. } \ q = \beta_3[\phi_{1,+1} d_m + (\psi_{1,+1} + \delta) d_a], \ d_m \le m_3, \ d_a \le a_3; \end{gathered}$$

and in an emergency meeting he chooses $(\hat{q}, \hat{d}_m, \hat{d}_a)$ to solve an analogous problem. The usual reasoning leads to

$$\begin{aligned} W_3(m_3, b_3, a_3) &= \sigma\{u_3(q) - p(d_a) + \beta_3 W_1(m_3 - d_m, b_3, a_3 - d_a)\} \\ &\quad + \hat{\sigma}\{\hat{u}_3(\hat{q}) - p(\hat{d}_a) + \beta_3 W_1(m_3 - \hat{d}_m, b_3, a_3 - \hat{d}_a)\} \\ &\quad + (1 - \sigma - \hat{\sigma})\beta_3 W_1(m_3, b_3, a_3). \end{aligned}$$

We look for an equilibrium where $d_m = \hat{d}_m = m_3$, while $d_a = 0$ and $\hat{d}_a > 0$ – i.e., buyers regularly spend all of m but do not touch a, and they liquidate a just in cases of emergency.

It should be clear that we can always pick the liquidation cost p so that we get $d_a = 0$ and $\hat{d}_a > 0$. As always, we have $q < q^*$. One can also show now that we can either have $\hat{q} < \hat{q}^*$ or $\hat{q} = \hat{q}^*$ depending on whether the asset supply A is below or above some cutoff value (see the method in Geromichalos et al. 2006). Let us assume A is low, so that $\hat{d}_a = a_3$ and $\hat{q} < \hat{q}^*$ (when you liquidate you spend all your a holdings, but even this is not enough to get $\hat{q}^*$). After some algebra, we have

$$
\begin{aligned}
W_{3b}(m_3, b_3, a_3) &= -\beta_3(1 + r_{1,+1}) \\
W_{3a}(m_3, b_3, a_3) &= \beta_3(\psi_{1,+1} + \delta)[\hat{\sigma}\hat{u}_3'(\hat{q}) + 1 - \hat{\sigma}] \\
W_{3m}(m_3, b_3, a_3) &= \beta_3\phi_{1,+1}[\sigma u_3'(q) + \hat{\sigma}\hat{u}_3'(\hat{q}) + 1 - \sigma - \hat{\sigma}].
\end{aligned}
$$

Given this, one can compute the rates of return on the assets as follows:

$$
\begin{aligned}
1 + i_b &= (1 + r_1)(1 + r_2) = 1 + \rho \\
1 + i_a &= \frac{\psi_{1,+1} + \delta}{\psi_1} = \frac{1 + \rho}{\hat{\sigma}u_3'(\hat{q}) + 1 - \hat{\sigma}} \\
1 + i_m &= \frac{\phi_{1,+1}}{\phi_1} = \frac{1 + \rho}{\sigma u_3'(q) + \hat{\sigma}\hat{u}_3'(\hat{q}) + 1 - \sigma - \hat{\sigma}}
\end{aligned}
$$

This yields *generalized rate of return dominance*: $i_b > i_a > i_m$. The reason is of course that b has no liquidity premium, m has a high liquidity premium, and a is somewhere in between. Consumers at $s = 2$ do not pay down debt even though they hold both m and a. They do not pay down debt with m at $s = 2$ because they know that with some probability they will find themselves in a situation at $s = 3$ where they want to consume, credit is not available, and a little bit of cash gets them by. They do not pay down debt with a at $s = 2$ because they know there is some probability they need a lot of money, and they value the fact that they can liquidate a in an emergency, as long as they pay the fixed cost p. So, as this three-asset extension makes clear, the fact that agents have high-interest debt while holding not only low-interest cash, but also other medium-interest assets, is not a problem in principle for liquidity-based theory.

4.2 Empirical Issues

Here we briefly comment on several empirical issues. First, recall the following: if one looks at the population in 2001 and considers those holding more than \$500 in liquid assets and more than \$500 in credit card debt, which Telyukova (2006) calls the puzzle group, they constitute 27% of the population. Gross and Souleles (2001) defined the puzzle group differently. First, they consider everyone who revolves *some* debt and holds *any* positive amount of liquidity, which in their 1995 data amounts to around 34% of the population. But they clearly recognize that people might need to keep some money on hand for transaction purposes, and attempt to control for this. To account for some liquidity demand they allow households to hold one month's income in liquid assets before counting them in their puzzle group, reducing its size to 11%. It is this 11% that they say cannot be explained with a transaction-based story.

The first thing to point out is that although Gross and Souleles suggest that allowing agents to hold one month's income in liquid assets is "arguably generous," one could also say it is "completely arbitrary." Why is a month's income the right number? Telyukova (2006) finds in the 2001 Consumer Expenditure Survey that the median household in her puzzle group, with \$3,800 in credit card debt and \$3,000 in the bank, purchased goods worth \$1,993 per month using liquid assets, amounting to around two-thirds of their monthly income. So one month's income allows for very little precautionary liquidity demand. As we said, precautionary demand can emerge from the possibility of shocks involving home or auto repairs, medical emergencies, etc., and it is not so much that these shocks necessarily happen often – although they might – but that they can be very costly when one is not financially prepared. Hence, it is by no means a foregone conclusion that the 11% puzzle group identified by Gross and Souleles could not be holding liquidity for precautionary reasons.[16]

[16] Income shocks also affect precautionary demand: even if expenses like mortgage payments are predictable, in case of a fall in income opportunities, you better have a little something in the bank to make the payments.

We adopt a broader notion of the puzzle group, amounting to 27% of the population, and take the position that the behavior of all these agents would be hard to understand based on simple economic theory, but not so hard based on a theory incorporating liquidity. Therefore liquidity-based models are useful. Now it is obviously difficult to measure exactly how much people desire liquidity based on actual expenditures out of liquid assets (unless they are completely risk neutral, which is not a very attractive assumption). The more liquidity matters, the more we can explain using a liquidity-based approach. Allowing it to matter less and focusing on a smaller puzzle group, as in Gross and Souleles, makes the phenomenon harder to explain but perhaps also diminishes its importance – would we worry if *one* individual in the data had an especially bizarre asset position? We are happy thinking there is a large group that is easier to understand once we introduce liquidity, even if there are some people we do not explain based on this approach.

From this perspective, one could ask *how much* we can explain, rather than if we explain everything. This is a fairly complicated quantitative question, and as such a serious attempt to answer it belongs in a different paper. But we can summarize some of the findings in Telyukova (2006), who calibrates a model in the same spirit as the one developed here, although she relaxes some of the assumptions, and introduces some others.[17] As compared to the data, her calibrated model generates almost but not quite as many households simultaneously holding credit card debt and liquid assets – i.e. the size of the puzzle group is about right. And for these households, the model easily accounts for at least half of their liquid assets. So liquidity-based explanation appears quantitatively relevant; one could say that it looks to be a sizable part of the puzzle, if not the whole story. If there is something left to be explained, that is interesting, and opens the door for other explanations to play some role – we do not insist that there is necessarily one

[17]Her specification is more general in the sense that it departs from quasi-linear utility, which may be quantitatively relevant and generates more interesting debt rollover, but since this complicates the analysis considerably she has to make sacrifices on other dimensions (e.g. interest rates are exogenous).

definitive explanation that works to the exclusion of all others.

Still, it is worth thinking about ways to discriminate between stories. Based on a cursory reading of Gross and Souleles and the analysis in this paper up to but not including the extensions in Section 4.1, one might think that a liquidity-based approach could be dismissed as follows. Our baseline model predicts agents may have debt and cash on hand, but they would never hold other assets with interest rates that are positive but less than those on debt, as in the data some people do. The puzzle is not just why agents with credit card debt hold cash, we also have to explain why they do not draw down other wealth, like home equity or retirement accounts. While our baseline model cannot explain this, the extension with three assets and a fixed cost generates exactly $m, a, b > 0$ with $i_m < i_a < i_b$. So one cannot rule out liquidity-based stories as easily as that. What is true is that the theory requires low-return assets to be more liquid than high-return assets. To the extent that liquidity can be measured, this is testable.

Recall that some alternative potential explanations of the credit card debt puzzle were mentioned in the Introduction. One candidate is that people roll over credit card debt to control spending by their spouses (Bertaut and Haliassos 2002; Haliassos and Reiter 2003). A problem with this theory, in addition to the fact that it is a fairly expensive way to keep one's house in order, is that single and married people do not behave very differently in the data in terms of holding significant debt and liquidity; see Telyukova (2006). So this idea obviously cannot be the biggest piece of the puzzle. Another candidate explanation is that people in the puzzle group are on the verge of bankruptcy (Lehnert and Maki 2001). Problems with this include the fact that these people actually do not go bankrupt with particularly high probability, and the fact that they often have sizeable wealth in other assets; again see Telyukova (2006). So, while this may also be a piece of the puzzle, it cannot be a large part.

There are other candidate explanations. As a referee put it, "The obvious alternative is that the puzzle is due to a failure of optimal portfolio choice or no-arbitrage conditions for whatever reason." This is not exactly what we would call a competing *theory* – it is more of a *non-theory*, although approaches like the one in Laibson et al. (2000) might help make it less imprecise – but we have to admit it may be true. At some level, of course, it must be true: no one takes economic theory literally. The pertinent questions are, how well can we do understanding a relatively large fraction of households with our economic models? And do these models match up reasonably well with other aspects of the data? The same referee made several excellent suggestions concerning ways to think about this, and while clearly an in-depth empirical analysis goes beyond the scope of this paper, we can discuss some of the issues briefly.

A big component of our overall view is that there are unexpected shocks that require liquid assets. This apparently suggests those who are more likely to experience such shocks ought to hold more liquidity. For instance, home owners, car owners, and possibly those in poor health, at least to the extent that they are not fully insured, ought to hold more liquidity, other things being equal. Defining the puzzle group as those with over $500 in both debt and liquid assets, the 2001 Survey of Consumer Finance indicates that 30% of homeowners and 22% of non-homeowners are in this category.[18] Also, renters are twice as likely as homeowners to hold less than $500 in liquid assets, and the average monthly liquidity of a median renter is $1,000 while for a mortgage-paying homeowner it is about $4,000, amounting to 94% and 44% of monthly income, respectively (these numbers are for all households, not only the puzzle group). It appears that homeowners do hold greater liquidity, consistent with our theory.

In terms of automobiles, 28% of car owners are in the puzzle group, as compared to only 21% of non-car owners, also consistent with our theory. There is a monotone relationship between the

[18]To go into slightly more detail, 32% of homeowners who are paying a mortgage are in the puzzle group, as compared to 22% of those who are no longer paying a mortgage and 22% of renters. This suggests that the commitment to mortgage is the important factor. Of course, it may also suggest that richer people, who have paid off their mortgages, are more able to repay credit card debt: 75% of them have no debt, versus 63% among those with mortgages.

number of cars a household owns and the likelihood of being in the puzzle group, although it is not strong (29% of those who have three cars are in the group, versus 27% of those who only have one car). There does not appear to be a systematic relationship between the age of the cars a household owns and the likelihood of being in the puzzle group: those whose car is from 1990 are about as likely to be in the group as those whose car is a 2002 model in 2001. However, perhaps this is because both predicted and precautionary needs play a role for automobile owners, since those with newer cars are often making payments while those with older cars are done with payments but need more liquidity due to a higher probability of non-warrantied repairs.

The results from a quick look at the health data are less clear, but this is perhaps due to the fact that some health expenditures can be paid by credit card or financed through the hospital directly. In any case, the fractions in the puzzle group are 28%, 28% and 22% for those respectively reporting good, average and poor health status. What is more striking, if not surprising, is that bad health prevents people from holding on to liquidity at all: 4% of those in good health, 8% of those in average health, and 11% of those in bad health hold less than $500 in liquid assets. In terms of health insurance, 12% of those without insurance, and 4% of those with insurance, report less than $500 in liquid assets. Of course, these observations may be due a variety of other factors, and in general we think that confronting the theory with the health data demands more time than we have available in this brief discussion.

Another issue we want to mention is this: a naive observer might dismiss our approach all too quickly based on the fact that one can often get cash advances on credit cards, which means that even if a purchase cannot be made on credit one need not have cash in hand. This argument neglects the fact that cash advances usually have strict limits and typically involve very high interest charges (much higher than interest on purchases). A good question is, *why* are interest rates so high for cash advances? We do not know, for sure, but offer a conjecture. Credit card companies want agents to hold liquid assets, rather than use them to pay down debt, since after all revolving debt is how they make most of their profit. By making cash advances costly, credit card companies increase the demand for liquidity, keep people from paying down debt, and in this way increase profit. Careful analysis of this idea, like all those in this subsection, must be relegated to other work.

Two final issues come to mind. First, although the formal model here is based on anonymity in some trading opportunities, this is not really central to the general idea and is not meant to be taken literally. Other motives for sellers not extending credit – including e.g. tax motives – could in principle replace anonymity between buyers and sellers. It would therefore be a mistake to reject the story of home repairs often requiring cash just because in reality the plumber knows your name and where you live, since there may be another issue of anonymity concerning him and Uncle Sam. Second, although our analysis predicts agents with cash may use debt even though it entails interest charges, we do not say much about just *how high* that interest should be. If one wants a model of the actual magnitudes of credit card interest, one should probably incorporate several features not in our setting, perhaps especially default risk. Although it would be interesting to do so, default would certainly clutter the analysis, and does not seem central to our question.[19]

5 Conclusion

The coexistence of assets with different returns has recently been dubbed the credit card debt puzzle. We pointed out that this is actually a special case of the rate of return dominance puzzle. We extended recent developments in monetary theory to construct a model where agents sometimes have the option to trade using credit and sometimes do not. The framework

[19]Recent papers that study this aspect of the market include Chatterjee, Corbae and Rios-Rull (2007) and Chatterjee, Corbae, Nakajima and Rios-Rull (2007); previous work includes Ausubel. (1991, 1999).

is tractable, and yields strong predictions. One prediction is that agents use credit even if it is costly in terms of interest and they have liquid assets at hand. The intuition for this is that they know they may need the liquidity later, when credit may not be available. One might say this is reasonable, perhaps even obvious; this does not mean it is incorrect or uninteresting. Aside from the particular application to credit card debt, the formal framework here is also novel, and as there are not many tractable models with a role for both money and credit, we think the approach may find many other applications.

Appendix

First we derive the bargaining solution in Lemma 1. The necessary and sufficient conditions for (22) are

$$\theta\left[\beta_3\phi_{1,+1}d - c_3(q)\right]u_3'(q) = (1-\theta)\left[u_3(q) - \beta_3\phi_{1,+1}d\right]c_3'(q) \tag{41}$$

$$\begin{aligned}\theta\left[\beta_3\phi_{1,+1}d - c_3(q)\right]\beta_3\phi_{1,+1} &= (1-\theta)\left[u_3(q) - \beta_3\phi_{1,+1}d\right]\beta_3\phi_{1,+1} \\ &\quad -\lambda\left[u_3(q) - \beta_3\phi_{1,+1}d\right]^{1-\theta}\left[\beta_3\phi_{1,+1}d - c_3(q)\right]^{\theta}\end{aligned} \tag{42}$$

where λ is the Lagrange multiplier on $d \leq m_3$. There are two possible cases: If the constraint does not bind, then $\lambda = 0$, $q = q^*$ and $d = m^*$. If the constraint binds then q is given by (41) with $d = m_3$, as claimed.

We now argue that $m_3 < m_3^*$. First, as is standard, in any equilibrium $\phi_{1,+1} \leq (1+\rho)\phi_1$ (the nominal interest rate i is nonnegative). In fact, again as is standard, although we allow $i \to 0$, we assume $i > 0$, so that $\phi_{1,+1} < (1+\rho)\phi_1$. Now suppose $m_3 > m_3^*$ at some date for some agent. Since the bargaining solution tells us he never spends more than m_3^*, he could reduce m_3 by reducing h_1 at t, then increase h_1 at $t+1$ and not change anything else. It is easy to check that this increases utility, so $m_3 > m_3^*$ cannot occur in equilibrium. Hence $m_3 \leq m_3^*$. To show strict inequality, suppose $m_3 = m_3^*$ for same agent. Again he can reduce h_1 at t and carry less money. If he is a buyer in subperiod 3, he gets a smaller q, but the continuation value is the same since by the bargaining solution he still spends all his money. If he does not buy then he can increase h_1 at $t+1$ so that he need not change anything else. It is easy to check that the net gain from carrying less money is positive, as in LW.

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