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Green and Lin study a version of the Diamond-Dybvig model with a finite number of agents, independence (independent determination of each agent's type), and sequential service. For special preferences, they show that the ex ante first-best allocation is the unique equilibrium outcome of the model with private information about types. Via a simple argument, it is shown that uniqueness of the truth-telling equilibrium holds for general preferences. and, in particular, for a constrained-efficient allocation whether first-best or not. The crucial assumption is independence.

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1 Introduction

Green and Lin [4] study a version of the Diamond-Dybvig [3] model with a finite number of agents, independent (across agents) determination of each agent's type (impatient or patient), and sequential service. For special preferences, they conclude that the *ex ante* first-best allocation—the allocation that maximizes expected utility when information about types is public—is the unique equilibrium outcome of the model when information about types is private. We show, via a very simple argument, that uniqueness of the truth-telling equilibrium holds for general preferences. In other words, the conclusion that there is no bank-run equilibrium is unrelated to whether or not the *ex ante* first-best allocation can be implemented. The critical assumption is their independence assumption.

2 Environment

There are N agents, two dates, 1 and 2, and there is one good per date. The economy is endowed with an amount $Y > 0$ of date-1 good and has a constant returns to scale technology with gross rate-of-return R. An agent of type $t \in T$ has utility $u(\cdot, \cdot, t)$, where the first argument is date-1 consumption and the second is date-2 consumption and $T = \{i, p\}$ is the set of types (*impatient* and *patient*, respectively). For a given t, u is increasing and concave.[1] Each agent maximizes expected utility.

There is an exogenous random process that determines the queue $t^N = (t_1, t_2, ..., t_N)$, where $t_j \in T$ is the type of the j-th agent in line and is private information. Let $P_k = \{t^N \in T^N : \#p \in t^N = k\}$; i.e., the set of queues with k patient people. The number of patient people is a drawing from the distribution $\pi = (\pi_0, \pi_1, ..., \pi_N)$, where $\pi_k > 0$ is the probability that k agents are patient. If $t^N \in P_k$, then the probability that t^N occurs is $\pi_k/\#P_k = \pi_k/\binom{N}{k}$. That is, conditional on k, all permutations that determine place in line are equally likely. Because $\pi_k > 0$ and all permutations are equally likely, any $t^N \in T^N$ occurs with positive probability. A special case of π is independence; namely, $\pi_k = \binom{N}{k}\alpha^k (1-\alpha)^{N-k}$ for some $\alpha \in (0, 1)$. As in

[1]Thus, our preferences are of the general sort introduced into the Diamond-Dybvig model by Jacklin [5].

Green and Lin, we assume that each agent knows his place in the queue.[2]

The sequence of events is as follows. First, nature selects a queue, an element of T^N. Then agents consume at date 1 subject to *sequential service:* the date-1 consumption of agent n can depend at most on the agent's own information and/or actions and on those of the agents with earlier places in line. Finally, agents consume at date 2 (after all actions have been taken).

3 Mechanisms

We consider direct and symmetric mechanisms in which each agent's strategy is to announce a type. By symmetric, we mean that mechanisms do not depend on the agents' (initial) identities. We also limit mechanisms to those for which the planner reveals to each agent the announcements made by earlier agents.[3] And, because truth-telling constraints need not be convex, we allow lotteries. Finally, we assume throughout that an agent who is indifferent between announcing truthfully and not doing so tells the truth. In doing this, we are implicitly appealing to results on *virtual implementation* (see, for example, [1] and [6]).

To describe the lotteries, we start by describing the set of deterministic mechanisms that constitute the domain for the lotteries. A strategy for agent n is $\mathbf{s}_n : T^{n-1} \times T \to T$, where the first argument is the vector of announced types of those in earlier places in line and the second is the true type of agent n, t_n. We let $\mathbf{s}^n = (\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_n)$. A deterministic mechanism is $c = (c_1^n, c_2^n)$, $n = 1, 2, ..., N$, where $c_1^n : T^n \to \mathbf{R}_+$ is date-1 consumption of agent n and $c_2^n : T^N \to \mathbf{R}_+$ is date-2 consumption of agent n. The domain of c is announcements.

We say that c is feasible if for all $t^N \in T^N$, $R(Y - \sum_{n=1}^N c_1^n) \geq \sum_{n=1}^N c_2^n$. (That is, we require that the resource constraint be satisfied *ex post*). Let C denote the set of all feasible c and let Ω denote the set of all measures on C. A lottery mechanism is $\omega \in \Omega$. The realization from ω is independent

[2] This contrasts with Peck and Shell [7], who assume that an agent does not know his place in line. Under their assumption it is possible that mechanisms that do not reveal place in line to the agents would achieve better outcomes than those that do. Peck and Shell do not investiage that question. They study only mechanisms that do not reveal place in line.

[3] As noted below, our result does not rely on that limitation.

of the realization of the queue. The outcome to the lottery is chosen before any agent makes an announcement, but is seen by each agent only after the agent makes an announcement.[4] It follows that the *ex ante* utility associated with the deterministic mechanism $c \in C$ and the strategy $\mathbf{s}^N$ is

$$W(c, \mathbf{s}^N) = \sum_{k=0}^{N} \frac{\pi_k}{\binom{N}{k}} \sum_{t^N \in P_k} \sum_{n=1}^{N} u[c_1^n\left(s^{n-1}, s_n\right), c_2^n\left(s^N\right), t_n], \tag{1}$$

where s_n denotes the announcement or play implied by $\mathbf{s}_n$. Then, the *ex ante* utility associated with $\omega \in \Omega$ and the strategy $\mathbf{s}^N$ is

$$W(\omega, \mathbf{s}^N) = E_\omega[W(c, \mathbf{s}^N)], \tag{2}$$

where for any function f, $E_\omega(f(c)) = \int_{c\in C} f(c) d\omega$.

We let $t_{n+1}^N = (t_{n+1}, t_{n+2}, ..., t_N) \in T^{N-n-1}$ denote the vector of *types* of those in line after n. Let $\phi(t_{n+1}^N \mid s^{n-1}, t_n)$ denote agent n's *beliefs*; the probability of the outcome t_{n+1}^N conditional on earlier announcements and n's type.

In terms of the above notation, we have the following definition of equilibrium.

Definition 1 *The strategy $\mathbf{s}^N$ and the belief ϕ is a perfect Bayesian equilibrium for $\omega \in \Omega$ if*

$$\begin{aligned} &E_\omega \sum_{t_{n+1}^N} \phi(t_{n+1}^N \mid s^{n-1}, t_n) u[c_1^n(s^{n-1}, s_n), c_2^n(s^{n-1}, s_n, \mathbf{s}_{n+1}^N), t] \geq \\ &E_\omega \sum_{t_{n+1}^N} \phi(t_{n+1}^N \mid s^{n-1}, t_n) u[c_1^n(s^{n-1}, \tilde{s}_n), c_2^n(s^{n-1}, \tilde{s}_n, \mathbf{s}_{n+1}^N), t] \end{aligned} \tag{3}$$

for all $t^n \in T^n$, $\tilde{s}_n \in T$, and n, and if $\phi(t_{n+1}^N \mid s^{n-1}, t_n)$ is consistent with Bayes' rule whenever possible.

In (3), a realization for t_{n+1}^N implies a realization for s_{n+1}^N. More generally, given knowledge of the strategy, $\mathbf{s}_n$, a realization of t^n implies a realization

[4] Thus, agent n cannot see the date-1 consumptions assigned to those earlier in line because such information would tend to reveal the realization from the lottery.

of the play, s_n. However, the converse is not in general true. In particular, an observation on s^{n-1} does not uniquely determine t^{n-1}. For example, the strategy of the first agent could be to announce *impatient* independent of that agent's true type. Hence, we must distinguish between ϕ and the distribution of t_{n+1}^N conditional on (t^{n-1}, t_n), which we denote $\tilde{\phi}(t_{n+1}^N \mid t^{n-1}, t_n)$. The distribution $\tilde{\phi}$ is implied by π, while the distribution ϕ depends on π and the strategies.

By appeal to the revelation principle, we have

Definition 2 *We say that ω is weakly implementable if $\mathbf{s}_n(t^{n-1}, t_n) = t_n$ and $\tilde{\phi}\left(t_{n+1}^N \mid t^{n-1}, t_n\right)$ is a perfect Bayesian equilibrium for ω.*

And, we also have

Definition 3 *We say that ω is strongly implementable if $\mathbf{s}_n(t^{n-1}, t_n) = t_n$ and $\tilde{\phi}\left(t_{n+1}^N \mid t^{n-1}, t_n\right)$ is the unique perfect Bayesian equilibrium for ω.*

4 Independence and uniqueness

If π satisfies independence, then $\tilde{\phi}\left(t_{n+1}^N \mid t^{n-1}, t_n\right) = \hat{\phi}(t_{n+1}^N)$. (In particular, if t_{n+1}^N has k patient types, then $\hat{\phi}(t_{n+1}^N) = \alpha^k(1-\alpha)^{N-n-1-k}$.) Moreover, it follows from independence and the Bayes' rule requirement in definition 1 that $\phi(t_{n+1}^N \mid \cdot, \cdot) \equiv \hat{\phi}(t_{n+1}^N)$. This plays a crucial role in the proof of the following proposition.

Proposition 1 *Let π satisfy independence. If ω is weakly implementable, then it is strongly implementable.*

Proof. Suppose that ω is weakly implementable and that $\left(\mathbf{s}^N, \phi\right)$ is a perfect Bayesian equilibrium for ω. We show via a backward induction argument, starting with $\mathbf{s}_N$, that $\mathbf{s}_n(\cdot, t_n) \equiv t_n$ for all n. (That $\phi(t_{n+1}^N \mid \cdot, \cdot) \equiv \hat{\phi}(t_{n+1}^N)$ has already been noted.)

By weak implementability of ω,

$$\begin{aligned} & E_\omega u[c_1^N(t^{N-1}, t), c_2^N(t^{N-1}, t), t] \\ \geq \quad & E_\omega u[c_1^N(t^{N-1}, \tilde{s}_N), c_2^n(t^{N-1}, \tilde{s}_N), t] \qquad (4) \end{aligned}$$

for all $t^{N-1} \in T^{N-1}$ and all $\tilde{s}_N \in T$. Because (4) holds for all $t^{N-1} \in T^{N-1}$ and because $s^{N-1} \in T^{N-1}$, it follows that

$$\begin{aligned} & E_\omega u[c_1^N(s^{N-1}, t), c_2^N(s^{N-1}, t), t] \\ \geq \quad & E_\omega u[c_1^N(s^{N-1}, \tilde{s}_N), c_2^n(s^{N-1}, \tilde{s}_N), t] \end{aligned} \tag{5}$$

for all $\tilde{s}_N \in T$**.** This condition says that truth-telling is a best response for agent N independent of the strategies used by the other $N-1$ agents. Hence, $\mathbf{s}_N(\cdot, t_N) \equiv t_N$. (Note that this conclusion does not depend on the assumed independence of π.)

Now we turn to the induction step, for which we have the hypothesis $s_j(\cdot, t_j) = t_j$ for $j = n+1, \ldots, N$; i.e., future announcements are truthful. By weak implementability of ω,

$$\begin{aligned} & E_\omega \sum_{t_{n+1}^N} \tilde{\phi}(t_{n+1}^N \quad | \quad t^{n-1}, t_n) u[c_1^n(t^{n-1}, t), c_2^n(t^{n-1}, t, t_{n+1}^N), t] \geq \\ & E_\omega \sum_{t_{n+1}^N} \tilde{\phi}(t_{n+1}^N \quad | \quad t^{n-1}, t_n) u[c_1^n(t^{n-1}, \tilde{s}_n, t_{n+1}^N), c_2^n(t^{n-1}, \tilde{s}_n, t_{n+1}^N), t] \end{aligned} \tag{6}$$

for all $t^{n-1} \in T^{n-1}$ and all $\tilde{s}_n \in T$. But under independence, (6) can be written as

$$\begin{aligned} & E_\omega \sum_{t_{n+1}^N} \hat{\phi}(t_{n+1}^N) u[c_1^n(t^{n-1}, t), c_2^n(t^{n-1}, t, t_{n+1}^N), t] \\ \geq \quad & E_\omega \sum_{t_{n+1}^N} \hat{\phi}(t_{n+1}^N) u[c_1^n(t^{n-1}, \tilde{s}_n, t_{n+1}^N), c_2^n(t^{n-1}, \tilde{s}_n, t_{n+1}^N), t] \end{aligned} \tag{7}$$

for all $t^{n-1} \in T^{n-1}$ and all $\tilde{s}_n \in T$. As above, because (7) holds for all $t^{n-1} \in T^{n-1}$ and because $s^{n-1} \in T^{n-1}$, it follows that

$$\begin{aligned} & E_\omega \sum_{t_{n+1}^N} \hat{\phi}(t_{n+1}^N) u[c_1^n(s^{n-1}, t), c_2^n(s^{n-1}, t, t_{n+1}^N), t] \\ \geq \quad & E_\omega \sum_{t_{n+1}^N} \hat{\phi}(t_{n+1}^N) u[c_1^n(s^{n-1}, \tilde{s}_n, t_{n+1}^N), c_2^n(s^{n-1}, \tilde{s}_n, t_{n+1}^N), t] \end{aligned} \tag{8}$$

for all $\tilde{s}_n \in T$. By the induction hypothesis (which says that later agents tell the truth) and $\phi(t_{n+1}^N \mid \cdot, \cdot) \equiv \hat{\phi}(t_{n+1}^N)$, (8) is the condition that assures that truth-telling for agent n is a best response. Therefore, $\mathbf{s}_n(\cdot, t_n) \equiv t_n$, which completes the proof. ■

Absent independence, the induction step in the proof would fail because (8) would not be the condition that assures that truth-telling is a weakly dominant strategy for agent t_n. Absent independence, that condition would be (8) with $\hat{\phi}(t_{n+1}^N)$ replaced by $\phi(t_{n+1}^N \mid s^{n-1}, t_n)$ and it would not be implied by (6). Finally, as asserted above, nothing in the proof depends on the assumption that the agent is informed about earlier announcements. (The implication $\phi(t_{n+1}^N \mid \cdot, \cdot) \equiv \hat{\phi}(t_{n+1}^N)$ says that the conditioning information is not relevant.) It is enough that each agent knows his place in line.

5 Non-independence and multiplicity

It is tempting to try to construct an example with non-independence and with a bank-run equilibrium. In particular, we considered searching for an example for which the mechanism that gives rise to the best truth-telling equilibrium can have another equilibrium—one in which, along the equilibrium path, some patient agents announce that they are impatient.[5] Such multiplicity would be costly in the following sense. Either some worse equilibrium could occur (perhaps, with some probability) or a different optimum problem is solved—one that includes additional constraints ensuring that the truth-telling equilibrium is the only equilibrium.

As is well-known, the best truth-telling equilibrium is a solution to

Problem 1 *Choose $\omega \in \Omega$ to maximize $W(\omega, \mathbf{t}^N)$ (see (2)) subject to*

$$E_\omega \sum_{t_{n+1}^N} \phi(t_{n+1}^N \mid t^{n-1}, t_n) u[c_1^n(t^{n-1}, t), c_2^n(t^{n-1}, t_n, t_{n+1}^N), t] \geq$$

$$E_\omega \sum_{t_{n+1}^N} \phi(t_{n+1}^N \mid t^{n-1}, t_n) u[c_1^n(t^{n-1}, \tilde{s}_n), c_2^n(t^{n-1}, \tilde{s}_n, t_{n+1}^N), t] \qquad (9)$$

[5] Peck and Shell accomplish that, but only by placing restrictions on the mechanism; their mechanism does not reveal any information to agents.

for all $t^n \in T^n$, $\tilde{s}_n \in T$, *and* n.

The constraint here is that each agent is willing to be truthful given that others are truthful.

An example with multiplicity might exist because the weak dominance of truth-telling in proposition 1 is not robust to even small departures from independence. As in most problems with private information, for most preference specifications some of the truth-telling constraints in problem 1 will bind. Suppose that is the case for a π that satisfies independence. Now, holding the rest of the model unchanged, consider a neighborhood of π, a neighborhood that consists of distributions that are close to π, but do not satisfy independence. Then, generically, the same truth-telling constraints are binding in that neighborhood. But weak dominance of truth-telling ought to fail in part of the neighborhood because the distribution of future types conditional on (s^{n-1}, t_n) can depend on agent n's beliefs about the relationship between s^{n-1} and t^{n-1}. Of course, such failure does not imply existence of another equilibrium. That is why an example is needed.

However, we became discouraged about the possibility of finding such an example. Although problem 1 is a linear programming problem, to keep it tractable, N must be small and the set of deterministic allocations must be approximated by a finite set. Moreover, for tractability, it helps greatly to have utility functions that are the special sort used by Diamond and Dybvig: impatient types care only about date-1 consumption, patient types view consumption at the two dates as perfect substitutes, and $R > 1$. For such preferences, (i) the solution to problem 1 assigns positive date-1 consumption only to impatient types and positive date-2 consumption only to patient types, (ii) impatient types necessarily announce truthfully, and (iii) binding truth-telling constraints for patient types arise if there is sufficient "discounting" of the patient type's payoff.[6]

For $N = 3$ (the smallest N that could conceivably produce the multiplicity), any bank-run equilibrium would along the equilibrium path have agents $n = 1$ and $n = 2$ announce that they are impatient and have agent $n = 3$ announce truthfully.[7] But, even if a perfect Bayesian equilibrium of this

[6] Green and Lin use such preferences but without the discounting that would imply binding constraints.

[7] As the proof of proposition 1 demonstrates, the last person in line tells the truth for any π.

sort exists, the equilibrium would be implausible. It would violate almost any sensible refinement about beliefs and, in particular, the Cho-Kreps [2] intuitive criterion.

To see this, consider agent p_1 in the above bank-run equilibrium, where p_1 means that the first agent in line is patient. How would agent p_2 react to a defection by p_1, a defection to truth-telling? Agent p_2 would conclude that the first agent is patient because an impatient agent under the special preferences never announces p. And, if the first agent announces truthfully and the third does, then p_2's best response is to announce truthfully. Hence, if p_1 defects from bank-run equilibrium play, then all other agents announce truthfully. Given such behavior, p_1 has an incentive to defect from the bank-run equilibrium play and announce p because p_1 gets a higher payoff in the truth-telling equilibrium than in the bank-run equilibrium.[8] Hence, for $N = 3$ and the Diamond and Dybvig preferences, the Cho-Kreps refinement eliminates any possible bank-run equilibrium.

Thus, any example that will have a bank-run equilibrium that satisfies reasonable refinements will not be of the simple sort described above. That conclusion discouraged us from trying to find an example because any such example would give rise to a very high-dimensional linear programming problem.

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