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Theory, Measurement, and Calibration of Macroeconomic Models

by Paul Gomme and Peter Rupert

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Calibration has become a standard tool of macroeconomics. This paper extends and refines the calibration methodology along several important dimensions. First, accounting for home production is important both in measuring calibration targets and in organizing the data in a model-consistent fashion. For this reason, thinking about home production is important even if the model under consideration does not include home production. Second, investment-specific technological change is included because of its strong balanced growth parameter restrictions. Third, the measurement strategy is laid out as transparently as possible so that others can easily replicate the underlying calculations. The data and calculations used in this paper are available on the web.

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1 Introduction

Measurement plays a critical role in determining the quantitative performance of dynamic stochastic general equilibrium models (DSGE). Refinements to the seminal model laid out by Kydland and Prescott (1982), such as Prescott (1986), King, Plosser, and Rebelo (1988), Cooley and Prescott (1995), Greenwood and Hercowitz (1991) and Benhabib, Rogerson, and Wright (1991), have greatly improved our understanding of many key features of the data.

Although variants of this framework are constructed to answer a broad range of questions, the underlying quantitative structure, i.e., the calibration procedure, is often quite similar across models. The similarity means that many of the same parameters and targets can be used, facilitating comparison of the findings across models. Consistency of measurement ensures that any differences in the findings of the models will emanate from differences across model specification rather than from different data used for the calibration.

The goal of this paper is to extend and refine the methodology for organizing the data as well as to provide a careful and detailed description of those data. In addition, access to the data is provided via the web. An available spreadsheet allows researchers to make adjustments to the series as dictated by their particular question under study.

Much of the paper is concerned with very detailed descriptions of the data and their construction. For the purposes of *calibration* and *measurement*, it is useful to include a home production sector *even if* the specific questions being studied do not explicitly call for a home sector. By way of example, distinguishing between home and market activity has implications for calibration targets like investment-output ratios and hours of work, and consequently for the deep structural parameters of the model. While it may cause some concern that including a home sector will change the results, there are special cases of the home production model in which home production "does not matter" in the sense that the home production model generates the same results as a model without home production; see Benhabib et al. (1991) and Greenwood, Rogerson, and Wright (1995).

The paper examines implications for functional forms in economies with a household production sector and characterized by balanced growth. It is shown that in those models, allowing for capital-embodied technological change implies that within the class of constant elasticity of substitution (CES) production functions, the market and home production functions must be Cobb-Douglas. While this result is not new (see King et al., 1988), it clarifies the point that consistency requires restrictions on functional forms as well as on the data.

Another important component of this framework concerns the properties of the Solow residual. In general, the calculation involves subtracting share-weighted changes in the logs of labor and capital from output. While most of the measurement for the real business cycle (RBC) literature is based on a quarterly frequency, capital stock data are only available at an annual frequency. Cooley and Prescott (1995, page 22) choose to ignore the capital input in computing their Solow residual, arguing that any interpolation scheme for constructing a quarterly capital stock "is essentially arbitrary and may add to the variability of both output and the residuals" and that quarterly variation in the capital stock is approximately zero. Several alternative procedures will be explored here. For the most part, the properties of the Solow residual are robust to how quarterly capital stock series are computed, or whether the capital stock is simply ignored. Further, the parameters governing the Solow residual are remarkably close to the *standard* values used in the RBC/DSGE literature.

Another issue related to the capital stock is its use as a calibration target. Early in RBC history,

it was common to calibrate to the investment-output ratio. Over time, however, many have begun to use the capital-output ratio as a calibration target. The problem with using the capital-output ratio as a target is that the capital stock data is not well measured. The manifestation of the mismeasurement is that the data are subject to substantial, periodic revision; see Herman (2000). As an example, in 1997 the Bureau of Economic Analysis (BEA) revised broadly defined categories of capital upward by as much as 30%. Maddison's (1995) "gross capital stock" data is even larger than the post-revision estimate of the BEA. In addition to measurement problems there is also an issue concerning what should be included in the capital stock. For example, should the value of land or inventories be included? To the extent that calibrated parameters and model predictions for business cycle moments are sensitive to the capital-output ratio – and results in Fisher (2001) suggest that they are – a return to the earlier practice of calibrating to investment-output ratios would seem justified. That is exactly what is advocated below.

This paper also addresses a claim, made by several authors (including Greenwood et al. (1995), Hornstein and Praschnik (1997) and Gomme, Kydland, and Rupert (2001)), that the tax rate on capital income must be inordinately high (between 70 and 80%) for the model to be consistent with the observed after-tax rate of return and capital share of income. This claim is shown to be erroneous. Apparently, the confusion arises because it is unclear whether it is the pre-tax or after-tax rate of return that is used for the calibration. Loosely speaking, the setting of the discount rate in these models relies on the after-tax rate of return. Therefore, when adjusting the tax rate on capital, the after-tax return that is used for the discount rate must also be adjusted. This issue is discussed in detail in Subsection 4.5

There are several issues, however, that remain unresolved, and are left for future investigation. The first is that the typical calibration procedure using the RBC framework produces a real interest rate that is substantially higher than the real interest rate found from other sources. The baseline parameters used in this paper (and similar to many other studies) imply a pre-tax real interest rate of 13.2% per annum. Poterba (1998), however, finds a real return on capital of 8.6% (1959–1996) using National Income and Product Accounts (NIPA) data on capital income flows and BEA capital stock data. Siegel (1992) computes a real return of 7.77% for the Standard & Poor 500 (1800–1990), a value that is only slightly lower than reported in Mehra and Prescott (1985) for the period 1880–1980.

Also, it appears that several of the series required in this line of research are not wholly consistent with the maintained hypothesis of balanced growth. More specifically, the investment share of housing has fallen somewhat over this time period while that of equipment and software has risen. In addition, over the entire sample, 1925-2001, all of the measured depreciation rates show a secular increase. This paper does not attempt to resolve this issue, and the calibration uses averages computed over the time period under study.

Finally, the capital stock is disaggregated into four major categories and the model allows for investment-specific technological change in these components. The implications of investment-specific technological change in the market sector has been investigated by Greenwood, Hercowitz, and Krusell (1997); the current paper extends this idea to the home sector as well. Most work in this area has used data from Gordon (1990) who carefully documented changes in the prices and quality of a range of capital goods. Since the Gordon data has not been updated recently and is not available quarterly, the approach used below is to use the relative prices of the investment goods implicit in the NIPA data. Unfortunately, this procedure implies stochastic processes for investment-specific technological change that lead to investment volatility that is grossly at odds

with the data.

The rest of the paper is organized as follows. The economic environment is described in Section 2. The equilibrium conditions are given in Section 3. Calibration and measurement issues are presented in Section 4. Section 5 discusses the findings and Section 6 concludes.

2   The Economic Environment

The model is a standard representation of the DSGE/RBC framework. A home production sector is included owing to its usefulness for the quantitative analysis that follows.

2.1   Households

The representative household derives utility from consuming market goods, c_{mt}, and home goods, c_{ht}, and disutility from working either in the market, h_{mt}, or at home, h_{ht}. The household's preferences are summarized by

$$E_0 \sum_{t=0}^{\infty} \beta^t U[C(c_{mt}, c_{ht}), h_{mt}, h_{ht}] \quad 0 < \beta < 1. \tag{1}$$

β is the discount factor. In (1), the consumption aggregator, $C(c_{mt}, c_{ht})$, is introduced to conform with the home production literature, and because the properties of this aggregator are important in calibrating the model. Leisure and the constraint on the household's use of time are embedded in the function U.

There are two types of capital used in the home: housing capital, k_{ht}, corresponding to structures, and consumer durables, k_{dt}, consisting of the remainder of the home capital stock. Along with time spent working at home, these capital stocks produce home consumption according to

$$c_{ht} = H(k_{ht}, k_{dt}, \gamma_x^t h_{ht}) \tag{2}$$

where γ_x is the (exogenous) rate of labor-embodied technological change. It is assumed throughout this paper that the rate of labor-embodied technological change affects the home and market labor inputs symmetrically.

Early papers in the home production business cycle literature included a shock to home productivity. There is little *direct* evidence on the stochastic properties of the home productivity shock. While there are measures of the relevant capital stocks, time series on home output are not collected, and there is only scant data on home hours. Consequently, it is not possible to directly compute a Solow residual for the home sector. However, McGrattan, Rogerson, and Wright (1997) provide indirect evidence based on maximum likelihood estimation of a home production model. Their findings indicate that the home productivity shock is less persistent than the market productivity shock. Whether the home shock is more or less volatile than the market shock depends on data detrending procedures.

Ingram, Kocherlakota, and Savin (1997) use an innovative approach to construct time series for hours worked at home, leisure and home consumption. Briefly, they use the first-order conditions of a RBC/DSGE model with home production to infer what these three time series must look like.

Using data on home capital, their approach could presumably be used to construct a home sector Solow residual.

Absent direct evidence on the home productivity shock, several approaches have been adopted. The first chooses parameter values such that the home productivity shock has no effect on market variables. This is achieved in the baseline parameterization by setting $\gamma = 1$ (logarithmic utility) and $\xi = 0$ (Cobb-Douglas consumption aggregator) along with the result that the home production function is Cobb-Douglas.

A second approach is to specify a nonstochastic home technology shock process. This approach does *not* imply that home production does not matter. What matters is the difference between market and home productivity; that is, as long as market productivity varies, home production matters.

Third, the market and home productivity shocks could be specified to be the same shock as in Greenwood and Hercowitz (1991). More generally, a related possibility would assume that the stochastic process for the home shock resembles that of the market shock (but does not require that the shocks be the same). That is, the two shocks share the same autoregressive parameter and have the same standard deviations. In this case, however, it is also necessary to take a stand on the correlation between the innovations to the two shocks.

Finally, the parameters governing the home productivity shock could be chosen to match certain observed business cycle phenomena. There is some resistance in the literature to this approach because it uses, in part, the phenomena to be explained (business cycle behavior) to choose model parameters.

There are also two types of market capital: nonresidential structures, k_{st}, and equipment and software, k_{et}. The home and market capital stocks evolve according to:

$$k_{st+1} = (1 - \delta_s)k_{st} + q_{st}x_{st} \tag{3a}$$
$$k_{et+1} = (1 - \delta_e)k_{et} + q_{et}x_{et} \tag{3b}$$
$$k_{ht+1} = (1 - \delta_h)k_{ht} + q_{ht}x_{ht} \tag{3c}$$
$$k_{dt+1} = (1 - \delta_d)k_{dt} + q_{dt}x_{dt}. \tag{3d}$$

The interpretation of these laws of motion is straightforward. Focus for the moment on equation (3a). Market structures depreciate at the rate δ_s. The factor multiplying investment, q_{st}, reflects the current state of technology for producing market structures. While current investment in market structures uses x_{st} units of current output, this investment yields $q_{st}x_{st}$ units of market structures next period. The evolution of the other capital stocks, (3b)–(3d), have similar interpretations.

The household rents its market capital, k_{st} and k_{et}, at the competitively determined prices r_{st} and r_{et}, respectively. Capital income is taxed at the rate τ_k. Recall that the household allocates h_{mt} hours to working in the market. For this time, the household receives the wage rate, w_t. The household's labor income is taxed at the rate τ_ℓ. The final source of household income is a lump-sum transfer, τ_t, received from the government. The household divides its current income between market consumption, c_{mt}, and investment in the four capital stocks. Thus, the household faces the budget constraint,

$$c_{mt} + x_{st} + x_{et} + x_{ht} + x_{dt} = (1 - \tau_\ell)w_t h_{mt} + (1 - \tau_k)[r_{st}k_{st} + r_{et}k_{et}] + \tau_t. \tag{4}$$

In (4), gross capital income is taxed at the rate τ_k and there is no capital consumption allowance. The conventional means of introducing the capital consumption allowance is to add

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$\tau_k[\delta_s k_{st} + \delta_e k_{et}]$ to the right-hand side of (4); doing so is inconsistent with balanced growth since each of the terms in (4) grows at the rate of output with the exception of the capital consumption allowance term. Alternatively, a capital consumption allowance could be introduced by instead adding $\sum_{j=1}^{T} \tau_k[q_{st}x_{st} + q_{et}x_{et}]$ to the right-hand side of (4). While this second approach is consistent with balanced growth and is more in keeping with how the capital consumption allowance is handled in practice, it has the disadvantage of increasing the dimension of the state space of the model since past investments must be tracked. The formulation in (4) is the same as adopted by Greenwood et al. (1997) and Greenwood, Hercowitz, and Krusell (2000); provided that care is taken to properly measure the tax rate on capital (see Subsection 4.6), these different formulations can be expected to give very similar results.

2.2 Goods Producing Firms

Firms have access to a constant-returns-to-scale production function for producing market output,

$$y_t = F(k_{st}, k_{et}, \gamma_x^t h_{mt}; z_{mt}). \tag{5}$$

The typical firm rents the factors of production, structures, k_{st}, equipment and software, k_{et}, and labor, h_{mt}, at competitively determined prices, r_{st}, r_{et} and w_t, respectively. The firm seeks to maximize

$$F(k_{st}, k_{et}, \gamma_x^t h_{mt}; z_{mt}) - r_{st}k_{st} - r_{et}k_{et} - w_t h_{mt} \tag{6}$$

where z_{mt} is a productivity shock specific to market production which follows the autoregressive process,

$$\ln z_{mt+1} = \rho_m \ln z_{mt} + \varepsilon_{mt}, \tag{7}$$

where ε_{mt} is an independently and identically distributed random variable.

2.3 Government

The government's role in this economy is to collect tax revenues and then rebate the tax proceeds (lump-sum) to households. That is, the government satisfies the budget constraint,

$$\tau_k[r_{st}k_{st} + r_{et}k_{et}] + \tau_\ell w_t h_{mt} = \tau_t. \tag{8}$$

3 Equilibrium Conditions

Since the equilibrium of these models is well known, a full characterization is not undertaken and the equilibrium conditions are presented with minimal discussion.

Combining the first-order conditions of the representative household and representative firm yields:

$$c_{mt} + x_{st} + x_{et} + x_{ht} + x_{dt} = F(k_{st}, k_{et}, \gamma_x^t h_{mt}; z_{mt}) \tag{9a}$$

$$c_{ht} = H(k_{ht}, k_{dt}, \gamma_x^t h_{ht}) \tag{9b}$$

$$(1 - \tau_\ell)\gamma_x^t F_3(t)U_1(t)C_1(t) + U_2(t) = 0 \tag{9c}$$

$$\gamma_x^t H_3(t)U_1(t)C_2(t) + U_3(t) = 0 \tag{9d}$$

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$$\frac{U_1(t)C_1(t)}{q_{st}} = \beta E_t \left\{ \frac{U_1(t+1)C_1(t+1)}{q_{st+1}} [(1-\tau_k)F_1(t+1)q_{st+1} + 1 - \delta_s] \right\} \tag{9e}$$

$$\frac{U_1(t)C_1(t)}{q_{et}} = \beta E_t \left\{ \frac{U_1(t+1)C_1(t+1)}{q_{et+1}} [(1-\tau_k)F_2(t+1)q_{et+1} + 1 - \delta_e] \right\} \tag{9f}$$

$$\frac{U_1(t)C_1(t)}{q_{ht}} = \beta E_t \left\{ \frac{U_1(t+1)}{q_{ht+1}} [H_1(t+1)C_2(t+1)q_{ht+1} + (1-\delta_h)C_1(t+1)] \right\} \tag{9g}$$

$$\frac{U_1(t)C_1(t)}{q_{dt}} = \beta E_t \left\{ \frac{U_1(t+1)}{q_{dt+1}} [H_2(t+1)C_2(t+1)q_{dt+1} + (1-\delta_d)C_1(t+1)] \right\} \tag{9h}$$

along with the laws of motion governing the capital stocks, (3). In the above, $U_i(t)$ denotes the partial derivative of the utility function, U, with respect to its ith argument, with all arguments dated t. $C_i(t)$, $F_i(t)$ and $H_i(t)$ have similar interpretations.

The unknowns are: c_{mt}, c_{ht}, x_{st}, x_{et}, x_{ht}, x_{dt}, h_{mt}, h_{ht}, k_{st+1}, k_{et+1}, k_{ht+1} and k_{dt+1}.

4 Calibration

The calibration procedure involves choosing functional forms for the utility and production functions, and assigning values to the parameters of the model based on either micro-evidence or long run growth facts. Cooley and Prescott (1995) provide an overview of the general strategy. The next several subsections explain the reasoning and consequences of some standard modeling choices and describe in detail the calculation and determination of various parameters or targets that are typically used in the real business cycle framework. Subsection 4.9 provides descriptions of the remaining parameters (presented with much less detail) as well as the actual values chosen for the calibration.

4.1 Balanced Growth

An initial step involves solving for the model's balanced growth path which imposes a great deal of structure on the calibration exercise. To start, assume that the production functions, F and H, are each homogeneous of degree one in their three arguments. Also assume that the consumption aggregator, C, is homogeneous of degree one in its two arguments. Next, assume that the utility function, U, is homogeneous of degree $1-\gamma$ in its first argument (aggregated consumption). King et al. (1988) provide arguments justifying these assumptions. Consequently, U_1 is homogeneous of degree $-\gamma$ in C while U_2 and U_3 are both homogeneous of degree $1-\gamma$ in C. Finally, since the current focus is on long run growth, assume that investment-specific technological change is constant,

$$q_{st} = \gamma_s^t, \quad q_{et} = \gamma_e^t, \quad q_{ht} = \gamma_h^t, \quad q_{dt} = \gamma_d^t \tag{10}$$

and the the technology shocks, z_{mt} and z_{ht}, are equal to their unconditional means, $\bar{z}_m$ and $\bar{z}_h$.

In the literature, *balanced growth* is often taken to mean that all variables with positive growth rates grow at a common rate along the nonstochastic balanced growth path. As shown in Greenwood et al. (1997), this notion of balanced growth is often too restrictive. For example, the capital stocks do not grow at the same rate as output. Here, balanced growth means that along the nonstochastic balanced growth path, all variables grow at some (not necessarily common) constant rate.

For what follows, it is helpful to rewrite (9a) in terms of output shares:

$$\frac{c_{mt}}{y_t} + \frac{x_{st}}{y_t} + \frac{x_{et}}{y_t} + \frac{x_{ht}}{y_t} + \frac{x_{dt}}{y_t} = 1. \tag{11}$$

Figure 4 presents data on the investment shares for the United States. Focusing on the post-World War II period, it seems that the share of investment in equipment and software has trended up slightly while that of housing (residential structures) and consumer durables have fallen somewhat; the investment-output ratios are discussed in more detail in Subsection 4.4. Presumably, over longer periods of time, these shares are bounded strictly between 0 and 1. If this is the case, then all of the investment series, as well as market consumption and market output, must grow at some common rate; denote this rate by g. That is,

$$\frac{y_{t+1}}{y_t} = \frac{c_{mt+1}}{c_{mt}} = \frac{x_{st+1}}{x_{st}} = \frac{x_{et+1}}{x_{et}} = \frac{x_{ht+1}}{x_{ht}} = \frac{x_{dt+1}}{x_{dt}} = g \tag{12}$$

It can now be shown that the capital stocks grow faster than investment (and so market output). Consider market structures. Rewrite (3a) as:

$$\frac{k_{st+1}}{k_{st}} = 1 - \delta_s + \frac{\gamma_s^t x_{st}}{k_{st}}. \tag{13}$$

The left-hand side of (13) is the gross growth rate of the stock of market structures; denote this growth rate by g_s. Rewrite (13) as

$$g_s = 1 - \delta_s + \frac{\gamma_s^t g^t \tilde{x}_{st}}{g_s^t \tilde{k}_{st}} \tag{14}$$

where $\tilde{x}_{st} \equiv x_{st}/g^t$ and $\tilde{k}_{st} \equiv k_{st}/g_s^t$. That is, ~ denotes a variable rendered stationary by transforming that variable by its long run growth rate. For g_s to be constant requires that the last term in (14) be stationary which implies

$$g_s = g\gamma_s. \tag{15}$$

From (15), it follows that the gross growth rate of structures, g_s, is strictly bigger than one, and further that growth rate exceeds the growth rate of output, g. Similarly,

$$g_e = g\gamma_e, \quad g_h = g\gamma_h, \quad g_d = g\gamma_d. \tag{16}$$

Suppose that the market production function is constant-elasticity-of-substitution (CES):

$$y_t = z_{mt} \left[\alpha_s k_{st}^\zeta + \alpha_e k_{et}^\zeta + (1 - \alpha_s - \alpha_e) \left(\gamma_x^t h_{mt} \right)^\zeta \right]^{1/\zeta}. \tag{17}$$

This equation can be rewritten in terms of detrended variables. Along the nonstochastic balanced growth path,

$$\tilde{y} = \bar{z}_m \left[\alpha_s \left[\left(\frac{g_s}{g} \right)^t \tilde{k}_s \right]^\zeta + \alpha_e \left[\left(\frac{g_e}{g} \right)^t \tilde{k}_e \right]^\zeta + (1 - \alpha_s - \alpha_e) \left[\left(\frac{\gamma_x}{g} \right)^t \bar{h}_m \right]^\zeta \right]^{1/\zeta}. \tag{18}$$

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From (15) and (16), $g_s > g$ and $g_e > g$. Regardless of the magnitude of g relative to γ_x, equation (18) implies that the general CES production function is inconsistent with balanced growth: As $t \to \infty$, the growth rate terms involving the capital stocks become arbitrarily large, implying that the detrended capital stocks are not stationary (but, by construction, they must be stationary). As pointed out in Greenwood et al. (1997), within the class of CES production functions, only the Cobb-Douglas case is consistent with balanced growth. In this case,

$$y_t = z_{mt} k_{st}^{\alpha_s} k_{et}^{\alpha_e} \left(\gamma_x^t h_{mt}\right)^{1-\alpha_s-\alpha_e}. \tag{19}$$

It can be shown that

$$g = g_s^{\alpha_s} g_e^{\alpha_e} \gamma_x^{1-\alpha_s-\alpha_e}. \tag{20}$$

Using the expressions for the growth rates of the capital stocks, (15) and (16), yields

$$g = \gamma_s^{\frac{\alpha_s}{1-\alpha_s-\alpha_e}} \gamma_e^{\frac{\alpha_e}{1-\alpha_s-\alpha_e}} \gamma_x. \tag{21}$$

The key feature of the Cobb-Douglas case is that all growth can, in effect, be expressed as labor-augmenting.

Regarding the consumption aggregator, there are two cases to consider. The first imposes no structure on C apart from the earlier homogeneity assumption. In this case, market and home output must grow at the *same* rate: $g = g_c$. To see this, write (9e) as

$$1 = \beta E_t \left\{ \frac{U_1(C_{t+1}, h_{mt+1}, h_{ht+1})}{U_1(C_t, h_{mt}, h_{ht})} \frac{C_1(c_{mt+1}, c_{ht+1})}{C_1(c_{mt}, c_{ht})} \frac{1}{\gamma_s} \left[(1-\tau_k)\alpha_s \frac{y_{t+1}}{k_{st+1}} \gamma_s^{t+1} + 1 - \delta_s \right] \right\}. \tag{22}$$

The homogeneity assumption on U implies that U_1 is homogeneous of degree $-\gamma$ in C; thus,

$$\frac{U_1(C_{t+1}, h_{mt+1}, h_{ht+1})}{U_1(C_t, h_{mt}, h_{ht})} = g_a^{-\gamma} \frac{U_1(\tilde{C}_{t+1}, h_{mt+1}, h_{ht+1})}{U_1(\tilde{C}_t, h_{mt}, h_{ht})}. \tag{23}$$

The fact that the consumption aggregator, C, is assumed to be homogeneous of degree one in its two arguments is of little help in managing the terms involving its partial derivative, C_1. However, if c_{mt} and c_{ht} grow at the *same* rate, then

$$C_1(c_{mt}, c_{ht}) = C_1(\tilde{c}_{mt}, \tilde{c}_{ht}) \tag{24}$$

by virtue of the fact that C_1 is homogeneous of degree zero in its two arguments. Since market consumption grows at the same rate as market output, it follows that $g_c = g$.

The second case restricts the consumption aggregator to be Cobb-Douglas:

$$C(c_m, c_h) = c_m^\psi c_h^{1-\psi}. \tag{25}$$

Now,

$$\frac{C_1(\tilde{c}_{mt+1}, \tilde{c}_{ht+1})}{C_1(\tilde{c}_{mt}, \tilde{c}_{ht})} = \frac{\psi c_{mt+1}^{\psi-1} c_{ht+1}^{1-\psi}}{\psi c_{mt}^{\psi-1} c_{ht}^{1-\psi}} = \left(\frac{g_c}{g}\right)^{1-\psi} \frac{\psi \tilde{c}_{mt+1}^{\psi-1} \tilde{c}_{ht+1}^{1-\psi}}{\psi \tilde{c}_{mt}^{\psi-1} \tilde{c}_{ht}^{1-\psi}} \tag{26}$$

which is stationary. In the Cobb-Douglas case, the growth rates of market and home consumption can be extracted from the consumption aggregator in a convenient fashion. The growth rate of

aggregated consumption, denoted g_a, is a geometric mean of the growth rates of market and home consumption:

$$g_a = g^{\Psi} g_c^{1-\Psi}. \tag{27}$$

Next, let the home production function be CES:

$$c_{ht} = z_{ht} \left[\theta_h k_{ht}^{\eta} + \theta_d k_{dt}^{\eta} + (1 - \theta_h - \theta_d) \left(\gamma_x^t h_{ht} \right)^{\eta} \right]^{1/\eta}. \tag{28}$$

As with the market production function, along the nonstochastic balanced growth path,

$$\tilde{c}_h = \bar{z}_h \left[\theta_h \left[\left(\frac{g_h}{g_c} \right)^t \tilde{k}_h \right]^{\eta} + \theta_d \left[\left(\frac{g_d}{g_c} \right)^t \tilde{k}_d \right]^{\eta} + (1 - \theta_h - \theta_d) \left[\left(\frac{\gamma_x}{g_c} \right)^t \bar{h}_m \right]^{\eta} \right]^{1/\eta} \tag{29}$$

where g_c is the growth rate of home consumption (home output). While the left-hand side is constant along the nonstochastic balanced growth path, in general the right-hand side is not (recall that by construction $\tilde{k}_h$ and $\tilde{k}_d$ are constant along the nonstochastic balanced growth path). Again, within the class of CES production functions, Cobb-Douglas is the only case consistent with balanced growth:

$$c_{ht} = k_{ht}^{\theta_h} k_{dt}^{\theta_d} \left(\gamma_x^t h_{ht} \right)^{1-\theta_h-\theta_d}. \tag{30}$$

In general, finding the parameter values involves solving for the model's nonstochastic balanced growth path. The relevant equations are collected in (31):

$$\tilde{c}_{mt} + \tilde{x}_{st} + \tilde{x}_{et} + \tilde{x}_{ht} + \tilde{x}_{dt} = F(\tilde{k}_{st}, \tilde{k}_{et}, h_{mt}; z_{mt}) \tag{31a}$$

$$\tilde{c}_{ht} = H(\tilde{k}_{ht}, \tilde{k}_{dt}, h_{ht}) \tag{31b}$$

$$(1 - \tau_\ell) F_3(\tilde{k}_{st}, \tilde{k}_{et}, h_{mt}; z_{mt}) U_1[C(\tilde{c}_{mt}, \tilde{c}_{ht}), h_{mt}, h_{ht}] C_1(\tilde{c}_{mt}, \tilde{c}_{ht}) + U_2[C(\tilde{c}_{mt}, \tilde{c}_{ht}), h_{mt}, h_{ht}] = 0 \tag{31c}$$

$$H_3(\tilde{k}_{ht}, \tilde{k}_{dt}, h_{ht}) U_1[C(\tilde{c}_{mt}, \tilde{c}_{ht}), h_{mt}, h_{ht}] C_2(\tilde{c}_{mt}, \tilde{c}_{ht}) + U_3[C(\tilde{c}_{mt}, \tilde{c}_{ht}), h_{mt}, h_{ht}] = 0 \tag{31d}$$

$$U_1[C(\tilde{c}_{mt}, \tilde{c}_{ht}), h_{mt}, h_{ht}] C_1(\tilde{c}_{mt}, \tilde{c}_{ht}) = \beta E_t \left\{ \frac{U_1[C(\tilde{c}_{mt+1}, \tilde{c}_{ht+1}), h_{mt+1}, h_{ht+1}]}{g^{\gamma} \gamma_s} \right.$$
$$\left. \times C_1(\tilde{c}_{mt+1}, \tilde{c}_{ht+1}) \left[(1 - \tau_k) F_1(\tilde{k}_{st+1}, \tilde{k}_{et+1}, h_{mt+1}; z_{mt+1}) + 1 - \delta_s \right] \right\} \tag{31e}$$

$$U_1[C(\tilde{c}_{mt}, \tilde{c}_{ht}), h_{mt}, h_{ht}] C_1(\tilde{c}_{mt}, \tilde{c}_{ht}) = \beta E_t \left\{ \frac{U_1[C(\tilde{c}_{mt+1}, \tilde{c}_{ht+1}), h_{mt+1}, h_{ht+1}]}{g^{\gamma} \gamma_e} \right.$$
$$\left. \times C_1(\tilde{c}_{mt+1}, \tilde{c}_{ht+1}) \left[(1 - \tau_k) F_2(\tilde{k}_{st+1}, \tilde{k}_{et+1}, h_{mt+1}; z_{mt+1}) + 1 - \delta_e \right] \right\} \tag{31f}$$

$$U_1[C(\tilde{c}_{mt}, \tilde{c}_{ht}), h_{mt}, h_{ht}] C_1(\tilde{c}_{mt}, \tilde{c}_{ht}) = \beta E_t \left\{ \frac{U_1[C(\tilde{c}_{mt+1}, \tilde{c}_{ht+1}), h_{mt+1}, h_{ht+1}]}{g^{\gamma} \gamma_h} \right.$$
$$\left. \times \left[H_1(\tilde{k}_{ht+1}, \tilde{k}_{dt+1}, h_{ht+1}) C_2(\tilde{c}_{mt+1}, \tilde{c}_{ht+1}) + (1 - \delta_h) C_1(\tilde{c}_{mt+1}, \tilde{c}_{ht+1}) \right] \right\} \tag{31g}$$

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$$U_1[C(\tilde{c}_{mt}, \tilde{c}_{ht}), h_{mt}, h_{ht}]C_1(\tilde{c}_{mt}, \tilde{c}_{ht}) = \beta E_t \left\{ \frac{U_1[C(\tilde{c}_{mt+1}, \tilde{c}_{ht+1}), h_{mt+1}, h_{ht+1}]}{g^{\gamma} \gamma_d} \right.$$

(31h)

$$\left. \left[H_2(\tilde{k}_{ht+1}, \tilde{k}_{dt+1}, h_{ht+1})C_2(\tilde{c}_{mt+1}, \tilde{c}_{ht+1}) + (1 - \delta_d)C_1(\tilde{c}_{mt+1}, \tilde{c}_{ht+1}) \right] \right\}$$

$$g_s \tilde{k}_{st+1} = (1 - \delta_s)\tilde{k}_{st} + \tilde{x}_{st} \tag{31i}$$

$$g_e \tilde{k}_{et+1} = (1 - \delta_e)\tilde{k}_{et} + \tilde{x}_{et} \tag{31j}$$

$$g_h \tilde{k}_{ht+1} = (1 - \delta_h)\tilde{k}_{ht} + \tilde{x}_{ht} \tag{31k}$$

$$g_d \tilde{k}_{dt+1} = (1 - \delta_d)\tilde{k}_{dt} + \tilde{x}_{dt} \tag{31l}$$

with g given by (21) and g_s, g_e, g_h and g_e then given by (14)–(15).

4.1.1 Discussion

An oft-cited justification for using a Cobb-Douglas market production function are the following facts: capital's share of output and the return to capital exhibit no secular trend while the real wage rate has; see, for example, Prescott (1986). Yet, as pointed out by King et al. (1988), *any* constant-returns-to-scale production function is consistent with these facts. It is the addition of investment-specific technological change that pushes more convincingly in the direction of Cobb-Douglas.

Restricting attention to CES production functions that are homogeneous of degree one may also be too restrictive. Whether this is true is left as an open question. For example, in the context of an endogenous growth model, Kongsamut, Rebelo, and Xie (2001) find an interesting "knife-edge" case involving preferences. More specifically, Kongsamut et al. construct a "generalized balanced growth path" along which the share of agriculture declines and the share of services rises – as seen in the data.

In many ways, the arguments developed above make more explicit arguments in favor of a Cobb-Douglas market production function presented in Kydland (1995). While Kydland's argument is couched in terms of a secular declining price of capital goods, as shown in Greenwood et al. (1997), a declining price of capital goods is equivalent to investment-specific technological change.

4.2 Calculating Labor's Share of Income

The value of the labor share parameter for market production, $1 - \alpha$, used in the calibration is 0.717. On the face of it, computing labor's share of income should be a fairly straightforward exercise: Under the maintained assumption that the aggregate production function is Cobb-Douglas, add up all sources of labor income and divide by output. In practice, however, it is more complicated. For example, Proprietor's Income has both labor and capital income components. Over time, a consensus has developed over how to treat Proprietor's Income: The fraction of Proprietor's Income that should be treated as labor income is the same as for the economy as a whole; see below for details. Another issue is how to measure output. Since Cooley and Prescott (1995) use a very broad measure of the capital stock – including, among other things, government capital and household capital – they need a broad measure of output. Since the NIPA include an imputed

capital income flow for owner occupied housing, they need to impute capital income flows for consumer durables and government capital.

There are at least two problems with the Cooley and Prescott (1995) approach. First, while their measure of output includes household capital income flows, it omits the corresponding labor income flows. This omission likely results in a downward bias in their estimate of labor's share of income. Second, their income imputation method relies on capital stock data that, as discussed above, is periodically subject to large revisions.

The approach adopted here differs from that of Cooley and Prescott (1995) as follows. First, since home production is explicitly modeled, there is no need to include home produced goods in aggregate output. Consequently, the NIPA data is adjusted by *removing* housing income flows, obviating the need to impute an income flow to consumer durables. Nor is there any need to try to measure the labor income flows associated with household capital. Second, GDP is used as the basis on which output is measured rather than GNP. Given the way in which hours of work and capital stock data are measured, GDP is conceptually the appropriate measure of output, not GNP. Third, in measuring output, government labor income is removed from GDP rather than adding an imputed flow to government capital. To see why this approach is sensible, write total income as:

$$Y = Y^{KP} + Y^{NP} + Y^{KG} + Y^{NG} \tag{32}$$

where K refers to capital, N to labor, P to the private sector, and G to the government sector. If private sector production is Cobb-Douglas,

$$Y^P = (K^P)^\alpha (N^P)^{1-\alpha}, \tag{33}$$

then if factor markets are competitive, labor's share of income is

$$1 - \alpha = \frac{Y^{NP}}{Y^{NP} + Y^{KP}}. \tag{34}$$

Recalling that GDP omits government capital income, the denominator of (34) can be obtained by subtracting government labor income from GDP. *If* the government production function has the same parameter values as the private production function (as maintained by Cooley and Prescott (1995)), and *if* the government capital stock and real rate of return are correctly measured, *then* the approach above and that of Cooley and Prescott (1995) should give the same number for labor's share of income. One benefit of not imputing government capital income flows is that there is no need to assume that the labor share parameter in the government sector production function is the same as for the private sector. In any event, in reporting business cycle moments, Cooley and Prescott (1995) dispense with their capital income imputations and focus on NIPA GNP.

As in Cooley and Prescott (1995), care must be taken in regard to the income flows that have both labor and capital income components. Full data descriptions are available in Table 1. Of particular interest at this juncture is private ambiguous income which consists of Proprietors' Income plus Indirect Taxes less Subsidies. As discussed earlier in this subsection, the accepted manner of handling ambiguous income is to apportion the fraction α to capital income and the remainder to labor income. Total labor income is, then, $Y^{LP} + (1-\alpha)Y^{AP}$. Since the aggregate production function is Cobb-Douglas, total private labor income is a fraction $(1-\alpha)$ of total private income: $(1-\alpha)(Y - Y^{LG})$. Equating these two expressions and simplifying gives

$$1 - \alpha = \frac{Y^{LP}}{Y - Y^{LG} - Y^{PA}}. \tag{35}$$

Over the period 1954–2001, $\alpha = 0.283$; see Figure 1. The full sample, 1929–2001, is not used in computing α in order to avoid any odd behavior in income shares associated with the Great Depression, World War II and the Korean War. Recall that in the model, the market capital stock is broken up between structures and equipment and software. NIPA data do not permit separate estimation of the share parameters, α_s and α_e. The results above do, however, impose the following restriction:

$$\alpha = \alpha_s + \alpha_e. \tag{36}$$

Table 1: Data Description

Y	Nominal market output: GROSS DOMESTIC PRODUCT less GROSS HOUSING PRODUCT
Y^P	Nominal private market output: GROSS DOMESTIC PRODUCT less GROSS HOUSING PRODUCT less GOVERNMENT COMPENSATION OF EMPLOYEES
C_m	Nominal market consumption: PERSONAL CONSUMPTION EXPENDITURES ON NONDURABLES and SERVICES less Gross HOUSING PRODUCT
X_m	Nominal market investment: NONRESIDENTIAL FIXED INVESTMENT plus CHANGE IN PRIVATE INVENTORIES
X_h	Nominal home investment: RESIDENTIAL FIXED INVESTMENT plus PERSONAL CONSUMPTION EXPENDITURES ON DURABLES
Y^{LP}	Nominal private unambiguous labor income: COMPENSATION OF EMPLOYEES less HOUSING COMPENSATION OF EMPLOYEES less GOVERNMENT COMPENSATION OF EMPLOYEES
Y^{KP}	Nominal private unambiguous capital income: RENTAL INCOME plus NET INTEREST INCOME plus CORPORATE PROFITS less HOUSING RENTAL INCOME less HOUSING NET INTEREST INCOME less HOUSING CORPORATE PROFITS
Y^{AP}	Nominal private ambiguous income: PROPRIETORS' INCOME less HOUSING PROPRIETORS' INCOME plus (NET NATIONAL PRODUCT less NET HOUSING PRODUCT) less (NATIONAL INCOME less HOUSING NATIONAL INCOME)
Y^{LG}	Nominal government labor income: GOVERNMENT COMPENSATION OF EMPLOYEES
POP	Population: Civilian noninstitutionalized population aged 16 and over

4.3 Depreciation Rates

The depreciation rates, δ_s, δ_e, δ_h and δ_d, are obtained from BEA data on fixed reproducible wealth. In particular, the BEA reports figures for both the capital stock and depreciation. The depreciation rate for, say, structures in year t is computed as depreciation of structures in year t divided by the stock of structures as of the end of year $t-1$. Both depreciation and the capital stock are mea-

Figure 1: Labor's Share of Income
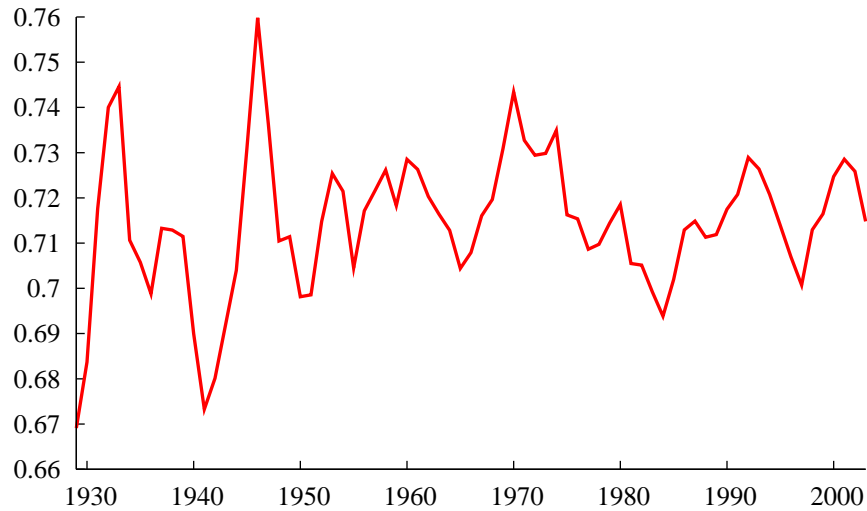
sured in current dollars.[1] This calculation overstates the depreciation rate *slightly* since the BEA treats investment over year t as having occurred exactly half way through the year. Consequently, depreciation for year t includes 6 months of depreciation on investments made in year t.

In a sense, this calculation of δ_s can be thought of as a weighted average of the depreciation rates actually used by the BEA. Specifically, there are lots of different "kinds" of capital that make up structures, and the BEA computes depreciation rates for each kind of capital. The weights correspond to the shares of each kind of capital in the total stock of market structures.

The other depreciation rates are similarly computed; all are plotted in Figure 2. Over the entire sample (1926–2001), all of the computed depreciation rates have trended upwards, although the rates on market structures and housing appear fairly flat since 1970. The depreciation rate on equipment and software has risen since 1960, with an acceleration since 1980. Much of this rise over the past 20–30 years can be attributed to the growing importance of information technology (IT) goods in equipment and software, and a rise in the depreciation rates of IT goods; see Figure 3. The annual depreciation rate on computing equipment has risen from 15% (1960-1980) to around 40% by the 1990s while the depreciation rate on software been steady at around 50% per year. There are a variety of important issues regarding software. For instance, it was not until 1997 that the BEA included software in its measure of the capital stock; previously, software purchases were expensed. For pre-packaged software, the BEA uses a service life of 3 years; see Herman (2000). From the perspective of the firms purchasing the software, this sort of service life is probably reasonable. However, to the extent that software purchases represent upgrades to software that firms already own, the service life of software is likely much longer than 3 years – after all, the firms creating these software packages build on their existing code when creating the latest version of their software.

From the perspective of calibrating the model to a balanced growth path, the nonstationary depreciation rates in Figure 2 are problematic. It is possible that the economy is on a transition to a balanced growth path, perhaps owing to a transition to an information technology-based econ-

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[1]A real chain-type quantity index is available for depreciation and the capital stock; using this data requires converting into comparable units.
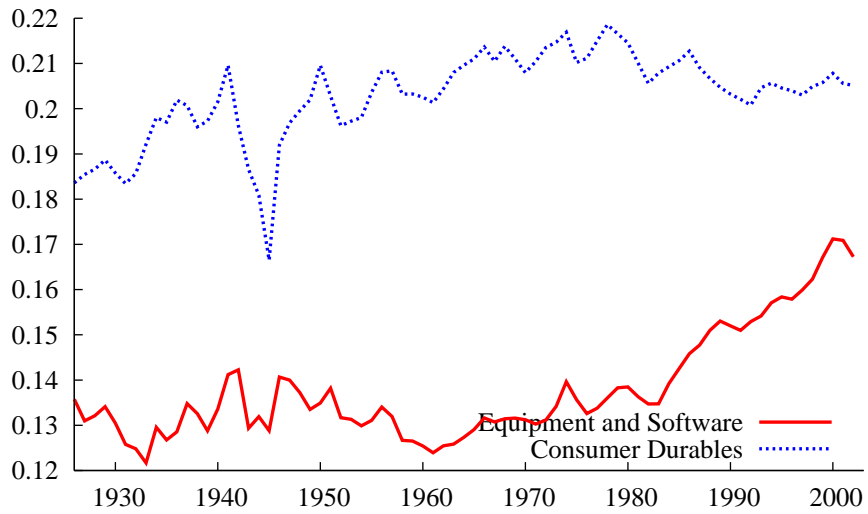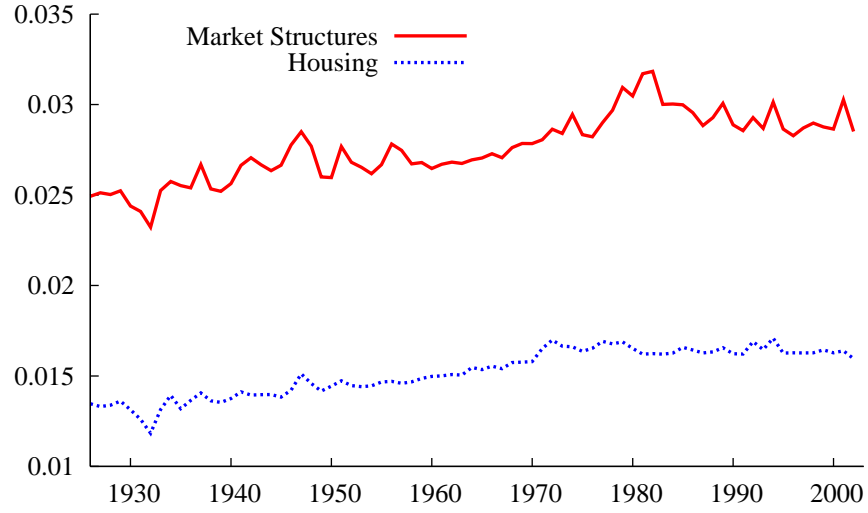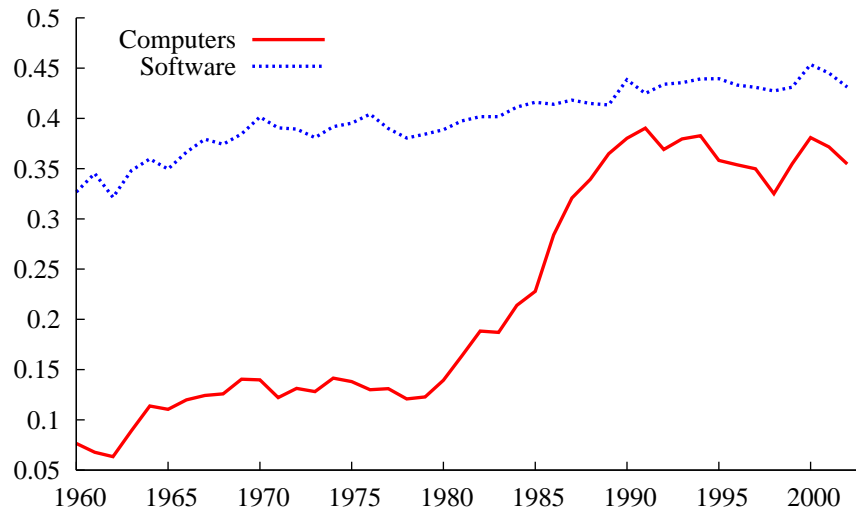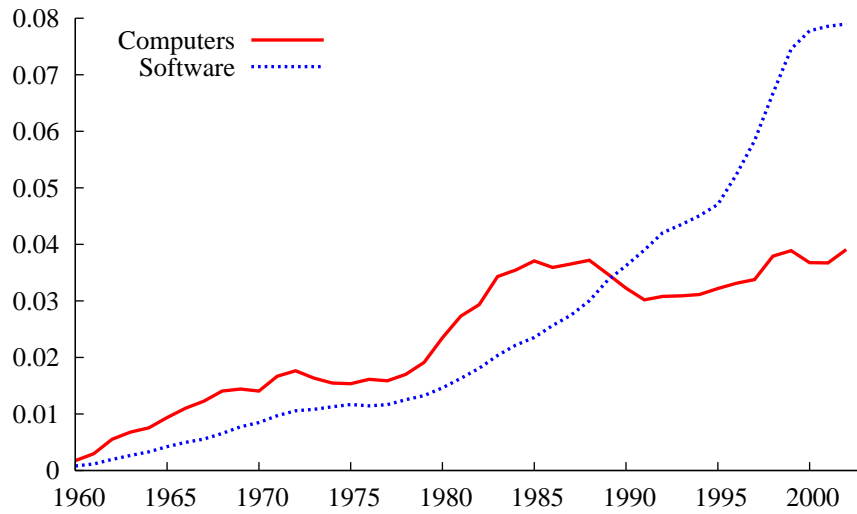
Figure 2: Implied Depreciation Rates

Figure 3: Information Technology Goods

(a) Depreciation Rates

(b) Shares of Stock of Equipment and Software

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omy. Or, perhaps the BEA will reconsider the depreciation schedules that it uses (as it did in the 1990s), and in the future the depreciation rates will look more stationary. For now, annual depreciation rates are computed for the period 1954–2001, just like the other data used in the calibration. Understanding the time series pattern on the implied depreciation rates is left to future research.

4.4 Investment-Output Ratios

Investment shares of output are plotted in Figure 4. Since World War II, the shares of market structures and consumer durables appear stationary. The investment share of housing has fallen somewhat over this time period while that of equipment and software has risen. As with the depreciation rate data, the apparent nonstationarity of two of the investment-output ratios is troublesome for the calibration procedure. As with the depreciation rate, this issue is left for future research, and average investment shares over the period 1954–2001 are used in calibrating the model.

Figure 4: Investment Shares of Output
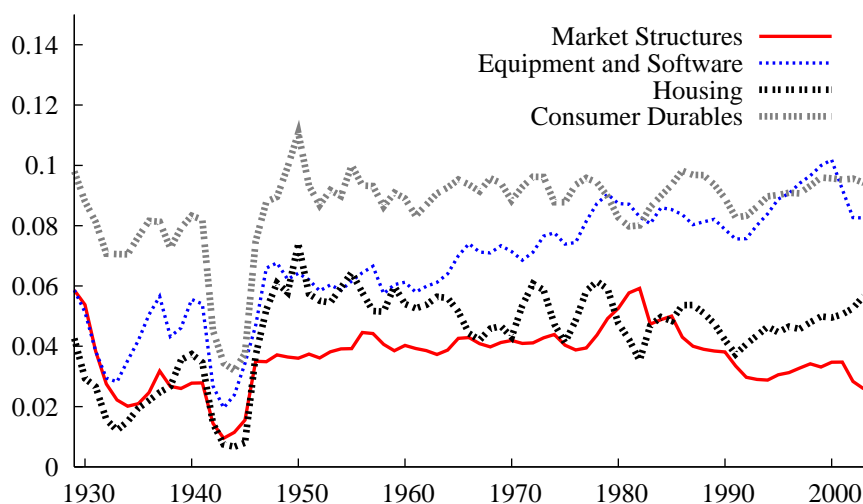
In the literature, it has become common to calibrate to capital-output ratios (measured here by the current cost capital stock divided by nominal GDP) than to investment shares. The connection between investment and capital is given by (31i)–(31l). Since the growth rates of the capital stocks as well as their depreciation rates are, at this point, given, knowing an investment-output ratio determines the corresponding capital-output ratio.

There are a number of reasons to prefer to calibrate to investment shares rather than to capital-output ratios. First, NIPA data is probably measured with more accuracy than the BEA's capital stock data. For instance, the BEA's 1997 revisions increased some broad measures of the capital stock by as much as 30%. Maddison (1995) has constructed "gross capital stock" series for the U.S. that are even larger than the current BEA estimates. Of course, our calibration procedure is not immune from data revision issues. In the case of the 1997 revisions mentioned above, much of the action on the capital stocks can be attributed to changes in the depreciation schedules used by the BEA. These changes would, presumably, be reflected in the depreciation rates derived above. It is our hope that by directly computing depreciation rates and using this evidence to set the

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depreciation rates in the model, that the effects of future revisions will be concentrated on those depreciation rates rather than contaminating other model parameters.

Second, what should be included in the capital stock? Cooley and Prescott (1995) consider a very broad measure of capital, including not only market structures, equipment, housing, and consumer durables, but also government capital, land, and inventories. They would have excluded software since, at the time Cooley and Prescott (1995) were writing, the BEA did not include software in any measure of the capital stock. The land data is no longer published by the Board of Governors owing to data quality issues. Evidently, this series was obtained as a residual from the Flow of Funds accounts, and was periodically negative. While it is plausible that a parcel of land may have a negative value (for example, it might be severely polluted and require extensive cleanup before it could be used), it is difficult to imagine that the stock of land for the U.S. as a whole has a negative value. The justification for including inventories in the capital stock is that they are a factor of production. Yet, few RBC/DSGE models actually model inventories. All this is to say that there are important and conceptually difficult issues in deriving the aggregate capital stock.

Figure 5 plots the capital-output ratios along with the inventories-output ratio. While market structures and housing-to-output ratios appear stationary in the post-World War II period, the other ratios do not. Most noticeable is the upward drift in the equipment and software-to-output ratio, and the downward drift in the inventory-output ratio. Since some of these ratios are not stationary, any attempt to calibrate to these ratios would face similar nonstationarity issues as raised above with respect to the depreciation rates and the investment shares of output.[2]

4.5 The Real Interest Rate

For the baseline parameterization – which imposes the restriction that market and home output grow at the same rate – there is no need to use the real interest rate to calibrate the model. One set of parameters (γ, γ_s, γ_e, γ_h, γ_d, τ_ℓ, τ_k, γ_s, γ_e, γ_h and γ_d) are set directly (and except for the coefficient of relative risk aversion, to match balanced growth facts) while the remaining parameters (β, ω, ψ, α_s, α_e, θ_h, θ_d and γ_x) are set so that along the model's nonstochastic balanced growth path, the model matches the time-use evidence, the investment shares of output, capital's share of income, and the per capita growth rate of output. An obvious way to incorporate evidence regarding the real interest rate is to drop the restriction that home output grows at the same rate as market output. Of course, this option would not be available for a general CES consumption aggregator ($\xi \neq 0$) since balanced growth considerations restrict these growth rates to be equal.

What does the baseline parameterization imply for the real interest rate? Along the nonstochastic balanced growth path, the gross pre-tax real interest rate, R, satisfies

$$g^\gamma = \beta[1 + (1 - \tau_k)R]. \tag{37}$$

The baseline parameters imply a pre-tax real interest rate of 13.2% per annum (an after-tax real return of 7.5%). This value is considerably higher than typical estimates of the pre-tax return on capital. For example, Poterba (1998) finds a (pre-tax) real return on capital of 8.6% (1959–1996) using NIPA data on capital income flows and BEA capital stock data. Siegel (1992) computes a

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[2]Feroli (2002) constructs a model tying together the secular decline in the relative price in equipment and software with the fall in the inventory-output ratio.

Figure 5: Capital-Output Ratios
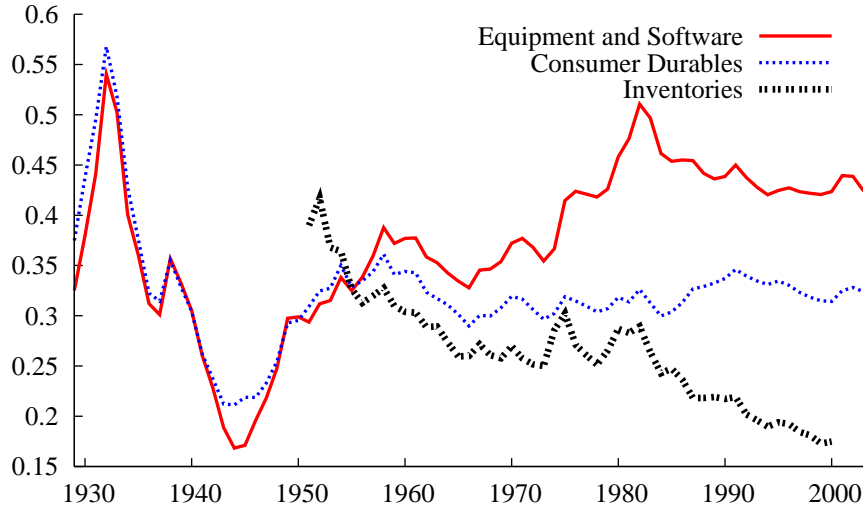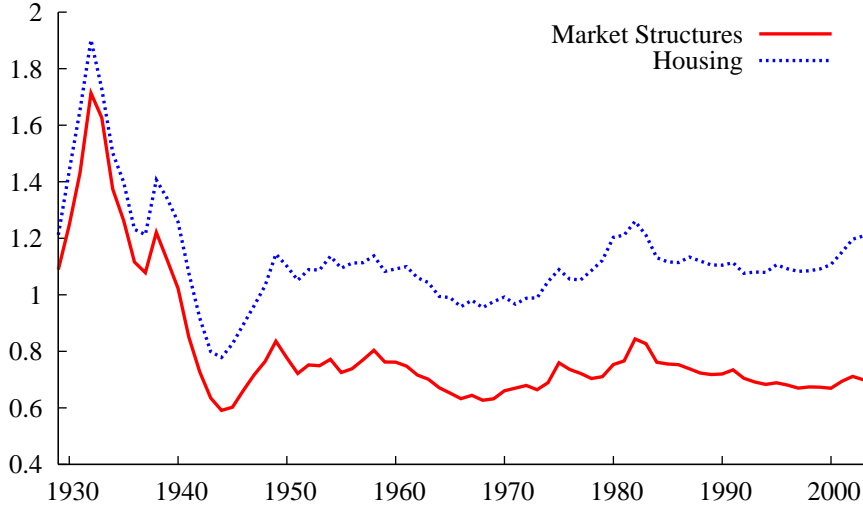
(pre-tax) real return of 7.77% for the Standard & Poor 500 (1800–1990), a value a little smaller than reported in Mehra and Prescott (1985) for the period 1880–1980.

The fact that the baseline calibration implies a very high pre-tax real interest rate is not unique to this paper. Greenwood et al. (1995) calibrate to an annual real growth rate of 1.88%, their annualized discount factor is 0.9598, and their capital income tax rate is 0.70. Together, these parameter values imply a pre-tax real return of 20.5%.

A related issue is the extremely high tax rate placed on capital income. In the home production literature it has become accepted wisdom that high capital income taxes must be included for the model to be consistent with various balanced growth facts; see Greenwood et al. (1995), Hornstein and Praschnik (1997), and Gomme et al. (2001), among others. This claim, however, is incorrect. The problem is most easily seen by suppressing investment-specific technological change and considering a single market capital stock, k_m. Along the nonstochastic balanced growth path, the following equation must be satisfied:

$$g^\gamma = \beta \left[1 - \delta_m + (1 - \tau_k)\alpha \frac{y}{k_m} \right] \tag{38}$$

where δ_m is the depreciation rate of market capital. The term $\alpha(y/k_m)$ is a convenient way of expressing the marginal product of capital when the market production function is Cobb-Douglas. Using Greenwood et al.'s (1995) parameter values ($\gamma = 1$, $g = 1.0188$, $\beta = 0.9598$, $\delta_m = 0.09$, $k_m/y = 1$ and $\alpha = 0.3$) implies $\tau_k = 0.50$.[3] In fact, Hornstein and Praschnik (1997) follow a strategy very similar to the one described above and arrive at a value for τ_k of approximately 0.80. The argument for such a high tax rate on capital is that if $\tau_k = 0$, then, using (38), either capital's share of income would be too low ($\alpha = 0.15$) or the market capital-output ratio would be too high ($k_m/y = 2$) in order to match the other facts.

In making inferences regarding the capital income tax rate based on (38), care must be taken to distinguish between pre-tax and after-tax real interest rates. The most readily available real return data is on pre-tax returns; again, see Poterba (1998) and Siegel (1992). However, the setting of β relies on an after-tax return. As τ_k is adjusted (presumably to satisfy equation (38)), so must the after-tax return that is used to set β. More specifically, the pre-tax real return evidence speaks to the return,

$$1 - \delta_m + \alpha \frac{y}{k_m}. \tag{39}$$

However, this equation cannot be used to make inferences about τ_k, as it does not appear in this expression. It is not clear whether the authors cited in the previous paragraph intended to match after-tax returns or pre-tax returns, thus causing the confusion.

Finally, in the literature, some papers include a capital consumption allowance while others do not. The current paper does not because the method used by Mendoza, Razin, and Tesar (1994) to compute average effective capital income tax rates already includes the effects of the capital consumption allowance.

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[3]Since Greenwood et al. (1995) include a capital consumption allowance, their calibration actually require $\tau_k = 0.70$.

4.6 Tax Rates

Several careful studies have calculated the average tax rate on individual income; see, in particular, Barro and Sahasakul (1983, 1986), Seater (1985), and Stephenson (1998). While there are slight differences in methods, the results show the tax rate between 1954 and 1994 to be (roughly) between 22% and 30%. Mendoza et al. (1994), calculate *average* effective tax rates on both labor and capital. They report labor income tax rates for the U.S. ranging from 17% to 30%, and capital income tax rates between 27% and 50%.

Average effective tax rates are computed following the basic methodology of Mendoza et al. (1994) and Carey and Tchilinguirian (2000). The procedure is to calculate all taxes received by the government for each category and divide by the total income that accrued to each. As described by Mendoza et al. (1994), a first step in is to find the tax rate on household income, τ_h:

$$\tau_h = \frac{\text{PERSONAL CURRENT TAXES}}{\begin{array}{c}\text{NET} \\ \text{INTEREST}\end{array} + \begin{array}{c}\text{PROPRIETORS'} \\ \text{INCOME}\end{array} + \begin{array}{c}\text{RENTAL} \\ \text{INCOME}\end{array} + \begin{array}{c}\text{COMPENSATION} \\ \text{OF} \\ \text{EMPLOYEES}\end{array}} \tag{40}$$

This tax rate is then used to determine the tax rate on labor income, τ_ℓ:

$$\tau_\ell = \frac{\tau_h(\text{COMPENSATION OF EMPLOYEES}) + \text{CONTRIBUTIONS FOR SOCIAL INSURANCE}}{(\text{COMPENSATION OF EMPLOYEES} + \text{EMPLOYER CONTRIBUTIONS OASDI})} \tag{41}$$

where OASDI stands for Old Age, Survivors and Disability Income.

Finally, the tax rate on capital, τ_k, can be computed. Start by summing all taxes paid:

$$\begin{aligned}\text{TAXES PAID} = {} & \tau_h(\text{NET INTEREST} + \alpha\text{PROPRIETOR'S INCOME} + \text{RENTAL INCOME}) \\ & + \text{TAXES ON CORPORATE INCOME} + \text{STATE AND LOCAL PROPERTY TAXES} \\ & + \text{STATE AND LOCAL OTHER TAXES}.\end{aligned} \tag{42}$$

Dividing these taxes paid by the income generated by those sources gives the desired tax rate:

$$\tau_k = \frac{\text{TAXES PAID}}{\text{NET OPERATING SURPLUS} + \text{CAPITAL CONSUMPTION} - (1-\alpha)\text{PROPRIETOR'S INCOME}} \tag{43}$$

where NET OPERATING SURPLUS is value added less depreciation (capital consumption) and payments to labor.

In the calculations in this subsection, no adjustments have been made to net out either the housing sector or government. Such adjustments are not necessary since the coverage for the numerator and denominator of each tax calculation is the same (e.g., total capital income taxes paid divided by total capital income).

The results for the tax rate on household income, τ_h, labor income, τ_ℓ, and capital, τ_k, are summarized in Figure 6. The tax rates on household income and labor income compare favorably with those of Mendoza et al. (1994); the tax on capital income does not since they measure the tax rate on *net* capital income while the tax rate constructed above is on *gross* capital income

Figure 6: Average Tax Rates
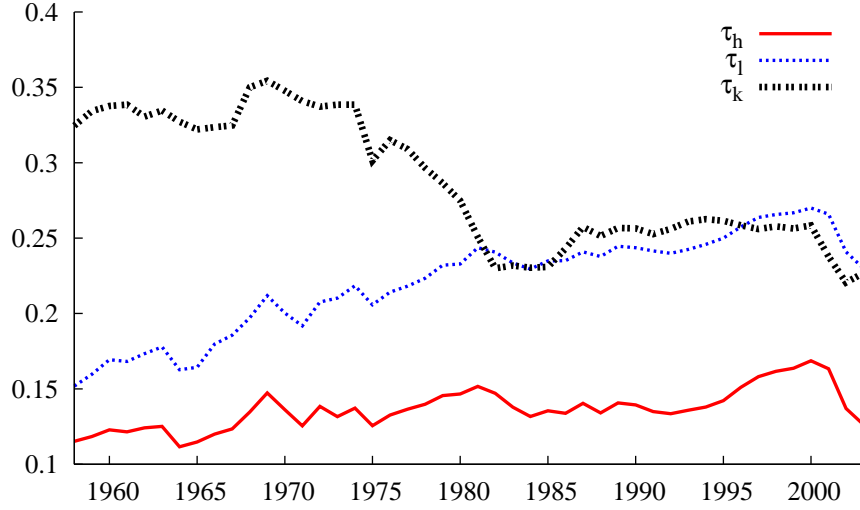
4.7 The Market Technology Shock

Before launching into an extended discussion of the time series properties of the U.S. Solow residual, it is perhaps useful to explain why such a discussion is warranted. Although Prescott (1986) is widely cited as saying that the autoregressive parameter of the U.S. Solow residual is 0.95 and the standard deviation of the innovation to the shock is 0.763%, there is very little formal analysis of the derivation. For example, the closest Prescott comes to saying anything about the autoregressive parameter is to say the Solow residual is highly persistent, and approximately a random walk. Evidently, it is common practice to simply impose an autoregressive coefficient of 0.95; see, among others, Kydland and Prescott (1982), Hansen (1985) and Cooley and Prescott (1995). The 0.763% figure for the standard deviation of the innovation to the Solow residual in Prescott actually refers to the growth rate of the Solow residual.

The main contribution of this subsection is to carefully document the time series properties of the U.S. Solow residual. In particular, the parameters of interest are the autoregressive parameter, ρ_m, and the standard deviation of the innovation to the shock, σ_m.

Assuming that the aggregate production function is Cobb-Douglas, the Solow residual can be computed as

$$z_{St} = \ln Y_t - \alpha_s \ln K_{st} - \alpha_e \ln K_{et} - (1 - \alpha_s - \alpha_e)H_{mt}. \qquad (44)$$

Given time series for output, Y_t, market structures, K_{st}, market equipment and software, K_{et}, and market hours, H_{mt}, a time series for the Solow residual, z_{St} can be constructed. This Solow residual will *grow* over time.

Next, substituting (19) into the autoregressive process for z_{mt}, (7), implies

$$z_{St} = \rho_m(1 - \alpha_s - \alpha_e)\ln \gamma_x + \rho_m z_{St-1} + (1 - \rho_m)(1 - \alpha_s - \alpha_e)(\ln \gamma_x)t + \varepsilon_{mt}. \qquad (45)$$

An important issue raised by equation (45) is how to handle the growth component of the Solow residual. One approach is to rewrite (45) as

$$z_{St} = \beta_0 + \beta_1 z_{St-1} + \beta_2 t + \varepsilon_{mt}. \qquad (46)$$

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The parameters governing the market technology shock, z_{mt}, can be obtained by running a regression of the Solow residual against its own lagged value and a time trend. Therefore, $\rho_m = \hat{\beta}_1$ with σ_m^2 is the variance of the error term of the regression. While this approach can deliver parameter estimates that are remarkably similar to those typically used in the RBC/DSGE literature, the parameter estimates are also extremely sensitive to the sample period. Adding one or two years of data can lead to a substantial change in the parameter estimates.

To address this problem of parameter sensitivity, the growth trend was, instead, removed via application of the Hodrick-Prescott filter, using a conventional value for the smoothing parameter (1600 as the data is quarterly). In this case, the relevant parameters are obtained from the regression equation,

$$\tilde{z}_{St} = \beta_0 + \beta_1 \tilde{z}_{St-1} + \varepsilon_{mt} \tag{47}$$

where $\tilde{z}_{St}$ is the deviation of the Solow residual from its Hodrick-Prescott filtered trend line. The parameter estimates were found to be quite stable across different time samples. Fairly similar results are obtained if, instead, the trend is removed by applying a band pass filter which keeps the frequencies between 6 and 32 quarters.

Since it is standard practice in the RBC framework to use quarterly data, ideally a quarterly capital stock should be used to construct a quarterly Solow residual. However, the BEA only produces annual estimates of the capital stock. One approach adopted in the literature has been to simply omit the capital stock when computing the Solow residual. Prescott (1986) justifies this approach on the basis that the aggregate capital stock is a fairly smooth series (since investment flows are small relative to the stock), and so its omission has little effect on the Solow residual at a business cycle frequency. Cooley and Prescott (1995) argue that any procedure used to construct a quarterly capital stock series will necessarily introduce additional noise into the measured Solow residual, and that a conservative approach is to omit capital when computing the Solow residual. Of course, omitting the capital stock may also introduce noise into the measured Solow residual.

One of the recurring themes in this paper is that the capital stock is subject to considerable measurement error. It would be tempting to use mismeasurement as another justification for omitting the capital stock from the calculation of the Solow residual. Yet, it may be that this mismeasurement primarily affects the *level* of the capital stock. If this is the case, then the level of the measured Solow residual will be affected, but not its time series properties. Rather than dogmatically take a stand one way or the other, properties for the Solow residual are reported with and without the capital stock. Moreover, calculations are also performed allowing the capital stock to be decomposed into market structures and equipment and software separately.

The derivation of quarterly capital stocks is based on the approach of Greenwood et al. (1997) (except that they construct annual capital stocks). Recall that real investment is computed by dividing nominal investment by the consumption deflator. Starting with an initial capital stock in 1947, the quarterly capital stocks are computed using the laws of motion (3). The initial capital stock (for the first quarter of 1947) is obtained as follows. Initialize the annual capital stock for 1929 to the value implied by the BEA's chain-type quantity index, converted to 1996 dollars (since the index is set equal to 100.0 in 1996) using the BEA's current cost estimates of the capital stock. Use the laws of motion for capital in (3) to obtain the capital stock as of the start of 1947; use this value to then compute the quarterly capital stocks.

Since nominal investment has been divided by the consumption deflator, the effects of investment-specific technological change are incorporated into measured real investment. For example, for

market structures, measured real investment gives $q_{st}x_{st}$, not just x_{st}, where q_{st} is the current state of investment-specific technological change, and x_{st} is investment as measured in terms of foregone output. As pointed out in Greenwood et al. (1997), there is an equivalence between investment-specific technological change (as modeled both in their paper and as above), and a declining relative price of capital goods.

The depreciation rates used are those computed in Subsection 4.3. Recall that some of these depreciation rates appear nonstationary. Since the sources of these nonstationarities are not modeled, it seems best to use the time series for the depreciation rates rather than their averages. That is, for year t, each capital stock is assumed to depreciate according to the depreciation rate computed for year t.

The constructed capital stocks will, in general, differ from the BEA's capital stock measures for three reasons. First, the BEA works with finer categories of capital and so applies depreciation rates that are more appropriate for each type of capital. Second, the BEA applies a different depreciation scheme than is imposed by (3). In particular, the BEA's depreciation rates decline with the age of the capital. Third, the BEA's adjustments of depreciation rates to account for technological obsolescence do not fully reflect how investment-specific technological change operates as modeled above.

Figure 7 compares the capital stocks as constructed above with series available from the BEA. As with the investment data, the BEA's current cost measures of capital are deflated by the consumption deflator. Their chain-type index measures are converted to 1996 dollars (the base year) by multiplying the index by the corresponding current cost measure of the capital stock in 1996. For market structures and housing, there is little difference between these three measures of the capital stock. For equipment and software and consumer durables, the constructed capital stock series most closely track the BEA's current cost measures of capital (deflated by the price of consumption goods).

Table 2 presents three sets of regression results based on equation (46). The first set computes the Solow residual as in equation (45), using the capital stocks for market structures and equipment and software separately. The second set of results uses a more standard Solow residual calculation by aggregating the two capital stocks. In this case,

$$z_{st} = \ln Y_t - (\alpha_s + \alpha_e)\ln(K_{st} + K_{et}) - (1 - \alpha_s - \alpha_e)\ln H_{mt}. \tag{48}$$

The final set of results omits capital as in Prescott (1986) and Cooley and Prescott (1995). That is,

$$z_{st} = \ln Y_t - (1 - \alpha_s - \alpha_e)\ln H_{mt}. \tag{49}$$

In general, the estimated parameters of the Solow residual process are quite similar across the three methods of computing the Solow residual, although the method that omits capital has a somewhat lower autoregressive parameter than the other two. Focus on the first set of results for which the Solow residual was computed using the two individual capital stocks. The estimated parameters of the Solow residual process are somewhat different from those typically used in the RBC/DSGE literature. Specifically, the autoregressive coefficient, 0.7518, is quite a bit smaller than the usual value of 0.95, and the standard deviation of the innovation is lower (0.0075 versus 0.00763). Together, these results imply that the standard deviation of the stationary portion of the Solow residual (that is, after removing the growth trend) is roughly 0.0172, compared to a value of 0.0244 for the parameters typically used in the RBC/DSGE literature. As will be seen in Section 5,

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Figure 7: Annual Capital Stocks
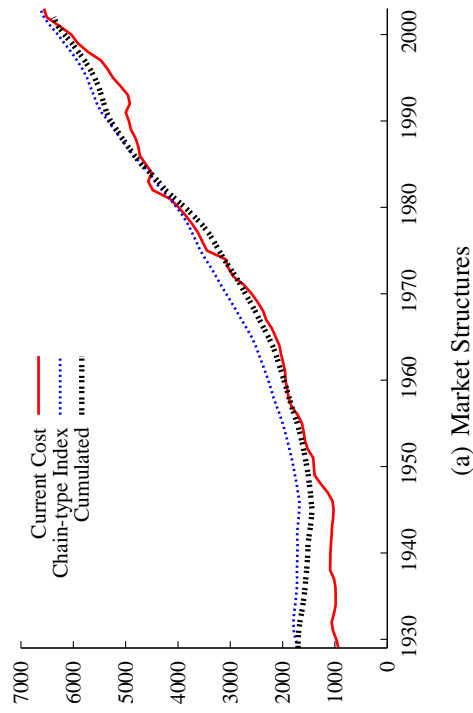
(a) Market Structures
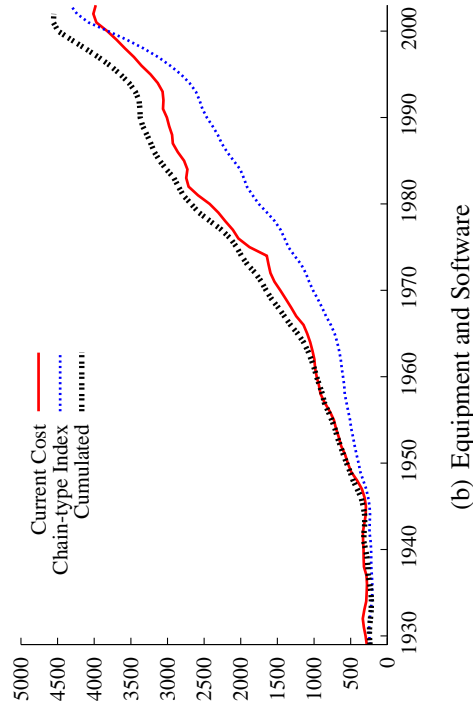
(b) Equipment and Software
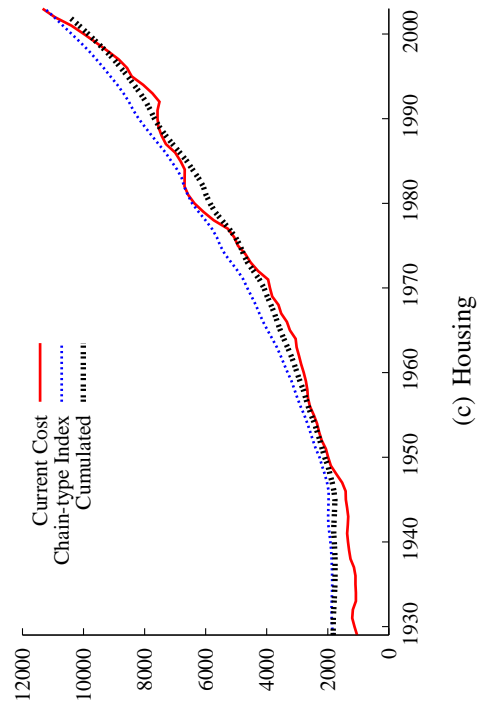
(c) Housing

(d) Consumer Durables

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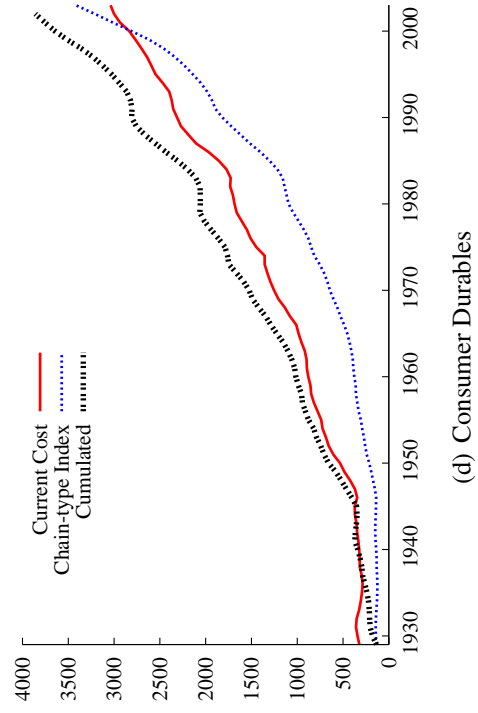
Table 2: Solow Residual Regressions

	Lag	Constant	SD(ε_m)
Two Capital Stocks	0.751613	0.000116	0.007493
	(0.045364)	(0.000528)	
One Capital Stock	0.750222	0.000122	0.007476
	(0.045421)	(0.000527)	
No Capital	0.735568	0.000105	0.007391
	(0.046866)	(0.000522)	

a model using the estimated market shock process continues to display fluctuations that are quite similar to those observed in the U.S. data.

Although it is standard in the literature to use a first-order autoregressive process to describe the Solow residual, four additional lags were added to the regression as a check on this practice. The null hypothesis that these higher order autoregressive coefficients are equal to zero cannot be rejected at conventional levels of significance. It appears that a first-order autoregressive process provides a good description of the data.

The regression results presented in Table 2 are representative of those obtained using the BEA's capital stock measures. These results are robust to using alternative measures of output or different sample periods.

4.8   Measurement of Investment-specific Technologies

The baseline calibration suppresses the stochastic nature of the investment-specific shocks; their importance in the baseline calibration is, in combination with balanced growth facts, to deliver key parameter restrictions – in particular, the Cobb-Douglas restriction on the home and market production functions. Starting with Greenwood et al. (1997), there is a growing literature exploring the role that investment-specific shocks play in accounting for business cycle fluctuations. This subsection discusses measurement of these shocks, and estimates their stochastic properties.

The seminal reference for measurement of investment-specific technologies is Gordon (1990). Gordon's data covers the period 1947–1983 and has not been updated. It has become common practice to extrapolate in various ways the Gordon data to the present; see, for example, Greenwood et al. (1997, 2000); Pakko (2002); Hornstein (1999); Cummins and Violante (2002). For the purposes of the current study, a further issue is that Gordon's data is annual while most business cycle studies focus on quarterly data. The approach adopted here is to use NIPA data to infer the stochastic properties of investment-specific technology. Previous studies have eschewed the use of NIPA data on the basis that NIPA understates the growth rate of investment-specific technological change; see Pakko (2002) for a discussion of these issues. While the NIPA data may understate the (long run) growth rate of investment-specific technological change, it may nonetheless be useful in inferring the shorter-run, business cycle properties of these shocks.

Greenwood et al. (1997) argue that a *common* price deflator should be used when converting nominal NIPA data into real terms, and that a natural choice is the price deflator for nondurables and services. They construct such a series by dividing nominal expenditures on nondurables and services by real expenditures. The same procedure is employed here, and the resulting price index

will be referred to simply as the "consumption deflator."[4] Having thus constructed real investment series, it is a straightforward matter to derive time series for the relative price of investment goods by dividing nominal investment by real investment. As shown in Greenwood et al. (1997), the state of investment-specific technology is the inverse of the relative price of investment goods.

The investment-specific technologies are plotted in Figure 8 for the four investment series, market structures, equipment & software, housing, and durables. The market structures technology and the housing technology are relatively stationary; the scale of the figures tends to exaggerate its movements. Notice, however, that both series decline from around 1.2 in the mid-1960s to a low of around 0.95 in the late 1970s. Equipment & software and durables technologies exhibit very similar trends with both growing throughout most of the sample.

The investment-specific technology series are detrended in the same way as the Solow residual: Hodrick-Prescott filtered with a smoothing parameter of 1600. The parameters describing the time series properties of these series are obtained by running regressions of each series against its own lag – just like the Solow residual was in equation (47). To account for simultaneity in these series, as well as the Solow residual, and to obtain a correlation matrix of the residuals, the parameters are estimated as SUR. Parameter estimates are summarized in Table 3. In all cases, the estimated autoregressive parameters are fairly high (above 0.75), and less than one. The market structures series is quite noisy relative to the other investment-specific technologies; implications of this fact are explored in Subsection 5.1. Given the similarities between equipment & software and durables, it probably is not too surprising that the strongest correlation among innovations is for these two series. By way of contrast, the innovations to the housing and market structures technologies are virtually uncorrelated.

The method of detrending the investment-specific technologies has important implications for their estimated time series properties. For example, detrending with a time trend leads to parameter estimates that are very close to one (in the case of durables, the point estimate exceeds one). Adding time-squared and time-cubed terms to the regression does little to the estimated autoregressive parameters. The reason why the Hodrick-Prescott filter is used to detrend the investment-specific technologies is to treat these technologies the same as the Solow residual. In the case of the Solow residual, detrending using time implied that the estimated time series properties of the Solow residual were rather sensitive to the sample period; Hodrick-Prescott filtering reduces this sensitivity.

4.9   Parameterization and Targets

Given the above arguments, within the class of CES production functions, balanced growth implies that the market and home production functions are Cobb-Douglas; see equations (19) and (30). However, balanced growth imposes no particular restrictions on the consumption aggregator; in

---

[4]In the late 1990s, the BEA adopted "chain weighting" in constructing both price indexes and real magnitudes. The BEA has pointed out that, strictly speaking, it is not appropriate to add real magnitudes. An alternative would be to use either the price index for the consumption of services, or the price index for the consumption of nondurables. Roughly speaking, the consumption deflator constructed above is a weighted average of the two individual price indexes.

Figure 8: State of Investment-specific Technology
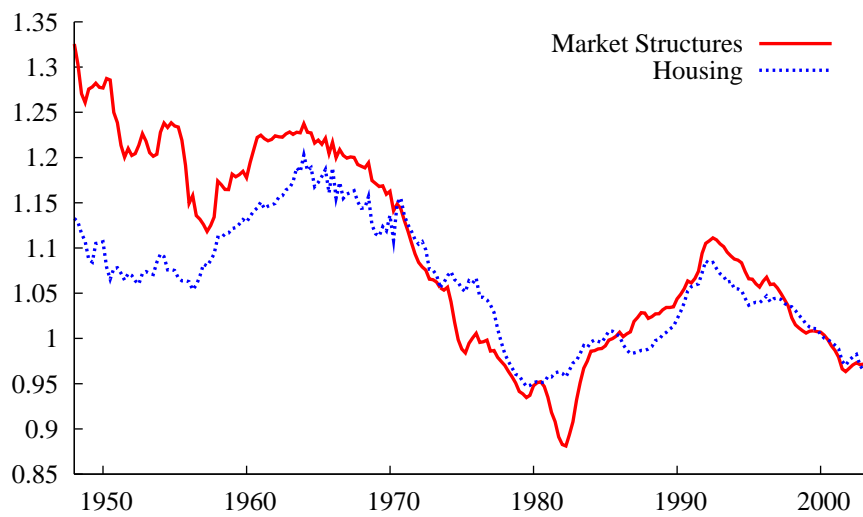
(a) Market Structures and Housing
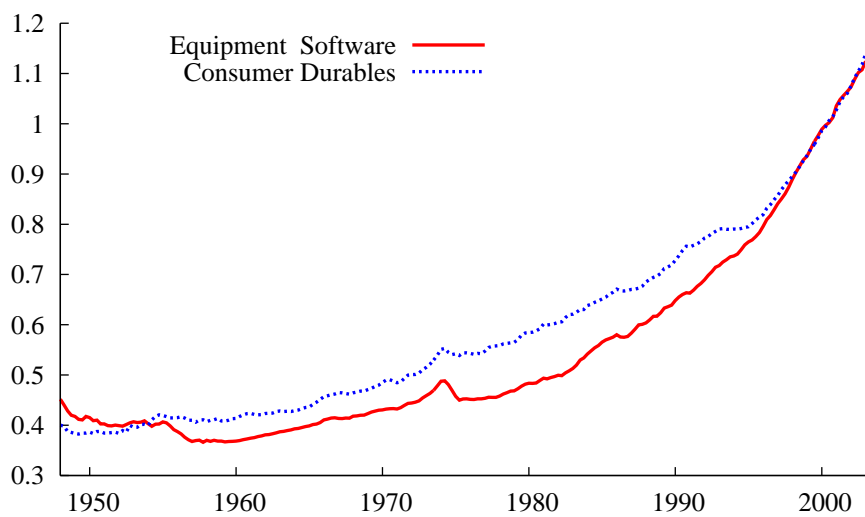
(b) Equipment & Software and Consumer Durables

Table 3: Solow Residual and Investment-specific Shock Regressions

	Lag	Constant	SD(ε)
Solow Residual	0.733754	0.000113	0.007496
	(0.044849)	(0.000526)	
Structures	0.870569	0.000067	0.007437
	(0.028926)	(0.000522)	
Equipment	0.870834	-0.000135	0.005738
	(0.028416)	(0.000403)	
Housing	0.751865	-0.000080	0.007184
	(0.038487)	(0.000504)	
Durables	0.854442	0.000068	0.005318
	(0.033352)	(0.000374)	

Correlation matrix of residuals:

$$
\begin{bmatrix}
1.0000 & & & & \\
-0.0775 & 1.0000 & & & \\
0.0098 & 0.3312 & 1.0000 & & \\
-0.0694 & 0.5233 & 0.0924 & 1.0000 & \\
-0.0590 & 0.1946 & 0.4790 & 0.0894 & 1.0000
\end{bmatrix}
$$

the literature both CES and Cobb-Douglas have been used:

$$
C(c_m, c_h) = \begin{cases} \left[\psi c_m^{\xi} + (1 - \psi) c_h^{\xi} \right]^{1/\xi} & \text{if } \xi \in (-\infty, 0) \cup (0, 1) \\ c_m^{\psi} c_h^{1-\psi} & \text{if } \xi = 0. \end{cases} \tag{50}
$$

In the baseline parameter settings, it is assumed that the consumption aggregator is Cobb-Douglas ($\xi = 0$). Although not required in this case, it is also assumed that home consumption grows at the same rate as market output ($g_c = g$).

The utility function is assumed to be of the constant relative risk aversion variety:

$$
U(C, h_m, h_h) = \begin{cases} \frac{(C(1 - h_m - h_h)^{\omega})^{1-\gamma}}{1 - \gamma} & \text{if } \gamma \in (0, 1) \cup (1, \infty) \\ \ln C + \omega \ln(1 - h_m - h_h) & \text{if } \gamma = 1 \end{cases} \tag{51}
$$

The technology shock, z_m, follows a first-order autoregressive process as specified in equation (7).

The set of parameters for which values must be assigned is summarized in Table 4. Ignoring for the moment the parameters governing the market productivity shock, there are 20 parameters. Consequently, 20 independent pieces of information, summarized in Table 5, are needed to (uniquely) pin down these parameter values. Unless stated otherwise, the calibration targets listed in this table are for the period 1954Q1–2000Q4.

The remainder of this subsection describes the setting of parameters that do not require extensive discussion.

Table 4: Parameters

Preferences		
β	0.9857	discount factor
ω	0.5043	leisure weight
γ	1	coefficient of relative risk aversion
ψ	0.5138	market-home consumption weight
ξ	0	CES parameter in consumption aggregator
Market Production		
α_s	0.1281	share of market structures
α_e	0.1549	share of equipment and software
δ_s	0.0073	depreciation rate of market structures
δ_e	0.0391	depreciation rate of equipment and software
Home Production		
θ_h	0.1958	share of housing
θ_d	0.1819	share of durables
δ_h	0.004	depreciation rate of housing
δ_d	0.058	depreciation rate of durables
Government		
τ_ℓ	0.22	tax rate on labor income
τ_K	0.2868	tax rate on capital income
Shock		
ρ_m	0.7518	market shock autocorrelation
σ_m	0.0075	standard deviation of market shock innovation
Growth		
γ_x	1.002	labor-embodied technological change
γ_s	1.0025	technological change embodied in market structures
γ_e	1.0079	technological change embodied in equipment and software
γ_h	1.0025	technological change embodied in housing
γ_d	1.0079	technological change embodied in consumer durables

Table 5: Calibration Targets

h_m	0.3333	Time-use survey
h_h	0.25	Time-use survey
γ	1.0-2.0	Microevidence
$\alpha_s + \alpha_e$	0.283	Capital's share; NIPA
x_s/y	0.0447	investment-output ratio, market structures; NIPA
x_e/y	0.0859	investment-output ratio, equipment and software; NIPA
x_h/y	0.0557	investment-output ratio, housing; NIPA
x_d/y	0.1014	investment-output ratio, consumer durables; NIPA
δ_s	0.0073	depreciation rate, market structures; BEA
δ_e	0.0391	depreciation rate, equipment and software; BEA
δ_h	0.004	depreciation rate, housing; BEA
δ_d	0.058	depreciation rate, consumer durables; BEA
g	1.0042	Gross real growth rate of per capita U.S. GDP; NIPA
γ_s	1.0025	technological change embodied in market structures; Gort, Greenwood, and Rupert (1999)
γ_e	1.0079	technological change embodied in equipment and software; Gordon (1990)
γ_h	1.0025	technological change embodied in housing; Gort et al. (1999)
γ_d	1.0079	technological change embodied in consumer durables; Gordon (1990)
$g = g_c$		balanced growth considerations.
τ_k	0.2868	tax on capital income; NIPA
τ_ℓ	0.22	tax on labor income; NIPA
ρ_m	0.7518	Time series properties of U.S. Solow residual
σ_m	0.0075%	Time series properties of U.S. Solow residual

Mehra and Prescott (1985) survey the micro estimates of the coefficient of relative risk aversion and find that the bulk of the evidence places its value between 1 and 2. For the baseline calibration, γ is set equal to 1, a value commonly used in the RBC literature, implying logarithmic preferences.

The *Time-Use Survey* asks households how they spend their time. Households are free to categorize different activities as they see fit. For example, some households might find lawn mowing an onerous task and designate it as household work. Other households may categorize lawn mowing as leisure. Analysis of the *Time-Use Survey* data reveals that households spend, on average, around 1/3 of their time working in the market, and around 1/4 of their time performing household work.

There is extensive work documenting the fact that the BEA price indexes inadequately account for quality changes; see, for example, Greenwood et al. (1997). Perhaps the most influential work in this area has been that of Gordon (1990) who carefully documented the quality changes in a variety of capital goods. Following Greenwood et al., the Gordon data is used to pin down the long run rate of investment-specific technological change in equipment and software: $\gamma_e = 1.0079$ at an annual rate. Gordon's data also seems suitable for establishing the rate of investment-specific technical change in consumer durables: $\gamma_d = 1.0079$. Gort et al. (1999) estimate the rate of investment-specific technological progress in market structures at $\gamma_s = 1.0025$. Absent any other evidence, and given the similarity between market structures and housing, it is assumed that the rate of technical change in housing is the same as that of market structures: $\gamma_h = 1.0025$.

Since the model abstracts from population growth, the growth rate of output needs to be expressed in per capita terms. The relevant population is taken to be those individuals who are potentially members of the labor force, namely the civilian noninstitutionalized population aged 16 and over. In order to be consistent with the measurement of labor's share of income and investment shares of output, output is measured as private market output (that is, excluding government compensation of employees). In NIPA, there is no income flow associated with government capital; the remainder of government expenditures amounts to income transfers of various types. The gross growth rate of per capita output is 1.0042 which is roughly 0.1 percentage points higher than the growth rate of total GDP per capita.

5   Business Cycle Moments

This section explores the business cycle properties of the baseline calibration – that is, with disembodied shocks to market and home production, but without investment-specific shocks. As is the usual practice in the RBC/DSGE literature, both the U.S. and model-simulated data are detrended by taking logarithms, then applying the Hodrick-Prescott filter (again, with a smoothing parameter of 1600). Summary business cycle moments for the post-Korean War U.S. data are reported in Table 6 where the lead and lag patterns are expressed relative to private GDP (see Table 1 for its definition); the patterns obtained using total GDP are virtually indistinguishable from those reported in Table 6. Corresponding moments for the baseline model can be found in Table 7; these moments are averages over 1000 replications of 204 observations (the same number of observations as are available for the U.S. economy).

Overall, the model's performance measures up quite favorably relative to previous work in the RBC/DSGE literature. This finding may be surprising since the autoregressive coefficient of the market technology shock is much lower than typically used in the literature, and consequently the volatility of the market technology shock is much lower; see the discussion of these estimates in

Table 6: U.S. Data: Selected Moments; Output = Private GDP

	Standard Deviation	Cross Correlation of Real Output With								
		x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}
Output	1.75	0.11	0.36	0.62	0.85	1.00	0.84	0.60	0.34	0.10
Output (private)	1.92	0.10	0.35	0.61	0.84	1.00	0.84	0.61	0.35	0.10
Consumption	0.85	0.19	0.39	0.58	0.74	0.79	0.70	0.55	0.37	0.18
Investment	4.61	0.18	0.38	0.61	0.81	0.92	0.82	0.63	0.39	0.13
Market	4.70	−0.20	−0.03	0.22	0.49	0.73	0.82	0.80	0.68	0.49
Structures	5.60	−0.38	−0.27	−0.08	0.13	0.37	0.53	0.62	0.63	0.55
Equipment & Software	5.04	−0.06	0.11	0.37	0.63	0.83	0.87	0.78	0.61	0.37
Home	6.29	0.39	0.55	0.70	0.81	0.80	0.61	0.36	0.11	−0.12
Housing	9.98	0.42	0.57	0.69	0.75	0.69	0.50	0.25	0.03	−0.16
Consumer Durables	4.85	0.31	0.46	0.63	0.77	0.82	0.66	0.44	0.18	−0.05
Hours	1.76	−0.13	0.09	0.35	0.62	0.84	0.89	0.80	0.63	0.42
Productivity	1.01	0.42	0.47	0.47	0.38	0.25	−0.12	−0.37	−0.52	−0.56
Productivity (private)	1.04	0.40	0.49	0.54	0.50	0.41	0.03	−0.25	−0.44	−0.52
Capital	0.83	−0.20	−0.21	−0.20	−0.19	−0.15	−0.13	−0.10	−0.06	0.00
Market	1.09	0.05	−0.04	−0.13	−0.19	−0.26	−0.30	−0.32	−0.30	−0.26
Structures	1.06	0.06	−0.03	−0.12	−0.21	−0.28	−0.32	−0.33	−0.30	−0.24
Equipment & Software	1.44	0.04	−0.03	−0.10	−0.16	−0.19	−0.22	−0.24	−0.25	−0.23
Home	0.88	−0.37	−0.30	−0.21	−0.11	−0.00	0.07	0.13	0.19	0.25
Housing	1.09	−0.43	−0.32	−0.19	−0.05	0.09	0.19	0.27	0.34	0.40
Consumer Durables	1.30	−0.10	−0.08	−0.04	0.02	0.09	0.11	0.13	0.14	0.15

Table 7: Baseline Model (No Investment-specific Shocks): Selected Moments

	Standard Deviation	Cross Correlation of Real Output With								
		x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}
Output	1.88	−0.07	0.10	0.35	0.72	1.00	0.72	0.35	0.10	−0.07
Consumption	0.21	−0.35	−0.44	−0.52	−0.60	−0.61	−0.37	0.12	0.41	0.55
Market	0.26	−0.39	−0.25	−0.01	0.40	0.77	0.69	0.59	0.48	0.36
Home	0.53	−0.09	−0.24	−0.44	−0.73	−0.94	−0.69	−0.22	0.08	0.26
Investment	6.11	−0.03	0.13	0.38	0.74	1.00	0.70	0.32	0.06	−0.11
Market	52.53	0.09	0.17	0.28	0.43	0.32	−0.29	−0.23	−0.18	−0.13
Structures	195.57	0.06	0.12	0.20	0.32	0.25	−0.17	−0.14	−0.12	−0.09
Equipment and Software	26.76	0.11	0.19	0.30	0.45	0.31	−0.38	−0.28	−0.20	−0.14
Home	27.98	−0.13	−0.14	−0.17	−0.19	0.06	0.66	0.42	0.24	0.11
Housing	367.00	−0.08	−0.10	−0.12	−0.14	0.02	0.45	0.28	0.16	0.08
Consumer Durables	1.89	−0.17	−0.10	0.02	0.20	0.57	0.94	0.56	0.29	0.10
Hours	0.40	−0.02	0.15	0.39	0.74	1.00	0.70	0.30	0.04	−0.13
Market	1.13	−0.02	0.15	0.39	0.74	1.00	0.70	0.30	0.04	−0.13
Home	0.56	0.02	−0.15	−0.39	−0.74	−1.00	−0.70	−0.30	−0.04	0.13
Productivity (market)	0.77	−0.14	0.02	0.29	0.68	0.99	0.75	0.42	0.19	0.03
Capital	0.46	−0.47	−0.38	−0.18	0.16	0.53	0.63	0.64	0.59	0.50
Market	1.13	−0.16	0.01	0.28	0.69	0.96	0.63	0.37	0.19	0.06
Structures	0.92	−0.25	−0.09	0.18	0.60	0.92	0.67	0.47	0.31	0.18
Equipment and Software	1.36	−0.09	0.08	0.34	0.73	0.97	0.58	0.30	0.11	−0.03
Home	0.74	−0.34	−0.44	−0.56	−0.69	−0.62	−0.08	0.25	0.43	0.50
Housing	1.02	−0.30	−0.41	−0.56	−0.73	−0.69	−0.15	0.19	0.38	0.46
Consumer Durables	0.30	−0.50	−0.52	−0.48	−0.36	−0.09	0.33	0.55	0.63	0.63

Table 8: U.S. Data: Investment Correlations

	Market	Structures	E&S	Home	Housing	Durables
Market	1.00					
Structures	0.83	1.00				
E&S	0.94	0.59	1.00			
Home	0.37	−0.02	0.56	1.00		
Housing	0.26	−0.10	0.44	0.95	1.00	
Durables	0.46	0.07	0.63	0.94	0.79	1.00

Table 9: Baseline Model: Investment Correlations

	Market	Structures	E&S	Home	Housing	Durables
Market	1.00					
Structures	0.74	1.00				
E&S	0.94	0.70	1.00			
Home	−0.74	−0.46	−0.86	1.00		
Housing	−0.45	−0.24	−0.55	0.76	1.00	
Durables	−0.49	−0.30	−0.59	0.83	0.57	1.00

Subsection 4.7. The model's prediction for the volatility of output is virtually the same as that of private GDP; in the literature, output volatility typically falls short of that seen in the data. Other findings generally match up well with those previously seen in the literature:

(1) The volatility of consumption is less than that of output. However, the model predicts that market consumption is too smooth relative to the data.

(2) Volatility of overall investment exceeds that of output, although the model predicts too much volatility in investment.

(3) The standard deviation of hours is roughly a quarter of that seen in the data; this is a well-known problem for RBC/DSGE models, and one standard solution is to introduce indivisible labor as in Hansen (1985)–Rogerson (1988).

The observations above are relatively insensitive to the 'nonstandard' model features included in the baseline model. For example, a model that combines nonresidential structures and equipment & software into a single market capital stock, and housing and durables into a single home capital stock, delivers very similar business cycle predictions. Likewise, removing home production from the model has little effect on the model's predicted business cycle moments. Of particular note is that using the estimated Solow residual process from Subsection 4.7 in an off-the-shelf RBC model delivers standard deviations and correlations that are quite similar to those obtained with a more conventional value for the autoregressive parameter of the market technology shock.

One area where the model's predictions are grossly at variance with the data is in the volatility

of the subcomponents of investment.[5] The model's prediction for the volatility of market investment is over 12 times that seen in the data while that of home investment is 4.5 times as variable. Increased disaggregation of these investment subcategories reveals further anomalies. With the exception of consumer durables, whose volatility the model severely *under*predicts, the model implies investment variability many times that seen in the U.S. data. The reason why the model's prediction for the standard deviation of total investment is only slightly higher than that seen in the data can be attributed to the large negative correlation between market and home investment; see Table 9 and contrast with the correlations in the U.S. data in Table 8. This negative correlation is common to many home production models; see, for example, Benhabib et al. (1991) and Greenwood et al. (1995). In response to a shock to market productivity, on impact market investment responds strongly positively while home investment responds strongly negatively; subsequently, both return close to their original paths. In the literature, solutions to the excess volatility of market and home investment include:

(1) Increased substitutability between market and home consumption goods, decreased substitutability between capital and labor in home production, and highly correlated market and home shocks; see Greenwood and Hercowitz (1991). Recall, though, the balanced growth analysis in Subsection 4.1 which favors Cobb-Douglas production functions.

(2) Introducing time-to-build for market capital smoothes out the spike in market investment following a market technology shock, and can generate a positive correlation between market and home investment; see Gomme et al. (2001). The motive for introducing time-to-build for market capital is the fact that market structures, in particular, take several quarters to finish whereas housing structures can be completed within one quarter. However, since investment in market structures is, on average, roughly half of investment in equipment & software, it is not clear how successful a model would be that imposes time-to-build on market structures, but not on equipment and software.

(3) A nonlinear transformation of output into consumption and investment as in Fisher (1997). Basically, this modification makes it increasingly costly to transform output into investment, thus spreading out the spike to market investment in the face of a market technology shock.

(4) Adding adjustment or installation costs of investment. Again, by increasing the cost of undertaking a lot of investment in one period, these adjustment/installation costs serve to smooth out the previously discussed spike in market investment.

While the model overpredicts the volatility of investment, its performance with regards to capital variability is quite favorable, except perhaps for consumer durables. Presumably, the fact that investment flows are small relative to the capital stocks explains why the excessive investment volatility does not spill over to the capital stock measures.

In the literature, lead-lag patterns usually do not garner much attention – Kydland and Prescott (1990) being a notable exception – although there are potentially important relationships to be

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[5]In simulating the model, individual investment components are periodically negative. To avoid taking the logarithm of a negative number (prior to applying the Hodrick-Prescott filter), investment series are restricted, during simulation, to be strictly positive (negative values are replaced by small but positive entries). While it would be preferable to handle the possibility of negative investment while solving the model, given the large number of state variables, linearization techniques are the most practical method of solving the model.

addressed. Here, attention will be focused on areas in which the theory deviates substantially from the data. Consider, first, investment. In the data, market investment *lags* the cycle by one quarter, and investment in nonresidential structures lags by two quarters. The baseline model predicts that market investment and its components *lead* the cycle by one quarter. That is to say, the model predicts leads where the data shows lags. The same is true of home investment: The model predicts that home investment and its components *lag* the cycle by a quarter while in the data, consumer durables are coincident-to-leading, residential structures (housing) leads by a quarter, and home investment overall leads by a quarter. At least some of the mechanisms used to reduce investment volatility have had some success in terms of the lead-lag patterns.

Turning next to labor market lead-lag patterns, most notable is the fact that average labor productivity *leads* the cycle by two quarters while hours *lag* by one quarter. The model predicts that both series are coincident with the cycle. The leading behavior of productivity is an important outstanding issue for RBC/DSGE theory.

5.1 With All Shocks

Table 10 reports the findings for the model with both disembodied and investment-specific shocks. In this case, the parameters are as reported in Table 3. Innovations to the home technology shock are assumed independent of the investment-specific shocks, but not the market technology shock. The parameters for the home shock are otherwise the same as for the market shock.

The first observation that stands out from Table 10 is that adding investment-specific shocks raises the model's prediction for volatility of *every* macroeconomic series reported in this table. For example, the model now predicts that the standard deviation of output is roughly 1.5 times higher than in the U.S. data while total investment is almost 1.9 times as volatile as in the data. The investment-specific shocks serve to boost the volatility of the individual investment categories: market structures, 121 times that in the U.S. data; equipment & software, 104 times; housing, 72 times; and durables, 4 times. As with the baseline model, the negative correlations among the investment series implies that aggregate investment series are less volatile than the individual components; see Table 11.

While the business cycle results in this subsection are almost entirely negative, some important lessons nonetheless emerge. First, incorporating empirically plausible investment-specific technology shocks are unlikely to resolve the baseline model's overpredictions for investment volatility, and are unlikely to resolve other anomalous investment behavior without introducing other problems; see Fisher (1997) for a related discussion couched in terms of the relative price of investment goods.

Second, the NIPA data may be a poor place to look for (direct) evidence regarding the business cycle behavior of investment-specific technology shocks. This observation is likely independent of whether or not the NIPA data adequately accounts for the longer term growth trends associated with investment-specific technological change. In particular, the shorter term movements in the relative price of investment goods may be dominated by factors other than pure changes in technology.

Together, these (tentative) conclusions pose another challenge to RBC/DSGE theory. First, it will be necessary to find mechanisms that can generate investment dynamics that conform with the data. Second, at some point it will be necessary to confront the relative price of investment goods data in order to understand the factors that drive these prices, and so why they are (apparently) a poor measure of investment-specific technology.

Table 10: Model with All Shocks: Selected Moments

	Standard Deviation	Cross Correlation of Real Output With								
		x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}
Output	2.56	−0.07	0.08	0.31	0.64	1.00	0.64	0.31	0.08	−0.07
Consumption	0.63	−0.13	−0.17	−0.21	−0.26	−0.54	−0.27	−0.01	0.15	0.24
Market	0.53	−0.20	−0.14	−0.03	0.15	0.26	0.29	0.28	0.25	0.21
Home	1.26	−0.04	−0.11	−0.21	−0.34	−0.68	−0.41	−0.14	0.04	0.15
Investment	8.76	−0.04	0.11	0.32	0.63	0.99	0.61	0.27	0.05	−0.10
Market	570.41	0.10	0.15	0.23	0.33	−0.13	−0.19	−0.15	−0.11	−0.08
Structures	676.26	0.08	0.12	0.18	0.25	−0.05	−0.14	−0.11	−0.09	−0.07
Equipment and Software	522.97	0.08	0.13	0.19	0.28	−0.17	−0.18	−0.13	−0.09	−0.06
Home	523.87	−0.09	−0.13	−0.16	−0.22	0.25	0.26	0.18	0.11	0.06
Housing	713.65	−0.11	−0.14	−0.18	−0.24	0.28	0.28	0.18	0.11	0.07
Consumer Durables	19.07	−0.05	−0.06	−0.08	−0.10	0.08	0.18	0.12	0.08	0.04
Hours	0.59	−0.03	0.12	0.32	0.62	0.98	0.60	0.26	0.03	−0.12
Market	1.65	−0.03	0.12	0.32	0.62	0.98	0.60	0.26	0.03	−0.12
Home	0.82	0.03	−0.12	−0.32	−0.62	−0.98	−0.60	−0.26	−0.03	0.12
Productivity (market)	1.01	−0.13	0.02	0.25	0.59	0.94	0.64	0.36	0.16	0.02
Capital	0.73	−0.31	−0.18	0.03	0.38	0.47	0.48	0.44	0.38	0.31
Market	3.46	0.01	0.18	0.43	0.79	0.59	0.35	0.17	0.04	−0.05
Structures	3.39	−0.02	0.09	0.26	0.51	0.45	0.31	0.19	0.10	0.03
Equipment and Software	5.33	0.03	0.18	0.40	0.72	0.49	0.26	0.09	−0.02	−0.09
Home	2.14	−0.19	−0.34	−0.52	−0.77	−0.47	−0.15	0.06	0.19	0.25
Housing	3.04	−0.18	−0.31	−0.50	−0.74	−0.44	−0.15	0.04	0.16	0.23
Consumer Durables	1.56	−0.10	−0.15	−0.20	−0.27	−0.19	−0.03	0.07	0.13	0.16

Table 11: Model with Investment-specific Shocks: Investment Correlations

	Market	Structures	E&S	Home	Housing	Durables
Market	1.00	0.60	0.57	−0.34	−0.49	−0.16
Structures	0.60	1.00	0.16	−0.39	−0.54	−0.04
E&S	0.57	0.16	1.00	−0.29	−0.40	−0.22
Home	−0.34	−0.39	−0.29	1.00	0.65	0.16
Housing	−0.49	−0.54	−0.40	0.65	1.00	0.07
Durables	−0.16	−0.04	−0.22	0.16	0.07	1.00

6 Conclusion

The major goal of this paper was to write down a 'recipe' for calibrating RBC/DSGE models that could be easily replicated. Several important features were added to an otherwise standard RBC model. First, home production was included because of its importance in the measurement of work time, investment, and capital. Second, investment-specific technology growth was incorporated because of its importance, along with balanced growth considerations, in delivering key parameter restrictions. Specifically, these factors suggest that market and home production are most likely Cobb-Douglas, not the more general CES production function. Third, having included investment-specific technological growth, it was appropriate to divide the market capital stock into nonresidential structures and equipment & software, and home capital between housing (residential structures) and consumer durables. This division was made because the growth of structures-specific technology is considerably lower than that embodied in equipment & software and durables.

Key aspects of the calibration are as follows:

(1) Capital's share of income, as measured using private measures of income, is 0.283. This value is toward the low end of values typically used in the RBC/DSGE literature.

(2) Depreciation rates were obtained by dividing depreciation, as reported by the BEA, by the corresponding stock of capital. The time series for these depreciation rates reveal that they have been rising over time – particularly for equipment & software which can be attributed to the growing importance of software and its high depreciation rate.

(3) This paper advocates the use of investment-output ratios rather than capital-output ratios. Simply put, the periodic revisions to the capital stock data are so large, and the conceptual questions about what should be included in the capital stock are so difficult to satisfactorily answer, that estimates of the capital-output ratios are too unreliable to use as calibration targets. The discussion in Subsection 4.4 raises important issues regarding the non-stationarity of at least some of the investment-output ratios. However, the same can be said of the capital-output ratios.

(4) The pre-tax real interest rate for the U.S. economy is fairly high: 7.77% according to Siegel (1992) who used a very long time series of stock market returns, or 8.6% according to Poterba (1998) using NIPA income data and BEA capital stock data. The real interest rate can be used

as a calibration target if the aggregator over market and home consumption goods is restricted to be Cobb-Douglas, in which case the growth rates of market and home capital can differ.

On a related issue, many home production papers have used very high income tax rates on the basis that such high tax rates are needed for the model to simultaneously deliver a reasonable value for capital's share of income and a reasonable figure for the market capital-output ratio. Roughly speaking, these calculations have been made by varying the capital income tax rate, holding fixed the after-tax real interest rate. The appropriate calculation is to hold the pre-tax real interest rate fixed, allowing the after-tax real interest rate to vary with the capital income tax rate. In this case, the capital income tax rate has *no effect* on either the capital-output ratio or capital's share of income.

(5) Calculations of the Solow residual reveal that it is best characterized by an autoregressive parameter of 0.7518 and a standard deviation of its innovation of 0.0075, compared to more standard values of 0.95 and 0.00763, respectively. These parameter estimates were obtained by detrending the Solow residual via the Hodrick-Prescott filter. When detrended in this way, the parameter estimates are not very sensitive to the sample period; nor are the estimates too sensitive to how the Solow residual is calculated (for example, whether or not the capital stock is used). By way of contrast, estimates obtained after removing a deterministic time trend are very sensitive to the sample period. The properties of the Solow residual are used to assign values to the market and home technology shocks. As shown in Section 5, the model continues to display the sort of business cycle phenomena that are familiar from the RBC/DSGE literature, despite the different values for the market technology shock.

(6) The stochastic processes for investment-specific shocks were estimated using relative prices of investment goods obtained from NIPA data. As with the Solow residual, the series were detrended via the Hodrick-Prescott filter. The investment-specific shocks were found to be highly persistent, and in the case of non-residential structures, quite variable.

Implications of the calibration were explored in Section 5. In general terms, the model's business cycle predictions compare favorably with prior work in the RBC/DSGE literature. Several anomalies were noted:

(1) While the variability of total investment compares favorably with that seen in the U.S. data, more disaggregated measures are far too volatile relative to the U.S. data. A related problem is that in the model, market and home investment are strongly negatively correlated whereas in the data, they are weakly positively correlated.

(2) The model fails to capture the observed lead-lag patterns in the investment series. In the data, market investment lags the cycle and home investment leads; the model generates the opposite pattern.

(3) In the data, average labor productivity leads the cycle by two quarters while hours lag by a quarter; the model predicts that both series are contemporaneous with the cycle.

(4) Introducing investment-specific technology shocks, as estimated from the data, caused the model to grossly overpredict the volatility of all macroeconomic variables with the exception of market consumption. The implied variability of the various investment series is grossly at

variance with the data. These findings suggest that introducing relative price changes (equivalently, investment-specific technology shocks) is unlikely to solve the investment volatility and lead-lag patterns identified above. Understanding these relative price movements poses a challenge to RBC/DSGE theory.

References

Barro, Robert, and Sahasakul, C. "Measuring the Average Marginal Tax Rate From the Individual Tax Rate." *Journal of Business* 56 (October 1983): 419–452.

Barro, Robert, and Sahasakul, C. "Average Marginal Tax Rates on Social Security and the Individual Income Tax." *Journal of Business* 59 (October 1986): 555–566.

Benhabib, Jess, Rogerson, Richard, and Wright, Randall. "Homework in Macroeconomics: Household Production and Aggregate Fluctuations." *The Journal of Political Economy* 99 (December 1991): 1166–1187.

Carey, David, and Tchilinguirian, Harry. "Average Effective Tax Rates on Capital, Labour and Consumption." Working Papers Series 258, OECD (2000).

Cooley, Thomas, and Prescott, Edward C. "Economic Growth and Business Cycles." In Thomas Cooley, editor, "Frontiers of Business Cycle Research," Princeton, N.J.: Princeton University Press (1995): 1–38.

Cummins, Jason G., and Violante, Giovanni L. "Investment-Specific Technical Change in the United States (1947-2000): Measurement and Macroeconomic Consequences." *Review of Economic Dynamics* 5 (April 2002): 243–284.

Feroli, Michael. "An Equilibrium Model of Inventories with Investment-Specific Technical Change." (2002). Mimeo, New York University.

Fisher, Jonas D.M. "Relative Prices, Complementarities and Comovement Among Components of Aggregate Expenditures." *Journal of Monetary Economics* 39 (August 1997): 449–474.

Fisher, Jonas D.M. "A Real Explanation for Heterogeneous Investment Dynamics." Working Paper 2001-14, Federal Reserve Bank of Chicago (2001).

Gomme, Paul, Kydland, Finn, and Rupert, Peter. "Home Production Meets Time-to-build." *Journal of Political Economy* 109 (October 2001): 1115–1131.

Gordon, Robert J. *The Measurment of Durable Goods Prices*. Chicago: University of Chicago Press (1990).

Gort, Michael, Greenwood, Jeremy, and Rupert, Peter. "Measuring the Rate of Technological Progress in Structures." *Review of Economic Dynamics* 2 (January 1999): 207–30.

Greenwood, Jeremy, and Hercowitz, Zvi. "The Allocation of Capital and Time over the Business Cycle." *The Journal of Political Economy* 99 (December 1991): 1188–1214.

Greenwood, Jeremy, Hercowitz, Zvi, and Krusell, Per. "Macroeconomic Implications of Capital-embodied Technological Change." *American Economic Review* 87 (June 1997): 342–362.

Greenwood, Jeremy, Hercowitz, Zvi, and Krusell, Per. "The Role of Investment-Specific Technological Change in the Business Cycle." *European Economic Review* 44 (January 2000): 91–115.

Greenwood, Jeremy, Rogerson, Richard, and Wright, Randall. "Household Production in Real Business Cycle Theory." In Thomas Cooley, editor, "Frontiers of Business Cycle Research," Princeton, N.J.: Princeton University Press (1995): 157–174.

Hansen, Gary D. "Indivisible Labor and the Business Cycle." *Journal of Monetary Economics* 16 (November 1985): 309–327.

Herman, Shelby W. "Fixed Assets and Consumer Goods for 1925–2000." *Survey of Current Business* 81 (April 2000): 26–38.

Hornstein, Andreas. "Growth Accounting with Technological Revolutions." *Federal Reserve Bank of Richmond Quarterly Review* 85 (Summer 1999): 1–22.

Hornstein, Andreas, and Praschnik, Jack. "Intermediate Inputs and Sectoral Comovement in the Business Cycle." *Journal of Monetary Economics* 40 (December 1997): 573–595.

Ingram, Beth F., Kocherlakota, Narayana R., and Savin, N. E. "Using Theory for Measurement: An Analysis of the Cyclical Behavior of Home Production." *Journal of Monetary Economics* 40 (December 1997): 435–456.

King, Robert G., Plosser, Charles I., and Rebelo, Sergio T. "Production, Growth and Business Cycles: I The Basic Neoclassical Model." *Journal of Monetary Economics* 21 (March/May 1988): 195–232.

Kongsamut, Piyabha, Rebelo, Sergio, and Xie, Danyang. "Beyond Balanced Growth." *Review of Economic Studies* 68 (October 2001): 869–882.

Kydland, Finn E. "Business Cycles and Aggregate Labor Market Fluctuations." In Thomas Cooley, editor, "Frontiers of Business Cycle Research," Princeton, N.J.: Princeton University Press (1995): 126–156.

Kydland, Finn E., and Prescott, Edward C. "Time to Build and Aggregate Fluctuations." *Econometrica* 50 (November 1982): 1345–1370.

Kydland, Finn E., and Prescott, Edward C. "Business Cycles: Real Facts and a Monetary Myth." *Federal Reserve Bank of Minneapolis Quarterly Review* 14 (Spring 1990): 3–18.

Maddison, Angus. *Monitoring The World Economy 1820–1992*. Paris and Washington, D.C.: Organisation for Economic Co-operation and Development (1995).

McGrattan, Ellen R., Rogerson, Richard, and Wright, Randall. "An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy." *International Economic Review* 38 (May 1997): 267–290.

Mehra, Rajnish, and Prescott, Edward C. "The Equity Premium: A Puzzle." *Journal of Monetary Economics* 15 (March 1985): 145–161.

Mendoza, Enrique G., Razin, Assaf, and Tesar, Linda L. "Effective Tax Rates in Macroeconomics: Cross-Country Estimates of Tax Rates on Factor Incomes and Consumption." *Journal of Monetary Economics* 34 (December 1994): 297–323.

Pakko, Michael R. "Investment-Specific Technology Growth: Concepts and Recent Estimates." *Federal Reserve Bank of St. Louis Review* 84 (November/December 2002): 37–48.

Poterba, James M. "Rate of Return to Corporate Capital and Factor Shares: New Estimates Using Revised National Income Accounts and Capital Stock Data." *Carnegie-Rochester Conference Series on Public Policy* 48 (June 1998): 211–246.

Prescott, Edward C. "Theory Ahead of Business Cycle Measurement." *Federal Reserve Bank of Minneapolis Quarterly Review* 10 (Fall 1986): 9–22.

Rogerson, Richard. "Indivisible Labor, Lotteries and Equilibrium." *Journal of Monetary Economics* 21 (January 1988): 1–16.

Seater, J. J. "On the Construction of Marginal Federal Personal and Social Security Tax Rates in the U.S." *Journal of Monetary Economics* 15 (1985): 121–135.

Siegel, Jeremy J. "The Real Rate of Interest from 1800-1990: A Study of the U.S. and the U.K." *Journal of Monetary Economics* 29 (April 1992): 227–252.

Stephenson, E. F. "Average Marginal Tax Rates Revisited." *Journal of Monetary Economics* 41 (1998): 389–409.