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Comments on Backward-Looking Interest-Rate Rules, Interest-Rate Smoothing, and Macroeconomic Instability

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Benhabib, Schmitt-Grohe, and Uribe (2003) argue that if you relied solely on local analysis you would be led to believe that aggressive, backward-looking interest rate rules are sufficient for determinacy. But from the perspective of global analysis, backward-looking rules do not guarantee uniqueness of equilibrium and indeed may lead to cyclic and even chaotic equilibria. This comment argues that this result is premature. We utilize a discrete time model and make two observations. First, compared to their continuous time model, the cyclic equilibria under a backward-looking rule are much *less* likely to arise in a discrete time model. Second, pure backward-looking rules are *less* likely to suffer from these global indeterminacy problems than rules that also include current or future inflation.

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I. Introduction

Benhabib, Schmitt-Grohe, and Uribe are to be commended for writing a clear and thoughtful paper. Most researchers simply look at local analysis when analyzing different monetary policy rules. The present authors, however, have consistently worried about the global properties of different interest rate rules. In this paper they argue that if you relied solely on local analysis you would be led to believe that aggressive, backward-looking interest rate rules are sufficient for determinacy. But from the perspective of global analysis, backward-looking rules do not guarantee uniqueness of equilibrium and indeed may lead to cyclic and even chaotic equilibria.

This comment argues that this result is premature. We utilize a discrete time model and make two observations. First, compared to the corresponding continuous time model, the cyclic equilibria under a backward-looking rule are much *less* likely to arise in a discrete time model. Second, pure backward-looking rules are *less* likely to suffer from these global indeterminacy problems than rules that also include current or future inflation. By "pure" we mean rules where *only* lagged inflation rates are in the interest-rate rule. This distinction between lagged and current or future inflation does not arise in the continuous time model of Benhabib et al. An interesting observation is that the discrete time interest rate rule that most closely mirrors the results of Benhabib et al is the rule that includes expected inflation.

We also show that these cycles arise because Benhabib et al. adopt a money in the production function (MIPF) framework. As shown by Feenstra (1986) in a flexible price environment, this is equivalent to a money in the utility function (MIUF) framework with a negative cross partial between consumption and real money balances ($U_{mc} < 0$). The existence of cycles arises because the cross partial is significantly negative ($U_{mc} << 0$). With discrete time a rule that does not include expected inflation has to be even more negative than it does for continuous time. It would be interesting to see whether the MIPF framework adopted by Benhabib et al. can generate cycles in discrete time for either the current or backward-looking rules. Similarly since other motives for holding money may lead to a positive cross partial between consumption and real money balances ($U_{mc} > 0$), cycles may not even emerge in continuous time if other motives for holding money were included in the model.

II. A Discrete Time Model

Assume that preferences are separable in the consumption-money composite and in labor (L):

$$U(c,m,1-L) \equiv V(c,m) - B\frac{L^{1+\gamma}}{1+\gamma},$$

and that production is linear in labor $y = L$. We follow Yun (1996) and utilize the assumption of staggered pricing in Calvo (1983). Specifically marginal cost (z) evolves according to the following log-linearized "Phillips curve"

$$\tilde{\pi}_t = K\tilde{z}_t + \beta\tilde{\pi}_{t+1},$$

where $\pi_t = (P_t/P_{t-1})$, denotes the inflation rate, the tildes denote log deviations, and K is a measure of price stickiness ($K \to \infty$ represents perfectly flexible prices). Along with this Phillips curve, the equilibrium conditions are given by

$$\frac{BL_t^{\gamma}}{U_c(t)} = z_t$$

$$\frac{U_c(t)}{P_t} = \beta \frac{U_c(t+1)}{P_{t+1}} R_t$$

$$\frac{U_m(t)}{U_c(t)} = \frac{R_t - 1}{R_t}$$

Finally to close the model we adopt either a *current-looking, backward-looking, or a forward-looking rule:*

$$\widetilde{R}_t = \left(\frac{1}{b}\right)\widetilde{R}_{t-1} + \tau \widetilde{\pi}_{t+i} \,,$$

where b > 1 and i = 0 for current-looking, i = -1 for backward-looking, and i = 1 for forward-looking interest rate rules. Letting $\tau = \theta\left(\frac{b-1}{b}\right)$ we solve these rules backwards to yield the equivalent expression

$$\widetilde{R}_t = \theta \widetilde{\pi}_t^{\,p} = \theta\left(\frac{b-1}{b}\right)\sum_{j=0}^{\infty}\left(\frac{1}{b}\right)^j \widetilde{\pi}_{t+i-j} \;.$$

Notice that if i = Δ or i = -Δ and Δ→0 all three rules converge to the same continuous time rule. Both the current rule and the backward rule are in the spirit of Benhabib et al. The forward case is included for completeness sake. As noted above, the results for the forward rule most closely mirror the results of Benhabib et al.

With all three rules the central bank is reacting to a weighted average of all past inflation rates. With the current rule this weighted average also includes the current inflation rate while with the forward rule it includes both the current and next period's (expected) inflation rate. The key difference between the rules is whether current or future inflation is part of the rule. In the case of the backward rule it is not, so the nominal interest rate is predetermined. In the case of the current and future rules, the nominal interest rate is not predetermined.

III. Current-looking Interest-Rate Rules

We first analyze the model if the policy rule is current-looking. Log linearizing the equilibrium conditions for the current looking rule we have

$$\begin{bmatrix} -1 & 0 & \theta & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta & 0 & -\theta A_{34} & 0 \end{bmatrix} \begin{bmatrix} \tilde{\pi}_{t+1} \\ \tilde{\pi}_t \\ \tilde{\pi}_t^p \\ \tilde{\lambda}_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ \frac{b-1}{b} & 0 & \frac{1}{b} & 0 \\ 1 & 0 & 0 & A_{44} \end{bmatrix} \begin{bmatrix} \tilde{\pi}_t \\ \tilde{\pi}_{t-1} \\ \tilde{\pi}_{t-1}^p \\ \tilde{\lambda}_t \end{bmatrix}$$

where

$$\lambda_t = U_c$$

$$A_{34} = \frac{K\gamma U_{mc}}{DR_{ss}}$$

$$A_{44} = K\left(1 + \frac{\gamma U_{mc}(R_{ss}-1)}{DR_{ss}} - \frac{\gamma U_{mm}R_{ss}}{DR_{ss}}\right)$$

$$D = \left(U_{cc}U_{mm} - U_{mc}^2\right)c_{ss} > 0$$

The characteristic equation for the above system is

$$J(q) = q^4 - \frac{\{\beta - \theta A_{34} + b(1+\beta+\theta A_{34} + A_{44})\}}{\beta b}q^3 + \frac{\{1+\beta+A_{44}+b+(b-1)\theta(A_{34}+A_{44}))\}}{\beta b}q^2 - \frac{1}{\beta b}q = 0$$

We are interested in situations in which the system is locally determinate, but is globally indeterminate because of a supercritical Hopf bifurcation. In this note we will not consider the issue of supercriticality, but leave this for future work. For present purposes we simply assume that any bifurcation is supercritical. We use the terminology

"Locally Determinate, Globally Indeterminate" (LDGI) for situations in which we have local determinacy, but a Hopf bifurcation exists. Following Benhabib et. al. we ask whether there exists a LDGI equilibrium as we vary b. Even when there cannot exist a LDGI with respect to b our analysis does not completely rule out the possibility of cycles that may emerge from varying other deep structural parameters such as β and γ.

The above system is locally determinate if two of the eigenvalues lie outside the unit circle while two lie inside the unit circle. By inspection, one root of J is zero, so that we are left with the following cubic:

$$J(q) = q^3 - \frac{\{\beta - \theta A_{34} + b(1+\beta+\theta A_{34} + A_{44})\}}{\beta b} q^2 + \frac{\{1+\beta+A_{44}+b+(b-1)\theta(A_{34}+A_{44}))\}}{\beta b} q - \frac{1}{\beta b} = 0$$

With discrete time a LDGI exists when one root of J is inside the unit circle and the other two are complex conjugates with a norm equal to one. We will henceforth make use of the following assumptions:

Assumptions A1: $A_{44} > 0$ and $J(-1) < 0$.

Note the assumption that $A_{44} > 0$ is the analogue of Benhabib et al.'s $A_{21} > 0$. The assumption that $J(-1) < 0$ is extremely weak and is equivalent to

$$b\big(2(1+\beta) + A_{44}(1+\theta) + 2\theta A_{34}\big) + 2(1+\beta) - 2\theta A_{34} + (1-\theta)A_{44} > 0\,.$$

Lemma 1: Suppose that assumptions A1 are satisfied. If $\theta > 1$, then one or three real roots of J lie in the unit circle. Thus, under A1, a necessary condition for determinacy is $\theta > 1$.

Proof: The above implies

$$J(1) = \frac{A_{44}\big((b-1)(\theta-1)\big)}{\beta b} > 0 .$$

Since by assumption J(-1) < 0, there are an odd number of real roots in (-1,1). QED

We follow Benhabib et al. and consider variations in the coefficient on the weighted average of current and past inflations. The necessary condition for determinacy if we varied the coefficient on current inflation, τ, in the current-looking Taylor rule is $\tau + 1/b > 1$.

Corollary 1: If J(-1) > 0 and $\theta > 1$ then the system is either over-determined or under-determined so that there cannot exist a LDGI.

Lemma 2: Under the assumptions A1, $J(\frac{1}{b\beta}) = 0$ and $b > 1/\beta$ is a necessary and sufficient condition for a LDGI with respect to b.

Proof:

Recall that the product of the three roots of J is equal to $1/b\beta$. This immediately establishes the necessity of this condition. As for sufficiency, suppose that $J(\frac{1}{b\beta}) = 0$. The other two roots have a norm of unity. If they are real, one must be within the unit circle and one must be outside the unit circle. But from Lemma 1, there cannot be an even number of roots in the unit circle. Hence, the remaining two roots must be complex. QED

Proposition 1: Assume A1 and $\theta > 1$. A necessary and sufficient condition for a LDGI with respect to b is $(1-\beta+\theta(A_{34}+A_{44}))<0$.

Proof: From Lemma 2, we need to analyze J(1/bβ).

$J(1/b\beta) \equiv \beta^3 b^3 j(b) =$

$$\beta(1-\beta+\theta A_{34}+A_{44}\theta)b^2-[\beta^2-1+A_{44}(\beta-1)-\beta\theta(A_{34}+A_{44})-\theta A_{34}]b+(1-\beta)+\theta A_{34}$$

A bifurcation exists if j(b) = 0 for some b > 1/β. Note that

$$j(1/\beta)=\frac{A_{44}\left((1-\beta)(\theta-1)\right)}{\beta b}>0.$$

Suppose that $(1-\beta+\theta(A_{34}+A_{44}))<0$. Then as b goes to infinity, j becomes negative. Hence, there exists a b such that j(b) = 0. Suppose instead that $(1-\beta+\theta(A_{34}+A_{44}))>0$. Since j is convex and j′(1/β) > 0, there does not exist a b > 1/β such that j(b) = 0.

QED.

Corollary 2: A necessary condition for a LDGI with respect to b is that

$$A_{34}+A_{44}=K\left(1+\frac{\gamma(U_{mc}-U_{mm})}{D}\right)<0.$$

Thus a further necessary condition is $U_{mc}<U_{mm}<0$.

IV. Backward-looking Interest-Rate Rules

We next show that a LDGI is less likely if the policy rule looks purely backward. Log linearizing the equilibrium conditions for the backward-looking rule yields

$$\begin{bmatrix} -1 & 0 & \theta & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta & 0 & -\theta A_{34} & 0 \end{bmatrix} \begin{bmatrix} \tilde{\pi}_{t+1} \\ \tilde{\pi}_t \\ \tilde{\pi}_t^p \\ \tilde{\lambda}_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & \frac{b-1}{b} & \frac{1}{b} & 0 \\ 1 & 0 & 0 & A_{44} \end{bmatrix} \begin{bmatrix} \tilde{\pi}_t \\ \tilde{\pi}_{t-1} \\ \tilde{\pi}_{t-1}^p \\ \tilde{\lambda}_t \end{bmatrix}$$

where A_{34} and A_{44} are as defined earlier. Once again one eigenvalue of the system is zero so that we are left with the following cubic:

$$H(q) = q^3 - \frac{\{\beta + b(1+\beta + A_{44})\}}{\beta b} q^2 + \frac{\{1 + \beta + A_{44} + \theta A_{34} + b(1-\theta A_{34})\}}{\beta b} q + \frac{\theta(A_{34} + A_{44})(b-1) - 1}{\beta b} = 0$$

As before, we will utilize assumptions A1: $A_{44} > 0$ and $H(-1) < 0$. Note that $H(-1) = J(-1)$. Since $H(-1) < 0$, $H(1) > 0$ is necessary for determinacy (as this implies an odd number of roots in the unit circle). Therefore a necessary condition for determinacy is $H(1) > 0$. Since

$$H(1) = \frac{A_{44}(b-1)(\theta - 1)}{\beta b}$$

and $b > 1$ we conclude that $\theta > 1$ (or $\tau + 1/b > 1$) is necessary for local determinacy.

Proposition 2: If $\theta > 1$ then there exists a LDGI with respect to b if and only if there exists an a in (-1,1) and a $b > 1$, such that the following conditions are satisfied:

$$f(a,b) \equiv \theta(b-1)(1-2a)(A_{44} + A_{34}) - (1 + \theta(b-1))A_{44} + 2(a-1) + (\beta - 1)(b-1) = 0$$

$$g(a,b) \equiv (\theta(A_{34} + A_{44}) + (1-\beta))(b-1) + bA_{44} + 2\beta b(1-a) = 0$$

$$b(\beta + \theta(A_{34} + A_{44})) > 1 + \theta(A_{34} + A_{44})$$

This implies that a necessary condition for a LDGI with respect to b is that

$(A_{44} + A_{34}) + (1 - \beta) < 0$.

Proof:

We can construct the following polynomial that has real root r and complex roots on the unit circle with a real part equal to a:

$$h(q) = q^3 - (2a + r)q^2 + (1 + 2ra)q \; - r = 0 .$$

Comparing h and H, there exists a LDGI if and only if there exists an a in the unit circle and a b > 1 such that the quadratic and linear coefficients coincide. This provides two linear restrictions:

$$f(a,b) \equiv \theta(b-1)(1-2a)(A_{44} + A_{34}) - \big(1 + \theta(b-1)\big)A_{44} + 2(a-1) + (\beta - 1)(b-1) = 0$$

$$g(a,b) \equiv \big(\theta(A_{34} + A_{44}) + (1-\beta)\big)(b-1) + bA_{44} + 2\beta b(1-a) = 0 .$$

Since b > 1 g(a,b) = 0 can only be satisfied if $(A_{44} + A_{34}) + (1 - \beta) < 0$. For it to be an LDGI the real root of H must be in the unit circle. As in Proposition 1, the real root must be the negative of the constant term in H:

$$0 < \frac{1 - \theta\big(A_{34} + A_{44}\big)(b-1)}{\beta b} < 1 .$$

Given that $S \equiv (A_{44} + A_{34}) < 0$ the lower bound is always satisfied. The upper bound is equivalent to the last condition in the proposition.

QED

Under the case of β = 1, we can prove a stronger result:

Corollary 3: Assume $\beta = 1$. There exists an LDGI with respect to b if and only if

$$1 < \theta < \frac{-\{A_{44}(S+1)+S\}}{S^2}$$

where $S \equiv (A_{44} + A_{34}) < 0$. *Hence, for any S, a sufficiently large* θ *eliminates the LDGI with respect to b.*

Proof:

With β = 1, we can sum f(a,b) = 0 and g(a,b) = 0 and find

$$(b-1)\{2(1-a)(\theta S+1) + A_{44}(1-\theta)\} = 0 .$$

Since b > 1, we can use this to solve for (1-a) and substitute this back into g(a,b) which will now be a function only of b:

$$g(b) \equiv b\theta\{\theta S^2 + S(1+A_{44}) + A_{44}\} - \theta S(\theta S+1) = 0 .$$

Since g(1) > 0, there exists a LDGI if and only if g′(b) < 0. This is only possible if

$$(\theta S^2 + S(1+A_{44}) + A_{44}) < 0 .$$

QED

Corollary 4: Assume $\beta = 1$. *A further necessary condition for an LDGI with respect to b is*

$$1 < \theta < \frac{(1+A_{44})^2}{4A_{44}}.$$

Hence a necessary condition for an LDGI to exist is if $A_{44} > 1$.

Proof:

From the previous corollary a necessary and sufficient condition for an LDGI is

$$f(S) = (\theta S^2 + S(1+A_{44}) + A_{44}) < 0 .$$

This reaches a minimum at

$$f'(S^*) = (1+A_{44}) + 2\theta S^* = 0 .$$

Solving for S and substituting this value into f(S) yields the following necessary condition for an LDGI

$$f(S^*) = \frac{-(1+A_{44})^2}{4\theta} + A_{44} < 0 \text{ or}$$

$$1 < \theta < \frac{(1+A_{44})^2}{4A_{44}}.$$

QED

This condition is in contrast to the conditions for a LDGI when policy includes the current inflation rate. With the current rule, β = 1, and $S \equiv (A_{44} + A_{34}) < 0$, there exists a LDGI with respect to b *for all* values of θ > 1. In the case of the backward rule, enough weight on past values of inflation rule out the existence of a LDGI. Thus backward-looking rules make these cycles less likely.

V. Forward-looking Interest-Rate Rules

We now consider the forward-looking rule and demonstrate that a LDGI exists under even weaker conditions, and that these conditions are essentially the same conditions that arise in the continuous time analysis of Benhabib et al. Log linearizing the equilibrium conditions for the forward-looking rule yields

$$\begin{bmatrix} -1 & \theta & 1 \\ \frac{1-b}{b} & 1 & 0 \\ \beta & -\theta A_{34} & 0 \end{bmatrix} \begin{bmatrix} \tilde{\pi}_{t+1} \\ \tilde{\pi}_t^p \\ \tilde{\lambda}_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & \frac{1}{b} & 0 \\ 1 & 0 & A_{44} \end{bmatrix} \begin{bmatrix} \tilde{\pi}_t \\ \tilde{\pi}_{t-1}^p \\ \tilde{\lambda}_t \end{bmatrix}$$

where A_{34} and A_{44} are as defined earlier. The characteristic equation is given by the following cubic:

$$H(q) = q^3 - \frac{\{\beta + \theta(A_{34} + A_{44}) + b(1 + \beta + A_{44} - \theta(A_{34} + A_{44}))\}}{\beta b - (b-1)\theta A_{34}} q^2 + \frac{\{1 + \beta + A_{44} + b\}}{\beta b - (b-1)\theta A_{34}} q - \frac{1}{\beta b - (b-1)\theta A_{34}} = 0$$

As before, we will utilize assumptions A1: $A_{44} > 0$ and $H(-1) < 0$. Once again $H(-1) = J(-1)$. The variables $\pi_t, \pi_{t-1}^p, \lambda_t$ are all seemingly jump variables. But since

$\pi_{t-1}^p = \left(\frac{1}{b}\right)\pi_{t-2}^p + \left(\frac{b-1}{b}\right)\pi_t$, π_{t-1}^p is predetermined except for its dependence on π_t so that we need only one explosive eigenvalue to determine both π_t and π_{t-1}^p. Hence, determinacy of the system requires that two roots of H lie outside the unit circle. As before $H(-1) < 0$, so that $H(1) < 0$ is necessary for determinacy. Since

$$H(1) = \frac{A_{44}(b-1)(\theta-1)}{\beta b - (b-1)\theta A_{34}}$$

and b > 1 we conclude that θ > 1 (or τ + 1/b > 1) is necessary for local determinacy. This assumes that $\beta b-(b-1)\theta A_{34}>0$. Proposition 3 shows that this is a necessary condition for an LDGI.

Proposition 3: If $\theta > 1$ then there exists a LDGI with respect to b if and only if there exists an a in (-1,1) and a b > 1, such that the following conditions are satisfied:

$$f(a,b)\equiv \theta(b-1)(1-2a)(A_{44}+A_{34})-\left(b-2a\theta(b-1)\right)A_{44}+(1-\beta)+\beta b(2a-1)-b=0$$

$$g(a,b)\equiv \left(\theta A_{34}+(1-\beta)\right)(b-1)+A_{44}+2(1-a)=0$$

$$b\left(\beta-\theta A_{34}\right)>\left(1-\theta A_{34}\right)$$

Proof:

The proof mirrors that of Proposition 2.

QED

Note that since b > 1, $\beta > \theta A_{34}$ is necessary for an LDGI.

Corollary 5: Assume A1, and $\theta > 1$. A necessary and sufficient condition for a LDGI with respect to b is $(1-\beta+\theta A_{34})<0$.

Proof:

Note that g(a,b) = 0 implies that $\left(\theta A_{34}+(1-\beta)\right)<0$ (which implies that $\beta > \theta A_{34}$) is necessary for a LDGI. As for sufficiency, solving g(a,b) for a and substituting this into f(a,b) yields the following necessary and sufficient conditions for a LDGI with respect to b:

$$h(b) \equiv \quad (\beta - \theta A_{34})(1 - \beta + \theta A_{34})b^2 +$$
$$\left(2\theta A_{34}(\theta A_{34} - \beta) - A_{44}(1 - \beta + \theta(A_{34} - 1)) + \beta(\beta - \theta A_{34}) - (1 - \theta A_{34})\right)b +$$
$$\left(1 - \beta + \theta A_{44}(A_{34} - 1) + \theta A_{34}(\beta - \theta A_{34})\right) = 0$$

and

$$b > \frac{(1 - \theta A_{34})}{(\beta - \theta A_{34})}.$$

Assuming $(1 - \beta + \theta A_{34}) < 0$ implies that h is concave. Since

$$h\left(\frac{(1 - \theta A_{34})}{(\beta - \theta A_{34})}\right) = \frac{A_{44}(\theta - 1)(1 - \beta)}{\beta - \theta A_{34}} > 0,$$

then $h(b) \to -\infty \ as \ b \to \infty$ so that a LDGI exists.

QED

This condition is in contrast to the conditions for a LDGI when policy includes the current inflation rate. With the current rule, $\beta = 1$ and $\theta > 1$implied that a LDGI was possible if and only if $S \equiv (A_{44} + A_{34}) < 0$. With the forward looking rule, $\beta = 1$ and $\theta > 1$, there exists an LDGI if and only if $A_{34} < 0$, a noticeably weaker condition. Thus forward elements makes an LDGI more likely.

VI. The Continuous Time Model

Our focus has been on the discrete time model, but for completeness we note how this MIUF model matches up with the MIPF model in Benbabib et al. In the case of continuous time, the log-linearized system is given by

$$\begin{pmatrix} \dot{\lambda} \\ \dot{\pi} \\ \dot{\pi}^p \end{pmatrix} = \begin{pmatrix} 0 & 1 & -\theta \\ A_{44} & r & \theta A_{34} \\ 0 & b & -b \end{pmatrix} \begin{pmatrix} \tilde{\lambda} \\ \tilde{\pi} \\ \tilde{\pi}^p \end{pmatrix}$$

Note that this system matches up identically with Benhabib et al, equations (29)-(30). Our θA_{34} (A_{44}) corresponds to their A_{23} (A_{21}). Following their results we have that a necessary condition for determinacy is $\theta > 1$.

As for bifurcations, in the continuous time model there exists a LDGI as we vary b if and only if $r+\theta A_{34} < 0$. Note that this is identical to the outcome in the discrete-time forward rule, but is in contrast to the discrete time current or backward rule model. In the case of a current rule, there exists a LDGI if and only if $(1-\beta+\theta(A_{34}+A_{44}))<0$. This latter condition is stronger by the term $A_{44} > 0$. As prices approach perfect flexibility ($K\rightarrow\infty$), an LDGI arises in the continuous model whenever $U_{cm} < 0$. But for the discrete-time-current rule, this condition is

$$\left(1+\frac{\gamma}{D}(U_{mc}-U_{mm})\right)<0,$$

a noticeably stronger condition.

We summarize these results in two observations. First, the condition for a LDGI to exist in continuous time is basically identical to the condition in the discrete-time forward-looking rule. Second, in the discrete time model, an LDGI is most likely to occur for a forward-looking rule, least likely for the pure backward-looking rule, with the current rule somewhere in between.

VII. Conclusions

Benhabib et al. suggest that another way to rule out LDGI is to adopt superinertial rules. Taking our current looking rule above and letting b < 1 we can solve forward to generate the equivalent reaction function.

$$\widetilde{R}_{t-1} = \theta\widetilde{\pi}_t^f = \theta(1-b)\sum_{i=0}^{\infty} b^i \widetilde{\pi}_{t+i} \ .$$

As the authors show a superinertial rule is equivalent to an infinitely looking forward rule. One would think that such a policy would generate indeterminacy but a weighted average of today's inflation plus all future inflation is actually predetermined since R_{t-1} is given. An important result of Benhabib et al is that such a rule is not be subject to bifurcations over b < 1.

Yet as noted earlier, there still may be bifurcations as we vary other structural parameters such as β and γ. Furthermore, if bifurcations arise for b > 1, then as we vary b below unity these cycles may come with us. More detailed simulations are necessary to determine whether or not superinertial rules cannot have cycles. Another concern with these superinertial rules is whether they are learnable in the sense of E-stability (see, for example, Evans and Honkapohja (2001)). A related concern is whether cyclic equilibria are learnable.

This comment demonstrates that in a discrete time model LDGI are much less likely than in a continuous time model. A continuous time model where the central bank reacts to a weighted average of all past inflation rates closely resembles a discrete time model where the central bank reacts to a weighted average of tomorrow's, today's, and all past inflation rates. It thus appears that Benhabib et al.'s policy conclusion that backward rules are subject to cycles is premature. It appears that rules with a forward-looking element are most susceptible to LDGI, while pure backward-looking rules are least susceptible to this problem.

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