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How Amenities Affect Job and Wage Choices over the Life Cycle

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Observing the current wage at a job may not fully reflect the "value" of that job. For example, a job with a low starting wage may be preferred to a high starting wage job if the growth rate of wages in the former exceeds the latter. In fact, differences in wage growth can potentially explain why a worker might want to quit a high paying job for a job with a lower initial wage. Job amenities are shown to be another important factor that not only influence the value of a job but also provide an independent rationale for why workers change jobs. The inclusion of a job amenity as part of the "value" can also generate a move from either high-paying to low-paying or low-paying to high-paying jobs as part of an optimal consumption plan over the life cycle. Both the direction of movement and the timing of a job change are shown to depend critically on the relationship between the worker's rate of time preference and the market interest rate.

JEL Classification: J2, J3, J6
Key Words: job changes, amenities, lifetime wage profile

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1 Introduction

Models of the labor market typically use the wage as a statistic that determines such things as whether to enter the labor force, what job to take, how many hours to work, and so on. In other contexts, it compensates for certain job characteristics, such as required hours of work, risk of injury, or other job specific amenities or disamenities.[1]

In this paper job amenities are explicitly included as part of a job to address several questions concerning the sequencing of job choices and wage changes over the life cycle. How does the presence of a job specific amenity affect initial job choice? How do amenities affect job mobility over the life cycle? How does the presence of a job amenity affect the observed wage profile?

To answer these questions a life cycle model is constructed that allows workers to choose their career *path* over various jobs, where a job is defined by a wage and amenity bundle. There is no uncertainty over the level of wages and amenities at all jobs. The extent to which wages and the level of the amenity are substitutable in preferences is shown to play a key role in the analysis. In a version of the model where the level of the amenity is fixed for a given job and wages and the amenity are not substitutes, workers will always want to change jobs over their lifetimes. Some workers will initially choose high paying jobs and will migrate to lower paying ones; other workers will follow precisely the opposite strategy. The key variable that determines the choice of jobs over time is the worker's rate of time preference. Workers who have a relatively high rate of time preference will move to higher paying jobs over their lifetimes, while those who have a relatively low rate of time preference will move to lower paying jobs. Note that even if the individual moves from a higher wage to a lower wage job, this move gives higher lifetime utility as compared to staying in the higher wage job. In another version of the model where wages and the amenity are perfect substitutes, workers may not necessarily want to change jobs. But when they do change jobs, as in the case where wages and amenities are not substitutes, the key variable that determines the pattern of job choices is the worker's rate of time preference.

As mentioned above, an intriguing finding in this paper is that it may be optimal for workers to move from higher to lower paying jobs even with full information over the set of potential

[1] For example, Altonji and Paxson (1988) show that wages are affected by hours constraints; Hwang, Mortensen, and Reed (1998) use a search model to show how estimates of compensating differentials may be biased.

jobs and who are not subject to employment or earnings shocks. Moreover, the movement from high to low paying jobs is consistent with the data. Using the Panel Study of Income Dynamics it is possible to identify workers who have changed employers *voluntarily*.[2] That is, workers who report they were neither fired nor subject to a layoff or shutdown. While the majority of the voluntary leavers move to new jobs that pay more than their previous job, a surprisingly substantial proportion (approximately 42%) move to new jobs where the wages are actually *lower*.[3] This paper provides a model that can account for workers moving from either high to low or low to high paying jobs where the direction of the movement depends upon the worker's (relative) rate of time preference.

Recently, there have been several papers that are related in spirit to the idea in this paper, namely that observing the initial wage at a job may not reflect the overall "value" of the job. Dey and Flinn (2003) examine the relationship between wages and health provision in a search model. They find that although some employers may not offer health insurance, workers essentially "pay" for health insurance in terms of lower wages. Although health insurance in their model plays the role of an amenity in that it is job specific, they do not analyze the sequencing of job choice over the life cycle.

Postel-Vinay and Robin (2002) and Connolly and Gottschalk (2002) where workers move from higher to lower paying jobs. The key to such movement in their models is that workers may move from higher to lower wage jobs if there exists the possibility of higher *future* wage growth in the lower paying job. The point here is that the current wage does not capture the "value" of a job, where the value of a job is given by the *lifetime* income that it generates. Hence, a job that has a current low wage, but high wage growth potential, may be preferred to a job that currently has a high wage but low wage growth. This implies that workers may willingly migrate from (current) high paying jobs to low paying ones. Our model is not inconsistent with this notion, but makes the additional point that the "value" of a job may also depend upon non-wage considerations, such as amenities–and these non-wage considerations may be an important determinant of job choice.

[2]Although in principle it is difficult to know whether a separation is voluntary or involuntary, the question in the PSID asks workers to choose from several reasons as to why they left their last job, one of which being that they chose to leave.

[3]The model makes no distinction between employer changes or job changes, though in the PSID the question concerns employer changes. However, in this paper, job and employer changes are used interchangeably.

In this paper a comparison of lifetime incomes across jobs, i.e., knowing the slope of the wage profile for each job, does not necessarily determine the initial choice of a job nor the direction of movement from low to high or high to low paying jobs.

The paper is organized as follows. Section 2 presents some stylized facts associated with job changes by workers. These data help to both motive and bring perspective to our model, which is presented in section 3. Section 4 analyzes the case where wages and amenities are not be substitutable within a job, while section 5 analyzes the case where they are perfect substitutes. Section 6 concludes.

2 Data Regarding Job Changes

Between 1984 and 1992 the Panel Study of Income Dynamics (PSID) asked individuals about their current and previous employer. For those workers who changed employers, a number of questions were asked: Their reasons for leaving the last employer; their wage with the last and current employer; when they left their last employer and when they began their current employment. The actual question for 1989 in the PSID and choice of response was:[4]

Question:

What happened with that employer—did the company go out of business, were you (HEAD) laid off, did you quit, or what?

Responses:

1. Company folded/changed hands/moved out of town; employer died/went out of business 1989
2. Strike; lockout
3. Laid off; fired
4. Quit; resigned; retired; pregnant; needed more money; just wanted a change in jobs; was self- employed before
5. Other; transfer; any mention of armed services
6. Job was completed; seasonal work; was a temporary job
7. NA; DK
8. Inap.: not working for money now; no other main-job employer during 1988; still working for other employer

[4]The question and responses are slightly different for some years.

After responding that an employer change took place, some follow-up questions were asked. In particular the worker was asked: How much their wage was when the job ended with their previous employer, how much they earned when they started with their new employer. In addition, the date of the ending of the last job and beginning of the current job was also asked. Reported wages were converted to real wages using the monthly CPI since the dates of job endings and beginnings are given as a month within the year.

For the nine years of data (1984-1992) containing the above question, there are 42,765 observations where the respondent had positive income, was either head of the household or spouse of the head, and between the ages 18 and 70. The numbers in Table 1 and Table 2 are averages using all employer changes throughout all of the years. That is, each job change is considered one observation and no account is taken of the fact that some individuals in the data change employers several times while others may change only once.

From that population there were 3,599 people who changed employers for any reason. Table 1 shows summary statistics for all employer changers in the PSID from 1984-1992.

Table 1: All Job Changers

	To Lower Wage		To Same Wage		To Higher Wage	
	mean	std.	mean	std.	mean	std.
% of Job Changers	0.421	0.494	0.084	0.277	0.495	0.500
Age	33.6	9.62	34.5	10.2	32.6	9.06
Months Between Jobs	1.49	2.03	0.003	.057	0.906	1.51
N	3,599					

However, as mentioned above, this paper is concerned with those workers who answered with response #4. Table 2 provides summary statistics for those who changed employers voluntarily. There were 2,313 observations of employer changes between 1984 and 1992.

Though the majority of voluntary job changers, 53%, move to higher paying jobs, a very large proportion of voluntary job changers, 42.5%, move to jobs that pay lower wages. There is very little difference in age between those moving to higher or lower paying jobs, around 32-33 years of

Table 2: Voluntary Job Changers

	To Lower Wage		To Same Wage		To Higher Wage	
	mean	std.	mean	std.	mean	std.
% of Job Changers	0.424	0.494	0.048	0.214	0.528	0.499
Age	32.7	9.13	33.4	10.8	32.0	8.58
Months Between Jobs	1.32	1.88	0.009	.095	0.920	1.50
N	2,313					

age. The median percentage change in real wages for those moving to lower paying jobs is -17.8%, while the median for those moving to higher paying jobs is nearly 20%, as can be seen in table 3. The challenge posed by this data is to provide a coherent theory of why a large fraction of workers move to higher paying jobs, while at the same time, a significant fraction of workers move to lower paying jobs.

Table 3: Wage Changes (%) for Voluntary Job Changers

	Quantiles				
	10%	25%	50%	75%	90%
moved to:					
lower wages	-2.03	-7.46	-17.8	-40.5	-72.5
higher wages	4.08	9.43	19.8	41.4	73.6

Postel-Vinay and Robin (2002) and Connolly and Gottschalk (2002) motivate a worker moving to a lower paying job by the potential for higher wage growth, compared to the worker's current job. Although it is possible to track wage growth before and after the switch, the PSID only asked these job change questions between 1984 and 1992, so that there are not many years before or after the job change. In any event, it is possible to look at those who changed jobs to lower paying jobs exactly in the middle year of the data, 1988, and examine their wage growth four years before and

four years after the job change. The results show very little difference between wage growth before and after the employer change.

3 The Model

Workers are born at date 0 and live for one period of continuous time. At each instant in time workers inelastically supply one unit of labor to a job. For simplicity, it is assumed that the there are only two jobs, job 1 and job 2. Each job $i \in 1,2$ is characterized by a wage, w_i, and a fixed level of the amenity, A_i. There is no uncertainty over the wage/amenity package at each job. At each time t the worker decides where to work. The indicator function $\alpha(t)$ describes the worker's job choice at date t. In particular, if $\alpha(t) = 1$, then the worker chooses job 1 at date t; if $\alpha(t) = 0$, then the worker chooses job 2 at date t.

Agent's preferences are represented by the momentary utility function $u(c(t), A_i)$, where $c(t)$ represents consumption of a private good at time t and A_i is the job amenity that the worker consumes at date t. Agents discount the future at rate δ and can borrow and lend at interest rate r. The worker's total savings (or stock of wealth) at date t is denoted by $a(t)$. The instantaneous change in the worker's wealth at date t is given by the sum of (i) interest income on existing wealth, $ra(t)$, and (ii) the difference between the date t wage, w_i, $i \in \{1,2\}$, and consumption at date t, $c(t)$. Below, an explicit structure on the momentary utility function will be imposed that will reflect the assumed substitutability between wages and amenities.

In order to motivate the incentive to change jobs, jobs are parameterized so that $w_1 > w_2$ and $A_1 < A_2$. One interpretation is that jobs where working conditions are not as pleasant pay a higher wage.

4 Wages and Amenities are not Substitutable

This section specifies a particular functional form for preferences where the wage alone does not embody all of the relevant aspects of the job. The specification assumes no within job substitutability between the private good and the amenity. A momentary utility function for a worker choosing job i that embodies this notion is given by:

$$u(c, A_i) = \ln(c) + A_i. \tag{1}$$

As will be seen below, with this specification of preferences, private consumption is independent of the level of amenity consumption in the sense that, for a given level of lifetime income, the optimal path of private consumption will not depend on the path of amenity consumption.

Obviously, if the instantaneous utility associated with job i, $\ln(w_i)+A_i$, is sufficiently larger than the instantaneous utility associated with job j, $\ln(w_j)+A_j$, then the worker will choose job i and will remain in that job for life. Hence, the decision to change jobs becomes meaningful only if the difference between the instantaneous utilities of the two jobs is not "too big." In fact, assuming that this difference is zero provides the main insights as well as making the problem simpler to solve; therefore, it is assumed that

$$\ln(w_1)+A_1=\ln(w_2)+A_2. \tag{2}$$

The worker will choose a consumption stream, $\{c(t)\}$, and where to work, $\{\alpha(t)\}$, in a manner that solves:

$$\max_{\{c(t),\alpha(t)\}} \int_0^1 e^{-\delta t}[\alpha(t)(\ln(c(t))+A_1)+(1-\alpha(t))(\ln(c(t))+A_2)]dt, \tag{3}$$

subject to

$$\dot{a}(t)=a(t)r+\alpha(t)(w_1-c(t))+(1-\alpha(t))(w_2-c(t)). \tag{4}$$

and

$$a(0)=a(1)=0 \tag{5}$$

The objective function (3) is the worker's lifetime utility. Equation (4) describes how wealth evolves over time. The equations contained in (5) simply say that the worker begins life with no wealth and (optimally) ends life with no wealth.

The (current value) Hamiltonian, $\mathcal{H}$, associated with the maximization problem (3)-(5) is,

$$\begin{aligned}\mathcal{H} &= \alpha(t)(\ln(c(t))+A_1)+(1-\alpha(t))(\ln(c(t))+A_2)+ \\ &\quad \lambda(t)(a(t)r+\alpha(t)w_1+(1-\alpha(t))w_2-c(t)).\end{aligned} \tag{6}$$

The solution to the worker's problem is given by,

$$\mathcal{H}_{c(t)}=\frac{1}{c(t)}-\lambda(t)=0, \tag{7}$$

$$\mathcal{H}_{\alpha(t)}=\begin{cases} A_1-A_2+\lambda(t)(w_1-w_2)>0 & \text{if} \quad \alpha(t)=1 \\ A_1-A_2+\lambda(t)(w_1-w_2)<0 & \text{if} \quad \alpha(t)=0 \\ A_1-A_2+\lambda(t)(w_1-w_2)=0 & \text{if} \quad \alpha(t)=1 \text{ or } \alpha(t)=0 \end{cases} \tag{8}$$

and

$$\dot{\lambda}(t)=-\mathcal{H}_a+\lambda(t)\delta \quad \text{or} \quad \frac{\dot{\lambda}(t)}{\lambda(t)}=-r+\delta. \tag{9}$$

Equations (7) and (9) imply that private consumption grows at the rate of $r-\delta$, i.e.,

$$\frac{\dot{c}(t)}{c(t)} = r-\delta. \tag{10}$$

The shape of the consumption profile is given by the sign of $r-\delta$; if $r-\delta>0$, then consumption is strictly increasing over the worker's life; if $r-\delta<0$, then consumption is strictly decreasing over the worker's life; and if $r-\delta=0$, then consumption is constant.

The worker's job choice, $\alpha(t)$, is determined by (8). Since $\lambda_t = 1/c(t)$, the worker's job choice can be simplified to

$$\alpha(t) = 1 \text{ if } c(t) < \frac{w_1-w_2}{A_2-A_1} \tag{11}$$

$$\alpha(t) = 0 \text{ if } c(t) > \frac{w_1-w_2}{A_2-A_1} \tag{12}$$

and

$$\alpha(t) = 0 \text{ or } \alpha(t) = 1 \text{ if } c(t) = \frac{w_1-w_2}{A_2-A_1} \tag{13}$$

Hence, if at date t the worker's level of consumption is less than $\frac{w_1-w_2}{A_2-A_1}$, then at date t it is optimal for the worker to be at job 1, (inequality (11)); if at date t the worker's level of consumption is greater than $\frac{w_1-w_2}{A_2-A_1}$, then at date t it is optimal for the worker to be at job 2, (inequality (12)).

Notice that the worker will *always* change jobs at least once. To see this suppose that the worker chooses job 1 and remains there for life. Then, equations (4), (5), and (10) imply that at some date $t \in [0,1]$ the worker's level of consumption must equal w_1. But if the worker spends all of life in job 1, then it must be the case, from (11), that

$$w_1 < \frac{w_1-w_2}{A_2-A_1}. \tag{14}$$

Recognizing that $A_2-A_1 = \ln(w_1)-\ln(w_2)$, inequality (14) can be rearranged to read

$$\ln(w_2) > \ln(w_1) + \frac{w_2-w_1}{w_1}. \tag{15}$$

The right hand side of (15) is simply a linear approximation of $\ln(w_2)$ taken at w_1. But $\ln(\cdot)$ is a strictly concave function, which means that the right hand side of (15) must be *strictly greater* than the left hand side, which is a contradiction of inequality (15). Therefore, it must be the case that $w_1 > \frac{w_1-w_2}{A_2-A_1}$, which implies, by (12), that the worker will *not* remain at job 1 forever.

Similarly, if we suppose that the worker spends all of life in job 2, then at some date $t \in [0,1]$, consumption will equal w_2. Hence, inequality (12) implies that

$$w_2 > \frac{w_1-w_2}{A_2-A_1} \tag{16}$$

which can be rewritten as

$$\ln(w_1) > \ln(w_2) + \frac{w_1 - w_2}{w_2}. \tag{17}$$

For exactly the same kind of reasoning as above—i.e., the right hand side of (17) is a linear approximation of $\ln(w_1)$ taken at w_2—inequality (17) can not possibly hold. Hence, $w_2 < \frac{w_1 - w_2}{A_2 - A_1}$, which, by (11), contradicts the assertion that the worker will spend all of life in job 2.

Given that the worker will always change jobs, the sequence of job choices depends on $r - \delta$, as shown in the following subsections.

4.1 $r > \delta$

When $r > \delta$, it is never optimal for the worker to move *from* job 2 *to* job 1. (This implies that the worker will change jobs only once.) If the worker did follow this job sequence, then, by (11), it must be the case that consumption falls after the job change. However, when $r > \delta$ the worker's optimal consumption stream, implicitly given by (10), is always *strictly increasing* over time. Hence, the only possible equilibrium job choice strategy for the worker is to spend the first part of life at job 1 and the second part in job 2. This sequence of job choices is consistent with a strictly increasing lifetime consumption profile, i.e., it is consistent with (11), (12) and (13). It is rather interesting to note, in light of the data presented in Section 2, that when $r > \delta$ the worker will actually move from a *high* wage job to a *low* wage job.

4.2 $r < \delta$

When $r < \delta$, equation (10) implies that the worker's lifetime consumption profile will be strictly decreasing over time. Hence, it is never optimal for the worker to change from job 1 to job 2, as this sequencing of job choices would not be consistent with a strictly decreasing consumption profile, i.e., see (11) and (12). The equilibrium job choice strategy for the worker will be to spend the first part of life at job 2 and the second part at job 1; this sequence of job choice *is* consistent with a strictly decreasing lifetime consumption profile and the worker will change jobs only once. When $r < \delta$ the worker will move from *low* wage jobs to *high* wage jobs.

4.3 $\delta = r$

When the discount rate equals the interest rate, equation (10) implies that the worker's lifetime consumption stream will be constant. Since the worker changes jobs at least once and lifetime consumption is constant, it must be the case that $c(t) = \frac{w_1 - w_2}{A_2 - A_1}$ for all $t \in [0,1]$.

The worker's initial job choice and the number of job changes will now be characterized. The following notation turns out to be helpful. Define $D \equiv \int_0^1 e^{-rt} dt$ and $d_{t_i}^{t_j} \equiv \int_{t_i}^{t_j} e^{-rt} dt$, where $t_j > t_i$. One can interpret both D and $d_{t_i}^{t_j}$ in terms of "discounted time." That is, D represents the discounted value of one unit of time starting at $t = 0$; $d_{t_i}^{t_j}$ represents the discounted value of $t_j - t_i$ units of time t_i units of time from now. The present value of lifetime consumption when $c(t) = \frac{w_1 - w_2}{A_2 - A_1}$ for all $t \in [0,1]$ is then simply $\frac{w_1 - w_2}{A_2 - A_1} D$.

If the worker's initial job choice is, say, job 1, and changes jobs n times, where the last job is, say, job 2, then lifetime income is

$$w_1 d_0^{t_1} + w_2 d_{t_1}^{t_2} + w_1 d_{t_2}^{t_3} + \cdots + w_2 d_{t_n}^1, \tag{18}$$

Note that $\sum_{i=0}^{n+1} d_{t_i}^{t_i} = D$, where $t_0 \equiv 0$ and $t_{n+1} \equiv 1$, and that for a given r, D is just a number. The present value of lifetime income must equal the present value of lifetime consumption, i.e.,

$$w_1 (d_0^{t_1} + \cdots + d_{t_{n-1}}^{t_n}) + w_2 (d_{t_1}^{t_2} + \cdots + d_{t_n}^1) = \frac{w_1 - w_2}{A_2 - A_1} D. \tag{19}$$

Hence, the worker spends $D_1 = d_0^{t_1} + \cdots + d_{t_{n-1}}^{t_n}$ units of discounted time at job 1 and $D_2 = d_{t_1}^{t_2} + \cdots + d_{t_n}^1$ units of discounted time at job 2. But, since $D_1 = D - D_2$, equation (19) is simply an equation in one unknown, D_1. Call the solution D_1^*. That is, D_1^* solves $w_1 D_1^* + w_2 (1 - D_1^*) = \frac{w_1 - w_2}{A_2 - A_1} D$ or

$$D_1^* = \frac{D}{A_2 - A_1} + \frac{w_2}{w_1 - w_2}. \tag{20}$$

Above, it was assumed that the worker's initial job choice was job 1, changed jobs n times and the last job was job 2. It turns out that there is nothing special about this sequencing of job choices. All that is required is that the worker spend D_1^* units of discounted time in job 1 and $D_2^* = D - D_1^*$ units of discounted time in job 2. It does not matter where the worker's initial job is, how many times he changes jobs or what his last job is; all that is required is that he spend the fraction D_1^*/D of discounted time in job 1 and the remainder in job 2.

To sum up, the sign of $r - \delta$ determines whether the worker moves from a high paying job to a low paying one or from the low paying job to a higher paying one. When $r > \delta$, the worker changes

jobs once and moves from the high to low paying job. When $r < \delta$, the worker also changes jobs once but moves from the low to high paying job. When $r = \delta$ the worker will change jobs at least once and is indifferent between job 1 and job 2 as an initial job.

4.4 Discussion

4.4.1 Initial Job Choice

The intuition behind the choice of an initial job is easiest to see by fixing the amount of time spent in each job. By choosing job 1, the higher wage job, first, lifetime income will be higher than if job 2 is choosen first; and, the higher the interest rate, r, the greater will be the difference in lifetime incomes. So, as the interest rate increases job 1 looks more and more attractive as a starting job. Conversely, job 2, as an initial job choice, will provide a higher lifetime value of amenities as will job 1 as an initial job choice; as the discount rate, δ, rises, the greater will be the difference in lifetime value of amenities. As a result, the higher the discount rate for a worker, the more attractive job 2 looks as a starting job.

When $r > \delta$ the "interest rate effect" associated with taking job 1 first dominates the "discount rate effect" of taking job 2 first. So, lifetime utility is higher when job 1 is chosen first. When $r < \delta$ the "discount rate effect" dominates the "interest rate effect," leading to higher lifetime utility by choosing job 2 first. When $r = \delta$, the "interest rate effect" associated with taking job 1 first exactly offsets the "discount rate effect" associated with taking job 2 first, implying that the worker is indifferent between choosing job 1 and job 2 at date $t = 0$.

4.4.2 Why Do Workers Change Jobs?

In order to gain some intuition as to *why* individuals change jobs, assume that a worker lives for only an instant of time. As a first approximation, this allows us to ignore discounting.[5] Imagine that in this instant unit of time the worker spends a fraction q in job 1 and $(1-q)$ in job 2. Over this instant of time financial markets permit the worker to smooth consumption of the market good, c, i.e., the worker can consume approximately $\bar{w} = qw_1 + (1-q)w_2$. But, of course, the worker is

[5]Discounting is important in terms of explaining *which* job the worker will initially take but is not that important in terms of explaining *why* workers change jobs. For example, when $r = \delta = 0$, although the worker is indifferent between which job to take at date $t = 0$, he is not indifferent between changing and not changing jobs; he strictly prefers to change jobs.

unable to smooth the consumption of the amenity since the amenity is job specific. Hence, if the worker smooths consumption of the market good, utility over the instant of time is (approximately) equal to $qu(\bar{w},A_1)+(1-q)u(\bar{w},A_2)$. If the worker spends the entire instant of time in either job 1 or job 2, i.e., the worker does not change jobs, then utility is equal to $u(w_1,A_1)=u(w_1,A_2)$. The worker will prefer changing jobs, compared to staying in the same job, if

$$\begin{aligned} \ln(\bar{w})+\bar{A} &> \ln(w_1)+A_1=\ln(w_2)+A_2 \\ &= q(\ln(w_1)+A_1)+(1-q)(\ln(w_2)+A_2), \end{aligned} \tag{21}$$

where $\bar{A}=qA_1+(1-q)A_2$. Since $\ln(\bar{w})>q\ln(w_1)+(1-q)\ln(w_2)$, inequality (21) holds. Hence, workers want to change jobs because they effectively get to consume "the average" of bundles (w_1,A_1) and (w_2,A_2) and the only way to consume an average of the bundles is by changing jobs.[6]

4.4.3 Many Jobs

Except for the knife-edge case where $r=\delta$, workers will change jobs exactly once in their lifetimes. In reality, however, some workers may never change jobs or other workers may change jobs more than once over their lifetimes.

The model can be generalized along two dimensions. Suppose first that the instantaneous utilities associated with each job need not be equal. [7] Second, suppose that instead of facing two possible job choices each worker is randomly given $n>2$ jobs to choose from. Without loss of generality, let job 1 be the "best" job and job n be the "worst" job in the following sense,

$$u(w_1,A_1)\geq u(w_2,A_2)\geq\cdots\geq u(w_n,A_n). \tag{22}$$

If it turns out that the instantaneous utility of job 1 is substantially higher than job 2, then the worker chooses job 1 at date $t=0$ and will never change jobs. The case considered in the body of the paper can be interpreted by having the instantaneous utilities of job 1 and job 2 not being significantly different from one another, but the instantaneous utility of job 2 substantially larger than job 3. In this situation, the worker will change jobs once: which job the worker chooses first

[6]Note that the inequality (22) is actually a statement about quasi-concavity. Recall that the notion of quasi-concavity is that a consumer can be made better off by consuming the average of two bundles that provide the same level of utility compared to consuming either one of the bundles.

[7]Recall that $u(w_1,A_1)=u(w_2,A_2)$ has been assumed for analytical reasons. By continuity, all of our results will go through if $u(w_1,A_1)\neq u(w_2,A_2)$ but are "close" in value to one another. Clearly, if the instantaneous utility associated with one job is significantly higher than another then the worker will choose the "high" utility job and will not change jobs.

will depend upon the sign of $r - \delta_i$. In general, if the instantaneous utilities associated with the first k jobs are not significantly different from one another, but there is a significant difference between the k^{th} and $k+1^{st}$ job, then the worker will change jobs k times. So by increasing the number of jobs available to workers and by relaxing the assumption that instantaneous utility of all jobs is equal, it is possible for the model to be consistent with observed outcomes in the data.

4.4.4 Worker Heterogeneity

In the data, some individuals move from lower to higher paying jobs, while other individuals move from higher to lower paying jobs. The model can be made consistent with these observed facts if workers are heterogeneous. For example, one simple form of heterogeneity is that different workers have different discount rates. Let δ_i be the discount rate for worker i. One can imagine that there is a population of workers and a distribution of discount rates over this population. All workers, i, that have discount rates greater than the interest rate, i.e., $\delta_i > r$, will spend the first part of life at the low paying job and the second part at the high paying job; all workers, i, characterized by $\delta_i < r$ will spend the first part of the life at the high paying job and the second part at the low paying job. Hence, heterogeneity along the worker discount rate dimension can generate flows of workers moving from low to high paying jobs and at the same time flows of workers moving from high to low paying jobs.

It might be interesting to know which workers starting at, say job 1 (workers with relatively low discount rates), will be the first to change jobs; the higher discount rate workers or the lower? Although it is not possible to get an analytical solution to this answer, numerical solutions are possible. The numerical solutions are performed for the case where the consumption good and amenity are not substitutable. It is also necessary to assign values to the parameters w_1, w_2, A_1, r and δ. Note that A_2 can not be chosen independently of w_1, w_2 and A_1, and is determined by (2).[8] The values chosen for the numerical solutions presented below are $w_1 = 10$, $w_2 = 7$, $A_1 = 5$, and then from (2), $A_2 = 5.36$. The interest rate, r is set equal to 0.05 and δ varies between 0.001 and 0.1. Qualitatively speaking, the numerical results for other parameter values are the same as those presented below as long as the difference in wages is not "too small" and δ is economically

[8]Or more to the point, only three of the four job parameters can be chosen independently, the fourth being determined by (2)

reasonable, i.e., values of δ corresponding to discount factors that are greater than 0.9. Roughly speaking this implies that $\delta \leq 0.1$.

Define q_{ij} as the fraction of time spent in the initial job where the initial job is i. For the parameters chosen, it turns out that, independent of the location of the starting job, workers with a higher discount rate will change jobs first, see figures 1 and 2. In fact, for economically relevant values of δ and, as long as the difference between the wages is not "too small", numerical simulations indicate that the time spent in the first job is a strictly decreasing monotonic function of δ. For values of δ that are not economically relevant, i.e., $\delta > .1$, then q_{21} may display a non-monotonicity. Specifically, as δ increases, it is possible that in some region q_{21} may increase. However, after this increase, q_{21} is again a monotonically decreasing function of (higher) δ's. When the difference between the wages is "not big" q_{12} may display a similar non-monotonicity over a range of δ's.

5 Wages and Amenities are "Perfect Substitutes"

The model developed so far makes the rather extreme assumption that wages and amenities are not at all substitutable. To see if such an extreme assumption is the driving force behind why a worker changes jobs and/or the sequencing of job choice over a worker's lifetime, this section analyzes the opposite extreme where wages and amenities are "perfect substitutes." Now, unlike the case when wages and amenities are not substitutable, the optimal consumption path of the private good will depend critically on the path of amenity consumption. It will be shown below that even when wages and amenities are perfect substitutes, workers may decide to change jobs and, as above, if the worker chooses to change jobs, the sequence of job choices is determined by the sign of $r - \delta$.

The momentary utility function is now assumed to take the form

$$u(c(t), A_i) = \ln(c(t) + A_i). \tag{23}$$

It is useful for what follows to define $w_i + A_i$ as the "aggregate wage" for job i and $c(t) + A_i$ as "aggregate consumption."

Suppose, but only for the time being, that the worker chooses a job at date 0 and remains at that job for life. If the worker chooses job i, then the lifetime consumption-saving decision is determined by the solution to the following maximization problem,

$$\max_{\{c(t)\}} \int_0^1 e^{-\delta t} \ln(c_i(t) + A_i) dt \tag{24}$$

subject to

$$\dot{a}_i(t) = ra_i(t) + w_i - c_i(t) \tag{25}$$

$$a_i(0) = a_i(1) = 0 \tag{26}$$

$$c_i(t) \geq 0 \tag{27}$$

Qualitatively speaking, the only difference between this maximization problem and the one studied in the previous section is to be found in constraint (27). This constraint says that private consumption can not be negative.[9] When this constraint binds it means the worker would, in some time periods, *prefer* to consume less than A_i, saving now to increase consumption in some other periods. Such a strategy, however, is not feasible because the amenity can not be saved—it must be "consumed" in its entirety at each point in time. Optimal consumption is characterized in the following subsections, first when constraint (27) does not bind, and then when it does bind. A discussion concerning optimal job choice follows.

5.1 Constraint (27) Does Not Bind

Under the assumption that constraint (27) *does not bind*, the (current value) Hamiltonian for the above maximization problem is,

$$\mathcal{H} = \ln(c_i(t) + A_i) + \lambda_i(t)(a_i(t)r - w_i - c_i(t)). \tag{28}$$

The solution to this problem is given by,

$$\mathcal{H}_{c_i(t)} = \frac{1}{c_i(t) + A_i} - \lambda_i(t) = 0, \tag{29}$$

$$\dot{\lambda}_i(t) = \mathcal{H}_{a_i(t)} - \lambda(t)r = -\lambda_i(t)(r - \delta). \tag{30}$$

Equations (29) and (30) imply that aggregate consumption, $c_i(t) + A_i$, grows at the rate of $r - \delta$, i.e.,

$$\frac{\frac{d(c_i(t)+A_i)}{dt}}{c_i(t) + A_i} = r - \delta. \tag{31}$$

Hence, when $r > \delta$, the worker's aggregate consumption is increasing over time; when $r < \delta$, it is decreasing; and when $r = \delta$, it is constant.

Equations (29) and (30), in conjunction with the lifetime budget constraint, $\int c_i(t)e^{-rt}dt =$

[9] Such a constraint was not required in the previous section's model because the marginal utility of private consumption is infinite when private consumption is zero.

$w_i \int e^{-rt} dt$, implies that the optimal aggregate consumption is given by,

$$c_i(t) + A_i = (w_i + A_i) \frac{\int_0^1 e^{-rt} dt}{\int_0^1 e^{-\delta t} dt} e^{(r-\delta)t}, \quad t \in [0,1]. \tag{32}$$

5.2 Constraint (27) Does Bind

Now, suppose constraint (27) does bind. Under the assumption (relaxed below) that the worker must keep for life the job he chooses at date 0, the Lagrangian for the worker's maximization problem (24)-(27) is given by,

$$\begin{aligned} \mathcal{L} &= \mathcal{H} + \gamma_i(t) c_i(t) \\ &= \ln(c_i(t) + A_i + \lambda_i(t)(a_i(t) r - w_i - c_i(t)) + \gamma_i(t) c_i(t) \end{aligned} \tag{33}$$

where $\gamma_i(t)$ is the multiplier associated with the constraint that consumption must be non-negative. Since the solution to the worker's problem is straightforward, if somewhat tedious, many of the technical details are omitted in what follows. Intuitively, if the worker chooses job i, then aggregate consumption will be A_i when constraint (27) binds; and when this constraint does not bind, aggregate consumption grows at the rate $r - \delta$. As in the previous section, it will be most convenient to describe the optimal consumption paths under three separate cases: (i) $r > \delta$; (ii) $r < \delta$; and (iii) $r = \delta$.

5.2.1 $r > \delta$

When $r > \delta$, the worker's optimal aggregate consumption path is non-decreasing. The worker's aggregate consumption from job i, $c_i(t) + A_i$, is given by

$$c_i(t) + A_i = \begin{cases} A_i & \text{for } t \in [0, t_i^*] \\ A_i e^{(r-\delta)(t-t_i^*)} & \text{for } t \in (t_i^*, 1] \end{cases}, \tag{34}$$

where $\gamma_i(t) > 0$ for $t \in [0, t_i^*]$ and $\gamma_i(t) = 0$ for $t \in (t_i^*, 1]$. Optimal aggregate consumption paths for the worker are depicted in Figure 3. As in the previous section, in order to facilitate comparison between the two jobs, assume for the time being that $w_1 + A_1 = w_2 + A_2$. The aggregate consumption path c^* in figure 3 assumes that it is possible to borrow and/or save the amenity, i.e., it is the optimal consumption plan when one *ignores* constraint (27). Note that for this consumption path $c^*(t) < A_i$ for some t; hence, for some t, constraint (27) *must* bind. If the worker takes job 1 and constraint (27) is not ignored, then the worker's consumption will be equal to A_1 for all $t \in [0, t_1^*]$, after which aggregate consumption grows at the rate of $r - \delta$. This consumption path is depicted

as c_1^* in Figure 3. Similarly, if the worker takes job 2 and constraint (27) is not ignored, then the worker's consumption will be equal to A_2 for all $t \in [0, t_2^*]$ after which aggregate consumption grows at the rate of $r - \delta$. This consumption path is depicted as c_2^* in Figure 3.

5.2.2 $r < \delta$

When $r > \delta$, the worker's optimal aggregate consumption path is non-increasing. The worker's optimal aggregate consumption, $c_i(t) + A_i$, is given by

$$c_i(t) + A_i = \begin{cases} (c_i(0) + A_i)e^{(r-\delta)t} & \text{for } t \in [0, t_i^*] \\ A_i & \text{for } t \in (t_i^*, 1] \end{cases}, \tag{35}$$

where $\gamma_i(t) = 0$ for $t \in [0, t_i^*]$ and $\gamma_i(t) > 0$ for $t \in (t_i^*, 1]$. The worker's aggregate consumption at date 0 is $c_i(0)$ at job i and can be shown to be equal to $A_i(e^{-(r-\delta)t_i^*} - 1)$.[10] The optimal aggregate consumption paths for the worker are depicted in Figure 4. As in Figure 3, it is assumed that $w_1 + A_1 = w_2 + A_2$ and the consumption path c^* allows the possibility to borrow and save the amenity, i.e., constraint (27) is ignored. If the worker chooses job 1 and constraint (27) is not ignored, then for $t \in [0, t_1^*]$, the worker's aggregate consumption grows at rate $r - \delta$ and for $t \in (t_1^*, 1]$ the worker's consumption is equal to A_1; see consumption profile c_1^* in Figure 4. Similarly, if the worker takes job 2 and constraint (27) is not ignored, then the worker's consumption will be equal to A_2 for all $t \in [0, t_2^*]$ after which aggregate consumption grows at the rate of $r - \delta$; see consumption profile c_2^* in Figure 4.

5.2.3 $r = \delta$

When $r = \delta$, the worker's optimal aggregate consumption path is constant. The worker's optimal aggregate consumption, $c_i(t) + A_i$, is given by

$$c_i(t) + A_i = \ w_i + A_i \quad \text{for all } t \in (0, 1]. \tag{37}$$

Note that constraint (27) can not bind when $r = \delta$ since, in equilibrium, $c_i(t) = w_i > 0$ for all $t \in (0, 1]$

[10] It can be shown that t_i^* is given by the solution to

$$w_i \int_0^1 e^{-rt} dt = A_i \left(\int_0^{t_i^*} e^{-(r-\delta)t_i^* - \delta t} dt - \int_0^{t_i^*} e^{-rt} dt \right). \tag{36}$$

5.3 Discussion

When constraint (27) *does not* bind, the optimal aggregate consumption equation, (32), implies that the aggregate wage is a "sufficient statistic" for the job. Sufficient statistic means that the worker will choose that job that has the highest aggregate wage and has no incentive to switch jobs later on. Since at each point in time aggregate consumption is higher for higher aggregate wage jobs, see equation (32), the worker will choose and remain in the job that has the higher aggregate wage, $w_1 + A_2$ or $w_2 + A_2$. In this situation, the worker has no strict incentive to change jobs. So unlike the case where private consumption and the amenity are not substitutable, when the instantaneous utilities of the two jobs are the same, the worker is indifferent between changing jobs or not. In fact, if there is an arbitrarily small cost associated with changing jobs, a worker will strictly prefer not to change jobs when the instantaneous utilities of the two jobs are the same. When private consumption and the amenity are not substitutable and the instantaneous utilities of the two jobs are the same, the worker will still strictly prefer to change jobs when there is an arbitrarily small cost associated with moving. Note that when $r = \delta$, constraint (27) never binds. In this situation, the aggregate wage is again a sufficient statistic for the job.

The remainder of the discussion assumes that constraint (27) *does* bind, which necessarily implies that either $r > \delta$ or $r < \delta$. When $w_1 + A_1 = w_2 + A_2$ the lifetime utility associated with job 1 will be strictly greater than the lifetime utility associated with job 2. The easiest way to see this is to note that since $w_1 + A_1 = w_2 + A_2$, it is possible to replicate the optimal job 2 aggregate consumption profile, c_2^*, when in job 1; however, since $A_2 > A_1$ it is not possible to replicate the optimal job 1 aggregate profile when in job 2, see Figures 3 and 4. Since a worker chooses not to replicate c_2^* when in job 1, it must imply that the lifetime utility associated with consumption profile c_1^* is higher than that associated with consumption profile c_2^*. Note also that the constraint (27) binds for a longer period of time in job 2, compared to job 1; loosely speaking, the consumption profile c_2^* is "farther away" from the unconstrained consumption profile, c^*, than is consumption profile c_1^* in Figures 3 and 4. Finally, when $w_1 + A_1 = w_2 + A_2$, the worker does not have a (strict) incentive to change jobs. If there is an arbitrarily some cost associated with moving, then the worker's optimal strategy is to take job 1 and remain at it for the rest of life.

Now suppose that $w_1 + A_1 \geq w_2 + A_2$. Then the worker will choose and will remain in job 1

for life. Clearly, if the worker prefers job 1 to job 2 when $w_1 + A_1 = w_2 + A_2$, job 1 will continue to be preferred to job 2 when $w_1 + A_1 > w_2 + A_2$. Hence when $w_1 + A_1 \geq w_2 + A_2$, as in the case where constraint (27) does not bind, the aggregate wage is a sufficient statistic for the job in the sense that the worker will choose and remain in the job that pays the highest (aggregate) wage.

Suppose now that $w_1 + A_1 < w_2 + A_2$. If the worker must stay at a job for life, then job 1 will be chosen over job 2 if the difference between the aggregate wages between jobs 1 and 2 is "small."[11] To see this, note that although job 2 provides the worker with a higher aggregate lifetime income, job 1 provides a "less constrained" consumption profile. So when the aggregate wage differences between the jobs are "small" the latter effect will dominate the former. More importantly, in this situation—where the aggregate wage in job 2 exceeds that of job 1 but the difference in aggregate wages is small—the worker will actually want to change jobs. To see this, and without loss of generality, assume that $r > \delta$. In Figure 5, c_1^* represents the consumption profile if the worker stays forever in job 1. Suppose that a worker chooses job 1 and follows the consumption profile c_1^* until time $\hat{t}$, where $c_1^*(\hat{t}) = A_2$, at which time switches to job 2 for the remainder of life, see Figure 5. Since $w_2 + A_2 > w_1 + A_1$, the worker will be able to consume more than $c_1^*(t)$ in all $t \geq \hat{t}$; hence it is optimal for the worker to change jobs. The *optimal* consumption profile for the individual, c_{12}^*, and the optimal time to change jobs, date t^*, are described in Figure 5. When $r > \delta$ and constraint (27) binds, the worker consumes A_1 for $t \in [0, t_1]$, after which aggregate consumption grows at the rate of $r - \delta$. The critical dates, t_1 and t^*, are determined by the equations:

$$c_{12}^*(t^*) = A_1 e^{(r-\delta)(t^* - t_1)} = A_2 \tag{38}$$

and

$$A_1\left(\int_0^{t_1} e^{-rt}dt + \int_{t_1}^{1} e^{\delta t}dt\right) = (w_1 + A_1)\int_0^{t^*} e^{-rt}dt + (w_2 + A_2)\int_{t^*}^{1} e^{-rt}dt. \tag{39}$$

The first equation says that the optimal aggregate consumption at the time the worker changes jobs, date t^*, is equal to A_2. The second equation simply says that lifetime aggregate consumption equals lifetime aggregate wages.

Assume that constraint (27) binds, $w_2 + A_2 > w_1 + A_1$ and the difference between aggregate wages is not "too big." Then, when $r > \delta$, the worker will initially choose job 1 and will ultimately

[11] When $w_1 + A_1 = w_2 + A_2$, the worker strictly prefers job 1 to job 2. If $(w_2 + A_2) - (w_1 + A_1) = \varepsilon$, where ε is arbitrarily small, then, by continuity, the worker will continue to strictly prefer job 1 to job 2.

switch to job 2. In this case the worker will be observed to move from a low paying job to a high paying job. And, when $r < \delta$, the worker will initially choose job 2 and will ultimately switch to job 1. In this case, the worker will be observed to move from a high paying job to a low paying one.

In sum, when amenities are not an important component of aggregate wages, then the aggregate wage will be a sufficient statistic for the job. The idea here is that since amenities are a small component of aggregate wages, constraint (27) will probably not bind. However, when amenities become a more important component of aggregate wages, then aggregate wages will not necessarily be a sufficient statistic for the job. This happens precisely when the aggregate wage of the high amenity job exceeds the aggregate wage of the low amenity job, but by not too much. In this situation the worker will change jobs so as to obtain the benefits associated with each job: job 1 offers a less constrained consumption profile and job 2 offers a higher lifetime aggregate wage. So just as in the case where the wage and amenity are not substitutable, a model where wages and amenities are perfectly substitutable can have workers moving from high to low or from low to high paying jobs. And, if workers do change jobs, the direction of the movement depends on the relative magnitude of the rate of time preference.

6 Conclusions

Individuals may rationally choose to move from high paying jobs to lower paying ones as part of an optimal lifetime plan. A key insight to this observation is that a job is more than just a wage; workers also care about non-wage dimensions of a job. In the data, the majority of workers who voluntarily change jobs, move to higher wages. Our model would identify these individual as having "relatively high" discount rates. The data also document that a large proportion of voluntary job changers move to lower paying jobs. Our model would identify these workers as patient, "relatively low" discount rate individuals.

The particular specification of preferences analyzed above are not the only ones that imply that the worker will want to change jobs. Using a CES instantaneous utility function,

$$u(w,A) = (aw^{\rho} + bA^{\rho})^{\frac{1}{\rho}}$$

for $-\infty < \rho < 1$, computational experiments show that as long as the difference in instantaneous

utility associated with each job is not too big, then worker's will want to change jobs. Note that as long as $-\infty < \rho < 1$, the marginal utility associated with zero consumption of the private good is infinite. So, just as the case when the wage and amenity are not substitutable, with CES preferences workers want to change jobs so that they can consume the "average" of both jobs bundles. The direction of job movement—from low to high or high to low paying jobs—is determined by the relative magnitude of the worker's discount rate.

Figure 1: Moving Time: Job 1 to Job 2

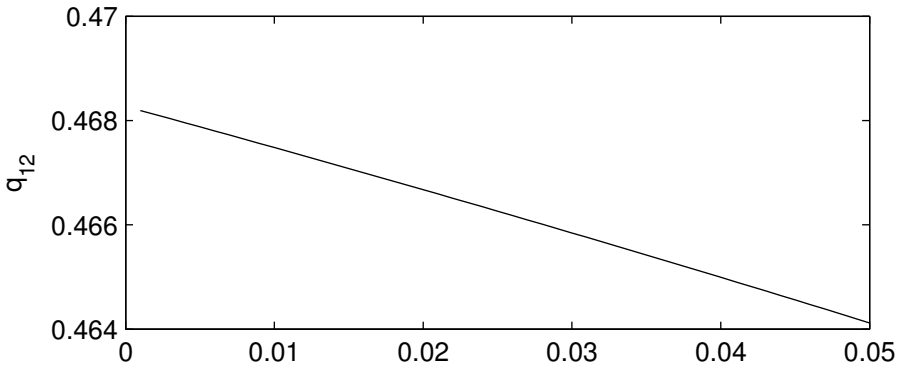


Figure 2: Moving Time: Job 2 to Job 1

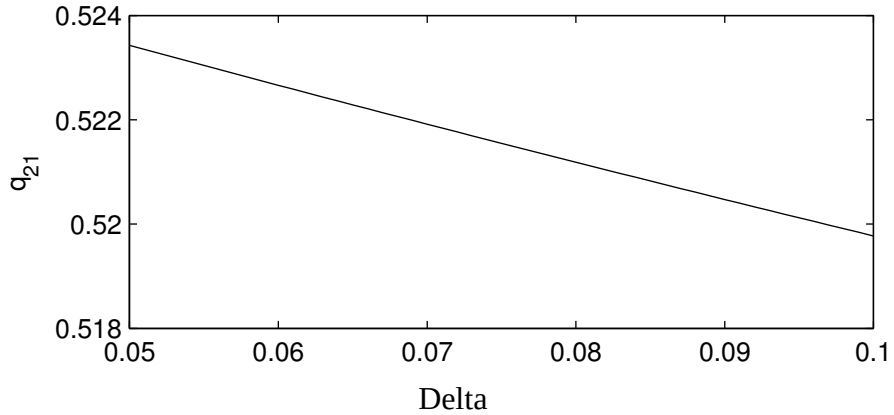


Figure 3: Consumption Profile when $r > \delta$

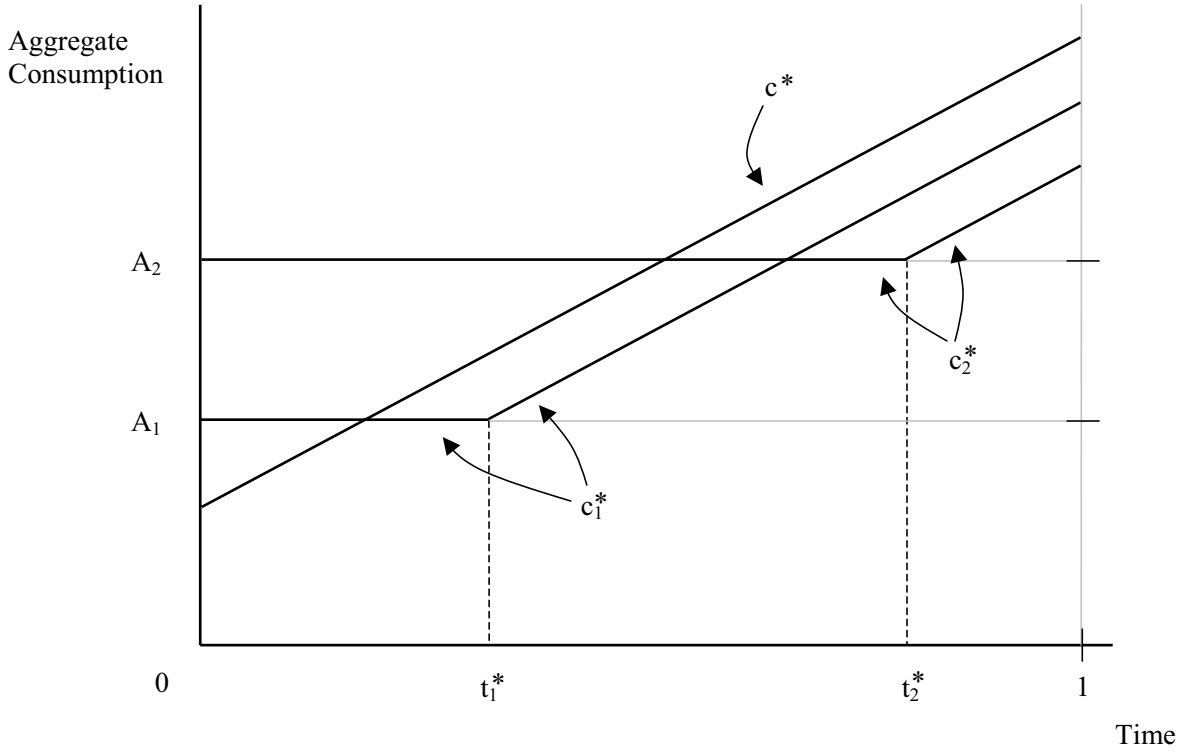


Figure 4: Consumption Profiles when $r < \delta$

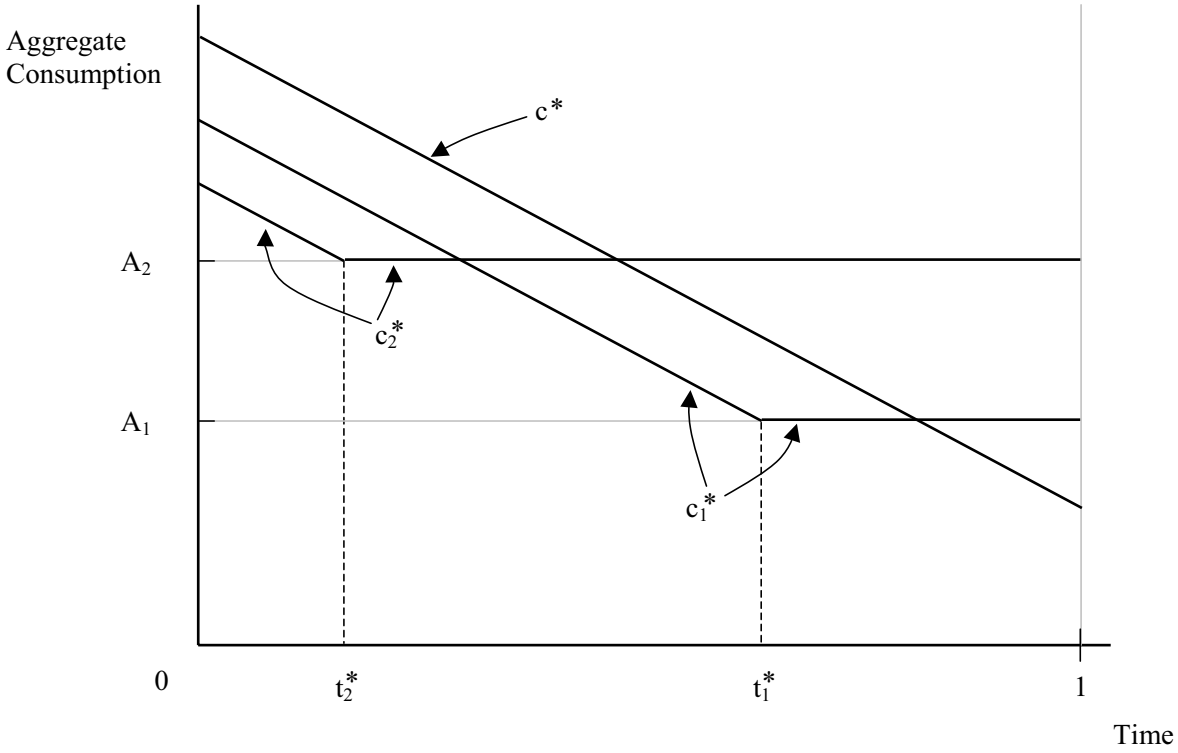
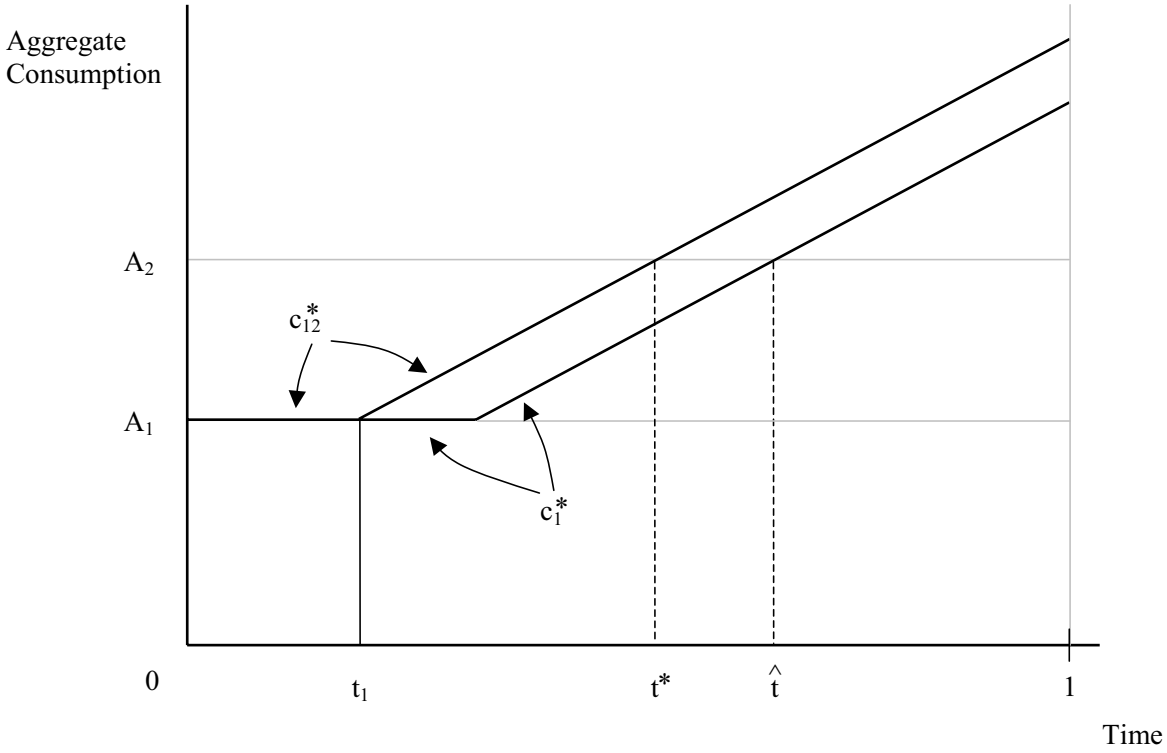


Figure 5: Optimal Consumption Profile and Job Change when $r > \delta$



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