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Earnings and Wealth Inequality and Income Taxation: Quantifying the Trade-offs of Switching to a Proportional Income Tax in the U.S.

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Ana Castañeda is with Intermoney; Javier Díaz-Giménez is at the Universidad Carlos III de Madrid; and José-Victor Ríos-Rull is at the University of Pennsylvania. The authors thank seminar participants at the University of Iowa, the Penn macro lunch group, The Federal Reserve Banks of Minneapolis and Cleveland (especially Ed Prescott and Dave Altig), the NBER Summer Institute, and the 1998 INCAE Conference in Costa Rica. Díaz-Giménez thanks the DGICYT for grant PB-940378. Ríos-Rull thanks the National Science Foundation for grant SBR-9309514, and the University of Pennsylvania Research Foundations.

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ABSTRACT

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1 Introduction

In this paper we ask what are the implications of switching from the current progressive income tax system to a proportional income tax system. The key consequences of this change are a reduction of distortions that result from the high marginal income tax rates and a reduction of the redistributive properties of the tax system. In short, such a change will bring about more inequality and more efficiency. In this paper we **measure** how much more.

In order to do this we need a quantitatively satisfactory theory of wealth and income inequality. In this paper, we provide such a theory. Our theory is based on uninsurable differences in earnings possibilities for life-cycle households that care for the wellbeing of their off-spring and that are subject to a social security system. Our theory does not use differences in patience across agents (not even temporary) to account for the large observed differences in assets holdings across households. Unlike other studies that generate a distribution of wealth endogenously using earnings shocks, (Aiyagari (1994), Krusell and Smith (1998), Krusell and Smith (1997), Castañeda, Díaz-Giménez, and Ríos-Rull (1998), Quadrini (1997), Huggett (1996), to name a few) we obtain a joint distribution of earnings and wealth that matches that in the U.S. data very closely even in the presence of the U.S. tax system. Unlike Hubbard, Skinner, and Zeldes (1995) who have already pointed out the importance of government policies in shaping the assets decisions of poor households, we use a general equilibrium model capable of accounting simultaneously for the asset holdings of rich and poor households.[1] Unlike Krusell and Smith (1998) our households do not face shocks to their time preference that produce cross-sectional differences in patience. The key features that allow us to match the data are the explicit consideration of *i.)* life cycle features, *ii.)* altruistic agents and *iii.)* a social security system that induces increases in the income of many people upon retirement.[2]

Our model economy is a heterogenous agent version of the neoclassical growth model which we calibrate to match the main macro aggregates, the central features of the public sector and the main inequality properties of the US economy. Once this is done we change some features of the tax policy and we compute the new steady state. Another important feature of our model economy is that we explicitly consider leisure. We think that this is important when trying to measure the distortionary effects of taxation since taxes distort the contemporaneous margin between consumption and leisure.

We find that our model economy generates the inequality in earnings and wealth observed

[1]See Quadrini and Ríos-Rull (1997) for a review of the previous successes and failures of models in accounting for wealth inequality.

[2]Gokhale, Kotlikoff, Sefton, and Weale (1998) construct an overlapping generation model with unintended bequests due to incomplete annuatization and mortality risk. They also find a key role played by social security in generation wealth inequality, although they only look at the inequality among those households whose head is around the retirement age.

in the U.S. We also find that a switch from the current tax system to one based on a proportional income tax implies *i.)* an increase in *efficiency* as measured by aggregate consumption of 4.1% and by aggregate output of 4.4%, due to an increase in the capital stock of 11.4% and to an increase in the labor input of .9%, *ii.)* no increase in inequality as measured by the Gini index of the earnings, and *iii.)* an increase in wealth inequality as measured by the Gini index of the wealth distribution of about 10.4%, which is quite large given the very high starting value of this index (it now stands at .78) and the fact that it has an upper bound of 1., and *iv.)* there is very little change in the mobility between the different earnings and wealth groups.

In our model all differences in earnings arise from differences in the realization of a stationary Markovian stochastic process for "wages" and on the decision of how many hours to work.[3] Agents cannot engage in activities that change the characteristics of this stochastic process. In particular, we abstract from such things as education acquisition both for the agent and for other members of his dynasty. We recognize that these issues might be important ingredients to understand the distribution of income and wealth, but we leave them for future research.

Other researchers have tried to measure the impact of similar tax changes in the context of different general equilibrium models. For example, Altig, Auerbach, Kotlikoff, Smetters, and Walliser (1997) use an overlapping generations model with multiple earnings classes.[4] In their model, a switch to a proportional income tax increases steady state consumption by 6.9%, capital by 7.6% and labor input by 4.8%. Their aggregate findings differ from ours mainly in the large response of the labor input to the tax change. Of course, the differences between the distributional implications of their model and ours are enormous: in their model all changes in assets are due to the life-cycle motive which reduces its ability to generate differences in inequality.

Section 2 describes the model economies. Section 3 the calibration targets and the details of how the benchmark model economy with progressive taxation is capable of accounting for the U.S. earnings and wealth inequality. Section 4 describes how the proportional income tax model economy compares with the progressive income tax benchmark model economy. Section 5 concludes.

[3]With wages we just mean earning opportunities per unit of time and not necessarily a specific contractual arrangement.

[4]To generate substantial wealth inequality, agents in their model are restricted to leave a certain inheritance to their off-spring that is a function of their earnings' class a la Fullerton and Rogers (1993).

2 The model economies

The model economies analyzed in this paper are modified versions of the stochastic neoclassical growth model with uninsured idiosyncratic risk and no aggregate uncertainty.[5] The key features of our model economies are the following: *i.)* they include a large number of heterogeneous households, *ii.)* these households face an uninsured, household-specific shock to their employment opportunities, *iii.)* households face a probability of dying and upon death the household is replaced by another household of the same dynasty. Households are altruistic towards future members of their dynasty.

2.1 The private sector

2.1.1 Population dynamics and information

We assume that at each point in time our model economy is inhabited by a large number, actually a measure one continuum, of households. Each period some households are born and some households die. We assume that the measure of the newly-born is the same as the measure of the deceased and, consequently, the measure of households remains constant.

Agents age stochastically. They are born as adults, this is to say in working age. With certain constant probability they retire. Upon retirement, each period they can die or stay retired, again with constant probability.[6] This way of modeling the demographics allows the introduction of a life-cycle element in a very parsimonious way but it still captures some features that we deem to be important: namely that the relative age distribution between working adults and retirees is the the same as in the data, and that periods are relatively short.

We also assume that each period, each household faces an idiosyncratic random disturbance, that determines its individual employment opportunities.

We use the compact notation s_t to denote jointly the age and employment opportunity of an agent. We assume that these disturbances are independent and identically distributed across households and that they follow a finite state Markov chain with conditional transition probabilities given by

$$\pi(s' \mid s) = Pr\{s_{t+1} = s' \mid s_t = s\}. \tag{1}$$

where $s, s' \in S = \{1, 2, \ldots, n_s\}$.[7] Finally, we assume that new-born households draw a shock from the stationary distribution of employment opportunities for the adults that have been

[5] Huggett (1993) and Aiyagari (1994) analyze model worlds that are similar to ours. We extend their model economies in two ways: *i.)* we include additional dimensions of household heterogeneity and *ii.)* we endogenize the choice between labor and leisure.

[6] When we calibrate the model economy we choose these probabilities so that they match the average durations of working age and of retirement.

[7] Note that $\pi(. \mid s)$ is the probability of being alive for one more period, which is smaller than one.

alive for more than one period which we denote $\Gamma^*(s)$.

2.1.2 Employment opportunities.

The household specific employment process takes values that belong to set $s \in S = \{\epsilon, r\}$, where ϵ is a J-dimensional vector that describes the earnings opportunities that the houeshold whose components are described below and r denotes that the agent is retired and incapable of working. When a household draws shock ϵ_j we say that the agent is not retired, and we assume that it receives an endowment of $\epsilon_j > 0$ efficiency labor units per unit of time which it can either allocate to the aggregate production technology or use as leisure. Variable Γ^* denotes the invariant distribution of shock s conditional on not being retired, *i.e.,* it denotes the invariant distribution of employment opportunities.

2.1.3 Preferences

We assume that households only derive utility from their consumption and leisure when they are alive and that they order their random streams of these goods according to

$$\sum_{t=0}^{\infty} \beta^t \left\{ \sum_{s_{t+1}} \pi(s_{t+1}|s_t) u(c_t, \ell - l_t, s_t) + \eta\beta \left(1 - \pi(.|s_t)\right) \sum_{s_{t+1} \in e} V(s_{t+1}, k_{t+1}) \Gamma^*(s_{t+1}) \right\} \quad (2)$$

where u is a continuous and strictly concave utility function; $0 < \beta < 1$ is the time-discount factor; $c_t \geq 0$ is household consumption, ℓ is the households' endowment of productive time, and l_t is labor. Hence, $\ell - l_t$ is time allocated by the household to non-market activities, which we call leisure. The second term in equation (2) describes the utility derived by a household from its bequests. Parameter η measures the households concern for the welfare of its off-spring, and to avoid a cumbersome representation of the utility of the household induced by its off-spring, we use the compact notation of $V(s_{t+1}, k_{t+1})$ to denote the utility of a newborn with idiosyncratic shock s_{t+1} and with wealth given by k_{t+1}.

2.1.4 Production possibilities.

We assume that aggregate output, Y_t, depends on aggregate capital, K_t, and on the aggregate labor input, L_t, through a constant returns to scale aggregate production function, $Y_t = f(K_t, L_t)$. We also assume that the capital stock depreciates at a constant rate δ.

2.2 The government sector

We assume that the government in our model economies taxes household income and estates and that it uses the proceeds of taxation to make real transfers to retired households and to finance government consumption. We assume that income taxes are described by function $\tau(y)$, where y denotes income, that estate taxes are described by function $\tau_E(k)$, where k

denotes wealth, and that transfers to the retirees are described by $\omega(s)$ where s denotes the realization of the household-specific shock. In our model economies, therefore, a government policy rule is a specification of $\{\tau(y), \tau_E(k), \omega(s)\}$ and the process on government consumption, G. Since we also assume that the government must balance its budget every period, these policies must satisfy the following restriction:

$$G_t + \Omega_t = T_t \tag{3}$$

where Ω_t and T_t denote, respectively, aggregate transfers and aggregate tax revenues.

Note that under these assumptions in our model economy social security takes the form of transfer to households with state $s = r$. This implies that the transfer does not depend on past contributions of the worker.[8]

2.3 Market arrangements

We assume that there are no insurance markets for the household-specific shock, s.[9] To buffer their streams of consumption against these shocks, households can accumulate assets in the form of real capital. Moreover, household capital asset holdings are restricted to belonging to a finite set $\mathcal{K}$. This restriction can be understood as a form of liquidity constraints.[10]

We also assume that firms rent factors of production from households in competitive spot markets. Consequently, factor prices are given by the corresponding marginal productivities.[11]

2.3.1 Initial endowment and liquidation of assets

We assume that the model economy households are born with an initial endowment of assets which they inherit from their parents. When their time comes to die, households do so overnight, after the current-period labor, consumption, investment, and savings have taken place. Early in the following morning, their estates are liquidated. A fraction $1 - \tau_E$ of their assets, if any, is inherited by the deceased agent's off-spring. The remaining assets are transformed into the current period composite good and are taxed away by the government.

[8] We make this assumption for technical reasons. To discriminate between households according to their past contributions to a social security system requires the inclusion of a second asset-type state variable in the individual problem which makes it very costly to solve. The computational costs are already very high in this paper due to the fact that we solve a very large number of model economies in our quest for the appropriate calibration.

[9] This is the key feature of this class of model worlds. When insurance markets are allowed to operate, this economy collapses to a standard representative agent model, as long as the right initial conditions hold.

[10] Aiyagari (1994) shows that in this class of incomplete market economies, the requirement that debt has to be repaid imposes a lower bound on the set of assets holdings endogenously.

[11] In this class of model economies firms do not play any intertemporal role for two main reasons: first, they do not make profits and, second, they cannot be used by the households who own them to substitute for insurance by choosing non-profit maximizing strategies.

2.4 Equilibrium

In this paper we consider recursive, *i.e.* stationary Markov, equilibria only. This equilibrium concept might exclude some other types of equilibria such as those that include arrangements that implement history dependent allocations such as those described, for instance, in Atkeson and Lucas (1992) and Atkeson and Lucas (1995). The reason for this assumption is that in this paper we are interested in the aggregate consequences of a specific set of market arrangements, but we do not attempt to account for the reasons that justify the existence of those markets. Furthermore, in this paper we consider steady states only.[12]

Each period, the economy-wide state is the measure of households, x, defined over $\mathcal{B}$, an appropriate family of subsets of $\{\mathcal{K} \times S\}$.[13]

2.4.1 The households decision problem

The household state variable is the pair (k, s) which includes the beginning-of-period capital stock, k, and the realization of the household-specific process, s. The dynamic program solved by a household alive in state (k, s) is the following:

$$v(k,s) = \max_{c \geq 0, k' \in \mathcal{K}, 0 \leq l \leq \ell} u(c, \ell - l, s) + \\ \beta \left\{ \sum_{s'} v(k', s') \pi(s'|s) + [1 - \pi(.|s)] \eta \sum_{s' \in e} V(k'(1 - \tau_E), s'), {}^*(s') \right\} \quad (4)$$

s.t.:

$$\begin{aligned} c + k' &= y - \tau(y) + k(1 - \delta) \\ y &= kr + l(k,s) w \epsilon_s + \omega(s) \end{aligned}$$

where v denotes the households' value function, and r and w denote the factor prices. Since the households' decision problem is a finite-state, discounted dynamic program, it can be shown that an optimal stationary Markov plan that solves the problem always exists.

2.4.2 Definition of equilibrium

A steady state equilibrium for this economy is a household policy, $\{c(k,s),\, k'(k,s),\, l(k,\, s)\}$, a pair of household value functions $v(k,s)$, and $V(k,s)$, a government policy, $\{\tau(y),\, \tau_E,\, \omega(s),$

[12]Moreover, in a recent paper Cole and Kocherlakota (1997) have shown that hidden storage makes the allocation implied by trades with one asset the one that arises in environment like the one in this paper. In other words, our market structure seems to be the "right one" under certain observability assumptions.

[13]Note that we do not need to keep track of household names since the decisions of households in the same individual state are always the same.

$G\}$, a stationary measure of households, x, a vector of time invariant prices, (r, w), and a vector of aggregates, (K, L, T, Ω) such that:

i.) Factor inputs, tax revenues and total transfers are obtained aggregating over households.

$$K = \int_{\mathcal{K},S} k \, dx, \qquad L = \int_{\mathcal{K},S} l(k,s) \, \epsilon_s \, dx \tag{5}$$

$$T = \int_{\mathcal{K},S} \tau(y) \, dx + \int_{\mathcal{K},S} \tau_E \, k'(k,s) \, [1 - \pi(. \mid s)] dx, \qquad \Omega = \int_{\mathcal{K},S} \omega(s) \, dx \tag{6}$$

where household income, $y(k, s)$, is defined above.

ii.) Given x, K, L, r and w, the household policy solves the households' decision problem described in (4), and factor prices are factor marginal productivities:

$$r = f_1 \left(K, L\right) + (1 - \delta) \text{ and } w = f_2 \left(K, L\right). \tag{7}$$

iii.) The utility of a newborn, $V(k, s)$, is the same as that of an older agent, $v(k, s)$.

$$V(k,s) = v(k,s). \tag{8}$$

iv.) The goods market clears:

$$\int_{\mathcal{K},S} (c(k,s) + k'(k,s)) \, dx + G \leq f \left(K, L\right) + (1 - \delta) K \tag{9}$$

v.) The government budget constraint is satisfied:

$$G + \Omega = T \tag{10}$$

vi.) The measure of households is stationary

$$x(\mathcal{K}_0, S_0) = \int_{\mathcal{K}_0,S_0} \left\{ \int_{\mathcal{K},S} \left(\xi_{k'=k'(k,s)} \ \pi(s'|s) + \xi_{k'=(1-\tau_E)k'(k,s)} \ (1 - \pi(.|s)) \, , \, ^*(s') \right) dx \right\} dk' ds' \tag{11}$$

for all $(\mathcal{K}_0, S_0) \in \mathcal{B}$, and where ξ denotes the indicator function. Appendix 1 describes the procedure that we use to compute this equilibrium.

3 Calibration

Recall that our strategy is to calibrate the economy to the current tax system, the current income and wealth distribution, and the key ratios from the national income and product accounts.

We start describing the calibration targets. Next we describe the functional forms that we choose, and the parameterization that implements our calibration targets. We end this section with a description of the quality of the calibration and a discussion of the key elements that allow us to be able to account so well for the distribution of earnings and wealth. Our definition of earnings include labor income only: it does not include either government transfers or capital income. The sources for the data and the definitions of all distributional variables can be found in Díaz-Gimenez, Quadrini, and Ríos-Rull (1997).

3.1 Quantitative targets

We want the model economy to satisfy certain characteristics that we describe below.

3.1.1 Model period

Time aggregation matters for the cross sectional distribution of flow variables such as earnings. On the other hand, the shorter the time period the larger the wealth to income ratios and, therefore, the larger the computational costs. The longest time period that is consistent with the data collection procedures is a year and it is the one we choose.

3.1.2 Main macroeconomic aggregates

We want our output shares in the model economy to mimic those in the U.S. economy. This means an investment I to output Y ratio, of 16%, a government expenditures G to output ratio of 19%, and a transfers Tr to output ratio of 9%. Unlike the previous two ratios which do not seem to have any trend in the post World War II U.S. data, transfers have been steadily growing. Our chosen value is a little lower than that of 14% that shows in the most recent years. We follow the logic of the analysis of Krusell and Ríos-Rull (1997) in identifying the size of the transfers. We also impose a value of .376 (see Castañeda et al. (1998) for details) to the capital share of output. We summarize these statistics in Table 1.

3.1.3 The distribution of earnings

The model economy Lorenz Curve for earnings mimics the U.S. Lorenz Curve for earnings as depicted in Table 2.

Table 1: The main aggregates of the U.S. economy

	Y	C	I	G	Tr
U.S.	1.00	.65	.16	.19	.09

3.1.4 The distribution of wealth

The model economy Lorenz Curve for wealth mimics the U.S. Lorenz Curve for wealth as depicted in Table 2.

Table 2: The earnings and wealth distributions in the U.S.

		Earnings	Wealth
Bottom	0–1	-.40	-.52
	1–5	.00	-.02
	5–10	.00	.01
Quintiles	0–20	-.40	-.39
	20–40	3.19	1.74
	40–60	12.49	5.72
	60–80	23.33	13.43
	80–100	61.39	79.49
Top	90–95	12.38	12.62
	95–99	16.37	23.95
	99–100	14.76	29.55
Gini Index		.63	.78

3.1.5 The U.S. tax system

The model economy tax system mimics key features of the U.S. tax system. We abstract from property taxes, consumption and excise taxes and from any differentiation between capital and labor income for taxation purposes.

Income taxes: Gouveia and Strauss (1994) have characterized the U.S. effective household income tax function for 1989 with the following functional form:

$$\tau(y) = a_0(y - (y^{-a_1} + a_2)^{-1/a_1}) \tag{12}$$

with parameter values a_0=.258; a_1=.768; a_2=.031. Figure 1 shows the implied average and marginal tax rates for this function.

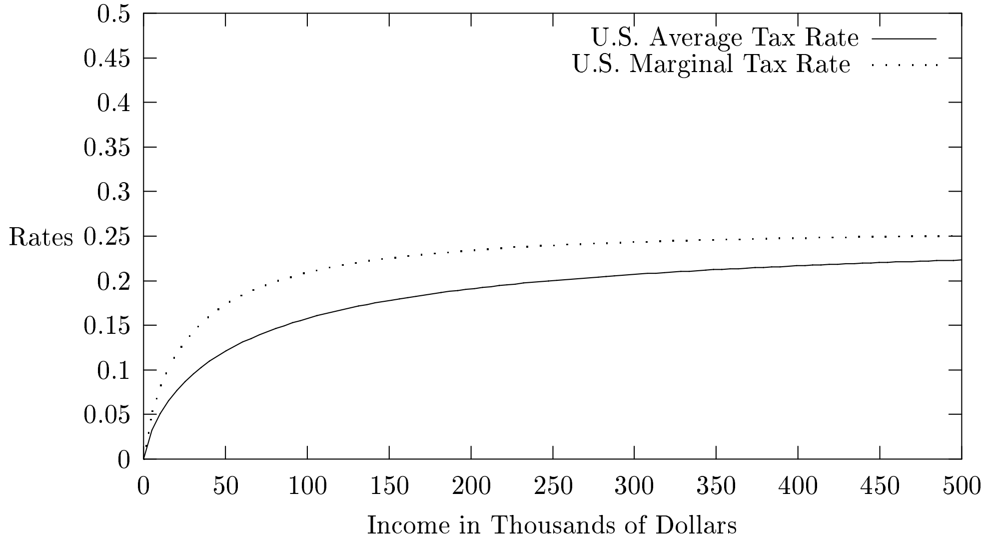

Figure 1: The U.S. in 1989 average and marginal tax functions rates according to Gouveia and Strauss (1994).

With this tax function the degree of progressivity is a function of the units (scale). To avoid this problem we compute the value of a_2 that equals the tax rates of average per capita income in the U.S. and in the model. This solves the problem of different units. To deal with the problem that the U.S. has also tax revenues from other sources, we add a source of proportional income taxation on top of that reported by Gouveia and Strauss (1994) which amounts to assume that all non income tax government revenue operates as a proportional tax on income.

Estate Taxes: In the model economy the government levies an estate tax, τ_E. We choose the model economy estate tax function, $\tau_E(k)$, to mimic estate taxes in the U.S. economy which we report in the first two columns of Table 3 below.

Table 3: The U.S. estate tax

Capital brackets	Tax Rates
0–\$600,000	.000
More than \$600,000	.5

Table 4: Fraction of households in each quintile that remain in the same quintile 5 years later in the U.S. between 1984 and 1989.

	Earnings	Wealth
1st	.86	.67
2nd	.41	.47
3rd	.47	.45
4th	.46	.50
5th	.66	.71

To mimic the U.S. estate taxes we have to translate those $600,000 to model units. Note that per household income was in 1990 around fifty thousand dollars making the limit on tax free inheritances about twelve times larger. In the model economy the maximum tax exempt capital, denote it $\bar{k}$, is set to be about twelve times average per household income. Next, we define the estate tax function in the following way:

$$\tau_E(k) = \begin{cases} 1/2(k-\bar{k}) & \text{for } k > \bar{k} \\ 0 & \text{for } k < \bar{k} \end{cases} \tag{13}$$

Estate taxes are an important issue and we will discuss them some more below.

3.1.6 Relative cross-sectional variance of hours and consumption

We want to match certain characteristics of the cross-sectional distribution of consumption and hours. In particular, we impose that the standard deviation of consumption is about four times that of hours work.

3.1.7 Mobility

We want to match the persistence of earnings and wealth as measured by the transition probabilities of going from certain quintiles in 1984 to others in 1989. Table 4 shows the mobility properties for households between 1984 and 1989 for both earnings and wealth. The table reports the fraction of households out of those in each quintile in 1984 that remained in the same quintile in 1989. Only those households that were present in both periods are taken into account when constructing the quintiles. The tables is extracted from Díaz-Gimenez et al. (1997) who used the PSID to perform this calculation.

The key property from this table is that the first and fifth quintiles are the most persistent, with the first being the most persistent for earnings and the fifth being the most persistent for wealth.

3.2 Functional forms

We now turn to the choice of functional forms for the utility and production functions. In an abuse of terminology, we include the choice of the length of the period as part of the choice of functional forms.

3.2.1 Preferences

To characterize the household decision problem described in equation (4), we must choose a form for the utility function. We use a constant relative risk aversion utility function which is separable in time and in the consumption-leisure decisions.

The utility function of private consumption for workers, $s = \epsilon$ (see below) is:

$$u(c, l, \epsilon) = \frac{c^{1-\gamma_1}}{1-\gamma_1} + B\frac{(\ell - l)^{1-\gamma_2}}{1-\gamma_2} \tag{14}$$

This choice is relatively non-standard. It is due to the fact that large shocks are needed to mimic the cross-sectional differences in earnings and wealth. With Cobb-Douglas preferences, the implied variability in hours worked would have been orders of magnitude larger than what is observed. For retirees, this utility function collapses to that of CRRA in consumption.

Finally, in this exercise we assume that households value the utility of their off-spring in the same way that they value their utility. Consequently, $\eta = 1$.

3.2.2 Technology

After World War II in the U.S., the real wage has increased at an approximately constant rate —at least until 1973— and factor income shares have displayed no trend. To account for these two properties we choose a Cobb-Douglas aggregate production function

$$f(K_t, L_t) = K_t^{\theta} L_t^{1-\theta} \tag{15}$$

with depreciation at a constant rate δ.

3.2.3 Government

We choose the same effective household income tax function used by Gouveia and Strauss (1994) described above. We use as functional form for estate taxes a proportional schedule that applies to all inheritances above a threshold $\hat{k}$. We assume that all government revenue comes from these two sources only. To accommodate this feature, we add assume that the taxation from other sources acts like a proportional income tax that is added to that described by Gouveia and Strauss (1994).

3.3 Parameterization of the benchmark model economy

The set of possible parameters that we are free to choose to match the calibration targets is very large. Also, the set of statistics to match is very large. Its specific size depends on how many statistics we want to match.

We can think of the calibration of this model economy as an exercise of solving a system with a large number of equations and of unknowns. Unfortunately, this is not as simple as it seems. In general, only linear systems are guaranteed to have (generically) a unique solution and our model is not linear. Another problem that arises when attempting to calibrate the model is that there is a large number of inequality constraints that the parameters have to satisfy. These include the fact that wages are non negative and that that require that the matrix π is indeed a transition matrix (its elements have to be between zero and one). Non linear equation solvers typically have problems with these type of constraints.

These considerations lead us to use a minimization procedure to calibrate the economy. We choose the parameters to minimize a weighted sum of the absolute value of the differences between our targets and the properties of the model economy where the weights are chosen to obtain similar relative differences. The aforementioned targets are those described in Section 3.1.

The actual parameters chosen are described in the next few Tables.

3.3.1 The employment process

The normalized endowments of efficiency labor units are reported in Table 5 and the transition probabilities of the household specific process are reported in Table 6. Table 5 also reports the stationary distribution of working-age households.

Table 5: The normalized calibrated endowments of efficiency labor units, $\epsilon(s)$. Relative productivity and sizes of the employed groups

	$s=\epsilon_1$	$s=\epsilon_2$	$s=\epsilon_3$	$s=\epsilon_4$	$s=\epsilon_5$
$\epsilon(s)$	1.	2.92	7.30	35.05	1810.22
, *	.236	.451	.279	.034	.00022

These two tables contain 14 independent parameters: the 4 endowments and 10 transition probabilities. Some transition probabilities are set to zero because the search algorithm attempted to set to make them negative.

Table 6: The transition probabilities for the household-specific process.

From s	To s' (in %) ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	r
ϵ_1	.970	0.121E-06	.819E-02	.00	.00	.222E-01
ϵ_2	.581E-02	.962	.100E-01	.00	.00	.222E-01
ϵ_3	.202E-02	.134E-01	.953	.970E-02	.672E-07	.222E-01
ϵ_4	.518E-01	.00	.278E-01	.897	.103E-02	.222E-01
ϵ_5	.101	.00	.571E-01	.335E-05	.820	.222E-01
r	.0	.00	.00	.00	.00	.944

3.3.2 Preferences, technology and government parameters

Table 7 shows the parameter values that we have chosen for preferences, for technology and for the government. That same table also reports the average income tax rate, and the marginal estate tax rate.

Table 7: The benchmark model economy: calibrated preferences and technology parameters (in yearly terms)

Preferences		Technology		Government	
Parameter	*Value*	*Parameter*	*Value*	*Parameter*	*Value*
β	.89	δ	.092	τ_y	.328
γ_1	1.50	θ	.376	τ_E	.500
γ_2	5.50			a_0	.258
ρ	1.70			a_1	.768
η	1.00			a_2	.424
χ	.10			a_3	.201

3.4 Calibration results

In this section we discuss the results of the calibration exercise.

3.4.1 The aggregate properties of the model economy

Table 8 shows the values of key macroeconomic aggregates of the U.S. economy and of the benchmark model economy. As it can be noticed, the values of the aggregates in the U.S.

and in the model economy are almost exactly identical.

Table 8: The main aggregates of the U.S. economy and the benchmark model economy

	Y	C	I	G	Tr
U.S.	1.00	.65	.16	.19	.09
Benchmark	1.00	.64	.17	.19	.09

3.4.2 The distributional properties

In Table 9 we report the key statistics of the earnings distribution in both the the the U.S (already reported in Table 2) and in the benchmark model economy.

Table 9: The earnings distributions in the U.S. and in the benchmark model economy

		U.S.	Model
	0–1	-.40	.00
Bottom	1–5	.00	.00
	5–10	.00	.00
	0–20	-.40	.00
	20–40	3.19	3.27
Quintiles	40–60	12.49	14.35
	60–80	23.33	20.73
	80–100	61.39	61.65
	90–95	12.38	10.25
Top	95–99	16.37	16.51
	99–100	14.76	15.01
Gini Index		.63	.615

They include eleven points in the Lorenz curve (including the quintiles) and the Gini index. Note that the distributions of earnings are very similar in both economies. The very mild differences arise in the third and fourth quintiles: the share of the third is higher in the U.S. than in the model economy while the reverse holds for the share of earnings of the fourth quintile. Also, there is a slightly higher share in the model economy than in the U.S. economy of those in the 80-90 percentiles. This happens at the expense of those in the 90-95 percentiles for whom the opposite relation holds.

As far as the wealth distribution is concerned, Table 10 reports the key statistics of the wealth distribution in both the U.S and in the benchmark model economy. These statistics are the same as those in Table 2. The only difference is that the fourth quintile has a larger share in the model economy than in the U.S. at the expense of the second and third quintiles. Overall though, we are able to replicate the very large concentration found in the U.S. distribution of wealth.

Table 10: The wealth distributions in the U.S. and in the benchmark model economy

		U.S.	Model
Bottom	0–1	-.52	.00
	1–5	-.02	.00
	5–10	.01	.00
Quintiles	0–20	-.39	.01
	20–40	1.74	.36
	40–60	5.72	3.78
	60–80	13.43	16.60
	80–100	79.49	79.26
Top	90–95	12.62	11.67
	95–99	23.95	22.81
	99–100	29.55	29.54
Gini Index		.78	.79

3.4.3 Government tax revenues

Figure 2 compares average tax rates in the model economy after normalization of the units with those reported by Gouveia and Strauss (1994). We can see that the amount of progressivity as indicated by the tax schedule is very close between the U.S. and the benchmark model economy.

3.4.4 The mobility properties

Table 11 reports the fractions of households of each quintile that remain in the same earnings and wealth quintiles five years later. We believe that these statistics capture the most important features of mobility. Of particular importance are the fractions of households that remain in the top and bottom quintile.

We see that the performance of the model economy is not as good with respect to mobility as it was with respect to the other properties that we are interested in. Still, relative large persistence is found for all quintiles. As in the data, the first and the fifth quintiles are the

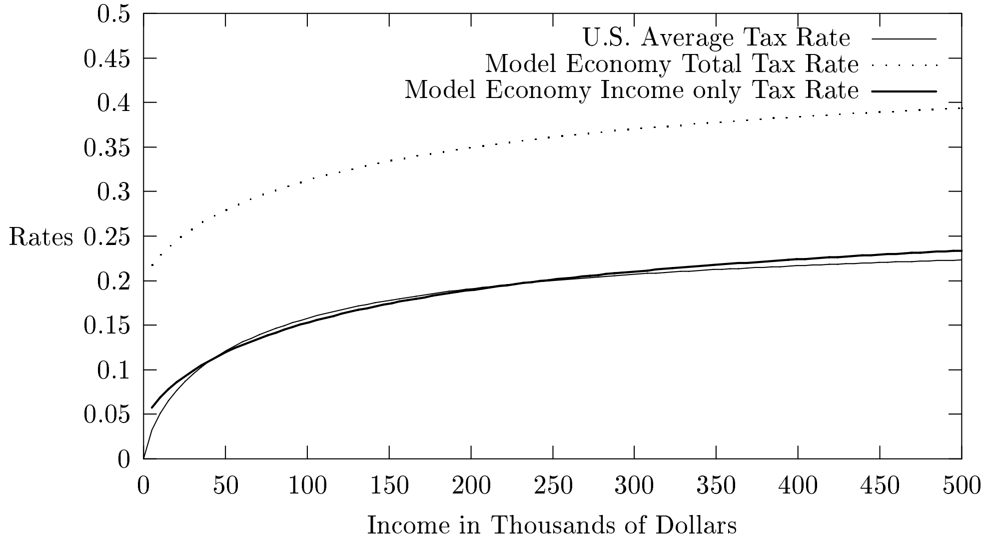

Figure 2: The U.S. and the model economy average tax functions with and without other sources of government revenue.

most persistent for both earnings and wealth. With respect to earnings both in the data and in the model we see that the first quintile is more persistent than the fifth quintile.

However, when we look in more detail, we start seeing differences between the model economy and the data. For example, with respect to earnings, the model economy shows too much persistence in the fifth quintile. Also, with respect to wealth the benchmark model economy is much more persistent than the U.S. data and the relative size of the persistence between the first and fifth quintiles is reversed: in the model the first quintile is more persistent than in the data while the opposite happens with respect to the fifth quintile.

The model used in this paper has some life-cycle properties (there is retirement) but these properties are very stylized. Mobility as recorded by the PSID has a strong life-cycle component (the age earnings profile has a very clear inverse U shape, see Auerbach and Kotlikoff (1987) or Ríos-Rull (1996) for example). Consequently, we should not expect the model economy to have mobility properties that are very close to those in the data. given the excessively parsimonious characterization that we have assumed for the life-cycle. We conjecture that versions of this model that include a more detailed specification of the life-cycle will be able to generate mobility statistics that mimic closer those found in the data.

3.4.5 A discussion

Overall, we think that the calibration of the model economy has been very successful. Unlike previous models, we succeed in replicating the different degrees of concentration of earnings and wealth found in the U.S. economy. There are still a couple of properties of the model

Table 11: Mobility in the U.S. and in the benchmark model economy: fraction of households that remain in the same quintile 5 years later

	Earnings		Wealth	
	U.S.	Benchmark Model	U.S.	Benchmark Model
1st	.86	.88	.67	.96
2nd	.41	.68	.47	.93
3rd	.47	.64	.45	.82
4th	.46	.75	.50	.75
5th	.66	.81	.71	.88

that should be improved. The most important of these relates to the relative share of wealth of the third and fourth quintiles. We have explored alternative calibrations where we put a lot more emphasis in replicating the relative share of wealth of the third and fourth quintiles at the expense of a worst match of other statistics. We found that for these alternative model economies the findings are very similar.

To better study mobility issues we think that we should move towards models that have a much more sophisticated implementation of the age of households. Those models will be able to simultaneously include mobility due to the life-cycle and to other motives such as the shocks that we have assumed in this paper.

Finally, there is an issue with the estate tax. Given its current parameterization, the model economy assumes that there are no ways to avoid inheritance taxes, while we know that the existence of trusts funds and inter vivos transfers reduces the amount that the government collects from this source. On the other hand, the model generates a wealth to output of slightly less than 2., too low relative to the one observed in the U.S. that is somewhere between 2.5 and 3. We conjecture that versions of the model with a less unavoidable estate tax are able to increase the wealth to output ratio without changing the other properties of the model economy.

4 The policy experiments

Once we have a satisfactory implementation of a model economy calibrated to the U.S. tax system, we proceed to compute the steady state of an alternative economy that has the following properties:

1. Preferences and technology (including the properties for the employment process) are identical to those in the benchmark model economy.

2. The size of government (expenditures and transfers), in absolute not relative terms, is identical to the benchmark model.

3. The tax system is different. In this model economy the government levies a proportional tax on income. The estate taxes are unchanged at about 12 times average household income.

Once we have computed the equilibrium with progressive taxation, computing the one with proportional taxation is simple. It amounts to solving a system of four equations and four unknowns. The four equations are the equilibrium condition for the capital labor ratio, the two conditions on the size of government, and the units condition that determines the units for the estate tax. The four unknowns are the capital labor ratio, the proportional tax rate, and the guesses for aggregate wealth and aggregate output.

4.1 The main aggregates in the model economies

Table 12 shows the main aggregates for the benchmark model economy and for the model economy with proportional income taxation. The table contains the macroeconomic aggre-

Table 12: The steady state aggregates of the benchmark and the proportional tax model economies

	Benchmark Model	Proportional Model	% Change
Y	1.000	1.044	4.4%
C	.635	.661	4.1%
I	.166	.185	11.4%
G	.189	.189	.0%
Tr	.087	.087	.0%
K	1.79	1.97	11.4%
Labor Input	1.00	1.009	.9%
Total Hours	1.00	1.016	1.6%
K/Y	1.79	1.89	5.6%
Government Revenues / Y	32.80	31.63	-3.7%
σ_c/c	2.627	3.535	34.5%
σ_l/l	.639	.641	.3%

gates used to calibrate the benchmark model economy and also total capital, the labor input, total hours worked in the market, the wealth to output ratio, the fraction of government revenues over GDP, and the coefficients of variation of both consumption and hours worked.

We find that switching to a proportional tax system increases the steady state level of output by 4.4%. This is due to a much higher steady state level of capital (11.4% more) and to a higher work effort that translates into a slight (less than 1%) increase in the labor input. Obviously, the difference in the levels of output between the two economies will not occur upon a switch in policy. They will happen many periods after the policy change once the transition to the new steady state has been completed (assuming that the new steady state is stable, a conjecture that cannot be verified with the tools at our disposal).

The increase in consumption after the shift to the proportional tax economy is 4.1%, slightly less than that of output. The rest of the extra output goes to increase investment, needed since the steady state stock of capital is much larger under proportional income taxation. Note that, by construction, there is no change in the level of government consumption and, therefore, its share of output is smaller in the economy with proportional income taxes. Note that given our assumptions about the behavior of the government, implies that public expenditures and transfers do not change in the proportional income tax model economy.

Note that there is a sizable increase in the stock of capital that is largely responsible for the increase of steady-state output. This is due to the very small increase of the labor input which is less than 1.%. As a consequence the wealth to output ratio is higher in the proportional income taxation, about 6% higher. Total hours worked in the market, on the other hand, shows a larger increase than the labor input. This can only happen if the increase in hours worked is done mostly by those with low labor efficiency. At first sight this seems a contradiction with a switch to proportional income taxation that should reduce the distortion against work effort more to high income households than to low income households. However, the general equilibrium effects turn out to be strong. There is a positive correlation between wealth and the best wage shocks. The higher total wealth (and as we will see the fact that it is more concentrated) induces a strong positive wealth effect of high wage households that partly counteracts the substitution effect arising from the switch to proportional income taxation.

The total reduction in the size of government as a fraction of output that arises is 3.7. This is smaller than the increase in output and is due to the fact that taxes are levied on Net National Product and not on Gross National Product and the increase in the former is smaller than in the latter due, again, to the small impact on total labor input.

Finally, we see an enormous increase, 35%, of the cross sectional coefficient of variation of household consumption. The hours worked counterpart shows, on the other hand, shows almost no change. This is a direct implication of the relatively small response of work effort to changes in the environment (recall the much larger curvature imposed on the leisure part of the instantaneous utility function relative to that of consumption) that was imposed in the calibration stage to obtain a much larger cross sectional variation of consumption than of hours worked.

4.2 Inequality in the model economies

Table 13 reports the key statistics of the distribution of earnings for both model economies. The central feature of this table is that it seems that there is no change in inequality of

Table 13: The Earnings Distributions in the Benchmark Model Economy and in the Proportional Taxation Model Economy.

		Benchmark Model	Proportional Model
Bottom	0–1	.00	.00
	1–5	.00	.00
	5–10	.00	.00
Quintiles	0–20	.00	.00
	20–40	3.27	3.31
	40–60	14.35	14.44
	60–80	20.73	20.87
	80–100	61.65	61.38
Top	90–95	10.25	10.24
	95–99	16.51	16.38
	99–100	15.01	14.80
Gini Index		.615	.613

earnings at all. In other words, the switch to a proportional taxation even though increases overall work effort by slightly less than 1.%, it does so in a manner that does not seem to affect inequality. This feature is consistent with the very small increase in both the labor input and total hours that is associated to the proportional income tax model economy. This means that most agents respond in a similar way to the new income tax. As another sign of this property of the model economies we can compare the cross sectional standard deviation of hours. It goes up by .3% in the proportional income tax model economy, an almost negligible amount. Again we see that the effect of the change of taxation is very small and it is also similar for all agents. For the low wage agents there is a mild increase in the income tax, (substitution effect going decreasing work effort) that is counterweighted by lower asset holdings (wealth effect increasing work effort). For the high wage guys (which are also wealthier), the opposite holds true. Overall, there is a mild increase in work effort that has almost no implications for earnings inequality.

Table 14 reports the key statistics of the distribution of wealth for both model economies. We find that inequality has increased dramatically. In the benchmark model economy the third quintile owns a non-negligible amount of wealth, while this is no longer the case in

Table 14: The wealth distributions in the benchmark model economy taxation and in the model economy with proportional taxes

		Benchmark Model	Proportional Model
	0–1	.00	.00
Bottom	1–5	.00	.00
	5–10	.00	.00
	0–20	.01	.00
	20–40	.36	.00
Quintiles	40–60	3.78	.11
	60–80	16.60	10.44
	80–100	79.26	89.46
	90–95	11.67	11.86
Top	95–99	22.81	25.77
	99–100	29.54	39.09
Gini Index		.791	.874

the model economy with proportional taxation, where the third quintile owns almost no wealth. In fact, under proportional taxation, the bottom 60% of asset holders have about 1 per thousand of total wealth. We also find that under proportional taxes only about 10% of the population increases their share of wealth, and this group is the group with the highest wealth. This does not mean that the remaining 90% of the population own less assets, it means that their share of wealth is smaller. Nevertheless, many households do in fact reduce their asset holdings (again this is across steady states). This is because the equilibrium interest rate is smaller and because for some of the low income households the marginal tax rate has increased. The increase in the Gini Index is enormous, a staggering 10.5%, especially if we realize that the maximum inequality[14] where one household holds everything would have implied an increase of 26%.

Given this large increase in wealth inequality and the fact that there is a positive correlation between employment opportunities and wealth it is easy to understand why there is no increase in earnings inequality. The income and wealth effects cancel each other for the labor choice. High wage people face lower marginal income tax rates but are wealthier inducing a very small effect in their willingness to work.

In the model economies one can easily calculate many distribution statistics. As an ex-

[14]It should be understood that we refer to the maximum Gini index that can be achieved with non-negative values of the variable of interest.

ample, Table 15 shows the key statistics of the distribution of consumption across groups. As

Table 15: The distribution of consumption in the benchmark model economy and in the model economy with proportional taxation

		Benchmark Model	Proportional Model	Proportional Model Levels
	0–1	.23	.22	.23
Bottom	1–5	.94	.86	.90
	5–10	1.32	1.08	1.13
	0–20	6.10	5.16	5.38
	20–40	9.41	7.60	7.93
Quintiles	40–60	13.66	12.95	13.51
	60–80	19.89	18.76	19.58
	80–100	50.94	55.54	57.96
	90–95	9.11	9.19	9.59
Top	95–99	13.45	14.82	15.46
	99–100	12.11	15.87	16.56

with the distribution of wealth, proportional taxes only bring about an increase in the share of consumption for the top 10% of households. A very small number of households experience an increase in their consumption levels. This can be seen by comparing the benchmark model in the table with the last column, which shows the distribution of consumption of the proportional income tax model economy measured in the same units as those used for the benchmark model economy (they are not shares). We see that, besides a tiny increase shown by the lowest one per cent, only those in the top 10% (plus a few others in the top 10 to 20% group) experience an actual increase in consumption. This finding has important implications. A switch to proportional taxation might reduce the steady state levels of consumption for a substantial majority of the population.

Still, we cannot, and will not make any normative statement about the desirability of either tax system. Note that we are comparing steady states, and to compare the welfare of different policies we should take into account the transition paths. The computation of the transition is a technically very demanding endeavor for relatively simple models (see for example Auerbach and Kotlikoff (1987) or Ríos-Rull (1994)). For the class of models in this paper, this type of transition paths exercises are still beyond our computational ability.

4.3 Taxation in the model economies

Figure 3 shows the marginal tax rates of the two economies. Note that mean household income is around $50,000 and that the graph also shows for comparison the fraction of income that the government collects in the benchmark model economy. Note that despise the increase in total output, many households (low income ones) face an increase in taxation in the new regime.

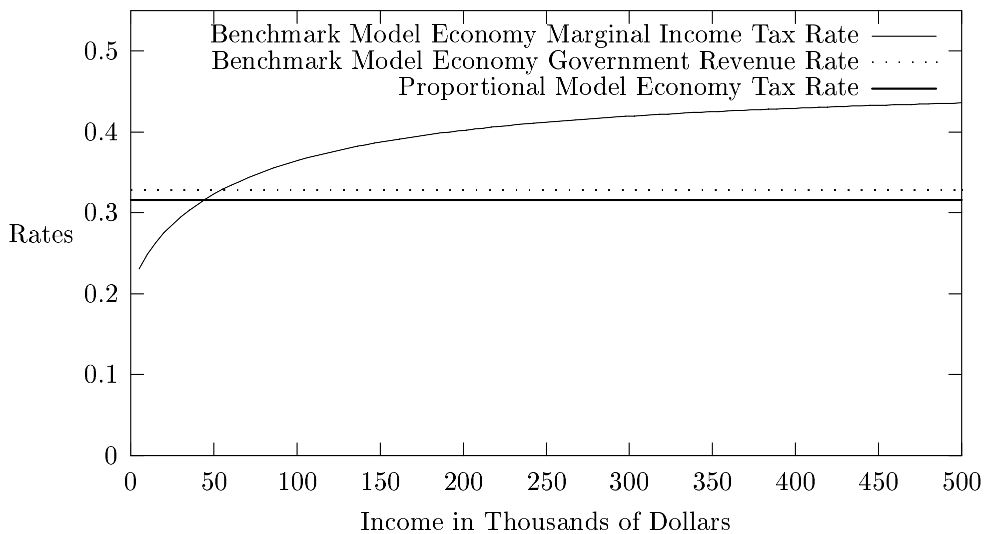

Figure 3: The benchmark model economy and the proportional income tax model economy marginal income tax rates.

4.4 Mobility in the model economies

Table 16 shows the fraction of households in each quintile that remain in the same quintile five years later. As discussed above, this is a dimension of the benchmark model that we do not stress. However, we find that the switch to a proportional taxation system does not change mobility by much. The only instance in which there is some change is in the fraction of households that are in the first earnings quintile in one year and remain in that quintile five years later. The table shoes a substantial decrease after a switch towards proportional income taxation. We believe, however, that this result is not very indicative. The first households that have some positive earnings are for both economies not far from the 20% per mark, and their earnings are minuscule since their hours worked are very tiny. Movements in and out of the first quintile are very sensitive to these small changes. As evidence that there is no much change in the mobility properties, just note that the fraction of households that remain in the lowest 40% of the model economies is .8366 for the benchmark model economy and .8358 for the proportional income taxation model economy. To summarize, we find that mobility is basically unaffected by the tax change. We also want to recall that this model was not designed with the purpose of accounting well for mobility so this findings, even though they are important, they are less important than those pertaining macroeconomic aggregates and earnings and wealth inequality.

Table 16: Fraction of households in each quintile that remain in the same quintile 5 years later in the benchmark model economy and in the model economy with proportional taxation

	Earnings		Wealth	
	Benchmark Model	Proportional Model	Benchmark Model	Proportional Model
1st	.88	.70	.96	.98
2nd	.66	.64	.93	.93
3rd	.64	.64	.82	.83
4th	.75	.75	.74	.78
5th	.81	.81	.88	.88

5 Concluding comments

This paper provides a theory of earnings and wealth inequality capable of accounting quantitatively for the inequality observed in the U.S. The key features of our ability to match the data are the explicit consideration of *i.)* life cycle features, *ii.)* altruistic agents and *iii.)* a social security system that induces increases in the income of many people upon retirement.

We use the resulting quantitative theory of inequality to measure the effects from switching from the current, progressive, tax system, to one where income is proportionally taxed. We compare the steady states of two economies that only differ in the tax system. We find that the policy switch implies some important changes. On the one hand, output, wealth and, to a very little extent, work effort are higher in the economy with proportional income taxation. On the other hand, in the economy with proportional income taxation, even though inequality in earnings remains basically unchanged, both wealth and consumption are much more unequally distributed.

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Appendix

A Computation

This appendix describes the algorithm that we have used to compute the equilibrium allocations of the economy. The outline of the algorithm used is the following:

We use a standard non-linear equation solver (specifically a modification of Powell hybrid method, subroutine DNSQ from the SLATEC package). We use it to search for a zero of a system of 24 equations and 24 unknowns. The equations include the steady state equilibrium condition as well as the differences between the model's statistics and the desired targets. The algorithm typically wants to choose values of the parameters that are not permissible such as negative elements of the transition matrix π. This means that the algorithm effectively worked as a minimization routine where the objective is to minimize a weighted sum of the deviations from the targets.

For each specification of the parameters computing equilibrium involves three different but important steps.

- *Step 1:* Solve for the decision rules of the agents. This we do by a piecewise linear decision rule method. The grid is not equally spaced. Given the very large range of possible asset holdings that we need to achieve the observed wealth concentration, we use a very large number of points. About 1000 grid points per realization of the shock, for a total of 6,000 grid points. At every iteration in every gridpoint a non-linear equation has to be solved. Monotonicity of this equation and the assumed piecewise linearity of the decision makes it easy. However, the model has leisure which adds one extra level of complexity. For every evaluation of the Euler equation another non-linear equation has to be solved, the contemporaneous first order condition. Occasionally this first order condition does not have a solution, this is the case only when the feasibility constraint on the upper bound of leisure binds. In this case work effort is set to zero. See Ríos-Rull (1997) for details.
- *Step 2:* Associated to the decision rules there is a Markov process for each agents which satisfies the necessary conditions (see Aiyagari (1994) or Huggett (1995)) for existence of a unique stationary distribution x^*. We approximate x^* with the aid of a piecewise linear approximation to its associated distribution function. Again, the grid for this approximation has many points (about 12,000) and they are particularly close to each other towards the origin. Again, see Ríos-Rull (1997) for details.
- *Step 3:* Compute the statistics (distributional and aggregate) of the model economy. This step effectively involves the computation of integrals with respect to the invariant measure x^*. We evaluate these integrals directly using our approximation to the distribution function for every statistic except those that measure mobility. In this latter case we computed the statistics using a large sample of agents drawn from x^*. Again see Ríos-Rull (1997) for details.