Self-Selection and Discrimination in Credit Markets

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Supplemental Proofs

LEMMA 3: *If* $\theta_A^m > \theta_B^m$*, then* $q_X(s) < q_A(s)$ *for all s.*

Proof of Lemma 3: If no group *B* applicants attempt to self-select toward bank *Y*,

$$q_A(s) - q_X(s) = \int_{\theta_A^m}^{1} \frac{\theta\, p(s|\theta)}{\int_{\theta_A^m}^{1} p(s|\theta)d\theta} d\theta - \int_{\theta_B^m}^{1} \frac{\theta\, p(s|\theta)}{\int_{\theta_B^m}^{1} p(s|\theta)d\theta} d\theta$$

$$\propto \int_{\theta_B^m}^{1} p(s|\theta)d\theta \int_{\theta_A^m}^{1} \theta\, p(s|\theta)d\theta - \int_{\theta_A^m}^{1} p(s|\theta)d\theta \int_{\theta_B^m}^{1} \theta\, p(s|\theta)d\theta \quad . \qquad (1)$$

$$= \int_{\theta_B^m}^{\theta_A^m} p(s|\theta)d\theta \int_{\theta_A^m}^{1} \theta\, p(s|\theta)d\theta - \int_{\theta_A^m}^{1} p(s|\theta)d\theta \int_{\theta_B^m}^{\theta_A^m} \theta\, p(s|\theta)d\theta \, .$$

Now,

$$\int_{\theta_B^m}^{\theta_A^m} p(s|\theta)d\theta \int_{\theta_A^m}^{1} \theta\, p(s|\theta)d\theta > \theta_A^m \int_{\theta_B^m}^{\theta_A^m} p(s|\theta)d\theta \int_{\theta_A^m}^{1} p(s|\theta)d\theta > \int_{\theta_A^m}^{1} p(s|\theta)d\theta \int_{\theta_B^m}^{\theta_A^m} \theta\, p(s|\theta)d\theta \, , \qquad (2)$$

.

implying that $q_A(s) > q_X(s)$ for all *s*.

If group low-θ group *B* applicants do attempt to self-select toward bank *X*, there are two cases to consider: $\theta_A^m < \theta^c$ and $\theta_A^m > \theta^c$. If $\theta_A^m < \theta^c$,

$$
\begin{aligned}
q_A(s)-q_X(s) &= \int_{\theta_A^m}^{1}\frac{\theta\, p(s\,|\,\theta)}{\int_{\theta_A^m}^{1} p(s\,|\,\theta)d\theta}d\theta-\frac{\int_{\theta_B^m}^{\theta^c}\theta\, p(s\,|\,\theta)\gamma\, d\theta+\int_{\theta^c}^{1}\theta\, p(s\,|\,\theta)\,\tfrac{1}{2}\,d\theta}{\int_{\theta_B^m}^{\theta^c} p(s\,|\,\theta)\gamma\, d\theta+\int_{\theta^c}^{1} p(s\,|\,\theta)\,\tfrac{1}{2}\,d\theta} \\
&\propto \left(\int_{\theta_A^m}^{\theta^c}\theta\, p(s\,|\,\theta)d\theta+\int_{\theta^c}^{1}\theta\, p(s\,|\,\theta)d\theta\right) \\
&\qquad\times\left(\int_{\theta_B^m}^{\theta_A^m} p(s\,|\,\theta)\gamma\, d\theta+\int_{\theta_A^m}^{\theta^c} p(s\,|\,\theta)\gamma\, d\theta+\int_{\theta^c}^{1} p(s\,|\,\theta)\,\tfrac{1}{2}\,d\theta\right) \qquad (3)\\
&\quad-\left(\int_{\theta_A^m}^{\theta^c} p(s\,|\,\theta)d\theta+\int_{\theta^c}^{1} p(s\,|\,\theta)d\theta\right) \\
&\qquad\times\left(\int_{\theta_B^m}^{\theta_A^m}\theta\, p(s\,|\,\theta)\gamma\, d\theta+\int_{\theta_A^m}^{\theta^c}\theta\, p(s\,|\,\theta)\gamma\, d\theta+\int_{\theta^c}^{1}\theta\, p(s\,|\,\theta)\,\tfrac{1}{2}\,d\theta\right).
\end{aligned}
$$

Simplifying this expression yields

$$
\begin{aligned}
q_A(s)-q_X(s) \propto (\gamma-\tfrac{1}{2})&\left(\int_{\theta_A^m}^{\theta^c} p(s\,|\,\theta)d\theta\int_{\theta^c}^{1}\theta\, p(s\,|\,\theta)d\theta-\int_{\theta_A^m}^{\theta^c}\theta\, p(s\,|\,\theta)d\theta\int_{\theta^c}^{1} p(s\,|\,\theta)d\theta\right) \\
+\gamma&\left(\int_{\theta_B^m}^{\theta_A^m} p(s\,|\,\theta)d\theta\int_{\theta_A^m}^{\theta^c}\theta\, p(s\,|\,\theta)d\theta-\int_{\theta_B^m}^{\theta_A^m}\theta\, p(s\,|\,\theta)d\theta\int_{\theta_A^m}^{\theta^c} p(s\,|\,\theta)d\theta\right) \qquad (4)\\
+\gamma&\left(\int_{\theta_B^m}^{\theta_A^m} p(s\,|\,\theta)d\theta\int_{\theta^c}^{1}\theta\, p(s\,|\,\theta)d\theta-\int_{\theta_B^m}^{\theta_A^m}\theta\, p(s\,|\,\theta)d\theta\int_{\theta^c}^{1} p(s\,|\,\theta)d\theta\right).
\end{aligned}
$$

Using the same bounding techniques employed in (2) above, each of the terms in this expression are positive, proving that $q_A(s) > q_X(s)$.

Finally, suppose that $\theta_A^m > \theta^c$. In this case,

$$
\begin{aligned}
q_A(s)-q_X(s) \propto & \int_{\theta_A^m}^{1}\theta\, p(s\,|\,\theta)d\theta\left(\int_{\theta_B^m}^{\theta^c} p(s\,|\,\theta)\gamma\, d\theta+\int_{\theta^c}^{\theta_A^m} p(s\,|\,\theta)\,{}^1\!/\!_2\, d\theta+\int_{\theta_A^m}^{1} p(s\,|\,\theta)\,{}^1\!/\!_2\, d\theta\right)\\
& -\int_{\theta_A^m}^{1} p(s\,|\,\theta)d\theta\left(\int_{\theta_B^m}^{\theta^c}\theta\, p(s\,|\,\theta)\gamma\, d\theta+\int_{\theta^c}^{\theta_A^m}\theta\, p(s\,|\,\theta)\,{}^1\!/\!_2\, d\theta+\int_{\theta_A^m}^{1}\theta\, p(s\,|\,\theta)\,{}^1\!/\!_2\, d\theta\right)\\
= & \;\gamma\left(\int_{\theta_A^m}^{1}\theta\, p(s\,|\,\theta)d\theta\int_{\theta_B^m}^{\theta^c} p(s\,|\,\theta)d\theta-\int_{\theta_A^m}^{1} p(s\,|\,\theta)d\theta\int_{\theta_B^m}^{\theta^c}\theta\, p(s\,|\,\theta)d\theta\right)\\
& -{}^1\!/\!_2\left(\int_{\theta_A^m}^{1}\theta\, p(s\,|\,\theta)d\theta\int_{\theta^c}^{\theta_A^m} p(s\,|\,\theta)d\theta-\int_{\theta_A^m}^{1} p(s\,|\,\theta)d\theta\int_{\theta^c}^{\theta_A^m}\theta\, p(s\,|\,\theta)d\theta\right).
\end{aligned}
\tag{5}
$$

Once again, bounding arguments similar to those used in (2) above can be used to verify that this expression must be positive, proving that $q_A(s) > q_X(s)$. ♠

<u>Proof that $q_Y(s) > q_A(s)$ for all *s* (Proposition 2, part 2)</u>: Since $\theta_B^m > \theta_A^m$, we can write

$$
\begin{aligned}
q_A(s)-q_Y(s) = & \int_{\theta_A^m}^{1}\frac{\theta\, p(s\,|\,\theta)}{\int_{\theta_A^m}^{1} p(s\,|\,\theta)d\theta}d\theta-\frac{\int_{\theta_B^m}^{\theta^c}\theta\, p(s\,|\,\theta)(1-\gamma)d\theta+\int_{\theta^c}^{1}\theta\, p(s\,|\,\theta)\,{}^1\!/\!_2\, d\theta}{\int_{\theta_B^m}^{\theta^c} p(s\,|\,\theta)(1-\gamma)d\theta+\int_{\theta^c}^{1} p(s\,|\,\theta)\,{}^1\!/\!_2\, d\theta}\\
\propto & \left(\int_{\theta_A^m}^{\theta_B^m}\theta\, p(s\,|\,\theta)d\theta+\int_{\theta_B^m}^{\theta^c}\theta\, p(s\,|\,\theta)d\theta+\int_{\theta^c}^{1}\theta\, p(s\,|\,\theta)d\theta\right)\\
& \times\left(\int_{\theta_B^m}^{\theta^c} p(s\,|\,\theta)(1-\gamma)d\theta+\int_{\theta^c}^{1} p(s\,|\,\theta)\,{}^1\!/\!_2\, d\theta\right)\\
& -\left(\int_{\theta_A^m}^{\theta_B^m} p(s\,|\,\theta)d\theta+\int_{\theta_B^m}^{\theta^c} p(s\,|\,\theta)d\theta+\int_{\theta^c}^{1} p(s\,|\,\theta)d\theta\right)\\
& \times\left(\int_{\theta_B^m}^{\theta^c}\theta\, p(s\,|\,\theta)(1-\gamma)d\theta+\int_{\theta^c}^{1}\theta\, p(s\,|\,\theta)\,{}^1\!/\!_2\, d\theta\right).
\end{aligned}
\tag{6}
$$

This can be simplified to

$$
\begin{aligned}
q_A(s) - q_Y(s) \propto\ & (1-\gamma)\left(\int_{\theta_A^m}^{\theta_B^m} \theta\, p(s\,|\,\theta) d\theta \int_{\theta_B^m}^{\theta^c} p(s\,|\,\theta) d\theta - \int_{\theta_A^m}^{\theta_B^m} p(s\,|\,\theta) d\theta \int_{\theta_B^m}^{\theta^c} \theta\, p(s\,|\,\theta) d\theta \right) \\
& + \tfrac{1}{2}\left(\int_{\theta_A^m}^{\theta_B^m} \theta\, p(s\,|\,\theta) d\theta \int_{\theta^c}^{1} p(s\,|\,\theta) d\theta - \int_{\theta_A^m}^{\theta_B^m} p(s\,|\,\theta) d\theta \int_{\theta^c}^{1} \theta\, p(s\,|\,\theta) d\theta \right) \\
& + (\gamma - \tfrac{1}{2})\left(\int_{\theta_B^m}^{\theta^c} \theta\, p(s\,|\,\theta) d\theta \int_{\theta^c}^{1} p(s\,|\,\theta) d\theta - \int_{\theta_B^m}^{\theta^c} p(s\,|\,\theta) d\theta \int_{\theta^c}^{1} \theta\, p(s\,|\,\theta) d\theta \right).
\end{aligned} \tag{7}
$$

Once again, using the bounding techniques applied in (2) above, we see that this expression must be negative, proving that $q_Y(s) > q_A(s)$. ♠