

## Reducing Working Hours: A General Equilibrium Analysis

 by Terry J. Fitzgerald

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#### Abstract

This paper examines the effects of restricting the weekly hours of workers in a heterogeneous-agent, general equilibrium framework. The framework presented here contains two types of workers, which allows the author to explore the consequences of a mismatch between the skills of the nonemployed and the skills of the employed. Previous studies of these issues have been based on partial equilibrium models.

The main findings are that restricting weekly hours increases employment substantially, but may also lead to large declines in wages, productivity, output, and consumption, and can result in an increased wage disparity between skilled and unskilled workers. In one case, the policy raises the wages and consumption of skilled workers, while lowering them for the unskilled. A skills mismatch is shown to have dramatic effects on the aggregate and distributional consequences of the policy.


# Reducing Working Hours: A General Equilibrium Analysis* 

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#### Abstract

This paper examines the effects of restricting the weekly hours of workers in a heterogeneous agent general equilibrium framework. The framework contains two types of workers, which allows me to explore the consequences of a mismatch between the skills of the nonemployed and the skills of the employed. Previous studies related to these issues have been based on partial equilibrium models. The main findings are that restricting weekly hours substantially increases employment, but may also lead to large declines in wages, productivity, output, and consumption, and can result in an increase in the wage disparity between skilled and unskilled workers. In one case the policy increases the wages and consumption of skilled workers, while lowering them for the unskilled. A skills mismatch is shown to have dramatic effects on both the aggregate and distributional consequences of the policy.


[^0]
## 1 Introduction

Over the last 20 years there has been considerable interest in policies intended to increase employment by reducing working hours per worker. These policies are frequently referred to as "work sharing" or "spreading the work." In Europe, where unemployment rates in several countries have risen to historically high levels, many countries have experienced campaigns by workers and/or politicians to reduce working hours. While work-sharing policies have received less attention in the United States, proposals to reduce hours are not uncommon there, particularly during periods of high unemployment. ${ }^{1}$

The basic intuition underlying these policies is that reducing hours per worker would encourage firms to hire additional workers as they seek to replace the labor services currently provided by those workers putting in longer hours. One important consideration, then, is whether the labor services of existing workers could be replaced easily from the ranks of the nonemployed. However, it is well known that the pool of nonemployed people tend to possess notably different skills than the pool of employed people. This mismatch in skills may make it difficult for firms to find new workers for some jobs, and thereby limit the effectiveness of work sharing policies for increasing employment while accentuating any negative impact on productivity and output.

In this paper I explore the effect of restricting working hours in a heterogenous agent general equilibrium model. More specifically, I examine the effect of reducing weekly hours per worker on employment, output, productivity, wages, consumption, and capital accumulation. The model contains two types of workers, which allows me to explore the potential importance of a skill mismatch between the employed and the nonemployed and to examine how the effects of the policy differ across types of workers.

An independent contribution of this paper is that it takes a step towards pro-

[^1]viding a satisfying general equilibrium framework which can be used to analyze the quantitative effects of restricting working hours. Hours per worker, the number of workers, and output are endogenously determined, as are the wage rates of the two types or workers. The model is dynamic and includes capital to allow for the possibility that firms may substitute capital services for labor services. The main objective of the analysis is to gain some insights into the general equilibrium effects of reducing working hours, and to identify those aspects of the model which play a quantitatively important role in determining those effects.

The literature on policies which effect the hours and employment decision of firms has focused largely on changes in the standard workweek length and/or the overtime premium. This literature has predominantly used partial equilibrium labor demand models where firms minimize cost, or maximize output, taking wage schedules as given. The theoretical research has emphasized the conditions under which these regulatory changes will, in fact, reduce working hours and increase employment, while the empirical research has attempted to estimate the quantitative magnitudes. ${ }^{2}$ In contrast, this paper examines the effect of an unspecified policy which effectively reduces weekly hours per worker, and bypasses the issue of whether such a policy can be crafted. This allows me to focus directly on the effects of reducing working hours.

The idea that a mismatch in the skills of the employed and the nonemployed may limit the effectiveness of labor market policies is hardly a new one. For example, in their empirical study of the effects of increasing overtime wages, Ehrenberg and Schumann (1982) provide evidence that skill mismatches between those working overtime and the unemployed may be substantial and may "seriously constrain the creation of new jobs". In this paper I examine the potential importance of a very stark skill mismatch where one type of worker is fully employed so that all nonemployed people are of the second type.

[^2]Other research on the effects of reducing working hours has used efficiency wage models where firms sets wages [Hole and Vale (1986)], and union bargaining models where wages are set directly by the union or through a bargaining problem [Calmfors (1985), Booth and Schiantarelli (1987)]. Much of this literature takes working hours as exogenous, and generally concludes that the effects of a reduction in working hours on employment are ambiguous, although a decrease in employment is generally reported to be more likely than an increase. As we will see, these findings stands in stark contrast to the results from the price-taking general equilibrium model presented here.

I conduct three experiments to investigate the effects of reducing weekly working hours. First, I consider a policy which restricts the hours of all workers, and I assume that both types of workers are less than fully employed. From the skill mismatch perspective, I view this as a best case scenario since there is an abundance of nonemployed people of both types. In the second and third experiments I assume a very stark mismatch between the skills of the employed and the nonemployed. In particular, I assume that type A workers are fully employed, so that only type B workers are available for additional employment. In the second experiment, I again assume that the policy restricts the hours of all workers. The third experiment is motivated by the fact that most work-sharing proposals, in fact, exclude some groups of workers with relatively high employment rates. ${ }^{3}$ In this final experiment I explore whether restricting only the hours of the type $B$ workers mitigates any negative effects of the skills mismatch.

The major results of the experiments are as follows. First, restricting hours per worker by 4 hours per week leads to a large increase in employment. Second, a skills mismatch between those types whose hours are restricted and the nonemployed has a substantial detrimental effect on capital accumulation and output. Third, increases in employment for either type are accompanied by large decreases in the wages for that type, both hourly and weekly. Fourth, if type A is fully employed, the hours

[^3]restrictions results in a substantial increase in the wage disparity between the two types. In fact, when the policy applies to only type B, the wages and consumption of type A people actually increase, while those of type B fall.

Before proceeding I note a few caveats. First, this paper focuses on the effects of an hours restriction on production decisions, and abstracts from numerous potentially important labor supply considerations. For example, the model makes use of employment lotteries, which greatly simplify the computational requirements of solving the model, and assumes there exists perfect unemployment insurance. In addition, factors such as moonlighting, part-time employment, job search and matching, family labor supply, and home production are absent. Second, an equilibrium of the model economy without an hours restriction is pareto optimal, so that imposing such a restriction cannot be pareto improving, although it may have redistributive effects. Measuring the benefits of increasing employment, incorporating important labor supply considerations, and introducing other types of heterogeneity into the analysis are left for future research.

### 1.1 A Little Data on the Skills of the Employed and the Nonemployed

Table 1 provides some basic evidence of a mismatch between the skill characteristics of the employed and the nonemployed in the U.S.. Table 1 shows unemployment rates broken down by some commonly used proxies for skill, namely age, occupation, and educational attainment, for March 1992, a year in which unemployment rates were relatively high. Not surprisingly, people in their prime working years, people in managerial and professional specialty occupations, and people with more education tend to have much lower unemployment rates than the young, the less educated, and those in occupations that generally requiring less training.

There is also evidence that the pool of people not in the labor force are relatively less skilled. For example, table 1 shows that labor force participation rates for college graduates and people in their prime working years are considerably higher than for people with less schooling and both younger and older people.

As a result of both lower unemployment rates and higher labor force participa-
tion, college graduates and people in their prime working years have much higher employment to population ratios. For example, while almost 86 percent of all college graduates aged 25 to 64 were employed in March 1992, only half of those with less than a high school diploma were employed.

While informative, this comparison between the employed and those not employed is not quite the relevant one for this paper since only those people working more than a certain number of hours would likely be affected by an hours restriction. The relevant comparison here, then, is between the nonemployed and those whose hours would be reduced. Using a similar breakdown as in table 1, Fitzgerald (1996) compares the population of unemployed workers with the population of workers putting in more than 40 hours in a week, and reports substantial differences in the skill characteristics of the two groups. Ehrenberg and Schumann (1982) provide a more thorough comparison and reach the same qualitative conclusion.

Although the data provided here are for very broadly defined groups, it is consistent with the view that, in the aggregate, the ability of firms to replace the hours of some, but certainly not all, skilled workers with the hours of additional similarly skilled workers may be limited. As we will see, the findings in this paper suggest that the magnitude of this skill mismatch is important in determining the effects of work-sharing policies. This implies that a much more thorough analysis of this magnitude, such as that done by Ehrenberg and Schumann, should be conducted when considering the effects of a specific policy.

The rest of this paper is organized as follows. Section 2 describes the model economy. Section 3 defines a competitive equilibrium and briefly discusses the determination of hours per worker in the model. Section 4 assigns parameter values to a benchmark model economy, while section 5 presents the results of the experiments.

## 2 The Model Economy

The model economy used here is an extension of the framework developed by Hornstein and Prescott (1993) in which both the employment and hours per worker of homogeneous workers are endogenously determined. I construct the model so that
an equilibrium displays balanced growth, and is consistent with the secular growth patterns displayed by postwar U.S. data. However, for simplicity I abstract from population and real output growth in the presentation. Introducing growth in such models is a standard exercise in the real business cycle literature [See Cooley (1995)].

This model can be viewed as imbedding the standard model used in the labor demand literature into a neoclassical growth framework. Key features of the labor demand model include explicitly modeling the interaction between hours per worker and employment in the provision of labor services, allowing for quasi-fixed labor costs, and wage schedules that are increasing in hours worked.

The economy is populated by a continuum of infinitely lived people. These people are either type A or type B , with measure $\lambda_{A}$ and $\lambda_{B}$ respectively, where $\lambda_{A}$ and $\lambda_{B}$ sum to 1 . Each person is endowed with 1 unit of time each period, which can be allocated to either work, $h$, or leisure, $\ell$. However, leisure time is not equal to 1 h. Instead, I assume that $\tau$ units of time are required for commuting to work each period that an individual is employed. Therefore,

$$
\begin{array}{rlr}
\ell(h)=1-h-\tau, & \text { for } 0<h<1-\tau,  \tag{1}\\
\ell(0) & =1 &
\end{array}
$$

The function $\ell(\mathrm{h})$ is discontinuous at zero.
Type A and type B people are also endowed with $\mathrm{k}_{A}$ and $\mathrm{k}_{B}$ units of capital respectively. Capital depreciates exponentially at the rate $\delta$ per period, and evolves according to the law of motion

$$
\begin{equation*}
k_{t+1}=(1-\delta) k_{t}+i_{t} \tag{2}
\end{equation*}
$$

where $\mathrm{i}_{t}$ is investment.
People types are distinguished by the following properties. First, type A and B people provide labor services which are different factors of production. Second, type A and B people have different preferences over leisure. This second assumption provides parameters which allow the equilibrium employment-population ratios and hours per worker of the two types to be set independently.

### 2.1 The Production Technology

The main objective in specifying the technology is to model the basic trade-offs which firms faces in deciding upon the working hours of its workers, the number of workers of different types to employ, and the amount of capital to rent. In selecting the functional forms which characterize the technology, I borrow the specifications commonly used in the labor demand literature. I also restrict my attention to functional forms which are consistent with a balanced growth equilibrium.

Inputs to production are the labor services of type A workers, the labor services of type B workers, and capital services. I assume identical labor services functions for each type of worker. Following Ehrenberg (1971), a labor service function is chosen that is multiplicatively separable in d , the number of workers, and h , working hours per worker. This function is given by

$$
\begin{equation*}
L(h, d)=g(h) d^{1-\theta} \tag{3}
\end{equation*}
$$

where $0<\theta<1$.
Following Barzel (1973) and Feldstein (1967), the function $\mathrm{g}(\mathrm{h})$ is assumed to have the following characteristics. For low values of $h, g^{\prime}(h)$ is small, reflecting initial set-up activities or warm-up time per worker. For high values of $h, g^{\prime}(h)$ is small and may be negative, reflecting worker fatigue and/or boredom. I follow Fitzroy and Hart (1985) and let

$$
\begin{align*}
g(h) & =(h-\mu)^{\psi}, & & h>\mu  \tag{4}\\
& =0 & & 0 \leq h \leq \mu
\end{align*}
$$

where $\mu \geq 0$ is the set-up time per worker and $\psi>0$.
If $\psi$ is set to 1 , then labor services provided are proportional to hours per worker above $\mu$, while if $\psi$ is less (greater) than 1 , labor services increase less (more) than proportionally with hours. This case is often interpreted as reflecting worker fatigue or boredom. When $\mu$ is set to $0, \psi$ is the elasticity of labor services with respect to hours.

The output of a firm hiring $\mathrm{d}_{A}$ type A workers to work $\mathrm{h}_{A}$ hours each, $\mathrm{d}_{B}$ type
$B$ workers to work $h_{B}$ hours, and renting $k$ units of capital is given by

$$
\begin{equation*}
F\left(k, h_{A}, d_{A}, h_{B}, d_{B}\right)=f\left(k, h_{A}, d_{A}, h_{B}, d_{B}\right)-\phi_{B} d_{B}-\phi_{A} d_{A} \tag{5}
\end{equation*}
$$

where

$$
f\left(k, h_{A}, d_{A}, h_{B}, d_{B}\right)=A k^{\theta} L\left(h_{A}, d_{A}\right)^{\alpha} L\left(h_{B}, d_{B}\right)^{1-\alpha}
$$

where $0<\alpha<1$ and $\phi \geq 0$. The parameters $\phi_{A}$ and $\phi_{B}$ represent the fixed costs per worker of each type which is required to maintain the job positions. In the labor demand literature these costs are often interpreted as training, hiring, and record keeping costs [See Oi (1961), Rosen (1968), Hart (1984)]. ${ }^{4}$

### 2.2 The Firm's Decision Problem

The firm rents capital and employs type A and type B people. In making their employment decisions, a firm faces two wage schedules, given by $\mathrm{w}_{A}:[0,1] \rightarrow \mathrm{R}_{+}$and $\mathrm{w}_{B}:[0,1] \rightarrow \mathrm{R}_{+}$, which gives the wages of each type of worker as a function of the number of hours they work. Firms choose how many people of each type to employ, and the specific number of hours each type will work, given the wage schedules. ${ }^{5}$ This stands in contrast to standard labor supply theory in which hours are determined solely by workers who take as given a fixed hourly wage rate. ${ }^{6}$

In this framework hiring 2 production workers to work 20 hours each is not the same as hiring 1 worker for 40 hours. Not only will these two configurations of workers produce different amounts of labor services, they will also require different total wage payments. In general the period wage will not increase proportionally with the number of hours worked.

[^4]The firm solves the following static optimization problem each period

$$
\begin{equation*}
\max _{k, h_{A}, h_{B}, d_{A}, d_{B}} f\left(k, h_{A}, d_{A}, h_{B}, d_{B}\right)-r k-d_{A}\left[w_{A}\left(h_{A}\right)+\phi_{A}\right]-d_{B}\left[w_{B}\left(h_{B}\right)+\phi_{B}\right] . \tag{F}
\end{equation*}
$$

### 2.3 Traded Commodities and Preferences

This economy contains an indivisibility in the choice set of individuals. I follow Prescott and Townsend (1984) and Rogerson (1988) by introducing lotteries and allowing people to randomize over working hours of different lengths. People in this economy cannot work $3 / 4$ of a 40 hour workweek, but they can work a 40 hour workweek with probability $3 / 4$ and not work with probability $1 / 4$. One interpretation of these lotteries, provided by Hansen (1985), is that people face uncertain employment prospects and are able to perfectly insure themselves each period against not being employed.

In each period the commodities being traded are given by $\left(\mathrm{c}, \mathrm{i}, \mathrm{k}, \mathrm{x}_{A}, \mathrm{x}_{B}\right)$, where c is the consumption good, i is the investment good, and k is the services of the capital stock. For individuals, $\mathrm{x}_{A}$ and $\mathrm{x}_{B}$ are the probabilities of working $\mathrm{h}_{A}$ and $\mathrm{h}_{B}$ hours respectively, and will be denoted $\mathrm{n}_{A}$ and $\mathrm{n}_{B}$ in the individual's decision problem. For firms, $\mathrm{x}_{A}$ and $\mathrm{x}_{B}$ are the number of workers of type A workers supplying $\mathrm{h}_{A}$ hours and type $B$ workers supplying $h_{B}$ hours, and are denoted $d_{A}$ and $d_{B}$ in the firm's decision problem. The prices of capital services and the labor market goods are given by $\left(\mathrm{r}, \mathrm{w}_{A}(\cdot), \mathrm{w}_{B}(\cdot)\right)$, where the consumption good is the numeraire. Recall that $\mathrm{w}_{A}$ and $\mathrm{w}_{B}$ are functions of hours worked.

The preference ordering of a type e person, $e \in\{A, B\}$, in the economy is given by their discounted lifetime utility

$$
\begin{equation*}
\sum_{t=0}^{\infty} \beta^{t}\left[\log \left(c_{t}\right)-n_{t} v_{e}\left(h_{t}\right)\right] \tag{6}
\end{equation*}
$$

where the disutility of working $h$ hours is given by

$$
\begin{align*}
v_{e}(h) & =-\gamma_{e}\left[\frac{(1-\tau-h)^{\sigma_{e}}-1}{\sigma_{e}}\right], & & h>0, \sigma_{e} \neq 0  \tag{7}\\
& =-\gamma_{e} \ln (1-\tau-h), & & h>0, \sigma_{e}=0 \\
v_{e}(0) & =0 & &
\end{align*}
$$

where $\sigma_{B}, \sigma_{A} \leq 1 .{ }^{7}$ Recall that $\tau$ is commuting time. The functions $\mathrm{v}_{A}$ and $\mathrm{v}_{B}$ are positive, increasing, and convex for $\mathrm{h}>0$. These functions give the disutility of working h hours (and commuting $\tau$ ), while n gives the probability of working. Thus the expected disutility for a type e person of working $h$ with probability $n$, and working 0 with probability $1-\mathrm{n}$, is given by

$$
n v_{e}(h)+(1-n) v_{e}(0)=n v_{e}(h) .
$$

### 2.4 The Individuals' Decision Problems

Individuals in this economy purchase the consumption and investment goods, and sell capital and labor services to the firm taking the wage schedule and rental rate of capital as given. I assume that each type can only supply working hours of their own type and cannot work two jobs in a period. That is, a worker cannot work 30 hours at one job and 20 hours at another, but he/she can work 50 hours at one job. A type e individual, $\mathrm{e} \in\{\mathrm{A}, \mathrm{B}\}$, chooses sequences $\left\{\mathrm{c}_{t}, \mathrm{k}_{t+1}, \mathrm{~h}_{t}, \mathrm{n}_{t}\right\}_{t=0}^{\infty}$ which solve the following optimization problem:

$$
\begin{equation*}
\max \sum_{t=0}^{\infty} \beta^{t}\left[\log \left(c_{t}\right)-n_{t} v_{e}\left(h_{t}\right)\right] \tag{H}
\end{equation*}
$$

subject to

$$
\begin{align*}
& c_{t}+i_{t} \leq r_{t} k_{t}+w_{e t}\left(h_{t}\right) n_{t}  \tag{8}\\
& k_{t+1}=(1-\delta) k_{t}+i_{t}  \tag{9}\\
& c_{t}, i_{t} \geq 0  \tag{10}\\
& 0 \leq h_{t} \leq 1-\tau  \tag{11}\\
& 0 \leq n_{t} \leq 1 \tag{12}
\end{align*}
$$

given $\mathrm{k}_{e 0}$ and the sequence of wage functions and interest rates $\left\{\mathrm{w}_{A_{t}}(\cdot), \mathrm{w}_{B_{t}}(\cdot), \mathrm{r}_{t}\right\}_{t=0}^{\infty}$.

## 3 Equilibrium and the Determination of Hours

In this section I define a competitive equilibrium and list steady state equilibrium conditions for this economy. This is followed by a discussion of the determinates of

[^5]equilibrium hours per worker.

### 3.1 Definition of Competitive Equilibrium

A competitive equilibrium is an allocation,

$$
\left\{c_{A t}, h_{A t}, n_{A t}, k_{A t}, c_{B t}, h_{B t}, n_{B t}, k_{B t}, d_{A t}, d_{B t}, k_{t}\right\}_{t=0}^{\infty}
$$

and a sequence of wage functions and interest rates $\left\{\mathrm{w}_{A t}(\cdot), \mathrm{w}_{B t}(\cdot), \mathrm{r}_{t}\right\}_{t=0}^{\infty}$ such that
i) Individuals maximize utility:
$\left\{\mathrm{c}_{e t}, \mathrm{~h}_{e t}, \mathrm{n}_{e t}, \mathrm{k}_{e t+1}\right\}_{t=0}^{\infty}$ solves (H) given the wage functions and interest rate sequences, $\mathrm{e} \in\{\mathrm{a}, \mathrm{b}\}$.
ii) Firm maximize profits:
$\left\{\mathrm{h}_{A t}, \mathrm{~h}_{B t}, \mathrm{~d}_{A t}, \mathrm{~d}_{B t}, \mathrm{k}_{t}\right\}_{t=0}^{\infty}$ solves ( F ) given the wage functions and interest rate sequences.
iii) Markets clear:

$$
\begin{aligned}
& \lambda_{A} n_{A t}=\mathrm{d}_{A t} \\
& \lambda_{B} n_{B t}=\mathrm{d}_{B t} \\
& \lambda_{B} k_{B t}+\lambda_{A} k_{A t}=\mathrm{k}_{t} \\
& \lambda_{B} c_{B t}+\lambda_{A} c_{A t}+\mathrm{k}_{t+1}-(1-\delta) \mathrm{k}_{t}=F\left(k_{t}, h_{A t}, d_{A t}, h_{B t}, d_{B t}\right)
\end{aligned}
$$

The benchmark economy in this paper has an interior solution for the hours and employment of both people types, and I provide the steady state conditions for this case. A steady state equilibrium can be characterized by a list of 14 numbers.

$$
\left\{c_{A}, h_{A}, n_{A}, k_{A}, c_{B}, h_{B}, n_{B}, k_{B}, d_{A}, d_{B}, k, w_{A}, w_{B}, r\right\}
$$

which satisfy the following conditions $\left[\mathrm{w}_{A}\right.$ and $\mathrm{w}_{B}$ are constants here]:

$$
\begin{align*}
v_{A}\left(h_{A}\right) & =\frac{w_{A}}{c_{A}}  \tag{s1}\\
v_{B}\left(h_{B}\right) & =\frac{w_{B}}{c_{B}}  \tag{s2}\\
c_{A} & =\rho k_{A}+w_{A} n_{A}  \tag{s3}\\
c_{B} & =\rho k_{B}+w_{B} n_{B}  \tag{s4}\\
r & =\rho+\delta \tag{s5}
\end{align*}
$$

$$
\begin{align*}
w_{A}+\phi_{A} & =\frac{\partial f\left(k, h_{A}, d_{A}, h_{B}, d_{B}\right)}{\partial d_{A}}  \tag{s6}\\
w_{B}+\phi_{A} & =\frac{\partial f\left(k, h_{A}, d_{A}, h_{B}, d_{B}\right)}{\partial d_{B}}  \tag{s7}\\
r & =\frac{\partial f\left(k, h_{A}, d_{A}, h_{B}, d_{B}\right)}{\partial k}  \tag{s8}\\
k & =\lambda_{B} k_{B}+\lambda_{A} k_{A}  \tag{s9}\\
\lambda_{A} n_{A} & =d_{A}  \tag{s10}\\
\lambda_{B} n_{B} & =d_{B}  \tag{s11}\\
\frac{(1-\theta) g\left(h_{A}\right)}{g^{\prime}\left(h_{A}\right)} & =\frac{c_{A} v_{A}\left(h_{A}\right)+\phi_{A}}{v_{A}\left(h_{A}\right)}  \tag{s12}\\
\frac{(1-\theta) g\left(h_{B}\right)}{g^{\prime}\left(h_{B}\right)} & =\frac{c_{B} v_{B}\left(h_{B}\right)+\phi_{B}}{v_{B}\left(h_{B}\right)}, \tag{s13}
\end{align*}
$$

where

$$
\rho=\frac{1}{\beta}-1
$$

Equations (s1-s4) come from the individuals' optimization problems. Eqns (s1-s2) are the marginal rate of substitution conditions between employment and consumption. Eqns (s3-s4) are the budget constraints. Equation (s5) is the standard condition on the rental rate of capital. Eqns ( s 6 -s8) are the conditions that capital services and workers are paid their marginal product. Eqns (s9-s11) are market clearing conditions for capital and workers. Finally, (s12-s13) are the conditions which guarantee that the steady state working hours, $\mathrm{h}_{A}$ and $\mathrm{h}_{B}$, satisfy both the firm's and the individuals' optimality conditions.

Notice that there are 13 equations in 14 unknowns. As is typical in models with two types of people, there are a continuum of steady states associated with different wealth positions of the two types. I focus on a specific steady state by adding a condition on the ratio of the steady state capital holdings of the two types.

### 3.2 The Determination of Hours

Next I discuss the the factors which determine equilibrium hours per worker. In determining the optimal hours to employ workers, the firm can solve a much easier subproblem. For any given level of labor services for type e workers, say $L_{e}^{*}$ where
$e \in\{A, B\}$, the firm solves

$$
\begin{equation*}
\min _{d_{e}, h_{e}} d_{e}\left[w_{e}\left(h_{e}\right)+\phi_{e}\right] \tag{13}
\end{equation*}
$$

subject to

$$
\begin{align*}
& g\left(h_{e}\right) d_{e}^{1-\theta} \geq L_{e}^{*}  \tag{14}\\
& h_{e} \leq 1-\tau \tag{15}
\end{align*}
$$

which can be rewritten

$$
\begin{equation*}
\min _{h_{e}} L_{e}^{* \frac{1}{1-\theta}} g\left(h_{e}\right)^{\frac{-1}{1-\theta}}\left[w_{e}\left(h_{e}\right)+\phi_{e}\right] \text { s.t. } h_{e} \leq 1-\tau \tag{16}
\end{equation*}
$$

Notice that the solution to this problem is independent of $\mathrm{L}_{e}^{*}$, which simply scales the objective function. This is a familiar result in the literature, and says that the optimal $h$ for a firm is independent of scale.

An interior solution to the firm's problem must satisfy the following condition

$$
\begin{equation*}
\frac{w_{e}(h)+\phi_{e}}{w_{e}^{\prime}(h)}=\frac{(1-\theta) g(h)}{g^{\prime}(h)} \tag{17}
\end{equation*}
$$

which states that the marginal cost of varying the number of workers is equated to the marginal cost of varying the number of hours per worker ${ }^{8}$. This condition allows equilibrium hours per worker to be solved for, given the wage schedules, without regard to the equilibrium levels of employment.

The shape of the equilibrium wage schedules are read off the employment optimality conditions from the individuals' problems, which are given by (assuming an interior solution)

$$
\begin{align*}
w_{A}\left(h_{A}\right) & =c_{A} v_{A}\left(h_{A}\right)  \tag{18}\\
w_{B}\left(h_{B}\right) & =c_{B} v_{B}\left(h_{B}\right) \tag{19}
\end{align*}
$$

Combining the equilibrium conditions of the firm and individuals', we obtain the following equilibrium conditions for hours per worker:

$$
\begin{align*}
& \frac{(1-\theta) g\left(h_{A}\right)}{g^{\prime}\left(h_{A}\right)}=\frac{c_{A} v_{A}\left(h_{A}\right)+\phi_{A}}{v_{A}\left(h_{A}\right)}  \tag{20}\\
& \frac{(1-\theta) g\left(h_{B}\right)}{g^{\prime}\left(h_{B}\right)}=\frac{c_{B} v_{B}\left(h_{B}\right)+\phi_{B}}{v_{B}\left(h_{B}\right)} . \tag{21}
\end{align*}
$$

[^6]Using the functional forms provided earlier, these conditions implicitly define hours per worker as a function of the model parameters and current period consumption

$$
\begin{align*}
h_{A} & =H^{*}\left(\theta, \psi, \mu, \sigma_{A}, \tau, \phi_{A}, \gamma_{A}, c_{A}\right)  \tag{22}\\
h_{B} & =H^{*}\left(\theta, \psi, \mu, \sigma_{B}, \tau, \phi_{B}, \gamma_{B}, c_{B}\right) \tag{23}
\end{align*}
$$

The derivatives of this function have the following signs.

$$
\begin{aligned}
H_{\theta}^{*}, H_{\psi}^{*}, H_{\mu}^{*}, H_{\phi}^{*} & >0 \\
H_{\gamma}^{*}, H_{c}^{*} & \leq 0(<0 \text { if } \phi>0)
\end{aligned}
$$

As expected, equilibrium hours for each type is an increasing function of fixed labor costs $\phi$, set-up time $\mu$, the elasticity of labor services with respect to hours $\psi$, and the curvature parameter $\theta$. Hours are a decreasing function of current period consumption and the disutility of work parameter $\gamma$. The derivatives with respect to $\tau$ and $\sigma$ cannot be signed.

## 4 Benchmark Model Economy

The preceding sections have laid out the model economy. To carry out the quantitative analysis, we must assign parameter values to a benchmark model economy. Where possible, parameter values are selected so that the steady state of the model economy reproduces prominent aspects of observed U.S. data or are drawn from empirical labor studies. Given the general robustness of the qualitative nature of the results to changes in the parameter values, I conjecture that the general findings would continue to hold were the model parameterized using data from most European countries. The sensitivity of the experiment results to the choice of parameter values is discussed after the results are presented for the benchmark economy. Note, however, that the goal of this paper is not to derive precise quantitative estimates of the impact of an hours restriction, but rather to illustrate the qualitative predictions of a general equilibrium model and to identify key features of the framework in determining the policy's quantitative impact.

Values must be assigned to 15 parameters, $\left\{\delta, \theta, \beta, \alpha, \mathrm{A}, \tau, \lambda_{A}, \sigma_{A}, \sigma_{B}, \gamma_{A}\right.$, $\left.\gamma_{B}, \mu, \psi, \phi_{A}, \phi_{B}\right\}$. A model period is taken to be one week. I assume there are 100 hours available for work or leisure and that commuting requires 4 hours per week, which implies $\tau$ is set to 0.04 . These numbers are consistent with the data reported in Juster and Stafford (1991). The parameters $\delta, \theta$, and $\beta$ are selected so that the annual capital-output ratio is roughly 2.1 , the annual investment-output ratio is 0.18 , and the real interest rate is 4 percent annually. The capital-output and investmentoutput ratios are somewhat lower than those typically seen in the real business cycle literature due to the fact that I exclude household capital, consumer durables and residential housing, and its services from my definitions of capital, investment, and output. ${ }^{9}$ The production constant, A, is chosen so that the sum of steady state output and quasi-fixed worker costs equals 1 .

For the benchmark model I set $\mu$ to 0.0 , so that no warm-up time per worker is required in production. I set $\psi$ to 1 , implying an elasticity of labor services with respect to hours worked of 1 , an estimate within the range of those reported in Hamermesh (1993). Finally I set $\phi_{A}$ and $\phi_{B}$, the per worker fixed costs, to 0.05 each. This implies that roughly 5 percent of total labor costs are devoted to maintaining job positions. Since it is difficult to find microeconomic studies which seek to directly measure the cost of maintaining job positions, these numbers reflect a back-of-the envelope calculation. Somewhat surprisingly, the qualitative nature of the experiment results are not sensitive to changes in the values of $\phi_{A}$ and $\phi_{B}$ between 0.0 and 0.10, nor to allowing $\phi_{A}$ to be greater than $\phi_{B}$ as empirical evidence suggests. The sensitivity of the experiment results to the values of $\mu$ and $\psi$ will be explored in the next section.

A major difficulty in assigning the remaining parameter values is that the model is not specific about the differences between the two types of people. However, the nature of the policy experiments provides some additional direction. In particular, type A people should comprise groups which both experience high employment rates

[^7]and whose skills would be difficult to replace from the nonemployed population. With that in mind, I make the following assumptions on the two types. Type A workers form 20 percent of total employment, somewhat below the fraction of U.S. workers in 1992 who were college graduates ( 27 percent) or whose occupation was classified as managerial or professional speciality ( 26 percent). The employment-population ratio of type A people is set to 0.84 , slightly less than the 1992 ratio for college graduates, versus 0.74 for the population aged 25 to 64 as a whole. ${ }^{10}$ The hourly wage rate of type A workers is set to 1.6 times the hourly wage rate of type B workers. The wage ratio of 1.6 is in line with empirical findings on education and earnings and with the ratio of the median weekly earnings of managerial and professional specialty workers to all other workers. ${ }^{11}$ Average hours per worker for both types is set to 0.43 , or 43 hours per week, which was the average hours at work for full-time workers in nonagricultural industries in 1992. These assumptions are used to determine the values of parameters $\lambda_{A}, \gamma_{A}, \gamma_{B}, \sigma_{A}, \sigma_{B}$, and $\alpha$.

The initial capital stock endowments of the two types are chosen to be the steady state values in which the ratio of the capital stocks equals the ratio of their labor incomes. The results are not sensitive to the selection of the relative wealth positions of the two types. Table 2 lists the parameter values for the benchmark economy, while table 3 lists the steady state allocation and prices.

## 5 Effects of an Hours Restriction

This section reports on the consequences of introducing a restriction that firms can employ one or both types of workers for at most 0.387 units of time, or 38.7 hours per week. This is a 10 percent from the benchmark level of 43 hours. Three experiments are conducted. In the first experiment, the hours constraint applies to both types

[^8]of workers. This is accomplished by adding the following constraints to the firm's decision problem:
\[

$$
\begin{align*}
h_{A} & \leq 0.387  \tag{RA}\\
h_{B} & \leq 0.387 \tag{RB}
\end{align*}
$$
\]

A key aspect of this experiment is that there is an abundance of nonemployed people of both types who are equally productive to those that are working. More specifically, the constraints that $\mathrm{n}_{A}$ and $\mathrm{n}_{B}$ are less than or equal to 1 are not binding when the hours restriction is imposed.

For the next two experiments I add the assumption that there are no additional type A workers available to employ. This can be done in two ways. First, the benchmark economy could be reparameterized so that all type A workers are employed, $\mathrm{n}_{A}$ is 1 . In order to make the experiments comparable with the first one, this would require, among other things, changing the number of people of each type so as to keep the number of workers of each type unchanged. A second and simpler way to proceed, and the one pursued here, is to assume that no more than 84 percent of type A people can work in any period. This amounts to adding the following constraint to type A's decision problem:

$$
\begin{equation*}
n_{A} \leq 0.84 \tag{RC}
\end{equation*}
$$

These two strategies produce virtually identical quantitative implications.
In the second experiment, the hours constraint again applies to both types. In the third experiment the hours constraint applies only to type B workers.

Recall that the model economy has many steady states, so that an issue arises as to which steady state the economy will converge when the hours restriction is imposed. To find the new steady state, the solution procedure makes use of the fact that the budget constraints of each person type must be satisfied, without transfers, along the transition path and in the new steady state. The steady state capital holdings in the benchmark economy provide the initial conditions. In practice, only one candidate steady state satisfied this requirement. A linear quadratic approximation solution procedure was used to solve the model.

### 5.1 Experiment 1

The results of the first experiment, where the hours restriction applies to both types and there nonemployed people of both types, are reported in the first column of table 4. One striking finding is that employment increases by so much that total hours worked actually increases by more than 3 percent. However, productivity falls by 3.6 percent, with the net effect being a 0.3 percent decline in output and a 0.5 percent decline in consumption. There is only a small increase in the capital stock as firms substitute capital services for labor services.

While the hours restriction is quite successful at increasing employment, this increase does not come without costs. Hourly wage rates for both types fall by more than 3 percent, and, since hours per worker declines by 10.0 percent, weekly wages fall by more than 13 percent. Because people effectively share the total labor income of their type, and both types have more people working, consumption per person falls by only 0.5 percent or less for each type. The effects of the policy are very similar for the types, although the wages and consumption of type B people fall by slightly more and their employment increases by slightly more.

### 5.2 Experiment 2

The second experiment is motivated by empirical evidence which suggests that finding people who are equally skilled to workers whose hours are being reduced may be difficult for some jobs. This experiment explores the effect of a stark skills mismatch where one type is fully employed.

I again assume that the hours restrictions applies to all workers, so that constraints RA and RB are again imposed on the firm's problem. In addition, I impose constraint RC on type A's decision problem. This implies that there are no additional people of type A available for firms to employ when the hours restriction is imposed, and is interpreted as full employment for type A.

The results of this experiment are reported in the second column of table 4. First, notice that the hours restrictive is as effective at increasing the employment of type B workers in this experiment as it was in the first experiment. However, in
contrast to the first experiment, the policy now leads to a relatively large 4.8 percent decline in output and a 4.3 percent decline in consumption. The larger output decline reflects both a larger decrease in productivity, due to the shift in the composition of the workforce toward the less productive type B workers, and a smaller increase in total hours worked. The policy also now has a substantial negative effect on capital accumulation, leading to a 3.6 percent decline in the capital stock.

With type A people fully employed, the policy has strikingly different effects on the wage rates of the two types. Wages per hour for type B workers now fall by 7.7 percent, almost 4 percent more than in the first experiment, while the hourly wage of type A workers increases by 7.0 percent. Weekly wages for type B workers fall by almost 17 percent, while type A workers experience a relatively minor 3.7 percent decline. Since type A workers receive higher wages in the benchmark economy, this implies an increase in the wage disparity across the types.

Both types also suffer much larger declines in their consumption compared to the first experiment, with type B experiencing the larger decline. Type B's consumption falls by more despite the fact that, as a group, they now work 3.4 percent more while type A people work 10 percent less.

Given the effect of the policy on the employment of the two types and the form of the production function, it is easy to see why the wage disparity increases by looking at the ratio of the firm's marginal product conditions for wages:

$$
\begin{equation*}
\frac{w_{A}+\phi_{A}}{w_{B}+\phi_{B}}=\text { constant } \times \frac{n_{B}}{n_{A}} . \tag{24}
\end{equation*}
$$

This condition states that the ratio of the per period wages plus fixed costs of the two types is inversely related to the ratio of the employment of each type. Thus, if the employment of type $B$ increases more than the employment of type $A$, the relative wage of type $B$ workers must fall.

### 5.3 Experiment 3

Given the results from the previous experiment, a natural question to ask is "what if we excluded type A workers from the hours restriction?". In fact, work-sharing policies and proposals typically do exclude the hours of some skilled groups of workers,
such as executives, managers, and professionals, whose labor services would likely be more difficult to replace. The third experiment examines the extent to which the negative effects attributed to the full employment of type A can be mitigated by excluding that type from the hours restriction.

Here I drop the hours constraint, RA, on type A workers from the firm's problem, while keeping the hours constraint, RB, on type B workers and the employment constraint, RC, on type A people. The results of this experiment are reported in the third column of table 4.

The major finding is that applying the hours restriction to only type B workers completely mitigates the additional negative consequences of the policy which resulted when type A people were fully employed. The effect on type B people is identical to that in the first experiment, and output falls by only 0.2 percent, even less than in the first experiment. The decline in output is smaller than in experiment 1 despite the fact that total hours worked increases by less, and is due to the fact that productivity also falls by less.

The policy effect on type A, however, is quite different than in either of the prior experiments. Type A workers now experience a slight increase in their hourly and weekly wages, and in their consumption, of 0.2 percent, while their weekly and total hours fall slightly. Thus the policy now has the effect of not only increasing the disparity in wages, it results in type A people strictly better off.

### 5.4 Sensitivity Analysis

This subsection reports on the sensitivity of the experiment results to the specific benchmark parameter values used. The emphasis is on identifying those observations and assumptions used in the parameter selection criteria which play an important role in determining the qualitative nature of the findings. Observations and parameters which are not discussed have little effect on the results. ${ }^{12}$

[^9]Parameter values which determine the technological trade-offs between the number of workers, weekly hours per worker, and capital services clearly play an important role in determining the effects of the policy. These parameters are central in determining how productivity is affected by changes in working hours, and include $\psi$, the elasticity of labor services with respect to hours, and $\mu$, the fixed set-up or warm-up time required per worker for production. Larger values for each of these parameters imply larger declines in productivity, or smaller increases, for a fixed number of workers and capital stock. Thus we would expect that larger (smaller) values of $\psi$ or $\mu$ would imply larger (smaller) declines in productivity, wages, output, and consumption.

Table 5 shows how the experiment results vary with the size of $\psi$. As expected, higher values of $\psi$ have a negative effect on productivity, wages, output, and consumption. Output declines are larger despite the fact that employment and total hours increase by much more. Increases in $\mu$ from 0 to 0.2 , corresponding to set-up time of 0 to 2 hours per day, have a similar quantitative effect as varying $\psi$ between 0.8 and 1.2.

While the technology parameters are important, the experiment results indicate that the most crucial assumptions which in determining the effects of the policy have to do with the ability of the firm to hire additional workers of both types. These factors include whether type A people are fully employed, whether type A workers can be excluded from the hours restriction, the relative number of type A workers, and the relative wage rates of the two groups.

The experiment results so far have shown that when type A people are fully employed and that group is not excluded from the hours restriction, the detrimental effect of the policy on output, productivity, and consumption greatly increases. In that case, the number of type A workers and their relative wage rates become important.

Table 6 shows how the effects of the policy change with the fraction of workers who are type A. Not surprisingly, the negative effect of type A's full employment
on output, productivity, consumption, and capital accumulation diminishes as the relative number of type A workers decreases. As the number of type A workers approaches 0 , the effects of the policy on the aggregate economy and on type $B$ workers converge more or less to those found in experiment 1. The effect on the fully employed type A people also improves as their numbers dwindle. In fact, when type A form only 1 percent of employment, the hours restriction on all workers raises their hourly wages by so much that their weekly wages and consumption are unchanged, despite the fact that they work 10 percent less.

As the relative wage rate of type A workers increases, the negative effects of the policy also increase, although less dramatically. For example, if the wage ratio of type A to type B is 2 rather than 1.6, the decline in output, productivity, wages, and consumption is about 0.6 percentage points larger. That is, the more productive is the fully employed type, the more detrimental is the policy when the hours of that group are restricted.

The important assumption on working hours is not that workers of both types work exactly 43 hours, but rather that working hours are reduced by 10 percent. If type A workers were assumed to have their hours reduced from 45 hours to 40.5 hours, while type B workers declined from 40 to 36 , the results would be quantitatively similar.

### 5.5 Comments on Experiment Results

A few additional comments on the experiment results are worth mentioning. First, the fact that the employment effects are implausibly large clearly reflects some shortcomings in the model. As mentioned in the introduction, one shortcoming is the labor supply aspect of the model. Identical workers within each type each work some fraction of the weeks in a year depending on the lottery outcome. Reducing weekly hours in this model results in people choosing to work more weeks in the year, in expectation. Incorporating a model of labor supply which incorporates some of the features discussed in the introduction may diminish the employment response and is an obvious next step.

Second, the assumption that type A people are fully employed in the second
and third experiments is not crucial for obtaining the results. In fact, the results for experiment 3 , in which the hours restriction applies only to type B workers, are essentially unchanged by assuming that type A people are much less than fully employed, since employment for type A changes only slightly in any case. The results for experiment 2 , in which the hours restriction applies to both types, would be quantitatively similar if I assumed that type A employment could increase by 1 or 2 percent. The more type A employment increases before the the full employment constraint binds, the more the experiment results would be similar to those found when the constraint never binds (experiment 1).

Third, one might be tempted to infer from these results that a policy which excludes broad occupational categories that contain skilled groups of workers may be an effective way to avoid the negative effects of possible shortages in skilled workers. For example, perhaps by excluding executives, managers, and professionals from the provisions of a work-sharing policy, as the Fair Labor Standards Act (FLSA) does, policy makers can largely mitigate the detrimental effects of a shortage of skilled workers. However, some evidence suggests that isolating the skilled worker types who may be in limited supply may not be quite so straightforward. For example, within each broadly defined occupational category, unemployment rates vary greatly across subcategories. Ehrenberg and Schumann (1982) argue that there is likely to be a substantial mismatch of skills between the unemployed and those working overtime within at least some of the broad occupational categories.

Finally, the Cobb-Douglas specification of the production function is too restrictive, implying that the elasticities of substitution between capital, type A labor services, and type B labor services, are all 1 . Incorporating a more general form of the production function which allows for differing degrees of capital-skill complementarity, such as the constant elasticity formulation used in Krusell, Ohanian, Rios-Rull, and Violante (1997) would be clearly desirable.

## 6 Concluding Remarks

This paper takes a step towards developing a general equilibrium framework useful for analyzing the impact of macroeconomic policies which seek to influence working hours and employment. While the model abstracts from many potentially important factors, the experiment results confirm some suspicions and provide several insights. First, the results suggest that the previous studies based on models which have abstracted from the general equilibrium effects on total hours worked, output, wages, and capital accumulation provide an incomplete, and potentially misleading, picture of the consequences of policies which influence working hours.

Second, the results serve to point out features of general equilibrium models which are likely to be central in determining the quantitative effects of the policy. Modeling worker heterogeneity and the differences in skills between the employed whose hours are reduced and the nonemployed was found to be crucial. That is, whether firms, in the aggregate, can hire additional workers with the same skills as those workers whose hours are reduced is of central importance. Not surprisingly, the results also indicate that other factors which determine how productivity is affected by changes in working hours are also important, such as set-up time for workers and the substitution possibilities between hours per worker, employment, and capital services.

Finally, the results provide a warning about some potential, perhaps unforeseen, effects of an hours restriction. For example, the hours restriction not only leads to a decline in output and productivity, but also results in a large decline in the wages of at least some workers, and greatly increases the wage disparity across workers when skilled workers are fully employed. In fact, the policy can result in an increase in the absolute wages and consumption of skilled workers, while lowering the wages and consumption of the others.

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Table $1^{13}$
Unemployment, Labor Force Participation, and Employment/Population Rates in the U.S. by Age, Occupation, and Educational Attainment (March 1992)

| Total (ages 16 + ) | 7.8 | 66.0 | 60.8 |
| :--- | :---: | :---: | :---: |
| Age |  |  |  |
| $16-24$ | 14.3 | 63.2 | 54.1 |
| $25-54$ | 6.8 | 83.6 | 77.9 |
| $55-64$ | 5.3 | 56.5 | 53.6 |
| 65 and over |  | 11.9 | 11.5 |
|  |  |  |  |
| Occupation (ages 16 + ) | 2.9 | - | - |
| Managerial and professional specialty | 5.7 | - | - |
| Technical, sales, administrative support | 8.1 | - | - |
| Service occupations | 11.2 | - | - |
| Precision, production, craft and repair | 12.6 |  |  |
| Operators, fabricators, and laborers |  |  |  |
|  |  | 79.0 | 73.7 |
|  | 6.7 |  |  |
|  |  | 60.3 | 52.2 |
| Total (ages 25-64 ) | 13.5 | 78.3 | 72.3 |
| Educational Attainment (ages 25-64) | 7.7 | 83.5 | 88.8 |
| Less than high school diploma | 5.9 | 88.4 | 85.8 |
| High school graduate | 2.9 |  |  |

[^10]Table 2
Benchmark Parameter Values

| Preferences | Technology |
| :--- | :--- |
| $\beta=0.9995$ | $\theta=0.224$ |
| $\sigma_{A}=-0.135$ | $\alpha=0.718$ |
| $\sigma_{B}=-0.200$ | $\psi=1.000$ |
| $\gamma_{A}=1.685$ | $\mu=0.000$ |
| $\gamma_{B}=1.929$ | $A=1.674$ |
| $\tau=0.040$ | $\phi_{A}=0.050$ |
| $\lambda_{A}=0.176$ | $\phi_{B}=0.050$ |
|  | $\delta=0.0017$ |

Table 3
$\xlongequal{\overline{\text { Steady State of Benchmark Model Economy }}}$

| Hours per worker | 0.430 |  |
| :--- | :--- | :--- |
| Type A $\left(\mathrm{h}_{A}\right)$ |  | 0.430 |
| Type B $\left(\mathrm{h}_{B}\right)$ |  | 0.430 |
| Employment | 0.740 |  |
| Type A $\left(\mathrm{n}_{A}\right)$ |  | 0.840 |
| Type B $\left(\mathrm{n}_{B}\right)$ |  | 0.719 |
| Capital | 103.5 |  |
| Output | 0.963 |  |
| Output per hour | 3.026 |  |
| Consumption | 0.788 |  |

Wages per hour
Type A $\left(\mathrm{w}_{A} / \mathrm{h}_{A}\right) \quad 3.318$
Type B $\left(\mathrm{w}_{B} / \mathrm{h}_{B}\right) \quad 2.074$

Wages per period
Type A $\left(\mathrm{w}_{A}\right) \quad 1.427$
Type B $\left(\mathrm{w}_{B}\right) \quad 0.892$

Capital Holdings
Type A $\left(\mathrm{k}_{A}\right)$
167.8

Type B $\left(\mathrm{k}_{B}\right) \quad 89.7$
Consumption
Type A $\left(\mathrm{c}_{A}\right) \quad 1.277$
Type B ( $\mathrm{c}_{B}$ ) 0.683

Table 4
Effects of Hours Restriction

|  | Experiment 1 | Experiment 2 | Experiment 3 |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{h}_{A} \leq 0.387$ | $\mathrm{h}_{A} \leq 0.387$ | $\mathrm{h}_{A} \leq 0.387$ |
|  | $\mathrm{h}_{B} \leq 0.387$ | $\mathrm{h}_{B} \leq 0.387$ | $\mathrm{h}_{B} \leq 1.00$ |
|  | $\mathrm{n}_{A} \leq 1.00$ | $\mathrm{n}_{A} \leq 0.84$ | $\mathrm{n}_{A} \leq 0.84$ |
|  | \% Change | \% Change | \% Change |
| Hours per worker | -10.0 | -10.0 | -8.0 |
| Type A ( $\mathrm{h}_{A}$ ) | -10.0 | -10.0 | -10.0 |
| Type B ( $\mathrm{h}_{B}$ ) | -10.0 | -10.0 | 0.0 |
| Employment | 14.8 | 11.9 | 11.8 |
| Type A ( $\mathrm{n}_{A}$ ) | 14.6 | 0.0 | 0.0 |
| Type B ( $\mathrm{n}_{B}$ ) | 14.9 | 14.8 | 14.9 |
| Total Hours Worked | 3.3 | 0.7 | 2.7 |
| Type A ( $\mathrm{h}_{A} \mathrm{n}_{A}$ ) | 3.1 | -10.0 | -0.004 |
| Type B ( $\mathrm{h}_{B} \mathrm{n}_{B}$ ) | 3.4 | 3.4 | 3.4 |
| Capital | 0.2 | -3.6 | 0.2 |
| Output | -0.3 | -4.2 | -0.2 |
| Output per hour | -3.6 | -4.8 | -2.9 |
| Consumption | -0.5 | -4.3 | -0.3 |
| Wages per hour |  |  |  |
| Type A ( $\mathrm{w}_{A} / \mathrm{h}_{A}$ ) | -3.3 | 7.0 | 0.2 |
| Type B ( $\mathrm{w}_{B} / \mathrm{h}_{B}$ ) | -3.9 | -7.7 | -3.9 |
| Wages per period |  |  |  |
| Type A ( $\mathrm{w}_{A}$ ) | -13.0 | -3.7 | 0.2 |
| Type B ( $\mathrm{w}_{B}$ ) | -13.5 | -16.9 | -13.5 |
| Capital Holdings |  |  |  |
| Type A $\left(\mathrm{k}_{A}\right)$ | 0.2 | -3.6 | 0.2 |
| Type B ( $\mathrm{k}_{B}$ ) | 0.2 | -3.5 | 0.2 |
| Consumption |  |  |  |
| Type A ( $\mathrm{c}_{A}$ ) | -0.3 | -3.7 | 0.2 |
| Type B ( $\mathrm{c}_{B}$ ) | -0.6 | -4.5 | -0.6 |

Table 5
Sensitivity Analysis: $\psi$

| $\psi=0.8$ | $\psi=1.0$ | $\psi=1.2$ |
| :--- | :--- | :--- |
|  |  |  |
| Experiment 1 | Experiment 1 | Experiment 1 |
| $\mathrm{h}_{A} \leq 0.387$ | $\mathrm{~h}_{A} \leq 0.387$ | $\mathrm{~h}_{A} \leq 0.387$ |
| $\mathrm{~h}_{B} \leq 0.387$ | $\mathrm{~h}_{B} \leq 0.387$ | $\mathrm{~h}_{B} \leq 0.387$ |
| $\mathrm{n}_{A} \leq 1.00$ | $\mathrm{n}_{A} \leq 1.00$ | $\mathrm{n}_{A} \leq 1.00$ |

\% Change \% Change \% Change

| Hours per worker | -10.0 | -10.0 | -10.0 |
| :---: | :---: | :---: | :---: |
| Type A $\left(\mathrm{h}_{A}\right)$ | -10.0 | -10.0 | -10.0 |
| Type B $\left(\mathrm{h}_{B}\right)$ | -10.0 | -10.0 | -10.0 |
| Employment | 11.9 | 14.8 | 17.8 |
| Type A $\left(\mathrm{n}_{A}\right)$ | 11.7 | 14.6 | 17.6 |
| Type B $\left(\mathrm{n}_{B}\right)$ | 11.9 | 14.9 | 17.9 |
| Total Hours Worked | 0.7 | 3.3 | 6.0 |
| Type A $\left(\mathrm{h}_{A} \mathrm{n}_{A}\right)$ | 0.6 | 3.1 | 5.8 |
| Type B $\left(\mathrm{h}_{B} \mathrm{n}_{B}\right)$ | 0.7 | 3.4 | 6.1 |
| Capital | 0.3 | 0.2 | 0.1 |
| Output | -0.1 | -0.3 | -0.6 |
| Output per hour | -0.8 | -3.6 | -6.3 |
| Consumption | -0.2 | -0.5 | -0.8 |

Wages per hour

| Type A $\left(\mathrm{w}_{A} / \mathrm{h}_{A}\right)$ | -0.6 | -3.3 | -6.0 |
| :---: | :---: | :---: | :---: |
| Type B $\left(\mathrm{w}_{B} / \mathrm{h}_{B}\right)$ | -1.0 | -3.9 | -6.6 |
| Wages per period |  |  |  |
| Type A $\left(\mathrm{w}_{A}\right)$ | -10.5 | -13.0 | -15.4 |
| Type B $\left(\mathrm{w}_{B}\right)$ | -10.9 | -13.5 | -15.9 |
| Capital Holdings |  |  |  |
| Type A $\left(\mathrm{k}_{A}\right)$ | 0.3 | 0.2 | 0.1 |
| Type B $\left(\mathrm{k}_{B}\right)$ | 0.3 | 0.2 | 0.1 |
| Consumption |  |  |  |
| Type A $\left(\mathrm{c}_{A}\right)$ | -0.04 | -0.3 | -0.5 |
| Type B $\left(\mathrm{c}_{B}\right)$ | -0.3 | -0.6 | -0.9 |

Table 6

| Sensitivity Analysis: type A fraction of total employment |  |  |  |
| :---: | :---: | :---: | :---: |
| Employment: |  |  |  |
| type A / total | 0.20 | 0.10 | 0.01 |
| $\lambda_{A}$ | 0.176 | 0.088 | 0.009 |
|  | Experiment 2 | Experiment 2 | Experiment 2 |
|  | $\mathrm{h}_{A} \leq 0.387$ | $\mathrm{h}_{A} \leq 0.387$ | $\mathrm{h}_{A} \leq 0.387$ |
|  | $\mathrm{h}_{B} \leq 0.387$ | $\mathrm{h}_{B} \leq 0.387$ | $\mathrm{h}_{B} \leq 0.387$ |
|  | $\mathrm{n}_{A} \leq 0.84$ | $\mathrm{n}_{A} \leq 0.84$ | $\mathrm{n}_{A} \leq 0.84$ |
|  | \% Change | \% Change | \% Change |
| Hours per worker | -10.0 | -10.0 | -10.0 |
| Type A ( $\mathrm{h}_{A}$ ) | -10.0 | -10.0 | -10.0 |
| Type B ( $\mathrm{h}_{B}$ ) | -10.0 | -10.0 | -10.0 |
| Employment | 11.9 | 13.3 | 14.6 |
| Type A ( $\mathrm{n}_{A}$ ) | 0.0 | 0.0 | 0.0 |
| Type B ( $\mathrm{n}_{B}$ ) | 14.8 | 14.8 | 14.8 |
| Total Hours Worked | 0.7 | 2.0 | 3.2 |
| Type A ( $\mathrm{h}_{A} \mathrm{n}_{A}$ ) | -10.0 | -10.0 | -10.0 |
| Type B ( $\mathrm{h}_{B} \mathrm{n}_{B}$ ) | 3.4 | 3.3 | 3.3 |
| Capital | -3.6 | -1.8 | 0.0 |
| Output | -4.2 | -2.4 | -0.6 |
| Output per hour | -4.8 | -4.3 | -3.6 |
| Consumption | -4.3 | -2.5 | -0.7 |
| Wages per hour |  |  |  |
| Type A ( $\mathrm{w}_{A} / \mathrm{h}_{A}$ ) | 7.0 | 9.0 | 11.1 |
| Type B $\left(\mathrm{w}_{B} / \mathrm{h}_{B}\right)$ | -7.7 | -5.8 | -3.9 |
| Wages per period |  |  |  |
| Type A ( $\mathrm{w}_{A}$ ) | -3.7 | -1.9 | 0.0 |
| Type B ( $\mathrm{w}_{B}$ ) | -16.9 | -15.2 | -13.5 |
| Capital Holdings |  |  |  |
| Type A ( $\mathrm{k}_{A}$ ) | -3.6 | -1.8 | 0.0 |
| Type B ( $\mathrm{k}_{B}$ ) | -3.5 | -1.8 | 0.0 |
| Consumption |  |  |  |
| Type A ( $\mathrm{c}_{A}$ ) | -3.7 | -1.9 | 0.0 |
| Type B ( $\mathrm{c}_{B}$ ) | -4.5 | -2.6 | -0.7 |


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[^1]:    ${ }^{1}$ For example, in 1993 the "Full Employment Act for Fiscal Year 1994" was proposed in Congress which would have amended the Fair Labor Standards Act (FLSA) by reducing the "standard" workweek from forty to thirty hours and increased the overtime premium to two times the base hourly wage. Similar proposals were introduced in 1985 and in 1979. Interest in work sharing policies also comes from doomsayers who predict massive future unemployment in the U.S. due to technological advancement.

[^2]:    ${ }^{2}$ Examples of the theoretical research include Ehrenberg (1971), Hart (1984,1987), Fitzroy and Hart (1985), Hoel (1986), Calmfors and Hoel (1988,1989), and Santamäki (1988). Hart (1987) contains an overview of this literature. Empirical studies on the effects of increasing the overtime premium have typically found positive employment effects, though estimates of the magnitude very substantially. These studies and some of their shortcomings, are briefly summarized in Ehrenberg and Smith (1994). The quantative importance of some of these shortcomings are analyzed in great detail by Ehrenberg and Schumann (1982).

[^3]:    ${ }^{3}$ For example, U.S. proposals to amend the Fair Labor Standards Act would apply only to production and nonsupervisory workers, and exclude executives, managers, and professionals. Also, work-sharing policies are commonly advocated by unions, whose memberships are drawn mostly from production and nonsupervisory workers.

[^4]:    ${ }^{4}$ Since doubling the number of workers, the hours per worker, and capital at a plant more than doubles the output of that plant, one might think that a nonconvexity has been introduced into the aggregate production possibility set. However, because of the way the commodity set is defined, this set is in fact convex and standard general equilibrium tools can be used to analyze this economy. See Hornstein and Prescott [1993] or Fitzgerald [1996] for details.
    ${ }^{5}$ In the general formulation of this economy, firms can hire workers of the same type to work different numbers of working hours. The firm then distributes workers and capital across plants which employ workers for different numbers of hours. For the economies used in this paper, an equilibrium result is that all workers of each type work the same number of hours. To simplify the presentation of the economy I impose this equilibrium condition from the start.
    ${ }^{6}$ This characterization of the labor market, where the traded goods in the labor market are fixed hours-wage packages, builds closely on the observations of H.G. Lewis (1969).

[^5]:    ${ }^{7}$ Simpler specifications for the disutility of work, such as $-\gamma \mathrm{h}^{\sigma}$ or $\gamma(1-\mathrm{h})^{\sigma}$, tends to result in corner solutions for n or h , while the specification used does not.

[^6]:    ${ }^{8}$ The identical condition is found in labor demand models [See Hart(1987), p. 75].

[^7]:    ${ }^{9}$ A version of the model economy with population and real output per capita growth was used for calibration purposes, which leads the depreciation rate and the discount factor to be slightly larger in the stationary economy without growth.

[^8]:    ${ }^{10}$ Data on total employment by education and occupation is from the U.S. Department of Labor, Bureau of Labor Statistics, and is for March 1992.
    ${ }^{11}$ For example, Rupert, Schweitzer, Severance-Lossin, and Turner (1996) report that for full-time workers in 1993, college graduates earn nearly 60 percent more than high school graduates with no college. In the fourth quarter of 1992, the ratio of the median weekly earnings of full-time workers in managerial and professional specialty occupations to all full-time workers was about 1.5. Excluding full-time managerial and professional specialty occupations from the denominator of this ratio would obviously result in a slight increase.

[^9]:    ${ }^{12}$ For each change in the observations and/or assumptions, I compute the new parameter values implied by the parameter selection criteria described. I then perform the three experiments on this new benchmark economy, and compare the results with those obtained using the original benchmark economy. Plausible changes in the following observations have little effect on the findings: i) an employment-population ratio of type A of 0.84 ; ii) an aggregate employment-population ratio of 0.74 ; iii) an annual capital-output ratio of 2.1 ; iv) an annual investment-output ratio of 0.18 ; v) a

[^10]:    ${ }^{13}$ Data is from the U.S. Bureau of Labor Statistics and the Statistical Abstract of the United States 1997 and is not seasonally adjusted.

