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INTEREST RATE RULES ARE INFEASIBLE AND FAIL TO COMPLETE MACROECONOMIC MODELS

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ABSTRACT

This article reconsiders the recently controversial issues of whether an interest rate rule is feasible and whether it leaves nominal magnitudes indeterminate. It is shown that interest rate rules are infeasible unless the policy authorities possess complete current information. Furthermore, interest rate rules do not complete prototype macroeconomic models under rational expectations with no money illusion. Some influential analyses due to McCallum (1981, 1986) are reinterpreted in a manner consistent with these propositions.

INTEREST RATE RULES ARE INFEASIBLE AND FAIL TO COMPLETE MACROECONOMIC MODELS

I. Introduction

Sargent and Wallace (1975) presented the first dynamic model showing that if an interest rate rule is substituted in place of a money supply rule, then nominal variables are indeterminate. This article revalidates the Sargent and Wallace indeterminacy result. McCallum (1981) sought to modify this result, contending that interest rate rules do not result in nominal indeterminacy if they are chosen with any degree of concern about the consequences for the money stock. Canzoneri, Henderson, and Rogoff (1983) then argued that the authorities can peg the interest rate, even at a constant level, and the nominal determinacy problem will not arise, so long as the authorities announce a trend for the money supply. McCallum (1986), considering this result, clarified or interpreted it to mean that private expectations of the money supply are not anchored by an interest rate rule unless it is a limiting case of an underlying money supply function, as the elasticity of the money supply with respect to the interest rate increases without limit.

A view has developed that interest rate rules are feasible and determine nominal magnitudes, if they are limiting cases of a well-specified money supply function or if they are directed toward the achievement of money targets. A number of papers rely critically on the notion of the feasibility of interest rate rules. For example, Dotsey and King (1983, 1986) find that interest rate rules have the disadvantage of removing information signals otherwise available to private agents, by eliminating any observable relation

between the interest rate, which is observable, and money, which is not. In their model, the elasticity of the money supply with respect to the interest rate is entirely irrelevant, unless that elasticity can be infinite, in which case the signals available to agents are reduced.

The widespread popular idea that policies designed to control money, prices, and/or other nominal variables can be formulated, described, and executed in terms of predetermined interest rates is vulnerable to both the infeasibility and indeterminacy problems. Unless and until these problems can be resolved, there can be no objective basis for such a policy designed in terms of interest rates. These problems can be avoided by placing feasible, meaningful constraints directly on the money stock via a money supply function.

II. Concepts and Definitions

Interest rate rules are often seen as descriptions of policy. This section presents the view that it is more precise to distinguish interest rate rules from decision rules, or truly structural equations for monetary policy. The latter are easier to motivate on a formal level than are interest rate rules, so discussion begins with them.

A money supply function is a structural equation. It specifies a decision by the authorities concerning the quantity of money to supply, contingent on the observed state of the economy. The observed state of the economy is the set of information available at the time of the decision about the quantity of money. Realistically, it includes the current nominal interest rate and lagged state vector. An example of a money supply function appropriate to standard log-linear macroeconomic models is:

$$m_t = qR_t + \mu S_{t-1} \qquad (1)$$

where m_t is the log of the money stock,

R_t is the log of one plus the interest rate,

S_{t-1} is the lagged state vector,

and q and μ are finite parameters representing policy choice.

q and μ must be finite or (1) is not a valid expression. Incidentally, the money supply function can be renormalized with the nominal interest rate on the left-hand side, as in:

$$R_t = q^{-1}m_t - (q^{-1}\mu)S_{t-1} \qquad (2)$$

so long as q is nonzero. Regardless of how the money supply function is written, the current money stock, m_t, must appear.

An interest rate rule is a strict relation between the current nominal interest rate and the lagged state vector. In other words, an interest rate rule is a requirement or restriction that the interest rate behave in a particular predetermined manner. Under the rule, the current interest rate is invariant with respect to current innovations in the state vector. The feature that distinguishes an interest rate rule from a money supply function is that the former excludes the current money stock as an argument. For example, the equation

$$R_t = \rho_0 + \rho_1 R_{t-1} + \rho_2 m_t + \rho_3 m_{t-1} \qquad (3)$$

represents a money supply function if ρ_2 is nonzero, but represents an interest rate rule if ρ_2 is zero.

Money supply functions and interest rate rules are fundamentally different kinds of relations. The money supply function is a structural equation: it is logically prior to the solution to the model. The analyst can write down the money supply function and the nonpolicy structural equations and from them derive solutions for endogenous variables, as will be illustrated below. On the other hand, an interest rate rule is a restriction on the behavior of the

interest rate. Unless the interest rate is a direct instrument of policy, then an interest rate rule cannot adequately represent an operational policy. What is needed is a decision rule for an actual choice variable. Among candidate variables in the structure, only money is appropriately conceived as a feasible choice variable. Then feasible policy can be described adequately only in terms of a rule for the behavior of the money stock as a function of the observed state. In other words, policy can be adequately and operationally described only in terms of a money supply function.

An interest rate rule can be interpreted only as a restriction on the solution or outcome of the model. Indeed, the interest rate rule, **if it** in fact holds true in an economy, is the reduced-form equation for the interest rate.

III. Nominal Determinacy in a Prototype Model

This section presents a simple model and shows how money and prices are determined by solving the money supply function and the nonpolicy structural equations simultaneously.

Consider the following illustrative macroeconomic model. Let aggregate demand be a function of the real rate of interest and a disturbance, while aggregate supply is a constant. Formally:

$$y_t^d = d_0 - d[R_t - (E_{t-1}p_{t+1} - p_t)] + u_t, \quad d>0 \tag{4}$$

$$y_t^s = y^f \tag{5}$$

where y_t^d and y_t^s are aggregate demand and supply, respectively; p_t is the log of the price level; y^f is a fixed output supply; and u_t is a nonautocorrelated disturbance to aggregate demand. The lagged expectation operator, E_{t-1}, when applied to p_{t+1}, returns the objective mathematical

expectation of the future price level conditioned on all lagged realizations of the state vector.[1] Then the commodity market equilibrium condition, or IS function, is

$$R_t = r_t + (E_{t-1}p_{t+1} - p_t) \tag{6}$$

where the (ex ante) real rate,

$$r_t = (1/d)[(d_0 - y^f) + u_t], \tag{7}$$

is exogenous. $(d_0 - y^f)$ will be taken as zero. This assumption is heuristic only; it has the effect of making the mean of the real rate equal to zero, thereby simplifying mathematical expressions that follow.

Let the money demand function be

$$m_t - p_t = \alpha_0 - \alpha R_t + e_t, \quad \alpha > 0, \tag{8}$$

where m_t is the quantity of money and e_t is a white-noise disturbance.

For the purpose at hand, it will suffice to consider a simplified version of the money supply function analyzed by McCallum (1986):

$$m_t = \lambda_0 + \lambda(R_t - R^*), \quad -(1+\alpha) < \lambda < \infty, \tag{9}$$

where R* is an interest rate target. R* is treated as a constant to simplify analysis.

The economic structure is composed of the two nonpolicy equations, the IS function, (6), and the money demand function, (8), plus the policy equation, or money supply function, (9). It is easy to show that these three equations are sufficient to provide solutions for the three endogenous variables, p_t, m_t, R_t, as well as the endogenous expectation $E_{t-1}p_{t+1}$. It is useful for what follows to show how the method of undetermined coefficients can be used to solve the model if an appropriate state vector can be identified.

Formally, a <u>state vector</u> is a set of dated variables, both predetermined and exogenous, that completely describe the position, or state, of a dynamic system. Current realizations of endogenous variables are strict functions of

the state vector. The concept of state vector is invoked during analysis of given systems in order to hypothesize a trial solution, whose arguments are the state vector and whose coefficients are functions of structural parameters. Given a (linear) trial solution--based on an adequate state vector--and given a consistent (linear) economic structure, the coefficients of the trial solution can be determined. Operationally, a state vector is a set of dated predetermined and exogenous variables that, when included as arguments in a trial solution, provide a consistent set of identities relating structural parameters of the system to the coefficients in the trial solution. If a trial solution is found to be inconsistent with the structure, then the state vector on which it is based is inadequate and must be expanded (assuming the structure itself is consistent).

Unfortunately, the state vector is nonunique; there are an indefinitely large set of vectors adequate to describe the current state. From among this adequate set, the analyst may choose a particular state vector for analytical convenience. McCallum (1983) has suggested the employment of a minimal state vector, which is the smallest-dimensioned vector adequate to describe the current state of the system. A state vector is a minimal state vector if none of its elements can be left out of the trial solution without making the trial solution inconsistent with the structure.

Exogenous and predetermined variables explicitly appearing in the structural equations are obvious candidates for inclusion in the minimal state vector. Sometimes, but not always, they comprise a minimal state vector. In general, however. trial and error must be employed to ensure that the state vector is adequate. In the illustrative model, the set of state variables explicitly appearing in it will constitute an adequate state vector that rules out bubbles, or explosive paths for real money balances,

but that otherwise does not restrict the outcome of the model beyond the restrictions already present in the structure.[2]

In the illustrative model, the exogenous and predetermined variables appearing in the structural representation are:

$$S_t=\{1,r_t,e_t\}, \tag{10}$$

including the unit "variable" that is implicit. A trial solution in terms of the undetermined coefficients, denoted as Π_{ij}s, is given in equations (11), (12), and (13).

$$p_t=\Pi_{10}+\Pi_{11}r_t+\Pi_{12}e_t \tag{11}$$

$$m_t=\Pi_{20}+\Pi_{21}r_t+\Pi_{22}e_t \tag{12}$$

$$R_t=\Pi_{30}+\Pi_{31}r_t+\Pi_{32}e_t \tag{13}$$

Under the information assumptions and using (11), the expectation of the future price must be

$$E_{t-1}p_{t+1}=\Pi_{10} \tag{14}$$

where use has been made of $E_t(r_t)=[d_0-y^f/d)]=0$ and $E_t(e_t)=0$.

The solution is obtained by substituting (11), (12), (13), and (14) into the structural equations (6), (8), and (9) and solving the resulting identities for all Π_{ij} coefficients. These identities are shown below.

$$\begin{aligned}
&\Pi_{30}=\Pi_{10}-\Pi_{10}\\
&\Pi_{31}=1-\Pi_{11}\\
&\Pi_{32}=-\Pi_{12}\\
\\
&\Pi_{20}-\Pi_{10}=\alpha_0-\alpha\Pi_{30}\\
&\Pi_{21}-\Pi_{11}=-\alpha\Pi_{31}\\
&\Pi_{22}-\Pi_{12}=-\alpha\Pi_{32}+1
\end{aligned} \tag{15}$$

$$\Pi_{20}=\lambda_0+\lambda\Pi_{30}-\lambda R^*$$
$$\Pi_{21}=\lambda\Pi_{31} \qquad \text{(15, cont.)}$$
$$\Pi_{22}=\lambda\Pi_{32}$$

These identities are consistent, so the state vector is adequate. The coefficients are then determined to be as shown below.

$$\Pi_{10}=-\alpha_0-\lambda R^*$$
$$\Pi_{11}=(\alpha+\lambda)/(\alpha+\lambda+1)$$
$$\Pi_{12}=-1/(\alpha+\lambda+1)$$

$$\Pi_{20}=\lambda_0-\lambda R^*$$
$$\Pi_{21}=\lambda/(\alpha+\lambda+1) \qquad (16)$$
$$\Pi_{22}=\lambda/(\alpha+\lambda+1)$$

$$\Pi_{30}=0$$
$$\Pi_{31}=1/(\alpha+\lambda+1)$$
$$\Pi_{32}=1/(\alpha+\lambda+1)$$

When these expressions for the Π_{ij}s are substituted into (11), (12), and (13), it is clear that the solutions for all endogenous variables are well-defined.

IV. Interest Rate Rules Are Infeasible

As argued above, an interest rate rule cannot be interpreted as a structural equation; it is a restriction on the behavior of the interest

rate. If the interest rate rule is to hold true for an economy, then the reduced form must be identical to the interest rate rule. This section shows that there is no feasible policy choice--no set of money supply function parameters--that satisfies the requirement that it renders the interest rate reduced form identical to the interest rate rule.

Consider the trivial interest rate rule

$$R_t = R^*. \tag{17}$$

If an economy obeys such an interest rate rule, then the reduced form coefficients in the interest rate equation of the trial solution, (13), must be

$$\begin{aligned} \Pi_{30} &= R^* \\ \Pi_{31} &= 0 \\ \Pi_{32} &= 0. \end{aligned} \tag{18}$$

The second and third of these restrictions render the interest rate predetermined; the first restriction ensures that the interest rate target R^* is achieved. In view of the solution values for the Π_{3j}s shown in (16), the interest rate rule requires

$$\begin{aligned} R^* &= 0 \\ 1/(\alpha+\lambda+1) &= 0. \end{aligned} \tag{19}$$

Then an interest rate rule is feasible if and only if there exists a policy equation such that (19) holds. But there is no allowable policy vector $\{\lambda_0, \lambda, R^*\}$ satisfying these equations. Specifically, there is no finite λ to satisfy $\Pi_{31} = \Pi_{32} = (\alpha+\lambda+1)^{-1} = 0$. While it is true that Π_{31} and Π_{32} approach zero as λ approaches infinity, Π_{31} and Π_{32} are undefined at $\lambda = \infty$ because if λ is not finite, then (9) is not a valid expression of the structural policy equation. And if (9) is not the policy equation, something else describing monetary policy must be supplied to

complete the model. The feasibility of an interest rate rule cannot be assessed without a complete model.

Of course, the argument against the feasibility of an interest rate rule depends, so far, on the assumption that the money supply function takes the form (9). Fortunately for the argument, important generalizations on (9) can be made without affecting the result that interest rate rules are infeasible. Simple inductive evidence, in the form of trying various candidates and observing the implications, suggests that lagged realizations of any variables appearing in the model can be added as arguments to the policy rule (these, of course, may augment the minimal state vector) without affecting the infeasibility result.

Apparently, there is only one money supply function that would make an interest rate rule a feasible solution for the interest rate. This money supply function is of the form:

$$m_t=\lambda_o+\lambda_1 r_t+\lambda_2 e_t \qquad (20)$$

(which is also the reduced form for money), where the Xs are policy choice parameters. Then the reduced form for the interest rate is:

$$R_t = [(1-\lambda_1)/(1+\alpha)]r_t+[(1-\lambda_2)/(1+\alpha)]e_t. \qquad (21)$$

In view of the latter, it is clear that if $\lambda_1=\lambda_2=1$, then $R_t=R^*=0$. Therefore, if policy can condition the money stock arbitrarily on the state vector, as in (20), then an interest rate rule is feasible--there exists $\{A, ,\lambda_1,\lambda_2\}$ such that R_t is predetermined.

This alternative money supply function, (20), could be made operational only if the policymaker had complete current information, that is, if the authorities could observe the complete, current state of the economy, $1 ,r_t,e_t\}$. But in the realistic case in which the authorities do not have full knowledge of the current underlying shocks, because of

information lags, no operational money supply function can be found that generates predetermined behavior for the interest rate. An interest rate rule is infeasible because it is inconsistent with the structural assumptions of the model (no money illusion) and the informational assumption (policy authorities have incomplete current information).

V. Interest Rate Rules Are Not Limiting Cases of Money Supply Functions

This section considers the idea that an interest rate rule is a limiting case of a money supply rule, an idea that has been attributed to McCallum.[3] This section disproves this idea and ends with an account of how this idea arises from a failure to distinguish between reduced form and structural equations.

McCallum proves that as λ approaches infinity, the reduced form coefficients, the Π_{ij}s, remain well-defined. The point here is that any finite X, no matter how large, is consistent with determinacy. This point is unexceptionable, but the next step in his argument is that, because the coefficients on current innovations of the reduced form for the interest rate, Π_{31} and Π_{32}, become arbitrarily close to zero as λ gets larger, that one can come arbitrarily close to achieving an interest rate rule. Furthermore, it then seems reasonable to say that, in the *limit*, the money supply function becomes the interest rate rule.

This idea is important to refute because it has apparently become influential. For example, Dotsey and King (1986) analyze the effects of interest rate rules by setting λ equal to infinity: "An interest rate peg is the limiting case of a contemporaneous response to interest rates (i.e., $[\lambda]=\infty$)." (p. 37).

To prove that interest rate rules or pegs are not limiting cases of money supply rules, it is helpful to recall the definition of a limit. A limit is a fixed point to which an infinite sequence converges. Not all infinite sequences have limits nor, if they exist, are limits necessarily elements of the sequence. Consider an infinite sequence of λ values.

$$\Lambda=\{ \lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_j, \ldots. \}$$

with $\lambda_1<\lambda_2< \ldots<\lambda_j<\ldots.$

The money supply equation (9) defines a transformation $T:\Lambda->M$ where M is the infinite sequence of money supply functions:

$$M=\{ m_t^s=\lambda_0+\lambda_1(R_t-R^*), m_t^s=\lambda_0+\lambda_2(R_1-R^*),$$

$$m_t^s=\lambda_0+\lambda_3(R_t-R^*), \ldots , m_t^s=\lambda_0+\lambda_j(R_t-R^*), \ldots.\}$$

If a limiting money supply function existed as λ_j increased without limit, it would be the expression for $m_t^s=\lambda_0+\lambda_j(R_t-R^*)$ to which the sequence M converged as λ_j increases without limit. Obviously, such a limit does not exist.

How, then, can a rule such as (17) have been mistaken for a limiting case of (9)? The error arose from a failure to distinguish between the policy rule (9), which is a structural equation, and the solution for the interest rate. A natural source of this confusion is that the latter does in fact possess a fixed point, or limiting function, as λ approaches infinity. The reduced form solution for R_t given by (13) and (16), has the limiting function or fixed point identical to the interest rate rule, $R_t=0$.

VI. Interest Rate Rules Fail to Complete the Model

An interest rate rule does not complete the model. In particular, the

nonpolicy structural equations (6) and (8), together with an interest rate rule such as (17), do not provide unique bubble-free solutions for the price level and money stock.

Consider the system comprised of (6), (8), and (17), shown below for convenience.

$$R_t = r_t + (E_{t-1}p_{t+1} - p_t) \tag{6}$$

$$m_t - p_t = \alpha_0 - \alpha R_t + e_t \tag{8}$$

$$R_t = R^* \tag{17}$$

The money demand equation, (8), is of no use in determining the price level in this system. (6) and (17) together are sufficient to determine the expected rate of inflation,

$$(E_{t-1}p_{t+1} - p_t) = R^* - r_t, \tag{22}$$

but the price level, p_t, is undetermined, being dependent on an unsupplied terminal condition, $E_{t-1}p_{t+1}$:

$$p_t = E_{t-1}p_{t+1} + R^* - r_t. \tag{23}$$

By substituting (23) and (17) into (8), it is seen that m_t is also dependent one-for-one on the unsupplied terminal condition and, hence, is also indeterminate. The system {(6),(8),(17)} is incomplete with respect to p, and m_t.

McCallum evaluated the completeness of systems such as {(6),(8),(17)} using a trial solution derived from the undetermined coefficients method illustrated above. Systems for which the trial solutions for m_t and p_t are nonunique were considered nominally indeterminate. McCallum focuses on the conditions under which the intercept coefficients of p_t and m_t, which are Π_{10} and Π_{20}, respectively, fail to be uniquely determined, because this is the manner in which indeterminacy reveals itself in trial solutions. Such analysis is invalid, however, because it depends upon assumptions about

the state vector that cannot be justified. The state vector can be determined only from analysis of the complete structural model. If the structural model or any of its components is unknown, then there is no assurance either that any particular state vector is adequate or that it is consistent with the structure.[4] Demonstrations that the reduced forms for money and prices are indeterminate in trial solutions are irrelevant in an analysis of the completeness of a model, because they assume knowledge--about the state vector--that cannot be assumed if the completeness of the model is in doubt.

It may seem that the state vector assumption is innocuous. McCallum intends it not as a restriction on the structure, but rather as a restriction on the reduced form--one that lets the structure speak for itself. Yet, it already assumes that an infinitude of potential state variables have been excluded as arguments in the money supply function necessary to complete the model. Consider the general formulation for the money supply:

$$
\begin{aligned}
m_t = {} & \lambda_0+\lambda_1 r_t+\lambda_2 e_t \\
& +\theta_1^m m_{,-1}+\theta_2^m m_{t-2}+\ldots+\theta_j^m m_{,-1}+\ldots \\
& +\theta_1^p p_{t-1}+\theta_2^p p_{t-2}+\ldots+\theta_j^p p_{t-j}+\ldots \\
& +\theta_1^r r_{t-1}+\theta_2^r r_{t-2}+\ldots+\theta_j^r t_{t-j}+\ldots \\
& +\theta_1^e e_{t-1}+\theta_2^e e_{t-2}+\ldots+\theta_j^e e_{t-j}+\ldots \\
& +\tau_1 t+\tau_2 t^2+\ldots+\tau_j t^J+\ldots.
\end{aligned}
$$

Notice that r_t and e_t--the current state variables--must be arguments in the money supply rule in order for the interest rate rule to be feasible. There is a feasible interest rate rule associated with any set of zero restrictions on the θs and τs. If all lags of state variables and all time trends are excluded arbitrarily, then the structure is being restricted

such that all θs and τs in the money supply function are zero. But if this restriction can be made, it is because the money supply function is known to be of the form shown and because it is further known that the relevant parameters are zero. The fact that variables such as p_{t-1}, m_{t-1}, r_{t-1}, e_{t-1}, or t do not appear in either the nonpolicy equations (6) and (8) or in the interest rate rule (17) does not mean that such variables might not be state variables; that issue can be answered only after the money supply function has been made explicit to complete the model.

VII. Reinterpretation of a Result Due to McCallum

McCallum's 1981 article sought to show how an interest rate rule could be consistent with determined values of nominal variables, so long as the interest rate was set in order to have some desired effect on the money stock. Because his analysis is influential, some reconciliation between his analysis and that of this article is called for.

McCallum assumed that "...the monetary authorities adopt a feedback rule for the interest rate...." (1981, p. 319) It is not clear whether this means that the authorities make the interest rate predetermined through choice of an appropriate money supply function, or whether it means that the interest rate itself is a policy instrument or a direct choice variable. In either case, the assumption runs into problems discussed above concerning the feasibility of an interest rate rule. But even though such a rule should not be considered a policy rule, and even though an interest rate rule is infeasible, McCallum's analysis can still be considered for what it says about whether interest rate rules (however they may come into force) complete the model in the sense of rendering nominal magnitudes

determinate. It was this issue that Sargent and Wallace explicitly addressed.

The following discussion presents McCallum's assumptions about monetary policy in a simplified form, but one essentially adequate to the issues at hand. The original analysis involved additional, nonessential dynamic elements, with money targets depending on the lagged state and with the interest rate rule calling for smoothing of interest rate movements.

McCallum motivated the interest rate rule in the following way. The authorities, wishing to exert effective control over the money stock, choose the interest rate in a manner consistent with a predetermined money target. Letting the money target be a fixed value, μ_0, for simplicity, the authorities then set R_t in such a way that

$$E_{t-1}m_t=\mu_0 \tag{24}$$

via some appropriately chosen interest rate rule. Such an interest rate rule can be found by setting the expectation of money demand equal to the money target, μ_0:

$$E_{t-1}m_t=E_{t-1}\{p_t+\alpha_0-\alpha R_t\}=\mu_0, \tag{25}$$

which implies

$$E_{t-1}R_t=(1/\alpha)\{E_{t-1}p_t+\alpha_0-\mu_0\}. \tag{26}$$

The interest rate rule that achieves this is then

$$R_t=(1/\alpha)\{E_{t-1}p_t+\alpha_0-\mu_0\}. \tag{27}$$

McCallum regarded this last equation as the specification of policy behavior.

Given these assumptions, there is a well-defined equilibrium for all endogenous variables in terms of a trial solution in the minimal state vector. From this, McCallum concludes that an interest rate rule is consistent with nominal determinacy.

However, this line of reasoning does not provide an argument that

interest rate rules are consistent with nominal determinacy, because the conclusion is essentially among the premises. McCallum's demonstration hypothesizes that a nominal variable--the money stock--is rendered determinate by appropriate choice of the interest rate rule without actually explaining how that could come about. Then it is shown that this assumption is adequate to completely determine the solution to the model, including a unique bubble-free price level.

Formally, McCallum's argument has the following essential structure.

Hypothesis:

(H1) There exists a solution in terms of the state vector S_t:

$$p_t=\Pi_{10}+\Pi_{11}r_t+\Pi_{12}e_t \tag{11}$$

$$m_t=\Pi_{20}+\Pi_{21}r_t+\Pi_{22}e_t \tag{12}$$

$$R_t=\Pi_{30}+\Pi_{31}r_t+\Pi_{32}e_t \tag{13}$$

$$E_{t-1}p_{t+1}=\Pi_{10} \tag{14}$$

(H2) The interest rate obeys an interest rate rule (a direct implication of (27)):

$$\Pi_{31}=\Pi_{32}=0$$

(H3) The prior expectation of money is predetermined (in this example, an exogenous constant):

$$E_{t-1}m_t=\mu_0 \tag{24}$$

(H4) The nonpolicy structure:

$$R_t=r_t+(E_{t-1}p_{t+1}-p_t) \tag{6}$$

$$m_t-p_t=\alpha_o-\alpha R_t+e_t \tag{8}$$

Conclusion:

(C1) All the Π_{ij}s are uniquely determined, including Π_{10} and Π_{20}. Since the Π_{ij}s are well-defined, no indeterminacies arise.

If it is intended to prove that an interest rate rule is feasible, and some might interpret McCallum's argument in that way, then it obviously should not make assumption H2 of the hypothesis. As stated above, however, McCallum's argument can also be examined for what it says about completeness and nominal determinacy, which is a conceptually distinct issue. But if the argument is intended to prove that an interest rate rule completes the model in the sense of rendering all nominal magnitudes determinate, and of course the argument is used for that purpose, then it should not make assumptions H1 or H3. Assumptions H1 and H3 directly imply $\Pi_{20}=\mu_0$. This, in conjunction with the money demand equation, (8), implies that $\Pi_{10}=\mu_0-\alpha_0$. Hence, if Π_{20} is determined -- which it is by the assumption H3--then Π_{10} is also determinate. But if nominal indeterminacy is to occur, it will apply to both money and prices. Hence, a convincing argument would not assume the determinacy of either m_t or p_t, but rather would show how their determinacy can come about under an interest rate rule.

The idea that the interest rate can be manipulated for the control of money--the idea of the interest rate as an instrument for the control of money--is based on an implicit confusion between real and nominal quantities. In view of the money demand function--which is a relation between real money balances, $\exp(m_t-p_t)$, the interest rate, and a demand shock--variations in the interest rate have their effect on the real quantity of money, not on the nominal quantity. The money demand function implies no necessary relation between the nominal interest rate and the nominal quantity of money. If such

a necessary relation did exist, then it would be possible to describe a method by which the assumption H3 could be implemented.

As it is, there is no explanation offered of how this control over the expectation of money is to be achieved. What, then, is the specification of policy underlying the determinate solution and the interest rate rule? If it is assumed that some money supply function does complete the model, then it is possible to infer that this money supply function is

$$m_t=\lambda_0+r_t+e_t. \tag{28}$$

Consider the "policy rule," (27). When the solution for $E_{t-1}p_t=\Pi_{10}$ is substituted into it, the result is

$$R_t = 0.^5 \tag{27'}$$

In other words, the "policy rule" boils down to a statement that the interest rate is predetermined. But it has already been argued in section IV that this result is feasible only if the authorities can observe the full current state and effect the money supply function (28).

Formally, if H5 is added to the hypothesis, then C2 can be added to the conclusion, where H5 and C2 are as follows:

(H5) The model is complete (a money supply function exists).

(C2) The money supply function is (28).

Buttressing this conclusion is that the Π_{ij}s in the solution are precisely those that occur under the money supply function (28).

It seems, then, that the idea that the interest rate is used as an instrument or lever for indirect control over money plays no effective role in rendering prices or money determinate. Instead, determinacy arises because sufficient side restrictions, for example, H1 and H3, have been made that substitute perfectly for the assumption that the money supply function is (28), thus completing the model and providing unique solutions for all

variables.

Canzoneri, Henderson, and Rogoff (1983) also make sufficient side assumptions--that money has a fixed, predetermined trend--to make the model complete. McCallum (1986) provided a demonstration that the fixed-trend assumption is not implied by the interest rate rule. This interesting point is a special case of the more general proposition that few restrictions on the money supply function or on the behavior of nominal variables are implied by the incomplete system {(6),(8),(17)}.

VIII. Summary of Conclusions

Interest rate rules are infeasible.

An interest rate rule predetermines the interest rate; that is, an interest rate rule makes the interest rate a strict function of the lagged state. If the interest rate is to be rendered predetermined, then the money supply must somehow be able to offset the effects of current shocks on the interest rate. The interest rate rule is infeasible because economic disturbances are not contemporaneously observable by the authorities, so that the interest-rate-stabilizing money supply response to those disturbances cannot be forthcoming on the timely basis required. Instead, the authorities must depend on movements in the interest rate itself to convey the underlying disturbances affecting the interest rate. Changes in the money supply can be conditioned on variations in the current interest rate to any arbitrary, finite degree, but these money changes cannot absolutely eliminate interest rate movements, for then there would be no movements in the interest rate upon which to condition money supply movements in the first place. Hence, interest rate rules are infeasible unless the authorities can condition money

supply movements directly on the current economic disturbances. But they cannot, because they do not possess complete current information.

Interest rate rules fail to complete prototype macroeconomic models with rational expectations and a real money demand function.

If the policy design is to reflect any degree of concern about nominal variables, then--with the present state of macroeconomic science--the policy must place direct constraints on the nominal money supply. More precisely, there must be direct constraints placed on the relation between the nominal money stock and the observed state of the economy. But any completely specified policy can be formulated in terms of such a money supply function.

Suppose, for argument, that an interest rate rule is feasible. An interest rate rule predetermines the interest rate. This means that the relative price term in the real money demand function is predetermined. There is no way to manipulate this relative price so as to control the nominal money stock; it can be used only to influence the expectation of the real money stock. If only the real money stock, or ratio between nominal money and the price level, is determined, then neither nominal money nor the price level is determined.

Footnotes

1. The issues of this article are not materially affected if private agents are given some or all current information in forming future price expectations, or even if they are given perfect foresight.

2. This is not generally the case. In dynamic models with rational expectations, a state vector adequate to describe all interesting (non-bubble) solutions for a given, well-specified model will generally include state variables not explicitly present in the structural form. This point is discussed in Hoehn (1986).

3. McCallum (1986) is ambiguous as to whether he means that an interest rate rule is a money supply function with $\lambda=\infty$, or that an interest rate rule is the limiting case of a money supply function as λ approaches ∞. He clearly means at least one of these statements, and possibly both. The previous section argued that if $\lambda=\infty$, the money supply function is undefined so that any expressions dependent on it, such as the reduced form for the interest rate, are also undefined.

4. There seems to be no assurance of the existence of any finitely dimensioned state vector that is adequate to describe the position of the system, unless some sort of restriction on the money supply rule is made.

5. Equations (27) and (27') are particularly difficult to think of as means for controlling money, because they call for a fixed interest rate. McCallum introduced nonessential dynamic elements into the description of policy and derived a determinate solution for nominal variables that was consistent with it. Dynamic elements were thereby introduced into the determination of money and prices as well as the interest rate. But whether or not dynamic elements are introduced, there is no sense in which the interest rate rule exerts control over money or prices or renders either determinate.

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