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UNIVARIATE AND MULTIVARIATE ARIMA VERSUS VECTOR AUTOREGRESSION FORECASTING

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ABSTRACT

Key words: univariate ARIMA, multivariate ARIMA, vector autoregressions, forecasting performance

The purposes of this study are two: 1) to compare the forecasting abilities of the three methods: univariate autoregressive integrated moving average (ARIMA), multivariate autoregressive integrated moving average (MARIMA), and vector autoregression (both unconstrained--VAR--and Bayesian--BVAR) and 2) to study the idea that one advantage of vector autoregressions is that the models can easily and inexpensively be reestimated after each additional data point. All of these methods have been shown to provide forecasts that are more accurate than many econometric methods, which require more resources to implement.

These methods were applied to seven economic variables: real GNP, annual inflation rates, unemployment rate, the money supply (M1), gross private domestic investment, the rate on four- to six-month commercial paper, and the change in business inventories. The major results of this study are: 1) on average, the method that performs best in terms of the root mean square error (RMSE) is the multivariate ARIMA model; 2) the univariate ARIMA and BVAR methods perform approximately the same on average; 3) reestimating the VAR model after each data point increases the accuracy of this method; 4) reestimating the BVAR model after each data point becomes available decreases the accuracy of this method; and 5) the VAR method using reestimation is approximately as accurate as the BVAR method.

UNIVARIATE AND MULTIVARIATE ARIMA VERSUS VECTOR AUTOREGRESSION FORECASTING

I. Introduction

The main purpose of this research is to compare forecasts from three popular time series methods of forecasting: ARIMA, MARIMA, and VAR-BVAR. As part of this effort, we examine the problem of whether the VAR and the BVAR models should be reestimated after every new data point. The ability to reestimate these models easily and inexpensively has been claimed as one advantage of these models compared to ARIMA and MARIMA models. However, there seems to be a lack of studies concerning this aspect of VAR and BVAR models.

The data used in this study are quarterly observations on the following variables: real GNP (RGNP), the GNP deflator (INFLA), the unemployment rate (UNEMP), the money supply (M1), gross private domestic investment (INVEST), the rate on four- to six-month commercial paper (CPRATE), and the change in business inventories (CBI). The models used in this study were actually estimated in the change in the log of RGNP, the change in the log of INFLA, UNEMP, the log of M1, the log of INVEST, CPRATE, and CBI over the time period 1948:IQ through 1979:IVQ. (The starting point of the estimation time period was adjusted for the lags in the corresponding models.) The models were then used to forecast the log of RGNP (LRGNP), the log of INFLA (LINFLA), the log of M1 (LM1), UNEMP, the log of INVEST (LINVEST), CPRATE, and CBI.

II. Time Series Models

The following is a very brief description of the time series models used in this study. The univariate ARIMA models are discussed in detail in Box and

Jenkins (1976). Tiao and Box (1981) provide a more detailed description of the multivariate ARIMA models, and Litterman (1986) and Doan, Litterman, and Sims (1984) discuss the VAR and BVAR models. All of these models are particular versions of the general time series model of order (p,q) given by:

(1) $$\underline{\Phi}_p(B)\underline{z}_t = \underline{\Theta}_q(B)\underline{a}_t + \underline{\theta}_0,$$

where

(2) $$\underline{\Phi}_p(B) = \underline{I} - \underline{\phi}_1 B - \ldots - \underline{\phi}_p B^p,$$
$$\underline{\Theta}_q(B) = \underline{I} - \underline{\theta}_1 B - \ldots - \underline{\theta}_q B^q,$$

and

B = backshift operator (e.g., $B^s z_{i,t} = z_{i,t-s}$),

$\underline{I}$ = k x k identity matrix,

$\underline{z}$ = vector of k variables in the model,

$\underline{\phi}_j$'s and $\underline{\theta}_j$'s = k x k matrixes of unknown parameters,

$\underline{\theta}_0$ = k x 1 vector of unknown parameters, and

$\underline{a}$ = k x 1 vector of random errors that are identically and independently distributed as $N(0,\underline{\Sigma})$.

Thus, it is assumed that the $a_{j,t}$'s at different points in time are independent, but not necessarily that the elements of $\underline{a}_t$ are independent at a given point in time.

The univariate models use only past history of the individual series being modeled. Thus, they do not use any information from other series that may be related to the series being forecast. The MARIMA, VAR, and BVAR models use information from other related series to attempt to obtain better forecasts. These models differ in how they model the relationships among the series. Both VAR and BVAR assume that the relationships can be approximated by using only autoregressive components of the more general autoregressive integrated moving average (ARIMA) models. The difference between the VAR and the BVAR models is in the method of estimating the models rather than in their form.

The n-period-ahead forecasts from these models at time t ($\underline{z}_t(n)$) are given by:

$$(3) \qquad \underline{z}_t(n) = \underline{\phi}_1[\underline{z}_{t+n-1}] + \ldots + \underline{\phi}_p[\underline{z}_{t+n-p}] + [\underline{a}_{t+n}] - \underline{\theta}_1[\underline{a}_{t+n-1}] - \ldots - \underline{\theta}_q[\underline{a}_{t+n-q}],$$

where, for any value of t,n,m, $[\underline{x}_{t+n-m}]$ implies the conditional expected values of the random variables $\underline{x}_{t+n-m}$ at time t. If n-m is less than or equal to zero, then the conditional expected values are the actual values of the random variables and the error terms. If n-m is greater than zero, then the expected values are the best forecasts available for these random variables and error terms at time t. Because the error terms are uncorrelated with present and past information, the best forecasts of the error terms for n-m greater than zero are their conditional means, which are zero. The forecasts can be generated iteratively with the one-period-ahead forecasts that depend only on known values of the variables and error terms. The longer-length forecasts, in turn, depend on the shorter-length forecasts.

III. Development of Models for Forecasting

Because we wish to test which method provides better forecasts, we divided the data into two periods. The data from 1947:IQ through 1979:IVQ were used to estimate the models for each method with adjustments in the starting period for the lags involved in the corresponding models. The last 26 observations (from 1980:IQ through 1986:IIQ) were used to test the forecast accuracy of these models in terms of root mean square error (RMSE) of the forecasts for one to eight quarters ahead.

The ARIMA models were developed using the method of Box and Jenkins (1976). This is an iterative method that involves: 1) tentatively identifying a model by examining autocorrelations of the series; 2) estimating the parameters of this model; and 3) applying diagnostic checks to the residuals. If the residuals do not pass the diagnostic checks, then the tentative model is modified, and steps two and three are repeated. This process continues until a satisfactory model is obtained.

For the MARIMA model, we developed a model by using the method of Tiao and Box (1981). This method is similar to that of the Box and Jenkins method for univariate models, except that cross-correlations between the series are added and modeled for.

For the VAR and BVAR models, we used six lags of each variable and a constant in each equation. In the BVAR models, several parameters specify the prior distribution used in the estimation process. To specify these parameters, we used the "Minnesota" prior as identified in the RATS program from VAR Econometrics.

Because of the large number of parameters involved in the models, we do not present the models in this paper. The models can be obtained from the author on request.

IV. Forecasting Results

The models developed for this study were used to forecast the seven variables for a forecast horizon of up to eight quarters over the period of 1980:IQ through 1986:IIQ. These were actual forecasts and did not use any information within the forecast horizon. Thus, the number of forecasts we have for each forecast length varies. For one-quarter-ahead forecasts, we have 26 observations; for two quarters ahead, we have 25 observations, etc. For the purposes of this study, we calculated the root mean square error (RMSE) as a measure of forecast accuracy. The results are presented in tables 1 through 7. Tables 8 through 14 present the corresponding ranks for the different methods.

LRGNP

For LRGNP, the best method for one- to six-quarters-ahead forecasting was the MARIMA. For seven and eight quarters, the best model was the static VAR. The BVAR model was second best for forecast horizons of one, three, seven, and eight. The UARIMA model was generally fifth or sixth except for the one-quarter-ahead forecast, in which it was third. Table 15 presents the average rank of the six methods by variable over all eight lags. From this table, we see that the MARIMA method has the smallest average rank, 1.88, with the static VAR model second at 2.38.

LINFLA

For LINFLA, the best method for all forecast lengths was the UARIMA model. The reestimated BVAR model was second best for one- to four-quarter and six-quarter forecasts. The reestimated VAR was second best for five-, seven-, and eight-quarter forecasts. The method with the best average rank was the UARIMA model, with the reestimated BVAR second best. For this variable, the reestimation for both the BVAR and the VAR models provided an increase in accuracy compared to the static models. The MARIMA was the worst predictor for this variable.

LM1

For LM1, the MARIMA method was best for one- to five-quarter forecasts, the UARIMA method was best for six- and seven-quarter forecasts, and the reestimated BVAR was best for eight-quarter forecasts. The MARIMA method was best in terms of average rank over the eight quarters, with the static BVAR second. Reestimation of the VAR and BVAR models lowered the accuracy of the forecast on average.

UNEMP

For UNEMP, the MARIMA method was best for one- to seven-quarter forecasts with the reestimated VAR best for eight-quarter forecasts. The best method in terms of average rank was the MARIMA method. Reestimation increased the accuracy of the VAR method but decreased the accuracy of the BVAR on average.

LINVEST

For LINVEST, the best methods were UARIMA for one-quarter forecasts, the reestimated VAR for two- and three-quarter forecasts, and MARIMA for four- to eight-quarter forecasts. On average, the MARIMA method was best. Reestimation improved the accuracy of both the VAR and BVAR methods. The reestimated VAR was more accurate than the reestimated BVAR method.

CPRATE

For CPRATE, the best methods were static VAR for one- to four-quarter forecasts, MARIMA for five-quarter forecasts, and UARIMA for six- to eight-quarter forecasts. On average, the best forecast method was the MARIMA method. Reestimation decreased the accuracy of both the VAR and BVAR methods.

CBI

For CBI, the best methods were MARIMA for one-quarter forecasts, static VAR for two- and five-quarter forecasts, and reestimated VAR for six- to eight-quarter forecasts. Reestimation increased the accuracy of the VAR method but decreased the accuracy of the BVAR method.

The overall average ranks for the six methods are:

UARIMA	3.62
MARIMA	2.75
VAR	
Static	4.00
Reestimated	3.31
BVAR	
Static	3.36
Reestimated	3.93

Thus, the best method in terms of average rank over all seven variables and eight forecast lengths was the MARIMA method. The worst was the static VAR.

Table 16 presents the average ranks by lag for the six methods. From this table, we see that the best method for one- and three- to six-quarter forecasts was the MARIMA method (with the reestimated VAR tied for first for seven-quarter forecasts). The reestimated VAR method was best for two-quarter forecasts. The MARIMA and reestimated VAR were tied for seven-quarter forecasts. The UARIMA method provided the best forecasts for eight-quarter forecasts.

V. Specific Comparisons

In this section, we compare the methods based on two criteria: 1) the number of forecast horizons in which one method forecasts better than the other by variable; and 2) the average rank (when only the two methods being compared are included in the ranking) over the seven variables by forecast horizons.

UARIMA versus MARIMA

For five of the seven variables (LRGNP, LM1, UNEMP, LINVEST, AND CPRATE), the MARIMA method provided better forecasts than the UARIMA for a majority of the forecast horizons. The MARIMA method also provided better forecasts for one- to five-quarter horizons than the UARIMA. Thus, on average, the MARIMA method did better than the UARIMA method.

UARIMA versus VAR (static)

The UARIMA method provided better forecasts than the static VAR method for three variables (LINFLA, LM1, AND LINVEST), while the static VAR provided better forecasts for three variables (LRGNP, CPRATE, and CBI) in terms of number of forecast horizons. The two methods tied for UNEMP. For the forecast horizons, the UARIMA method provided better forecasts for one through four and six through eight quarters, while the static VAR method provided better forecasts for only the five-quarter horizon. Thus, when compared by variable, the two methods tied; but when compared by forecast horizon, the UARIMA method was better.

UARIMA versus VAR (reestimated)

The reestimated VAR method provided better forecasts for four of the variables (LRGNP, UNEMP, LINVEST, AND CBI), the UARIMA method was better for two (LINFLA and LM1), and the two methods tied for CPRATE. In terms of forecast horizons, the reestimated VAR method provided better forecasts for one through seven quarters. The reestimated VAR was thus a better forecast method, on average, than the UARIMA method.

UARIMA versus BVAR (static)

The UARIMA method forecasted better than the static BVAR method for four variables (LINFLA, LM1, LINVEST, and CBI) and for four- through eight-quarter horizons. These two methods thus provided very similar performance, with the UARIMA being slightly better.

UARIMA versus BVAR (reestimated)

The UARIMA method forecasted better than the reestimated BVAR method for four variables (LINFLA, LM1, CPRATE, AND CBI) and for two- through eight-quarter horizons. The UARIMA method was thus slightly better than the reestimated BVAR method.

MARIMA versus VAR (static)

The MARIMA method forecasted better than the static VAR for four variables (LRGNP, LM1, UNEMP, and LINVEST), and there was a tie for CPRATE.

The MARIMA method provided better forecasts for all forecast horizons. Thus, the MARIMA method was a better forecast method than the static VAR.

MARIMA versus VAR (reestimated)

The MARIMA method forecasted better than the reestimated VAR for four variables (LRGNP, LM1, UNEMP, and LINVEST), tied for CPRATE, and forecasted better for one-quarter and three- through seven-quarter horizons. The MARIMA method was thus the better forecast method.

MARIMA versus BVAR (static)

The MARIMA method forecasted better than the static BVAR for six of the seven variables (all but LINFLA) and for one- through six-quarter horizons. Thus, the MARIMA method was substantially better than the static BVAR method.

MARIMA versus BVAR (reestimated)

The MARIMA method forecasted better than the reestimated BVAR for six of the seven variables (all but LINFLA) and for all forecast horizons. Thus, the MARIMA method was substantially better than the reestimated BVAR method.

VAR (static) versus VAR (reestimated)

The reestimated VAR method forecasted better than the static VAR method for three variables (LINFLA, UNEMP, and LINVEST), tied for CPRATE, and forecasted better for one- through four-quarter and six- through eight-quarter forecast horizons. Thus, the reestimated VAR was slightly better than the static VAR method.

VAR (static) versus BVAR (static)

The static BVAR method forecasted better than the static VAR method for four variables (LINFLA, LM1, UNEMP, and LINVEST) and for one- through five-quarter and seven- through eight-quarter forecast horizons, with a tie for the six-quarter horizon. Thus, the static BVAR method provided better forecasts than the static VAR method.

VAR (static) versus BVAR (reestimated)

The reestimated BVAR method forecasted better than the static VAR method for four variables (LINFLA, LM1, UNEMP, and LINVEST) and for all forecast horizons, with a tie for the six-quarter horizon. Thus, the reestimated BVAR method provided better forecasts than the static VAR method.

VAR (reestimated) versus BVAR (static)

The reestimated VAR forecasted better than the static BVAR for three variables (LINFLA, LINVEST, and CBI), tied for LINVEST, and forecasted better for two- through four-quarter and six- through eight-quarter horizons. Thus, these two methods provided similar forecast performance, with the reestimated VAR slightly better in terms of forecast horizons.

VAR (reestimated) versus BVAR (reestimated)

The reestimated VAR forecasted better than the reestimated BVAR for five variables (LRGNP, UNEMP, LINVEST, CPRATE, and CBI) and for all forecast horizons. Thus, the reestimated VAR was a better forecast method than the reestimated BVAR.

BVAR (static) versus BVAR (reestimated)

The static BVAR method forecasted better than the reestimated BVAR method for five variables (LRGNP, LM1, UNEMP, CPRATE, and CBI) and for all forecast horizons. Thus, the static BVAR method provided better forecasts than the reestimated BVAR method.

VI. Summary

For the variables used in this study, we have obtained the following results: 1) the MARIMA method provided better forecasts than any of the methods considered in this study; 2) the UARIMA method provided better forecasts than the static VAR method; 3) the reestimated methods provided better forecasts than the UARIMA method; 4) the UARIMA method provided slightly better forecasts than either the static or reestimated BVAR methods; 5) the reestimated VAR provided slightly better forecasts than the static VAR; and 6) the reestimated BVAR method provided worse forecasts than the static BVAR method.

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Table 1. Comparison of Root Mean Square Forecast Errors - LRGNP

Method	Forecast Horizon in Quarters (number of observations)							
	1 (26)	2 (25)	3 (24)	4 (23)	5 (22)	6 (21)	7 (20)	8 (19)
Univariate ARIMA	.0105	.0182	.0249	.0308	.0369	.0422	.0471	.0520
Multivariate ARIMA	.0089	.0155	.0148	.0179	.0231	.0321	.0421	.0527
Vector Autoregression								
Static	.0131	.0159	.0222	.0264	.0290	.0328	.0364	.0416
Reestimated	.0113	.0163	.0257	.0295	.0342	.0366	.0421	.0454
Bayesian Vector Autoregression								
Static	.0104	.0165	.0220	.0265	.0318	.0357	.0390	.0432
Reestimated	.0107	.0175	.0239	.0294	.0358	.0406	.0449	.0502

Table 2. Comparison of Root Mean Square Forecast Errors - LINFLA

Method	Forecast Horizon in Quarters (number of observations)							
	1 (26)	2 (25)	3 (24)	4 (23)	5 (22)	6 (21)	7 (20)	8 (19)
Univariate ARIMA	.0034	.0065	.0099	.0140	.0177	.0208	.0242	.0271
Multivariate ARIMA	.0100	.0233	.0380	.0542	.0716	.0895	.1083	.1266
Vector Autoregression								
Static	.0118	.0194	.0315	.0409	.0559	.0680	.0800	.0891
Reestimated	.0091	.0155	.0225	.0295	.0379	.0465	.0573	.0649
Bayesian Vector Autoregression								
Static	.0068	.0165	.0279	.0408	.0543	.0680	.0822	.0963
Reestimated	.0045	.0108	.0185	.0281	.0388	.0503	.0629	.0763

Table 3. Comparison of Root Mean Square Forecast Errors - LM1

Method	Forecast Horizon in Quarters (number of observations)							
	1 (26)	2 (25)	3 (24)	4 (23)	5 (22)	6 (21)	7 (20)	8 (19)
Univariate ARIMA	.0121	.0173	.0186	.0236	.0260	.0274	.0291	.0282
Multivariate ARIMA	.0093	.0135	.0162	.0208	.0253	.0281	.0307	.0327
Vector Autoregression								
Static	.0113	.0214	.0285	.0316	.0345	.0378	.0405	.0435
Reestimated	.0116	.0186	.0240	.0290	.0353	.0425	.0495	.0517
Bayesian Vector Autoregression								
Static	.0111	.0162	.0188	.0238	.0264	.0282	.0294	.0277
Reestimated	.0114	.0168	.0198	.0252	.0279	.0295	.0301	.0272

Table 4. Comparison of Root Mean Square Forecast Errors - UNEMP

Method	Forecast Horizon in Quarters (number of observations) 1 (26)	2 (25)	3 (24)	4 (23)	5 (22)	6 (21)	7 (20)	8 (19)
Univariate ARIMA	.3948	.8400	1.2821	1.7236	2.1464	2.4978	2.7907	3.0399
Multivariate ARIMA	.2375	.3828	.4946	.6317	.8139	.9682	1.1847	1.3963
Vector Autoregression								
Static	.6130	1.0561	1.5024	1.9118	2.0114	2.1124	2.0428	1.7813
Reestimated	.3922	.6765	.9507	1.1807	1.2778	1.2691	1.2963	1.1841
Bayesian Vector Autoregression								
Static	.3561	.6566	.9059	1.1110	1.2734	1.3833	1.4726	1.5441
Reestimated	.3657	.6936	.9882	1.2472	1.4605	1.6128	1.7388	1.8387

Table 5. Comparison of Root Mean Square Forecast Errors - LINVEST

Method	Forecast Horizon in Quarters (number of observations)							
	1 (26)	2 (25)	3 (24)	4 (23)	5 (22)	6 (21)	7 (20)	8 (19)
Univariate ARIMA	.0617	.1080	.1390	.1681	.1924	.2067	.2197	.2297
Multivariate ARIMA	.0693	.1105	.1255	.1221	.1211	.1389	.1651	.1915
Vector Autoregression								
Static	.0804	.1385	.1801	.2039	.2385	.2913	.3416	.3903
Reestimated	.0701	.0997	.1181	.1264	.1299	.1600	.1928	.2405
Bayesian Vector Autoregression								
Static	.0628	.1118	.1441	.1710	.1939	.2134	.2310	.2466
Reestimated	.0624	.1061	.1354	.1617	.1834	.1985	.2118	.2219

Table 6. Comparison of Root Mean Square Forecast Errors - CPRATE

Method	Forecast Horizon in Quarters (number of observations)							
	1 (26)	2 (25)	3 (24)	4 (23)	5 (22)	6 (21)	7 (20)	8 (19)
Univariate ARIMA	1.8513	2.7046	2.7298	3.1601	3.3749	3.3921	3.5630	3.3606
Multivariate ARIMA	1.6583	2.4154	2.5584	2.9660	3.3179	3.4421	3.6318	3.5465
Vector Autoregression								
Static	1.4220	1.8478	1.8916	2.7020	3.3343	3.4835	3.7579	3.9126
Reestimated	1.5863	2.1981	2.2703	2.8068	3.5251	3.7273	4.3016	4.7924
Bayesian Vector Autoregression								
Static	1.6710	2.4037	2.7153	3.1013	3.3328	3.4908	3.6093	3.5020
Reestimated	1.7037	2.4957	2.8300	3.2836	3.5925	3.8103	3.9987	3.8622

Table 7. Comparison of Root Mean Square Forecast Errors - CBI

Method	Forecast Horizon in Quarters (number of observations)							
	1 (26)	2 (25)	3 (24)	4 (23)	5 (22)	6 (21)	7 (20)	8 (19)
Univariate ARIMA	25.8302	32.5999	33.1308	32.4058	32.8064	33.5075	34.3202	34.9580
Multivariate ARIMA	22.9790	30.0738	33.2789	33.3005	33.3776	34.1595	35.0898	35.9638
Vector Autoregression								
Static	27.8589	26.5161	26.7075	25.5477	24.1644	27.1594	31.5004	32.9816
Reestimated	24.7857	27.5193	28.3064	29.1183	26.9541	24.9368	24.9646	28.4324
Bayesian Vector Autoregression								
Static	23.8216	29.6996	31.9864	33.9975	34.7556	34.7826	35.5538	35.9068
Reestimated	24.5126	31.4291	34.4877	37.8798	39.1500	38.2419	38.5923	37.3361

Table 8. Rankings of the Different Methods in Terms of RMSE for LRGNP

Method	Forecast Horizon in Quarters (number of observations)							
	1 (26)	2 (25)	3 (24)	4 (23)	5 (22)	6 (21)	7 (20)	8 (19)
Univariate ARIMA	3	6	5	6	6	6	6	5
Multivariate ARIMA	1	1	1	1	1	1	3	6
Vector Autoregression								
Static	6	2	3	2	2	2	1	1
Reestimated	5	3	6	5	4	4	3	3
Bayesian Vector Autoregression								
Static	2	4	2	3	3	3	2	2
Reestimated	4	5	4	4	5	5	5	4

Table 9. Rankings of the Different Methods in Terms of RMSE for LINFLA

Method	Forecast Horizon in Quarters (number of observations) 1 (26)	2 (25)	3 (24)	4 (23)	5 (22)	6 (21)	7 (20)	8 (19)
Univariate ARIMA	1	1	1	1	1	1	1	1
Multivariate ARIMA	5	6	6	6	6	6	6	6
Vector Autoregression								
Static	6	5	5	5	5	4	4	4
Reestimated	4	3	3	3	2	3	2	2
Bayesian Vector Autoregression								
Static	3	4	4	4	4	4	5	5
Reestimated	2	2	2	2	3	2	3	3

Table 10. Rankings of the Different Methods in Terms of RMSE for LM1

Method	Forecast Horizon in Quarters (number of observations) 1 (26)	2 (25)	3 (24)	4 (23)	5 (22)	6 (21)	7 (20)	8 (19)
Univariate ARIMA	6	4	2	2	2	1	1	3
Multivariate ARIMA	1	1	1	1	1	2	4	4
Vector Autoregression								
Static	3	6	6	6	5	5	5	5
Reestimated	5	5	5	5	6	6	6	6
Bayesian Vector Autoregression								
Static	2	2	3	3	3	3	2	2
Reestimated	4	3	4	4	4	4	3	1

Table 11. **Rankings** of the Different Methods in Terms of RMSE for UNEMP

Method	Forecast Horizon in Quarters (number of observations)							
	1 (26)	2 (25)	3 (24)	4 (23)	5 (22)	6 (21)	7 (20)	8 (19)
Univariate ARIMA	5	5	5	5	6	6	6	6
Multivariate ARIMA	1	1	1	1	1	1	1	2
Vector Autoregression								
Static	6	6	6	6	5	5	5	4
Reestimated	4	4	3	3	3	2	2	1
Bayesian Vector Autoregression								
Static	2	2	2	2	2	3	3	3
Reestimated	3	3	4	4	4	4	4	5

Table 12. Rankings of the Different Methods in Terms of RMSE for LINVEST

Method	Forecast Horizon in Quarters (number of observations)							
	1 (26)	2 (25)	3 (24)	4 (23)	5 (22)	6 (21)	7 (20)	8 (19)
Univariate ARIMA	1	3	4	4	4	4	5	3
Multivariate ARIMA	4	4	2	1	1	1	1	1
Vector Autoregression								
Static	6	6	6	6	6	6	6	6
Reestimated	5	1	1	2	2	2	2	4
Bayesian Vector Autoregression								
Static	3	5	5	5	5	5	4	5
Reestimated	2	2	3	3	3	3	3	2

Table 13. Rankings of the Different Methods in Terms of RMSE for CPRATE

Method	Forecast Horizon in Quarters (number of observations)							
	1 (26)	2 (25)	3 (24)	4 (23)	5 (22)	6 (21)	7 (20)	8 (19)
Univariate ARIMA	6	6	5	5	4	1	1	1
Multivariate ARIMA	3	4	3	3	1	2	3	3
Vector Autoregression								
Static	1	1	1	1	3	3	4	5
Reestimated	2	2	2	2	5	5	6	6
Bayesian Vector Autoregression								
Static	4	3	4	4	2	4	2	2
Reestimated	5	5	6	6	6	6	5	4

Table 14. Rankings of the Different Methods in Terms of RMSE for CBI

Method	Forecast Horizon in Quarters (number of observations) 1 (26)	2 (25)	3 (24)	4 (23)	5 (22)	6 (21)	7 (20)	8 (19)
Univariate ARIMA	5	6	4	3	3	3	3	3
Multivariate ARIMA	1	4	5	4	4	4	4	5
Vector Autoregression								
Static	6	1	1	1	1	2	2	2
Reestimated	4	2	2	2	2	1	1	1
Bayesian Vector Autoregression								
Static	2	3	3	6	5	5	5	4
Reestimated	3	5	6	5	6	6	6	6

Table 15. Average Rank by Variable

Method	LRGNP	LINFLA	LM1	UNEMP	LINVEST	CPRATE	CBI
Univariate ARIMA	5.38	1.00	2.63	5.50	3.50	3.63	3.75
Multivariate ARIMA	1.88	5.88	1.88	1.13	1.88	2.75	3.88
Vector Autoregression							
Static	2.38	4.75	5.13	5.38	6.00	2.38	2.00
Reestimated	4.13	2.75	5.50	2.75	2.38	3.75	1.88
Bayesian Vector Autoregression							
Static	2.63	4.13	2.50	2.38	4.63	3.13	4.13
Reestimated	4.50	2.38	3.38	3.88	2.63	5.38	5.38

Table 16. Average Rank by Lag

	Forecast Horizon in Quarters							
Method	1	2	3	4	5	6	7	8
Univariate ARIMA	3.86	4.43	3.71	3.71	3.71	3.14	3.29	3.14
Multivariate ARIMA	2.29	3.00	2.71	2.43	2.14	2.43	3.14	3.86
Vector Autoregression								
Static	4.86	3.86	4.00	3.86	3.86	3.86	3.86	3.86
Reestimated	4.14	2.86	3.14	3.14	3.43	3.29	3.14	3.29
Bayesian Vector Autoregression								
Static	2.57	3.29	3.29	3.86	3.43	3.86	3.29	3.29
Reestimated	3.29	3.57	4.14	4.00	4.43	4.29	4.14	3.57