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ESTIMATING TOTAL FACTOR PRODUCTIVITY
IN A GENERALIZED COST SYSTEM

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I. Introduction

Measuring the impact of regulation on electric utilities is of considerable interest to economists and regulators. Courville (1974), Spann (1974), Peterson (1975), Cowing (1978), and Nelson and Wohar (1983), for example, find evidence of an overcapitalization bias using variations of the Averch and Johnson (1962) model. These models are criticized by Atkinson and Halvorsen (1984) because the impact of additional regulatory constraints is ignored. Atkinson and Halvorsen (A-H) estimate a generalized cost model with a cross-section of electric utilities and find a regulatory bias on the utilization of noncapital factor inputs. Moreover, Fare and Logan (1983) show that a rate-of-return constraint alone invalidates the use of Shephard's Lemma for noncapital inputs.

Joskow (1974) argues that regulation may also bias the rate of technical change implemented by utilities. The only empirical paper to have considered such an impact is Nelson and Wohar. They find a regulatory impact on the total factor productivity (TFP) experienced by electric utilities during the late 1970s. However, in addition to considering only a rate-of-return constraint, their approach can be criticized because it considers the regulatory impact on TFP to be independent of the returns-to-scale and

technical-change components of TFP.¹ Because TFP is the sum of returns-to-scale and technical-change terms in a cost function framework, the Nelson and Wohar model implies a deus ex machina regulatory impact on TFP. Obviously, the authors cannot test for a regulatory impact on technical change.

In this paper we develop a model that can be used with time-series data to test for a regulatory impact on all the components of TFP. The foundation of our model is the A-H generalized cost function, modified with time variables to capture the dynamic aspect of TFP. To this, we add an equation for TFP. Although this equation is not necessary to test for the impact of regulation on TFP and its components, its use will presumably increase the efficiency of parameter estimates because it is additional behavioral information and because TFP is measured as a rate of change. Two alternative TFP equations are derived; one is considerably easier to estimate than the other.

In the next section of this paper, an alternative derivation of the A-H model is given. After that, the two TFP equations are formulated and discussed. Finally, the translog version of our generalized cost model is given.

1. The TFP equation in the Nelson and Wohar paper is:

$$\dot{W} = v_T + v_Q \dot{Q} + \lambda [\sum M_i \dot{P}_i - (sX_n/C) \dot{s}],$$

where \dot{W} = rate of change of TFP
 v_T = technical change
 v_Q = returns to scale
 Q = output
 M_i = i-th input share
 P_i = input price
 s = rate of return
 X = capital input
 C = cost.

II. The Atkinson and Halvorsen Model

Atkinson and Halvorsen show that general regulatory constraints alter the nature of the cost-minimizing, first-order conditions. Instead of equating the marginal costs of each factor input to its market price, a regulated firm finds it optimal to equate the marginal costs of each factor input to its shadow price. These shadow prices are market prices adjusted for the impact of regulation, and their specification depends on the exact nature of the regulatory constraints. Atkinson and Halvorsen approximate these shadow prices, P_i^* , with simple proportional relationships to market prices, $P_i^* = k_i P_i$, for each input i . The generalized, or shadow, cost function is simply the actual cost function, but with shadow prices instead of market prices. Because shadow costs are not observed, the shadow-cost function must be rewritten in terms of observable variables.

Accounting identities for the actual (C^a) and shadow (C^s) total costs are respectively:

$$(1) \quad C^a = \sum P_i X_i \text{ and}$$

$$C^s = \sum k_i P_i X_i,$$

where X_i is an input factor and P_i is its market price. The shadow-cost-share equations are:

$$M_i^s = \frac{k_i P_i X_i}{C^s} \quad \text{for each } i.$$

Instead of the traditional system of share equations, the following system can be derived:

$$(2) \quad P_i X_i = \frac{C^s M_i^s}{k_i}, \quad \text{for each } i.$$

The sum of these equations is:

$$(3) \quad C^a = C^s \sum (M_i^s / k_i),$$

and taking the logarithm of both sides yields:

$$(3') \quad \ln C^a = \ln C^s + \ln \sum (M_i^s / k_i).$$

Equation (3') is estimable after substituting the shadow translog cost function for $\ln C^s$ and the derived shadow cost shares $MQ = \partial \ln C^s / \partial \ln(k_i P_i)$.

Observable cost-share equations can be derived by dividing both sides of equation (2) by C^a :

$$(4) \quad M_i^a = \frac{C^s M_i^s}{C^a k_i},$$

where M_i^a is the actual cost share for input i , and by substituting equation (3) into equation (4) to obtain:

$$(5) \quad M_i^a = k_i^{-1} M_i^s / \sum k_i^{-1} M_i^s.$$

The A-H shadow-cost system is (3') and (5). Because the sum of the actual cost shares is one, one of the actual cost-share equations in equation (5) can be dropped.

III. The Total Factor Productivity Equations

The cost function employed by Atkinson and Halvorsen uses only shadow input factor prices and output as arguments. In order to estimate TFP using the A-H model, time must be added as an additional explanatory variable. The derivative of shadow cost with respect to time is then:

$$\frac{dC^s}{dT} = \sum \frac{\partial C^s}{\partial P_i^*} \frac{dP_i^*}{dT} + \frac{\partial C^s}{\partial Q} \frac{dQ}{dT} + \frac{\partial C^s}{\partial T}.$$

The first term on the right-hand side can be simplified by dividing both sides of the equation by C^s , by using Shephard's Lemma for the shadow cost

function (i.e., $(\partial C^s / \partial P_i^*) = X_i$), and by noting that k_i is constant.

Then multiply and divide the second term by Q to obtain:

$$(6) \quad \dot{C}^s = \sum M_i^s \dot{P}_i + v_Q^s \dot{Q} + v_T^s,$$

where the superscript \circ indicates the rate of change, for example $\dot{C}^s = (d \ln C^s / dT)$. v_Q^s is the shadow elasticity of shadow cost with respect to output. v_T^s is usually called technical change. It is the rate of change of shadow cost, holding constant all cost function arguments except time.

The rate of change in shadow TFP (W^s) can be defined similarly to that of actual TFP using a Divisia index of factor-input-shadow shares

$$W^s = \sum M_i^s \dot{X}_i.$$

Using the accounting identity (1)

$$(6') \quad \dot{C}^s = \sum M_i^s \dot{X}_i + \sum M_i^s \dot{P}_i,$$

W^s can be expressed as:

$$(7) \quad W^s = \dot{C}^s - \sum M_i^s \dot{P}_i = v_Q^s \dot{Q} + v_T^s.$$

Equation (7) is analogous to the equation derived by Nelson and Wohar for the rate of change in actual TFP (W^a):

$$(7') \quad W^a = \dot{C}^a - \sum M_i^a \dot{P}_i = v_Q^a \dot{Q} + v_T^a.$$

The important difference between equations (7) and (7') is that equation (7) was derived by applying Shephard's Lemma to shadow cost, while (7') uses Shephard's Lemma for actual cost.

The difference between W^a , defined in (7'), and W^s can be derived analytically. Substituting the identity $\dot{C}^a = \dot{C}^s + (C^a/C^s) \dot{C}^s$ into (7')

W^s . The advantage of using (12') is that it is considerably easier to estimate than (9). The disadvantage of using (12') is that, in case of $k_i = 1$, it does not use an actual value of W^a as in equation (7'). Instead, W^a derived from (12'), with $k_i = 1$, is $W^a = \hat{C}^a - \sum \hat{M}_i^a P_i$, where the \hat{M}_i^a are estimated actual cost shares.

IV. The Translog Specification of the Generalized Cost System

The translog form of the shadow-cost function is:

$$(13) \quad \ln C^s = \alpha_0 + \sum \beta_i \ln k_i P_i + \beta_Q \ln Q + \beta_T T \\ + .5 \sum \sum \gamma_{ij} \ln k_i P_i \ln k_j P_j + \sum \gamma_{iQ} \ln k_i P_i \ln Q \\ + \sum \gamma_{iT} (\ln k_i P_i) T + .5 \gamma_{QQ} (\ln Q)^2 + \gamma_{QT} (\ln Q) T + .5 \gamma_{TT} T^2.$$

The shadow-cost function is restricted to be linearly homogeneous with respect to shadow prices using the coefficient restrictions:

$$(13') \quad \sum \beta_i = 1, \quad \sum \gamma_{iQ} = 0, \quad \sum \gamma_{iT} = 0, \\ \sum \gamma_{ij} = 0, \text{ and } \gamma_{ij} = \gamma_{ji}.$$

The logarithmic partial derivative of equation (13) with respect to $\ln k_i P_i$, using the modified Shephard's Lemma, yields the translog cost-share equations:

$$(14) \quad M_i^s = (\partial \ln C^s) / \partial \ln (k_i P_i) \\ = \beta_i + \sum_j \gamma_{ij} \ln k_j P_j + \gamma_{iQ} \ln Q + \gamma_{iT} T.$$

Substituting equations (13) and (14) into (3') yields the translog version of the cost function:

$$\begin{aligned}
(15) \quad \ln C^a = & \alpha_0 + \sum \beta_i \ln k_i P_i + \beta_Q \ln Q + \beta_T T \\
& + .5 \sum \gamma_{ij} \ln k_i P_i \ln k_j P_j + \sum \gamma_{iQ} \ln k_i P_i \ln Q \\
& + \sum \gamma_{iT} (\ln k_i P_i) T + .5 \gamma_{QQ} (\ln Q)^2 + \gamma_{QT} (\ln Q) T \\
& + .5 \gamma_{TT} T^2 \\
& + \ln [\Sigma k_i^{-1} (\beta_i + \sum \gamma_{ij} \ln k_j P_j + \gamma_{iQ} \ln Q + \gamma_{iT} T)].
\end{aligned}$$

The translog cost-share equations are obtained by substituting equation (14) into (5):

$$\begin{aligned}
(16) \quad M_i^a = & [k_i^{-1} (\beta_i + \sum \gamma_{ij} \ln k_j P_j + \gamma_{iQ} \ln Q + \gamma_{iT} T)] / \\
& [\Sigma k_i^{-1} (\beta_i + \sum \gamma_{ij} \ln k_j P_j + \gamma_{iQ} \ln Q + \gamma_{iT} T)].
\end{aligned}$$

The returns to scale (v_Q^s), technical change (v_T^s), and $d\Sigma(M_{i,s}/k_i)/dT$ expression for the translog shadow TFP equations (9) and (12') are:

$$\begin{aligned}
(17) \quad v_Q^s = & (\partial \ln C^s) / \partial \ln Q \\
= & \beta_Q + \sum \gamma_{iQ} \ln k_i P_i + \gamma_{QT} T + \gamma_{QQ} \ln Q,
\end{aligned}$$

$$\begin{aligned}
(17') \quad v_T^s = & (\partial \ln C^s) / \partial T \\
= & \beta_T + \sum \gamma_{iT} \ln k_i P_i + \gamma_{QT} \ln Q + \gamma_{TT} T,
\end{aligned}$$

and

$$\begin{aligned}
(17'') \quad d\Sigma(M_{i,s}/k_i)/dT = & \Sigma [k_i^{-1} (\partial^2 \ln C^s) / \partial \ln(k_i P_i) \partial T] \\
& + \Sigma [k_i^{-1} (\partial^2 \ln C^s) / \partial \ln(k_i P_i) \partial \ln Q] \dot{Q} + \Sigma [k_i^{-1} (\partial^2 \ln C^s) / \partial^2 \ln(k_i P_i)] \dot{P}_i \\
& \frac{\Sigma k_i^{-1} \gamma_{iT} + \Sigma k_i^{-1} \gamma_{iQ} \dot{Q} + \Sigma \Sigma k_i^{-1} \gamma_{ij} \dot{P}_i}{\Sigma k_i^{-1} M_i^s} \\
= & \frac{\Sigma k_i^{-1} \gamma_{iT} + \Sigma k_i^{-1} \gamma_{iQ} \dot{Q}}{\Sigma k_i^{-1} M_i^s}.
\end{aligned}$$

Also, for the following discussion, note that:

$$(18) \quad v_Q^a = (\partial \ln C^a) / \partial \ln Q = v_Q^s + (\Sigma k_i^{-1} \gamma_{iQ}) / \Sigma k_i^{-1} M_i^s, \text{ and}$$

$$(19) \quad v_T^a = (\partial \ln C^a) / \partial T = v_T^s + (\Sigma k_i^{-1} \gamma_{iT}) / \Sigma k_i^{-1} M_i^T.$$

Thus, equation (9) can be rewritten as:

$$(20) \quad \dot{C}^a = v_Q^a \dot{Q} + v_T^a + \Sigma M_i^s \dot{P}_i.$$

Similarly, the **translog** form of the second shadow TFP equation is obtained by substituting (17), (17'), and (14) into (12'), which is obviously a much shorter expression than equation (20) in translog form.

V. Estimating the Regulatory Bias

Atkinson and Halvorsen showed that equations (15) and (16) are homogeneous of degree zero (h.d.z.) with respect to the k_i . Therefore, one of k_i can be chosen arbitrarily, and one is a natural and convenient normalization value. An estimate of the effect of regulation on total cost and other components is obtained by comparing the fitted values of the desired variable generated by the estimated model, with all of the k_i equal to their estimated values (estimated regulatory impact included), with the fitted values of the same variable but with all of the k_i set to one (no regulation).

It is important to note that this procedure works only for variables whose equations are h.d.z. with respect to the k_i ; otherwise, the magnitude of the regulatory bias depends on the value of the k_i normalization.

The $\ln C^s$ equation (13) is not h.d.z. in the k_i . All of the terms associated with the γ_i coefficients are h.d.z. with respect to k_i , from the coefficient restrictions (13'), but the terms related to the β_i

coefficients are not; if the k_i are multiplied by some constant t , then $\Sigma \beta_i \ln t = \ln t$ using (13').

The shadow share equations (14) are h.d.z. in the k_i because they have no terms involving both the β_i and k_i . Hence, the equations (16) are h.d.z. with respect to the k_i because the t factors for the k_i^{-1} will cancel out in the numerator and denominator. Actual cost (equation (15)) is h.d.z. in the k_i because the $\ln t$ term for the nonhomogeneous component of $\ln C^s$ will cancel out with the $-\ln t$ term of the nonhomogeneous component of $\ln \Sigma (M_{i,s}/k_i)$. Both v_0^s and v_T^s are h.d.z. in the k_i because they have no terms involving both the β_i and k_i ; v_0^a and v_T^a (equations (18) and (19)) are h.d.z. in the k_i because the terms added to v_0^s and v_T^s are h.d.z. in the k_i .

Both TFP equations, (20) and (12'), are h.d.z. in the k_i . The first two components of (20) are h.d.z. in k_i ; the third term also is h.d.z. in the k_i because M_i^s is h.d.z. in the k_i . All of the terms in equation (10) have already been shown to be h.d.z. in the k_i . Thus, the effect of regulation on TFP is computed by adding the individual regulatory effects on returns to scale, technical change, and shadow shares. This is an improvement over the Nelson and Wohar approach, which assumed that returns-to-scale and technical-change components are independent of any regulatory effect.

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