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STOCHASTIC INTEREST RATES IN THE AGGREGATE LIFE CYCLE/PERMANENT INCOME CUM RATIONAL EXPECTATIONS MODEL

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LIFE CYCLE/PERMANENT INCOME CUM RATIONAL EXPECTATIONS MODEL

Barro's neutrality hypothesis states that the impacts of federal government spending and tax policies do not depend on how the policies are financed. An important assumption of this hypothesis is that private agent spending horizons--the time span over which a permanent increase in life cycle wealth/permanent income is consumed--have essentailly infinite length, so that (expected) changes in life cycle wealth/permanent income do not affect current spending. If horizons are shorter, then monetary and fiscal stabilization policies may affect real variables at least in the short run.

In a world of proportional income taxes, a necessary condition for infinite horizon lengths is the existence of perfect capital markets. Perfect capital markets contain no transactions or other costs that drive a wedge between borrowing and lending rates of interest. They contain no informational asymmetries or other imperfections that are controlled by down payments, collateral requirements and security interests, collateral maintenence provisions, quantity limitations on the amount of debt extended to a single borrower, and other non-price loan provisions. All of these capital market imperfections constrain the liquidity of consumer life cycle wealth1 permanent income to be less than its full value; consumers cannot borrow against the full value of their life cycle wealth, or can do so only at a penalty rate of interest. The absence of perfect capital markets thus allows the possibility that horizons are shortened.

A number of recent studies have examined consumption behavior for evidence about the length of consumer spending horizons. The approach taken by most recent studies is to test some variant of the life cycle/permanent income cum rational expectations (RE-LC/PI) model assuming perfect capital markets. Rejection of the RE-LC/PI model incorporating perfect capital markets is taken to mean that horizon lengths may not be long enough to diminish the power of stabilization policies. Hall (1978), Flavin (1981,1985), Hayashi (1982), Muellbauer (1983), Wickens and Molana (1984), Bernanke (1982), Mankiw (1983), DeLong and Summers (1984), Boskin and Kotlikoff (1984), Kotlikoff and Pakes (1984), and Mankiw, Rotemberg, and Summers (1985) test the model with aggregate time series data, while Hall and Mishkin (1982), Bernanke (1984), and Hayashi (1985) use cross-section or panel data on individual households. Of the studies employing micro-data, only Bernanke (1984) cannot reject the model. Of the studies that employ aggregate time series data, Hall (1978), Hayashi (1982), Mankiw (1983), Bernanke (1984), and Delong and Summers (1984) cannot reject the model during the post-World War II period. Kotlikoff and Pakes (1984) can reject the model, but conclude that the differences from the model are not large enough to matter in practice.

None of the recent models estimated with U.S. aggregate time series data allows for uncertain real interest rates. All of the models except Bernanke (1982) and Mankiw (1983) assume that the real interest rate is constant. Bernanke (1982) and Mankiw (1983) allow real interest rates to vary, but assume that consumers know all future real interest rates. It is rather curious that stochastic real interest rates have been ignored, because the real interest rate is a key variable in the LC/PI models (and many New Classical models). Changes in interest rates, expected or unexpected, should lead to a reallocation of consumption spending across time. Thus, an allowance for stochastic real interest rates should provide a more powerful test of the RE-LCIPI model and indirectly of the length of the representative consumer's spending horizon.

The purpose of this paper is to estimate a RE-LCIPI model that allows for uncertain future interest rates. The model is developed by Muellbauer (1983) and Wickens and Molana (1984), who used U.K. data. In addition, the Hall (1978) and Flavin (1981) models are reestimated through 1984:IVQ. Updating the Hall and Flavin models serves at least four purposes. First, the updates help put the results from Muellbauer's model in perspective. Second, by estimating the models through 1984, the stability of these models may be examined. Third, it is interesting to know how the 1980s data fit these models. Real output and prices moved over wide latitudes during the 1980s and, hence, offer macroeconometricians a rich set of high influence data to help them estimate coefficients more precisely. It is likely that the 1980s data provide even stronger evidence against the RE-LCIPI model than was found by Flavin. Finally, the different models are estimated with different information sets (reduced forms) and different sample periods. It is reasonable to wonder if either the content of the information set or the estimation period has a large influence on the estimates. The interest in these models is not only that the RE-LCIPI model is accepted or rejected, though it is a very important consideration. If these models are to be useful for policymaking and forecasting, they should be robust to different assumptions about the underlying structure used to derive the reduced forms.

The first section of this paper reviews the RE-LCIPI model and derives the model with stochastic interest rates. The second section outlines the procedures followed in estimating the three models and explains the results. The last section concludes the paper.

I. THE LIFE CYCLEIPERMANENT INCOME MODEL WITH RATIONAL EXPECTATIONS

Tests of the RE-LCIPI model begin with Hall (1978). The consumer is assumed to maximize the expected present discounted value of current and future utility. Income is exogenous and known in the current period, but unknown thereafter; the consumer's choice variable is the level of consumption each period. The horizon begins with the current period and ends at the (known) last period of the consumer's lifetime. There are no bequests and no capital market imperfections. Expectations are rational--functions of all information available in the current period; real interest rates and rates of time preference are assumed constant. The model is:

(1) $$\max_{C_t} E_t \sum [\delta^i U(C_{t+i})]$$

subject to $$\sum_{i=0}^{T-t} \left\{ [R^i C_{t+i}] - [R^i y_{t+i}] \right\} = A,$$

where

δ is the inverse of 1 plus the pure rate of time preference, assumed constant,

R is the inverse of 1 plus the real, after-tax rate of interest r also assumed constant, $(\delta \geq R)$,

C is real life cycle consumption (not NIA personal consumption expenditures),

y is real labor income,

A is current real nonhuman wealth,

$U(\bullet)$ is the instantaneous utility function, and

E_t is the expectations operator, conditioned on the information available at time t (variables dated t-1 and earlier).

The first order conditions for this problem are:

(2a) $E_t U'(C_{t+i}) = (R/\delta) E_t U'(C_{t+i-1})$ i = 1 to T-t; in particular, for i=1,

(2b) $E_t U'(C_{t+1}) = (R/\delta) U'(C_t)$.

There are two things to note about (2b). First, C_t can be thought of as a sufficient statistic for C_{t+1}; that is, no variable except C_t helps predict future marginal utility of consumption $U'(C_{t+1})$. Second, with the assumption of rational expectations, marginal utility follows the regression relation:

(3) $$U'(C_{t+1}) = \gamma U'(C_t) + \varepsilon_{t+1}.$$

ε_{t+1} represents the impact on marginal utility of all new information that becomes available in period t+1 about the consumer's lifetime well-being. Under rational expectations, $E_t\varepsilon_{t+1} = 0$ and ε_{t+1} is orthogonal to $U'(C_t)$. Moreover, ε should be white noise, that is, unpredictable using variables in the information set.

If the utility function is quadratic, or "the change in marginal utility from one period to the next is small, both because the interest rate is close to the rate of time preference and because the stochastic change is small"'

(4) $$C_t = \gamma C_{t-1} + \varepsilon_t.$$

That is, life cycle consumption follows an AR(1) process--no other variables dated t-1 or earlier affect C_t. If $\gamma = 1$ then consumption follows a random walk. It is important to notice that (4) is not a structural model of life cycle consumption behavior. Because it is only the first order condition for utility maximization, it is only an implication of the life cycle model under rational expectations. Indeed, it is only a necessary condition for this RE-LC model to be true.

Hall also shows that lifetime resources evolve as a random walk with trend. First, nonhuman wealth follows the relation:

(5) $$A_t = R^{-1}(A_{t-1} + y_{t-1} - C_{t-1}).$$

Second, human wealth, H_t, is the sum of current labor income and the expected present discount value of future labor income:

(6) $$H_t = \sum_{i=0}^{T-t} [R^{-i}E_t y_{t+i}], \text{ where } E_t y_t = y_t,$$

from which it follows that:

(7a) $H_t = R^{-1}(H_{t-1} - y_{t-1}) + \mu_t$,

where μ_t represents the present value of the changes in expectations of future income that occur between period t-1 and t:

(7b) $\mu_t = \sum_{i=0}^{L-t} [R^i(E_t y_{t+i} - E_{t-1} y_{t+i})]$.

Again under rational expectations, $E_{t-1}\mu_t$ =.-0, and μ_t should be white noise. Under certainty equivalence, $\epsilon_t = \alpha_t \mu_t$, where α_t is an annuity factor modified to take account of the fact that the consumer plans to make consumption grow at a proportional rate γ over his remaining lifetime. Then the equation for total wealth is:

(8) $A_t + H_t = R^{-1}(1-\alpha_{t-1})(A_{t-1} + H_{t-1}) + \mu_t$.

Flavin (1981) estimates a different version of the permanent income model using the insight from (7) to eliminate the unobserved H_t. Flavin starts with the definition that current consumption is the sum of permanent and transitory consumption. By equating permanent consumption with permanent income (y_t^p), she has

(9) $C_t = y_t^p + \varepsilon_{2t}$,

where ε_{2t} is transitory consumption. Thus permanent income is defined to be the annuity value of the expected present discounted value of human and nonhuman wealth ($A_t + H_t$), assuming the real, after-tax rate of interest, r, is constant:

(10) $y_t^p = r(A_t + \sum_{i=0}^{\infty} [R^{i+1} E_t y_{t+i}])$.

Flavin shows that $E_t y_{t+1}^p = y_t^p$ using the insight behind (7b). Substituting (10) into (9) and using the nonhuman wealth constraint;

(11) $A_{t+1} = R^{-1}A_t + y_t - C_t$.

Note that unlike equation (5), current period saving does not earn interest in equation (11).

Equation (9) can be used to solve for C_{t+1} in terms of C_t:

$$(12)\quad C_{t+1} = C_t + r\sum_{i=0}^{\infty} [R^{i+1}(E_{t+1} - E_t)y_{t+i+1}] - R^{-1}\varepsilon_{2t} + \varepsilon_{2t+1}.$$

Flavin notes that because the coefficient of ε_{2t} is not -1, C will not evolve as a random walk, unless the transitory consumption term ε_{at} is zero for all t.

Equation (12) contains revisions in expectations of future real labor income. Flavin (1981, p. 998) notes that "(a)s an empirical matter, however, unanticipated capital gains and losses on nonhuman wealth probably constitute a significant fraction of the revisions in permanent income which this model is trying to capture. She defines unanticipated capital gains as the present value of the revision in the expected earnings associated with the current nonhuman wealth position. By then assuming "that changes in the rate of return to capital...are quantitatively more important than the endogenous changes [in nonhuman wealth] in determining the time-series properties of the observed path of nonlabor income", unanticipated capital gains can be approximated as the present value of the revision in expected future nonlabor income. This permits her to use disposable personal income (YD) in place of labor income (y) in equation (12).[3]

Flavin next derives an expression for the revision in expectations of future YD by assuming that YD follows an ARMA process. She shows that the revision in the expectation of YD_{t+s} (s>0) between periods t and t-1 is the product of the moving average error of YD in period t (u_t) and the s^{th}

coefficient from the corresponding moving average representation for YD (B_s). Then the present discounted value of the set of revisions is:

$$(13)\qquad (\sum_{s=0}^{\infty}[R^{s} B_s])u_t.$$

Thus she demonstrates that the revision in income expectations is white noise.

The ARMA model for YD plus the equation formed by substituting (13) into (12) is Flavin's permanent income consumption model. Note that (13) still contains an unobserved variable u_t. This term is included with the other error terms in estimation, making her consumption equation very similar to Hall's. The difference is that Hall's model can be viewed as a reduced form of Flavin's structural model. Flavin argues that the error terms in the two equations are correlated because her model is incomplete. The income equation error will contain additional terms, because the information set probably contains variables other than past income. These omitted information set variables will also appear in the consumption equation error through (13), thus producing the correlation between the two equation errors. She dismisses this apparent specification bias away by assuming that these omitted information set variables are serially uncorrelated and uncorrelated with the lagged income terms.

When the interest rate is random, the R term in the first order conditions (2a) cannot be taken outside of the expectations operator. Assuming as does Muellbauer (1983) a cash flow constraint $A_t = (1/R_{t-1})\ A_{t-1}+y_t-C_t$, the first order conditions become:

(14) $E_tU'(C_{t+i-1}) = \delta E_t[(1/R_{t+i-1})\ U'(C_{t+i})],\ i>=0.$

Using Muellbauer's utility function $U(C_t)=[(C_t)^{1-b}-1]/(1-b)$, where $b>0$, and setting $i=1$, (14) becomes:

(15) $C_t^{-b} = \delta E_t[(1/R_t)\ (C_{t+1})^{-b}]$

At this point, Muellbauer assumes there are point expectations about $1/R_t$ conditional on the information set at time t, allowing him to pull the interest-rate term outside the expectations operator:

(16) $Ct^b = \delta(1+r_t^t)\ E_t[(C_{t+1})^{-b}]$

where $(1+r_t^t)$ is the point expectation on $1/r_t$. This may be an important assumption, because it means that income and the interest rate are independent variables from the consumer's point of view. [4]

Taking logarithms of both sides and rearranging terms yields:

(17a) $E_t[\ln C_{t+1}] - \ln C_t = (1/b)\ln\delta + (1/b)\ln(1+r_t^t)$

and eliminating the expectations operator with an additive error yields:

(17b) $\Delta\ln C_{t+1} = (1/b)\ln\delta + (1/b)\ln(1+r_t^t) + \theta_{t+1}$.

The error term θ_{t+1} represents new information (innovations) about labor income σ_{1t+1} and the interest rate σ_{2t+1} as well as white noise ϵ_{t+1}.

Using this and the approximation $\ln(1+r_t^t) = r_t^t$ for small r_t^t, the model estimated by Muellbauer is:

(18) $\Delta\ln C_{t=1} = \mu_0 + \delta_3 r_t^t + \delta_1\sigma_{1t+1} + \delta_2\sigma_{2t+1} + \epsilon_{t+1}$.

The Wickens and Molana model varies only slightly from this because of a minor difference in the dating of the interest rate in the cash flow constraint.

The innovation and r_t^t terms are given by simple equations relating income and the interest rate to the information set variables, which are all lagged variables. The fitted value from the interest rate equation serves as the expected r term, and the residuals from these equations serve as the innovation terms. Thus (18) plus the forecasting equations for income and the interest rate is Muellbauer's RE-LC model. The idea for this formulation of the expected values and innovations originates with Barro (1977).

The RE-LC/PI models are tested by adding variables from the information set to the right-hand side of the consumption equation. Hall uses various lagged income and consumption terms and lagged values of Standard and Poor's stock price index. Flavin uses the current and first seven lagged changes in real per capita YD, and Muellbauer uses the explanatory variables from the income and interest-rate equations. If the model is correct, then no other variable in the information set except C_{t-1} will help forecast C_t. If other variables are significantly correlated with consumption, then either the assumption of rational expectations or the LC/PI model (or both) can be rejected. Unfortunately, it is not possible to know which assumption is incorrect. Only the joint hypothesis of both the LCIPI model and the rational expectations assumption can be tested.

II. UPDATES OF THE AGGREGATE LIFE CYCLE CUM RATIONAL EXPECTATIONS MODEL

This section updates the estimates and tests of the Hall and Flavin (1981) models and presents estimates and tests of the Muellbauer model using post-World War II, U.S. aggregate time series data.[5] The Hall and Flavin models are updated with their original samples, specifications, and estimation techniques. Then, to facilitate the comparison of the three models, we attempt is made to put them on an equal footing.

To do this, at least four decisions need to be made. One is the specification of the dependent and independent variables. Hall uses per capita PCE-nondurables and services, Flavin uses the change in per capita PCE-nondurables, and Muellbauer uses the change in the logarithm of per capita (U.K.) PCE-nondurables and services. The consumption definition used in the tests reported here is per capita PCE-nondurables and services. Flavin argues that only PCE-nondurables should be used as the consumption variable, because consumption of durable services should react to changes in permanent income with a lag due to the costs of adjusting durable stocks. The same is true of housing services, which form a large part of PCE-services. Althoughthese reasons may be valid, Flavin detrends the consumption and income data. This should eliminate, or at least greatly diminish, the lagged adjustment problem. It does not matter which functional form is used because the theory can be expressed in terms of levels or log levels; the change in the logarithm is used here to minimize heteroskedasticity problems. The income definition is real disposable income per capita. The log real per capita income and consumption data are detrended by their average growth trends from the 1947:IQ to 1984:IVQ.

When the same dependent variable is used, Flavin's consumption equation is for all practical purposes, the same as Muellbauer's with constant interest rates. The difference is the specification of the variables used to test the models. This difference is important, because it means that Flavin must estimate her model differently than Muellbauer does. Flavin's model is simultaneous because she adds the current and first seven lags of AYD to her consumption equation. When deriving the reduced form of her two-equation system, the equation for YD is used to substitute out the current YD term in AYD$_t$; the revision to permanent income due to the new information provided by current YD (13) cannot be identified and is thus thrown into the error term. Because Muellbauer uses only lagged variables to test the model, the income and interest-rate innovations remain identified by the income and interest-rate equations and their coefficients can be estimated.

Ignoring the interest-rate terms in Muellbauer's model, it is not clear that his test is more powerful than Flavin's. The presence of ΔYD_t in the consumption equation gives Flavin a direct test of the impact of current income on current consumption. If the RE-LC/PI model is rejected, there is some knowledge about what direction the search for the correct alternative might take. But Flavin cannot test for the impact of the income innovation term, an important variable of the null hypothesis. By not adding any current income terms, Muellbauer cannot test for a direct effect of current income on current consumption, but he does have a direct test of the impact of income innovations.

A second choice concerns seasonal adjustment of the data. Muellbauer uses seasonally unadjusted data, while Hall and Flavin use seasonally adjusted data. Although there are good reasons for using seasonally unadjusted data, it was decided to use seasonally adjusted data in order to maintain comparability with other U.S. consumption results.

A third choice concerns estimation technique. Hall uses ordinary least squares (OLS), Flavin uses maximum likelihood to estimate her consumption equation jointly with her income forecasting equation, and Muellbauer uses a two-step OLS procedure popularized by Barro (1977). The original estimation techniques used by Hall and Flavin will be used to update their models with the most recent data. However, the preferred estimation technique for Muellbauer's model is maximum likelihood because, as in Flavin's model, there are nonlinear cross-equation parameter constraints, and because maximum likelihood is asymptotically efficient. Moreover, Pagan (1984) shows that the coefficients from a two-step procedure may not have the same limiting distribution as those from maximum likelihood, meaning that the two-step coefficients may not be asymptotically efficient. More importantly, he also shows that the correct OLS coefficient standard error formulas may differ from the usual OLS formulas employed by computer regression programs, meaning that computed t-statistics may be incorrect.

Unfortunately, Pagan does not examine the properties of two-step estimators for Muellbauer's model. Extending Pagan's Theorem 4 to Muellbauer's model without the interest rate terms in the consumption equation, shows that the two-step estimator will have the same limiting distribution as maximum likelihood if the test variables used in the consumption equation are included in the information set, which is the case. However, extending Pagan's Theorem 5 shows that the usual OLS formula for the standard errors is wrong. However, the usual OLS standard errors are greater than the correct OLS errors, so that the usual OLS t-statistics are understated. Similar results are obtained for Muellbauer's model, including the interest-rate terms in the consumption equation by an extension of Pagan's Theorems 2 and 8.

These problems can be avoided by estimating Muellbauer's model with maximum likelihood using a stacked nonlinear least squares technique similar to Mishkin (1983).

A fourth choice is that of the definition of the real interest-rate. Instead of using an ex post real interest rate, Muellbauer uses something like an ex ante rate--a nominal interest rate minus an expected inflation rate. He computes this real rate by subtracting from the nominal rate a fitted value from an inflation equation. This choice of real rate is rather odd, for it means that instead of using an expected real interest rate as his theory requires, he is using something like an expected expected real interest rate in his consumption equation. It also means that he is using a three-step estimation process, with the estimation of the inflation equation as the first step. Moreover, the inflation equation uses a information set different from that used for the income and interest-rate equations. A logical extension and correction of his model would be to specify separate forecasting equations for the nominal rate and the inflation rate, use the same information set for all of the equations, and use the fitted values and residuals from both equations to compute the expected real rate and its innovation.

An equivalent technique is the use of an ex post rate, as in Wickens and Molana (1984). This requires only one forecasting equation. The ex post real three-month U.S. Treasury bill rate (nominal rate minus current quarter compounded annual actual growth rate in the PCE-nondurables and services deflator) is used as the real interest rate in the estimations of Muellbauer's model shown below.

Because there is no reason to think that U.S. real interest rates have behaved as random walks during the post-World War II period, an assumption Muellbauer made for U.K. real rates, the real interest-rate equation for Muellbauer's model estimated here will have information set variables as regressors. These will be the same as those used for the income equation--the first two lags of income, the first two lags of the real interest rate, and the first lag of consumption. This is simple extension of Muellbauer's original information set, which consisted of the first two lags of income and the first lag of consumption.

The estimation results are shown in tables 1 to 5. The data used for the computations contain revisions through the second revised estimates for 1984:IVQ, dated March 1985. The models in tables 1 to 3 were estimated over their original samples and over 1949:IIIQ to 1984:IVQ. For the re-estimates of Hall's model, the data were not detrended. For the re-estimates of Flavin's model, the consumption and income data were detrended, using their average growth rates from the 1947:IQ to 1979:IQ. When the two models are updated with the data through 1984:IVQ, the consumption and income data are detrended using their average growth rates from the 1947:IQ to 1984:IVQ, and a dummy variable is added to control for the credit controls of 1980:IIQ. Detrending biases the test in favor of the random walk hypothesis, because it removes the main source of correlation from these variables. It also may remove structural correlation between C and YD, again favoring the random walk hypothesis. It unfortunately leaves the trend unexplained. The dummy variable is part of the maintained hypothesis and is not included among the variables in the test of the RE-LC/PI model.

The first table contains OLS estimates of Hall's model. These equations were estimated with the OLSQ option in PEC version 9.1. The first equation shows the reestimates of Hall's model, with only one lagged income term. The coefficients, though different from Hall's published numbers, yield the same apparent inference: the RE-LC/PI model cannot be rejected. The next equation shows the original Hall model updated through 1984:IVQ. Note that the addition of the 1980s data did not change the conclusion of the hypothesis test--the coefficient on lagged personal income is small, has the wrong sign, and is statistically insignificant. However, the Durbin h-statistic rejects the hypothesis of positive serially uncorrelated errors at better than a 5 percent significance level using a one-tailed test. Because the theory predicts that the error should be white noise, the addition of the 1980s data may be signaling a breakdown of the model.[6] The third equation contains the change in the detrended log of per capita PCE-nondurables and services as the dependent variable and the detrended logarithm of real per capita disposable personal income as the income variable. The estimation period is 1948:IQ to 1977:IQ. Neither coefficient is large, the t-statistics are very low, and the adjusted R^2 is negative. The results change very little when the estimation period is extended through 1984:IVQ; all of the explanatory power of the right-hand side variables comes from the dummy variable. Thus Hall's model can find no evidence to reject the RE-LC/PI model.

The results for Flavin's model are shown in tables 2 and 3. The general specification is:

$$(12a) \quad YD_t = \mu_1 + \alpha_1 YD_{t-1} + \alpha_2 YD_{t-2} + \cdots + \alpha_8 YD_{t-8} + \eta_{1t}$$

$$\Delta C_t = \mu_2 + \beta_0(U_1 + (\alpha_1 - 1)YD_{t-1} + \alpha_2 YD_{t-2} + \cdots + \alpha_8 YD_{t-8})$$

$$+ \beta_1 \Delta YD_{t-1} + \beta_2 \Delta YD_{t-2} + \cdots + \beta_7 \Delta YD_{t-7} + \eta_{2t} \quad (13)$$

where η_{2t} contains ε_{2t} and (13). The ß's are "measures of the 'excess sensitivity' of consumption to current income, that is, sensitivity in excess of the response attributable to the new information contained in current income. Flavin (1981, p. 990). Thus, a test of the joint statistical significance of the R's is a test of the RE-PI model. The results in table 2 are the re-estimates of Flavin's model, using her definitions of consumption and income. Table 3 contains the results of her model using the change in the log of PCE-nondurables and services. The multivariate regression technique in TSP version 4.0E was used to estimate these equations. This is the same estimator Flavin used. It allows joint estimation of nonlinear equations with nonlinear cross-equation constraints and with contemporaneous correlation across equation error terms. It should yield results close to Flavin's indirect least squares estimates apart from the impact of data revisions. The first equation in table 2 shows the re-estimates of her original specification. Like the updates of Hall's model, these coefficients are not quantitatively the same as the original estimates but, qualitatively they are very similar. The coefficient on ΔY_t, β_0, though fairly large, has a very low t-statistic of the R's, only β_1 is significant at better than 5 percent using a one-tailed test. The second equation drops the AYD terms to test their joint significance. Recall that these terms must be jointly statistically different from zero in order to reject the model. Surprisingly the RE-LC/PI model cannot be rejected at the original significance level. Flavin's original likelihood ratio statistic (LRS), which is asymptotically distributed as $X^2(8)$, is 27.0, significant at better than 0.5 percent. The LRS for the test using equations (1) and (2) is only 11.8--significant at slightly better than 25.0 percent. Identical test results are obtained by estimating only the consumption reduced form equation with OLS and testing for the joint significance of the lagged income terms.[7] Apparently the results are sensitive to revisions in the data and the use of different trend values for PCE-nondurables and YD.[8]

Equations (3) and (4) in table 2 update Flavin's original model through 1984:IVQ. The 1947:IQ to 1979:IQ trend values are used to detrend the post-1979:IQ data. Interestingly, the model can now be rejected at better than a 5.0 percent significance level. The LRS is 17.1, while the $X^2(8)$ cut-off value is 15.5, at 5.0 percent. The coefficient β_0 is now smaller, but its t-statistic is larger; the coefficient and t-statistic on $\Delta \ln YD_{t-1}$ are larger also. Moreover, the fit of the two equations is improved over the longer period; the standard errors of the two equations are smaller in the longer sample. Thus, as was expected, the 1980s data appear to tighten up coefficient standard errors and help reject the RE-LCIPI model.

Equations (5) through (8) in table 3 use the change in the logarithm of per capita real PCE-nondurables and services as the dependent variable, and the log per capita consumption and income data are detrended over the 1947:IQ to 1984:IVQ period. They compare to the Hall equations (3) and (4) in table 1. The fifth equation shows the unconstrained results over the 1949:IIIQ to 1979:IQ sample period. Notice that they are qualitatively similar to those of equation (1); β_0 is about 0.3 and statistically insignificant, and β_1 is large and statistically significant. Testing this equation against the sixth equation, which drops the ΔlnYD terms, yields an LRS of 27.1, which is significant at better than 0.5 percent, Flavin's original significance level. Note that this result is much stronger than Flavin's original result, because the consumption variable includes PCE-services, which Flavin argued would bias the results against the RE-LCIPI model.

Equations (7) and (8) show the estimation results over the 1949:IIIQ to 1984:IVQ sample. Qualitatively, these results are similar to those of equations (5) and (6). The LRS of the test of the lagged ΔlnYD terms is now 29.4, greater than the LRS over the 1949:IIIQ to 1979:IQ sample; the standard errors of the equations also are smaller in the longer sample. Again it appears that the 1980s data provide additional stronger evidence against the RE-LC/PI model.

Tables 4 and 5 contain the estimates of Muellbauer's model. All of the equations are estimated with the stacked maximum likelihood technique using the LSQ option of PEC version 9.1. The dependent variable is the change in the logarithm of real per capita PCE-nondurables and services; detrending of the log real per capita consumption and income data occurs over the 1947:IQ to

1984:IVQ period. Table 4 contains the estimates of (18) without the interest rate terms r_t^e and σ_{2t}. The estimated model is:

$$\ln Y_t = \mu_1 + \alpha_1 \ln Y_{t-1} + \alpha_2 \ln Y_{t-2} + \alpha_3 \ln C_{t-1} + \alpha_9 DUM802_t + \epsilon_{1t}$$

$$\Delta \ln C_t = \mu_2 + \delta_1(\ln Y_t - \mu_1 - \alpha_1 \ln Y_{t-1} - \alpha_2 \ln Y_{t-2} - \alpha_3 \ln C_{t-1} - \alpha_9 DUM802_t) + \beta_1 \ln Y_{t-1} + \beta_2 \ln Y_{t-2} + \beta_3 \ln C_{t-1} + \beta_9 DUM802_t + \epsilon_{2t}.$$

The coefficient δ_1 should be positive because positive innovations in current income should lead to upward revisions in life cycle wealth/permanent income and hence in consumption. The first two equations show the results using the 1949:IIIQ to 1979:IQ sample. These equations compare to Flavin's equations (5) and (6) in table 3. The coefficient δ_1 is positive and statistically significant. Surprisingly, the RE-LC/PI model cannot be rejected by this form of Muellbauer's model, even though Flavin's model could. The LRS is only 3.8, significant at slightly less than 30 percent. Apparently the results are sensitive to the specification of the test.

The third and fourth equations in table 4 update Muellbauer's model without the interest rate terms over the 1949:IIIQ to 1984:IVQ sample. As was true of Flavin's model, Muellbauer's model without the interest rate terms fits better with the 1980s data. Moreover, the LRS is now 14.2, significant at better than 1 percent. Again the 1980s data lead to a convincing rejection of the RE-LC/PI model. Note that the β coefficients are of the same order of magnitude and statistical significance in equations (1) and (3). The difference is that the models fit better with the 1980s data.

Table 5 contains the estimates of Muellbauer's model including the interest rate terms. The model is:

$$\ln Y_t = \mu_1 + \alpha_1 \ln Y_{t-1} + \alpha_2 \ln Y_{t-2} + \alpha_3 r_{t-1} + \alpha_4 r_{t-2} + \alpha_5 \ln C_{t-1} + \alpha_9 DUM802_t + \varepsilon_{1t}$$

$$r_t = \gamma_0 + \gamma_1 \ln Y_{t-1} + \gamma_2 \ln Y_{t-2} + \gamma_3 r_{t-1} + \gamma_4 r_{t-2} + \gamma_5 \ln C_{t-1} + \gamma_9 DUM802_t + \epsilon_{2t}$$

$$\Delta \ln C_t = \mu_2 + \delta_1(\ln Y_t - \mu_1 - \alpha_1 \ln Y_{t-1} - \alpha_2 \ln Y_{t-2} - \alpha_3 r_{t-1} - \alpha_4 r_{t-2}$$
$$- \alpha_5 \ln C_{t-1} - \alpha_9 DUM802_t) + \delta_2(r_t - \gamma_0 - {}_1 \ln Y_{t-1} - \gamma_2 \ln Y_{t-2}$$
$$- \gamma_3 r_{t-1} - \gamma_4 r_{t-2} - \gamma_5 \ln C_{t-1} - \gamma_9 DUM802_t) + \delta_3(G_0 + \gamma_1 \ln Y_{t-2}$$
$$+ \gamma_2 \ln Y_{t-3} + \gamma_3 r_{t-2} + \gamma_4 r_{t-3} + \gamma_5 \ln C_{t-2} - \gamma_9 DUM802_{t-1})$$
$$+ \beta_1 \ln Y_{t-1} + \beta_2 \ln Y_{t-2} + \beta_3 r_{t-1} + \beta_4 r_{t-2} + \beta_5 \ln C_{t-1} + \beta_9 DUM802_t$$
$$+ \epsilon_{3t}$$

Recall from (18) that δ_3 is a positive function of the ratio of 1 plus the interest rate to 1 plus the rate of time preference, and hence, should be positive. Presumably the coefficient 62 on the interest-rate innovation is negative, since a higher-than-expected interest rate should cause consumers to save more in the current period. Equations (1) and (2) are the results over the 1949:IIIQ to 1979:IQ sample. The two interest-rate coefficients appear to be small in magnitude, but this is simply a scaling difference because interest rates are measured in percentage points; the interest rate innovation coefficient δ_2 is statistically insignificant in both equations, while δ_3 is significant at slightly better than 10 percent in equation (1) and better than 5 percent in equation (2), using two-tailed tests. The LRS for the test of the RE-LC/PI model is 27.8, which is asymptotically distributed as $X^2(5)$, and is significant at better than 1 percent. Compared with equations (1) and (2) in table 4, the allowance for stochastic interest rates now leads to the rejection of the RE-LC/PI model. Again, the specification of the test has an important effect on the results.

The third and fourth equations in table 5 show the estimates of Muellbauer's model with the interest-rate terms from 1949:IIIQ to 1984:IVQ. All of the coefficients are estimated more precisely, but unlike the previous results, both equations fit the longer period less well. The coefficients δ_2 and δ_3 have the correct signs, but are the same magnitude or smaller and mostly statistically insignificant. The LRS statistic for the test of the RE-LCIPI model is 26.2, rejecting the model at better than a 1 percent significance level, but it is a bit smaller than the LRS from the shorter sample period.

The worse fit using the 1980s data is explained by the interest-rate equation fitting less well in the longer period. This is not surprising, since interest rates behaved so differently in the 1980s than before.[9] Does this mean that the test is invalid because the equation generating the interest-rate expectations is wrong? This does not seem likely. Although the t-statistics on δ_2 and δ_3 are mostly very low, the LRS of the joint significance of the two interest-rate terms in equation 7 is 46.1. Thus, the interest-rate terms are undoubtedly important, even if they are poorly computed. Moreover, it is not clear how quickly interest-rate forecasting models were adjusted in the 1980s. Given the lag in the learning process, the number of quarters for which the interest-rate equation may be wrong is probably smaller than 20. Even if the interest-rate equation is wrong, it is not necessarily irrational. Finally, the fit of the model did not worsen so much that this is likely to be the sole reason the RE-LCIPI model is rejected.

III. WHAT HAS BEEN LEARNED?

The estimation results provide ample evidence to reject this form of the RE-LCIPI model during the post-war period, especially when the 1980s data are included. Even though Hall's specification cannot reject the model, minor generalizations of Flavin and Muellbauer can, and Muellbauer's specification including uncertain interest rates can reject the model with or without the 1980s data. It would appear that an important assumption for Barro's neutrality hypothesis does not hold.

Unfortunately this rejection of the RE-LCIPI model does not offer an explicit alternative as a replacement. As mentioned earlier, these tests cannot distinguish the assumption of rational expectations from that of the life cycle/permanent income model. All that can be inferred from these tests is that the joint hypothesis can be rejected. Flavin (1985) attempts to distinguish whether the rejection of the RE-LCIPI model is due to the assumption of perfect capital markets or to that of the permanent income model. She uses her original model augmented with an equation for the unemployment rate, which is a proxy for the number of liquidity-constrained consumers. However, there are many problems using such a crude variable for such a complex hypothesis; her tests undoubtedly have little power.

Nor do these tests provide many clues about the exact length of consumer spending horizons, or how the distribution of horizon lengths changes as interest rates, the distribution of income, or the supply of consumer credit changes. That the distribution of consumer horizon lengths may vary over time is suggested by the increased significance of the likelihood ratio tests when the 1980s data are included. In the early 1980s, apparently, the distribution of horizons lengths was skewed toward the shorter end, increasing the correlation of aggregate consumption to current disposable income. Additional evidence about changes in the distribution of consumer spending horizons is provided by Kowalewski (1982), who studies the time series behavior of aggregate personal bankruptcy filings in the United States. Personal bankruptcy filings are countercyclical, increasing in recessions and falling in recoveries. For a variety of reasons discussed in the article, it is likely that just before they file for bankruptcy, personal bankrupts have about the shortest spending horizons of all consumers. Thus increases in the number of personal bankruptcy filings may indicate a shift in the distribution of consumer spending horizons towards shorter lengths. In a regression explaining per capita personal bankruptcy filings, transitory income had a much larger impact than permanent income, suggesting that liquidity is very important for these financially distressed consumers. The composition of consumer portfolios was also significantly related to the behavior of personal bankruptcy filings. Unfortunately, this is only evidence about one tail of the distribution. It is clear that much work remains to be done before the time series behavior of aggregate consumption is understood.

TABLE 1 Hall Estimates

$$C_t = \alpha_0 + \alpha_1 C_{t-1} + \alpha_2 Y_{t-1} + \alpha_3 DUM802 + \varepsilon_t$$

	#1	#2	#3	#4
C_t	NDS/POP	NDS/POP	Chg in detrended log of NDS/POP	Chg in detrended log of NDS/POP
Y_{t-1}	YD72/POP	YD72/POP	Detrended log of YD72/POP	Detrended log of YD72/POP
Sample	48Q1-77Q1	48Q1-84Q4	48Q1-77Q1	49Q3-84Q4
α_0	-0.0376	-0.0059	0.0007	0.0005
	-2.2620	-0.5835	1.1492	0.8562
α_1	1.0811	1.0081		
	24.8721	31.3779		
α_2	-0.0480	-0.0008	-0.0063	-0.0060
	-1.6283	-0.0341	-0.4627	-0.4902
		-0.0441		-0.0131
		-3.0626		-2.3346
adj R^2	0.9989	0.9994	-0.0068	0.0263
Durbin h	1.358	1.7327	1.7752[a]	1.7460[a]
SER	0.01 36	0.0290	0.0058	0.0056

a. Durbin - Watson statistics

Variables:

NDS = PCE-nondurables + services, 1972 dollars
YD72 = disposable personal income, 1972 dollars
POP = noninstitutionalized, civilian population

Notes: The variables (3) and (4) are detrended from 1947:IQ to 1984:IVQ. T-statistics are shown below the coefficient estimates.

Table 2 Flavin Reestimates

$$Y_t = \mu_1 + \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \cdots + \alpha_8 Y_{t-8} + \alpha_9 DUM802 + \varepsilon_{1t}$$

$$\Delta C_t = \mu_2 + \beta_0(\mu_1 + (\alpha_1 - 1)Y_{t-1} + \alpha_2 Y_{t-2} + \cdots + \alpha_8 Y_{t-8} + \alpha_9 DUM802) + \beta_1 \Delta Y_{t-1} + \beta_2 \Delta Y_{t-2} + \cdots + \beta_7 \Delta Y_{t+7} + \beta_9 DUM802 + \varepsilon_{2t}$$

	#1	#2	#3	#4
Sample	49Q3-79Q1	49Q3-79Q1	49Q3-84Q4	49Q3-84Q4
μ_2	-0.0000	0.0004	0.0002	0.0005
	-0.0240	0.3750	0.2460	0.4949
β_0	0.3194		0.2712	
	1.1164		1.4596	
μ_1	-0.0036	-0.0017	-0.0060	-0.0039
	-0.7294	-0.3627	-1.3229	-0.8976
α_1	0.9606	0.9022	1.0055	0.9307
	10.4875	10.6577	12.3514	12.2531
α_2	0.0093	0.0698	0.0011	0.0851
	0.0740	0.5976	0.0093	0.7948
α_3	0.1539	0.1842	0.1051	0.1221
	1.2328	1.5971	0.9254	1.1516
α_4	-0.2861	-0.3060	-0.2565	-0.2537
	-2.2890	-2.6488	-2.2608	-2.3969
α_5	-0.0253	0.0451	-0.0597	-0.0169
	-0.2021	0.3889	-0.5249	-0.1595
α_6	0.1771	0.0900	0.1974	0.1222
	1.4208	0.7811	1.7414	1.1560
α_7	-0.0232	-0.0149	-0.0531	-0.0502
	-0.1859	-0.1296	-0.4672	-0.4740
α_8	-0.0053	0.0058	0.0074	0.0241
	-0.0594	0.0706	0.0914	0.3199
α_9			-0.0977	-0.0990
			-2.9660	-2.9866
β_1	0.0605		0.0650	
	1.8388		2.3574	
β_2	0.0079		-0.0099	
	0.2493		-0.3659	
β_3	-0.0662		-0.0535	
	-1.2940		-1.4499	
β_4	0.0415		0.0136	
	0.8088		0.3915	
β_5	-0.0081		-0.0082	
	-0.1410		-0.1908	
β_6	0.0068		0.0050	
	0.2163		0.1834	
β_7	0.0074		0.0169	
	0.2381		0.6146	
β_9			-0.0010	-0.0286
			-0.0436	-2.5935
C SER	0.0103	0.0108	0.0104	0.0110
C DW	2.0101	1.8921	2.0521	1.8748
Y SER	0.0329	0.0332	0.0325	0.0327
Y DW	2.0008	1.8834	2.0009	1.8461
Log Lklhood	622.413	616.536	742.652	734.078

ΔC=chg in detrend per capita PCE-Nondurables 1972$

Y = detrended per capita disposable personal income, 1972$

DUM802 = 1, 1980Q2
0, else

Detrending over 1947Q1-1979Q1

Table 3 Flavin Estimates Using Logs

$$\ln Yt = \mu_1 + \alpha_1 \ln Y_{t-1} + \alpha_2 \ln Y_{t-2} + \cdots + \alpha_8 \ln Y_{t-8} + \alpha_9 DUM802 + \varepsilon_{1t}$$

$$\Delta \ln Ct = \mu_2 + \beta_0(\mu_1 + (\alpha_1 - 1)\ln Y_{t-1} + \alpha_2 \ln Y_{t-2} + \cdots + \alpha_8 \ln Y_{t-8} + \alpha_9 DUM802) +$$

$$\beta_1 \Delta \ln Y_{t-1} + \beta_2 \Delta \ln Y_{t-2} + \cdots + \beta_7 \Delta \ln Y_{t-7} + \beta_9 DUM802 + \varepsilon_{2t}$$

	#5	#6	#7	#8
Sample	49Q3-79Q1	49Q3-79Q1	49Q3-84Q4	49Q3-84Q4
μ_2	0.0004	0.0007	0.0002	0.0003
	0.8143	1.3651	0.4725	0.7163
β_0	0.2794		0.2652	
	0.8282		0.9903	
μ_1	0.0016	0.0015	0.0012	0.0011
	1.4863	1.3563	1.3132	1.1620
α_1	0.9645	0.8657	0.9956	0.8926
	10.5459	10.5608	12.1313	11.9653
α_2	-0.0060	0.0399	-0.0024	0.0535
	-0.0479	0.3534	-0.0213	0.5116
α_3	0.1473	0.2608	0.1147	0.2003
	1.1772	2.3246	1.0029	1.9272
α_4	-0.3556	-0.4054	-0.3329	-0.3603
	-2.8830	-3.6677	-2.9472	-3.5083
α_5	0.1484	0.2783	0.1231	0.2226
	1.1955	2.5009	1.0839	2.1562
α_6	0.0883	-0.0886	0.1083	-0.0411
	0.7115	-0.7961	0.9553	-0.3992
α_7	0.0291	0.0469	0.0029	0.0053
	0.2381	0.4278	0.0262	0.0518
	-0.0495	-0.0227	-0.0494	-0.0045
	-0.5778	-0.2957	-0.6373	-0.0633
α_9			-0.0241	-0.0248
			-2.4375	-2.4614
β_1	0.1208		0.1280	
	2.9091		3.3398	
β_2	0.0709		0.0597	
	1.7267		1.6026	
β^3	-0.0977		-0.0762	
	-1.7462		-1.5329	
β_4	0.0577		0.0457	
	0.6548		0.6887	
β_5	-0.1296		-0.1095	
	-2.7135		-2.5909	
β_6	0.0444		0.0459	
	1.1380		1.2202	
β_7	0.0162		0.0423	
	0.4045		1.1439	
β_9			-0.0061	-0.0133
			-0.7364	-2.3817
C SER	0.0051	0.0057	0.0050	0.0056
C D-W	1.8636	1.7396	1.9003	1.7458
Y SER	0.0102	0.0104	0.0098	0.0100
Y D-W	1.9942	1.8075	2.0022	1.8012
Log Lklhood	850.000	836.462	1020.09	1005.41

$\Delta \ln C$=chg, detrend log per capita PCE-Nondur+Serv 1972$

$\ln Y$ = detrend log per capita disposable personal income, 1972$

DUM802 = 1, 1980Q2
0, else

Detrending over 1947Q1-1984Q4

Table 4 Muellbauer Estimates Without Interest Rate

	#1	#2	#3	#4
Sample	49Q3-79Q1	49Q3-79Q1	49Q3-84Q4	49Q3-84Q4
μ_2	0.0016	0.0003	0.0004	0.0003
	1.7656	0.5767	0.6641	0.7065
δ_1	0.2185	0.2400	0.2191	0.2378
	4.7533	5.1243	5.3016	5.5957
μ_1	0.0066	0.0053	0.0025	0.0024
	3.8964	3.0562	2.1604	2.1985
α_1	0.7841	0.6961	0.9413	0.8199
	7.9635	7.7702	10.9720	10.5383
α_2	-0.0578	0.0770	-0.0645	0.0590
	-0.6454	0.9425	-0.7672	0.7739
α_3	0.2820	0.2599	0.1050	0.1137
	3.7723	3.4917	2.0287	2.4260
α_9			-0.0255	-0.0265
			-2.4682	-2.5614
β_1	0.1207		0.1557	
	2.2775		3.4233	
β_2	-0.1683		-0.1585	
	-3.4897		-3.5539	
β_3	0.0418		-0.0112	
	1.0391		-0.4075	
β_9			-0.0120	-0.0133
			-2.1926	-2.3478
C SER	0.0053	0.0053	0.0050	0.0052
Y SER	0.0104	0.0104	0.0104	0.0103
Log (Lklhood)	1175.6	1173.7	1412.3	1405.2

AC = detrended per capita PCE-Nondur+Serv 1972$

Y = detrended per capita disposable personal income, 1972$

$$DUM802 = \begin{cases} 1, & 1980Q2 \\ 0, & \text{else} \end{cases}$$

The log likelihood statistic is computed using $-(n/2)\ln(\det\Sigma)$ where n is sample size and Σ is the system covariance matrix.

Table 5 Muellbauer Estimates Interest Rate

Sample	#5 49Q3-79Q1	#6 49Q3-79Q1	#7 49Q3-84Q4	#8 49Q3-84Q4
μ_1	-0.0013	0.0055	0.0038	0.0035
	-0.7621	3.0940	2.6821	2.6569
δ_1	0.2318	0.2702	0.2431	0.2611
	5.2208	5.7400	5.8538	6.0006
α_2	0.0054	0.0002	-0.0021	0.0003
	2.8113	0.3210	-1.3121	0.6088
α_1	0.8229	0.7484	0.9548	0.8480
	7.8555	8.1004	10.7637	10.7122
α_2	-0.0866	-0.0072	-0.1319	-0.0156
	-0.8650	-0.0802	-1.4426	-0.1905
α_3	-0.0005	-0.0011	-0.0008	-0.0011
	-0.8484	-2.2655	-1.6859	-2.6439
α_4	0.0003	0.0011	0.0001	0.0006
	0.6258	2.4303	0.2438	1.3820
α_5	0.2739	0.2798	0.1579	0.1600
	3.1864	3.6245	2.5750	2.8916
α_9			-0.0255	-0.0264
			-2.4839	-2.5537
δ_2	0.0001	-0.0001	-0.0001	-0.0001
	0.2042	-0.2759	-0.3660	-0.4529
δ_3	0.0042	0.0009	0.0026	0.0000
	1.8764	2.1543	2.1257	0.1300
β_1	0.0871		0.1433	
	1.6383		2.9831	
β_2	-0.0000		-0.0419	
	-0.0005		-0.6306	
β_3	0.0006		0.0003	
	2.2494		1.0161	
β_4	-0.0024		-0.0017	
	-2.1887		-2.6248	
β_5	-0.0738		-0.1109	
	-1.0183		-1.7685	
β_9			-0.0121	-0.0133
			-2.2398	-2.3229
	0.5295	0.6511	1.1284	1.1061
	1.7024	1.9722	4.4771	4.2313
	-12.7571	-24.2743	-25.6595	-35.4166
	-0.9715	-1.3773	-1.9296	-2.1664
	-22.3833	-16.8059	-22.2808	-11.2209
	-1.8363	-1.0012	-1.7618	-0.6660
	0.3406	0.2517	0.3851	0.4042
	3.5675	2.6924	4.5570	4.7133
	0.1288	0.2091	0.1548	0.1505
	1.7721	2.4253	2.1543	1.8454
	27.2926	33.7915	46.6558	45.7480
	2.0225	2.3161	4.3260	4.0485
			-0.9530	-2.1921
			-0.7022	-1.1565
C SER	0.0049	0.0052	0.0050	0.0053
Y SER	0.0111	0.0110	0.0108	0.0107
r SER	1.9737	1.9235	2.0102	1.9616
Log Lklhood	1123.8	1109.9	1336.4	1323.3

ΔlnC=chg, detrend log per capita PCE-Nondur+Serv 1972$

lnY = detrended log per capita disposable personal income, 1972$

DUM802 = {1, 1980Q2; 0, else

r = real, ex post 3-mo T-bill rate

Appendix A

A different technique can be used to obtain an estimable equation from (15). The expectations operator can be removed by including the error term $EXP[e_{t+1}]$, the exponential of e_{t+1}, on the left-hand side of (15):

$$(16) \quad (C_t^{-b})\, EXP[e_{t+1}] = \delta[(1/R_t)\,(C_{t+1})^{-b}].$$

This error term represents the ratio of the right-hand side term of (16a), the actual value of the product, to that of (15), the expected value of the product. Taking logarithms of both sides, using the approximation that $\ln(1/R_t) = r_t$ for small r_t, and rearranging terms, (16a) becomes:

$$(17c) \quad \Delta \ln C_{t+1} = (1/b)\,[\ln\delta + \ln r_t] - (1/b)\, e_{t+1}.$$

The formulation (17c) has the advantage of avoiding the assumption of point expectations for the interest rate; (17c) is identical to (17b) except that the interest-rate term in (17c) is not the point expectation of r_t, but the actual value of r_t. In a way (17c) makes more sense than (17b), because (17b) says that consumption next period depends on what consumers expected the interest rate to be this period. Usually future actions depend upon what is expected in the future, not what was expected in the past, unless there are specific learning behavior assumptions in the model.

The error term $EXP[e_{t+1}]$ is a complicated function of the innovations in income and interest rates through the joint distribution of income and the interest rate. Assuming that e_{t+1} can be approximated by $e_{yt+1} + e_{rt+1} + e_{yrt+1} + \epsilon_{t+1}$, where e_{yt+1} is the income innovation, e_{rt+1} is the interest-rate innovation, e_{yrt+1} is the interaction term, and ϵ_{t+1} is the remainder of the error and is white noise, (17c) is similar to (18) without the point expectation on r_t, but with e_{yrt+1} term.

The one major problem for estimating (17c) is the estimation of e_{yrt+1}. Muellbauer uses an interaction term "because of the way interest rates and incomes are intertwined in human capital" (Muellbauer [1983, p. 451) and estimates it with the product of the income and interest-rate innovations. The reason for an interaction term here is to account for the joint distribution of income and interest rates used by consumers to form their expectations. Muellbauer's strategy is easy to implement when OLS is used. However, it was argued in the text that using OLS with a model that does not include an interaction term is incorrect because the t-statistics are biased downward; an interaction term may only reinforce this result and complicate the direction of bias. Estimating the model with maximum likelihood is very difficult when the product of the errors is included because the cross-equation coefficient constraints add 21 messy terms to the consumption equation. An easier procedure would be to estimate an equation for the product $(\ln Y_t)r_t$ and use the residual for the interaction term in the consumption equation.

FOOTNOTES

1. Muellbauer (1983) and Wickens and Molana (1984) reject the model, using U. K. consumption and income data.

2. See Hall (1978, p. 975).

3. This assumption is not unreasonable, given that her model is explaining short-run changes in consumption. However, in her later paper, Flavin (1985), uses annual data, and it seems less likely that changes in the rate of return to capital dominate endogenous changes in wealth accumulation.

4. Muellbauer says that this assumption was not important for his findings on the U. K. economy. See appendix for the alternative derivation of Muellbauer's model.

5. The Wickens and Molana model was not updated, because it is similar to Muellbauer's, apart from some additional terms that complicate the estimation procedure.

6. Serially correlated errors may not signal a breakdown of the model if consumers take more than one quarter to assimilate new information and act upon a changed expectation of life cycle wealth.

7. The equivalence of these two procedures is proved in Flavin (1981, Appendix II).

8. When the consumption and income variables are detrended with their average growth rates between 1947:IQ and 1984:IVQ, the LRS for the joint test of the $\Delta \ln YD$ terms becomes 13.5, which implies the rejection of the null hypothesis at about a 10 percent significance level.

9. The standard error of the consumption equation also increased, but this is probably due to the poorer fit of the interest-rate equation through the cross-equation constraints.

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