Financial Services | WORKING PAPER

0497

The Evolution of Cash Transactions: Some Implications for Monetary Policy

by Stacey L. Schreft and Bruce D. Smith

FEDERAL RESERVE
FINANCIAL SERVICES

Financial Services Working Paper Series No. 04-97

THE EVOLUTION OF CASH TRANSACTIONS: SOME IMPLICATIONS FOR MONETARY POLICY

by Stacey L. Schreft and Bruce D. Smith

Stacey L. Schreft is a senior economist at the Federal Reserve Bank of Kansas City. Bruce D. Smith is a professor of economics at the University of Texas–Austin.

November 1997

Abstract

This paper considers the implications of a decreasing demand for cash transactions under several monetary policy regimes. A policy of nominal-interest-rate targeting implies that a secular decline in the volume of cash transactions unambiguously leads to accelerating inflation. A policy of maintaining a fixed composition of government liabilities leads to accelerating (decelerating) inflation if agents have sufficiently high (low) levels of risk aversion. A policy of inflation targeting produces falling nominal and real interest rates, while a policy of fixing the rate of money growth can easily lead to indeterminacy and endogenous oscillation in interest rates.

The Evolution of Cash Transactions: Some Implications for Monetary Policy

STACEY L. SCHREFT AND BRUCE D. SMITH*

November 4, 1997

Abstract

This paper considers the implications of a decreasing demand for cash transactions under several monetary policy regimes. A policy of nominal-interest-rate targeting implies that a secular decline in the volume of cash transactions unambiguously leads to accelerating inflation. A policy of maintaining a fixed composition of government liabilities leads to accelerating (decelerating) inflation if agents have sufficiently high (low) levels of risk aversion. A policy of inflation targeting produces falling nominal and real interest rates, while a policy of fixing the rate of money growth can easily lead to indeterminacy and endogenous oscillation in interest rates.

* Schreft, Research Department, Federal Reserve Bank of Kansas City, Kansas City, MO 64198, 816-881-2581, sschreft@frbkc.org; Smith, Department of Economics, University of Texas, Austin, TX 78712, 512-475-8548, bsmith@mundo.eco.utexas.edu. The views expressed in this paper are not necessarily those of the Federal Reserve Bank of Kansas City or the Federal Reserve System.

Continuous technological improvement in electronics and communication systems has led to the development of several new, noncash means of payment and to a shift away from the use of cash in transactions. According to *The Nilson Report* (1996), cash accounted for 20 percent of the dollar volume of U.S. payments made in 1990 and 18 percent in 1995, and is projected to account for only 14 percent in 2000 and 11 percent in 2005.[1]

This trend raises several questions for central bankers. Specifically, what effect will the declining use of cash for transactions have on the demand for base money? In light of the effect on the demand for base money, what will be the impact on interest rates and inflation? Will the effects depend on the way monetary policy is conducted? And do these effects suggest that monetary policy should be conducted in a particular way?

This paper addresses these questions using a pure-exchange, overlapping-generations model. To generate a demand for cash transactions, the model incorporates spatial separation and limited communication along the lines of Townsend (1987). To generate a role for banks, the model includes shocks to liquidity needs along the lines of Diamond and Dybvig (1983). Together these features imply a derived demand for base money that depends on the need for currency in payments and the demand by banks for cash reserves.[2] Finally, the analysis allows the volume of cash transactions to evolve over time in a way that affects the total demand for base money.[3]

There are two assets in the model: government-issued fiat currency and government bonds. The central bank can conduct monetary policy either by setting the mix of government bonds and money outstanding or by setting the money-supply growth rate. Alternatively, it can set either a nominal-interest-rate or an inflation-rate target.

The effect of a declining demand for cash transactions varies with the policy regime in place. In addition, under some—but not all—of these policy regimes, the consequences of a

[1] Cash is used, and will continue to be used, for most small transactions. Such transactions constitute the majority of all transactions, but their total dollar value is relatively small (see Nilson 1996).
[2] The model is in fact a pure-exchange version of that in Schreft and Smith (1997a, b).
[3] A related paper is Dow (1995), which studies the effect on the price level of an exogenous decline in the need for cash in a cash-in-advance model without banks. Monetary policy in his model consists of adjustments in the quantity of money accomplished through lump-sum transfers.

decline in the use of currency in transactions depend strongly on the risk aversion of consumers. The degree of risk aversion determines both the elasticity of the demand for base money with respect to the volume of cash transactions and the response of reserve demand to a change in the nominal interest rate. Both factors are important in determining how equilibria are affected by a declining role for currency in payments.

The results of the analysis are as follows. There are only two policies that yield unambiguous predictions about the behavior of the nominal interest rate and the inflation rate in equilibrium. With a nominal-interest rate target, the inflation rate necessarily rises as the use of cash in transactions shrinks, and the central bank must continuously drain base money from the economy through open market sales to maintain the target. With an inflation target, the nominal interest rate necessarily declines as the use of cash diminishes, and—as is true with an interest-rate target—the central bank must drain base money from the economy in each period. Under all other policy regimes, the degree of risk aversion matters. Specifically, when the central bank controls the bond-to-money ratio and agents are sufficiently risk averse, a declining demand for cash transactions results in a rising nominal interest rate and accelerating inflation. The reverse is true when agents have low levels of risk aversion.

Finally, the effect of a declining demand for cash transactions is more complicated when the government fixes the money growth rate. While there is a unique equilibrium under all the other policies considered, a large set of equilibria are possible under a policy of fixing the money growth rate, and many of the equilibria can exhibit endogenous volatility. More specifically, if consumers are more risk averse than with logarithmic utility and if the real interest rate is sufficiently low, then there are potentially many oscillatory equilibrium paths approaching the steady state. That is, even though the demand for cash transactions falls monotonically, the nominal interest rate need not adjust monotonically along paths approaching a steady state. And indeed, the economy may not converge to a steady state at all. In contrast, if consumers are less risk averse than with logarithmic utility, then there is a unique equilibrium that monotonically converges to the steady state.

The remainder of the paper proceeds as follows. Section 1 describes the environment, and section 2 presents the model's equilibrium conditions. Sections 3 through 6 analyze the various monetary policies considered, while section 7 compares the properties of the different policy regimes. Finally, Section 8 concludes by discussing the role that some of the assumptions play in generating the results obtained.

1. Environment

Consider an infinite-horizon economy, with t = 1,2,... indexing time. The economy consists of two identical islands that are inhabited by an infinite sequence of two-period-lived overlapping generations. Each island has, at the beginning of each date, a continuum of ex ante identical young agents of measure one. At the initial date, each region also has an identical old generation.

Agents are endowed with $w > 0$ units of the economy's single nonstorable consumption good when young. They have no endowment when old. In addition, they derive utility from consumption only when old, denoted by c. The utility function common to all agents is $u(c)=c^{1-r}/(1-r)$, with $r > 0$. Because agents care only about consumption when old, they save their entire endowment when young. They hold all their savings, either directly or indirectly, in the economy's primary assets.

Two primary assets are available to agents in the economy: money (fiat currency) and one-period, default-free government bonds. M_t denotes the per capita value of the monetary base on each island, and B_t denotes the nominal per capita supply of government bonds. Each bond issued at t is a claim to I_t units of currency at t+1. Thus, I_t is the gross nominal interest rate at t. The time t price level is p_t, which is common across islands. Thus, in real terms, the per capita supplies of money and bonds are $m_t \equiv M_t/p_t$ and $b_t \equiv B_t/p_t$, respectively.

The government, through its central bank, has a variety of monetary policy options. It can fix either the bond-to-money ratio or the rate of money creation. Alternatively, it could target either the nominal interest rate or the inflation rate. Subsequent sections of this article consider each of these policy options. Regardless of what option the government chooses, policy must be conducted so that the government budget constraint is satisfied:

$$R_{t-1}b_{t-1} = (M_t - M_{t-1})/p_t + b_t \,, \tag{1}$$

where $R_{t-1} \equiv I_{t-1}p_{t-1}/p_t$ is the gross real interest rate.

The division of the economy into islands introduces the economically important feature of spatial separation into transactions. There is no communication across islands while transactions are being conducted. Limited communication and spatial separation imply that agents cannot exchange privately issued claims across islands. In addition, money is assumed to be the only asset that can be carried across islands and thus is the only asset that can be used in interlocation exchange. This gives money an advantage over bonds in terms of liquidity, which permits money to be dominated in rate of return.[4]

Each period, after portfolio-allocation decisions are made, a fraction $\pi_t \in (0,1)$ of the young agents from each island discover that they have to relocate to the other island before the period ends. Those agents facing relocation have to hold all their wealth in the form of currency when they move or they will be unable to transact on their new island. To capture the evolution of cash transactions in the economy, π_t is assumed to satisfy

$$\pi_t - \overline{\pi} = \mu(\pi_{t-1} - \overline{\pi}) \,, \tag{2}$$

where $\overline{\pi}$ is the long-run value of π_t and $\mu \in (0,1]$ is a known constant. Thus, π_t is known at the beginning of period t, although the identities of the specific agents facing relocation are not known.

Equation (2) captures a number of possibilities regarding the role of currency in exchange over time. When $\mu = 1$, π_t is constant and the economy is stationary. When $\mu < 1$ and $\pi_1 > \overline{\pi}$, the volume of transactions that must be undertaken with currency shrinks over time. Indeed, when $\overline{\pi} = 0$, cash may, and with positive nominal interest rates will, eventually go out of use altogether.

[4] The notion that bonds are not useful in interlocation exchange could be motivated by the realistic assumption that they must be issued in relatively large denominations.

While p_t is defined as the fraction of agents who are relocated, a secular reduction in p_t can proxy for a number of scenarios in which improvements in communication and recordkeeping reduce the need for cash in transacting. For example, one such scenario involves all young agents being relocated between periods, with a fraction p_t of them relocated to a part of their destination island where communication is costly enough to preclude the use of checks in private exchange. In this scenario, a declining value of p_t corresponds to sustained improvements in communication that increase the use of checks or other instruments in interlocation exchange.

In any event, the model's assumptions on interlocation exchange imply that agents who learn they will be relocated will want to convert all their assets into currency. Thus, random relocations play the same role here that liquidity-preference shocks play in Diamond and Dybvig (1983). And as in Diamond-Dybvig, agents will want to insure against premature asset liquidation. This insurance can be provided efficiently (see Greenwood and Smith 1997) through a bank that takes deposits, holds the primary assets directly, and offers returns on deposits that depend on the date of withdrawal (that is, agents' deposit returns depend on whether they must relocate). As in Diamond-Dybvig, banks of this type intermediate all savings.

Once banks receive young agents' deposits, they choose how to allocate the deposited funds between money (that is, cash reserves) and government bonds. At the same time, banks must announce deposit-return schedules that depend on depositor-withdrawal dates (i.e., relocation status). There is free entry into banking, so competition ensures that bank profits are zero in equilibrium.

Now consider the determination of deposit-return schedules. Let $r_m(p_t)$ ($r_n(p_t)$) denote the state-contingent gross real return on deposits offered by a typical bank to agents who are (are not) relocated at t. Banks announce these returns, taking the returns offered by other banks as given. A Nash equilibrium is a deposit-return schedule ($r_m(p_t)$, $r_n(p_t)$) for each bank such that, given this return schedule, no other bank has an incentive to alter its set of announced return schedules. Competition among banks for depositors implies that, in equilibrium, banks choose deposit-return schedules to maximize the expected utility of a representative depositor subject to a set of resource constraints, which are described below. Given this behavior by banks, young agents choose to deposit their entire endowment *w*, implying that all savings are intermediated.

Next, consider the representative bank's portfolio-allocation decision. Let m_t denote the reserves (i.e., real balances) a representative bank chooses to hold per depositor at *t*, and let b_t denote the real value of bonds that the bank holds per depositor at *t*. Then m_t and b_t must satisfy

$$m_t + b_t \leq w, \quad t \geq 0. \tag{3}$$

By the law of large numbers, a fraction p_t of a bank's depositors must relocate at *t*. Thus, the representative bank must pay $p_t r_m(p_t)w$ to those agents at *t*. Relocated agents must be given currency, so the bank's payments to agents who move are constrained by its holdings of reserves:

$$p_t r_m(p_t)w \leq m_t p_t / p_{t+1}, \quad t \geq 0. \tag{4}$$

Because agents who move at *t* carry into *t*+1 the currency they receive upon withdrawing their deposits, the promised return on deposits in (4) incorporates the gross real return on money, p_t / p_{t+1}.

To the fraction $1-p_t$ of a bank's depositors who do not relocate at *t*, the bank must pay $(1-p_t)r_n(p_t)w$ upon withdrawal. Assuming that $I_t > 1$, money is dominated in rate of return, and the bank does not carry cash balances between periods. Payments to nonmovers, then, are financed solely with the bank's holdings of bonds; that is,

$$(1-p_t)r_n(p_t)w \leq R_t b_t, \quad t \geq 0. \tag{5}$$

Let $g \geq 0$ denote the reserve-deposit ratio. Then equations (4) and (5) can be written as

$$r_m(p_t) \leq g_t(p_t / p_{t+1})/p_t, \quad t \geq 0, \tag{6}$$

$$r_n(p_t) \leq R_t(1-g_t)/(1-p_t), \quad t \geq 0. \tag{7}$$

In a Nash equilibrium, $r_m(p_t)$, $r_n(p_t)$, and g are chosen to maximize

$$\left\{ p_t (r_m(p_t)w)^{1-r} + (1-p_t)(r_n(p_t)w)^{1-r} \right\} \Big/ (1-r)$$

subject to (6) and (7). The optimal reserve-deposit ratio for this problem is given by

$$g_t = \frac{1}{1+\left(\frac{1-p_t}{p_t}\right) I_t^{(1-r)/r}} \equiv g(I_t, p_t). \tag{8}$$

Some properties of the function $\gamma(I_t, p_t)$ will be useful for future reference. Differentiation of (8) establishes that the interest elasticity of reserve demand is

$$I_t\, \gamma_1(I_t, p_t)/\gamma(I_t, p_t) = ((\rho - 1)/\rho)[1 - \gamma(I_t, p_t)].$$

Clearly, $\gamma_1(I_t, p_t) \le (>) 0$ as $\rho \le (>)$ 1. The ambiguity in the sign of $\gamma_1(I_t, p_t)$ derives from conventional income and substitution effects. A higher value of I_t, ceteris paribus, increases the opportunity cost of holding reserves. The substitution effect causes banks, acting on behalf of depositors, to substitute away from low-yielding assets. A higher value of I_t, however, also increases the income a bank earns on its bond holdings. Standard income effects cause the bank to want to raise the consumption of relocated agents, which it can do only by holding more reserves. The substitution (income) effect dominates if $\rho < (>)$ 1.

In addition, the elasticity of the demand for reserves with respect to the volume of cash transactions is

$$p_t\, \gamma_2(I_t, p_t)/\gamma(I_t, p_t) = [1 - \gamma(I_t, p_t)]/(1 - p_t) > 0.$$

Thus, higher relocation probabilities (i.e., a larger volume of transactions that require cash) induce banks to hold higher levels of reserves, other things equal.

Finally, if $\rho \le (>)$ 1, then $\gamma(I_t, p_t) \in$ [0,p] ($\in$ [p,1]) for all $I_t > 1$. It immediately follows that $p_t\, \gamma_2(I_t, p_t)/\gamma(I_t, p_t) \ge (<) 1$ if $\rho \le (>)$ 1. In other words, the elasticity of reserve demand with respect to p_t is no less than (less than) one if agents are no more (more) risk averse than they would be with logarithmic utility.[5]

2. General Equilibrium

An equilibrium for the economy described above satisfies three conditions. First, the money market must clear. Given that all beginning-of-period demand for base money derives from banks, this requirement implies that

---

[5] Suppose that the environment described above is replaced with the simpler assumption that a cash-in-advance constraint applies to a fraction p_t of each agent's purchases, with p_t evolving as in the text. Then the elasticity of the demand for base money with respect to p_t will be one. As will be apparent from the subsequent discussion, this will substantially reduce the set of possible equilibrium outcomes. See Dow (1995) for a formulation of this type.

$$g(I_t, p_t)w = M_t / p_t \equiv m_t, \quad t \geq 1. \tag{9}$$

Second, the government-bond market must clear:

$$[1 - g(I_t, p_t)]w = B_t / p_t \equiv b_t, \quad t \geq 1 \tag{10}$$

Let $b_t \equiv b_t / m_t$ denote the ratio of bonds to money outstanding. Then (9) and (10) imply that I_t and b_t must satisfy the condition

$$b_t = [1 - g(I_t, p_t)] / g(I_t, p_t), \quad t \geq 1. \tag{11}$$

Finally, the government budget constraint, (1), must be satisfied at all dates. Using the definition of b_t, (1) can be written as

$$m_t(1 + b_t) = m_{t-1}[R_{t-1}b_{t-1} + (p_{t-1}/p_t)], \quad t \geq 2\,. \tag{12}$$

Substituting (9) into (12) and using the relationship $R_{t-1} \equiv I_{t-1}p_{t-1} / p_t$ yields the equivalent equilibrium condition[6]

$$(1 + b_t)\, g(I_t, p_t) = (p_{t-1}/p_t)\, g(I_{t-1}, p_{t-1})(1 + b_{t-1}I_{t-1}), \quad t \geq 2\,. \tag{13}$$

For a given specification of government policy, an equilibrium is a sequence $\{I_t, p_t, b_t\}$ that satisfies (11), (12), and (13) at all dates. The remaining sections of this article characterize the properties of equilibria under alternative assumptions about the conduct of monetary policy.

3. An Exogenous Bond-to-Money Ratio

The government could choose to conduct monetary policy by fixing once and for all a value of b, the bond-to-money ratio. Different choices of b correspond to different open market stances: high (low) values of b are associated with tight (loose) monetary policy, as conventionally conceived.

With b given, equations (1) and (11) determine the sequence of nominal interest rates, $\{I_t\}$, and (13) then determines the sequence of inflation rates, $\{p_{t+1} / p_t\}$. From equations (1) and (11) it follows that the nominal interest rate evolves according to

$$I_t = \left[b\left(p_t/(1 - p_t)\right)\right]^{r/(1-r)}, \quad t \geq 1. \tag{14}$$

---

[6] The appropriate version of this condition for the initial date, $t = 1$, is $(1 + b_1)\, g(I_1, p_1) = M_0 / p_1$, with $M_0 > 0$ exogenously given.

In addition, equations (11) and (13) imply that the rate of inflation is given by

$$p_t / p_{t-1} = (1 + bI_{t-1})/(1+b), \quad t \geq 2. \tag{15}$$

Substituting (14) into (15) yields the following closed-form expression for p_{t+1} / p_t:

$$p_{t+1}/p_t = \frac{1 + b^{1/(1-r)}(p_t/(1-p_t))^{r/(1-r)}}{(1+b)}, \quad t \geq 1. \tag{16}$$

Furthermore, the real rate of interest, R_t, evolves according to

$$R_t = I_t p_t / p_{t+1} = (1+b)I_t/(1+bI_t).$$

Evidently, the behavior of the real interest rate over time inherits the behavior of the nominal interest rate. The evolution of I_t and p_{t+1} / p_t depends on the value of r. There are two cases.

Case 1: $r < 1$

From (14) it is clear that, when $r < 1$, a declining role for currency in the payments system must lead to a nominal rate of interest that is falling over time. If, in addition, $b\overline{p}/(1-\overline{p}) < 1$, so that the asymptotic use of currency in payments is sufficiently small, then (14) implies that there exists a finite date, denoted by T, such that $I_t = 1$ for all $t \geq T$. Once the nominal interest rate ceases to be positive, the derivation of equilibrium above is no longer valid. However, it is not hard to show that, with $b\overline{p}/(1-\overline{p}) < 1$, $I_t = 1 = p_t / p_{t+1}$ must obtain for all $t \geq T$. Thus, both the nominal interest rate and the inflation rate decline over time until, at some finite date, the opportunity cost of holding money is zero. At that point, agents have no incentive to economize further on the use of currency. The price level stabilizes at its date T value.

In contrast, when $b\overline{p}/(1-\overline{p}) > 1$, so the asymptotic use of currency in payments remains sufficiently large, equation (14) implies that $I_t > 1$ holds for all t. Equation (14) also implies that I_t declines monotonically, as does the rate of inflation. Equation (16) implies that p_{t+1} / p_t asymptotically approaches

$$\left[1 + b^{1/(1-r)}(\overline{p}/(1-\overline{p}))^{r/(1-r)}\right]/(1+b) > 1.$$

Thus, whatever the asymptotic use of currency, a diminishing role for currency in transactions leads to a declining nominal interest rate and inflation rate.

Case 2: r > 1

When $r > 1$ holds, equation (14) implies that I_t must be rising over time. Equation (16) implies that the rate of inflation must also be rising. Thus, in this case a declining demand for currency in transactions leads to monotonically increasing nominal interest rates and accelerating inflation.

To summarize, when the central bank fixes the bond-to-money ratio, the effects of a diminishing role for currency in transactions depend heavily on the magnitude of r. When $r <$ (>) 1, the elasticity of demand for base money with respect to p_t exceeds (is less than) one. That is, $r <$ (>) 1 implies that a decrease in p_t induces a more than (less than) proportional decline in the demand for reserves. This, in turn, leads to declining (rising) values of the nominal and real interest rates and the inflation rate.

4. Nominal-Interest-Rate Targeting

Alternatively, the central bank could conduct monetary policy so as to maintain a constant value $I > 1$ for the gross nominal interest rate. Under this policy, equation (11) describes how the monetary authority must adjust the bond-to-money ratio to attain its target interest rate in the face of a declining demand for currency in transactions. Equations (1) and (11) imply that, with I constant, b_t must satisfy

$$b_t = \left((1-p_t)/p_t\right) I^{(1-r)/r} . \qquad (17)$$

Thus, as p_t declines, the central bank must raise the bond-to-money ratio to keep the nominal interest rate constant. In effect, then, the central bank must drain base money from the economy via open market sales. This is true regardless of the magnitude of r.

In addition, when the central bank holds I constant, equation (11) with $I_t = I$ describes the evolution of the demand for reserves. As noted previously, when the nominal interest rate is constant, a secular decline in the value of p_t induces a less (more) than proportional decline in reserve demand if r > (<) 1. Thus, the demand for base money may decline either more rapidly or more slowly than the volume of cash transactions. In addition, differentiation of (17) establishes that $(p_t/b_t)(db_t/dp_t) = -(1-p_t)^{-1}$. Thus, the central bank must increase the bond-to-money ratio more rapidly than the volume of cash transactions contracts if it is to successfully peg the nominal interest rate.

With respect to the rate of inflation, equations (17) and (13) imply that

$$p_t/p_{t-1} = I-(I-1)\,g(I,p_{t-1}) = \frac{1+(1-p_{t-1}/p_{t-1})I^{1/r}}{1+(1-p_{t-1}/p_{t-1})I^{(1-r)/r}}, \quad t \geq 2. \tag{18}$$

Since $g_2(I,p_{t-1}) > 0$, it follows from (18) that the equilibrium inflation rate must be rising over time as p_{t-1} declines. With I constant, it therefore follows that the real interest rate declines over time along with p_{t-1}. Both statements are clearly true regardless of the magnitude of r.

To summarize, when monetary policy is conducted by pegging the nominal interest rate, a declining transactions role for currency requires a secular expansion in the bond-to-money ratio. This expansion must proceed at a greater rate than the contraction in the demand for reserves. Finally, an interest-rate peg commits the economy to secularly rising inflation. This result is in contrast to the nonincreasing inflation rate that arises if $r < 1$ under a policy that fixes the bond-to-money ratio. It raises the possibility that the central bank might choose to directly target the inflation rate over time.

5. Inflation Targeting

Suppose that the central bank follows the policy of setting the inflation rate, p_t/p_{t-1}, equal to f for all $t \geq 1$. Assume that $f \geq 1$, so the central bank does not attempt to generate deflation. Then equations (11) and (13) imply that the nominal interest rate must satisfy the following condition at each date:

$$1 = (1/f)[I_t - (I_t - 1)\,g(I_t, p_t)]. \tag{19}$$

If $f > 1$ and $\bar{p} > 0$, then (19) necessarily has a unique solution with $I_t > 1$ in every period.

Now consider the effect of changes in the demand for currency in transactions on the equilibrium evolution of the nominal interest rate. Differentiation of (19) yields

$$dI_t/dp_t = \frac{(I_t - 1)\, g_2(I_t, p_t)}{1 - g(I_t, p_t) - (I_t - 1)\, g_1(I_t, p_t)}$$

$$= \frac{(I_t - 1)\, g_2(I_t, p_t)}{[1 - g(I_t, p_t)]\left[1 - \left(\frac{r-1}{r}\right)\left(\frac{I_t - 1}{I_t}\right) g(I_t, p_t)\right]}. \tag{20}$$

Since $g_2(I_t, p_t) > 0$, and since $1 > [(r-1)/r][(I_t-1)/I_t]\, g(I_t, p_t)$, the following proposition is immediate.

Proposition 1 $dI_t / dp_t > 0$.

Proposition 1 also has a corollary. With a fixed inflation rate, as p_t declines, the real interest rate necessarily falls over time. This consequence of a diminishing role of currency in transactions was also observed under an inflation target.

It remains to consider how the bond-to-money ratio must evolve to keep the inflation rate at its target level. The following result is easily established.

Proposition 2 $db_t / dp_t < 0$.

This proposition indicates that the central bank must continuously raise the bond-to-money ratio in order to maintain its inflation target.

6. A Constant Rate of Money Creation

Instead of targeting the inflation rate, the central bank could set, once and for all, a rate of growth for the money supply. In particular, in this section, the monetary base evolves according to

$$M_{t+1} = sM_t, \quad t \geq 2,$$

with $M_1 > 0$ given and with $s > 1$.

With a constant rate of money creation, the real return on reserve holdings is given by

$$\frac{p_{t-1}}{p_t} = \frac{m_t}{sm_{t-1}} = \frac{g(I_t, p_t)}{s\, g(I_{t-1}, p_{t-1})}, \quad t \geq 2. \tag{21}$$

Using (21) and (11) in (13) gives the equilibrium law of motion for I_t:

$$\frac{s}{g(I_t, p_t)} = 1 + I_{t-1}\left[\frac{1 - g(I_{t-1}, p_{t-1})}{g(I_{t-1}, p_{t-1})}\right] = 1 + \left(\frac{1 - p_{t-1}}{p_{t-1}}\right) I_{t-1}^{1/r}, \quad t \geq 2. \tag{22}$$

Equations (2) and (22) govern the evolution of the sequence $\{I_t, p_t\}$.

A. Steady-State Equilibria

In a steady state, clearly $p_t = \bar{p}$.[7] Then, imposing $p_t = p_{t-1} = \bar{p}$ and $I_t = I_{t-1} = I$ in (22) yields the condition that determines the steady-state nominal interest rate:

$$\left(\frac{\bar{p}}{1 - \bar{p}}\right)(s - 1) = I^{1/r}\left[1 - \left(\frac{s}{I}\right)\right]. \tag{23}$$

This equation has a unique solution satisfying $I > s > 1$. In addition, higher values of s lead to higher values of the steady-state nominal interest rate. The steady-state real interest rate rises when s is increased iff $(1-r)s > 1$.

B. Dynamics

Simple algebraic manipulation establishes that equation (22) has the equivalent representation

$$\begin{aligned} I_t^{(1-r)/r} &= \left(\frac{p_t}{1 - p_t}\right)\left[\left(\frac{1}{s}\right)\left(\frac{1 - p_{t-1}}{p_{t-1}}\right) I_{t-1}^{1/r} - \left(\frac{s - 1}{s}\right)\right] \\ &= \left[\frac{mp_{t-1} + (1-m)\bar{p}}{1 - mp_{t-1} - (1-m)\bar{p}}\right]\left[\left(\frac{1}{s}\right)\left(\frac{1 - p_{t-1}}{p_{t-1}}\right) I_{t-1}^{1/r} - \left(\frac{s - 1}{s}\right)\right], \quad t \geq 2. \end{aligned} \tag{24}$$

---

[7] The existence of a steady state with positive nominal interest rates requires that $\bar{p} > 0$.

Linearizing (24) in a neighborhood of the steady state yields the dynamical system

$$\left(I_t - I, p_t - \bar{p}\right)' = J\left(I_{t-1} - I, p_{t-1} - \bar{p}\right)',$$

where J is the Jacobian matrix

$$J = \begin{vmatrix} \dfrac{\partial I_t}{\partial I_{t-1}} & \dfrac{\partial I_t}{\partial p_{t-1}} \\ 0 & m \end{vmatrix},$$

with all partial derivatives evaluated at the steady state. Clearly the eigenvalues of J are m and

$$\left.\frac{\partial I_t}{\partial I_{t-1}}\right|_{I_{t-1}=I} = \left(\frac{1}{1-r}\right)\left(\frac{I}{s}\right).$$

Since $I/s > 1$ must hold in a steady state, there are two possibilities regarding equilibrium dynamics.

Case 1: $r < 1$

When $r < 1$, $\partial I_t/\partial I_{t-1} = [1/(1-r)](I/s) > 1$. It follows that the steady state is a saddle. Since I_1 is an endogenous initial condition, the equilibrium value I_1 must place the economy on its stable manifold. Consequently, there is a unique equilibrium, and that equilibrium displays monotonic convergence to the steady state.

Case 2: $r > 1$

When $r > 1$, $\partial I_t/\partial I_{t-1} = [1/(1-r)](I/s) < 0$. The steady state is a sink (source) if $I/s < (>)\ r-1$. Moreover, using equation (23), $I/s < (>)\ r-1$ iff

$$(\bar{p}/1-\bar{p})((s-1)/s) < (>)(r-2)[s(r-1)]^{(1-r)/r}.$$

Clearly, the steady state is a sink if $r > 2$, and if either $\bar{p}$ is sufficiently close to zero or s is sufficiently close to one. Note that if $r > 2$ is satisfied, the steady state will be a sink if the asymptotic use of currency in transactions ($\bar{p}$) is very small. Also, low, positive rates of money creation are conducive to the steady state being a sink.

When $I/s < r-1$, then any choice of I_1 sufficiently close to I allows the steady state to be approached. Thus dynamical equilibria are indeterminate. Moreover, equilibrium paths approaching the steady state will display oscillations. In other words, endogenously arising volatility will be observed. That is, whenever $r > 2$, and whenever the asymptotic use of currency is sufficiently small, a fixed money growth rate must lead to a situation of indeterminacy and endogenous volatility. Both the nominal rate of interest and the price level must fluctuate along any equilibrium path.

Note that the endogenous volatility that emerges might not vanish asymptotically. Indeed, for certain values of $\bar{p}$, and for fixed values of s, there will exist equilibria displaying two-period cycles. Thus, the policy of fixing a rate of money growth can easily lead to the existence of permanent fluctuations. Such fluctuations are impossible under the other methods described for the conduct of policy.

7. Comparison of Policy Regimes

The policy regimes considered above differ along several dimensions. First, they imply much different equilibrium time paths for the interest rate, the inflation rate, and the composition of government liabilities. Second, some methods for conducting policy allow considerable scope for the indeterminacy of equilibrium and for endogenously generated volatility, while others do not.

In general, the nature of the differences in equilibrium paths across regimes hinge on r, the degree of risk aversion. Similarly, the potential for multiple equilibria and endogenous oscillation under certain policy regimes depends on whether r is less than or equal to (exceeds) one. As noted previously, if $r \leq (>) 1$, then the elasticity of the demand for base money with respect to the volume of cash transactions is greater than or equal to (less than) unity. In addition, when $r \leq (>) 1$, substitution (income) effects dominate the consequences of changes in I for reserve demand. These factors are central in determining the nature of equilibria under alternative policy regimes.

When the central bank holds the composition of government liabilities (b) constant, the nominal and real interest rates and the inflation rate must all decrease (increase) over time if $r <$ (>) 1 is satisfied. Thus a declining demand for currency in transactions is not (is) inflationary under this regime when $r <$ (>) 1.

When the central bank targets the nominal interest rate, equation (17) implies that the ratio of bonds to money must continually rise to keep the interest rate at its target level. And regardless of the magnitude of r, a policy of interest-rate targeting implies that the inflation rate must be rising over time. By implication, it is also the case that the real interest rate must be falling secularly.

A policy of inflation targeting has been shown to imply that the nominal and real interest rates must decline along with the volume of cash transactions. And, to hit its inflation target, the central bank must continuously drain reserves from the banking system. (As already noted, this is also true under an interest-rate target.)

When policy is conducted by maintaining a constant rate of money creation, the nature of equilibria again depends strongly on the magnitude of r. If $r < 1$, then there is a unique equilibrium in which I_t declines monotonically to its steady state level. However, when $r > 1$, matters are substantially more complicated.

Conducting monetary policy so as to maintain a constant bond-to-money ratio or a constant inflation rate always results in a unique equilibrium. This need not be the case, though, when the money growth rate is held constant and when $r > 1$. Indeed, when $r > 2$ and either $s - 1$ or $\bar{p}$ are sufficiently small, there will necessarily be an open set of equilibria, all of which have the feature that the nominal interest rate and the inflation rate fluctuate over time. And, depending on the values of s and $\bar{p}$, there may be equilibria in which the inflation rate and the nominal interest rate asymptotically follow a two-period cycle, so that fluctuations do not vanish over time.

Why does the policy of holding the money growth rate fixed differ so much in this regard from the other policies examined here? The answer has to do with the fact that under this policy regime—and only this policy regime—the reserve-deposit ratio, the nominal interest rate, and the inflation rate are all simultaneously determined. When income effects are sufficiently large (how large they must be depends on the magnitudes of s and $\bar{p}$) the result can be equilibria in which all three variables fluctuate as a result of self-fulfilling prophecies.

A final conclusion that emerges from a comparison of alternative policies is that only under nominal-interest-rate targeting or inflation targeting is it possible to make totally unambiguous predictions about how all equilibrium values will behave as p_t declines. That is, the predictions about the interest-rate and inflation-rate paths under a nominal-interest-rate target, or under an inflation target, do not depend on any parameter values. Thus, a central bank might prefer to use such targets when some parameter values are not known to it because such a policy would reduce the uncertainty associated with the potential effects of its policy actions. The drawback, however, is that nominal-interest-rate targeting leads to secularly rising inflation, and both nominal-interest-rate targeting and inflation targeting lead to secularly declining real interest rates.

8. Concluding Remarks

The preceding analysis abstracts from several features that are relevant to modern-day payments systems. For example, the focus on a pure-exchange economy makes it impossible for developments in the technology of payments to affect the level of production in an economy. Endogenizing production levels would allow the evolution of cash transactions to affect real activity, which in turn might modify some conclusions about how this evolution affects the behavior of the price level or the behavior of real and nominal interest rates. And by allowing for capital accumulation, the model would intrinsically have much richer dynamics.

A second feature from which the model abstracts is the mechanism by which the demand for cash evolves. One can regard the specification of an exogenous law of motion for p_t as a "reduced-form" approach that implicitly takes no stand on the economic forces governing the use of cash in transactions. Clearly, it would be more satisfying to model these forces explicitly.

Likewise, it would be better to model the choice between cash and other instruments to make payments. Modeling that choice (for example, as done in Schreft 1992 and Ireland 1994) would endogenize the evolution of cash use and might not alter any of the results derived above.

Third, the model abstracts from the existence of active markets in which reserves can be borrowed and lent. The presence of such a market has, at least in principle, the potential to substantially affect the demand by banks for reserves. And reserve demand is at the heart of the analysis here.

Introducing a market for reserves into the model is quite straightforward. The most natural way to do so is to assume that each individual bank faces a stochastic demand for cash withdrawals (that is, a random value of p_t), but that there is no aggregate randomness. This introduces an additional feature into a bank's decision regarding its reserve holdings: banks face uncertainty regarding withdrawal demand. If banks must choose their reserve-deposit ratio before observing their withdrawal demand, then ex post some banks will have more, and some will have fewer, reserves than they need to pay off depositors. This fact leads naturally to the introduction of a market in which banks with a reserve surplus (deficit) can lend (borrow) reserves.

While the introduction of a stochastic withdrawal demand and a market for reserves that resembles today's federal funds market adds some notational complexity, it does not alter the fundamental behavior of the model. Indeed, all the results reported above have close analogs when these additional features are added. Thus, abstracting from a market for reserves does not affect any qualitative conclusions about how an evolving demand for cash transactions affects the economy under alternative methods of conducting monetary policy.

Finally, in this economy all financial transactions are conducted through banks. If the model were modified to allow for a richer set of financial institutions, agents could very well have access to financial instruments that help them overcome the spatial separation and limited communication critical to their demand for cash for transactions. The introduction into the economy of such new financial instruments likely would lead to a faster reduction in the use of cash over time, although the implications of this observation for the conduct of monetary policy are by no means clear.

References

Diamond, Douglas, and Philip Dybvig, 1983. "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, June, 91(3), 401-19.

Dow, James P., Jr., 1995. "A Model of the Transition to a World without Money," Mimeo, October.

Greenwood, Jeremy, and Bruce D. Smith, 1997. "Financial Markets in Development, and the Development of Financial Markets," *Journal of Economic Dynamics and Control*, January, vol. 21, 145-81.

Ireland, Peter N., 1994. "Economic Growth, Financial Evolution, and the Long-Run Behavior of Velocity," *Journal of Economic Dynamics and Control*, vol. 18, 815-48.

The Nilson Report, November 1996, p. 6.

Schreft, Stacey L., 1992. "Transaction Costs and the Use of Cash and Credit," *Economic Theory*, vol. 2, 283-96.

Schreft, Stacey L., and Bruce D. Smith, 1997a. "Money, Banking, and Capital Formation," *Journal of Economic Theory*, March, vol. 73(1), 157-82.

Schreft, Stacey L., and Bruce D. Smith, 1997b. "The Effects of Open Market Operations in a Model of Intermediation and Growth," *Review of Economic Studies*, forthcoming.

Townsend, Robert M., 1987. "Economic Organization with Limited Communication," *American Economic Review*, vol. 77, 954-71.