# Appendices to Adjusting Median and Trimmed-Mean Inflation Rates for Bias Based on Skewness

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## Appendix A1: Description of Skewness Measures

In this paper, we use the following two robust skewness statistics.

### Bowley Skew Statistic

$$Bowley = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

where $Q_i$ is the $i^{th}$ quartile of the distribution of component price changes (in a given month).

In constructing skew, Bowley statistic uses observations in the middle 50 percent of the distribution; that is, it excludes 25 percent of observations from each tail.

### Kelly Skew Statistic

$$Kelly = \frac{P_{90} + P_{10} - 2P_{50}}{P_{90} - P_{10}}$$

where $P_i$ is the $i^{th}$ percentile of the distribution of component price changes (in a given month).
In constructing skew, Kelly statistic excludes 10 percent of observations from each tail, that is, a total of 20 percent of observations.

### Symmetrical versus Skewed Distributions

A distribution is symmetrical if one can draw a line through the center of the distribution such that the shape to the right and to the left of that line are mirror images. In a perfectly symmetrical distribution, the mean equals the median.

A distribution is left-skewed if there are more extreme points to the left of the median than there are to the right of the median, in other words, if the left side has a longer tail. In a left-skewed distribution, skewness is negative, and the mean is "pulled down" by those extreme points relative to the median; that is, the mean is below (less than) the median.

A distribution is right-skewed if there are more extreme points to the right of the median than there are to the left of the median, in other words, if the right side has a longer tail. In a right-skewed distribution, skewness is positive, and the mean is above (greater than) the median.

## Appendix A2: Skewness and Bias-Adjusted Measures of Trimmed-Mean and Median CPI Inflation

For the CPI analysis, we restrict our attention to the post-2000 period.[1] CPI skewness is generally similar to PCE skewness, but in the recent pandemic period, CPI skewness has become notably greater and is at a historically high level; see Figure A.1.

**Figure A.1: Skewness of Distribution of PCE Component Inflation Rates and CPI Component Inflation Rates (12-Month Moving Average), 2000:1–2021:12**
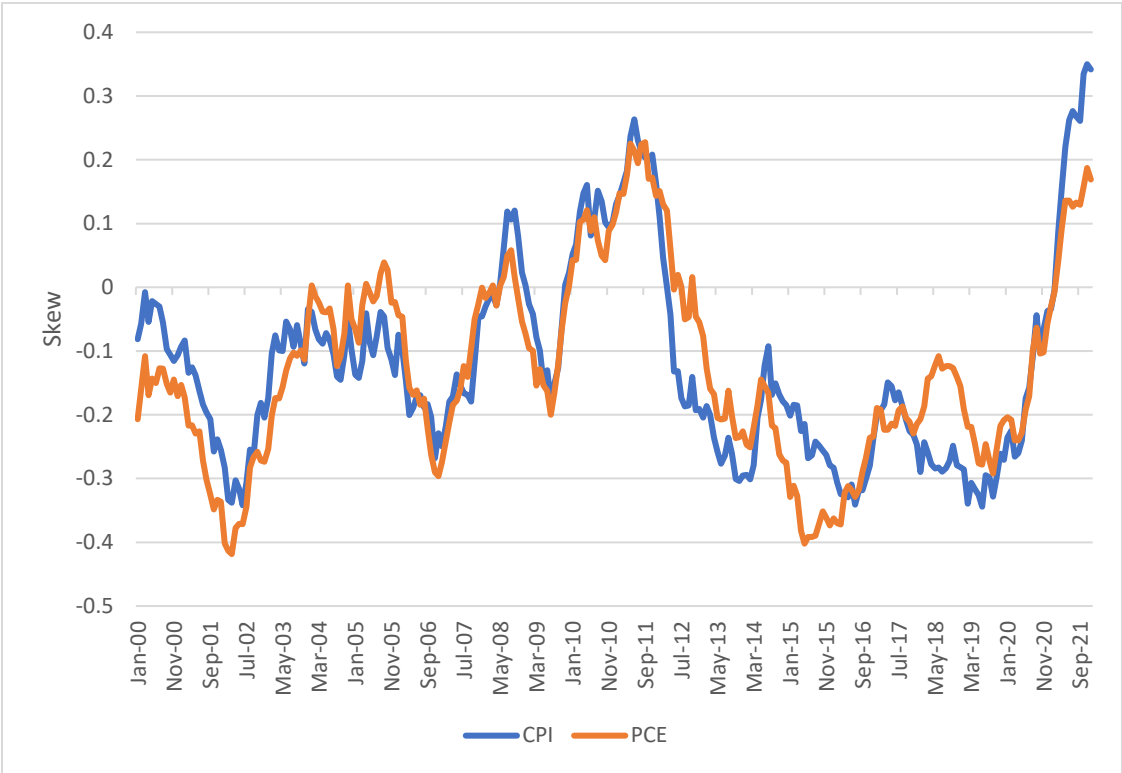


Sources: Bureau of Economic Analysis and Bureau of Labor Statistics via Haver Analytics, authors' calculations

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[1] While CPI component data are available for a longer time span, the regional owners' equivalent rent (OER) weights, which are constructed by the Federal Reserve Bank of Cleveland for the median CPI, are only available starting in 1998. Because the bias statistics are constructed using a (two-sided) 36-month window, we report the results starting in 2000.

We define the gap between monthly trimmed-mean CPI inflation $\left(\pi_{m,t}^{TM}\right)$ and monthly headline CPI inflation $\left(\pi_{m,t}\right)$ as

$$\left(gap_{m,t}^{TM}\right) = \left(\pi_{m,t}^{TM} - \pi_{m,t}\right) \tag{A.1}$$

We define the monthly gap of median CPI, $\left(gap_{m.t}^{Median}\right)$, in a similar manner. As in the case of median PCE inflation and trimmed-mean PCE inflation, we use a 36-month trimmed-mean moving average of the gaps to measure bias in a given month. These bias estimates are shown in Figure A.2.

**Figure A.2: Monthly Bias of Median CPI and Trimmed-Mean CPI Inflation Rates (36-Month Moving Average), 2000:1–2021:12**



Sources: Bureau of Labor Statistics via Haver Analytics, Federal Reserve Bank of Cleveland, authors' calculations

We estimate the following regression equation for trimmed-mean CPI inflation as

$$bias_{m,t}^{TM} = \alpha + \beta\left(skew_{m,t}\right) + \varepsilon_{m,t} \tag{A.2}$$

where $skew_{m,t}$ is the 12-month one-sided moving average of skewness of CPI inflation in Figure A.1. The regression equation for median CPI inflation is specified in an analogous manner.

Table A.1 reports the estimation results in which the numbers in parentheses indicate t-statistics formed from robust standard error estimates, and the regression is from January 2000 to May 2020.[2] As shown, there is a highly statistically significant negative relationship between bias and skewness for both trimmed-mean CPI inflation and median CPI inflation. Skewness explains about 30 percent to 40 percent of the total variation in bias as evidenced by the $R^2$ values.

**Table A.1: Coefficient Estimates for Trimmed-Mean CPI and Median CPI Inflation Rates**

|  | Trimmed-Mean CPI Inflation | Median CPI Inflation |
|---|---|---|
| $\alpha$ | -0.35 (-5.56) | -0.23 (3.11) |
| $\beta$ | -1.68 (-5.47) | -2.71 (-7.28) |
| $R^2$ | 0.28 | 0.42 |

The coefficient estimates can then be used to bias-adjust either limited-influence estimator. For example, the bias-adjusted trimmed-mean CPI inflation measure $\left( \tilde{\pi}_{m,t}^{TM} \right)$ is given by

$$\tilde{\pi}_{m,t}^{TM} = \pi_{m,t}^{TM} + 0.35 + 1.7 \left( skew_{m,t} \right) \tag{A.3}$$

The corresponding bias-adjusted median CPI inflation measure can be derived in an analogous manner using the relevant coefficient estimates in Table A.1.

Figure A.3 plots the adjustments implied by equation (A.3) and its analogue for median CPI inflation. As noted in the main text, the current pandemic period is associated with unusual inflation dynamics. The current right-skew of CPI inflation, depicted in Figure A.1, is at a historically high level. Accordingly, adjustment magnitudes in the last month (December 2021) are also at historically high levels. Moreover, since April 2021, the average upward adjustment to trimmed-mean CPI inflation is +0.82 percentage points, while the average upward adjustment to median CPI inflation over this period is even larger at +0.98 percentage points. For the most recent time period (December 2021), our bias-adjustment procedure boosts the current reading of trimmed-mean CPI inflation by 0.93 basis points to 5.76 percent and boosts the current reading of median PCE inflation by 116 basis points to 4.99 percent.

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[2] We use heteroscedasticity and autocorrelation consistent standard error estimates with three lags.
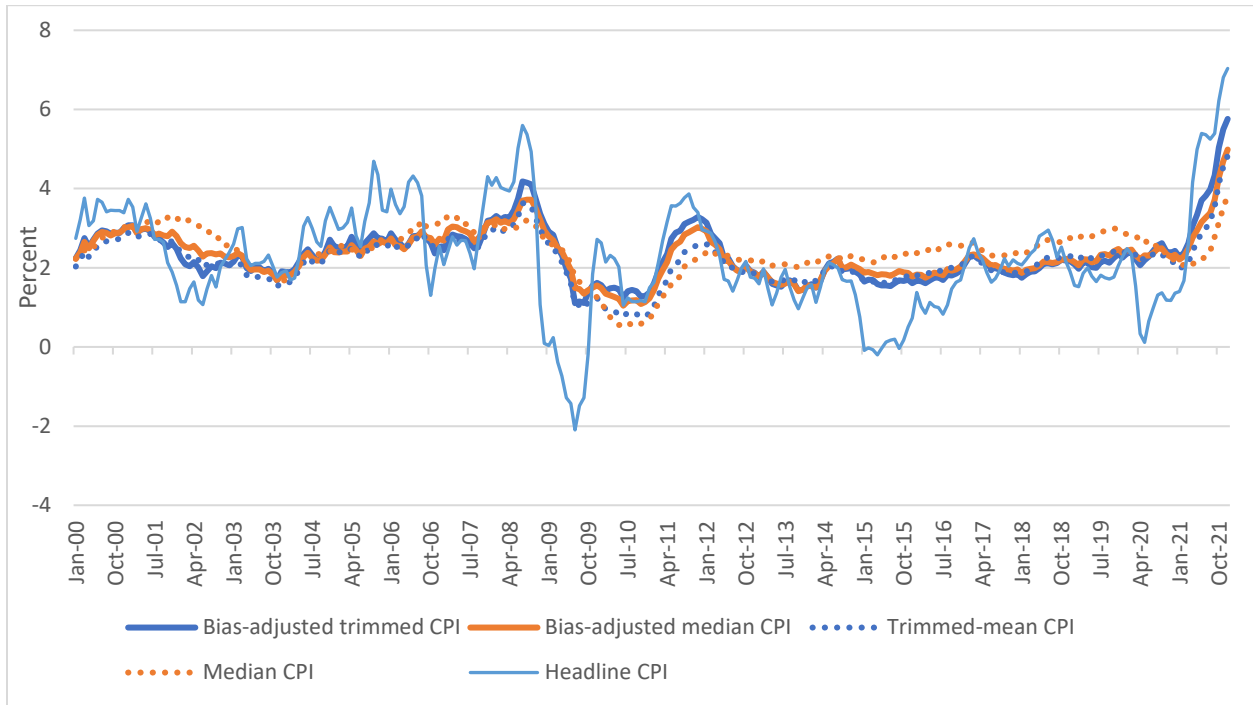
**Figure A.3: Trimmed-Mean CPI and Median CPI Bias Adjustments, 2000:1–2021:12**
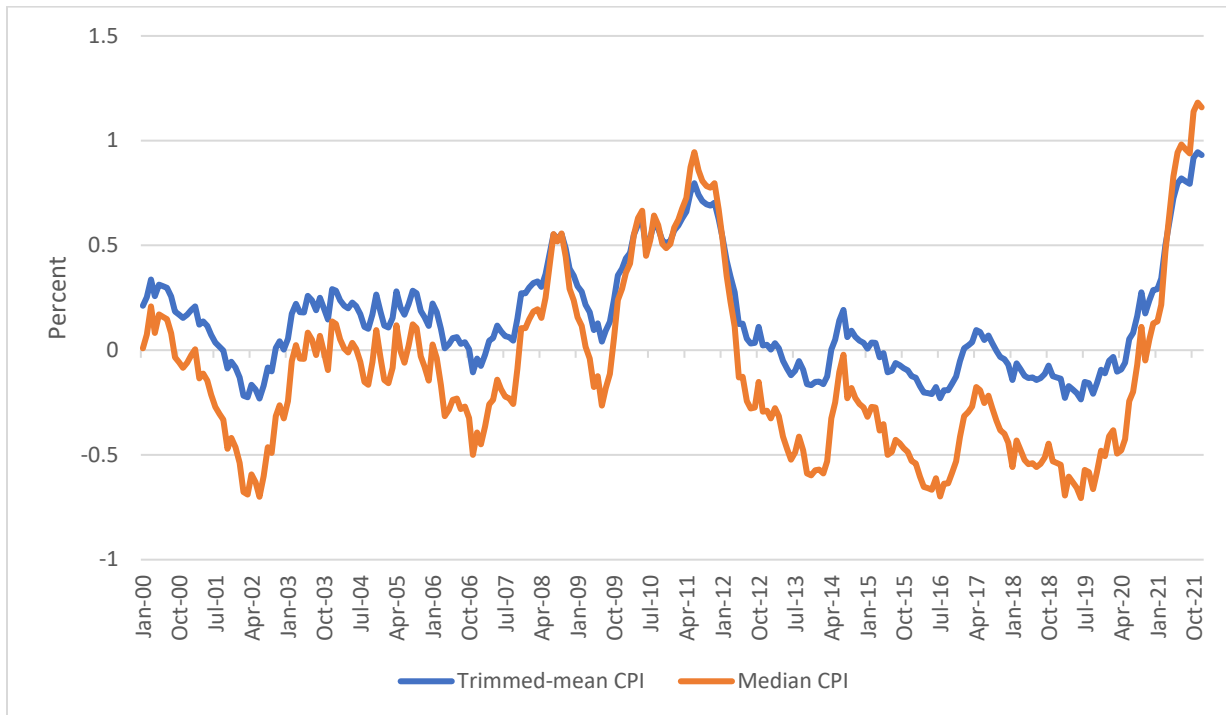


Sources: Bureau of Labor Statistics via Haver Analytics, Federal Reserve Bank of Cleveland, authors' calculations

Figure A.4 plots the unadjusted and bias-adjusted series along with headline CPI inflation. There are two takeaways that emerge from the figure. First, as in the case of the limited-influence estimators of PCE inflation, median CPI inflation and trimmed-mean CPI inflation display very similar behavior after bias adjustment. Second, bias adjustment almost invariably brings median CPI inflation and trimmed-mean CPI inflation closer to headline CPI inflation; as in the case of the analysis of PCE inflation, this is visually evident in the recovery from the Great Recession, from about 2010 onward through to the present pandemic period.

**Figure A.4: Bias-Adjusted Trimmed-Mean CPI and Median CPI versus Trimmed-Mean CPI, Median CPI, and Headline CPI, 2000:1–2021:12**



Sources: Bureau of Labor Statistics via Haver Analytics, Federal Reserve Bank of Cleveland, authors' calculations