Appendix to

"The Effect of the 2017 Tax Reform on Investment"

by Filippo Occhino

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To see how the corporate income tax can work as an investment subsidy, think of a company that has to decide how much to invest in a year to maximize its profits in the next year. By investing k in the first year, the company is able to produce and sell $f(k)$ in the second year. The production function, $f(k)$, is increasing and concave, so the marginal product of capital, $f'(k)$, is positive and decreasing. Capital fully depreciates after production.[1] The company finances its investment by debt, so the cash flow is equal to zero in the first year. To invest k, the company has to borrow $b = k$ in the first year. The borrowing rate is equal to r, so the company has to repay $(1 + r)b$ in the second year.

Let k^* denote the investment level that the company would choose in the absence of corporate taxation. In this case, the company's profits in the second year would be equal to the difference between revenue and debt repayment: $f(k) - (1 + r)b = f(k) - (1 + r)k$. The level of k^* would be determined by the first-order condition $f'(k^*)-(1+r)=0$, or

$$f'(k^*) = 1 + r. \tag{1}$$

In other words, the company would choose an investment level that makes the marginal product of capital (the left-hand side of equation 1) equal to the user cost of capital (the right-hand side of equation 1).

Now, let's introduce corporate taxation. Taxable income, Y, is obtained after subtracting any allowed deduction from revenue, $f(k)$. The tax liability is equal to the product of the tax rate, t, and taxable income, Y. The company's profits are obtained after subtracting debt repayment and tax liability from revenue: $f(k) - (1 + r)b - tY$.

To begin, suppose the tax system did not allow any interest deductibility or accelerated depreciation. The company would be able to deduct investment expenses, k, in the second year, when capital depreciates. Taxable income would be equal to the difference between revenue and depreciation: $Y = f(k) - k$. The company's profits would be equal to $f(k) - (1 + r)b - tY = f(k) - (1 + r)k - t[f(k) - k]$. The investment level would be determined by the first-order condition $f'(k) - (1 + r) - t[f'(k) - 1]=0$, or

$$f'(k) = 1 + r/(1 - t). \tag{2}$$

The user cost of capital (the right-hand side of equation 2) would be higher than in the case without corporate taxation (the right-hand side of equation 1). Then, the marginal product of capital, $f'(k)$, would be higher as well. Since $f'(k)$ is a decreasing function, the investment level,

[1] See Appendix A in Occhino (2019) for an infinite-horizon example where capital depreciates slowly over time.

k, would be less than k^*. In other words, if interest expenses could not be deducted, corporate taxation would raise the user cost of capital and discourage investment.

In reality, however, the tax system allows companies to deduct their interest expenses, equal to rb, from their taxable income. With interest deductibility, taxable income is equal to $Y = f(k) - k - rb$, and profits are equal to $f(k) - (1 + r)b - tY = \mathrm{f(k)} - (1 + r)k - t[\mathrm{f}(k) - k - rk]$. The investment level is determined by the first-order condition $f'(k) - (1 + \mathrm{r}) - t[f'(k) - 1 - r] = 0$, or

$$f'(k) = 1 + r. \tag{3}$$

This is the same as equation (1). With interest deductibility, then, the company chooses the same investment level as if there weren't any corporate taxation. In other words, interest deductibility lowers the user cost of capital enough to make corporate taxation neutral.

In addition, the tax system allows companies to deduct their investment expenses at a faster rate than the economic depreciation rate of capital. To model accelerated depreciation, suppose that the company can deduct a fraction z of investment expenses immediately, so it can deduct zk in the first year and $k - zk$ in the second year. The case $z = 1$ would represent full immediate expensing of investment. The company uses its first-year tax savings, equal to tzk, to reduce the amount that it borrows, so the level of debt in this case is $b = k - tzk$, less than in the cases that we have considered so far. Taxable income is equal to $Y = f(k) - (k - zk) - rb$, and profits are equal to $f(k) - (1 + r)b - tY = (1 - t)[f(k) - k - r(k - tzk)]$.[2] The investment level is determined by the first-order condition $(1 - t)[f'(k) - 1 - r(1 - tz)] = 0$, or:

$$f'(k) = 1 + r(1 - tz) \tag{4}$$

The user cost of capital (the right-hand side of equation 4) is lower than in the case without corporate taxation (the right-hand side of equation 1). Then, the marginal product of capital, $f'(k)$, is lower as well, and the investment level, k, is greater than k^*. The higher the corporate tax rate, t, the lower the user cost of capital and the higher the investment level. In other words, the combination of interest deductibility and accelerated depreciation ends up lowering the user cost of capital and working as an investment subsidy. The higher the tax rate, the higher the investment subsidy and the investment level.

To see why corporate taxation works as an investment subsidy, notice that the present value of the company's tax liabilities is equal to $-tzk + tY/(1 + r) + T$, where the first term is (minus) the first-year tax savings, the second term is the present value of the second-year tax liability, and T

[2] The steps to derive the last equation are:

$$\begin{aligned}
&f(k) - (1 + r)b - tY = \\
&f(k) - (1 + r)b - t[f(k) - (k - zk) - rb] = \\
&f(k) - b - rb - tf(k) + t(k - zk) + trb = \\
&(1 - t)f(k) - b - (1 - t)rb + t(k - zk) = \\
&(1 - t)f(k) - (k - tzk) - (1 - t)r(k - tzk) + t(k - zk) = \\
&(1 - t)f(k) - k + tzk - (1 - t)r(k - tzk) + tk - tzk = \\
&(1 - t)f(k) - (1 - t)k - (1 - t)r(k - tzk) = \\
&(1 - t)[f(k) - k - r(k - tzk)].
\end{aligned}$$

represents any other term that does not depend on k. Then, with a few substitutions and steps,[3] one can show that, at the investment level chosen by the company, the derivative of the present value with respect to k is negative; that is, the present value of the company's tax liabilities decreases as the company expands its investment.

References

Occhino, Filippo. 2019. "The Macroeconomic Effects of the Tax Cuts and Jobs Act." Federal Reserve Bank of Cleveland, Working Paper No. 19-28. doi:10.26509/frbc-wp-201928.

[3] The present value of the tax liabilities is equal to:

$$T - tzk + tY/(1 + r) = T - tzk + t[f(k) - (k - zk) - rb]/(1 + r) = T - tzk + t[f(k) - (k - zk) - r(k - tzk)]/(1 + r).$$

The derivative of the present value with respect to k is equal to:

$$t[f'(k) - (1 - z) - r(1 - tz)]/(1+r) - tz = t[1 + r(1 - tz) - (1 - z) - r(1 - tz)]/(1+r) - tz = tz/(1+r) - tz = -rtz/(1+r),$$

which is negative.