**Appendix to**

**“Lessons on the Economics of Pandemics from Recent Research,”**

**by Sewon Hur and Michael Jenuwine, Federal Reserve Bank of Cleveland, *Economic Commentary*, 2020-12**

This appendix provides details and equations used in the models referred to in the *Commentary*.

**Basic SIR model from epidemiology**

The SIR model, originally developed by Kermack and McKendrick (1927), simulates the course of a disease by tracking changes over time in three populations of interest: susceptible (*S*; those who are at risk of contracting the disease), infected (*I*; capable of transmitting the disease), and removed (*R*; those who recover or die from the disease). The total population is then denoted as $N = S + I + R$.

The SIR model takes the form of three equations characterizing the weekly change in the *S*, *I*, and *R* populations at time $t$ as functions of the week’s populations and two parameters, $\beta$, the rate at which infected individuals encounter susceptible individuals and successfully transmit the virus, and $\gamma$, the rate at which infected people recover or die:

$$S_{t+1} - S_t = \underbrace{-\beta S_t \frac{I_t}{N_t}}_{\text{new infections}} \tag{1}$$

$$I_{t+1} - I_t = \underbrace{\beta S_t \frac{I_t}{N_t}}_{\text{new infections}} - \underbrace{\gamma I_t}_{\substack{\text{recovery}\\ \text{or death}}} \tag{2}$$

$$R_{t+1} - R_t = \underbrace{\gamma I_t}_{\substack{\text{recovery}\\ \text{or death}}} . \tag{3}$$

The first equation shows that the susceptible population falls by the number of newly infected. The second equation shows that the infected population rises by the number of newly infected but falls by the number of newly removed. The third equation represents the removed as increasing with the share recovering or dying from infection.

An important number in the SIR class of models is known as the "basic reproduction number" (denoted $R_0$), which corresponds to how many people the average infected person passes the disease to while infectious and in the absence of mitigation efforts. In these models, $R_0$ is determined by the ratio of the transmission and recovery rates, that is, $\beta / \gamma$.

**SEIR model**

The SEIR model adds the category of exposed ($E_t$) individuals to the basic SIR model to represent those who are infected but not yet infectious themselves. In the SEIR model, people have to have been exposed to the virus before they become infected; that is, exposure is a necessary first step for infection. The rate at which exposed individuals start showing symptoms and become infectious is denoted by an additional parameter $\sigma$; medically speaking, this rate is related to the virus's incubation period. The SEIR model can be represented by equations of the SIR model modified to change the equation (2) for changes in the number of infected individuals to

$$I_{t+1} - I_t = \underbrace{\sigma E_t}_{\substack{\text{develop} \\ \text{symptoms}}} - \underbrace{\gamma I_t}_{\substack{\text{recovery} \\ \text{or death}}} \tag{4}$$

and adding an equation for changes in the number of exposed individuals of the form

$$E_{t+1} - E_t = \underbrace{\beta S_t \frac{I_t}{N_t}}_{\text{new exposures}} - \underbrace{\sigma E_t}_{\substack{\text{develop} \\ \text{symptoms}}} . \tag{5}$$

**SIR-based models with additional economics**

**Eichenbaum et al. (2020)** attempts to account for the relationship between the spread of disease and the economy as a whole by tying the likelihood of infection to participation in labor and consumption markets. In addition to the baseline level of transmission, modeled in the basic SIR model by $\beta S_t \frac{I_t}{N_t}$, individuals becoming sick each period, transmission can occur between susceptible and infected individuals while consuming (such as while out shopping) or while working. In terms of the model, this means changing equation (2) to

$$I_{t+1} - I_t = \underbrace{\beta_C (C_t^S S_t) \left( C_t^I \frac{I_t}{N_t} \right)}_{\text{new infections while consuming}} + \underbrace{\beta_N (N_t^S S_t) \left( N_t^I \frac{I_t}{N_t} \right)}_{\text{new infections while working}} + \underbrace{\beta S_t \frac{I_t}{N_t}}_{\text{other new infections}} - \underbrace{\gamma I_t}_{\substack{\text{recovery} \\ \text{or death}}} , \tag{6}$$

where $C^S$ and $C^I$ are consumption by susceptible and infected individuals, $N^S$ and $N^I$ are hours worked of susceptible and infected individuals, and $\beta_C$ $\beta_N$ are transmission rates that govern the spread of the disease related to consumption and labor, respectively. The terms $\beta_c(C_t^S S_t)(C_t^I I_t / N_t)$ and $\beta_N(N_t^S S_t)(N_t^I I_t / N_t)$ count the number of susceptible people who become infected while consuming and while working, respectively. Note that setting $\beta_C = \beta_N = 0$ reduces the equation to the basic SIR model.

On the economic side of the model, individuals choose how much to consume and how much to work. In doing so, they want to balance the utility they receive from consumption against the utility they receive from not working. The choices an individual makes change based on whether that person is susceptible, infected, or recovered. In particular, susceptible individuals may choose to consume and work less than others to lower the risk of becoming infected. The authors solve for optimal containment measures, modeled as a consumption tax and lump sum rebate, that maximize the discounted sum of utility from consumption and leisure for all agents in the economy.

**Alvarez et al. (2020)** extend the basic SIR model to include lockdown measures and solve for an optimal lockdown policy. The key addition to their model is that a fraction of the population, $L$, can be placed into lockdown or quarantine. When in lockdown, susceptible people cannot meet or become infected by any contagious people. However, people are less able to work when in lockdown, so there is an economic cost. An optimal lockdown policy over time, $L_t$, is chosen to balance the loss of lives from COVID-19 against the loss of economic output because of the lockdown. Changes in the number of infected individuals are determined by

$$I_{t+1} - I_t = \underbrace{\beta S_t (1-\theta L_t) \frac{I_t}{N_t} (1-\theta L_t)}_{\text{new infections out of lockdown}} - \underbrace{\gamma I_t}_{\substack{\text{recovery} \\ \text{or death}}}, \tag{7}$$

where $\theta$ is a parameter governing the effectiveness of the lockdown. This is nearly the same as the basic SIR model, except only the fraction of individuals out of lockdown $(1-\theta L_t)$ can pass along the disease. When $\theta = 0$ and lockdown is completely ineffective, the disease dynamics boil down to the basic SIR model. Another small change is that infected individuals die at rate $\delta(I_t)$, which is a function increasing in the number of infected people, $I_t$. This reflects that as more people are infected, the capacity to treat them is reduced, and the death rate increases.

## References

Alvarez, Fernando, David Argente, and Francesco Lippi. 2020. “A Simple Planning Problem for COVID-19 Lockdown.” University of Chicago, Becker Friedman Institute for Economics, Working Paper (April 6). https://bfi.uchicago.edu/working-paper/a-simple-planning-problem-for-covid-19-lockdown/.

Eichenbaum, Martin S., Sergio Rebelo, and Mathias Trabandt. 2020. “The Macroeconomics of Epidemics.” National Bureau of Economic Research, Working Paper No. 26882. https://doi.org/10.3386/w26882.

Kermack, William Ogilvy, and Anderson G. McKendrick. 1927. “A Contribution to the Mathematical Theory of Epidemics.” *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, 115 (772): 700–721.