Appendix to "Can Yield Curve Inversions Be Predicted?" by Kurt G. Lunsford

This appendix accompanies the Federal Reserve Bank of Cleveland *Economic Commentary* entitled "Can Yield Curve Inversions Be Predicted?" by Kurt G. Lunsford. This appendix provides the forecasting results using the Diebold and Li (2006) model.

Let $y(\tau)$ denote the continuously compounded zero-coupon nominal yield of a discount bond with a maturity of τ months. Then, the Diebold and Li model uses a dynamic Nelson and Siegel (1987) structure given by

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3,t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right).$$

I estimate the model in two steps. First, I estimate the values of $\beta_{1,t}$, $\beta_{2,t}$, and $\beta_{3,t}$ for each month t. To do this, I follow Diebold and Li (2006) by setting $\lambda = 0.0609$. I use monthly unsmoothed Fama and Bliss (1987) yields for maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108, and 120 months from 1983 to 2008 as my data. For a given month t, I define $y_t = [y_t(3), \dots, y_t(120)]'$, $\beta_t = \left[\beta_{1,t}, \beta_{2,t}, \beta_{3,t}\right]'$, and

$$X = \begin{bmatrix} 1 & \left(\frac{1-e^{-3\lambda}}{3\lambda}\right) & \left(\frac{1-e^{-3\lambda}}{3\lambda} - e^{-3\lambda}\right) \\ \vdots & \vdots & \vdots \\ 1 & \left(\frac{1-e^{-120\lambda}}{120\lambda}\right) & \left(\frac{1-e^{-120\lambda}}{120\lambda} - e^{-120\lambda}\right) \end{bmatrix}.$$

Then, the estimates in period t are given by $\hat{\beta}_t = (X'X)^{-1}X'y_t$.

In the second step, I estimate a dynamic relationship for $\hat{\beta}_t$ using an AR(1) model for each element of $\hat{\beta}_t$. An AR(2) model produces very similar results.

The forecasting procedure is as follows. I use an initial sample of 1983:01 to 1987:12 to estimate $\hat{\beta}_t$ for $t = 1983{:}01, \dots, 1987{:}12$ and to estimate an AR(1) for each element of $\hat{\beta}_t$. Using the AR(1) models, I recursively produce the forecasts $\beta_{T+1|T}, \beta_{T+2|T}, \dots \beta_{T+h|T}$, where T denotes 1987:12. Given these forecasts, I then produce forecasts of the yields with $y_{T+j|T} = X\beta_{T+j|T}$ for $j = 1, \dots, h$. Using this procedure, I successively add one month to the end of the sample and produce the forecasts until the last month of my sample, 2008:12.

This procedure produces monthly forecasts. To get quarterly forecasts comparable to the Blue Chip forecasts in the *Commentary*, I simply average the monthly forecasts over each quarter. To keep the information set in my Diebold and Li model similar to the information used to make the Blue Chip forecasts presented in the *Commentary*, I make forecasts taking the first two months of the quarter as given. That is, if I am making a nowcast in the first quarter of the year, I use the average of y_{Jan}, y_{Feb}, and $y_{Mar|Feb}$. Then, the forecast of the second quarter is the average of $y_{Apr|Feb}$, $y_{May|Feb}$, and $y_{Jun|Feb}$, and so on. Forecasts of the term spread take the forecasts of the 120-month yield and subtract the forecast of the 12-month yield.

Figure A.1 shows the Treasury term spread along with two forecasts. The top panel shows the forecast for a given quarter made two quarters prior. The bottom panel shows the forecast for a given quarter made four quarters prior. Grey bars in both panels indicate a yield curve inversion. This parallels Figure 3 in the *Commentary*.

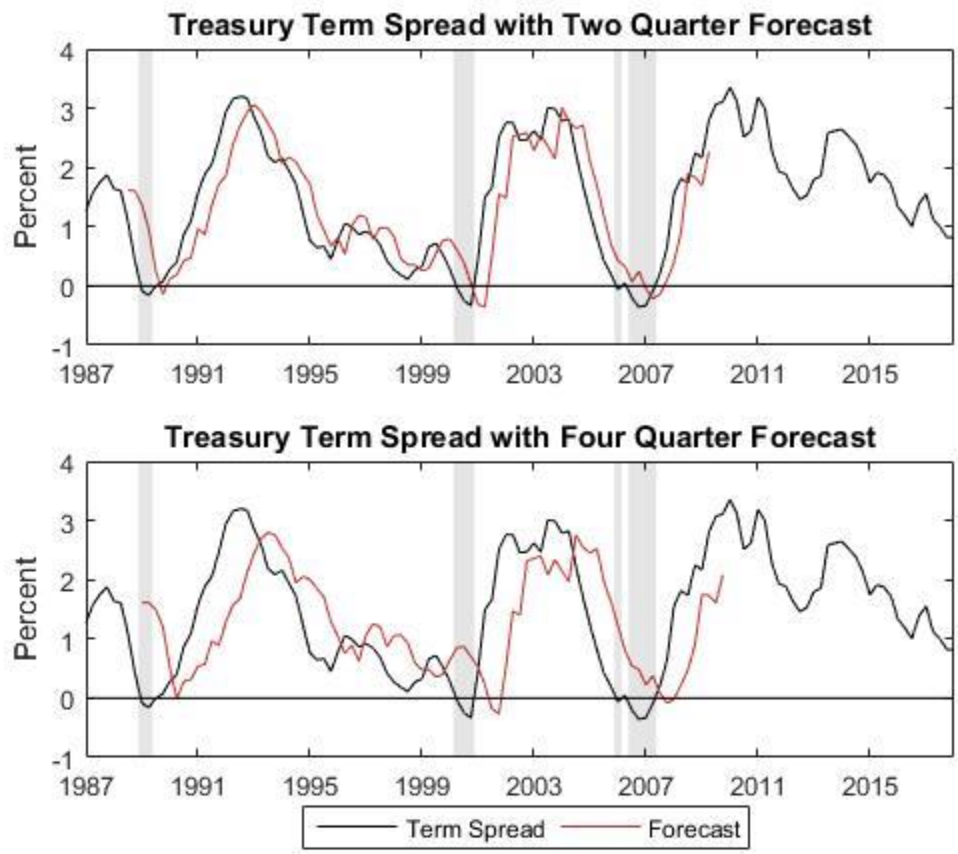

Figure A.1: The top panel shows the Treasury term spread and the forecast of the term spread made two quarters prior. The bottom panel shows the Treasury term spread and the forecast made four quarters prior. Grey bars indicate yield curve inversions.

Figure A.1 shows a similar pattern to Figure 3 in the *Commentary*. The beginnings of yield curve inversions are not predicted. Further, yield curve inversions are only predicted once an inversion has occurred.

Figure A.2 shows the 1-year and 10-year Treasury interest rates during each yield curve inversion episode along with the corresponding forecasts from the Diebold and Li model. Grey bars indicate yield curve inversions. This figure parallels Figure 4 of the *Commentary*.

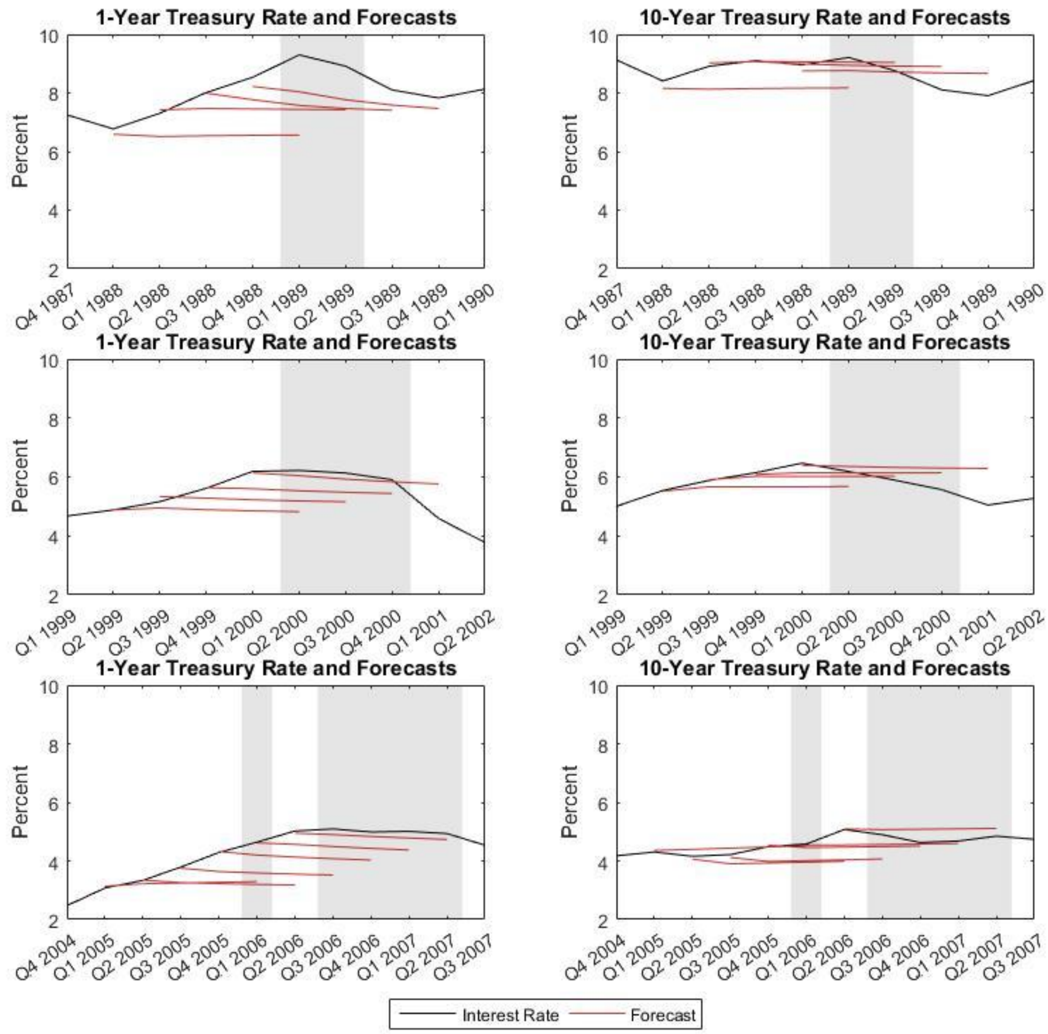

Figure A.2: Left panels show the 1-year Treasury interest rates and the corresponding forecasts. The right panels show the 10-year Treasury interest rates and the corresponding forecasts. Grey bars indicate yield curve inversions.

The left panels of Figure A.2 show the 1-year interest rates and forecasts. As with the professional forecasts in the *Commentary*, forecasts from the Diebold and Li model fail to forecast the magnitude of the rise in 1-year Treasury rates in each inversion episode. However, the professional forecasts are notably more accurate in 2005 and 2006.

The right panels of Figure A.2 show the 10-year interest rates and forecasts. The forecasts have mixed forecast errors in the Q1 and Q2 1989 inversion and in the Q2 to Q4 2000 inversion. Unlike the professional forecasts, the Diebold and Li forecasts do not systematically overpredict 10-year rates for Q1 2006 and for Q3 2006 to Q2 2007.

References

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Fama, Eugene F. and Robert R. Bliss. 1987. “The Information in Long-Maturity Forward Rates,” *American Economic Review*, 77(4): 680-692.

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