Appendix: Decomposing Changes in Average Inflation Expectations

Let ρ_t^h be share of high-uncertainty consumers at time *t* (and thus, $\left(1-\rho_t^h\right)$ is the share of low-uncertainty consumers at time *t*.) Let π_t^h and π_t^l denote the average long-run expected inflation of the high-uncertainty type and the low-uncertainty type, respectively, at time *t*. Then the average long-run expected inflation π_t in the UM Survey is given by

$$\pi_t = \rho_t^h \pi_t^h + \left(1-\rho_t^h\right)\pi_t^l$$

This implies that any change over time in π_t over time has the following decomposition:

$$\begin{aligned}\Rightarrow \frac{d\pi}{dt} &= \frac{d\rho_t^h}{dt}\pi_t^h + \frac{d\pi_t^h}{dt}\rho_t^h - \frac{d\rho_t^h}{dt}\pi_t^l + \left(1-\rho_t^h\right)\frac{d\pi_t^l}{dt} \\ &= \frac{d\rho_t^h}{dt}\left(\pi_t^h - \pi_t^l\right) + \frac{d\pi_t^h}{dt}\rho_t^h + \left(1-\rho_t^h\right)\frac{d\pi_t^l}{dt}\end{aligned}$$

Using the midpoint rule to approximate the change in average expectations from time *t* to time *t'*, we obtain the approximation:

$$\Delta\pi_t \approx \Delta\rho_t^h \frac{\pi_{t'}^h - \pi_{t'}^l + \pi_t^h - \pi_t^l}{2} + \Delta\pi_t^h \frac{\left(\rho_{t'}^h + \rho_t^h\right)}{2} + \Delta\pi_t^l\left(1 - \left\{\frac{\left(\rho_{t'}^h + \rho_t^h\right)}{2}\right\}\right)$$

The first term represents the part of the change in average inflation expectations that results from a change in the level of uncertainty. The second term represents the part that results from a change in the inflation expectations of the uncertain type. The final term represents the part that results from a change in the inflation expectations of the less uncertain type.

Table A1 summarizes the mean estimates of ρ_t^h, π_t^h and π_t^l for the second half of 2012, the first half of 2014, and the last six months considered in the Commentary (February – July 2016).

Table A1

	2012H2	2014H1	2016
ρ^h	0.16	0.10	0.07
π^h	6.53	8.96	6.90
π^l	2.52	2.85	2.50
π	3.16	3.46	2.82

If we consider the change from 2014H1 to 2016, for example, our formula gives the following numbers:

$$-0.64 = -0.03\frac{6.11 + 4.4}{2} - 2.06\frac{0.10 + 0.07}{2} - 0.35\left(1 - \left\{\frac{0.10 + 0.07}{2}\right\}\right)$$

$$= -0.14 - 0.18 - 0.32$$

Average expectations fell by 0.64 percentage points. This change came from a modest reduction in the overall level of uncertainty (the fraction of uncertain fell from 0.10 to 0.07), from a large reduction in the average expected inflation of the highly uncertain (that is multiplied by the small fraction of the respondents that are highly uncertain, reflecting their size in the sample), and from a modest reduction in the average expected inflation of the less uncertain (that is multiplied by the large fraction of the respondents that are less uncertain, reflecting their size in the sample).

Using the same formula, our formula gives the following for the 2012-2016 change:

$$-0.34 = -0.09\frac{4.01 + 4.4}{2} + 0.37\frac{0.16 + 0.07}{2} - 0.02\left(1 - \left\{\frac{0.16 + 0.07}{2}\right\}\right)$$

$$= -0.38 + 0.04 - 0.02$$

The two sides of the equality are slightly different, in this case, owing to rounding and approximation error. In words, this equation says that average expectations fell by 0.34 percentage points. By far, the largest part of this decline came from a decline in the share of the uncertain type, partially offset by a small increase in the average expectations of the high uncertainty type (+0.04), with a nearly negligible contribution from the small decrease in the average expectations of the low uncertainty type (-0.02).

Approximation error can be reduced by reducing the time period over which comparisons are made, and then cumulating those differences. If one uses six months increments, for instance, then the decomposition agrees to at least three significant digits:[1]

$$-0.346 = -0.484 + 0.169 - 0.031$$

This more accurate decomposition indicates that the decline in uncertainty, by itself, would have reduced average inflation expectations by nearly -0.49, but the movements in the average inflation expectations of the highly uncertain have (over this period) increased average inflation expectations by about 0.17. (The modest decline in inflation expectations of the less uncertain resulted in a very small reduction in the average inflation expectation, -0.03.) Interestingly, the accurate decomposition above is fairly close to the one that results from merely computing the 2012-2014 change, and the 2014-2016 change, and adding.

[1] We do not here provide the data necessary to do this computation. But we offer a simpler alternative below, which gives nearly the same answer.